

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.4-f-x-
 $^m-d+e-x^2-q-a+b-x^2+c-x^4-p$

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [300]. This is test number [28].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (300)	0.00 (0)
Mathematica	100.00 (300)	0.00 (0)
Maple	100.00 (300)	0.00 (0)
Giac	88.00 (264)	12.00 (36)
Fricas	85.33 (256)	14.67 (44)
Mupad	71.33 (214)	28.67 (86)
Maxima	57.67 (173)	42.33 (127)
IntegrateAlgebraic	49.67 (149)	50.33 (151)
Sympy	35.33 (106)	% 64.67 (194)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

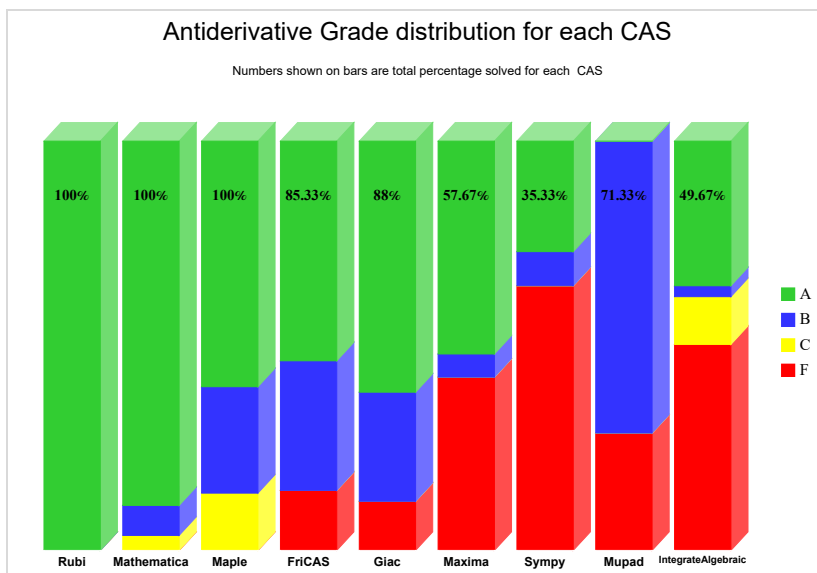
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

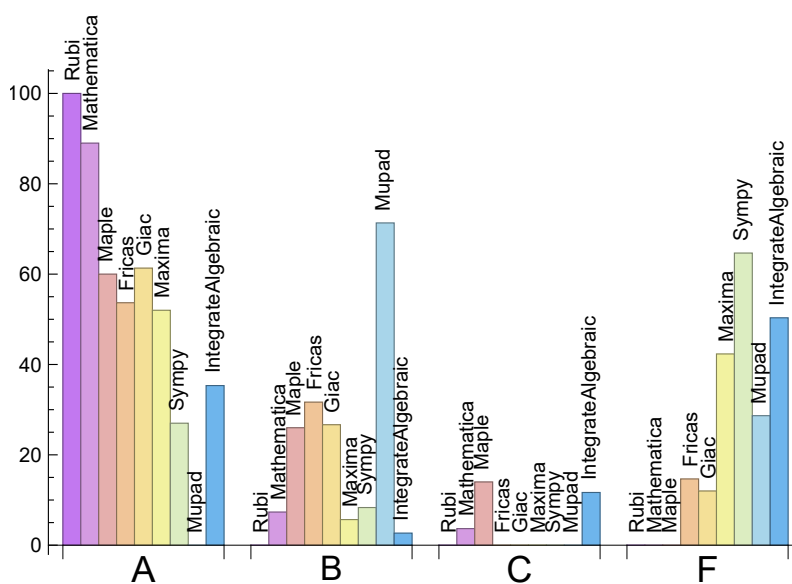
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	89.00	7.33	3.67	0.00
Giac	61.33	26.67	0.00	12.00
Maple	60.00	26.00	14.00	0.00
Fricas	53.67	31.67	0.00	14.67
Maxima	52.00	5.67	0.00	42.33
IntegrateAlgebraic	35.33	2.67	11.67	50.33
Sympy	27.00	8.33	0.00	64.67
Mupad	N/A	71.33	0.00	28.67

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	44	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	151	99.34 %	0.66 %	0.00 %
Giac	36	11.11 %	30.56 %	58.33 %
Maxima	127	68.50 %	0.00 %	31.50 %
Sympy	194	56.70 %	43.30 %	0.00 %
Mupad	86	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

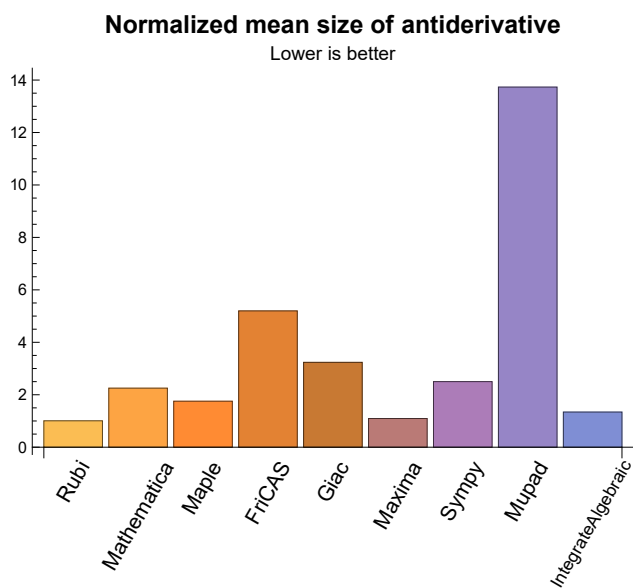
1.3 Performance

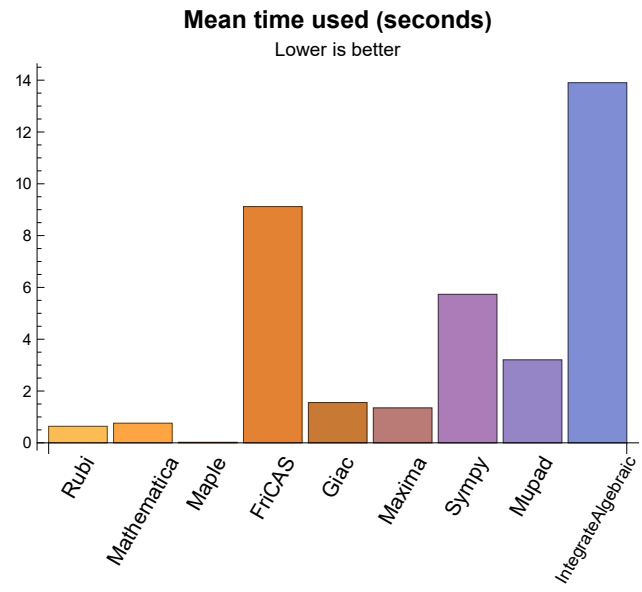
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.64	203.02	1.01	153.00	1.00
Mathematica	0.76	663.39	2.25	142.50	0.99
Maple	0.02	411.12	1.75	187.00	1.16
Maxima	1.35	134.41	1.09	107.00	0.98
Fricas	9.12	1503.34	5.20	215.50	2.06
Sympy	5.73	316.00	2.50	132.00	1.33
Giac	1.56	834.20	3.24	144.00	1.08
Mupad	3.20	4618.84	13.73	175.00	2.20
IntegrateAlgebraic	13.90	315.46	1.34	141.00	1.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {254, 261, 262, 264, 265, 266, 271, 272, 273, 274, 275, 281, 283, 284, 285, 294, 295, 296, 297, 298, 299}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

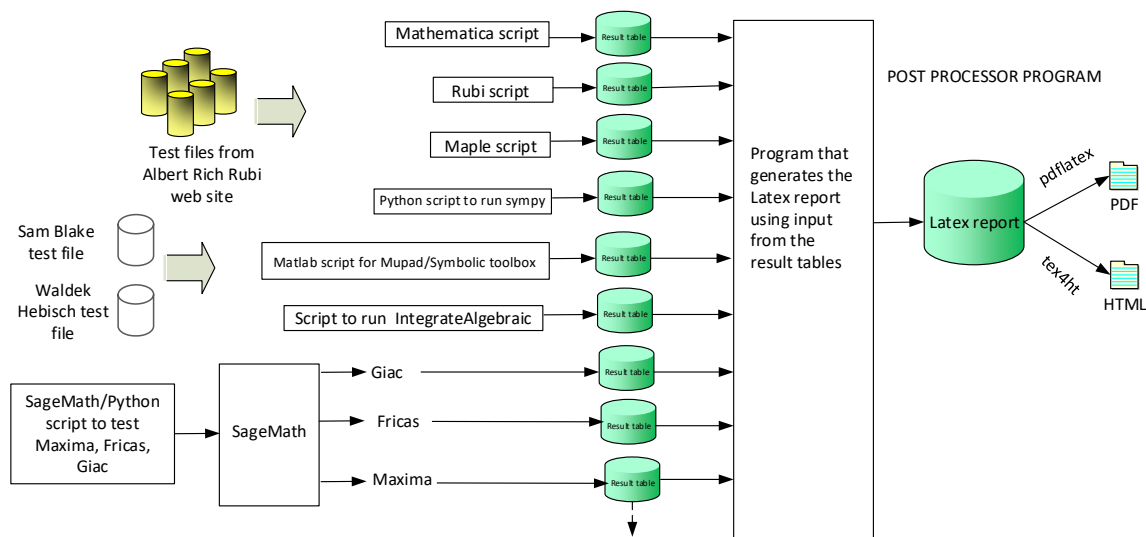
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.
The following field present only in Rubi and Mathematica Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 267, 268, 269, 270, 276, 277, 278, 279, 280, 286, 287, 288, 289, 290, 291, 292, 293, 300 }

B grade: { 36, 38, 40, 46, 48, 254, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 281, 282, 283, 284, 294 }

C grade: { 19, 20, 21, 34, 232, 285, 295, 296, 297, 298, 299 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 92, 93, 107, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 243, 244, 246, 247 }

B grade: { 35, 36, 38, 40, 45, 46, 48, 50, 67, 68, 80, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 136, 137, 152, 156, 157, 158, 217, 218, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 248, 249, 250, 251, 252, 253, 276, 277, 278, 279, 280, 285 }

C grade: { 232, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 41, 42, 43, 44, 45, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216 }

B grade: { 8, 9, 10, 11, 16, 17, 18, 24, 25, 26, 36, 38, 40, 46, 48, 50, 70 }

C grade: { }

F grade: { 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 119, 136, 137, 138, 139, 140, 141, 142, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 159, 160, 161, 162, 163, 164, 165, 174, 175, 176, 177, 178, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 234, 235, 237, 247 }

B grade: { 35, 36, 38, 40, 45, 46, 48, 50, 67, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 153, 154, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 182, 183, 184, 185, 186, 187, 227, 228, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 256, 257, 261, 262, 263, 264, 265, 266, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 288, 289, 290, 291, 300 }

C grade: { }

F grade: { 166, 179, 180, 181, 188, 217, 218, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 236, 238, 239, 240, 241, 242, 254, 255, 258, 259, 260, 267, 268, 269, 270, 271, 272, 286, 287, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 73, 74, 75, 76, 77, 78, 79, 87, 115, 116, 117, 118, 119, 156, 157, 158, 195, 196, 197, 198, 199, 200, 201, 203, 204, 206, 207, 210, 211, 212, 213, 214, 215 }

B grade: { 16, 17, 32, 33, 34, 36, 38, 40, 46, 48, 50, 80, 81, 82, 92, 93, 104, 105, 106, 107, 202, 205, 208, 209, 216 }

C grade: { }

F grade: { 35, 45, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 83, 84, 85, 86, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 29, 30, 31, 32, 33, 34, 37, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 64, 66, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 94, 95, 102, 103, 105, 106, 107, 108, 109, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 131, 132, 133, 134, 136, 137, 138, 140, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 237, 245, 247, 257, 261, 262, 270, 271, 272, 273, 286, 287, 288, 294, 300 }

B grade: { 13, 14, 20, 21, 26, 27, 28, 35, 36, 38, 40, 45, 46, 48, 50, 67, 68, 69, 71, 72, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 104, 110, 111, 112, 113, 114, 125, 126, 127, 128, 130, 135, 141, 142, 148, 149, 150, 156, 157, 158, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 253, 254, 255, 256, 258, 260, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

C grade: { }

F grade: { 61, 63, 65, 139, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 246, 248, 252, 259, 263, 264, 265, 266, 274, 275, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 60, 62, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 131, 138, 139, 140, 146, 147, 148, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 254, 255, 256, 257, 258, 259, 260, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

C grade: { }

F grade: { 55, 57, 59, 61, 63, 64, 65, 66, 67, 68, 69, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 149, 150, 151, 154, 155, 189, 190, 191, 192, 193, 194, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

2.1.9 IntegrateAlgebraic

A grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 55, 57, 59, 61, 63, 64, 65, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 189, 190, 191, 192, 193, 194, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 276, 277, 278, 279, 280 }

B grade: { 56, 58, 60, 62, 66, 238, 285, 300 }

C grade: { 232, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 271 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	149	126	125	125	151	131	125	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.01	0.88	0.84	0.00
time (sec)	N/A	0.220	0.016	0.010	0.431	0.554	0.145	0.261	0.306	0.000
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	149	126	125	125	155	131	125	0
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.04	0.88	0.84	0.00
time (sec)	N/A	0.098	0.004	0.000	0.463	0.571	0.088	0.193	0.069	0.000
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	146	125	124	124	150	130	124	0
N.S.	1	1.00	1.64	1.40	1.39	1.39	1.69	1.46	1.39	0.00
time (sec)	N/A	0.077	0.003	0.003	0.441	0.790	0.089	0.180	0.071	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	141	122	121	121	148	127	121	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.05	0.90	0.86	0.00
time (sec)	N/A	0.082	0.003	0.001	0.465	0.567	0.089	0.221	0.070	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	142	123	125	122	150	131	122	0
N.S.	1	1.00	1.00	0.87	0.88	0.86	1.06	0.92	0.86	0.00
time (sec)	N/A	0.111	0.012	0.014	0.518	0.504	0.250	0.197	0.109	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	122	121	127	143	127	121	0
N.S.	1	1.00	1.00	0.88	0.87	0.91	1.03	0.91	0.87	0.00
time (sec)	N/A	0.082	0.007	0.017	0.522	0.527	0.244	0.195	0.094	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	142	123	125	129	150	142	122	0
N.S.	1	1.00	1.00	0.87	0.88	0.91	1.06	1.00	0.86	0.00
time (sec)	N/A	0.121	0.008	0.009	0.504	0.646	0.279	0.208	0.072	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	50	53	102	48	97	54	42	56
N.S.	1	1.00	0.75	0.79	1.52	0.72	1.45	0.81	0.63	0.84
time (sec)	N/A	0.050	0.053	0.046	1.012	0.611	6.015	0.233	0.370	0.131

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	46	93	43	70	45	37	51
N.S.	1	1.00	0.86	0.90	1.82	0.84	1.37	0.88	0.73	1.00
time (sec)	N/A	0.031	0.026	0.006	1.016	0.746	4.307	0.233	0.392	0.111

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	36	34	67	34	53	38	32	42
N.S.	1	1.00	0.82	0.77	1.52	0.77	1.20	0.86	0.73	0.95
time (sec)	N/A	0.021	0.030	0.008	1.325	0.639	3.083	0.197	0.137	0.112

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	49	99	56	83	76	45	75
N.S.	1	1.00	0.98	0.84	1.71	0.97	1.43	1.31	0.78	1.29
time (sec)	N/A	0.055	0.059	0.016	1.561	0.630	15.588	0.213	0.146	0.157

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	59	61	88	72	83	91	51	76
N.S.	1	1.00	1.00	1.03	1.49	1.22	1.41	1.54	0.86	1.29
time (sec)	N/A	0.056	0.044	0.013	1.408	0.665	7.460	0.245	0.787	0.175

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	75	91	72	76	129	56	77
N.S.	1	1.00	0.94	1.19	1.44	1.14	1.21	2.05	0.89	1.22
time (sec)	N/A	0.055	0.067	0.016	1.237	0.614	6.042	0.244	0.423	0.261
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	72	52	59	59	63	116	43	65
N.S.	1	1.00	1.24	0.90	1.02	1.02	1.09	2.00	0.74	1.12
time (sec)	N/A	0.047	0.034	0.013	1.612	0.728	5.956	0.232	0.681	0.248
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	72	73	127	58	131	80	52	66
N.S.	1	1.00	0.87	0.88	1.53	0.70	1.58	0.96	0.63	0.80
time (sec)	N/A	0.059	0.049	0.025	1.349	0.631	14.314	0.247	0.319	0.184
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	54	58	118	53	124	71	47	61
N.S.	1	1.00	0.81	0.87	1.76	0.79	1.85	1.06	0.70	0.91
time (sec)	N/A	0.040	0.036	0.009	1.133	0.648	11.572	0.205	0.430	0.168

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	46	95	48	109	57	42	56
N.S.	1	1.00	0.93	0.77	1.58	0.80	1.82	0.95	0.70	0.93
time (sec)	N/A	0.030	0.027	0.014	1.597	0.824	8.193	0.242	0.177	0.170

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	67	75	138	67	114	90	55	85
N.S.	1	1.00	0.86	0.96	1.77	0.86	1.46	1.15	0.71	1.09
time (sec)	N/A	0.075	0.034	0.020	1.187	0.537	33.672	0.261	0.180	0.220

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	71	75	122	78	114	102	64	84
N.S.	1	1.00	0.88	0.93	1.51	0.96	1.41	1.26	0.79	1.04
time (sec)	N/A	0.075	0.053	0.016	1.330	0.604	11.391	0.258	0.765	0.252

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	60	73	123	82	133	146	71	88
N.S.	1	1.00	0.70	0.85	1.43	0.95	1.55	1.70	0.83	1.02
time (sec)	N/A	0.077	0.027	0.020	1.372	0.586	12.755	0.255	0.552	0.305

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	60	73	112	82	148	158	82	88
N.S.	1	1.00	0.73	0.89	1.37	1.00	1.80	1.93	1.00	1.07
time (sec)	N/A	0.075	0.025	0.023	1.346	0.486	12.558	0.270	0.946	0.332
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	44	51	104	43	85	46	38	51
N.S.	1	1.00	0.66	0.76	1.55	0.64	1.27	0.69	0.57	0.76
time (sec)	N/A	0.058	0.028	0.018	1.314	0.548	7.072	0.189	0.590	0.129
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	35	39	76	34	66	37	32	42
N.S.	1	1.00	0.69	0.76	1.49	0.67	1.29	0.73	0.63	0.82
time (sec)	N/A	0.042	0.022	0.012	1.218	0.742	5.519	0.216	0.309	0.148
Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	34	32	65	33	53	33	27	41
N.S.	1	1.00	0.97	0.91	1.86	0.94	1.51	0.94	0.77	1.17
time (sec)	N/A	0.026	0.017	0.007	1.158	0.616	4.041	0.216	0.491	0.131

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	20	42	26	22	26	19	32
N.S.	1	1.00	1.00	0.83	1.75	1.08	0.92	1.08	0.79	1.33
time (sec)	N/A	0.017	0.019	0.010	1.118	0.547	2.131	0.196	0.294	0.131
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	30	67	41	31	61	30	55
N.S.	1	1.00	1.00	0.79	1.76	1.08	0.82	1.61	0.79	1.45
time (sec)	N/A	0.038	0.016	0.013	1.173	0.540	6.008	0.205	0.615	0.151
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	31	47	47	31	66	31	51
N.S.	1	1.00	1.00	0.74	1.12	1.12	0.74	1.57	0.74	1.21
time (sec)	N/A	0.037	0.021	0.014	1.257	0.612	3.597	0.216	0.329	0.166
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	49	43	59	50	88	114	43	60
N.S.	1	1.00	0.84	0.74	1.02	0.86	1.52	1.97	0.74	1.03
time (sec)	N/A	0.051	0.023	0.019	1.118	0.724	14.329	0.217	0.694	0.235

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	51	50	89	62	66	45	97	51
N.S.	1	1.00	0.88	0.86	1.53	1.07	1.14	0.78	1.67	0.88
time (sec)	N/A	0.046	0.024	0.021	1.211	0.873	14.277	0.207	1.108	0.179
Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	46	37	63	58	48	39	89	44
N.S.	1	1.00	1.02	0.82	1.40	1.29	1.07	0.87	1.98	0.98
time (sec)	N/A	0.039	0.021	0.014	1.414	0.750	12.336	0.232	0.894	0.208
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	41	34	54	52	39	33	82	41
N.S.	1	1.00	1.17	0.97	1.54	1.49	1.11	0.94	2.34	1.17
time (sec)	N/A	0.027	0.017	0.007	1.227	0.697	10.675	0.254	0.841	0.195
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	22	31	31	16	16	20
N.S.	1	1.00	1.00	0.85	1.10	1.55	1.55	0.80	0.80	1.00
time (sec)	N/A	0.016	0.009	0.003	1.486	0.536	7.856	0.245	0.164	0.183

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	40	56	61	212	61	40	57
N.S.	1	1.00	1.00	0.87	1.22	1.33	4.61	1.33	0.87	1.24
time (sec)	N/A	0.043	0.032	0.020	1.405	0.540	19.618	0.253	0.475	0.210

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	45	47	68	77	228	82	47	65
N.S.	1	1.00	0.69	0.72	1.05	1.18	3.51	1.26	0.72	1.00
time (sec)	N/A	0.055	0.025	0.014	1.098	0.629	12.982	0.243	0.537	0.224

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	189	2295	372	1571	0	3752	1539	0
N.S.	1	1.00	0.70	8.53	1.38	5.84	0.00	13.95	5.72	0.00
time (sec)	N/A	0.161	0.557	0.027	0.885	0.819	0.000	0.459	1.777	1.202

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	153	130	129	132	134	143	121	0
N.S.	1	1.00	2.43	2.06	2.05	2.10	2.13	2.27	1.92	0.00
time (sec)	N/A	0.197	0.022	0.002	0.591	0.506	0.096	0.278	0.094	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	153	130	129	133	141	144	123	0
N.S.	1	1.00	1.00	0.85	0.84	0.87	0.92	0.94	0.80	0.00
time (sec)	N/A	0.117	0.023	0.000	0.667	0.865	0.095	0.296	0.121	0.000
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	151	130	129	133	136	144	123	0
N.S.	1	1.00	3.36	2.89	2.87	2.96	3.02	3.20	2.73	0.00
time (sec)	N/A	0.123	0.017	0.000	0.654	0.482	0.098	0.400	0.078	0.000
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	153	130	129	133	139	144	123	0
N.S.	1	1.00	1.00	0.85	0.84	0.87	0.91	0.94	0.80	0.00
time (sec)	N/A	0.085	0.017	0.002	0.499	0.813	0.097	0.288	0.081	0.000
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	149	130	129	133	133	144	123	0
N.S.	1	1.00	5.14	4.48	4.45	4.59	4.59	4.97	4.24	0.00
time (sec)	N/A	0.049	0.012	0.002	0.645	0.836	0.096	0.314	0.077	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	143	127	125	130	134	141	120	0
N.S.	1	1.00	1.00	0.89	0.87	0.91	0.94	0.99	0.84	0.00
time (sec)	N/A	0.074	0.016	0.001	0.576	0.564	0.098	0.362	0.079	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	149	132	130	127	131	145	121	0
N.S.	1	1.00	1.60	1.42	1.40	1.37	1.41	1.56	1.30	0.00
time (sec)	N/A	0.055	0.025	0.003	0.478	0.758	0.328	0.267	0.129	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	141	129	125	131	124	139	119	0
N.S.	1	1.00	1.00	0.91	0.89	0.93	0.88	0.99	0.84	0.00
time (sec)	N/A	0.082	0.026	0.005	0.504	0.880	0.321	0.361	0.081	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	147	131	130	133	131	156	120	0
N.S.	1	1.00	1.00	0.89	0.88	0.90	0.89	1.06	0.82	0.00
time (sec)	N/A	0.136	0.034	0.007	0.698	0.784	0.386	0.263	0.084	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	122	1121	192	759	0	1848	1483	0
N.S.	1	1.00	0.60	5.52	0.95	3.74	0.00	9.10	7.31	0.00
time (sec)	N/A	0.073	0.044	0.013	0.972	0.819	0.000	0.619	1.251	0.559
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	85	62	61	61	76	61	61	0
N.S.	1	1.00	2.50	1.82	1.79	1.79	2.24	1.79	1.79	0.00
time (sec)	N/A	0.047	0.002	0.001	0.637	0.560	0.072	0.339	0.062	0.000
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	62	61	61	75	61	61	0
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73	0.00
time (sec)	N/A	0.031	0.002	0.003	0.762	0.765	0.071	0.414	0.059	0.000
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	83	62	61	61	75	61	61	0
N.S.	1	1.00	3.61	2.70	2.65	2.65	3.26	2.65	2.65	0.00
time (sec)	N/A	0.022	0.002	0.001	0.563	0.859	0.073	0.362	0.060	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	62	61	61	75	61	61	0
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73	0.00
time (sec)	N/A	0.027	0.001	0.000	0.880	0.566	0.071	0.304	0.059	0.000
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	62	61	61	71	76	61	0
N.S.	1	1.00	1.00	5.64	5.55	5.55	6.45	6.91	5.55	0.00
time (sec)	N/A	0.002	0.002	0.001	0.782	0.512	0.070	0.314	0.058	0.000
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	58	57	57	68	57	57	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78	0.00
time (sec)	N/A	0.022	0.001	0.001	0.977	1.103	0.074	0.274	0.058	0.000
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	59	62	58	75	62	58	0
N.S.	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72	0.00
time (sec)	N/A	0.033	0.003	0.003	0.821	0.846	0.107	0.230	0.061	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	60	59	62	66	59	59	0
N.S.	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81	0.00
time (sec)	N/A	0.025	0.003	0.004	0.826	0.786	0.101	0.297	0.060	0.001
Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	61	62	64	75	69	60	0
N.S.	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75	0.00
time (sec)	N/A	0.040	0.003	0.005	0.736	0.504	0.112	0.357	0.061	0.000
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	80	90	54	129	90	101	-1	81
N.S.	1	1.00	0.55	0.62	0.37	0.89	0.62	0.70	-0.01	0.56
time (sec)	N/A	0.090	0.083	0.039	1.519	0.770	0.365	0.430	0.000	9.872
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	51	55	31	29	27	42	103	208
N.S.	1	1.00	0.61	0.66	0.37	0.35	0.33	0.51	1.24	2.51
time (sec)	N/A	0.072	0.021	0.010	0.779	0.834	0.277	0.383	0.876	0.461

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	69	62	33	98	82	59	-1	60
N.S.	1	1.00	0.71	0.64	0.34	1.01	0.85	0.61	-0.01	0.62
time (sec)	N/A	0.047	0.027	0.007	1.507	0.593	0.317	0.265	0.000	8.951

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	54	57	35	33	26	61	83	198
N.S.	1	1.00	0.59	0.62	0.38	0.36	0.28	0.66	0.90	2.15
time (sec)	N/A	0.072	0.021	0.010	0.734	0.780	0.709	0.381	0.766	0.442

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	72	67	37	105	82	62	-1	63
N.S.	1	1.00	0.71	0.66	0.37	1.04	0.81	0.61	-0.01	0.62
time (sec)	N/A	0.063	0.030	0.010	1.216	0.702	0.372	0.343	0.000	6.376

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	70	79	48	48	41	131	125	713
N.S.	1	1.00	0.51	0.58	0.35	0.35	0.30	0.96	0.91	5.20
time (sec)	N/A	0.098	0.033	0.013	0.784	0.712	0.726	0.386	0.810	1.170

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	108	188	125	300	0	0	-1	105
N.S.	1	1.00	0.71	1.23	0.82	1.96	0.00	0.00	-0.01	0.69
time (sec)	N/A	0.119	0.060	0.019	1.489	0.934	0.000	0.000	0.000	15.923

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	45	38	65	42	0	40	48	192
N.S.	1	1.00	0.58	0.49	0.84	0.55	0.00	0.52	0.62	2.49
time (sec)	N/A	0.066	0.019	0.007	0.636	0.847	0.000	0.507	0.179	0.851

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	108	186	124	301	0	0	-1	105
N.S.	1	1.00	0.69	1.19	0.79	1.93	0.00	0.00	-0.01	0.67
time (sec)	N/A	0.089	0.051	0.014	1.670	0.782	0.000	0.000	0.000	15.777

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	92	133	88	119	0	96	-1	258
N.S.	1	1.00	0.57	0.83	0.55	0.74	0.00	0.60	-0.01	1.60
time (sec)	N/A	0.118	0.041	0.021	0.888	0.645	0.000	0.528	0.000	0.885

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	124	206	134	334	0	0	-1	117
N.S.	1	1.00	0.65	1.08	0.71	1.76	0.00	0.00	-0.01	0.62
time (sec)	N/A	0.185	0.069	0.023	1.403	0.964	0.000	0.000	0.000	19.381

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	130	249	138	205	0	144	-1	1473
N.S.	1	1.00	0.58	1.12	0.62	0.92	0.00	0.65	-0.00	6.61
time (sec)	N/A	0.184	0.070	0.025	0.841	0.666	0.000	0.403	0.000	6.044

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	400	400	160	1099	491	853	0	2213	-1	0
N.S.	1	1.00	0.40	2.75	1.23	2.13	0.00	5.53	-0.00	0.00
time (sec)	N/A	0.243	0.220	0.010	0.815	0.859	0.000	0.687	0.000	5.116

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	112	495	243	381	0	1013	-1	0
N.S.	1	1.00	0.41	1.79	0.88	1.38	0.00	3.67	-0.00	0.00
time (sec)	N/A	0.153	0.110	0.009	0.964	0.693	0.000	0.524	0.000	3.138

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	86	131	75	94	0	269	-1	0
N.S.	1	1.00	0.56	0.86	0.49	0.61	0.00	1.76	-0.01	0.00
time (sec)	N/A	0.076	0.055	0.006	0.811	0.726	0.000	0.322	0.000	1.600
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	25	40	86	47	165	32	59	0
N.S.	1	1.00	0.74	1.18	2.53	1.38	4.85	0.94	1.74	0.00
time (sec)	N/A	0.029	0.008	0.002	0.722	0.709	9.597	0.289	0.142	0.132
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	45	62	135	92	0	196	108	0
N.S.	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26	0.00
time (sec)	N/A	0.089	0.024	0.007	0.737	0.664	0.000	0.427	0.171	0.285
Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	68	99	196	140	0	331	169	0
N.S.	1	1.00	0.53	0.77	1.53	1.09	0.00	2.59	1.32	0.00
time (sec)	N/A	0.129	0.035	0.006	0.698	0.774	0.000	0.385	0.202	0.420

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	226	166	193	202	193	169	0
N.S.	1	1.00	1.00	1.36	1.00	1.16	1.22	1.16	1.02	0.00
time (sec)	N/A	0.393	0.050	0.000	0.604	0.714	0.102	0.338	0.076	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	226	166	193	204	193	169	0
N.S.	1	1.00	1.00	1.36	1.00	1.16	1.23	1.16	1.02	0.00
time (sec)	N/A	0.154	0.054	0.000	0.726	0.615	0.101	0.270	0.096	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	154	226	166	193	199	193	169	0
N.S.	1	1.00	0.93	1.36	1.00	1.16	1.20	1.16	1.02	0.00
time (sec)	N/A	0.287	0.056	0.000	0.768	0.596	0.102	0.287	0.048	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	161	223	163	189	199	189	165	0
N.S.	1	1.00	1.00	1.39	1.01	1.17	1.24	1.17	1.02	0.00
time (sec)	N/A	0.118	0.046	0.000	0.730	0.764	0.102	0.264	0.049	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	162	191	167	164	199	193	166	0
N.S.	1	1.00	1.00	1.18	1.03	1.01	1.23	1.19	1.02	0.00
time (sec)	N/A	0.227	0.056	0.005	0.787	0.818	0.309	0.335	0.103	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	156	186	162	168	185	185	163	0
N.S.	1	1.00	1.00	1.19	1.04	1.08	1.19	1.19	1.04	0.00
time (sec)	N/A	0.108	0.082	0.004	0.707	0.955	0.307	0.306	0.051	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	162	190	167	170	197	212	166	0
N.S.	1	1.00	1.00	1.17	1.03	1.05	1.22	1.31	1.02	0.00
time (sec)	N/A	0.226	0.071	0.007	0.785	0.842	0.401	0.404	0.056	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	126	261	0	421	620	126	1343	0
N.S.	1	1.00	0.95	1.96	0.00	3.17	4.66	0.95	10.10	0.00
time (sec)	N/A	0.207	0.064	0.007	0.000	1.586	43.892	1.873	0.458	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	93	175	0	312	434	91	979	0
N.S.	1	1.00	0.96	1.80	0.00	3.22	4.47	0.94	10.09	0.00
time (sec)	N/A	0.116	0.068	0.004	0.000	0.772	10.545	1.812	0.646	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	98	0	219	287	67	606	0
N.S.	1	1.00	1.00	1.38	0.00	3.08	4.04	0.94	8.54	0.00
time (sec)	N/A	0.070	0.046	0.003	0.000	0.827	3.589	1.715	0.498	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	128	105	0	249	0	78	2424	0
N.S.	1	1.00	1.64	1.35	0.00	3.19	0.00	1.00	31.08	0.00
time (sec)	N/A	0.139	0.103	0.007	0.000	1.145	0.000	1.610	4.476	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	186	191	0	385	0	124	3729	0
N.S.	1	1.00	1.66	1.71	0.00	3.44	0.00	1.11	33.29	0.00
time (sec)	N/A	0.245	0.218	0.010	0.000	1.075	0.000	1.874	4.855	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	327	825	0	5140	0	4391	10177	0
N.S.	1	1.00	1.25	3.16	0.00	19.69	0.00	16.82	38.99	0.00
time (sec)	N/A	1.489	0.405	0.053	0.000	3.519	0.000	3.789	1.594	0.001
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	251	560	0	2632	0	3179	6366	0
N.S.	1	1.00	1.21	2.69	0.00	12.65	0.00	15.28	30.61	0.00
time (sec)	N/A	0.528	0.163	0.026	0.000	1.296	0.000	3.498	1.256	0.001
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	173	328	0	1569	314	1400	4109	0
N.S.	1	1.00	1.01	1.91	0.00	9.12	1.83	8.14	23.89	0.00
time (sec)	N/A	0.201	0.094	0.020	0.000	1.327	16.959	2.416	0.997	0.001
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	206	353	0	2914	0	2805	6335	0
N.S.	1	1.00	1.09	1.87	0.00	15.42	0.00	14.84	33.52	0.00
time (sec)	N/A	0.400	0.291	0.030	0.000	1.475	0.000	3.568	1.354	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	267	611	0	5442	0	2870	10101	0
N.S.	1	1.00	0.99	2.25	0.00	20.08	0.00	10.59	37.27	0.00
time (sec)	N/A	0.653	0.316	0.033	0.000	3.554	0.000	3.346	2.187	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	208	689	0	1323	0	239	2282	0
N.S.	1	1.00	0.98	3.25	0.00	6.24	0.00	1.13	10.76	0.00
time (sec)	N/A	0.381	0.282	0.020	0.000	0.779	0.000	1.649	0.834	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	160	286	0	849	0	194	1527	0
N.S.	1	1.00	1.09	1.95	0.00	5.78	0.00	1.32	10.39	0.00
time (sec)	N/A	0.175	0.186	0.014	0.000	1.008	0.000	1.617	1.217	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	111	158	0	538	394	120	283	0
N.S.	1	1.00	1.04	1.48	0.00	5.03	3.68	1.12	2.64	0.00
time (sec)	N/A	0.113	0.080	0.012	0.000	0.800	5.380	1.723	0.323	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	101	127	0	474	374	102	264	0
N.S.	1	1.00	1.07	1.35	0.00	5.04	3.98	1.09	2.81	0.00
time (sec)	N/A	0.088	0.074	0.007	0.000	0.691	3.363	1.367	0.301	0.001
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	243	361	0	1014	0	201	7119	0
N.S.	1	1.00	1.62	2.41	0.00	6.76	0.00	1.34	47.46	0.00
time (sec)	N/A	0.331	0.330	0.018	0.000	1.817	0.000	1.710	7.884	0.001
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	379	622	0	1635	0	250	10034	0
N.S.	1	1.00	1.70	2.79	0.00	7.33	0.00	1.12	45.00	0.00
time (sec)	N/A	0.419	0.564	0.023	0.000	3.863	0.000	1.647	9.088	0.001
Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	425	425	455	1507	0	7252	0	5681	16604	0
N.S.	1	1.00	1.07	3.55	0.00	17.06	0.00	13.37	39.07	0.00
time (sec)	N/A	3.675	1.202	0.043	0.000	8.558	0.000	6.490	4.358	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	362	1030	0	4658	0	4538	12396	0
N.S.	1	1.00	1.08	3.07	0.00	13.86	0.00	13.51	36.89	0.00
time (sec)	N/A	1.717	0.853	0.036	0.000	2.817	0.000	6.288	5.184	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	298	733	0	3467	0	3776	9444	0
N.S.	1	1.00	1.08	2.66	0.00	12.56	0.00	13.68	34.22	0.00
time (sec)	N/A	0.553	0.664	0.029	0.000	1.916	0.000	4.777	4.411	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	304	1761	0	4885	0	4426	12349	0
N.S.	1	1.00	1.04	6.01	0.00	16.67	0.00	15.11	42.15	0.00
time (sec)	N/A	0.846	0.786	0.110	0.000	3.721	0.000	5.965	4.839	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	389	389	382	1252	0	7583	0	5408	17591	0
N.S.	1	1.00	0.98	3.22	0.00	19.49	0.00	13.90	45.22	0.00
time (sec)	N/A	1.220	1.039	0.040	0.000	9.541	0.000	6.160	5.380	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	522	522	487	1653	0	10190	0	6327	21554	0
N.S.	1	1.00	0.93	3.17	0.00	19.52	0.00	12.12	41.29	0.00
time (sec)	N/A	1.365	1.199	0.047	0.000	22.200	0.000	8.150	5.698	0.001
Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	435	2054	0	3196	0	598	4501	0
N.S.	1	1.00	1.19	5.63	0.00	8.76	0.00	1.64	12.33	0.00
time (sec)	N/A	1.455	0.663	0.031	0.000	1.408	0.000	5.883	4.660	0.001
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	354	723	0	2167	0	466	3062	0
N.S.	1	1.00	1.39	2.85	0.00	8.53	0.00	1.83	12.06	0.00
time (sec)	N/A	0.404	0.479	0.026	0.000	1.163	0.000	6.311	5.151	0.001
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	261	398	0	1378	775	318	593	0
N.S.	1	1.00	1.79	2.73	0.00	9.44	5.31	2.18	4.06	0.00
time (sec)	N/A	0.139	0.268	0.017	0.000	0.948	102.043	6.463	0.694	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	233	411	0	1369	833	268	625	0
N.S.	1	1.00	1.26	2.22	0.00	7.40	4.50	1.45	3.38	0.00
time (sec)	N/A	0.262	0.236	0.020	0.000	0.953	44.844	6.587	0.680	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	172	379	0	1226	789	228	587	0
N.S.	1	1.00	1.01	2.23	0.00	7.21	4.64	1.34	3.45	0.00
time (sec)	N/A	0.163	0.204	0.017	0.000	0.928	21.367	6.057	0.660	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	142	262	0	1109	661	208	517	0
N.S.	1	1.00	1.02	1.88	0.00	7.98	4.76	1.50	3.72	0.00
time (sec)	N/A	0.124	0.135	0.010	0.000	0.688	12.404	5.570	0.587	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	396	1161	0	2494	0	421	11674	0
N.S.	1	1.00	1.57	4.61	0.00	9.90	0.00	1.67	46.33	0.00
time (sec)	N/A	0.543	0.691	0.029	0.000	7.448	0.000	6.445	11.572	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	642	1862	0	3956	0	648	16265	0
N.S.	1	1.00	1.77	5.13	0.00	10.90	0.00	1.79	44.81	0.00
time (sec)	N/A	0.771	1.500	0.039	0.000	14.441	0.000	6.371	15.906	0.001
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	554	554	644	2015	0	9636	0	3987	22911	0
N.S.	1	1.00	1.16	3.64	0.00	17.39	0.00	7.20	41.36	0.00
time (sec)	N/A	11.195	2.398	0.069	0.000	20.777	0.000	8.280	5.047	0.001
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	461	461	543	1631	0	7060	0	7578	19041	0
N.S.	1	1.00	1.18	3.54	0.00	15.31	0.00	16.44	41.30	0.00
time (sec)	N/A	4.622	2.021	0.048	0.000	6.935	0.000	11.662	3.951	0.001
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	380	380	447	1283	0	5650	0	3162	16688	0
N.S.	1	1.00	1.18	3.38	0.00	14.87	0.00	8.32	43.92	0.00
time (sec)	N/A	1.415	1.696	0.049	0.000	5.874	0.000	8.261	3.487	0.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	438	438	436	1335	0	7270	0	7267	18992	0
N.S.	1	1.00	1.00	3.05	0.00	16.60	0.00	16.59	43.36	0.00
time (sec)	N/A	1.090	1.646	0.046	0.000	9.419	0.000	11.773	3.916	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	516	11936	0	9909	0	4609	22914	0
N.S.	1	1.00	1.12	25.95	0.00	21.54	0.00	10.02	49.81	0.00
time (sec)	N/A	1.353	2.189	0.281	0.000	23.684	0.000	8.537	4.614	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	17	19	17	0
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.76	0.68	0.00
time (sec)	N/A	0.018	0.009	0.006	0.718	1.144	0.121	0.305	0.057	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	25	17	17	19	17	0
N.S.	1	1.00	1.00	0.72	1.00	0.68	0.68	0.76	0.68	0.00
time (sec)	N/A	0.029	0.006	0.004	0.739	0.665	0.115	0.283	0.029	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.035	0.013	0.003	1.401	0.944	0.120	0.282	0.212	0.000
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	53	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	1.43	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.042	0.006	0.003	1.597	1.189	0.121	0.372	0.034	0.000
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	41	0	47	44	38	41	0
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.91	0.00
time (sec)	N/A	0.049	0.026	0.006	0.000	0.830	0.152	0.944	0.052	0.001
Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	71	91	104	61	0	102	102	69
N.S.	1	1.00	0.70	0.89	1.02	0.60	0.00	1.00	1.00	0.68
time (sec)	N/A	0.079	0.029	0.029	0.672	1.361	0.000	0.465	0.490	0.178

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	66	74	87	56	0	88	85	64
N.S.	1	1.00	0.81	0.91	1.07	0.69	0.00	1.09	1.05	0.79
time (sec)	N/A	0.056	0.021	0.015	0.593	1.283	0.000	0.363	0.433	0.146

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	61	57	70	51	0	74	67	59
N.S.	1	1.00	0.82	0.77	0.95	0.69	0.00	1.00	0.91	0.80
time (sec)	N/A	0.044	0.017	0.011	0.653	1.283	0.000	0.342	0.287	0.148

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	92	85	89	95	0	98	86	93
N.S.	1	1.00	0.98	0.90	0.95	1.01	0.00	1.04	0.91	0.99
time (sec)	N/A	0.083	0.038	0.015	1.496	1.261	0.000	0.478	0.426	0.204

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	104	89	112	0	138	84	96
N.S.	1	1.00	1.00	1.07	0.92	1.15	0.00	1.42	0.87	0.99
time (sec)	N/A	0.083	0.041	0.016	1.350	0.711	0.000	0.515	0.878	0.224

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	97	121	106	112	0	169	-1	98
N.S.	1	1.00	0.98	1.22	1.07	1.13	0.00	1.71	-0.01	0.99
time (sec)	N/A	0.083	0.039	0.016	1.426	0.924	0.000	0.572	0.000	0.272
Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	74	118	99	90	0	189	-1	75
N.S.	1	1.00	0.82	1.31	1.10	1.00	0.00	2.10	-0.01	0.83
time (sec)	N/A	0.068	0.026	0.015	1.453	0.754	0.000	0.532	0.000	0.326
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	82	135	116	95	0	233	-1	80
N.S.	1	1.00	0.74	1.22	1.05	0.86	0.00	2.10	-0.01	0.72
time (sec)	N/A	0.086	0.027	0.017	1.736	1.217	0.000	0.457	0.000	0.368
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	84	152	133	100	0	255	-1	85
N.S.	1	1.00	0.64	1.15	1.01	0.76	0.00	1.93	-0.01	0.64
time (sec)	N/A	0.109	0.037	0.019	1.730	0.932	0.000	0.563	0.000	0.448

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	81	138	135	71	0	207	-1	79
N.S.	1	1.00	0.64	1.09	1.06	0.56	0.00	1.63	-0.01	0.62
time (sec)	N/A	0.096	0.035	0.033	0.585	0.996	0.000	0.541	0.000	0.303

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	76	121	118	66	0	179	-1	74
N.S.	1	1.00	0.72	1.14	1.11	0.62	0.00	1.69	-0.01	0.70
time (sec)	N/A	0.072	0.031	0.016	0.878	1.022	0.000	0.587	0.000	0.245

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	71	104	101	61	0	151	127	69
N.S.	1	1.00	0.72	1.05	1.02	0.62	0.00	1.53	1.28	0.70
time (sec)	N/A	0.058	0.025	0.016	0.666	0.765	0.000	0.452	0.530	0.242

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	104	117	120	106	0	113	-1	103
N.S.	1	1.00	0.87	0.98	1.01	0.89	0.00	0.95	-0.01	0.87
time (sec)	N/A	0.106	0.058	0.015	1.396	1.232	0.000	0.498	0.000	0.320

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	107	117	120	122	0	153	-1	106
N.S.	1	1.00	0.88	0.96	0.98	1.00	0.00	1.25	-0.01	0.87
time (sec)	N/A	0.107	0.054	0.021	1.746	1.035	0.000	0.559	0.000	0.346
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	107	117	137	122	0	190	-1	108
N.S.	1	1.00	0.84	0.92	1.08	0.96	0.00	1.50	-0.01	0.85
time (sec)	N/A	0.109	0.062	0.020	1.461	1.155	0.000	0.654	0.000	0.392
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	107	117	154	122	0	227	-1	108
N.S.	1	1.00	0.84	0.92	1.21	0.96	0.00	1.79	-0.01	0.85
time (sec)	N/A	0.107	0.051	0.022	1.585	1.021	0.000	0.681	0.000	0.534
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	139	286	0	315	0	138	-1	135
N.S.	1	1.00	0.91	1.87	0.00	2.06	0.00	0.90	-0.01	0.88
time (sec)	N/A	0.203	0.111	0.030	0.000	0.727	0.000	0.544	0.000	0.540

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	101	176	0	233	0	98	-1	104
N.S.	1	1.00	1.01	1.76	0.00	2.33	0.00	0.98	-0.01	1.04
time (sec)	N/A	0.093	0.047	0.015	0.000	1.093	0.000	0.449	0.000	0.473
Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	78	93	0	178	0	69	92	78
N.S.	1	1.00	1.03	1.22	0.00	2.34	0.00	0.91	1.21	1.03
time (sec)	N/A	0.064	0.025	0.012	0.000	1.300	0.000	0.461	1.046	0.385
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	89	76	0	517	0	0	81	89
N.S.	1	1.00	0.99	0.84	0.00	5.74	0.00	0.00	0.90	0.99
time (sec)	N/A	0.094	0.028	0.013	0.000	1.250	0.000	0.000	0.760	0.335
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	82	104	0	197	0	124	103	81
N.S.	1	1.00	1.02	1.30	0.00	2.46	0.00	1.55	1.29	1.01
time (sec)	N/A	0.083	0.035	0.018	0.000	1.330	0.000	0.538	0.776	0.392

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	107	194	0	255	0	339	-1	148
N.S.	1	1.00	0.86	1.56	0.00	2.06	0.00	2.73	-0.01	1.19
time (sec)	N/A	0.145	0.075	0.019	0.000	1.592	0.000	0.516	0.000	0.637
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	148	311	0	339	0	571	-1	188
N.S.	1	1.00	0.84	1.76	0.00	1.92	0.00	3.23	-0.01	1.06
time (sec)	N/A	0.239	0.108	0.020	0.000	1.884	0.000	0.639	0.000	0.905
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	66	87	90	56	0	60	-1	64
N.S.	1	1.00	0.67	0.89	0.92	0.57	0.00	0.61	-0.01	0.65
time (sec)	N/A	0.087	0.028	0.019	0.926	0.620	0.000	0.370	0.000	0.201
Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	61	70	73	51	0	53	-1	59
N.S.	1	1.00	0.79	0.91	0.95	0.66	0.00	0.69	-0.01	0.77
time (sec)	N/A	0.066	0.020	0.014	0.988	0.968	0.000	0.344	0.000	0.185

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	53	56	46	0	46	-1	54
N.S.	1	1.00	1.00	0.95	1.00	0.82	0.00	0.82	-0.02	0.96
time (sec)	N/A	0.045	0.016	0.013	1.040	0.984	0.000	0.348	0.000	0.182
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	36	39	39	0	39	35	47
N.S.	1	1.00	1.00	0.73	0.80	0.80	0.00	0.80	0.71	0.96
time (sec)	N/A	0.032	0.009	0.012	0.923	0.748	0.000	0.468	0.523	0.170
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	52	58	75	0	78	56	68
N.S.	1	1.00	1.00	0.75	0.84	1.09	0.00	1.13	0.81	0.99
time (sec)	N/A	0.062	0.013	0.012	1.929	0.804	0.000	0.435	1.009	0.193
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	62	49	51	78	0	101	83	63
N.S.	1	1.00	1.00	0.79	0.82	1.26	0.00	1.63	1.34	1.02
time (sec)	N/A	0.050	0.015	0.015	2.021	0.782	0.000	0.386	0.663	0.251

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	67	66	68	83	0	145	-1	70
N.S.	1	1.00	0.81	0.80	0.82	1.00	0.00	1.75	-0.01	0.84
time (sec)	N/A	0.070	0.022	0.013	2.000	0.812	0.000	0.521	0.000	0.284
Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	77	83	85	90	0	167	-1	75
N.S.	1	1.00	0.74	0.80	0.82	0.87	0.00	1.61	-0.01	0.72
time (sec)	N/A	0.088	0.024	0.017	1.981	0.860	0.000	0.507	0.000	0.339
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	72	91	73	86	0	52	-1	59
N.S.	1	1.00	0.94	1.18	0.95	1.12	0.00	0.68	-0.01	0.77
time (sec)	N/A	0.058	0.018	0.022	0.923	1.007	0.000	0.383	0.000	0.338
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	54	95	56	81	0	46	52	54
N.S.	1	1.00	0.96	1.70	1.00	1.45	0.00	0.82	0.93	0.96
time (sec)	N/A	0.043	0.121	0.013	0.933	0.990	0.000	0.512	0.312	0.273

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	32	46	0	21	21	25
N.S.	1	1.00	1.00	0.88	1.28	1.84	0.00	0.84	0.84	1.00
time (sec)	N/A	0.019	0.092	0.006	0.938	0.904	0.000	0.363	0.238	0.285
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	67	65	107	0	78	-1	67
N.S.	1	1.00	1.00	1.02	0.98	1.62	0.00	1.18	-0.02	1.02
time (sec)	N/A	0.057	0.024	0.019	2.006	0.935	0.000	0.556	0.000	0.364
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	88	84	82	124	0	122	-1	75
N.S.	1	1.00	0.98	0.93	0.91	1.38	0.00	1.36	-0.01	0.83
time (sec)	N/A	0.071	0.018	0.016	2.088	1.094	0.000	0.492	0.000	0.418
Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	191	1935	408	1357	11538	2816	769	0
N.S.	1	1.00	0.79	7.96	1.68	5.58	47.48	11.59	3.16	0.00
time (sec)	N/A	0.176	0.298	0.009	1.388	0.860	12.378	0.750	1.058	1.208

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	117	783	230	573	4190	1178	429	0
N.S.	1	1.00	0.75	5.05	1.48	3.70	27.03	7.60	2.77	0.00
time (sec)	N/A	0.099	0.123	0.008	1.221	0.704	5.444	0.398	0.598	0.817
Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	59	221	104	171	1056	350	171	0
N.S.	1	1.00	0.71	2.66	1.25	2.06	12.72	4.22	2.06	0.00
time (sec)	N/A	0.047	0.049	0.004	1.058	0.717	1.862	0.419	0.339	0.118
Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	134	122	120	277	0	121	181	0
N.S.	1	1.00	1.00	0.91	0.90	2.07	0.00	0.90	1.35	0.00
time (sec)	N/A	0.179	0.063	0.014	1.998	11.001	0.000	0.309	0.873	0.001
Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	99	108	107	212	0	105	166	0
N.S.	1	1.00	0.84	0.92	0.91	1.80	0.00	0.89	1.41	0.00
time (sec)	N/A	0.152	0.094	0.010	2.018	5.176	0.000	0.307	0.722	0.001

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	77	92	89	170	0	90	138	0
N.S.	1	1.00	0.73	0.88	0.85	1.62	0.00	0.86	1.31	0.00
time (sec)	N/A	0.137	0.034	0.009	1.998	2.920	0.000	0.356	0.989	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	66	83	82	145	0	86	944	0
N.S.	1	1.00	0.69	0.86	0.85	1.51	0.00	0.90	9.83	0.00
time (sec)	N/A	0.094	0.040	0.007	1.985	1.332	0.000	0.393	1.936	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	67	83	82	146	0	85	328	0
N.S.	1	1.00	0.70	0.86	0.85	1.52	0.00	0.89	3.42	0.00
time (sec)	N/A	0.064	0.037	0.007	1.985	1.500	0.000	0.354	1.020	0.001

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	134	101	101	201	0	102	527	0
N.S.	1	1.00	1.18	0.89	0.89	1.76	0.00	0.89	4.62	0.00
time (sec)	N/A	0.125	0.071	0.010	1.966	16.718	0.000	0.289	0.961	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	169	119	120	265	0	132	820	0
N.S.	1	1.00	1.31	0.92	0.93	2.05	0.00	1.02	6.36	0.00
time (sec)	N/A	0.150	0.101	0.013	1.997	101.424	0.000	0.302	1.381	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	209	145	145	0	0	168	1017	0
N.S.	1	1.00	1.34	0.93	0.93	0.00	0.00	1.08	6.52	0.00
time (sec)	N/A	0.183	0.093	0.016	2.050	0.000	0.000	0.342	1.868	0.001

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	344	405	294	4414	0	363	6097	0
N.S.	1	1.00	0.96	1.13	0.82	12.30	0.00	1.01	16.98	0.00
time (sec)	N/A	0.346	0.360	0.017	2.048	32.470	0.000	0.541	2.056	0.001

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	345	345	373	387	289	4354	0	333	5908	0
N.S.	1	1.00	1.08	1.12	0.84	12.62	0.00	0.97	17.12	0.00
time (sec)	N/A	0.302	0.238	0.008	2.089	4.864	0.000	0.444	1.827	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	233	363	268	4040	0	327	5111	0
N.S.	1	1.00	0.69	1.08	0.80	12.02	0.00	0.97	15.21	0.00
time (sec)	N/A	0.274	0.151	0.009	2.627	1.764	0.000	0.434	2.199	0.001

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	232	351	275	3892	0	336	4720	0
N.S.	1	1.00	0.69	1.04	0.82	11.55	0.00	1.00	14.01	0.00
time (sec)	N/A	0.267	0.126	0.009	1.427	1.776	0.000	0.377	1.589	0.001

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	234	363	268	4084	0	339	4802	0
N.S.	1	1.00	0.70	1.08	0.80	12.15	0.00	1.01	14.29	0.00
time (sec)	N/A	0.273	0.145	0.008	1.090	3.200	0.000	0.416	1.672	0.001

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	389	390	292	4362	0	348	5761	0
N.S.	1	1.00	1.12	1.12	0.84	12.53	0.00	1.00	16.55	0.00
time (sec)	N/A	0.300	0.253	0.012	1.274	9.080	0.000	0.398	1.999	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	367	406	297	4442	0	364	5972	0
N.S.	1	1.00	1.02	1.13	0.82	12.34	0.00	1.01	16.59	0.00
time (sec)	N/A	0.301	0.401	0.012	2.123	26.625	0.000	0.395	2.257	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	135	305	220	555	0	251	305	0
N.S.	1	1.00	0.80	1.80	1.30	3.28	0.00	1.49	1.80	0.00
time (sec)	N/A	0.367	0.210	0.020	2.075	35.277	0.000	0.364	1.302	0.001

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	142	260	197	457	0	223	647	0
N.S.	1	1.00	0.95	1.73	1.31	3.05	0.00	1.49	4.31	0.00
time (sec)	N/A	0.247	0.112	0.019	2.047	17.039	0.000	0.351	1.487	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	153	120	252	192	487	0	220	528	0
N.S.	1	0.99	0.77	1.63	1.24	3.14	0.00	1.42	3.41	0.00
time (sec)	N/A	0.248	0.153	0.016	2.077	6.195	0.000	0.331	1.520	0.001

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	148	114	247	186	492	0	188	527	0
N.S.	1	0.99	0.77	1.66	1.25	3.30	0.00	1.26	3.54	0.00
time (sec)	N/A	0.187	0.141	0.015	2.027	7.088	0.000	0.281	1.411	0.001

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	117	255	196	458	0	199	649	0
N.S.	1	1.00	0.77	1.69	1.30	3.03	0.00	1.32	4.30	0.00
time (sec)	N/A	0.181	0.133	0.017	2.030	16.308	0.000	0.383	1.493	0.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	241	309	228	0	0	279	1082	0
N.S.	1	1.00	1.15	1.48	1.09	0.00	0.00	1.33	5.18	0.00
time (sec)	N/A	0.239	0.183	0.023	2.054	0.000	0.000	0.368	2.577	0.001

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	248	332	278	0	0	344	1337	0
N.S.	1	1.00	1.05	1.41	1.18	0.00	0.00	1.46	5.67	0.00
time (sec)	N/A	0.261	0.427	0.025	2.007	0.000	0.000	0.353	2.942	0.001

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	265	278	363	332	0	0	350	1545	0
N.S.	1	1.00	1.05	1.37	1.25	0.00	0.00	1.32	5.83	0.00
time (sec)	N/A	0.327	0.391	0.025	2.082	0.000	0.000	0.367	3.477	0.001
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	712	712	431	873	504	9856	0	581	18343	0
N.S.	1	1.00	0.61	1.23	0.71	13.84	0.00	0.82	25.76	0.00
time (sec)	N/A	0.671	0.304	0.017	2.064	43.130	0.000	0.573	2.864	0.001
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	687	687	428	852	476	9822	0	595	17909	0
N.S.	1	1.00	0.62	1.24	0.69	14.30	0.00	0.87	26.07	0.00
time (sec)	N/A	0.602	0.369	0.019	2.079	27.277	0.000	0.587	2.825	0.001
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	685	685	423	848	490	9678	0	586	17180	0
N.S.	1	1.00	0.62	1.24	0.72	14.13	0.00	0.86	25.08	0.00
time (sec)	N/A	0.609	0.272	0.016	2.095	24.280	0.000	0.466	4.873	0.001

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	685	685	428	852	472	9774	0	603	17812	0
N.S.	1	1.00	0.62	1.24	0.69	14.27	0.00	0.88	26.00	0.00
time (sec)	N/A	0.560	0.283	0.017	2.148	26.340	0.000	0.502	2.867	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	689	689	429	873	506	9892	0	603	17945	0
N.S.	1	1.00	0.62	1.27	0.73	14.36	0.00	0.88	26.04	0.00
time (sec)	N/A	0.601	0.291	0.018	2.060	41.284	0.000	0.466	2.732	0.001

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	745	745	499	911	521	10188	0	639	24015	0
N.S.	1	1.00	0.67	1.22	0.70	13.68	0.00	0.86	32.23	0.00
time (sec)	N/A	0.772	0.370	0.020	2.111	111.346	0.000	0.452	5.161	0.001

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	751	751	513	932	543	0	0	628	20828	0
N.S.	1	1.00	0.68	1.24	0.72	0.00	0.00	0.84	27.73	0.00
time (sec)	N/A	0.686	0.386	0.021	2.110	0.000	0.000	0.497	5.219	0.001

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	142	159	124	206	0	156	-1	126
N.S.	1	1.00	0.58	0.65	0.51	0.85	0.00	0.64	-0.00	0.52
time (sec)	N/A	0.130	0.184	0.011	0.918	0.855	0.000	0.376	0.000	0.197
Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	56	51	50	50	0	68	-1	56
N.S.	1	1.00	0.52	0.47	0.46	0.46	0.00	0.63	-0.01	0.52
time (sec)	N/A	0.101	0.030	0.004	0.898	0.604	0.000	0.336	0.000	0.086
Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	121	119	81	155	0	109	-1	100
N.S.	1	1.00	0.68	0.67	0.46	0.87	0.00	0.61	-0.01	0.56
time (sec)	N/A	0.076	0.113	0.007	1.080	0.942	0.000	0.461	0.000	0.157
Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	83	80	45	123	0	84	-1	82
N.S.	1	1.00	0.55	0.53	0.30	0.81	0.00	0.55	-0.01	0.54
time (sec)	N/A	0.095	0.046	0.007	1.453	0.950	0.000	0.445	0.000	0.109

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	122	128	59	134	0	116	-1	90
N.S.	1	1.00	0.69	0.72	0.33	0.76	0.00	0.66	-0.01	0.51
time (sec)	N/A	0.091	0.109	0.013	1.080	0.883	0.000	0.471	0.000	0.198

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	90	133	83	141	0	100	-1	88
N.S.	1	1.00	0.51	0.75	0.47	0.80	0.00	0.56	-0.01	0.50
time (sec)	N/A	0.119	0.045	0.012	1.225	0.869	0.000	0.443	0.000	0.153

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	72	73	72	79	76	79	73	0
N.S.	1	1.00	0.92	0.94	0.92	1.01	0.97	1.01	0.94	0.00
time (sec)	N/A	0.135	0.026	0.001	1.106	0.831	0.079	0.267	0.038	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	73	72	79	82	79	73	0
N.S.	1	1.00	1.00	0.94	0.92	1.01	1.05	1.01	0.94	0.00
time (sec)	N/A	0.064	0.016	0.001	1.189	0.790	0.079	0.388	0.028	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	72	73	72	79	76	79	73	0
N.S.	1	1.00	0.96	0.97	0.96	1.05	1.01	1.05	0.97	0.00
time (sec)	N/A	0.131	0.022	0.000	1.211	0.809	0.077	0.269	0.029	0.000
Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	70	69	76	78	76	70	0
N.S.	1	1.00	1.00	0.96	0.95	1.04	1.07	1.04	0.96	0.00
time (sec)	N/A	0.044	0.015	0.002	1.173	0.518	0.077	0.354	0.029	0.000
Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	77	73	70	73	79	70	0
N.S.	1	1.00	1.00	1.04	0.99	0.95	0.99	1.07	0.95	0.00
time (sec)	N/A	0.090	0.020	0.003	1.120	0.897	0.171	0.263	0.034	0.001
Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	71	75	69	74	73	74	70	0
N.S.	1	1.00	1.00	1.06	0.97	1.04	1.03	1.04	0.99	0.00
time (sec)	N/A	0.048	0.032	0.003	1.070	0.845	0.164	0.361	0.033	0.001

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	71	76	73	76	71	97	70	0
N.S.	1	1.00	0.96	1.03	0.99	1.03	0.96	1.31	0.95	0.00
time (sec)	N/A	0.096	0.042	0.007	1.094	0.891	0.263	0.379	0.038	0.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	165	214	165	426	320	160	251	0
N.S.	1	1.00	0.98	1.27	0.98	2.54	1.90	0.95	1.49	0.00
time (sec)	N/A	0.234	0.138	0.013	2.487	0.986	1.177	0.321	0.325	0.001

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	133	176	130	350	189	125	179	0
N.S.	1	1.00	0.99	1.30	0.96	2.59	1.40	0.93	1.33	0.00
time (sec)	N/A	0.159	0.078	0.012	2.451	0.993	1.084	0.277	0.322	0.001

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	102	141	95	302	162	91	95	0
N.S.	1	1.00	0.96	1.33	0.90	2.85	1.53	0.86	0.90	0.00
time (sec)	N/A	0.106	0.065	0.010	2.517	1.029	0.966	0.335	0.342	0.001

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	88	118	84	268	153	75	77	0
N.S.	1	1.00	1.06	1.42	1.01	3.23	1.84	0.90	0.93	0.00
time (sec)	N/A	0.093	0.054	0.010	2.335	1.085	0.770	0.406	0.357	0.001
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	86	89	121	87	267	155	83	81	0
N.S.	1	0.97	1.00	1.36	0.98	3.00	1.74	0.93	0.91	0.00
time (sec)	N/A	0.118	0.061	0.013	2.462	0.899	1.118	0.293	0.367	0.001
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	105	146	103	316	167	94	98	0
N.S.	1	1.00	0.99	1.38	0.97	2.98	1.58	0.89	0.92	0.00
time (sec)	N/A	0.137	0.063	0.013	2.385	0.882	1.527	0.261	0.357	0.001
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	135	183	139	360	284	131	128	0
N.S.	1	1.00	0.99	1.35	1.02	2.65	2.09	0.96	0.94	0.00
time (sec)	N/A	0.252	0.088	0.015	2.463	0.858	2.133	0.327	0.383	0.001

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	166	221	174	436	328	164	156	0
N.S.	1	1.00	0.99	1.32	1.04	2.61	1.96	0.98	0.93	0.00
time (sec)	N/A	0.330	0.095	0.016	2.570	0.917	2.679	0.417	0.403	0.001

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	170	239	175	504	235	160	223	0
N.S.	1	1.00	0.98	1.38	1.01	2.91	1.36	0.92	1.29	0.00
time (sec)	N/A	0.321	0.111	0.015	2.466	0.727	3.583	0.359	0.354	0.001

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	141	202	139	462	212	125	137	0
N.S.	1	1.00	0.99	1.41	0.97	3.23	1.48	0.87	0.96	0.00
time (sec)	N/A	0.210	0.087	0.013	2.509	0.608	3.370	0.471	0.344	0.001

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	122	179	126	421	201	107	118	0
N.S.	1	1.00	0.98	1.44	1.02	3.40	1.62	0.86	0.95	0.00
time (sec)	N/A	0.138	0.104	0.011	2.461	0.579	2.621	0.263	0.386	0.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	110	131	121	391	196	101	112	0
N.S.	1	1.00	0.96	1.14	1.05	3.40	1.70	0.88	0.97	0.00
time (sec)	N/A	0.116	0.096	0.010	2.462	0.647	1.501	0.312	0.379	0.001
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	124	124	182	129	421	202	110	118	0
N.S.	1	0.98	0.98	1.43	1.02	3.31	1.59	0.87	0.93	0.00
time (sec)	N/A	0.204	0.135	0.013	2.650	0.666	2.142	0.316	0.390	0.001
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	141	207	147	476	214	128	138	0
N.S.	1	1.00	0.99	1.46	1.04	3.35	1.51	0.90	0.97	0.00
time (sec)	N/A	0.217	0.085	0.016	2.594	0.781	2.924	0.340	0.398	0.001
Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	173	245	183	514	330	164	168	0
N.S.	1	1.00	1.01	1.43	1.07	3.01	1.93	0.96	0.98	0.00
time (sec)	N/A	0.373	0.114	0.016	2.453	0.703	3.757	0.347	0.412	0.001

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	228	538	0	0	0	236	7024	0
N.S.	1	1.00	0.99	2.34	0.00	0.00	0.00	1.03	30.54	0.00
time (sec)	N/A	0.487	0.243	0.016	0.000	0.000	0.000	1.728	69.941	0.001

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	186	408	0	0	0	194	2304	0
N.S.	1	1.00	0.98	2.16	0.00	0.00	0.00	1.03	12.19	0.00
time (sec)	N/A	0.329	0.187	0.010	0.000	0.000	0.000	2.185	15.207	0.001

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	139	289	0	421	0	157	1853	0
N.S.	1	1.00	0.88	1.83	0.00	2.66	0.00	0.99	11.73	0.00
time (sec)	N/A	0.261	0.106	0.010	0.000	128.564	0.000	1.843	11.051	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	114	176	0	321	0	133	3704	0
N.S.	1	1.00	0.86	1.33	0.00	2.43	0.00	1.01	28.06	0.00
time (sec)	N/A	0.157	0.070	0.008	0.000	37.369	0.000	1.720	9.751	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	112	176	0	321	0	134	2434	0
N.S.	1	1.00	0.84	1.32	0.00	2.41	0.00	1.01	18.30	0.00
time (sec)	N/A	0.123	0.068	0.010	0.000	26.755	0.000	1.908	8.707	0.001

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	242	298	0	0	0	172	6285	0
N.S.	1	1.00	1.45	1.78	0.00	0.00	0.00	1.03	37.63	0.00
time (sec)	N/A	0.306	0.321	0.011	0.000	0.000	0.000	1.917	17.199	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	331	430	0	0	0	237	5368	0
N.S.	1	1.00	1.61	2.10	0.00	0.00	0.00	1.16	26.19	0.00
time (sec)	N/A	0.470	0.343	0.019	0.000	0.000	0.000	1.962	62.948	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	426	584	0	0	0	332	10300	0
N.S.	1	1.00	1.59	2.18	0.00	0.00	0.00	1.24	38.43	0.00
time (sec)	N/A	0.597	0.434	0.020	0.000	0.000	0.000	1.462	144.755	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	387	387	463	1449	0	0	0	12506	41755	0
N.S.	1	1.00	1.20	3.74	0.00	0.00	0.00	32.32	107.89	0.00
time (sec)	N/A	4.032	0.614	0.042	0.000	0.000	0.000	12.649	7.134	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	323	323	385	1098	0	0	0	11030	33892	0
N.S.	1	1.00	1.19	3.40	0.00	0.00	0.00	34.15	104.93	0.00
time (sec)	N/A	1.366	0.520	0.036	0.000	0.000	0.000	13.868	6.446	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	323	764	0	15553	0	8658	25202	0
N.S.	1	1.00	1.15	2.73	0.00	55.55	0.00	30.92	90.01	0.00
time (sec)	N/A	0.894	0.334	0.026	0.000	9.248	0.000	11.236	5.800	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	277	478	0	12269	0	6921	19401	0
N.S.	1	1.00	1.10	1.90	0.00	48.88	0.00	27.57	77.29	0.00
time (sec)	N/A	0.451	0.498	0.024	0.000	3.760	0.000	8.453	4.959	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	274	480	0	0	0	7650	23640	0
N.S.	1	1.00	1.08	1.89	0.00	0.00	0.00	30.12	93.07	0.00
time (sec)	N/A	0.520	0.248	0.027	0.000	0.000	0.000	9.134	5.614	0.001
Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	340	817	0	0	0	10058	33644	0
N.S.	1	1.00	1.14	2.74	0.00	0.00	0.00	33.75	112.90	0.00
time (sec)	N/A	0.960	0.399	0.030	0.000	0.000	0.000	12.818	5.890	0.001
Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	410	1160	0	0	0	12268	42882	0
N.S.	1	1.00	1.18	3.33	0.00	0.00	0.00	35.25	123.22	0.00
time (sec)	N/A	1.549	0.525	0.040	0.000	0.000	0.000	12.000	6.726	0.001
Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	866	866	267	336	0	0	0	0	43112	308
N.S.	1	1.00	0.31	0.39	0.00	0.00	0.00	0.00	49.78	0.36
time (sec)	N/A	2.509	0.377	0.096	0.000	0.000	0.000	0.000	6.836	0.397

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	350	350	251	1270	0	0	0	0	-1	267
N.S.	1	1.00	0.72	3.63	0.00	0.00	0.00	0.00	-0.00	0.76
time (sec)	N/A	0.573	0.534	0.036	0.000	0.000	0.000	0.000	0.000	1.284
Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	562	562	240	1207	0	0	0	0	-1	261
N.S.	1	1.00	0.43	2.15	0.00	0.00	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.924	0.503	0.033	0.000	0.000	0.000	0.000	0.000	1.285
Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	171	267	0	1364	0	0	-1	230
N.S.	1	1.00	0.99	1.54	0.00	7.88	0.00	0.00	-0.01	1.33
time (sec)	N/A	0.314	0.367	0.020	0.000	58.184	0.000	0.000	0.000	0.697
Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	133	204	0	1084	0	0	-1	195
N.S.	1	1.00	0.97	1.49	0.00	7.91	0.00	0.00	-0.01	1.42
time (sec)	N/A	0.157	0.118	0.010	0.000	4.113	0.000	0.000	0.000	0.541

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	87	165	0	357	0	75	-1	141
N.S.	1	1.00	1.01	1.92	0.00	4.15	0.00	0.87	-0.01	1.64
time (sec)	N/A	0.084	0.018	0.007	0.000	1.190	0.000	0.486	0.000	0.382
Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	134	207	0	1097	0	0	-1	195
N.S.	1	1.00	0.97	1.50	0.00	7.95	0.00	0.00	-0.01	1.41
time (sec)	N/A	0.195	0.140	0.010	0.000	1.742	0.000	0.000	0.000	0.518
Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	175	276	0	1414	0	208	-1	237
N.S.	1	1.00	0.80	1.27	0.00	6.49	0.00	0.95	-0.00	1.09
time (sec)	N/A	0.266	0.367	0.011	0.000	2.781	0.000	0.488	0.000	0.815
Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	271	720	0	4901	0	0	-1	308
N.S.	1	1.00	1.15	3.05	0.00	20.77	0.00	0.00	-0.00	1.31
time (sec)	N/A	0.474	0.821	0.063	0.000	172.624	0.000	0.000	0.000	1.207

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	204	613	0	1381	0	458	-1	226
N.S.	1	1.00	1.22	3.67	0.00	8.27	0.00	2.74	-0.01	1.35
time (sec)	N/A	0.294	0.616	0.015	0.000	2.201	0.000	0.720	0.000	0.784
Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	162	506	0	1349	0	441	-1	218
N.S.	1	1.00	1.02	3.18	0.00	8.48	0.00	2.77	-0.01	1.37
time (sec)	N/A	0.191	0.172	0.011	0.000	2.245	0.000	0.641	0.000	0.753
Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	167	454	0	1379	0	454	-1	224
N.S.	1	1.00	1.01	2.73	0.00	8.31	0.00	2.73	-0.01	1.35
time (sec)	N/A	0.171	0.149	0.008	0.000	2.491	0.000	0.618	0.000	0.746
Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	236	612	0	4909	0	0	-1	324
N.S.	1	1.00	0.89	2.30	0.00	18.45	0.00	0.00	-0.00	1.22
time (sec)	N/A	0.388	0.731	0.024	0.000	10.494	0.000	0.000	0.000	1.491

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	419	419	350	863	0	6486	0	762	-1	479
N.S.	1	1.00	0.84	2.06	0.00	15.48	0.00	1.82	-0.00	1.14
time (sec)	N/A	0.564	1.477	0.028	0.000	22.941	0.000	2.736	0.000	2.074
Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	406	406	943	496	0	0	0	928	11195	502
N.S.	1	1.00	2.32	1.22	0.00	0.00	0.00	2.29	27.57	1.24
time (sec)	N/A	8.593	10.842	0.059	0.000	0.000	0.000	0.650	2.476	1.650
Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	324	324	591	332	0	0	0	745	8222	454
N.S.	1	1.00	1.82	1.02	0.00	0.00	0.00	2.30	25.38	1.40
time (sec)	N/A	3.529	7.251	0.033	0.000	0.000	0.000	0.819	1.994	1.233
Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	292	308	275	0	2435	0	619	5705	352
N.S.	1	1.00	1.05	0.94	0.00	8.34	0.00	2.12	19.54	1.21
time (sec)	N/A	3.600	0.547	0.026	0.000	138.367	0.000	0.782	2.342	1.112

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	179	177	0	1085	0	228	717	279
N.S.	1	1.00	0.89	0.88	0.00	5.37	0.00	1.13	3.55	1.38
time (sec)	N/A	0.363	0.328	0.016	0.000	21.449	0.000	0.555	1.723	0.733

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	241	294	0	0	0	717	10964	280
N.S.	1	1.00	0.86	1.05	0.00	0.00	0.00	2.55	39.02	1.00
time (sec)	N/A	1.350	0.814	0.030	0.000	0.000	0.000	0.675	6.875	0.735

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	382	370	349	401	0	0	0	0	19959	398
N.S.	1	0.97	0.91	1.05	0.00	0.00	0.00	0.00	52.25	1.04
time (sec)	N/A	4.134	1.389	0.030	0.000	0.000	0.000	0.000	5.460	1.500

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	552	552	466	655	0	0	0	1055	33925	579
N.S.	1	1.00	0.84	1.19	0.00	0.00	0.00	1.91	61.46	1.05
time (sec)	N/A	4.244	1.967	0.035	0.000	0.000	0.000	0.905	7.300	2.392

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	390	390	10915	290	0	6534	0	53	-1	566
N.S.	1	1.00	27.99	0.74	0.00	16.75	0.00	0.14	-0.00	1.45
time (sec)	N/A	2.919	6.397	0.040	0.000	104.324	0.000	2.038	0.000	91.937

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	324	324	7768	224	0	3260	0	27	-1	425
N.S.	1	1.00	23.98	0.69	0.00	10.06	0.00	0.08	-0.00	1.31
time (sec)	N/A	1.517	6.171	0.027	0.000	13.844	0.000	1.832	0.000	57.344

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	2585	161	0	985	0	0	-1	250
N.S.	1	1.00	10.77	0.67	0.00	4.10	0.00	0.00	-0.00	1.04
time (sec)	N/A	0.318	5.125	0.018	0.000	3.343	0.000	0.000	0.000	17.048

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	291	291	4644	272	0	2402	0	0	-1	348
N.S.	1	1.00	15.96	0.93	0.00	8.25	0.00	0.00	-0.00	1.20
time (sec)	N/A	0.671	6.322	0.029	0.000	7.224	0.000	0.000	0.000	70.513

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	373	373	7777	322	0	4095	0	0	-1	479
N.S.	1	1.00	20.85	0.86	0.00	10.98	0.00	0.00	-0.00	1.28
time (sec)	N/A	2.527	6.391	0.038	0.000	31.157	0.000	0.000	0.000	111.742

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	512	512	10933	503	0	5773	0	0	-1	687
N.S.	1	1.00	21.35	0.98	0.00	11.28	0.00	0.00	-0.00	1.34
time (sec)	N/A	4.943	6.586	0.038	0.000	77.192	0.000	0.000	0.000	160.945

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	460	457	490	0	0	0	857	16951	524
N.S.	1	1.00	0.99	1.07	0.00	0.00	0.00	1.86	36.85	1.14
time (sec)	N/A	5.084	0.966	0.033	0.000	0.000	0.000	1.246	3.413	2.124

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	324	279	0	0	0	649	12392	447
N.S.	1	1.00	0.99	0.85	0.00	0.00	0.00	1.98	37.90	1.37
time (sec)	N/A	1.456	0.568	0.021	0.000	0.000	0.000	1.089	4.129	1.553

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	333	388	0	0	0	827	28434	389
N.S.	1	1.00	0.96	1.12	0.00	0.00	0.00	2.39	82.18	1.12
time (sec)	N/A	1.744	1.379	0.029	0.000	0.000	0.000	1.024	7.673	1.531
Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	416	380	555	0	0	0	433	35855	548
N.S.	1	1.00	0.91	1.33	0.00	0.00	0.00	1.04	85.98	1.31
time (sec)	N/A	3.243	1.600	0.043	0.000	0.000	0.000	0.725	6.097	2.117
Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	595	595	18689	516	0	0	0	104	-1	0
N.S.	1	1.00	31.41	0.87	0.00	0.00	0.00	0.17	-0.00	0.00
time (sec)	N/A	3.285	6.489	0.039	0.000	0.000	0.000	1.869	0.000	180.173
Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	491	491	14032	382	0	0	0	58	-1	692
N.S.	1	1.00	28.58	0.78	0.00	0.00	0.00	0.12	-0.00	1.41
time (sec)	N/A	1.802	6.261	0.033	0.000	0.000	0.000	1.975	0.000	154.730

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	487	487	9290	217	0	7721	0	27	-1	392
N.S.	1	1.00	19.08	0.45	0.00	15.85	0.00	0.06	-0.00	0.80
time (sec)	N/A	1.572	6.160	0.027	0.000	99.051	0.000	1.968	0.000	121.925

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	260	432	7789	360	0	4059	0	0	-1	423
N.S.	1	1.66	29.96	1.38	0.00	15.61	0.00	0.00	-0.00	1.63
time (sec)	N/A	0.855	6.302	0.036	0.000	37.774	0.000	0.000	0.000	121.559

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F(-1)	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	523	523	9321	511	0	7830	0	0	-1	484
N.S.	1	1.00	17.82	0.98	0.00	14.97	0.00	0.00	-0.00	0.93
time (sec)	N/A	2.617	6.408	0.036	0.000	149.195	0.000	0.000	0.000	179.824

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	354	2134	0	3615	0	4637	917	425
N.S.	1	1.00	1.26	7.59	0.00	12.86	0.00	16.50	3.26	1.51
time (sec)	N/A	7.336	0.542	0.099	0.000	16.821	0.000	4.484	1.450	0.975

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	229	276	1223	0	2053	0	4060	776	285
N.S.	1	1.00	1.21	5.34	0.00	8.97	0.00	17.73	3.39	1.24
time (sec)	N/A	1.751	0.387	0.063	0.000	6.457	0.000	4.104	1.311	0.677

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	169	1167	0	871	0	591	649	190
N.S.	1	1.00	0.93	6.41	0.00	4.79	0.00	3.25	3.57	1.04
time (sec)	N/A	0.268	0.243	0.051	0.000	3.123	0.000	4.242	1.288	0.311

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	212	2099	0	1232	0	3639	669	277
N.S.	1	1.00	0.88	8.71	0.00	5.11	0.00	15.10	2.78	1.15
time (sec)	N/A	1.645	0.432	0.064	0.000	11.648	0.000	3.674	1.298	0.645

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	290	292	2770	0	2799	0	1675	825	360
N.S.	1	1.00	1.01	9.55	0.00	9.65	0.00	5.78	2.84	1.24
time (sec)	N/A	2.362	0.762	0.085	0.000	32.217	0.000	7.120	1.410	2.087

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	325	325	10606	222	0	2860	0	1710	1024	594
N.S.	1	1.00	32.63	0.68	0.00	8.80	0.00	5.26	3.15	1.83
time (sec)	N/A	5.391	6.307	0.042	0.000	4.252	0.000	6.686	1.300	0.966

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	7543	175	0	1430	0	3580	870	415
N.S.	1	1.00	28.68	0.67	0.00	5.44	0.00	13.61	3.31	1.58
time (sec)	N/A	2.135	6.140	0.023	0.000	1.982	0.000	4.734	1.272	0.631

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	220	220	2266	130	0	759	0	641	989	155
N.S.	1	1.00	10.30	0.59	0.00	3.45	0.00	2.91	4.50	0.70
time (sec)	N/A	0.294	5.418	0.013	0.000	0.925	0.000	5.122	1.268	0.171

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	B	F	B	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	265	265	2661	217	0	1998	0	3965	1234	471
N.S.	1	1.00	10.04	0.82	0.00	7.54	0.00	14.96	4.66	1.78
time (sec)	N/A	0.780	4.946	0.027	0.000	1.418	0.000	5.037	1.205	0.690

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	96	96	743	160	0	290	0	209	383	202
N.S.	1	1.00	7.74	1.67	0.00	3.02	0.00	2.18	3.99	2.10
time (sec)	N/A	0.200	0.496	0.088	0.000	1.072	0.000	0.718	1.500	0.591
Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	479	479	461	377	0	0	0	105	-1	538
N.S.	1	1.00	0.96	0.79	0.00	0.00	0.00	0.22	-0.00	1.12
time (sec)	N/A	1.858	1.868	0.034	0.000	0.000	0.000	2.027	0.000	60.185
Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	366	366	355	269	0	0	0	55	-1	458
N.S.	1	1.00	0.97	0.73	0.00	0.00	0.00	0.15	-0.00	1.25
time (sec)	N/A	1.173	1.030	0.026	0.000	0.000	0.000	2.140	0.000	45.098
Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	292	200	0	11094	0	27	-1	320
N.S.	1	1.00	0.98	0.67	0.00	37.23	0.00	0.09	-0.00	1.07
time (sec)	N/A	0.724	0.588	0.022	0.000	51.774	0.000	2.049	0.000	30.969

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	341	341	2112	246	0	0	0	0	-1	342
N.S.	1	1.00	6.19	0.72	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.774	7.152	0.024	0.000	0.000	0.000	0.000	0.000	62.476
Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	339	462	2158	387	0	0	0	0	-1	546
N.S.	1	1.36	6.37	1.14	0.00	0.00	0.00	0.00	-0.00	1.61
time (sec)	N/A	2.837	6.774	0.037	0.000	0.000	0.000	0.000	0.000	109.355
Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	419	647	2218	541	0	0	0	0	-1	762
N.S.	1	1.54	5.29	1.29	0.00	0.00	0.00	0.00	-0.00	1.82
time (sec)	N/A	5.566	6.796	0.042	0.000	0.000	0.000	0.000	0.000	144.272
Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	44	44	101	0	120	0	42	-1	107
N.S.	1	1.10	1.10	2.52	0.00	3.00	0.00	1.05	-0.02	2.68
time (sec)	N/A	0.072	0.052	0.059	0.000	0.937	0.000	0.222	0.000	1.087

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	20	0.100
2	A	2	1	1.00	20	0.050
3	A	4	3	1.00	18	0.167
4	A	2	1	1.00	17	0.059
5	A	3	2	1.00	20	0.100
6	A	2	1	1.00	20	0.050
7	A	3	2	1.00	20	0.100
8	A	5	5	1.00	20	0.250
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	18	0.222
11	A	7	7	1.00	20	0.350
12	A	7	7	1.00	20	0.350
13	A	7	7	1.00	20	0.350
14	A	6	6	1.00	20	0.300
15	A	6	5	1.00	20	0.250
16	A	5	4	1.00	20	0.200
17	A	5	4	1.00	18	0.222
18	A	8	7	1.00	20	0.350
19	A	8	8	1.00	20	0.400
20	A	8	7	1.00	20	0.350
21	A	8	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	20	0.150
25	A	3	3	1.00	18	0.167
26	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	5	5	1.00	20	0.250
28	A	6	6	1.00	20	0.300
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	20	0.150
32	A	2	2	1.00	18	0.111
33	A	6	6	1.00	20	0.300
34	A	6	6	1.00	20	0.300
35	A	3	2	1.00	25	0.080
36	A	4	3	1.00	23	0.130
37	A	3	2	1.00	23	0.087
38	A	4	3	1.00	23	0.130
39	A	3	2	1.00	23	0.087
40	A	4	3	1.00	21	0.143
41	A	3	2	1.00	20	0.100
42	A	5	4	1.00	23	0.174
43	A	3	2	1.00	23	0.087
44	A	4	3	1.00	23	0.130
45	A	3	2	1.00	23	0.087
46	A	4	3	1.00	21	0.143
47	A	3	2	1.00	21	0.095
48	A	4	3	1.00	21	0.143
49	A	3	2	1.00	21	0.095
50	A	2	2	1.00	19	0.105
51	A	3	2	1.00	18	0.111
52	A	4	3	1.00	21	0.143
53	A	3	2	1.00	21	0.095
54	A	4	3	1.00	21	0.143
55	A	4	4	1.00	33	0.121
56	A	4	4	1.00	31	0.129
57	A	3	3	1.00	30	0.100
58	A	4	3	1.00	33	0.091
59	A	3	3	1.00	33	0.091
60	A	4	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	33	0.121
62	A	3	3	1.00	31	0.097
63	A	4	4	1.00	30	0.133
64	A	4	3	1.00	33	0.091
65	A	5	4	1.00	33	0.121
66	A	4	3	1.00	33	0.091
67	A	3	2	1.00	35	0.057
68	A	3	2	1.00	35	0.057
69	A	3	2	1.00	35	0.057
70	A	2	2	1.00	29	0.069
71	A	5	4	1.00	31	0.129
72	A	5	4	1.00	31	0.129
73	A	3	2	1.00	25	0.080
74	A	2	1	1.00	25	0.040
75	A	3	2	1.00	23	0.087
76	A	2	1	1.00	22	0.045
77	A	3	2	1.00	25	0.080
78	A	2	1	1.00	25	0.040
79	A	3	2	1.00	25	0.080
80	A	7	6	1.00	25	0.240
81	A	6	6	1.00	25	0.240
82	A	5	5	1.00	23	0.217
83	A	7	6	1.00	25	0.240
84	A	7	6	1.00	25	0.240
85	A	5	3	1.00	25	0.120
86	A	4	3	1.00	25	0.120
87	A	3	2	1.00	22	0.091
88	A	4	3	1.00	25	0.120
89	A	5	3	1.00	25	0.120
90	A	7	7	1.00	25	0.280
91	A	6	6	1.00	25	0.240
92	A	4	4	1.00	25	0.160
93	A	4	4	1.00	23	0.174
94	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	7	1.00	25	0.280
96	A	6	4	1.00	25	0.160
97	A	5	4	1.00	25	0.160
98	A	4	3	1.00	25	0.120
99	A	4	3	1.00	22	0.136
100	A	5	4	1.00	25	0.160
101	A	6	4	1.00	25	0.160
102	A	8	7	1.00	25	0.280
103	A	7	6	1.00	25	0.240
104	A	5	5	1.00	25	0.200
105	A	5	5	1.00	25	0.200
106	A	5	5	1.00	25	0.200
107	A	5	5	1.00	23	0.217
108	A	9	7	1.00	25	0.280
109	A	9	7	1.00	25	0.280
110	A	7	4	1.00	25	0.160
111	A	6	4	1.00	25	0.160
112	A	5	3	1.00	25	0.120
113	A	5	4	1.00	25	0.160
114	A	5	3	1.00	22	0.136
115	A	4	3	1.00	21	0.143
116	A	5	4	1.00	22	0.182
117	A	5	5	1.00	17	0.294
118	A	6	6	1.00	18	0.333
119	A	5	5	1.00	22	0.227
120	A	6	6	1.00	25	0.240
121	A	5	5	1.00	25	0.200
122	A	5	5	1.00	23	0.217
123	A	7	6	1.00	25	0.240
124	A	7	6	1.00	25	0.240
125	A	7	6	1.00	25	0.240
126	A	5	5	1.00	25	0.200
127	A	6	6	1.00	25	0.240
128	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	7	6	1.00	25	0.240
130	A	6	5	1.00	25	0.200
131	A	6	5	1.00	23	0.217
132	A	8	6	1.00	25	0.240
133	A	8	7	1.00	25	0.280
134	A	8	6	1.00	25	0.240
135	A	8	7	1.00	25	0.280
136	A	5	5	1.00	27	0.185
137	A	4	4	1.00	27	0.148
138	A	4	4	1.00	25	0.160
139	A	6	5	1.00	27	0.185
140	A	4	4	1.00	27	0.148
141	A	5	5	1.00	27	0.185
142	A	6	5	1.00	27	0.185
143	A	6	5	1.00	25	0.200
144	A	5	5	1.00	25	0.200
145	A	4	4	1.00	25	0.160
146	A	4	4	1.00	23	0.174
147	A	6	5	1.00	25	0.200
148	A	4	4	1.00	25	0.160
149	A	5	5	1.00	25	0.200
150	A	6	5	1.00	25	0.200
151	A	5	5	1.00	25	0.200
152	A	4	4	1.00	25	0.160
153	A	2	2	1.00	23	0.087
154	A	5	5	1.00	25	0.200
155	A	5	5	1.00	25	0.200
156	A	2	1	1.00	27	0.037
157	A	2	1	1.00	27	0.037
158	A	2	1	1.00	25	0.040
159	A	6	5	1.00	22	0.227
160	A	6	5	1.00	22	0.227
161	A	6	5	1.00	22	0.227
162	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	6	6	1.00	20	0.300
164	A	6	5	1.00	22	0.227
165	A	6	5	1.00	22	0.227
166	A	6	5	1.00	22	0.227
167	A	12	8	1.00	22	0.364
168	A	12	8	1.00	22	0.364
169	A	12	8	1.00	22	0.364
170	A	12	8	1.00	22	0.364
171	A	12	8	1.00	19	0.421
172	A	12	8	1.00	22	0.364
173	A	12	8	1.00	22	0.364
174	A	7	6	1.00	22	0.273
175	A	7	6	1.00	22	0.273
176	A	7	6	0.99	22	0.273
177	A	7	6	0.99	22	0.273
178	A	7	6	1.00	20	0.300
179	A	8	6	1.00	22	0.273
180	A	8	6	1.00	22	0.273
181	A	8	6	1.00	22	0.273
182	A	24	11	1.00	22	0.500
183	A	23	10	1.00	22	0.454
184	A	23	10	1.00	22	0.454
185	A	23	10	1.00	22	0.454
186	A	22	9	1.00	19	0.474
187	A	22	9	1.00	22	0.409
188	A	22	9	1.00	22	0.409
189	A	6	6	1.00	37	0.162
190	A	4	3	1.00	35	0.086
191	A	5	5	1.00	34	0.147
192	A	6	6	1.00	37	0.162
193	A	5	5	1.00	37	0.135
194	A	6	6	1.00	37	0.162
195	A	3	2	1.00	25	0.080
196	A	2	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	3	2	1.00	23	0.087
198	A	2	1	1.00	22	0.045
199	A	3	2	1.00	25	0.080
200	A	2	1	1.00	25	0.040
201	A	3	2	1.00	25	0.080
202	A	4	3	1.00	25	0.120
203	A	4	3	1.00	25	0.120
204	A	4	3	1.00	25	0.120
205	A	3	3	1.00	22	0.136
206	A	3	3	0.97	25	0.120
207	A	4	3	1.00	25	0.120
208	A	4	3	1.00	25	0.120
209	A	4	3	1.00	25	0.120
210	A	5	4	1.00	25	0.160
211	A	5	4	1.00	25	0.160
212	A	4	4	1.00	25	0.160
213	A	3	3	1.00	22	0.136
214	A	4	4	0.98	25	0.160
215	A	5	3	1.00	25	0.120
216	A	5	4	1.00	25	0.160
217	A	7	6	1.00	27	0.222
218	A	7	6	1.00	27	0.222
219	A	7	6	1.00	27	0.222
220	A	7	6	1.00	27	0.222
221	A	7	7	1.00	25	0.280
222	A	7	6	1.00	27	0.222
223	A	7	6	1.00	27	0.222
224	A	7	6	1.00	27	0.222
225	A	6	3	1.00	27	0.111
226	A	6	3	1.00	27	0.111
227	A	6	3	1.00	27	0.111
228	A	6	3	1.00	27	0.111
229	A	6	3	1.00	24	0.125
230	A	6	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	6	3	1.00	27	0.111
232	A	19	12	1.00	31	0.387
233	A	8	7	1.00	29	0.241
234	A	7	6	1.00	29	0.207
235	A	7	6	1.00	27	0.222
236	A	9	6	1.00	29	0.207
237	A	21	8	1.00	29	0.276
238	A	9	7	1.00	29	0.241
239	A	8	6	1.00	29	0.207
240	A	8	7	1.00	27	0.259
241	A	14	8	1.00	29	0.276
242	A	24	9	1.00	29	0.310
243	A	7	6	1.00	29	0.207
244	A	6	5	1.00	29	0.172
245	A	3	3	1.00	27	0.111
246	A	7	4	1.00	29	0.138
247	A	10	5	1.00	29	0.172
248	A	7	6	1.00	29	0.207
249	A	5	5	1.00	29	0.172
250	A	5	5	1.00	29	0.172
251	A	5	5	1.00	27	0.185
252	A	11	6	1.00	29	0.207
253	A	15	7	1.00	29	0.241
254	A	7	5	1.00	29	0.172
255	A	7	5	1.00	29	0.172
256	A	6	5	1.00	29	0.172
257	A	5	4	1.00	27	0.148
258	A	8	6	1.00	29	0.207
259	A	10	7	0.97	29	0.241
260	A	13	7	1.00	29	0.241
261	A	10	7	1.00	29	0.241
262	A	9	6	1.00	29	0.207
263	A	11	6	1.00	26	0.231
264	A	8	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	12	7	1.00	29	0.241
266	A	15	7	1.00	29	0.241
267	A	7	5	1.00	29	0.172
268	A	6	5	1.00	27	0.185
269	A	8	6	1.00	29	0.207
270	A	10	7	1.00	29	0.241
271	A	17	9	1.00	29	0.310
272	A	16	8	1.00	29	0.276
273	A	13	7	1.00	26	0.269
274	A	16	8	1.66	29	0.276
275	A	19	10	1.00	29	0.345
276	A	7	5	1.00	29	0.172
277	A	6	5	1.00	29	0.172
278	A	5	4	1.00	27	0.148
279	A	8	6	1.00	29	0.207
280	A	8	6	1.00	29	0.207
281	A	9	6	1.00	29	0.207
282	A	8	5	1.00	29	0.172
283	A	9	5	1.00	26	0.192
284	A	8	5	1.00	29	0.172
285	A	8	6	1.00	25	0.240
286	A	17	7	1.00	29	0.241
287	A	13	7	1.00	29	0.241
288	A	10	6	1.00	29	0.207
289	A	6	3	1.00	29	0.103
290	A	5	3	1.00	26	0.115
291	A	9	5	1.00	29	0.172
292	A	11	6	1.00	29	0.207
293	A	14	6	1.00	29	0.207
294	A	14	7	1.45	29	0.241
295	A	8	5	1.00	29	0.172
296	A	8	5	1.00	29	0.172
297	A	8	5	1.00	26	0.192
298	A	12	8	1.36	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	15	8	1.54	29	0.276
300	A	5	5	1.10	28	0.179

Chapter 3

Listing of integrals

Local contents

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3.8	$\int x^5 (2 + 3x^2) \sqrt{5 + x^4} dx$	148
3.9	$\int x^3 (2 + 3x^2) \sqrt{5 + x^4} dx$	152
3.10	$\int x (2 + 3x^2) \sqrt{5 + x^4} dx$	156
3.11	$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x} dx$	160
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3.14	$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7} dx$	175
3.15	$\int x^5 (2 + 3x^2) (5 + x^4)^{3/2} dx$	180
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3.22	$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}} dx$	212
3.23	$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}} dx$	216
3.24	$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} dx$	220
3.25	$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}} dx$	224
3.26	$\int \frac{2+3x^2}{x\sqrt{5+x^4}} dx$	227
3.27	$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}} dx$	231
3.28	$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}} dx$	235
3.29	$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}} dx$	240
3.30	$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}} dx$	244
3.31	$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}} dx$	248
3.32	$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} dx$	252
3.33	$\int \frac{2+3x^2}{x(5+x^4)^{3/2}} dx$	255
3.34	$\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}} dx$	260
3.35	$\int (fx)^m (d+ex^2)(1+2x^2+x^4)^5 dx$	265
3.36	$\int x^5 (d+ex^2)(1+2x^2+x^4)^5 dx$	275
3.37	$\int x^4 (d+ex^2)(1+2x^2+x^4)^5 dx$	279
3.38	$\int x^3 (d+ex^2)(1+2x^2+x^4)^5 dx$	283
3.39	$\int x^2 (d+ex^2)(1+2x^2+x^4)^5 dx$	287
3.40	$\int x (d+ex^2)(1+2x^2+x^4)^5 dx$	291
3.41	$\int (d+ex^2)(1+2x^2+x^4)^5 dx$	295

3.42	$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx$	299
3.43	$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$	303
3.44	$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx$	307
3.45	$\int (fx)^m (1+x^2)(1+2x^2+x^4)^5 dx$	311
3.46	$\int x^5 (1+x^2)(1+2x^2+x^4)^5 dx$	318
3.47	$\int x^4 (1+x^2)(1+2x^2+x^4)^5 dx$	322
3.48	$\int x^3 (1+x^2)(1+2x^2+x^4)^5 dx$	325
3.49	$\int x^2 (1+x^2)(1+2x^2+x^4)^5 dx$	329
3.50	$\int x (1+x^2)(1+2x^2+x^4)^5 dx$	332
3.51	$\int (1+x^2)(1+2x^2+x^4)^5 dx$	335
3.52	$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} dx$	338
3.53	$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx$	342
3.54	$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx$	345
3.55	$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx$	349
3.56	$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx$	353
3.57	$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}} dx$	357
3.58	$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}} dx$	361
3.59	$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}} dx$	365
3.60	$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}} dx$	369
3.61	$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$	373
3.62	$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$	377
3.63	$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$	381
3.64	$\int \frac{d+ex^2}{x(a^2+2abx^2+b^2x^4)^{3/2}} dx$	385
3.65	$\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}} dx$	389

3.66	$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}} dx$	394
3.67	$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx$	399
3.68	$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{3/2} dx$	405
3.69	$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4} dx$	410
3.70	$\int x (a+bx^2) (a^2+2abx^2+b^2x^4)^p dx$	414
3.71	$\int x^3 (a+bx^2) (a^2+2abx^2+b^2x^4)^p dx$	418
3.72	$\int x^5 (a+bx^2) (a^2+2abx^2+b^2x^4)^p dx$	422
3.73	$\int x^3 (A+Bx^2) (a+bx^2+cx^4)^3 dx$	427
3.74	$\int x^2 (A+Bx^2) (a+bx^2+cx^4)^3 dx$	431
3.75	$\int x (A+Bx^2) (a+bx^2+cx^4)^3 dx$	434
3.76	$\int (A+Bx^2) (a+bx^2+cx^4)^3 dx$	438
3.77	$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} dx$	441
3.78	$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2} dx$	445
3.79	$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} dx$	448
3.80	$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$	452
3.81	$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4} dx$	458
3.82	$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} dx$	463
3.83	$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)} dx$	468
3.84	$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)} dx$	474
3.85	$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4} dx$	480
3.86	$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4} dx$	493
3.87	$\int \frac{A+Bx^2}{a+bx^2+cx^4} dx$	503
3.88	$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)} dx$	510
3.89	$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)} dx$	520
3.90	$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	533
3.91	$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	540

3.92	$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	546
3.93	$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	551
3.94	$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^2} dx$	556
3.95	$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2} dx$	565
3.96	$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	576
3.97	$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	596
3.98	$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$	611
3.99	$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2} dx$	623
3.100	$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)^2} dx$	638
3.101	$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2} dx$	658
3.102	$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	683
3.103	$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	693
3.104	$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	701
3.105	$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	707
3.106	$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	713
3.107	$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	719
3.108	$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3} dx$	725
3.109	$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3} dx$	738
3.110	$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	754
3.111	$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	779

3.112	$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	802
3.113	$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$	820
3.114	$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3} dx$	842
3.115	$\int \frac{x(-7+4x^2)}{4-5x^2+x^4} dx$	866
3.116	$\int \frac{-7x+4x^3}{4-5x^2+x^4} dx$	869
3.117	$\int \frac{x(2+x^2)}{1+x^2+x^4} dx$	872
3.118	$\int \frac{2x+x^3}{1+x^2+x^4} dx$	876
3.119	$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2} dx$	880
3.120	$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4} dx$	884
3.121	$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4} dx$	888
3.122	$\int x(2+3x^2)\sqrt{3+5x^2+x^4} dx$	892
3.123	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} dx$	896
3.124	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3} dx$	901
3.125	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} dx$	906
3.126	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7} dx$	911
3.127	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9} dx$	915
3.128	$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}} dx$	920
3.129	$\int x^5(2+3x^2)(3+5x^2+x^4)^{3/2} dx$	925
3.130	$\int x^3(2+3x^2)(3+5x^2+x^4)^{3/2} dx$	930
3.131	$\int x(2+3x^2)(3+5x^2+x^4)^{3/2} dx$	934
3.132	$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx$	938
3.133	$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx$	943
3.134	$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx$	948
3.135	$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx$	953
3.136	$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$	958

3.137	$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$	963
3.138	$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$	967
3.139	$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}} dx$	971
3.140	$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}} dx$	975
3.141	$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}} dx$	979
3.142	$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}} dx$	984
3.143	$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$	989
3.144	$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$	993
3.145	$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$	997
3.146	$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$	1001
3.147	$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}} dx$	1005
3.148	$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}} dx$	1009
3.149	$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}} dx$	1013
3.150	$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}} dx$	1017
3.151	$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$	1022
3.152	$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$	1026
3.153	$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$	1030
3.154	$\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}} dx$	1033
3.155	$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}} dx$	1037
3.156	$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 dx$	1042
3.157	$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 dx$	1057
3.158	$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) dx$	1064
3.159	$\int \frac{x^9}{(d+ex^2)(a+cx^4)} dx$	1068
3.160	$\int \frac{x^7}{(d+ex^2)(a+cx^4)} dx$	1072

3.161	$\int \frac{x^5}{(d+ex^2)(a+cx^4)} dx$.1076
3.162	$\int \frac{x^3}{(d+ex^2)(a+cx^4)} dx$.1080
3.163	$\int \frac{x}{(d+ex^2)(a+cx^4)} dx$.1085
3.164	$\int \frac{1}{x(d+ex^2)(a+cx^4)} dx$.1089
3.165	$\int \frac{1}{x^3(d+ex^2)(a+cx^4)} dx$.1094
3.166	$\int \frac{1}{x^5(d+ex^2)(a+cx^4)} dx$.1099
3.167	$\int \frac{x^8}{(d+ex^2)(a+cx^4)} dx$.1104
3.168	$\int \frac{x^6}{(d+ex^2)(a+cx^4)} dx$.1114
3.169	$\int \frac{x^4}{(d+ex^2)(a+cx^4)} dx$.1124
3.170	$\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx$.1133
3.171	$\int \frac{1}{(d+ex^2)(a+cx^4)} dx$.1142
3.172	$\int \frac{1}{x^2(d+ex^2)(a+cx^4)} dx$.1151
3.173	$\int \frac{1}{x^4(d+ex^2)(a+cx^4)} dx$.1161
3.174	$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx$.1171
3.175	$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2} dx$.1176
3.176	$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2} dx$.1182
3.177	$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$.1188
3.178	$\int \frac{x}{(d+ex^2)(a+cx^4)^2} dx$.1194
3.179	$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$.1199
3.180	$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx$.1204
3.181	$\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2} dx$.1209
3.182	$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx$.1215
3.183	$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2} dx$.1235

3.184	$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx$	1255
3.185	$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx$	1274
3.186	$\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$	1294
3.187	$\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2} dx$	1313
3.188	$\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2} dx$	1336
3.189	$\int x^2 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4} dx$	1353
3.190	$\int x \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4} dx$	1358
3.191	$\int \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4} dx$	1362
3.192	$\int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x} dx$	1366
3.193	$\int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx$	1371
3.194	$\int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^3} dx$	1375
3.195	$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4) dx$	1380
3.196	$\int x^2 (d+ex^2)^2 (a+bx^2+cx^4) dx$	1383
3.197	$\int x (d+ex^2)^2 (a+bx^2+cx^4) dx$	1386
3.198	$\int (d+ex^2)^2 (a+bx^2+cx^4) dx$	1390
3.199	$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x} dx$	1393
3.200	$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^2} dx$	1397
3.201	$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^3} dx$	1400
3.202	$\int \frac{x^6 (a+bx^2+cx^4)}{(d+ex^2)^2} dx$	1404
3.203	$\int \frac{x^4 (a+bx^2+cx^4)}{(d+ex^2)^2} dx$	1408
3.204	$\int \frac{x^2 (a+bx^2+cx^4)}{(d+ex^2)^2} dx$	1412
3.205	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$	1416
3.206	$\int \frac{a+bx^2+cx^4}{x^2 (d+ex^2)^2} dx$	1420
3.207	$\int \frac{a+bx^2+cx^4}{x^4 (d+ex^2)^2} dx$	1424

3.208	$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2} dx$.1428
3.209	$\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2} dx$.1432
3.210	$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3} dx$.1436
3.211	$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx$.1441
3.212	$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx$.1446
3.213	$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$.1451
3.214	$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3} dx$.1455
3.215	$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3} dx$.1460
3.216	$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3} dx$.1465
3.217	$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)} dx$.1470
3.218	$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)} dx$.1478
3.219	$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)} dx$.1484
3.220	$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)} dx$.1490
3.221	$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)} dx$.1496
3.222	$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)} dx$.1502
3.223	$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)} dx$.1510
3.224	$\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)} dx$.1518
3.225	$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)} dx$.1528
3.226	$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx$.1558
3.227	$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)} dx$.1583
3.228	$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx$.1610
3.229	$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$.1632

3.230	$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)} dx$.1651
3.231	$\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)} dx$.1676
3.232	$\int \frac{1}{\sqrt{fx}(d+ex^2)(a+bx^2+cx^4)} dx$.1707
3.233	$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2} dx$.1736
3.234	$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2} dx$.1741
3.235	$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2} dx$.1747
3.236	$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)} dx$.1752
3.237	$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)} dx$.1757
3.238	$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$.1763
3.239	$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$.1769
3.240	$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$.1775
3.241	$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)} dx$.1781
3.242	$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)} dx$.1787
3.243	$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$.1794
3.244	$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$.1800
3.245	$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$.1805
3.246	$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$.1809
3.247	$\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$.1814
3.248	$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1819
3.249	$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1826
3.250	$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1831
3.251	$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1836
3.252	$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1841

3.253	$\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$.1849
3.254	$\int \frac{x^7 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1858
3.255	$\int \frac{x^5 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1869
3.256	$\int \frac{x^3 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1879
3.257	$\int \frac{x \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1888
3.258	$\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)} dx$.1893
3.259	$\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)} dx$.1904
3.260	$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)} dx$.1919
3.261	$\int \frac{x^4 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1941
3.262	$\int \frac{x^2 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1949
3.263	$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$.1955
3.264	$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)} dx$.1963
3.265	$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)} dx$.1971
3.266	$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)} dx$.1978
3.267	$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$.1986
3.268	$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$.1999
3.269	$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)} dx$.2010
3.270	$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)} dx$.2029
3.271	$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$.2051
3.272	$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$.2056
3.273	$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$.2061
3.274	$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)} dx$.2069
3.275	$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)} dx$.2076

3.276	$\int \frac{x^5 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2085
3.277	$\int \frac{x^3 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2095
3.278	$\int \frac{x \sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2103
3.279	$\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)} dx$.2109
3.280	$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)} dx$.2118
3.281	$\int \frac{x^4 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2127
3.282	$\int \frac{x^2 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2135
3.283	$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} dx$.2142
3.284	$\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)} dx$.2148
3.285	$\int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} dx$.2157
3.286	$\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2162
3.287	$\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2168
3.288	$\int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2173
3.289	$\int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2183
3.290	$\int \frac{1}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2191
3.291	$\int \frac{1}{x^2 \sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2197
3.292	$\int \frac{1}{x^4 \sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2205
3.293	$\int \frac{1}{x^6 \sqrt{d+ex^2}(a+bx^2+cx^4)} dx$.2210
3.294	$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2215
3.295	$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2220
3.296	$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2228
3.297	$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2235
3.298	$\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2242

3.299	$\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$.2248
3.300	$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}} dx$.2255

3.1 $\int x^3 (d + ex^2) (a + cx^4)^5 dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}$$

Rubi [A] time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1252, 766}

$$\frac{5}{8}a^2c^3dx^{16} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{9}a^2c^3ex^{18} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26

Rule 766

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2) (a + cx^4)^5 dx &= \frac{1}{2} \text{Subst} \left(\int x(d + ex) (a + cx^2)^5 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (a^5 dx + a^5 ex^2 + 5a^4 cdx^3 + 5a^4 cex^4 + 10a^3 c^2 dx^5 + 10a^3 c^2 ex^6 + 10a^2 c^3 dx^7 + 10a^2 c^3 ex^8 + 5a^2 c^3 dx^9 + 5a^2 c^3 ex^{10} + 5a^2 c^3 dx^{11} + 5a^2 c^3 ex^{12} + 5a^2 c^3 dx^{13} + 5a^2 c^3 ex^{14} + 5a^2 c^3 dx^{15} + 5a^2 c^3 ex^{16} + 5a^2 c^3 dx^{17} + 5a^2 c^3 ex^{18} + 5a^2 c^3 dx^{19} + 5a^2 c^3 ex^{20} + 5a^2 c^3 dx^{21} + 5a^2 c^3 ex^{22} + 5a^2 c^3 dx^{23} + 5a^2 c^3 ex^{24} + 5a^2 c^3 dx^{25} + 5a^2 c^3 ex^{26} + 5a^2 c^3 dx^{27} + 5a^2 c^3 ex^{28} + 5a^2 c^3 dx^{29} + 5a^2 c^3 ex^{30} + 5a^2 c^3 dx^{31} + 5a^2 c^3 ex^{32} + 5a^2 c^3 dx^{33} + 5a^2 c^3 ex^{34} + 5a^2 c^3 dx^{35} + 5a^2 c^3 ex^{36} + 5a^2 c^3 dx^{37} + 5a^2 c^3 ex^{38} + 5a^2 c^3 dx^{39} + 5a^2 c^3 ex^{40} + 5a^2 c^3 dx^{41} + 5a^2 c^3 ex^{42} + 5a^2 c^3 dx^{43} + 5a^2 c^3 ex^{44} + 5a^2 c^3 dx^{45} + 5a^2 c^3 ex^{46} + 5a^2 c^3 dx^{47} + 5a^2 c^3 ex^{48} + 5a^2 c^3 dx^{49} + 5a^2 c^3 ex^{50} + 5a^2 c^3 dx^{51} + 5a^2 c^3 ex^{52} + 5a^2 c^3 dx^{53} + 5a^2 c^3 ex^{54} + 5a^2 c^3 dx^{55} + 5a^2 c^3 ex^{56} + 5a^2 c^3 dx^{57} + 5a^2 c^3 ex^{58} + 5a^2 c^3 dx^{59} + 5a^2 c^3 ex^{60} + 5a^2 c^3 dx^{61} + 5a^2 c^3 ex^{62} + 5a^2 c^3 dx^{63} + 5a^2 c^3 ex^{64} + 5a^2 c^3 dx^{65} + 5a^2 c^3 ex^{66} + 5a^2 c^3 dx^{67} + 5a^2 c^3 ex^{68} + 5a^2 c^3 dx^{69} + 5a^2 c^3 ex^{70} + 5a^2 c^3 dx^{71} + 5a^2 c^3 ex^{72} + 5a^2 c^3 dx^{73} + 5a^2 c^3 ex^{74} + 5a^2 c^3 dx^{75} + 5a^2 c^3 ex^{76} + 5a^2 c^3 dx^{77} + 5a^2 c^3 ex^{78} + 5a^2 c^3 dx^{79} + 5a^2 c^3 ex^{80} + 5a^2 c^3 dx^{81} + 5a^2 c^3 ex^{82} + 5a^2 c^3 dx^{83} + 5a^2 c^3 ex^{84} + 5a^2 c^3 dx^{85} + 5a^2 c^3 ex^{86} + 5a^2 c^3 dx^{87} + 5a^2 c^3 ex^{88} + 5a^2 c^3 dx^{89} + 5a^2 c^3 ex^{90} + 5a^2 c^3 dx^{91} + 5a^2 c^3 ex^{92} + 5a^2 c^3 dx^{93} + 5a^2 c^3 ex^{94} + 5a^2 c^3 dx^{95} + 5a^2 c^3 ex^{96} + 5a^2 c^3 dx^{97} + 5a^2 c^3 ex^{98} + 5a^2 c^3 dx^{99} + 5a^2 c^3 ex^{100} dx, x, x^2 \right) \\ &= \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26} \end{aligned}$$

Mathematica [A] time = 0.02, size = 149, normalized size = 1.00

$$\frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (d + ex^2) (a + cx^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] IntegrateAlgebraic[x^3*(d + e*x^2)*(a + c*x^4)^5, x]

fricas [A] time = 0.55, size = 125, normalized size = 0.84

$$\frac{1}{26}x^{26}ec^5 + \frac{1}{24}x^{24}dc^5 + \frac{5}{22}x^{22}ec^4a + \frac{1}{4}x^{20}dc^4a + \frac{5}{9}x^{18}ec^3a^2 + \frac{5}{8}x^{16}dc^3a^2 + \frac{5}{7}x^{14}ec^2a^3 + \frac{5}{6}x^{12}dc^2a^3 + \frac{1}{2}x^{10}eca^4 + \frac{5}{8}x^8dca^4 + \frac{1}{6}x^6ea^5 + \frac{1}{4}x^4da^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")

[Out] 1/26*x^26*e*c^5 + 1/24*x^24*d*c^5 + 5/22*x^22*e*c^4*a + 1/4*x^20*d*c^4*a + 5/9*x^18*e*c^3*a^2 + 5/8*x^16*d*c^3*a^2 + 5/7*x^14*e*c^2*a^3 + 5/6*x^12*d*c^2*a^3 + 1/2*x^10*e*c*a^4 + 5/8*x^8*d*c*a^4 + 1/6*x^6*e*a^5 + 1/4*x^4*d*a^5

giac [A] time = 0.26, size = 131, normalized size = 0.88

$$\frac{1}{26}c^5x^{26}e + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4x^{22}e + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3x^{18}e + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2x^{14}e + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cx^{10}e + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5x^6e + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")

[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 + 5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4

maple [A] time = 0.01, size = 126, normalized size = 0.85

$$\frac{1}{26}c^5ex^{26} + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4ex^{22} + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3ex^{18} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cex^{10} + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5ex^6 + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x)

[Out] $\frac{1}{4}a^5d*x^4 + \frac{1}{6}a^5e*x^6 + \frac{5}{8}a^4c*d*x^8 + \frac{1}{2}a^4c*e*x^{10} + \frac{5}{6}a^3c^2*d*x^{12} + \frac{5}{7}a^3c^2*e*x^{14} + \frac{5}{8}a^2c^3*d*x^{16} + \frac{5}{9}a^2c^3*e*x^{18} + \frac{1}{4}a*c^4*d*x^{20} + \frac{5}{22}a*c^4*e*x^{22} + \frac{1}{24}c^5*d*x^{24} + \frac{1}{26}c^5*e*x^{26}$

maxima [A] time = 0.43, size = 125, normalized size = 0.84

$$\frac{1}{26}c^5ex^{26} + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4ex^{22} + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3ex^{18} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cex^{10} + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5ex^6 + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")

[Out] $\frac{1}{26}c^5e*x^{26} + \frac{1}{24}c^5d*x^{24} + \frac{5}{22}a*c^4e*x^{22} + \frac{1}{4}a*c^4d*x^{20} + \frac{5}{9}a^2c^3e*x^{18} + \frac{5}{8}a^2c^3d*x^{16} + \frac{5}{7}a^3c^2e*x^{14} + \frac{5}{6}a^3c^2d*x^{12} + \frac{1}{2}a^4c*e*x^{10} + \frac{5}{8}a^4c*d*x^8 + \frac{1}{6}a^5e*x^6 + \frac{1}{4}a^5d*x^4$

mupad [B] time = 0.31, size = 125, normalized size = 0.84

$$\frac{ea^5x^6}{6} + \frac{da^5x^4}{4} + \frac{ea^4cx^{10}}{2} + \frac{5da^4cx^8}{8} + \frac{5ea^3c^2x^{14}}{7} + \frac{5da^3c^2x^{12}}{6} + \frac{5ea^2c^3x^{18}}{9} + \frac{5da^2c^3x^{16}}{8} + \frac{5eac^4x^{22}}{22} + \frac{dac^4x^{20}}{4} + \frac{ec^5x^{26}}{26} + \frac{dc^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + c*x^4)^5*(d + e*x^2),x)

[Out] $\frac{(a^5d*x^4)}{4} + \frac{(a^5e*x^6)}{6} + \frac{(c^5d*x^{24})}{24} + \frac{(c^5e*x^{26})}{26} + \frac{(5a^3c^2d*x^{12})}{6} + \frac{(5a^2c^3d*x^{16})}{8} + \frac{(5a^3c^2e*x^{14})}{7} + \frac{(5a^2c^3e*x^{18})}{9} + \frac{(5a^4c*d*x^8)}{8} + \frac{(a*c^4d*x^{20})}{4} + \frac{(a^4c*e*x^{10})}{2} + \frac{(5a*c^4e*x^{22})}{22}$

sympy [A] time = 0.15, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4} + \frac{a^5ex^6}{6} + \frac{5a^4cdx^8}{8} + \frac{a^4cex^{10}}{2} + \frac{5a^3c^2dx^{12}}{6} + \frac{5a^3c^2ex^{14}}{7} + \frac{5a^2c^3dx^{16}}{8} + \frac{5a^2c^3ex^{18}}{9} + \frac{ac^4dx^{20}}{4} + \frac{5ac^4ex^{22}}{22} + \frac{c^5dx^{24}}{24} + \frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)

[Out] $a**5d*x**4/4 + a**5e*x**6/6 + 5a**4c*d*x**8/8 + a**4c*e*x**10/2 + 5a**3c**2d*x**12/6 + 5a**3c**2e*x**14/7 + 5a**2c**3d*x**16/8 + 5a**2c**3e*x**18/9 + a*c**4d*x**20/4 + 5a*c**4e*x**22/22 + c**5d*x**24/24 + c**5e*x**26/26$

3.2 $\int x^2 (d + ex^2) (a + cx^4)^5 dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^5dx^3 + \frac{1}{5}a^5ex^5 + \frac{5}{7}a^4cdx^7 + \frac{5}{9}a^4cex^9 + \frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19} + \frac{5}{21}ac^4ex^{21} + \frac{1}{23}c^5dx^{23} + \frac{1}{25}c^5ex^{25}$$

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1262}

$$\frac{2}{3}a^2c^3dx^{15} + \frac{10}{11}a^3c^2dx^{11} + \frac{10}{17}a^2c^3ex^{17} + \frac{10}{13}a^3c^2ex^{13} + \frac{5}{7}a^4cdx^7 + \frac{5}{9}a^4cex^9 + \frac{1}{3}a^5dx^3 + \frac{1}{5}a^5ex^5 + \frac{5}{19}ac^4dx^{19} + \frac{5}{21}ac^4ex^{21} + \frac{1}{23}c^5dx^{23} + \frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25

Rule 1262

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2) (a + cx^4)^5 dx &= \int (a^5 dx^2 + a^5 ex^4 + 5a^4 cdx^6 + 5a^4 cex^8 + 10a^3 c^2 dx^{10} + 10a^3 c^2 ex^{12} + 10a^2 c^3 dx^{14} \\ &+ \frac{1}{3}a^5 dx^3 + \frac{1}{5}a^5 ex^5 + \frac{5}{7}a^4 cdx^7 + \frac{5}{9}a^4 cex^9 + \frac{10}{11}a^3 c^2 dx^{11} + \frac{10}{13}a^3 c^2 ex^{13} + \frac{2}{3}a^2 c^3 dx^{15} \end{aligned}$$

Mathematica [A] time = 0.00, size = 149, normalized size = 1.00

$$\frac{1}{3}a^5dx^3 + \frac{1}{5}a^5ex^5 + \frac{5}{7}a^4cdx^7 + \frac{5}{9}a^4cex^9 + \frac{10}{11}a^3c^2dx^{11} + \frac{10}{13}a^3c^2ex^{13} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{17}a^2c^3ex^{17} + \frac{5}{19}ac^4dx^{19} + \frac{5}{21}ac^4ex^{21} + \frac{1}{23}c^5dx^{23} + \frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] $(a^5 d x^3)/3 + (a^5 e x^5)/5 + (5 a^4 c d x^7)/7 + (5 a^4 c e x^9)/9 + (10 a^3 c^2 d x^{11})/11 + (10 a^3 c^2 e x^{13})/13 + (2 a^2 c^3 d x^{15})/3 + (10 a^2 c^3 e x^{17})/17 + (5 a c^4 d x^{19})/19 + (5 a c^4 e x^{21})/21 + (c^5 d x^{23})/23 + (c^5 e x^{25})/25$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (d + ex^2) (a + cx^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(d + e*x^2)*(a + c*x^4)^5, x]

[Out] IntegrateAlgebraic[x^2*(d + e*x^2)*(a + c*x^4)^5, x]

fricas [A] time = 0.57, size = 125, normalized size = 0.84

$$\frac{1}{25}x^{25}ec^5 + \frac{1}{23}x^{23}dc^5 + \frac{5}{21}x^{21}ec^4a + \frac{5}{19}x^{19}dc^4a + \frac{10}{17}x^{17}ec^3a^2 + \frac{2}{3}x^{15}dc^3a^2 + \frac{10}{13}x^{13}ec^2a^3 + \frac{10}{11}x^{11}dc^2a^3 + \frac{5}{9}x^9eca^4 + \frac{5}{7}x^7dca^4 + \frac{1}{5}x^5ea^5 + \frac{1}{3}x^3da^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5, x, algorithm="fricas")

[Out] $1/25*x^{25}*e*c^5 + 1/23*x^{23}*d*c^5 + 5/21*x^{21}*e*c^4*a + 5/19*x^{19}*d*c^4*a + 10/17*x^{17}*e*c^3*a^2 + 2/3*x^{15}*d*c^3*a^2 + 10/13*x^{13}*e*c^2*a^3 + 10/11*x^{11}*d*c^2*a^3 + 5/9*x^9*e*c*a^4 + 5/7*x^7*d*c*a^4 + 1/5*x^5*e*a^5 + 1/3*x^3*d*a^5$

giac [A] time = 0.19, size = 131, normalized size = 0.88

$$\frac{1}{25}c^5x^{25}e + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4x^{21}e + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3x^{17}e + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2x^{13}e + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cx^9e + \frac{5}{7}a^4cdx^7 + \frac{1}{5}a^5x^5e + \frac{1}{3}a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5, x, algorithm="giac")

[Out] $1/25*c^5*x^{25}*e + 1/23*c^5*d*x^{23} + 5/21*a*c^4*x^{21}*e + 5/19*a*c^4*d*x^{19} + 10/17*a^2*c^3*x^{17}*e + 2/3*a^2*c^3*d*x^{15} + 10/13*a^3*c^2*x^{13}*e + 10/11*a^3*c^2*d*x^{11} + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5*d*x^3$

maple [A] time = 0.00, size = 126, normalized size = 0.85

$$\frac{1}{25}c^5ex^{25} + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4ex^{21} + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3ex^{17} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2ex^{13} + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cex^9 + \frac{5}{7}a^4cdx^7 + \frac{1}{5}a^5ex^5 + \frac{1}{3}a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)*(c*x^4+a)^5,x)`

[Out] $\frac{1}{3}a^5d*x^3 + \frac{1}{5}a^5e*x^5 + \frac{5}{7}a^4c*d*x^7 + \frac{5}{9}a^4c*e*x^9 + \frac{10}{11}a^3c^2*d*x^{11} + \frac{10}{13}a^3c^2e*x^{13} + \frac{2}{3}a^2c^3*d*x^{15} + \frac{10}{17}a^2c^3e*x^{17} + \frac{5}{19}a*c^4*d*x^{19} + \frac{5}{21}a*c^4e*x^{21} + \frac{1}{23}c^5*d*x^{23} + \frac{1}{25}c^5e*x^{25}$

maxima [A] time = 0.46, size = 125, normalized size = 0.84

$$\frac{1}{25}c^5ex^{25} + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4ex^{21} + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3ex^{17} + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2ex^{13} + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cex^9 + \frac{5}{7}a^4cdx^7 + \frac{1}{5}a^5ex^5 + \frac{1}{3}a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")`

[Out] $\frac{1}{25}c^5e*x^{25} + \frac{1}{23}c^5d*x^{23} + \frac{5}{21}a*c^4e*x^{21} + \frac{5}{19}a*c^4d*x^{19} + \frac{10}{17}a^2c^3e*x^{17} + \frac{2}{3}a^2c^3d*x^{15} + \frac{10}{13}a^3c^2e*x^{13} + \frac{10}{11}a^3c^2d*x^{11} + \frac{5}{9}a^4c*e*x^9 + \frac{5}{7}a^4c*d*x^7 + \frac{1}{5}a^5e*x^5 + \frac{1}{3}a^5d*x^3$

mupad [B] time = 0.07, size = 125, normalized size = 0.84

$$\frac{ea^5x^5}{5} + \frac{da^5x^3}{3} + \frac{5ea^4cx^9}{9} + \frac{5da^4cx^7}{7} + \frac{10ea^3c^2x^{13}}{13} + \frac{10da^3c^2x^{11}}{11} + \frac{10ea^2c^3x^{17}}{17} + \frac{2da^2c^3x^{15}}{3} + \frac{5ea^4x^{21}}{21} + \frac{5dac^4x^{19}}{19} + \frac{e^5x^{25}}{25} + \frac{d^5x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + c*x^4)^5*(d + e*x^2),x)`

[Out] $\frac{a^5d*x^3}{3} + \frac{a^5e*x^5}{5} + \frac{c^5d*x^{23}}{23} + \frac{c^5e*x^{25}}{25} + \frac{10a^3c^2d*x^{11}}{11} + \frac{2a^2c^3d*x^{15}}{3} + \frac{10a^3c^2e*x^{13}}{13} + \frac{10a^2c^3e*x^{17}}{17} + \frac{5a^4c*d*x^7}{7} + \frac{5a^4c^4d*x^{19}}{19} + \frac{5a^4c^4e*x^9}{9} + \frac{5a^4c^4e*x^{21}}{21}$

sympy [A] time = 0.09, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3} + \frac{a^5ex^5}{5} + \frac{5a^4cdx^7}{7} + \frac{5a^4cex^9}{9} + \frac{10a^3c^2dx^{11}}{11} + \frac{10a^3c^2ex^{13}}{13} + \frac{2a^2c^3dx^{15}}{3} + \frac{10a^2c^3ex^{17}}{17} + \frac{5ac^4dx^{19}}{19} + \frac{5ac^4ex^{21}}{21} + \frac{c^5dx^{23}}{23} + \frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)`

[Out] $a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*x**23/23 + c**5*e*x**25/25$

3.3 $\int x (d + ex^2) (a + cx^4)^5 dx$

Optimal. Leaf size=89

$$\frac{1}{2}a^5dx^2 + \frac{5}{6}a^4cdx^6 + a^3c^2dx^{10} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{18}ac^4dx^{18} + \frac{e(a+cx^4)^6}{24c} + \frac{1}{22}c^5dx^{22}$$

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1248, 641, 194}

$$\frac{5}{7}a^2c^3dx^{14} + a^3c^2dx^{10} + \frac{5}{6}a^4cdx^6 + \frac{1}{2}a^5dx^2 + \frac{5}{18}ac^4dx^{18} + \frac{e(a+cx^4)^6}{24c} + \frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int x(d+ex^2)(a+cx^4)^5 dx &= \frac{1}{2} \text{Subst}\left(\int (d+ex)(a+cx^2)^5 dx, x, x^2\right) \\
&= \frac{e(a+cx^4)^6}{24c} + \frac{1}{2}d \text{Subst}\left(\int (a+cx^2)^5 dx, x, x^2\right) \\
&= \frac{e(a+cx^4)^6}{24c} + \frac{1}{2}d \text{Subst}\left(\int (a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c^5x^{10}) dx, x, x^2\right) \\
&= \frac{1}{2}a^5dx^2 + \frac{5}{6}a^4cdx^6 + a^3c^2dx^{10} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{18}ac^4dx^{18} + \frac{1}{22}c^5dx^{22} + \frac{e(a+cx^4)^6}{24c}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 146, normalized size = 1.64

$$\frac{1}{2}a^5dx^2 + \frac{1}{4}a^5ex^4 + \frac{5}{6}a^4cdx^6 + \frac{5}{8}a^4cex^8 + a^3c^2dx^{10} + \frac{5}{6}a^3c^2ex^{12} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{8}a^2c^3ex^{16} + \frac{5}{18}ac^4dx^{18} + \frac{1}{4}ac^4ex^{20} + \frac{1}{22}c^5dx^{22} + \frac{1}{24}c^5ex^{24}$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(d+ex^2)(a+cx^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(d + e*x^2)*(a + c*x^4)^5,x]

[Out] IntegrateAlgebraic[x*(d + e*x^2)*(a + c*x^4)^5, x]

fricas [A] time = 0.79, size = 124, normalized size = 1.39

$$\frac{1}{24}x^{24}ec^5 + \frac{1}{22}x^{22}dc^5 + \frac{1}{4}x^{20}ec^4a + \frac{5}{18}x^{18}dc^4a + \frac{5}{8}x^{16}ec^3a^2 + \frac{5}{7}x^{14}dc^3a^2 + \frac{5}{6}x^{12}ec^2a^3 + x^{10}dc^2a^3 + \frac{5}{8}x^8eca^4 + \frac{5}{6}x^6dca^4 + \frac{1}{4}x^4ea^5 + \frac{1}{2}x^2da^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")

[Out] $\frac{1}{24}x^{24}e^{c^5} + \frac{1}{22}x^{22}d^{c^5} + \frac{1}{4}x^{20}e^{c^4}a + \frac{5}{18}x^{18}d^{c^4}a + \frac{5}{8}x^{16}e^{c^3}a^2 + \frac{5}{7}x^{14}d^{c^3}a^2 + \frac{5}{6}x^{12}e^{c^2}a^3 + x^{10}d^{c^2}a^3 + \frac{5}{8}x^8e^{c^1}a^4 + \frac{5}{6}x^6d^{c^1}a^4 + \frac{1}{4}x^4e^{a^5} + \frac{1}{2}x^2d^{a^5}$

giac [A] time = 0.18, size = 130, normalized size = 1.46

$$\frac{1}{24}c^5x^{24}e + \frac{1}{22}c^5dx^{22} + \frac{1}{4}ac^4x^{20}e + \frac{5}{18}ac^4dx^{18} + \frac{5}{8}a^2c^3x^{16}e + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{6}a^3c^2x^{12}e + a^3c^2dx^{10} + \frac{5}{8}a^4cx^8e + \frac{5}{6}a^4cdx^6 + \frac{1}{4}a^5x^4e + \frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")

[Out] $\frac{1}{24}c^5x^{24}e + \frac{1}{22}c^5d^{c^5}x^{22} + \frac{1}{4}a^{c^4}x^{20}e + \frac{5}{18}a^{c^4}d^{c^4}x^{18} + \frac{5}{8}a^2c^3x^{16}e + \frac{5}{7}a^2c^3d^{c^3}x^{14} + \frac{5}{6}a^3c^2x^{12}e + a^3c^2d^{c^2}x^{10} + \frac{5}{8}a^4cx^8e + \frac{5}{6}a^4cd^{c^1}x^6 + \frac{1}{4}a^5x^4e + \frac{1}{2}a^5d^{c^1}x^2$

maple [A] time = 0.00, size = 125, normalized size = 1.40

$$\frac{1}{24}c^5e^{x^{24}} + \frac{1}{22}c^5dx^{22} + \frac{1}{4}ac^4ex^{20} + \frac{5}{18}ac^4dx^{18} + \frac{5}{8}a^2c^3ex^{16} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{6}a^3c^2ex^{12} + a^3c^2dx^{10} + \frac{5}{8}a^4cex^8 + \frac{5}{6}a^4cdx^6 + \frac{1}{4}a^5ex^4 + \frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x)

[Out] $\frac{1}{24}c^5e^{x^{24}} + \frac{1}{22}c^5d^{c^5}x^{22} + \frac{1}{4}a^{c^4}e^{x^{20}} + \frac{5}{18}a^{c^4}d^{c^4}x^{18} + \frac{5}{8}a^2c^3e^{x^{16}} + \frac{5}{7}a^2c^3d^{c^3}x^{14} + \frac{5}{6}a^3c^2e^{x^{12}} + a^3c^2d^{c^2}x^{10} + \frac{5}{8}a^4cex^8 + \frac{5}{6}a^4cd^{c^1}x^6 + \frac{1}{4}a^5ex^4 + \frac{1}{2}a^5d^{c^1}x^2$

maxima [A] time = 0.44, size = 124, normalized size = 1.39

$$\frac{1}{24}c^5ex^{24} + \frac{1}{22}c^5dx^{22} + \frac{1}{4}ac^4ex^{20} + \frac{5}{18}ac^4dx^{18} + \frac{5}{8}a^2c^3ex^{16} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{6}a^3c^2ex^{12} + a^3c^2dx^{10} + \frac{5}{8}a^4cex^8 + \frac{5}{6}a^4cdx^6 + \frac{1}{4}a^5ex^4 + \frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")

[Out] $\frac{1}{24}c^5e^{x^{24}} + \frac{1}{22}c^5d^{c^5}x^{22} + \frac{1}{4}a^{c^4}e^{x^{20}} + \frac{5}{18}a^{c^4}d^{c^4}x^{18} + \frac{5}{8}a^2c^3e^{x^{16}} + \frac{5}{7}a^2c^3d^{c^3}x^{14} + \frac{5}{6}a^3c^2e^{x^{12}} + a^3c^2d^{c^2}x^{10} + \frac{5}{8}a^4cex^8 + \frac{5}{6}a^4cd^{c^1}x^6 + \frac{1}{4}a^5e^{x^4} + \frac{1}{2}a^5d^{c^1}x^2$

mupad [B] time = 0.07, size = 124, normalized size = 1.39

$$\frac{e^{a^5}x^4}{4} + \frac{da^5x^2}{2} + \frac{5ea^4cx^8}{8} + \frac{5da^4cx^6}{6} + \frac{5ea^3c^2x^{12}}{6} + da^3c^2x^{10} + \frac{5ea^2c^3x^{16}}{8} + \frac{5da^2c^3x^{14}}{7} + \frac{eac^4x^{20}}{4} + \frac{5dac^4x^{18}}{18} + \frac{ec^5x^{24}}{24} + \frac{dc^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + c*x^4)^5*(d + e*x^2),x)

[Out] $(a^5 d x^2)/2 + (a^5 e x^4)/4 + (c^5 d x^{22})/22 + (c^5 e x^{24})/24 + a^3 c^2 d x^{10} + (5 a^2 c^3 d x^{14})/7 + (5 a^3 c^2 e x^{12})/6 + (5 a^2 c^3 e x^{16})/8 + (5 a^4 c d x^6)/6 + (5 a c^4 d x^{18})/18 + (5 a^4 c e x^8)/8 + (a c^4 e x^{20})/4$

sympy [A] time = 0.09, size = 150, normalized size = 1.69

$$\frac{a^5 dx^2}{2} + \frac{a^5 ex^4}{4} + \frac{5a^4 c dx^6}{6} + \frac{5a^4 c ex^8}{8} + a^3 c^2 dx^{10} + \frac{5a^3 c^2 ex^{12}}{6} + \frac{5a^2 c^3 dx^{14}}{7} + \frac{5a^2 c^3 ex^{16}}{8} + \frac{5ac^4 dx^{18}}{18} + \frac{ac^4 ex^{20}}{4} + \frac{c^5 dx^{22}}{22} + \frac{c^5 ex^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)`

[Out] $a^5 d x^2/2 + a^5 e x^4/4 + 5 a^4 c d x^6/6 + 5 a^4 c e x^8/8 + a^3 c^2 d x^{10} + 5 a^3 c^2 e x^{12}/6 + 5 a^2 c^3 d x^{14}/7 + 5 a^2 c^3 e x^{16}/8 + 5 a c^4 d x^{18}/18 + a c^4 e x^{20}/4 + c^5 d x^{22}/22 + c^5 e x^{24}/24$

3.4 $\int (d + ex^2)(a + cx^4)^5 dx$

Optimal. Leaf size=141

$$a^5 dx + \frac{1}{3} a^5 ex^3 + a^4 c dx^5 + \frac{5}{7} a^4 c ex^7 + \frac{10}{9} a^3 c^2 dx^9 + \frac{10}{11} a^3 c^2 ex^{11} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{2}{3} a^2 c^3 ex^{15} + \frac{5}{17} a c^4 dx^{17} + \frac{5}{19} a c^4 ex^{19} + \frac{1}{21} c^5 dx^{21} + \frac{1}{23} c^5 ex^{23}$$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1154}

$$\frac{10}{13} a^2 c^3 dx^{13} + \frac{10}{9} a^3 c^2 dx^9 + \frac{2}{3} a^2 c^3 ex^{15} + \frac{10}{11} a^3 c^2 ex^{11} + a^4 c dx^5 + \frac{5}{7} a^4 c ex^7 + a^5 dx + \frac{1}{3} a^5 ex^3 + \frac{5}{17} a c^4 dx^{17} + \frac{5}{19} a c^4 ex^{19} + \frac{1}{21} c^5 dx^{21} + \frac{1}{23} c^5 ex^{23}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]

[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23

Rule 1154

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)(a + cx^4)^5 dx &= \int (a^5 d + a^5 ex^2 + 5a^4 c dx^4 + 5a^4 c ex^6 + 10a^3 c^2 dx^8 + 10a^3 c^2 ex^{10} + 10a^2 c^3 dx^{12} + 10a^2 c^3 ex^{14} + 5a c^4 dx^{16} + 5a c^4 ex^{18} + c^5 dx^{20} + c^5 ex^{22}) dx \\ &= a^5 dx + \frac{1}{3} a^5 ex^3 + a^4 c dx^5 + \frac{5}{7} a^4 c ex^7 + \frac{10}{9} a^3 c^2 dx^9 + \frac{10}{11} a^3 c^2 ex^{11} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{2}{3} a^2 c^3 ex^{15} + \frac{5}{17} a c^4 dx^{17} + \frac{5}{19} a c^4 ex^{19} + \frac{1}{21} c^5 dx^{21} + \frac{1}{23} c^5 ex^{23} \end{aligned}$$

Mathematica [A] time = 0.00, size = 141, normalized size = 1.00

$$a^5 dx + \frac{1}{3} a^5 ex^3 + a^4 c dx^5 + \frac{5}{7} a^4 c ex^7 + \frac{10}{9} a^3 c^2 dx^9 + \frac{10}{11} a^3 c^2 ex^{11} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{2}{3} a^2 c^3 ex^{15} + \frac{5}{17} a c^4 dx^{17} + \frac{5}{19} a c^4 ex^{19} + \frac{1}{21} c^5 dx^{21} + \frac{1}{23} c^5 ex^{23}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]

[Out] $a^5 d x + (a^5 e x^3)/3 + a^4 c d x^5 + (5 a^4 c e x^7)/7 + (10 a^3 c^2 d x^9)/9 + (10 a^3 c^2 e x^{11})/11 + (10 a^2 c^3 d x^{13})/13 + (2 a^2 c^3 e x^{15})/3 + (5 a c^4 d x^{17})/17 + (5 a c^4 e x^{19})/19 + (c^5 d x^{21})/21 + (c^5 e x^{23})/23$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)(a + cx^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2)*(a + c*x^4)^5,x]

[Out] IntegrateAlgebraic[(d + e*x^2)*(a + c*x^4)^5, x]

fricas [A] time = 0.57, size = 121, normalized size = 0.86

$$\frac{1}{23}x^{23}ec^5 + \frac{1}{21}x^{21}dc^5 + \frac{5}{19}x^{19}ec^4a + \frac{5}{17}x^{17}dc^4a + \frac{2}{3}x^{15}ec^3a^2 + \frac{10}{13}x^{13}dc^3a^2 + \frac{10}{11}x^{11}ec^2a^3 + \frac{10}{9}x^9dc^2a^3 + \frac{5}{7}x^7eca^4 + x^5dca^4 + \frac{1}{3}x^3ea^5 + xda^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")

[Out] $\frac{1}{23}x^{23}e*c^5 + \frac{1}{21}x^{21}d*c^5 + \frac{5}{19}x^{19}e*c^4*a + \frac{5}{17}x^{17}d*c^4*a + \frac{2}{3}x^{15}e*c^3*a^2 + \frac{10}{13}x^{13}d*c^3*a^2 + \frac{10}{11}x^{11}e*c^2*a^3 + \frac{10}{9}x^9d*c^2*a^3 + \frac{5}{7}x^7e*c*a^4 + x^5d*c*a^4 + \frac{1}{3}x^3e*a^5 + x*d*a^5$

giac [A] time = 0.22, size = 127, normalized size = 0.90

$$\frac{1}{23}c^5x^{23}e + \frac{1}{21}c^5dx^{21} + \frac{5}{19}ac^4x^{19}e + \frac{5}{17}ac^4dx^{17} + \frac{2}{3}a^2c^3x^{15}e + \frac{10}{13}a^2c^3dx^{13} + \frac{10}{11}a^3c^2x^{11}e + \frac{10}{9}a^3c^2dx^9 + \frac{5}{7}a^4cx^7e + a^4cdx^5 + \frac{1}{3}a^5x^3e + a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")

[Out] $\frac{1}{23}c^5x^{23}e + \frac{1}{21}c^5d*x^{21} + \frac{5}{19}a*c^4*x^{19}e + \frac{5}{17}a*c^4*d*x^{17} + \frac{2}{3}a^2*c^3*x^{15}e + \frac{10}{13}a^2*c^3*d*x^{13} + \frac{10}{11}a^3*c^2*x^{11}e + \frac{10}{9}a^3*c^2*d*x^9 + \frac{5}{7}a^4*c*x^7e + a^4*c*d*x^5 + \frac{1}{3}a^5*x^3e + a^5*d*x$

maple [A] time = 0.00, size = 122, normalized size = 0.87

$$\frac{1}{23}c^5ex^{23} + \frac{1}{21}c^5dx^{21} + \frac{5}{19}ac^4ex^{19} + \frac{5}{17}ac^4dx^{17} + \frac{2}{3}a^2c^3ex^{15} + \frac{10}{13}a^2c^3dx^{13} + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{9}a^3c^2dx^9 + \frac{5}{7}a^4cex^7 + a^4cdx^5 + \frac{1}{3}a^5ex^3 + a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(c*x^4+a)^5,x)

[Out] $a^5 dx + \frac{1}{3} a^5 e^x x^3 + a^4 c dx^5 + \frac{5}{7} a^4 c e^x x^7 + \frac{10}{9} a^3 c^2 dx^9 + \frac{10}{11} a^3 c^2 e^x x^{11} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{2}{3} a^2 c^3 e^x x^{15} + \frac{5}{17} a^2 c^4 dx^{17} + \frac{5}{19} a^2 c^4 e^x x^{19} + \frac{1}{21} c^5 dx^{21} + \frac{1}{23} c^5 e^x x^{23}$

maxima [A] time = 0.46, size = 121, normalized size = 0.86

$$\frac{1}{23} c^5 e^x x^{23} + \frac{1}{21} c^5 dx^{21} + \frac{5}{19} a^2 c^4 e^x x^{19} + \frac{5}{17} a^2 c^4 dx^{17} + \frac{2}{3} a^2 c^3 e^x x^{15} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{10}{11} a^2 c^3 e^x x^{11} + \frac{10}{9} a^2 c^2 dx^9 + \frac{5}{7} a^2 c^2 e^x x^7 + a^4 c dx^5 + \frac{1}{3} a^5 e^x x^3 + a^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")

[Out] $\frac{1}{23} c^5 e^x x^{23} + \frac{1}{21} c^5 dx^{21} + \frac{5}{19} a^2 c^4 e^x x^{19} + \frac{5}{17} a^2 c^4 dx^{17} + \frac{2}{3} a^2 c^3 e^x x^{15} + \frac{10}{13} a^2 c^3 dx^{13} + \frac{10}{11} a^2 c^3 e^x x^{11} + \frac{10}{9} a^2 c^2 dx^9 + \frac{5}{7} a^2 c^2 e^x x^7 + a^4 c dx^5 + \frac{1}{3} a^5 e^x x^3 + a^5 dx$

mupad [B] time = 0.07, size = 121, normalized size = 0.86

$$\frac{e a^5 x^3}{3} + d a^5 x + \frac{5 e a^4 c x^7}{7} + d a^4 c x^5 + \frac{10 e a^3 c^2 x^{11}}{11} + \frac{10 d a^3 c^2 x^9}{9} + \frac{2 e a^2 c^3 x^{15}}{3} + \frac{10 d a^2 c^3 x^{13}}{13} + \frac{5 e a c^4 x^{19}}{19} + \frac{5 d a c^4 x^{17}}{17} + \frac{e c^5 x^{23}}{23} + \frac{d c^5 x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^4)^5*(d + e*x^2),x)

[Out] $\frac{a^5 e^x x^3}{3} + \frac{c^5 dx^{21}}{21} + \frac{c^5 e^x x^{23}}{23} + a^5 dx + \frac{10 a^3 c^2 dx^9}{9} + \frac{10 a^2 c^3 dx^{13}}{13} + \frac{10 a^3 c^2 e^x x^{11}}{11} + \frac{2 a^2 c^3 e^x x^{15}}{3} + \frac{a^4 c dx^5}{5} + \frac{5 a^2 c^4 dx^{17}}{17} + \frac{5 a^4 c e^x x^7}{7} + \frac{5 a^2 c^4 e^x x^{19}}{19}$

sympy [A] time = 0.09, size = 148, normalized size = 1.05

$$a^5 dx + \frac{a^5 e^x x^3}{3} + a^4 c dx^5 + \frac{5 a^4 c e^x x^7}{7} + \frac{10 a^3 c^2 dx^9}{9} + \frac{10 a^3 c^2 e^x x^{11}}{11} + \frac{10 a^2 c^3 dx^{13}}{13} + \frac{2 a^2 c^3 e^x x^{15}}{3} + \frac{5 a c^4 dx^{17}}{17} + \frac{5 a c^4 e^x x^{19}}{19} + \frac{c^5 dx^{21}}{21} + \frac{c^5 e^x x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)

[Out] $a^5 dx + a^5 e^x x^3 / 3 + a^4 c dx^5 + 5 a^4 c e^x x^7 / 7 + 10 a^3 c^2 dx^9 / 9 + 10 a^3 c^2 e^x x^{11} / 11 + 10 a^2 c^3 dx^{13} / 13 + 2 a^2 c^3 e^x x^{15} / 3 + 5 a^2 c^4 dx^{17} / 17 + 5 a^4 c e^x x^7 / 7 + 5 a^2 c^4 e^x x^{19} / 19 + c^5 dx^{21} / 21 + c^5 e^x x^{23} / 23$

$$3.5 \quad \int \frac{(d+ex^2)(a+cx^4)^5}{x} dx$$

Optimal. Leaf size=142

$$a^5 d \log(x) + \frac{1}{2} a^5 e x^2 + \frac{5}{4} a^4 c d x^4 + \frac{5}{6} a^4 c e x^6 + \frac{5}{4} a^3 c^2 d x^8 + a^3 c^2 e x^{10} + \frac{5}{6} a^2 c^3 d x^{12} + \frac{5}{7} a^2 c^3 e x^{14} + \frac{5}{16} a c^4 d x^{16} + \frac{5}{18} a c^4 e x^{18} + \frac{1}{20} c^5 d x^{20} + \frac{1}{22} c^5 e x^{22}$$

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1252, 766}

$$\frac{5}{6} a^2 c^3 d x^{12} + \frac{5}{4} a^3 c^2 d x^8 + \frac{5}{7} a^2 c^3 e x^{14} + a^3 c^2 e x^{10} + \frac{5}{4} a^4 c d x^4 + \frac{5}{6} a^4 c e x^6 + a^5 d \log(x) + \frac{1}{2} a^5 e x^2 + \frac{5}{16} a c^4 d x^{16} + \frac{5}{18} a c^4 e x^{18} + \frac{1}{20} c^5 d x^{20} + \frac{1}{22} c^5 e x^{22}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]

[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*Log[x]

Rule 766

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)(a+cx^4)^5}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)(a+cx^2)^5}{x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(a^5 e + \frac{a^5 d}{x} + 5a^4 c d x + 5a^4 c e x^2 + 10a^3 c^2 d x^3 + 10a^3 c^2 e x^4 + 10a^2 c^3 d x^5 + \right. \right. \\ &\quad \left. \left. + \frac{5}{2} a^5 e x^2 + \frac{5}{4} a^4 c d x^4 + \frac{5}{6} a^4 c e x^6 + \frac{5}{4} a^3 c^2 d x^8 + a^3 c^2 e x^{10} + \frac{5}{6} a^2 c^3 d x^{12} + \frac{5}{7} a^2 c^3 e x^{14} + \frac{5}{16} a c^4 d x^{16} + \frac{5}{18} a c^4 e x^{18} + \frac{1}{20} c^5 d x^{20} + \frac{1}{22} c^5 e x^{22} \right) dx, x, x^2 \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$a^5 d \log(x) + \frac{1}{2} a^5 e x^2 + \frac{5}{4} a^4 c d x^4 + \frac{5}{6} a^4 c e x^6 + \frac{5}{4} a^3 c^2 d x^8 + a^3 c^2 e x^{10} + \frac{5}{6} a^2 c^3 d x^{12} + \frac{5}{7} a^2 c^3 e x^{14} + \frac{5}{16} a c^4 d x^{16} + \frac{5}{18} a c^4 e x^{18} + \frac{1}{20} c^5 d x^{20} + \frac{1}{22} c^5 e x^{22}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]

[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(a + cx^4)^5}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x,x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x, x]

fricas [A] time = 0.50, size = 122, normalized size = 0.86

$$\frac{1}{22} c^5 e x^{22} + \frac{1}{20} c^5 d x^{20} + \frac{5}{18} a c^4 e x^{18} + \frac{5}{16} a c^4 d x^{16} + \frac{5}{7} a^2 c^3 e x^{14} + \frac{5}{6} a^2 c^3 d x^{12} + a^3 c^2 e x^{10} + \frac{5}{4} a^3 c^2 d x^8 + \frac{5}{6} a^4 c e x^6 + \frac{5}{4} a^4 c d x^4 + \frac{1}{2} a^5 e x^2 + a^5 d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")

[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 + 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)

giac [A] time = 0.20, size = 131, normalized size = 0.92

$$\frac{1}{22} c^5 x^{22} e + \frac{1}{20} c^5 d x^{20} + \frac{5}{18} a c^4 x^{18} e + \frac{5}{16} a c^4 d x^{16} + \frac{5}{7} a^2 c^3 x^{14} e + \frac{5}{6} a^2 c^3 d x^{12} + a^3 c^2 x^{10} e + \frac{5}{4} a^3 c^2 d x^8 + \frac{5}{6} a^4 c x^6 e + \frac{5}{4} a^4 c d x^4 + \frac{1}{2} a^5 x^2 e + \frac{1}{2} a^5 d \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")

[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 + 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5 e x^{22}}{22} + \frac{c^5 d x^{20}}{20} + \frac{5 a c^4 e x^{18}}{18} + \frac{5 a c^4 d x^{16}}{16} + \frac{5 a^2 c^3 e x^{14}}{7} + \frac{5 a^2 c^3 d x^{12}}{6} + a^3 c^2 e x^{10} + \frac{5 a^3 c^2 d x^8}{4} + \frac{5 a^4 c e x^6}{6} + \frac{5 a^4 c d x^4}{4} + \frac{a^5 e x^2}{2} + a^5 d \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(c*x^4+a)^5/x,x)

[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)

maxima [A] time = 0.52, size = 125, normalized size = 0.88

$$\frac{1}{22} c^5 e x^{22} + \frac{1}{20} c^5 d x^{20} + \frac{5}{18} a c^4 e x^{18} + \frac{5}{16} a c^4 d x^{16} + \frac{5}{7} a^2 c^3 e x^{14} + \frac{5}{6} a^2 c^3 d x^{12} + a^3 c^2 e x^{10} + \frac{5}{4} a^3 c^2 d x^8 + \frac{5}{6} a^4 c e x^6 + \frac{5}{4} a^4 c d x^4 + \frac{1}{2} a^5 e x^2 + \frac{1}{2} a^5 d \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")

[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 + 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)

mupad [B] time = 0.11, size = 122, normalized size = 0.86

$$\frac{a^5 e x^2}{2} + \frac{c^5 d x^{20}}{20} + \frac{c^5 e x^{22}}{22} + a^5 d \ln(x) + \frac{5 a^3 c^2 d x^8}{4} + \frac{5 a^2 c^3 d x^{12}}{6} + a^3 c^2 e x^{10} + \frac{5 a^2 c^3 e x^{14}}{7} + \frac{5 a^4 c d x^4}{4} + \frac{5 a c^4 d x^{16}}{16} + \frac{5 a^4 c e x^6}{6} + \frac{5 a c^4 e x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^4)^5*(d + e*x^2))/x,x)

[Out] (a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*c^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/7 + (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4*e*x^18)/18

sympy [A] time = 0.25, size = 150, normalized size = 1.06

$$a^5 d \log(x) + \frac{a^5 e x^2}{2} + \frac{5 a^4 c d x^4}{4} + \frac{5 a^4 c e x^6}{6} + \frac{5 a^3 c^2 d x^8}{4} + a^3 c^2 e x^{10} + \frac{5 a^2 c^3 d x^{12}}{6} + \frac{5 a^2 c^3 e x^{14}}{7} + \frac{5 a c^4 d x^{16}}{16} + \frac{5 a c^4 e x^{18}}{18} + \frac{c^5 d x^{20}}{20} + \frac{c^5 e x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)

[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 + c**5*e*x**22/22

$$3.6 \quad \int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx$$

Optimal. Leaf size=139

$$-\frac{a^5d}{x} + a^5ex + \frac{5}{3}a^4cdx^3 + a^4cex^5 + \frac{10}{7}a^3c^2dx^7 + \frac{10}{9}a^3c^2ex^9 + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{13}a^2c^3ex^{13} + \frac{1}{3}ac^4dx^{15} + \frac{5}{17}ac^4ex^{17} + \frac{1}{19}c^5dx^{19} + \frac{1}{21}c^5ex^{21}$$

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1262}

$$\frac{10}{11}a^2c^3dx^{11} + \frac{10}{7}a^3c^2dx^7 + \frac{10}{13}a^2c^3ex^{13} + \frac{10}{9}a^3c^2ex^9 + \frac{5}{3}a^4cdx^3 + a^4cex^5 - \frac{a^5d}{x} + a^5ex + \frac{1}{3}ac^4dx^{15} + \frac{5}{17}ac^4ex^{17} + \frac{1}{19}c^5dx^{19} + \frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2, x]

[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21)/21

Rule 1262

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx &= \int \left(a^5e + \frac{a^5d}{x^2} + 5a^4cdx^2 + 5a^4cex^4 + 10a^3c^2dx^6 + 10a^3c^2ex^8 + 10a^2c^3dx^{10} + 10a^2c^3ex^{12} \right. \\ &\quad \left. + \frac{a^5d}{x} + a^5ex + \frac{5}{3}a^4cdx^3 + a^4cex^5 + \frac{10}{7}a^3c^2dx^7 + \frac{10}{9}a^3c^2ex^9 + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{13}a^2c^3ex^{13} \right) dx \end{aligned}$$

Mathematica [A] time = 0.01, size = 139, normalized size = 1.00

$$-\frac{a^5d}{x} + a^5ex + \frac{5}{3}a^4cdx^3 + a^4cex^5 + \frac{10}{7}a^3c^2dx^7 + \frac{10}{9}a^3c^2ex^9 + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{13}a^2c^3ex^{13} + \frac{1}{3}ac^4dx^{15} + \frac{5}{17}ac^4ex^{17} + \frac{1}{19}c^5dx^{19} + \frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]

[Out] $-\frac{a^5 d}{x} + a^5 e x + \frac{5 a^4 c d x^3}{3} + a^4 c e x^5 + \frac{10 a^3 c^2 d x^7}{7} + \frac{10 a^3 c^2 e x^9}{9} + \frac{10 a^2 c^3 d x^{11}}{11} + \frac{10 a^2 c^3 e x^{13}}{13} + \frac{a c^4 d x^{15}}{3} + \frac{5 a c^4 e x^{17}}{17} + \frac{c^5 d x^{19}}{19} + \frac{c^5 e x^{21}}{21}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(a + cx^4)^5}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^2, x]

fricas [A] time = 0.53, size = 127, normalized size = 0.91

138567 c^5 e x^22 + 153153 c^5 d x^20 + 855855 a c^4 e x^18 + 969969 a c^4 d x^16 + 2238390 a^2 c^3 e x^14 + 2645370 a^2 c^3 d x^12 + 3233230 a^2 c^2 e x^10 + 4157010 a^2 c^2 d x^8 + 2909907 a^4 c e x^6 + 4849845 a^4 c d x^4 + 2909907 a^5 e x^2 - 2909907 a^5 d
2909907 x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")

[Out] $\frac{1}{2909907} * (138567 * c^5 * e * x^{22} + 153153 * c^5 * d * x^{20} + 855855 * a * c^4 * e * x^{18} + 969969 * a * c^4 * d * x^{16} + 2238390 * a^2 * c^3 * e * x^{14} + 2645370 * a^2 * c^3 * d * x^{12} + 3233230 * a^2 * c^2 * e * x^{10} + 4157010 * a^2 * c^2 * d * x^8 + 2909907 * a^4 * c * e * x^6 + 4849845 * a^4 * c * d * x^4 + 2909907 * a^5 * e * x^2 - 2909907 * a^5 * d) / x$

giac [A] time = 0.20, size = 127, normalized size = 0.91

$$\frac{1}{21} c^5 x^{21} e + \frac{1}{19} c^5 d x^{19} + \frac{5}{17} a c^4 x^{17} e + \frac{1}{3} a c^4 d x^{15} + \frac{10}{13} a^2 c^3 x^{13} e + \frac{10}{11} a^2 c^3 d x^{11} + \frac{10}{9} a^3 c^2 x^9 e + \frac{10}{7} a^3 c^2 d x^7 + a^4 c x^5 e + \frac{5}{3} a^4 c d x^3 + a^5 x e - \frac{a^5 d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")

[Out] $\frac{1}{21} * c^5 * x^{21} * e + \frac{1}{19} * c^5 * d * x^{19} + \frac{5}{17} * a * c^4 * x^{17} * e + \frac{1}{3} * a * c^4 * d * x^{15} + \frac{10}{13} * a^2 * c^3 * x^{13} * e + \frac{10}{11} * a^2 * c^3 * d * x^{11} + \frac{10}{9} * a^3 * c^2 * x^9 * e + \frac{10}{7} * a^3 * c^2 * d * x^7 + a^4 * c * x^5 * e + \frac{5}{3} * a^4 * c * d * x^3 + a^5 * x * e - \frac{a^5 * d}{x}$

maple [A] time = 0.02, size = 122, normalized size = 0.88

$$\frac{c^5 e x^{21}}{21} + \frac{c^5 d x^{19}}{19} + \frac{5 a c^4 e x^{17}}{17} + \frac{a c^4 d x^{15}}{3} + \frac{10 a^2 c^3 e x^{13}}{13} + \frac{10 a^2 c^3 d x^{11}}{11} + \frac{10 a^3 c^2 e x^9}{9} + \frac{10 a^3 c^2 d x^7}{7} + a^4 c e x^5 + \frac{5 a^4 c d x^3}{3} + a^5 e x - \frac{a^5 d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(c*x^4+a)^5/x^2,x)`

[Out] $-a^5d/x + a^5e*x + 5/3*a^4*c*d*x^3 + a^4*c*e*x^5 + 10/7*a^3*c^2*d*x^7 + 10/9*a^3*c^2*e*x^9 + 10/11*a^2*c^3*d*x^11 + 10/13*a^2*c^3*e*x^13 + 1/3*a*c^4*d*x^15 + 5/17*a*c^4*e*x^17 + 1/19*c^5*d*x^19 + 1/21*c^5*e*x^21$

maxima [A] time = 0.52, size = 121, normalized size = 0.87

$$\frac{1}{21}c^5ex^{21} + \frac{1}{19}c^5dx^{19} + \frac{5}{17}ac^4ex^{17} + \frac{1}{3}ac^4dx^{15} + \frac{10}{13}a^2c^3ex^{13} + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{9}a^3c^2ex^9 + \frac{10}{7}a^3c^2dx^7 + a^4cex^5 + \frac{5}{3}a^4cdx^3 + a^5ex - \frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")`

[Out] $1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 + 10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x$

mupad [B] time = 0.09, size = 121, normalized size = 0.87

$$\frac{c^5dx^{19}}{19} - \frac{a^5d}{x} + \frac{c^5ex^{21}}{21} + a^5ex + \frac{10a^3c^2dx^7}{7} + \frac{10a^2c^3dx^{11}}{11} + \frac{10a^3c^2ex^9}{9} + \frac{10a^2c^3ex^{13}}{13} + \frac{5a^4cdx^3}{3} + \frac{ac^4dx^{15}}{3} + a^4cex^5 + \frac{5ac^4ex^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)`

[Out] $(c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^7)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)/13 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17)/17$

sympy [A] time = 0.24, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x} + a^5ex + \frac{5a^4cdx^3}{3} + a^4cex^5 + \frac{10a^3c^2dx^7}{7} + \frac{10a^2c^3dx^{11}}{11} + \frac{10a^3c^2ex^9}{9} + \frac{10a^2c^3ex^{13}}{13} + \frac{ac^4dx^{15}}{3} + \frac{5ac^4ex^{17}}{17} + \frac{c^5dx^{19}}{19} + \frac{c^5ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)`

[Out] $-a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x**13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*x**21/21$

$$3.7 \quad \int \frac{(d+ex^2)(a+cx^4)^5}{x^3} dx$$

Optimal. Leaf size=142

$$-\frac{a^5d}{2x^2} + a^5e \log(x) + \frac{5}{2}a^4cdx^2 + \frac{5}{4}a^4cex^4 + \frac{5}{3}a^3c^2dx^6 + \frac{5}{4}a^3c^2ex^8 + a^2c^3dx^{10} + \frac{5}{6}a^2c^3ex^{12} + \frac{5}{14}ac^4dx^{14} + \frac{5}{16}ac^4ex^{16} + \frac{1}{18}c^5dx^{18} + \frac{1}{20}c^5ex^{20}$$

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1252, 766}

$$a^2c^3dx^{10} + \frac{5}{3}a^3c^2dx^6 + \frac{5}{6}a^2c^3ex^{12} + \frac{5}{4}a^3c^2ex^8 + \frac{5}{2}a^4cdx^2 + \frac{5}{4}a^4cex^4 - \frac{a^5d}{2x^2} + a^5e \log(x) + \frac{5}{14}ac^4dx^{14} + \frac{5}{16}ac^4ex^{16} + \frac{1}{18}c^5dx^{18} + \frac{1}{20}c^5ex^{20}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]

[Out] $-(a^5d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^{10} + (5*a^2*c^3*e*x^{12})/6 + (5*a*c^4*d*x^{14})/14 + (5*a*c^4*e*x^{16})/16 + (c^5*d*x^{18})/18 + (c^5*e*x^{20})/20 + a^5*e*\text{Log}[x]$

Rule 766

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m+1)/2]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)(a+cx^4)^5}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)(a+cx^2)^5}{x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(5a^4cd + \frac{a^5d}{x^2} + \frac{a^5e}{x} + 5a^4cex + 10a^3c^2dx^2 + 10a^3c^2ex^3 + 10a^2c^3dx^4 + \dots \right) dx, x, x^2 \right) \\ &= -\frac{a^5d}{2x^2} + \frac{5}{2}a^4cdx^2 + \frac{5}{4}a^4cex^4 + \frac{5}{3}a^3c^2dx^6 + \frac{5}{4}a^3c^2ex^8 + a^2c^3dx^{10} + \frac{5}{6}a^2c^3ex^{12} + \frac{5}{14}ac^4dx^{14} + \frac{5}{16}ac^4ex^{16} + \frac{1}{18}c^5dx^{18} + \frac{1}{20}c^5ex^{20} + a^5e \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 142, normalized size = 1.00

$$-\frac{a^5 d}{2x^2} + a^5 e \log(x) + \frac{5}{2} a^4 c d x^2 + \frac{5}{4} a^4 c e x^4 + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^3 c^2 e x^8 + a^2 c^3 d x^{10} + \frac{5}{6} a^2 c^3 e x^{12} + \frac{5}{14} a c^4 d x^{14} + \frac{5}{16} a c^4 e x^{16} + \frac{1}{18} c^5 d x^{18} + \frac{1}{20} c^5 e x^{20}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]

[Out] $-\frac{1}{2} \frac{a^5 d}{x^2} + \frac{5 a^4 c d x^2}{2} + \frac{5 a^4 c e x^4}{4} + \frac{5 a^3 c^2 d x^6}{3} + \frac{5 a^3 c^2 e x^8}{4} + a^2 c^3 d x^{10} + \frac{5 a^2 c^3 e x^{12}}{6} + \frac{5 a c^4 d x^{14}}{14} + \frac{5 a c^4 e x^{16}}{16} + \frac{c^5 d x^{18}}{18} + \frac{c^5 e x^{20}}{20} + a^5 e \log[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(a + cx^4)^5}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(a + c*x^4)^5)/x^3, x]

fricas [A] time = 0.65, size = 129, normalized size = 0.91

$$\frac{252 c^5 e x^{22} + 280 c^5 d x^{20} + 1575 a c^4 e x^{18} + 1800 a c^4 d x^{16} + 4200 a^2 c^3 e x^{14} + 5040 a^2 c^3 d x^{12} + 6300 a^3 c^2 e x^{10} + 8400 a^3 c^2 d x^8 + 6300 a^4 c e x^6 + 12600 a^4 c d x^4 + 5040 a^5 e x^2 \log(x) - 2520 a^5 d}{5040 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")

[Out] $\frac{1}{5040} (252 c^5 e x^{22} + 280 c^5 d x^{20} + 1575 a c^4 e x^{18} + 1800 a c^4 d x^{16} + 4200 a^2 c^3 e x^{14} + 5040 a^2 c^3 d x^{12} + 6300 a^3 c^2 e x^{10} + 8400 a^3 c^2 d x^8 + 6300 a^4 c e x^6 + 12600 a^4 c d x^4 + 5040 a^5 e x^2 \log(x) - 2520 a^5 d) / x^2$

giac [A] time = 0.21, size = 142, normalized size = 1.00

$$\frac{1}{20} c^5 x^{20} e + \frac{1}{18} c^5 d x^{18} + \frac{5}{16} a c^4 x^{16} e + \frac{5}{14} a c^4 d x^{14} + \frac{5}{6} a^2 c^3 x^{12} e + a^2 c^3 d x^{10} + \frac{5}{4} a^3 c^2 x^8 e + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^4 c x^4 e + \frac{5}{2} a^4 c d x^2 + \frac{1}{2} a^5 e \log(x^2) - \frac{a^5 x^2 e + a^5 d}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")

[Out] $\frac{1}{20} c^5 x^{20} e + \frac{1}{18} c^5 d x^{18} + \frac{5}{16} a c^4 x^{16} e + \frac{5}{14} a c^4 d x^{14} + \frac{5}{6} a^2 c^3 x^{12} e + a^2 c^3 d x^{10} + \frac{5}{4} a^3 c^2 x^8 e + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^4 c x^4 e + \frac{5}{2} a^4 c d x^2 + \frac{1}{2} a^5 e \log(x^2) - \frac{a^5 x^2 e + a^5 d}{2 x^2}$

$$6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*x^2*e + a^5*d)/x^2$$

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{c^5 e x^{20}}{20} + \frac{c^5 d x^{18}}{18} + \frac{5 a c^4 e x^{16}}{16} + \frac{5 a c^4 d x^{14}}{14} + \frac{5 a^2 c^3 e x^{12}}{6} + a^2 c^3 d x^{10} + \frac{5 a^3 c^2 e x^8}{4} + \frac{5 a^3 c^2 d x^6}{3} + \frac{5 a^4 c e x^4}{4} + \frac{5 a^4 c d x^2}{2} + a^5 e \ln(x) - \frac{a^5 d}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x)

[Out] $-1/2*a^5*d/x^2 + 5/2*a^4*c*d*x^2 + 5/4*a^4*c*e*x^4 + 5/3*a^3*c^2*d*x^6 + 5/4*a^3*c^2*e*x^8 + a^2*c^3*d*x^{10} + 5/6*a^2*c^3*e*x^{12} + 5/14*a*c^4*d*x^{14} + 5/16*a*c^4*e*x^{16} + 1/18*c^5*d*x^{18} + 1/20*c^5*e*x^{20} + a^5*e*\ln(x)$

maxima [A] time = 0.50, size = 125, normalized size = 0.88

$$\frac{1}{20} c^5 e x^{20} + \frac{1}{18} c^5 d x^{18} + \frac{5}{16} a c^4 e x^{16} + \frac{5}{14} a c^4 d x^{14} + \frac{5}{6} a^2 c^3 e x^{12} + a^2 c^3 d x^{10} + \frac{5}{4} a^3 c^2 e x^8 + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^4 c e x^4 + \frac{5}{2} a^4 c d x^2 + \frac{1}{2} a^5 e \log(x^2) - \frac{a^5 d}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")

[Out] $1/20*c^5*e*x^{20} + 1/18*c^5*d*x^{18} + 5/16*a*c^4*e*x^{16} + 5/14*a*c^4*d*x^{14} + 5/6*a^2*c^3*e*x^{12} + a^2*c^3*d*x^{10} + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2$

mupad [B] time = 0.07, size = 122, normalized size = 0.86

$$\frac{c^5 d x^{18}}{18} - \frac{a^5 d}{2 x^2} + \frac{c^5 e x^{20}}{20} + a^5 e \ln(x) + \frac{5 a^3 c^2 d x^6}{3} + a^2 c^3 d x^{10} + \frac{5 a^3 c^2 e x^8}{4} + \frac{5 a^2 c^3 e x^{12}}{6} + \frac{5 a^4 c d x^2}{2} + \frac{5 a c^4 d x^{14}}{14} + \frac{5 a^4 c e x^4}{4} + \frac{5 a c^4 e x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)

[Out] $(c^5*d*x^{18})/18 - (a^5*d)/(2*x^2) + (c^5*e*x^{20})/20 + a^5*e*log(x) + (5*a^3*c^2*d*x^6)/3 + a^2*c^3*d*x^{10} + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^{12})/6 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^{14})/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^4*e*x^{16})/16$

sympy [A] time = 0.28, size = 150, normalized size = 1.06

$$-\frac{a^5 d}{2 x^2} + a^5 e \log(x) + \frac{5 a^4 c d x^2}{2} + \frac{5 a^4 c e x^4}{4} + \frac{5 a^3 c^2 d x^6}{3} + \frac{5 a^3 c^2 e x^8}{4} + a^2 c^3 d x^{10} + \frac{5 a^2 c^3 e x^{12}}{6} + \frac{5 a c^4 d x^{14}}{14} + \frac{5 a c^4 e x^{16}}{16} + \frac{c^5 d x^{18}}{18} + \frac{c^5 e x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +  
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*  
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18  
+ c**5*e*x**20/20
```

3.8 $\int x^5 (2 + 3x^2) \sqrt{5 + x^4} dx$

Optimal. Leaf size=67

$$\frac{3}{10} (x^4 + 5)^{3/2} x^4 - \frac{25}{8} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{5}{8} \sqrt{x^4 + 5} x^2 - \frac{1}{4} (4 - x^2) (x^4 + 5)^{3/2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{10} (x^4 + 5)^{3/2} x^4 - \frac{5}{8} \sqrt{x^4 + 5} x^2 - \frac{1}{4} (4 - x^2) (x^4 + 5)^{3/2} - \frac{25}{8} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]

[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)

), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int x^5 (2 + 3x^2) \sqrt{5 + x^4} dx &= \frac{1}{2} \text{Subst} \left(\int x^2 (2 + 3x) \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= \frac{3}{10} x^4 (5 + x^4)^{3/2} + \frac{1}{10} \text{Subst} \left(\int x(-30 + 10x) \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= \frac{3}{10} x^4 (5 + x^4)^{3/2} - \frac{1}{4} (4 - x^2) (5 + x^4)^{3/2} - \frac{5}{4} \text{Subst} \left(\int \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= -\frac{5}{8} x^2 \sqrt{5 + x^4} + \frac{3}{10} x^4 (5 + x^4)^{3/2} - \frac{1}{4} (4 - x^2) (5 + x^4)^{3/2} - \frac{25}{8} \text{Subst} \left(\int \frac{1}{\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= -\frac{5}{8} x^2 \sqrt{5 + x^4} + \frac{3}{10} x^4 (5 + x^4)^{3/2} - \frac{1}{4} (4 - x^2) (5 + x^4)^{3/2} - \frac{25}{8} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.75

$$\frac{1}{40} \sqrt{x^4 + 5} (12x^8 + 10x^6 + 20x^4 + 25x^2 - 200) - \frac{25}{8} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 - (25*ArcSinh[x^2/Sqrt[5]])/8

IntegrateAlgebraic [A] time = 0.13, size = 56, normalized size = 0.84

$$\frac{25}{8} \log \left(\sqrt{x^4 + 5} - x^2 \right) + \frac{1}{40} \sqrt{x^4 + 5} (12x^8 + 10x^6 + 20x^4 + 25x^2 - 200)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]

[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 + (25*Log[-x^2 + Sqrt[5 + x^4]])/8

fricas [A] time = 0.61, size = 48, normalized size = 0.72

$$\frac{1}{40} (12x^8 + 10x^6 + 20x^4 + 25x^2 - 200)\sqrt{x^4 + 5} + \frac{25}{8} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.23, size = 54, normalized size = 0.81

$$\frac{1}{8} (2x^4 + 5)\sqrt{x^4 + 5}x^2 + \frac{3}{10} (x^4 + 5)^{\frac{5}{2}} - \frac{5}{2} (x^4 + 5)^{\frac{3}{2}} + \frac{25}{8} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.05, size = 53, normalized size = 0.79

$$\frac{(x^4 + 5)^{\frac{3}{2}} x^2}{4} - \frac{5\sqrt{x^4 + 5} x^2}{8} - \frac{25 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{8} + \frac{(x^4 + 5)^{\frac{3}{2}} (3x^4 - 10)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x)

[Out] 1/10*(x^4+5)^(3/2)*(3*x^4-10)+1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-5/8*arcsinh(1/5*x^2*5^(1/2))

maxima [B] time = 1.01, size = 102, normalized size = 1.52

$$\frac{3}{10} (x^4 + 5)^{\frac{5}{2}} - \frac{5}{2} (x^4 + 5)^{\frac{3}{2}} - \frac{25 \left(\frac{\sqrt{x^4 + 5}}{x^2} + \frac{(x^4 + 5)^{\frac{3}{2}}}{x^6} \right)}{8 \left(\frac{2(x^4 + 5)}{x^4} - \frac{(x^4 + 5)^2}{x^8} - 1 \right)} - \frac{25}{16} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) + \frac{25}{16} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")`

[Out] $3/10*(x^4 + 5)^{(5/2)} - 5/2*(x^4 + 5)^{(3/2)} - 25/8*(\sqrt{x^4 + 5})/x^2 + (x^4 + 5)^{(3/2)}/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*\log(\sqrt{x^4 + 5}/x^2 + 1) + 25/16*\log(\sqrt{x^4 + 5}/x^2 - 1)$

mupad [B] time = 0.37, size = 42, normalized size = 0.63

$$\sqrt{x^4 + 5} \left(\frac{3x^8}{10} + \frac{x^6}{4} + \frac{x^4}{2} + \frac{5x^2}{8} - 5 \right) - \frac{25 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)`

[Out] $(x^4 + 5)^{(1/2)}*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*\operatorname{asinh}((5^{1/2}*x^2)/5))/8$

sympy [A] time = 6.01, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^4 + 5}} + \frac{3x^8\sqrt{x^4 + 5}}{10} + \frac{15x^6}{8\sqrt{x^4 + 5}} + \frac{x^4\sqrt{x^4 + 5}}{2} + \frac{25x^2}{8\sqrt{x^4 + 5}} - 5\sqrt{x^4 + 5} - \frac{25 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)`

[Out] $x^{10}/(4*\sqrt{x^{4} + 5}) + 3*x^{8}*\sqrt{x^{4} + 5}/10 + 15*x^{6}/(8*\sqrt{x^{4} + 5}) + x^{4}*\sqrt{x^{4} + 5}/2 + 25*x^{2}/(8*\sqrt{x^{4} + 5}) - 5*\sqrt{x^{4} + 5} - 25*\operatorname{asinh}(\sqrt{5}*x^{2}/5)/8$

3.9 $\int x^3 (2 + 3x^2) \sqrt{5 + x^4} dx$

Optimal. Leaf size=51

$$-\frac{75}{16} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{16} \sqrt{x^4 + 5} x^2 + \frac{1}{24} (9x^2 + 8) (x^4 + 5)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 780, 195, 215}

$$-\frac{15}{16} \sqrt{x^4 + 5} x^2 + \frac{1}{24} (9x^2 + 8) (x^4 + 5)^{3/2} - \frac{75}{16} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]

[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh[x^2/Sqrt[5]])/16

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m + 1)/2]$

Rubi steps

$$\begin{aligned} \int x^3 (2 + 3x^2) \sqrt{5 + x^4} dx &= \frac{1}{2} \text{Subst} \left(\int x(2 + 3x) \sqrt{5 + x^2} dx, x, x^2 \right) \\ &= \frac{1}{24} (8 + 9x^2) (5 + x^4)^{3/2} - \frac{15}{8} \text{Subst} \left(\int \sqrt{5 + x^2} dx, x, x^2 \right) \\ &= -\frac{15}{16} x^2 \sqrt{5 + x^4} + \frac{1}{24} (8 + 9x^2) (5 + x^4)^{3/2} - \frac{75}{16} \text{Subst} \left(\int \frac{1}{\sqrt{5 + x^2}} dx, x, x^2 \right) \\ &= -\frac{15}{16} x^2 \sqrt{5 + x^4} + \frac{1}{24} (8 + 9x^2) (5 + x^4)^{3/2} - \frac{75}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.86

$$\frac{1}{48} \left(\sqrt{x^4 + 5} (18x^6 + 16x^4 + 45x^2 + 80) - 225 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6) - 225*ArcSinh[x^2/Sqrt[5]])/48

IntegrateAlgebraic [A] time = 0.11, size = 51, normalized size = 1.00

$$\frac{75}{16} \log \left(\sqrt{x^4 + 5} - x^2 \right) + \frac{1}{48} \sqrt{x^4 + 5} (18x^6 + 16x^4 + 45x^2 + 80)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6))/48 + (75*Log[-x^2 + Sqrt[5 + x^4]])/16

fricas [A] time = 0.75, size = 43, normalized size = 0.84

$$\frac{1}{48} (18x^6 + 16x^4 + 45x^2 + 80) \sqrt{x^4 + 5} + \frac{75}{16} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.23, size = 45, normalized size = 0.88

$$\frac{3}{16} (2x^4 + 5)\sqrt{x^4 + 5}x^2 + \frac{1}{3} (x^4 + 5)^{\frac{3}{2}} + \frac{75}{16} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")

[Out] 3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{3(x^4 + 5)^{\frac{3}{2}}x^2}{8} - \frac{15\sqrt{x^4 + 5}x^2}{16} - \frac{75 \operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16} + \frac{(x^4 + 5)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x)

[Out] 3/8*(x^4+5)^(3/2)*x^2-15/16*(x^4+5)^(1/2)*x^2-75/16*arcsinh(1/5*5^(1/2)*x^2)+1/3*(x^4+5)^(3/2)

maxima [B] time = 1.02, size = 93, normalized size = 1.82

$$\frac{1}{3} (x^4 + 5)^{\frac{3}{2}} - \frac{75 \left(\frac{\sqrt{x^4 + 5}}{x^2} + \frac{(x^4 + 5)^{\frac{3}{2}}}{x^6} \right)}{16 \left(\frac{2(x^4 + 5)}{x^4} - \frac{(x^4 + 5)^2}{x^8} - 1 \right)} - \frac{75}{32} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} + 1 \right) + \frac{75}{32} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")

[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/32*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.39, size = 37, normalized size = 0.73

$$\sqrt{x^4 + 5} \left(\frac{3x^6}{8} + \frac{x^4}{3} + \frac{15x^2}{16} + \frac{5}{3} \right) - \frac{75 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)`

[Out] $(x^4 + 5)^{(1/2)} * ((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*\operatorname{asinh}((5^{(1/2)})*x^2)/5))/16$

sympy [A] time = 4.31, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^4 + 5}} + \frac{45x^6}{16\sqrt{x^4 + 5}} + \frac{75x^2}{16\sqrt{x^4 + 5}} + \frac{(x^4 + 5)^{3/2}}{3} - \frac{75 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2), x)`

[Out] $3*x^{10}/(8*\sqrt{x^4 + 5}) + 45*x^6/(16*\sqrt{x^4 + 5}) + 75*x^2/(16*\sqrt{x^4 + 5}) + (x^4 + 5)**(3/2)/3 - 75*\operatorname{asinh}(\sqrt{5}*x^2/5)/16$

3.10 $\int x(2 + 3x^2) \sqrt{5 + x^4} dx$

Optimal. Leaf size=44

$$\frac{1}{2}(x^4 + 5)^{3/2} + \frac{5}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\sqrt{x^4 + 5}x^2$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1248, 641, 195, 215}

$$\frac{1}{2}\sqrt{x^4 + 5}x^2 + \frac{1}{2}(x^4 + 5)^{3/2} + \frac{5}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int x(2+3x^2)\sqrt{5+x^4} dx &= \frac{1}{2} \text{Subst}\left(\int(2+3x)\sqrt{5+x^2} dx, x, x^2\right) \\
&= \frac{1}{2}(5+x^4)^{3/2} + \text{Subst}\left(\int\sqrt{5+x^2} dx, x, x^2\right) \\
&= \frac{1}{2}x^2\sqrt{5+x^4} + \frac{1}{2}(5+x^4)^{3/2} + \frac{5}{2} \text{Subst}\left(\int\frac{1}{\sqrt{5+x^2}} dx, x, x^2\right) \\
&= \frac{1}{2}x^2\sqrt{5+x^4} + \frac{1}{2}(5+x^4)^{3/2} + \frac{5}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{5}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2} \sqrt{x^4+5} (x^4+x^2+5)$$

Antiderivative was successfully verified.

[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcSinh[x^2/Sqrt[5]])/2

IntegrateAlgebraic [A] time = 0.11, size = 42, normalized size = 0.95

$$\frac{1}{2} \sqrt{x^4+5} (x^4+x^2+5) - \frac{5}{2} \log(\sqrt{x^4+5} - x^2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(2 + 3*x^2)*Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 - (5*Log[-x^2 + Sqrt[5 + x^4]])/2

fricas [A] time = 0.64, size = 34, normalized size = 0.77

$$\frac{1}{2} (x^4+x^2+5) \sqrt{x^4+5} - \frac{5}{2} \log(-x^2 + \sqrt{x^4+5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2), x, algorithm="fricas")

[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.20, size = 38, normalized size = 0.86

$$\frac{1}{2} \sqrt{x^4 + 5} x^2 + \frac{1}{2} (x^4 + 5)^{\frac{3}{2}} - \frac{5}{2} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{x^4 + 5} x^2}{2} + \frac{5 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} + \frac{(x^4 + 5)^{\frac{3}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x)

[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*5^(1/2)*x^2)+1/2*(x^4+5)^(1/2)*x^2

maxima [B] time = 1.32, size = 67, normalized size = 1.52

$$\frac{1}{2} (x^4 + 5)^{\frac{3}{2}} + \frac{5 \sqrt{x^4 + 5}}{2 x^2 \left(\frac{x^4 + 5}{x^4} - 1\right)} + \frac{5}{4} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) - \frac{5}{4} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")

[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.14, size = 32, normalized size = 0.73

$$\frac{5 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} + \sqrt{x^4 + 5} \left(\frac{x^4}{2} + \frac{x^2}{2} + \frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)

[Out] (5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)

sympy [A] time = 3.08, size = 53, normalized size = 1.20

$$\frac{x^6}{2\sqrt{x^4+5}} + \frac{5x^2}{2\sqrt{x^4+5}} + \frac{(x^4+5)^{\frac{3}{2}}}{2} + \frac{5 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)

[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 + 5*asinh(sqrt(5)*x**2/5)/2

$$3.11 \quad \int \frac{(2+3x^2)\sqrt{5+x^4}}{x} dx$$

Optimal. Leaf size=58

$$-\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{15}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{4}\sqrt{x^4+5}(3x^2+4)$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{4}\sqrt{x^4+5}(3x^2+4) + \frac{15}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 815

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p]/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{5+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} + \frac{1}{4} \text{Subst} \left(\int \frac{20+15x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} + \frac{15}{4} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) + 5 \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} + \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{5}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x}} dx, x, x^4 \right) \\
&= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} + \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + 5 \text{Subst} \left(\int \frac{1}{-5+x^2} dx, x, \sqrt{5+x^4} \right) \\
&= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} + \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \sqrt{5} \tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.98

$$\frac{1}{4} \left(-4\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4+5}}{\sqrt{5}} \right) + 15 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \sqrt{x^4+5} (3x^2+4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 15*ArcSinh[x^2/Sqrt[5]] - 4*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4

IntegrateAlgebraic [A] time = 0.16, size = 75, normalized size = 1.29

$$\frac{1}{4} \sqrt{x^4+5} (3x^2+4) - \frac{15}{4} \log \left(\sqrt{x^4+5} - x^2 \right) + 2\sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + 2*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (15*Log[-x^2 + Sqrt[5 + x^4]])/4

fricas [A] time = 0.63, size = 56, normalized size = 0.97

$$\frac{1}{4} \sqrt{x^4+5} (3x^2+4) + \sqrt{5} \log \left(-\frac{\sqrt{5} - \sqrt{x^4+5}}{x^2} \right) - \frac{15}{4} \log \left(-x^2 + \sqrt{x^4+5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")

[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15/4*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.21, size = 76, normalized size = 1.31

$$\frac{1}{4} \sqrt{x^4 + 5} (3x^2 + 4) + \sqrt{5} \log \left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4 + 5}}{x^2 - \sqrt{5} - \sqrt{x^4 + 5}} \right) - \frac{15}{4} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")

[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{3\sqrt{x^4 + 5} x^2}{4} + \frac{15 \operatorname{arcsinh} \left(\frac{\sqrt{5} x^2}{5} \right)}{4} - \sqrt{5} \operatorname{arctanh} \left(\frac{\sqrt{5}}{\sqrt{x^4 + 5}} \right) + \sqrt{x^4 + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x)

[Out] 3/4*(x^4+5)^(1/2)*x^2+15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))

maxima [B] time = 1.56, size = 99, normalized size = 1.71

$$\frac{1}{2} \sqrt{5} \log \left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{\sqrt{5} + \sqrt{x^4 + 5}} \right) + \sqrt{x^4 + 5} + \frac{15 \sqrt{x^4 + 5}}{4x^2 \left(\frac{x^4 + 5}{x^4} - 1 \right)} + \frac{15}{8} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} + 1 \right) - \frac{15}{8} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")

[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.15, size = 45, normalized size = 0.78

$$\frac{15 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} - \sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right) + \sqrt{x^4+5} \left(\frac{3x^2}{4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)`

[Out] `(15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5) + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)`

sympy [A] time = 15.59, size = 83, normalized size = 1.43

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} + \frac{\sqrt{5} \log(x^4)}{2} - \sqrt{5} \log\left(\sqrt{\frac{x^4}{5}+1} + 1\right) + \frac{15 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)`

[Out] `3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + sqrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 15*asinh(sqrt(5)*x**2/5)/4`

$$3.12 \quad \int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3} dx$$

Optimal. Leaf size=59

$$-\frac{3}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\sqrt{x^4+5}(2-3x^2)}{2x^2}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}(2-3x^2)}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3, x]

[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
) + e*g*(m + 1)*x*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{-30-4x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \frac{15}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) + \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{15}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x}} dx, x, x^4 \right) \\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{15}{2} \text{Subst} \left(\int \frac{1}{-5+x^2} dx, x, \sqrt{5+x^4} \right) \\
&= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{3}{2} \sqrt{5} \tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 1.00

$$\sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{1}{2} \left(\frac{(3x^2-2)\sqrt{x^4+5}}{x^2} - 3\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4+5}}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3, x]

[Out] ArcSinh[x^2/Sqrt[5]] + (((-2 + 3*x^2)*Sqrt[5 + x^4])/x^2 - 3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2

IntegrateAlgebraic [A] time = 0.17, size = 76, normalized size = 1.29

$$\frac{\sqrt{x^4+5}(3x^2-2)}{2x^2} - \log(\sqrt{x^4+5} - x^2) + 3\sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3, x]

[Out] (((-2 + 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + 3*Sqrt[5]*ArcTanh[x^2/Sqrt[5]] - Sqrt[5 + x^4]/Sqrt[5]) - Log[-x^2 + Sqrt[5 + x^4]]

fricas [A] time = 0.66, size = 72, normalized size = 1.22

$$\frac{3\sqrt{5}x^2 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2x^2 \log\left(-x^2 + \sqrt{x^4+5}\right) - 2x^2 + \sqrt{x^4+5}(3x^2 - 2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")

[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + sqrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2

giac [A] time = 0.25, size = 91, normalized size = 1.54

$$\frac{3}{2}\sqrt{5} \log\left(\frac{x^2 + \sqrt{5} - \sqrt{x^4+5}}{x^2 - \sqrt{5} - \sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} + \frac{10}{(x^2 - \sqrt{x^4+5})^2 - 5} - \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")

[Out] 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 61, normalized size = 1.03

$$\frac{\sqrt{x^4+5} x^2}{5} + \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right) - \frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2} - \frac{(x^4+5)^{\frac{3}{2}}}{5x^2} + \frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x)

[Out] 3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5/x^2*(x^4+5)^(3/2)+1/5*(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.41, size = 88, normalized size = 1.49

$$\frac{3}{4}\sqrt{5} \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} + \frac{1}{2} \log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) - \frac{1}{2} \log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")

[Out] $\frac{3}{4}\sqrt{5}\log\left(\frac{-\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2+1}\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2-1}\right)$

mupad [B] time = 0.79, size = 51, normalized size = 0.86

$$\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{x^4+5}}{x^2} + \frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}i}{5}\right)3i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)

[Out] $\operatorname{asinh}\left(\frac{5^{1/2}x^2}{5}\right) + \frac{5^{1/2}\operatorname{atan}\left(\frac{5^{1/2}(x^4+5)^{1/2}i}{5}\right)3i}{2} + \frac{3(x^4+5)^{1/2}}{2} - \frac{(x^4+5)^{1/2}}{x^2}$

sympy [A] time = 7.46, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}} + \frac{3\sqrt{x^4+5}}{2} + \frac{3\sqrt{5}\log(x^4)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)

[Out] $-x^2/\sqrt{x^4+5} + 3\sqrt{x^4+5}/2 + 3\sqrt{5}\log(x^4)/4 - 3\sqrt{5}\log(\sqrt{x^4/5+1}+1)/2 + \operatorname{asinh}(\sqrt{5}x^2/5) - 5/(x^2\sqrt{x^4+5})$

$$3.13 \quad \int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} dx$$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}} + \frac{3}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\sqrt{x^4+5}(3x^2+1)}{2x^4}$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1252, 811, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}(3x^2+1)}{2x^4} + \frac{3}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]

[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
2*c*d*p*(e*f - d*g)*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
+ 2*p + 2)) - 2*a*e^2*g*(m + 1)]*x, x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
&& !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3} dx, x, x^2 \right) \\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} - \frac{1}{40} \text{Subst} \left(\int \frac{-20-60x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x}} dx, x, x^4 \right) \\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-5+x^2} dx, x, \sqrt{5+x^4} \right) \\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.94

$$\frac{1}{10} \left(-\sqrt{5} \tanh^{-1} \left(\sqrt{\frac{x^4}{5} + 1} \right) + 15 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{5\sqrt{x^4+5} (3x^2+1)}{x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]

[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + 15*ArcSinh[x^2/Sqrt[5]] - Sqrt[5]*ArcTanh[Sqrt[1 + x^4/5]])/10

IntegrateAlgebraic [A] time = 0.26, size = 77, normalized size = 1.22

$$\frac{\sqrt{x^4+5} (-3x^2-1)}{2x^4} - \frac{3}{2} \log \left(\sqrt{x^4+5} - x^2 \right) + \frac{\tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]

[Out] ((-1 - 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 + x^4]])/2

fricas [A] time = 0.61, size = 72, normalized size = 1.14

$$\frac{\sqrt{5}x^4 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 15x^4 \log\left(-x^2 + \sqrt{x^4+5}\right) - 15x^4 - 5\sqrt{x^4+5}(3x^2+1)}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")

[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + sqrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4

giac [B] time = 0.24, size = 129, normalized size = 2.05

$$\frac{1}{10}\sqrt{5}\log\left(\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{(x^2-\sqrt{x^4+5})^3 + 15(x^2-\sqrt{x^4+5})^2 + 5x^2 - 5\sqrt{x^4+5} - 75}{((x^2-\sqrt{x^4+5})^2 - 5)^2} - \frac{3}{2}\log(-x^2 + \sqrt{x^4+5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")

[Out] 1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 75, normalized size = 1.19

$$\frac{3\sqrt{x^4+5}x^2}{10} + \frac{3\operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10} - \frac{3(x^4+5)^{\frac{3}{2}}}{10x^2} - \frac{(x^4+5)^{\frac{3}{2}}}{10x^4} + \frac{\sqrt{x^4+5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x)

[Out] -1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/10*(x^4+5)^(3/2)/x^2+3/10*(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.24, size = 91, normalized size = 1.44

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{2x^2} - \frac{\sqrt{x^4+5}}{2x^4} + \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) - \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")

[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.42, size = 56, normalized size = 0.89

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} - \frac{3 \sqrt{x^4 + 5}}{2 x^2} - \frac{\sqrt{x^4 + 5}}{2 x^4} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5} \sqrt{x^4 + 5} i}{5}\right) i}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)

[Out] (3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)

sympy [A] time = 6.04, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4 + 5}} - \frac{\sqrt{5} \operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10} + \frac{3 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} - \frac{\sqrt{1 + \frac{5}{x^4}}}{2x^2} - \frac{15}{2x^2\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)

[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))

$$3.14 \quad \int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7} dx$$

Optimal. Leaf size=58

$$\frac{3\sqrt{x^4+5}}{4x^4} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}} - \frac{(x^4+5)^{3/2}}{15x^6}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 807, 266, 47, 63, 207}

$$-\frac{(x^4+5)^{3/2}}{15x^6} - \frac{3\sqrt{x^4+5}}{4x^4} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7, x]

[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(4*Sqrt[5])

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 807

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)) / (2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x^2)\sqrt{5 + x^4}}{x^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2 + 3x)\sqrt{5 + x^2}}{x^4} dx, x, x^2 \right) \\
 &= -\frac{(5 + x^4)^{3/2}}{15x^6} + \frac{3}{2} \text{Subst} \left(\int \frac{\sqrt{5 + x^2}}{x^3} dx, x, x^2 \right) \\
 &= -\frac{(5 + x^4)^{3/2}}{15x^6} + \frac{3}{4} \text{Subst} \left(\int \frac{\sqrt{5 + x}}{x^2} dx, x, x^4 \right) \\
 &= -\frac{3\sqrt{5 + x^4}}{4x^4} - \frac{(5 + x^4)^{3/2}}{15x^6} + \frac{3}{8} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x}} dx, x, x^4 \right) \\
 &= -\frac{3\sqrt{5 + x^4}}{4x^4} - \frac{(5 + x^4)^{3/2}}{15x^6} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right) \\
 &= -\frac{3\sqrt{5 + x^4}}{4x^4} - \frac{(5 + x^4)^{3/2}}{15x^6} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{4\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.24

$$\frac{3 \left(5x^4 + \sqrt{5} \sqrt{x^4 + 5} x^4 \tanh^{-1} \left(\sqrt{\frac{x^4}{5} + 1} \right) + 25 \right)}{20x^4 \sqrt{x^4 + 5}} - \frac{(x^4 + 5)^{3/2}}{15x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7, x]

[Out] -1/15*(5 + x^4)^(3/2)/x^6 - (3*(25 + 5*x^4 + Sqrt[5]*x^4*Sqrt[5 + x^4]*ArcTanh[Sqrt[1 + x^4/5]]))/(20*x^4*Sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.25, size = 65, normalized size = 1.12

$$\frac{3 \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right)}{2\sqrt{5}} + \frac{\sqrt{x^4+5} (-4x^4 - 45x^2 - 20)}{60x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7, x]

[Out] ((-20 - 45*x^2 - 4*x^4)*Sqrt[5 + x^4])/(60*x^6) + (3*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])

fricas [A] time = 0.73, size = 59, normalized size = 1.02

$$\frac{9\sqrt{5}x^6 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 4x^6 - (4x^4 + 45x^2 + 20)\sqrt{x^4+5}}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7, x, algorithm="fricas")

[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 + 45*x^2 + 20)*sqrt(x^4 + 5))/x^6

giac [B] time = 0.23, size = 116, normalized size = 2.00

$$\frac{3}{20} \sqrt{5} \log\left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4 + 5}}{x^2 - \sqrt{5} - \sqrt{x^4 + 5}}\right) + \frac{9(x^2 - \sqrt{x^4 + 5})^5 + 12(x^2 - \sqrt{x^4 + 5})^4 - 225x^2 + 225\sqrt{x^4 + 5} + 100}{6((x^2 - \sqrt{x^4 + 5})^2 - 5)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")

[Out] $\frac{3\sqrt{5}\log(-x^2 + \sqrt{5} - \sqrt{x^4 + 5})}{(x^2 - \sqrt{5} - \sqrt{x^4 + 5})} + \frac{1}{6}(9(x^2 - \sqrt{x^4 + 5})^5 + 12(x^2 - \sqrt{x^4 + 5})^4 - 225x^2 + 225\sqrt{x^4 + 5} + 100)/((x^2 - \sqrt{x^4 + 5})^2 - 5)^3$

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20} - \frac{3(x^4+5)^{\frac{3}{2}}}{20x^4} - \frac{(x^4+5)^{\frac{3}{2}}}{15x^6} + \frac{3\sqrt{x^4+5}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x)

[Out] $-1/15*(x^4+5)^{(3/2)}/x^6-3/20*(x^4+5)^{(3/2)}/x^4+3/20*(x^4+5)^{(1/2)}-3/20*5^{(1/2)}*\operatorname{arctanh}(5^{(1/2)}/(x^4+5)^{(1/2)})$

maxima [A] time = 1.61, size = 59, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{(x^4+5)^{\frac{3}{2}}}{15x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")

[Out] $\frac{3}{40}\sqrt{5}\log(-(\sqrt{5}-\sqrt{x^4+5})/(\sqrt{5}+\sqrt{x^4+5}))-\frac{3}{4}\sqrt{x^4+5}/x^4-\frac{1}{15}(x^4+5)^{(3/2)}/x^6$

mupad [B] time = 0.68, size = 43, normalized size = 0.74

$$-\frac{3\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{20} - \frac{3\sqrt{x^4+5}}{4x^4} - \frac{(x^4+5)^{3/2}}{15x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)

[Out] $-\frac{(3*5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*(x^4+5)^{(1/2)})/5))/20-(3*(x^4+5)^{(1/2)})/(4*x^4)-(x^4+5)^{(3/2)/(15*x^6)}$

sympy [A] time = 5.96, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15} - \frac{3\sqrt{5} \operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20} - \frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2} - \frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

$$3.15 \quad \int x^5 (2 + 3x^2) (5 + x^4)^{3/2} dx$$

Optimal. Leaf size=83

$$\frac{3}{14} (x^4 + 5)^{5/2} x^4 - \frac{125}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{5}{24} (x^4 + 5)^{3/2} x^2 - \frac{25}{16} \sqrt{x^4 + 5} x^2 - \frac{1}{42} (18 - 7x^2) (x^4 + 5)^{5/2}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{14} (x^4 + 5)^{5/2} x^4 - \frac{5}{24} (x^4 + 5)^{3/2} x^2 - \frac{25}{16} \sqrt{x^4 + 5} x^2 - \frac{1}{42} (18 - 7x^2) (x^4 + 5)^{5/2} - \frac{125}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2), x]

[Out] (-25*x^2*sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/16

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int x^5 (2 + 3x^2) (5 + x^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left(\int x^2 (2 + 3x) (5 + x^2)^{3/2} dx, x, x^2 \right) \\
 &= \frac{3}{14} x^4 (5 + x^4)^{5/2} + \frac{1}{14} \text{Subst} \left(\int x(-30 + 14x) (5 + x^2)^{3/2} dx, x, x^2 \right) \\
 &= \frac{3}{14} x^4 (5 + x^4)^{5/2} - \frac{1}{42} (18 - 7x^2) (5 + x^4)^{5/2} - \frac{5}{6} \text{Subst} \left(\int (5 + x^2)^{3/2} dx, x, x^2 \right) \\
 &= -\frac{5}{24} x^2 (5 + x^4)^{3/2} + \frac{3}{14} x^4 (5 + x^4)^{5/2} - \frac{1}{42} (18 - 7x^2) (5 + x^4)^{5/2} - \frac{25}{8} \text{Subst} \left(\int \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= -\frac{25}{16} x^2 \sqrt{5 + x^4} - \frac{5}{24} x^2 (5 + x^4)^{3/2} + \frac{3}{14} x^4 (5 + x^4)^{5/2} - \frac{1}{42} (18 - 7x^2) (5 + x^4)^{5/2} \\
 &= -\frac{25}{16} x^2 \sqrt{5 + x^4} - \frac{5}{24} x^2 (5 + x^4)^{3/2} + \frac{3}{14} x^4 (5 + x^4)^{5/2} - \frac{1}{42} (18 - 7x^2) (5 + x^4)^{5/2}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.87

$$\frac{3}{14} (x^4 - 2) (x^4 + 5)^{5/2} + \frac{1}{6} x^2 (x^4 + 5)^{5/2} - \frac{5}{48} \left(75 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \sqrt{x^4 + 5} (2x^4 + 25) x^2 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2), x]
```

```
[Out] (x^2*(5 + x^4)^(5/2))/6 + (3*(-2 + x^4)*(5 + x^4)^(5/2))/14 - (5*(x^2*Sqrt[5 + x^4]*(25 + 2*x^4) + 75*ArcSinh[x^2/Sqrt[5]]))/48
```

IntegrateAlgebraic [A] time = 0.18, size = 66, normalized size = 0.80

$$\frac{125}{16} \log\left(\sqrt{x^4+5} - x^2\right) + \frac{1}{336} \sqrt{x^4+5} (72x^{12} + 56x^{10} + 576x^8 + 490x^6 + 360x^4 + 525x^2 - 3600)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]

[Out] (Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 56*x^10 + 72*x^12))/336 + (125*Log[-x^2 + Sqrt[5 + x^4]])/16

fricas [A] time = 0.63, size = 58, normalized size = 0.70

$$\frac{1}{336} (72x^{12} + 56x^{10} + 576x^8 + 490x^6 + 360x^4 + 525x^2 - 3600) \sqrt{x^4+5} + \frac{125}{16} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")

[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sqrt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.25, size = 80, normalized size = 0.96

$$\frac{3}{14} (x^4+5)^{\frac{7}{2}} + \frac{1}{48} (2(4x^4+5)x^4 - 75) \sqrt{x^4+5} x^2 + \frac{5}{8} (2x^4+5) \sqrt{x^4+5} x^2 - \frac{3}{2} (x^4+5)^{\frac{5}{2}} + \frac{125}{16} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")

[Out] 3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 73, normalized size = 0.88

$$\frac{\sqrt{x^4+5} x^{10}}{6} + \frac{35\sqrt{x^4+5} x^6}{24} + \frac{25\sqrt{x^4+5} x^2}{16} - \frac{125 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16} + \frac{3\sqrt{x^4+5} (x^4-2)(x^8+10x^4+25)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x)

[Out] 3/14*(x^4+5)^(1/2)*(x^4-2)*(x^8+10*x^4+25)+1/6*x^10*(x^4+5)^(1/2)+35/24*x^6*(x^4+5)^(1/2)+25/16*(x^4+5)^(1/2)*x^2-125/16*arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.35, size = 127, normalized size = 1.53

$$\frac{3}{14} (x^4 + 5)^{\frac{7}{2}} - \frac{3}{2} (x^4 + 5)^{\frac{5}{2}} - \frac{125 \left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{8(x^4+5)^{\frac{3}{2}}}{x^6} - \frac{3(x^4+5)^{\frac{5}{2}}}{x^{10}} \right)}{48 \left(\frac{3(x^4+5)}{x^4} - \frac{3(x^4+5)^2}{x^8} + \frac{(x^4+5)^3}{x^{12}} - 1 \right)} - \frac{125}{32} \log \left(\frac{\sqrt{x^4+5}}{x^2} + 1 \right) + \frac{125}{32} \log \left(\frac{\sqrt{x^4+5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")

[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125/32*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.32, size = 52, normalized size = 0.63

$$\sqrt{x^4 + 5} \left(\frac{3x^{12}}{14} + \frac{x^{10}}{6} + \frac{12x^8}{7} + \frac{35x^6}{24} + \frac{15x^4}{14} + \frac{25x^2}{16} - \frac{75}{7} \right) - \frac{125 \operatorname{asinh} \left(\frac{\sqrt{5}x^2}{5} \right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)

[Out] (x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^10/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16

sympy [A] time = 14.31, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^4+5}} + \frac{3x^{12}\sqrt{x^4+5}}{14} + \frac{55x^{10}}{24\sqrt{x^4+5}} + \frac{12x^8\sqrt{x^4+5}}{7} + \frac{425x^6}{48\sqrt{x^4+5}} + \frac{15x^4\sqrt{x^4+5}}{14} + \frac{125x^2}{16\sqrt{x^4+5}} - \frac{75\sqrt{x^4+5}}{7} - \frac{125 \operatorname{asinh} \left(\frac{\sqrt{5}x^2}{5} \right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)

[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(24*sqrt(x**4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt(x**4 + 5)) + 15*x**4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4 + 5)) - 75*sqrt(x**4 + 5)/7 - 125*asinh(sqrt(5)*x**2/5)/16

$$3.16 \quad \int x^3 (2 + 3x^2) (5 + x^4)^{3/2} dx$$

Optimal. Leaf size=67

$$-\frac{375}{32} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{1}{20} (5x^2 + 4) (x^4 + 5)^{5/2} - \frac{5}{16} x^2 (x^4 + 5)^{3/2} - \frac{75}{32} x^2 \sqrt{x^4 + 5}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 780, 195, 215}

$$\frac{1}{20} (5x^2 + 4) (x^4 + 5)^{5/2} - \frac{5}{16} x^2 (x^4 + 5)^{3/2} - \frac{75}{32} x^2 \sqrt{x^4 + 5} - \frac{375}{32} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]

[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 + x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1)/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int x^3 (2 + 3x^2) (5 + x^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left(\int x(2 + 3x) (5 + x^2)^{3/2} dx, x, x^2 \right) \\
 &= \frac{1}{20} (4 + 5x^2) (5 + x^4)^{5/2} - \frac{5}{4} \text{Subst} \left(\int (5 + x^2)^{3/2} dx, x, x^2 \right) \\
 &= -\frac{5}{16} x^2 (5 + x^4)^{3/2} + \frac{1}{20} (4 + 5x^2) (5 + x^4)^{5/2} - \frac{75}{16} \text{Subst} \left(\int \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= -\frac{75}{32} x^2 \sqrt{5 + x^4} - \frac{5}{16} x^2 (5 + x^4)^{3/2} + \frac{1}{20} (4 + 5x^2) (5 + x^4)^{5/2} - \frac{375}{32} \text{Subst} \left(\int \sqrt{5 + x^2} dx, x, x^2 \right) \\
 &= -\frac{75}{32} x^2 \sqrt{5 + x^4} - \frac{5}{16} x^2 (5 + x^4)^{3/2} + \frac{1}{20} (4 + 5x^2) (5 + x^4)^{5/2} - \frac{375}{32} \sinh^{-1} \left(\frac{x}{\sqrt{5}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.81

$$\frac{1}{160} \left(\sqrt{x^4 + 5} (40x^{10} + 32x^8 + 350x^6 + 320x^4 + 375x^2 + 800) - 1875 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2), x]

[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10) - 1875*ArcSinh[x^2/Sqrt[5]])/160

IntegrateAlgebraic [A] time = 0.17, size = 61, normalized size = 0.91

$$\frac{375}{32} \log \left(\sqrt{x^4 + 5} - x^2 \right) + \frac{1}{160} \sqrt{x^4 + 5} (40x^{10} + 32x^8 + 350x^6 + 320x^4 + 375x^2 + 800)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2), x]

[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10))/160 + (375*Log[-x^2 + Sqrt[5 + x^4]])/32

fricas [A] time = 0.65, size = 53, normalized size = 0.79

$$\frac{1}{160} (40x^{10} + 32x^8 + 350x^6 + 320x^4 + 375x^2 + 800)\sqrt{x^4 + 5} + \frac{375}{32} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")

[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5) + 375/32*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.20, size = 71, normalized size = 1.06

$$\frac{1}{32} (2(4x^4 + 5)x^4 - 75)\sqrt{x^4 + 5}x^2 + \frac{15}{16} (2x^4 + 5)\sqrt{x^4 + 5}x^2 + \frac{1}{5} (x^4 + 5)^{\frac{5}{2}} + \frac{375}{32} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")

[Out] 1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$\frac{\sqrt{x^4 + 5} x^{10}}{4} + \frac{35\sqrt{x^4 + 5} x^6}{16} + \frac{75\sqrt{x^4 + 5} x^2}{32} - \frac{375 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{32} + \frac{(x^4 + 5)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x)

[Out] 1/4*(x^4+5)^(1/2)*x^10+35/16*(x^4+5)^(1/2)*x^6+75/32*(x^4+5)^(1/2)*x^2-375/32*arcsinh(1/5*5^(1/2)*x^2)+1/5*(x^4+5)^(5/2)

maxima [B] time = 1.13, size = 118, normalized size = 1.76

$$\frac{1}{5} (x^4 + 5)^{\frac{5}{2}} - \frac{125 \left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{8(x^4+5)^{\frac{3}{2}}}{x^6} - \frac{3(x^4+5)^{\frac{5}{2}}}{x^{10}} \right)}{32 \left(\frac{3(x^4+5)}{x^4} - \frac{3(x^4+5)^2}{x^8} + \frac{(x^4+5)^3}{x^{12}} - 1 \right)} - \frac{375}{64} \log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) + \frac{375}{64} \log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{5}(x^4 + 5)^{5/2} - \frac{125}{32}(3\sqrt{x^4 + 5}/x^2 - 8(x^4 + 5)^{3/2}/x^6 - 3(x^4 + 5)^{5/2}/x^{10}) / (3(x^4 + 5)/x^4 - 3(x^4 + 5)^2/x^8 + (x^4 + 5)^3/x^{12} - 1) - \frac{375}{64}\log(\sqrt{x^4 + 5}/x^2 + 1) + \frac{375}{64}\log(\sqrt{x^4 + 5}/x^2 - 1)$

mupad [B] time = 0.43, size = 47, normalized size = 0.70

$$\sqrt{x^4 + 5} \left(\frac{x^{10}}{4} + \frac{x^8}{5} + \frac{35x^6}{16} + 2x^4 + \frac{75x^2}{32} + 5 \right) - \frac{375 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)`

[Out] $(x^4 + 5)^{1/2} * ((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^{10}/4 + 5) - (375*\operatorname{asinh}((5^{1/2}*x^2)/5))/32$

sympy [B] time = 11.57, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^4 + 5}} + \frac{55x^{10}}{16\sqrt{x^4 + 5}} + \frac{x^8\sqrt{x^4 + 5}}{5} + \frac{425x^6}{32\sqrt{x^4 + 5}} + \frac{x^4\sqrt{x^4 + 5}}{3} + \frac{375x^2}{32\sqrt{x^4 + 5}} + \frac{5(x^4 + 5)^{3/2}}{3} - \frac{10\sqrt{x^4 + 5}}{3} - \frac{375 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2), x)`

[Out] $x^{14}/(4*\sqrt{x^4 + 5}) + 55*x^{10}/(16*\sqrt{x^4 + 5}) + x^8*\sqrt{x^4 + 5}/5 + 425*x^6/(32*\sqrt{x^4 + 5}) + x^4*\sqrt{x^4 + 5}/3 + 375*x^2/(32*\sqrt{x^4 + 5}) + 5*(x^4 + 5)**(3/2)/3 - 10*\sqrt{x^4 + 5}/3 - 375*\operatorname{asinh}(\sqrt{5}*x^2/5)/32$

$$3.17 \quad \int x(2 + 3x^2)(5 + x^4)^{3/2} dx$$

Optimal. Leaf size=60

$$\frac{3}{10}(x^4 + 5)^{5/2} + \frac{75}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{4}x^2(x^4 + 5)^{3/2} + \frac{15}{8}x^2\sqrt{x^4 + 5}$$

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1248, 641, 195, 215}

$$\frac{3}{10}(x^4 + 5)^{5/2} + \frac{1}{4}x^2(x^4 + 5)^{3/2} + \frac{15}{8}x^2\sqrt{x^4 + 5} + \frac{75}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]

[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10 + (75*ArcSinh[x^2/Sqrt[5]])/8

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ

[{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int x(2+3x^2)(5+x^4)^{3/2} dx &= \frac{1}{2} \text{Subst}\left(\int (2+3x)(5+x^2)^{3/2} dx, x, x^2\right) \\
 &= \frac{3}{10}(5+x^4)^{5/2} + \text{Subst}\left(\int (5+x^2)^{3/2} dx, x, x^2\right) \\
 &= \frac{1}{4}x^2(5+x^4)^{3/2} + \frac{3}{10}(5+x^4)^{5/2} + \frac{15}{4} \text{Subst}\left(\int \sqrt{5+x^2} dx, x, x^2\right) \\
 &= \frac{15}{8}x^2\sqrt{5+x^4} + \frac{1}{4}x^2(5+x^4)^{3/2} + \frac{3}{10}(5+x^4)^{5/2} + \frac{75}{8} \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2\right) \\
 &= \frac{15}{8}x^2\sqrt{5+x^4} + \frac{1}{4}x^2(5+x^4)^{3/2} + \frac{3}{10}(5+x^4)^{5/2} + \frac{75}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.93

$$\frac{75}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\sqrt{x^4+5} \left(\frac{3x^8}{5} + \frac{x^6}{2} + 6x^4 + \frac{25x^2}{4} + 15\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2), x]

[Out] (Sqrt[5 + x^4]*(15 + (25*x^2)/4 + 6*x^4 + x^6/2 + (3*x^8)/5))/2 + (75*ArcSinh[x^2/Sqrt[5]])/8

IntegrateAlgebraic [A] time = 0.17, size = 56, normalized size = 0.93

$$\frac{1}{40}\sqrt{x^4+5} (12x^8 + 10x^6 + 120x^4 + 125x^2 + 300) - \frac{75}{8} \log\left(\sqrt{x^4+5} - x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(2 + 3*x^2)*(5 + x^4)^(3/2), x]

[Out] (Sqrt[5 + x^4]*(300 + 125*x^2 + 120*x^4 + 10*x^6 + 12*x^8))/40 - (75*Log[-x^2 + Sqrt[5 + x^4]])/8

fricas [A] time = 0.82, size = 48, normalized size = 0.80

$$\frac{1}{40} (12x^8 + 10x^6 + 120x^4 + 125x^2 + 300)\sqrt{x^4+5} - \frac{75}{8} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")

[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.24, size = 57, normalized size = 0.95

$$\frac{1}{8} (2x^4 + 5)\sqrt{x^4 + 5}x^2 + \frac{3}{10} (x^4 + 5)^{\frac{5}{2}} + \frac{5}{2} \sqrt{x^4 + 5}x^2 - \frac{75}{8} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")

[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 + 5)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 46, normalized size = 0.77

$$\frac{\sqrt{x^4 + 5} x^6}{4} + \frac{25\sqrt{x^4 + 5} x^2}{8} + \frac{75 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{8} + \frac{3(x^4 + 5)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x)

[Out] 3/10*(x^4+5)^(5/2)+1/4*(x^4+5)^(1/2)*x^6+25/8*(x^4+5)^(1/2)*x^2+75/8*arcsinh(1/5*5^(1/2)*x^2)

maxima [B] time = 1.60, size = 95, normalized size = 1.58

$$\frac{3}{10} (x^4 + 5)^{\frac{5}{2}} + \frac{25 \left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{5(x^4+5)^{\frac{3}{2}}}{x^6} \right)}{8 \left(\frac{2(x^4+5)}{x^4} - \frac{(x^4+5)^2}{x^8} - 1 \right)} + \frac{75}{16} \log \left(\frac{\sqrt{x^4+5}}{x^2} + 1 \right) - \frac{75}{16} \log \left(\frac{\sqrt{x^4+5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")

[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) - 75/16*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.18, size = 42, normalized size = 0.70

$$\frac{75 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8} + \sqrt{x^4+5} \left(\frac{3x^8}{10} + \frac{x^6}{4} + 3x^4 + \frac{25x^2}{8} + \frac{15}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)`

[Out] $(75*\operatorname{asinh}((5^{1/2}*x^2)/5))/8 + (x^4 + 5)^{1/2}*((25*x^2)/8 + 3*x^4 + x^6/4 + (3*x^8)/10 + 15/2)$

sympy [B] time = 8.19, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^4+5}} + \frac{3x^8\sqrt{x^4+5}}{10} + \frac{35x^6}{8\sqrt{x^4+5}} + \frac{x^4\sqrt{x^4+5}}{2} + \frac{125x^2}{8\sqrt{x^4+5}} + \frac{5(x^4+5)^{3/2}}{2} - 5\sqrt{x^4+5} + \frac{75 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)`

[Out] $x^{10}/(4*\sqrt{x^4 + 5}) + 3*x^{8}*\sqrt{x^4 + 5}/10 + 35*x^{6}/(8*\sqrt{x^4 + 5}) + x^{4}*\sqrt{x^4 + 5}/2 + 125*x^{2}/(8*\sqrt{x^4 + 5}) + 5*(x^4 + 5)^{3/2}/2 - 5*\sqrt{x^4 + 5} + 75*\operatorname{asinh}(\sqrt{5}*x^{2}/5)/8$

$$3.18 \quad \int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} dx$$

Optimal. Leaf size=78

$$-5\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{225}{16} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{24} (9x^2+8)(x^4+5)^{3/2} + \frac{5}{16} (9x^2+16)\sqrt{x^4+5}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{24} (9x^2+8)(x^4+5)^{3/2} + \frac{5}{16} (9x^2+16)\sqrt{x^4+5} + \frac{225}{16} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]

[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225 *ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 815

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1))]*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1252

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{1}{8} \text{Subst} \left(\int \frac{(40+45x)\sqrt{5+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{5}{16} (16+9x^2)\sqrt{5+x^4} + \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{1}{16} \text{Subst} \left(\int \frac{400+225x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{5}{16} (16+9x^2)\sqrt{5+x^4} + \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{225}{16} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{5}{16} (16+9x^2)\sqrt{5+x^4} + \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{225}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{25}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{5}{16} (16+9x^2)\sqrt{5+x^4} + \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{225}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + 25 \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{5}{16} (16+9x^2)\sqrt{5+x^4} + \frac{1}{24} (8+9x^2)(5+x^4)^{3/2} + \frac{225}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - 5\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4+5}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.86

$$\frac{1}{48} \left(-240\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4+5}}{\sqrt{5}} \right) + 675 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \sqrt{x^4+5} (18x^6 + 16x^4 + 225x^2 + 320) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]

[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]] - 240*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/48

IntegrateAlgebraic [A] time = 0.22, size = 85, normalized size = 1.09

$$-\frac{225}{16} \log \left(\sqrt{x^4+5} - x^2 \right) + 10\sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right) + \frac{1}{48} \sqrt{x^4+5} (18x^6 + 16x^4 + 225x^2 + 320)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]

[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6))/48 + 10*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (225*Log[-x^2 + Sqrt[5 + x^4]])/16

fricas [A] time = 0.54, size = 67, normalized size = 0.86

$$\frac{1}{48} (18x^6 + 16x^4 + 225x^2 + 320)\sqrt{x^4 + 5} + 5\sqrt{5} \log\left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{x^2}\right) - \frac{225}{16} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")

[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.26, size = 90, normalized size = 1.15

$$\frac{1}{48} \sqrt{x^4 + 5} ((2(9x^2 + 8)x^2 + 225)x^2 + 320) + 5\sqrt{5} \log\left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4 + 5}}{x^2 - \sqrt{5} - \sqrt{x^4 + 5}}\right) - \frac{225}{16} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")

[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 75, normalized size = 0.96

$$\frac{3\sqrt{x^4 + 5} x^6}{8} + \frac{\sqrt{x^4 + 5} x^4}{3} + \frac{75\sqrt{x^4 + 5} x^2}{16} + \frac{225 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16} - 5\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4 + 5}}\right) + \frac{20\sqrt{x^4 + 5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x)

[Out] 3/8*(x^4+5)^(1/2)*x^6+75/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^2)+1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))

maxima [B] time = 1.19, size = 138, normalized size = 1.77

$$\frac{1}{3}(x^4 + 5)^{\frac{3}{2}} + \frac{5}{2}\sqrt{5} \log\left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{\sqrt{5} + \sqrt{x^4 + 5}}\right) + 5\sqrt{x^4 + 5} + \frac{75\left(\frac{3\sqrt{x^4 + 5}}{x^2} - \frac{5(x^4 + 5)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2(x^4 + 5)}{x^4} - \frac{(x^4 + 5)^2}{x^8} - 1\right)} + \frac{225}{32} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) - \frac{225}{32} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")

[Out] $\frac{1}{3}(x^4 + 5)^{3/2} + \frac{5}{2}\sqrt{5}\log(-(\sqrt{5} - \sqrt{x^4 + 5})) / (\sqrt{5} + \sqrt{x^4 + 5}) + 5\sqrt{x^4 + 5} + \frac{75}{16}(3\sqrt{x^4 + 5}/x^2 - 5(x^4 + 5)^{3/2}/x^6) / (2(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + \frac{225}{32}\log(\sqrt{x^4 + 5}/x^2 + 1) - \frac{225}{32}\log(\sqrt{x^4 + 5}/x^2 - 1)$

mupad [B] time = 0.18, size = 55, normalized size = 0.71

$$\frac{225 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16} - 5\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5} \sqrt{x^4 + 5}}{5}\right) + \sqrt{x^4 + 5} \left(\frac{3x^6}{8} + \frac{x^4}{3} + \frac{75x^2}{16} + \frac{20}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)

[Out] $(225*\operatorname{asinh}((5^{1/2}*x^2)/5))/16 - 5*5^{1/2}*\operatorname{atanh}((5^{1/2}*(x^4 + 5)^{1/2})/5) + (x^4 + 5)^{1/2}*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)$

sympy [A] time = 33.67, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4 + 5}} + \frac{105x^6}{16\sqrt{x^4 + 5}} + \frac{375x^2}{16\sqrt{x^4 + 5}} + \frac{(x^4 + 5)^{3/2}}{3} + 5\sqrt{x^4 + 5} + \frac{5\sqrt{5} \log(x^4)}{2} - 5\sqrt{5} \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right) + \frac{225 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)

[Out] $3*x^{10}/(8*\sqrt{x^4 + 5}) + 105*x^6/(16*\sqrt{x^4 + 5}) + 375*x^2/(16*\sqrt{x^4 + 5}) + (x^4 + 5)^{3/2}/3 + 5*\sqrt{x^4 + 5} + 5*\sqrt{5}*\log(x^4)/2 - 5*\sqrt{5}*\log(\sqrt{x^4/5 + 1} + 1) + 225*\operatorname{asinh}(\sqrt{5}*x^2/5)/16$

$$3.19 \quad \int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} dx$$

Optimal. Leaf size=81

$$-\frac{15}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{15}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(2-x^2)(x^4+5)^{3/2}}{2x^2} + \frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1252, 813, 815, 844, 215, 266, 63, 207}

$$-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2} + \frac{3}{2}(x^2+5)\sqrt{x^4+5} + \frac{15}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]

[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*ArcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 813

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 815

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{3}{2} (5+x^2) \sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{8} \text{Subst} \left(\int \frac{-300-60x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{3}{2} (5+x^2) \sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) + \frac{7}{2} \\
&= \frac{3}{2} (5+x^2) \sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{75}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{3}{2} (5+x^2) \sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{75}{2} \text{Subst} \left(\int \frac{1}{-5} dx, x, x^2 \right) \\
&= \frac{3}{2} (5+x^2) \sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{15}{2} \sqrt{5} \tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.88

$$\frac{1}{2} \left(\sqrt{x^4+5} (x^4+20) - 15\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4+5}}{\sqrt{5}} \right) \right) - \frac{5\sqrt{5} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{x^4}{5} \right)}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3, x]

[Out] (Sqrt[5 + x^4]*(20 + x^4) - 15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2 - (5*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/x^2

IntegrateAlgebraic [A] time = 0.25, size = 84, normalized size = 1.04

$$-\frac{15}{2} \log \left(\sqrt{x^4+5} - x^2 \right) + 15\sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right) + \frac{\sqrt{x^4+5} (x^6 + x^4 + 20x^2 - 10)}{2x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]

[Out] (Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/(2*x^2) + 15*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (15*Log[-x^2 + Sqrt[5 + x^4]])/2

fricas [A] time = 0.60, size = 78, normalized size = 0.96

$$\frac{15\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 15x^2\log\left(-x^2 + \sqrt{x^4+5}\right) - 10x^2 + (x^6 + x^4 + 20x^2 - 10)\sqrt{x^4+5}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")

[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 + sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2

giac [A] time = 0.26, size = 102, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^4+5}((x^2+1)x^2+20) + \frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{50}{(x^2-\sqrt{x^4+5})^2-5} - \frac{15}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 + 5))^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 75, normalized size = 0.93

$$\frac{\sqrt{x^4+5}x^4}{2} + \frac{\sqrt{x^4+5}x^2}{2} + \frac{15\operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{15\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2} - \frac{5\sqrt{x^4+5}}{x^2} + 10\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x)

[Out] 1/2*(x^4+5)^(1/2)*x^4+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+1/2*(x^4+5)^(1/2)*x^2+15/2*arcsinh(1/5*5^(1/2)*x^2)-5*(x^4+5)^(1/2)/x^2

maxima [A] time = 1.33, size = 122, normalized size = 1.51

$$\frac{1}{2}(x^4+5)^{\frac{3}{2}} + \frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{15}{2}\sqrt{x^4+5} - \frac{5\sqrt{x^4+5}}{x^2} + \frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")

[Out] $\frac{1}{2}*(x^4 + 5)^{(3/2)} + \frac{15}{4}*\sqrt{5}*\log(-(\sqrt{5} - \sqrt{x^4 + 5})) / (\sqrt{5} + \sqrt{x^4 + 5}) + \frac{15}{2}*\sqrt{x^4 + 5} - \frac{5*\sqrt{x^4 + 5}}{x^2} + \frac{5}{2}*\sqrt{x^4 + 5} / (x^2*((x^4 + 5)/x^4 - 1)) + \frac{15}{4}*\log(\sqrt{x^4 + 5}/x^2 + 1) - \frac{15}{4}*\log(\sqrt{x^4 + 5}/x^2 - 1)$

mupad [B] time = 0.76, size = 64, normalized size = 0.79

$$\frac{15 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} + \sqrt{x^4 + 5} \left(\frac{x^4}{2} + \frac{x^2}{2} + 10\right) - \frac{5 \sqrt{x^4 + 5}}{x^2} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5} \sqrt{x^4 + 5} 1i}{5}\right)}{2} 15i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)

[Out] $(15*\operatorname{asinh}((5^{1/2}*x^2)/5))/2 + (5^{1/2}*\operatorname{atan}((5^{1/2}*(x^4 + 5)^{(1/2)}*1i)/5)*15i)/2 + (x^4 + 5)^{(1/2)}*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^{(1/2)})/x^2$

sympy [A] time = 11.39, size = 114, normalized size = 1.41

$$\frac{x^6}{2\sqrt{x^4 + 5}} - \frac{5x^2}{2\sqrt{x^4 + 5}} + \frac{(x^4 + 5)^{\frac{3}{2}}}{2} + \frac{15\sqrt{x^4 + 5}}{2} + \frac{15\sqrt{5} \log(x^4)}{4} - \frac{15\sqrt{5} \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right)}{2} + \frac{15 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} - \frac{25}{x^2 \sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)

[Out] $x^{**6}/(2*\sqrt{x^{**4} + 5}) - 5*x^{**2}/(2*\sqrt{x^{**4} + 5}) + (x^{**4} + 5)^{(3/2)}/2 + 15*\sqrt{x^{**4} + 5}/2 + 15*\sqrt{5}*\log(x^{**4})/4 - 15*\sqrt{5}*\log(\sqrt{x^{**4}/5 + 1} + 1)/2 + 15*\operatorname{asinh}(\sqrt{5}*x^{**2}/5)/2 - 25/(x^{**2}*\sqrt{x^{**4} + 5})$

$$3.20 \quad \int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} dx$$

Optimal. Leaf size=86

$$-\frac{3}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \frac{45}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4} - \frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4} - \frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2} + \frac{45}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]

[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^3} dx, x, x^2 \right) \\
&= -\frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} - \frac{3}{16} \text{Subst} \left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{3}{32} \text{Subst} \left(\int \frac{80+120x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{15}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{15}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{15}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{3}{2} \sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5+x^4}} \right)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.70

$$\frac{1}{125} (x^4 + 5)^{5/2} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{x^4}{5} + 1 \right) - \frac{15\sqrt{5} {}_2F_1 \left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{x^4}{5} \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]

[Out] (-15*sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/5*x^4])/(2*x^2) + ((5 + x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/125

IntegrateAlgebraic [A] time = 0.31, size = 88, normalized size = 1.02

$$-\frac{45}{4} \log \left(\sqrt{x^4 + 5} - x^2 \right) + 3\sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right) + \frac{\sqrt{x^4 + 5} (3x^6 + 4x^4 - 30x^2 - 10)}{4x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]

[Out] (Sqrt[5 + x^4]*(-10 - 30*x^2 + 4*x^4 + 3*x^6))/(4*x^4) + 3*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]] - (45*Log[-x^2 + Sqrt[5 + x^4]])/4

fricas [A] time = 0.59, size = 82, normalized size = 0.95

$$\frac{6\sqrt{5}x^4 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 45x^4 \log(-x^2 + \sqrt{x^4+5}) - 30x^4 + (3x^6 + 4x^4 - 30x^2 - 10)\sqrt{x^4+5}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")

[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 + sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4

giac [B] time = 0.25, size = 146, normalized size = 1.70

$$\frac{1}{4}\sqrt{x^4+5}(3x^2+4) + \frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{5\left((x^2-\sqrt{x^4+5})^3+15(x^2-\sqrt{x^4+5})^2+5x^2-5\sqrt{x^4+5}-75\right)}{\left((x^2-\sqrt{x^4+5})^2-5\right)^2} - \frac{45}{4}\log(-x^2+\sqrt{x^4+5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")

[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 73, normalized size = 0.85

$$\frac{3\sqrt{x^4+5}x^2}{4} + \frac{45\operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} - \frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2} - \frac{15\sqrt{x^4+5}}{2x^2} - \frac{5\sqrt{x^4+5}}{2x^4} + \sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x)

[Out] (x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-5/2*(x^4+5)^(1/2)/x^4+3/4*(x^4+5)^(1/2)*x^2+45/4*arcsinh(1/5*5^(1/2)*x^2)-15/2*(x^4+5)^(1/2)/x^2

maxima [A] time = 1.37, size = 123, normalized size = 1.43

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \sqrt{x^4+5} - \frac{15\sqrt{x^4+5}}{2x^2} + \frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5\sqrt{x^4+5}}{2x^4} + \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")

[Out] $\frac{3}{4}\sqrt{5}\log(-(\sqrt{5} - \sqrt{x^4 + 5})/(\sqrt{5} + \sqrt{x^4 + 5})) + \sqrt{x^4 + 5} - \frac{15}{2}\sqrt{x^4 + 5}/x^2 + \frac{15}{4}\sqrt{x^4 + 5}/(x^2*((x^4 + 5)/x^4 - 1)) - \frac{5}{2}\sqrt{x^4 + 5}/x^4 + \frac{45}{8}\log(\sqrt{x^4 + 5}/x^2 + 1) - \frac{45}{8}\log(\sqrt{x^4 + 5}/x^2 - 1)$

mpad [B] time = 0.55, size = 71, normalized size = 0.83

$$\frac{45 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{4} + \sqrt{x^4 + 5} \left(\frac{3x^2}{4} + 1\right) - \frac{15\sqrt{x^4 + 5}}{2x^2} - \frac{5\sqrt{x^4 + 5}}{2x^4} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4 + 5} + 1i}{5}\right) 3i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)

[Out] $\frac{45\operatorname{asinh}((5^{1/2}*x^2)/5)}{4} + (5^{1/2}*\operatorname{atan}((5^{1/2}*(x^4 + 5)^{1/2}*1i)/5)*3i)/2 + (x^4 + 5)^{1/2}*((3*x^2)/4 + 1) - (15*(x^4 + 5)^{1/2})/(2*x^2) - (5*(x^4 + 5)^{1/2})/(2*x^4)$

sympy [A] time = 12.75, size = 133, normalized size = 1.55

$$\frac{3x^6}{4\sqrt{x^4 + 5}} - \frac{15x^2}{4\sqrt{x^4 + 5}} + \sqrt{x^4 + 5} + \frac{\sqrt{5} \log(x^4)}{2} - \sqrt{5} \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right) - \frac{\sqrt{5} \operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2} + \frac{45 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{4} - \frac{5\sqrt{1 + \frac{5}{x^4}}}{2x^2} - \frac{75}{2x^2\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)

[Out] $3*x**6/(4*\sqrt{x**4 + 5}) - 15*x**2/(4*\sqrt{x**4 + 5}) + \sqrt{x**4 + 5} + \sqrt{5}\log(x**4)/2 - \sqrt{5}\log(\sqrt{x**4/5 + 1} + 1) - \sqrt{5}\operatorname{asinh}(\sqrt{5}/x**2)/2 + 45*\operatorname{asinh}(\sqrt{5}*x**2/5)/4 - 5*\sqrt{1 + 5/x**4}/(2*x**2) - 75/(2*x**2*\sqrt{x**4 + 5})$

$$3.21 \quad \int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7} dx$$

Optimal. Leaf size=82

$$-\frac{9}{4}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right) + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(4-9x^2)\sqrt{x^4+5}}{4x^2} - \frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1252, 811, 813, 844, 215, 266, 63, 207}

$$-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6} - \frac{(4-9x^2)\sqrt{x^4+5}}{4x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5} \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7, x]

[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
+ 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
&& !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^4} dx, x, x^2 \right) \\
&= -\frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} - \frac{1}{40} \text{Subst} \left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{1}{80} \text{Subst} \left(\int \frac{900+80x}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{45}{4} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{45}{8} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{45}{4} \text{Subst} \left(\int \frac{1}{-5-x^2} dx, x, x^2 \right) \\
&= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{9}{4} \sqrt{5} \tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.73

$$\frac{3}{250} (x^4 + 5)^{5/2} {}_2F_1 \left(2, \frac{5}{2}; \frac{7}{2}; \frac{x^4}{5} + 1 \right) - \frac{5\sqrt{5} {}_2F_1 \left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{x^4}{5} \right)}{3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7, x]

[Out] (-5*Sqrt[5]*Hypergeometric2F1[-3/2, -3/2, -1/2, -1/5*x^4])/(3*x^6) + (3*(5 + x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/250

IntegrateAlgebraic [A] time = 0.33, size = 88, normalized size = 1.07

$$-\log \left(\sqrt{x^4 + 5} - x^2 \right) + \frac{9}{2} \sqrt{5} \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right) + \frac{\sqrt{x^4 + 5} (18x^6 - 16x^4 - 45x^2 - 20)}{12x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]

[Out] (Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + (9*Sqrt[5]*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/2 - Log[-x^2 + Sqrt[5 + x^4]]

fricas [A] time = 0.49, size = 82, normalized size = 1.00

$$\frac{27\sqrt{5}x^6 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 12x^6 \log\left(-x^2 + \sqrt{x^4+5}\right) - 16x^6 + (18x^6 - 16x^4 - 45x^2 - 20)\sqrt{x^4+5}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2 + sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/x^6

giac [B] time = 0.27, size = 158, normalized size = 1.93

$$\frac{9}{4}\sqrt{5} \log\left(\frac{x^2 + \sqrt{5} - \sqrt{x^4+5}}{x^2 - \sqrt{5} - \sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} + \frac{5(9(x^2 - \sqrt{x^4+5})^5 + 24(x^2 - \sqrt{x^4+5})^4 - 120(x^2 - \sqrt{x^4+5})^2 - 225x^2 + 225\sqrt{x^4+5} + 400)}{6((x^2 - \sqrt{x^4+5})^2 - 5)^3} - \log(-x^2 + \sqrt{x^4+5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")

[Out] 9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - sqrt(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5) + 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 73, normalized size = 0.89

$$\operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{9\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4} - \frac{4\sqrt{x^4+5}}{3x^2} - \frac{15\sqrt{x^4+5}}{4x^4} - \frac{5\sqrt{x^4+5}}{3x^6} + \frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x)

[Out] 3/2*(x^4+5)^(1/2)-9/4*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-15/4*(x^4+5)^(1/2)/x^4+arcsinh(1/5*5^(1/2)*x^2)-4/3*(x^4+5)^(1/2)/x^2-5/3*(x^4+5)^(1/2)/x^6

maxima [A] time = 1.35, size = 112, normalized size = 1.37

$$\frac{9}{8}\sqrt{5}\log\left(\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{(x^4+5)^{\frac{3}{2}}}{3x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")

[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2 *sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.95, size = 82, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\left(\frac{2}{3x^2}-\frac{5}{3x^6}\right)-\frac{2\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4x^4}+\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}i}{5}\right)9i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)

[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)/4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*(x^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)

sympy [A] time = 12.56, size = 148, normalized size = 1.80

$$-\frac{x^2}{\sqrt{x^4+5}}-\frac{\sqrt{1+\frac{5}{x^4}}}{3}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log(x^4)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{15\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{5}{x^2\sqrt{x^4+5}}-\frac{5\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)

[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)

$$3.22 \quad \int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}} dx$$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x^4+5}x^4 + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{3}{8}\sqrt{x^4+5}x^6 - \frac{5}{48}(27x^2+32)\sqrt{x^4+5}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 833, 780, 215}

$$\frac{3}{8}\sqrt{x^4+5}x^6 + \frac{1}{3}\sqrt{x^4+5}x^4 - \frac{5}{48}(27x^2+32)\sqrt{x^4+5} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 + x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{3}{8} x^6 \sqrt{5+x^4} + \frac{1}{8} \text{Subst} \left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{3} x^4 \sqrt{5+x^4} + \frac{3}{8} x^6 \sqrt{5+x^4} + \frac{1}{24} \text{Subst} \left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{3} x^4 \sqrt{5+x^4} + \frac{3}{8} x^6 \sqrt{5+x^4} - \frac{5}{48} (32+27x^2) \sqrt{5+x^4} + \frac{225}{16} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{3} x^4 \sqrt{5+x^4} + \frac{3}{8} x^6 \sqrt{5+x^4} - \frac{5}{48} (32+27x^2) \sqrt{5+x^4} + \frac{225}{16} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.66

$$\frac{1}{48} \left(675 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \sqrt{x^4 + 5} (18x^6 + 16x^4 - 135x^2 - 160) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]])/48

IntegrateAlgebraic [A] time = 0.13, size = 51, normalized size = 0.76

$$\frac{1}{48} \sqrt{x^4 + 5} (18x^6 + 16x^4 - 135x^2 - 160) - \frac{225}{16} \log \left(\sqrt{x^4 + 5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6))/48 - (225*Log[-x^2 + Sqrt[5 + x^4]])/16

fricas [A] time = 0.55, size = 43, normalized size = 0.64

$$\frac{1}{48} (18x^6 + 16x^4 - 135x^2 - 160)\sqrt{x^4 + 5} - \frac{225}{16} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.19, size = 46, normalized size = 0.69

$$\frac{1}{48} \sqrt{x^4 + 5} \left((2(9x^2 + 8)x^2 - 135)x^2 - 160 \right) - \frac{225}{16} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 51, normalized size = 0.76

$$\frac{3\sqrt{x^4 + 5} x^6}{8} - \frac{45\sqrt{x^4 + 5} x^2}{16} + \frac{225 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16} + \frac{\sqrt{x^4 + 5} (x^4 - 10)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x)

[Out] 3/8*(x^4+5)^(1/2)*x^6-45/16*(x^4+5)^(1/2)*x^2+225/16*arcsinh(1/5*5^(1/2)*x^2)+1/3*(x^4+5)^(1/2)*(x^4-10)

maxima [A] time = 1.31, size = 104, normalized size = 1.55

$$\frac{1}{3} (x^4 + 5)^{\frac{3}{2}} - 5\sqrt{x^4 + 5} - \frac{75 \left(\frac{5\sqrt{x^4 + 5}}{x^2} - \frac{3(x^4 + 5)^{\frac{3}{2}}}{x^6} \right)}{16 \left(\frac{2(x^4 + 5)}{x^4} - \frac{(x^4 + 5)^2}{x^8} - 1 \right)} + \frac{225}{32} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} + 1 \right) - \frac{225}{32} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] $1/3*(x^4 + 5)^{(3/2)} - 5*\sqrt{x^4 + 5} - 75/16*(5*\sqrt{x^4 + 5})/x^2 - 3*(x^4 + 5)^{(3/2)}/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*\log(\sqrt{x^4 + 5})/x^2 + 1) - 225/32*\log(\sqrt{x^4 + 5})/x^2 - 1)$

mupad [B] time = 0.59, size = 38, normalized size = 0.57

$$\frac{225 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16} - \sqrt{x^4 + 5} \left(-\frac{3 x^6}{8} - \frac{x^4}{3} + \frac{45 x^2}{16} + \frac{10}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)`

[Out] $(225*\operatorname{asinh}((5^{1/2}*x^2)/5))/16 - (x^4 + 5)^{(1/2)}*((45*x^2)/16 - x^4/3 - (3*x^6)/8 + 10/3)$

sympy [A] time = 7.07, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^4 + 5}} - \frac{15x^6}{16\sqrt{x^4 + 5}} + \frac{x^4\sqrt{x^4 + 5}}{3} - \frac{225x^2}{16\sqrt{x^4 + 5}} - \frac{10\sqrt{x^4 + 5}}{3} + \frac{225 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2), x)`

[Out] $3*x^{10}/(8*\sqrt{x^4 + 5}) - 15*x^6/(16*\sqrt{x^4 + 5}) + x^4*\sqrt{x^4 + 5}/3 - 225*x^2/(16*\sqrt{x^4 + 5}) - 10*\sqrt{x^4 + 5}/3 + 225*\operatorname{asinh}(\sqrt{5}*x^2/5)/16$

$$3.23 \quad \int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}} dx$$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{x^4+5}x^4 - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{1}{2}(10-x^2)\sqrt{x^4+5}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 833, 780, 215}

$$\frac{1}{2}\sqrt{x^4+5}x^4 - \frac{1}{2}(10-x^2)\sqrt{x^4+5} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[5]])/2

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} x^4 \sqrt{5+x^4} + \frac{1}{6} \text{Subst} \left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} x^4 \sqrt{5+x^4} - \frac{1}{2} (10-x^2) \sqrt{5+x^4} - \frac{5}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} x^4 \sqrt{5+x^4} - \frac{1}{2} (10-x^2) \sqrt{5+x^4} - \frac{5}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{1}{2} \left(\sqrt{x^4+5} (x^4+x^2-10) - 5 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4) - 5*ArcSinh[x^2/Sqrt[5]])/2

IntegrateAlgebraic [A] time = 0.15, size = 42, normalized size = 0.82

$$\frac{1}{2} \sqrt{x^4+5} (x^4+x^2-10) + \frac{5}{2} \log \left(\sqrt{x^4+5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 + (5*Log[-x^2 + Sqrt[5 + x^4]])/2

fricas [A] time = 0.74, size = 34, normalized size = 0.67

$$\frac{1}{2} (x^4 + x^2 - 10) \sqrt{x^4 + 5} + \frac{5}{2} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.22, size = 37, normalized size = 0.73

$$\frac{1}{2} \sqrt{x^4 + 5} \left((x^2 + 1)x^2 - 10 \right) + \frac{5}{2} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{\sqrt{x^4 + 5} x^2}{2} - \frac{5 \operatorname{arcsinh} \left(\frac{\sqrt{5} x^2}{5} \right)}{2} + \frac{\sqrt{x^4 + 5} (x^4 - 10)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x)

[Out] 1/2*(x^4+5)^(1/2)*(x^4-10)+1/2*(x^4+5)^(1/2)*x^2-5/2*arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.22, size = 76, normalized size = 1.49

$$\frac{1}{2} (x^4 + 5)^{\frac{3}{2}} - \frac{15}{2} \sqrt{x^4 + 5} + \frac{5 \sqrt{x^4 + 5}}{2 x^2 \left(\frac{x^4 + 5}{x^4} - 1 \right)} - \frac{5}{4} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} + 1 \right) + \frac{5}{4} \log \left(\frac{\sqrt{x^4 + 5}}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.31, size = 32, normalized size = 0.63

$$\sqrt{x^4 + 5} \left(\frac{x^4}{2} + \frac{x^2}{2} - 5 \right) - \frac{5 \operatorname{asinh} \left(\frac{\sqrt{5} x^2}{5} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)`

[Out] $(x^4 + 5)^{(1/2)} * (x^2/2 + x^4/2 - 5) - (5 * \operatorname{asinh}((5^{(1/2)} * x^2)/5))/2$

sympy [A] time = 5.52, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}} + \frac{x^4\sqrt{x^4+5}}{2} + \frac{5x^2}{2\sqrt{x^4+5}} - 5\sqrt{x^4+5} - \frac{5 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2), x)`

[Out] $x^{**6}/(2*\operatorname{sqrt}(x^{**4} + 5)) + x^{**4}*\operatorname{sqrt}(x^{**4} + 5)/2 + 5*x^{**2}/(2*\operatorname{sqrt}(x^{**4} + 5)) - 5*\operatorname{sqrt}(x^{**4} + 5) - 5*\operatorname{asinh}(\operatorname{sqrt}(5)*x^{**2}/5)/2$

$$3.24 \quad \int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} dx$$

Optimal. Leaf size=35

$$\frac{1}{4} (3x^2 + 4) \sqrt{x^4 + 5} - \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1252, 780, 215}

$$\frac{1}{4} (3x^2 + 4) \sqrt{x^4 + 5} - \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(2+3x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} - \frac{15}{4} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= \frac{1}{4} (4+3x^2) \sqrt{5+x^4} - \frac{15}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.97

$$\frac{1}{4} \left((3x^2 + 4) \sqrt{x^4 + 5} - 15 \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/4

IntegrateAlgebraic [A] time = 0.13, size = 41, normalized size = 1.17

$$\frac{1}{4} \sqrt{x^4 + 5} (3x^2 + 4) + \frac{15}{4} \log \left(\sqrt{x^4 + 5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*Log[-x^2 + Sqrt[5 + x^4]])/4

fricas [A] time = 0.62, size = 33, normalized size = 0.94

$$\frac{1}{4} \sqrt{x^4 + 5} (3x^2 + 4) + \frac{15}{4} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2), x, algorithm="fricas")

[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.22, size = 33, normalized size = 0.94

$$\frac{1}{4} \sqrt{x^4 + 5} (3x^2 + 4) + \frac{15}{4} \log \left(-x^2 + \sqrt{x^4 + 5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{3\sqrt{x^4 + 5} x^2}{4} - \frac{15 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{4} + \sqrt{x^4 + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x)

[Out] 3/4*(x^4+5)^(1/2)*x^2-15/4*arcsinh(1/5*5^(1/2)*x^2)+(x^4+5)^(1/2)

maxima [B] time = 1.16, size = 65, normalized size = 1.86

$$\sqrt{x^4 + 5} + \frac{15 \sqrt{x^4 + 5}}{4 x^2 \left(\frac{x^4 + 5}{x^4} - 1\right)} - \frac{15}{8} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) + \frac{15}{8} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqrt(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.49, size = 27, normalized size = 0.77

$$\sqrt{x^4 + 5} \left(\frac{3x^2}{4} + 1\right) - \frac{15 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)

[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4

sympy [A] time = 4.04, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4 + 5}} + \frac{15x^2}{4\sqrt{x^4 + 5}} + \sqrt{x^4 + 5} - \frac{15 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) - 1  
5*asinh(sqrt(5)*x**2/5)/4
```

$$3.25 \quad \int \frac{x(2+3x^2)}{\sqrt{5+x^4}} dx$$

Optimal. Leaf size=24

$$\frac{3\sqrt{x^4+5}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1248, 641, 215}

$$\frac{3\sqrt{x^4+5}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x(2+3x^2)}{\sqrt{5+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+3x}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{3\sqrt{5+x^4}}{2} + \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{3\sqrt{5+x^4}}{2} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{3\sqrt{x^4+5}}{2} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]

IntegrateAlgebraic [A] time = 0.13, size = 32, normalized size = 1.33

$$\frac{3\sqrt{x^4+5}}{2} - \log \left(\sqrt{x^4+5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/Sqrt[5 + x^4], x]

[Out] (3*Sqrt[5 + x^4])/2 - Log[-x^2 + Sqrt[5 + x^4]]

fricas [A] time = 0.55, size = 26, normalized size = 1.08

$$\frac{3}{2} \sqrt{x^4+5} - \log \left(-x^2 + \sqrt{x^4+5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2), x, algorithm="fricas")

[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))

giac [A] time = 0.20, size = 26, normalized size = 1.08

$$\frac{3}{2} \sqrt{x^4+5} - \log \left(-x^2 + \sqrt{x^4+5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 20, normalized size = 0.83

$$\operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right) + \frac{3\sqrt{x^4 + 5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x)

[Out] arcsinh(1/5*5^(1/2)*x^2)+3/2*(x^4+5)^(1/2)

maxima [B] time = 1.12, size = 42, normalized size = 1.75

$$\frac{3}{2} \sqrt{x^4 + 5} + \frac{1}{2} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) - \frac{1}{2} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.29, size = 19, normalized size = 0.79

$$\operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right) + \frac{3\sqrt{x^4 + 5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)

[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2

sympy [A] time = 2.13, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4 + 5}}{2} + \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)

[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)

$$3.26 \quad \int \frac{2+3x^2}{x\sqrt{5+x^4}} dx$$

Optimal. Leaf size=38

$$\frac{3}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{3}{2} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x*sqrt[5 + x^4]),x]

[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x\sqrt{5 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5 + x^2}} dx, x, x^2 \right) + \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x}} dx, x, x^4 \right) \\
 &= \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right) \\
 &= \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]), x]

[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]

IntegrateAlgebraic [A] time = 0.15, size = 55, normalized size = 1.45

$$\frac{2 \tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{3}{2} \log\left(\sqrt{x^4+5} - x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]

[Out] (2*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 + x^4]]))/2

fricas [A] time = 0.54, size = 41, normalized size = 1.08

$$\frac{1}{5} \sqrt{5} \log\left(-\frac{\sqrt{5} - \sqrt{x^4+5}}{x^2}\right) - \frac{3}{2} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 + 5))

giac [B] time = 0.20, size = 61, normalized size = 1.61

$$\frac{1}{5} \sqrt{5} \log\left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4+5}}{x^2 - \sqrt{5} - \sqrt{x^4+5}}\right) - \frac{3}{2} \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 30, normalized size = 0.79

$$\frac{3 \operatorname{arcsinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x)

[Out] $3/2*\operatorname{arcsinh}(1/5*5^{(1/2)}*x^2)-1/5*5^{(1/2)}*\operatorname{arctanh}(5^{(1/2)}/(x^4+5)^{(1/2)})$

maxima [B] time = 1.17, size = 67, normalized size = 1.76

$$\frac{1}{10} \sqrt{5} \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{4} \log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4} \log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")`

[Out] $1/10*\sqrt{5}*\log(-(\sqrt{5}-\sqrt{x^4+5})/(\sqrt{5}+\sqrt{x^4+5})) + 3/4*\log(\sqrt{x^4+5}/x^2+1) - 3/4*\log(\sqrt{x^4+5}/x^2-1)$

mupad [B] time = 0.61, size = 30, normalized size = 0.79

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2} - \frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5} \sqrt{x^4+5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/(x*(x^4+5)^(1/2)),x)`

[Out] $(3*\operatorname{asinh}((5^{(1/2)}*x^2)/5))/2 - (5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*(x^4+5)^{(1/2)})/5))/5$

sympy [A] time = 6.01, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5} \operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5} + \frac{3 \operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)`

[Out] $-\sqrt{5}*\operatorname{asinh}(\sqrt{5}/x**2)/5 + 3*\operatorname{asinh}(\sqrt{5}*x**2/5)/2$

$$3.27 \quad \int \frac{2+3x^2}{x^3 \sqrt{5+x^4}} dx$$

Optimal. Leaf size=42

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}} - \frac{\sqrt{x^4+5}}{5x^2}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1252, 807, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}}{5x^2} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]

[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 807

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))

$\int \frac{2 + 3x^2}{x^3 \sqrt{5 + x^4}} dx = \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^2 \sqrt{5 + x^2}} dx, x, x^2 \right)$
 $= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{x \sqrt{5 + x^2}} dx, x, x^2 \right)$
 $= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{x \sqrt{5 + x}} dx, x, x^4 \right)$
 $= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right)$
 $= -\frac{\sqrt{5 + x^4}}{5x^2} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{2\sqrt{5}}$

Rule 1252

$\text{Int}[(x_)^{(m_.)} * ((d_) + (e_.) * (x_)^2)^{(q_.)} * ((a_) + (c_.) * (x_)^4)^{(p_.)}, x_S$
 $\text{ymbol}] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m - 1)/2)} * (d + e*x)^q * (a + c*x^2)^p, x],$
 $x, x^2], x] /; \text{FreeQ}[\{a, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m + 1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{2 + 3x^2}{x^3 \sqrt{5 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^2 \sqrt{5 + x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{x \sqrt{5 + x^2}} dx, x, x^2 \right) \\ &= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{x \sqrt{5 + x}} dx, x, x^4 \right) \\ &= -\frac{\sqrt{5 + x^4}}{5x^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right) \\ &= -\frac{\sqrt{5 + x^4}}{5x^2} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{3 \tanh^{-1} \left(\frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right)}{2\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{5x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]

[Out] -1/5*Sqrt[5 + x^4]/x^2 - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 1.21

$$\frac{3 \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right)}{\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{5x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]

[Out] -1/5*Sqrt[5 + x^4]/x^2 + (3*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5]

fricas [A] time = 0.61, size = 47, normalized size = 1.12

$$\frac{3\sqrt{5}x^2 \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2x^2 - 2\sqrt{x^4+5}}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")

[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^4 + 5))/x^2

giac [B] time = 0.22, size = 66, normalized size = 1.57

$$\frac{3}{10}\sqrt{5} \log\left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4 + 5}}{x^2 - \sqrt{5} - \sqrt{x^4 + 5}}\right) + \frac{2}{(x^2 - \sqrt{x^4 + 5})^2 - 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")

[Out] 3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10} - \frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x)

[Out] -3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2

maxima [A] time = 1.26, size = 47, normalized size = 1.12

$$\frac{3}{20}\sqrt{5} \log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] $\frac{3}{20}\sqrt{5}\log\left(\frac{-(\sqrt{5}-\sqrt{x^4+5})}{(\sqrt{5}+\sqrt{x^4+5})}\right) - \frac{1}{5}\sqrt{x^4+5}/x^2$

mupad [B] time = 0.33, size = 31, normalized size = 0.74

$$-\frac{3\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{10} - \frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)

[Out] $-\frac{(3\sqrt{5}\operatorname{atanh}((\sqrt{5}(x^4+5)^{1/2})/5))}{10} - \frac{(x^4+5)^{1/2}}{(5x^2)}$

sympy [A] time = 3.60, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)

[Out] $-\sqrt{1+5/x^4}/5 - 3\sqrt{5}\operatorname{asinh}(\sqrt{5}/x^2)/10$

$$3.28 \quad \int \frac{2+3x^2}{x^5 \sqrt{5+x^4}} dx$$

Optimal. Leaf size=58

$$-\frac{\sqrt{x^4+5}}{10x^4} + \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{3\sqrt{x^4+5}}{10x^2}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 835, 807, 266, 63, 207}

$$-\frac{3\sqrt{x^4+5}}{10x^2} - \frac{\sqrt{x^4+5}}{10x^4} + \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]

[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(10*Sqrt[5])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x^5\sqrt{5 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^3\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{5 + x^4}}{10x^4} - \frac{1}{20} \text{Subst} \left(\int \frac{-30 + 2x}{x^2\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{5 + x^4}}{10x^4} - \frac{3\sqrt{5 + x^4}}{10x^2} - \frac{1}{10} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{5 + x^4}}{10x^4} - \frac{3\sqrt{5 + x^4}}{10x^2} - \frac{1}{20} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x}} dx, x, x^4 \right) \\
 &= -\frac{\sqrt{5 + x^4}}{10x^4} - \frac{3\sqrt{5 + x^4}}{10x^2} - \frac{1}{10} \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right) \\
 &= -\frac{\sqrt{5 + x^4}}{10x^4} - \frac{3\sqrt{5 + x^4}}{10x^2} + \frac{\tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{10\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.84

$$\frac{\sqrt{5}x^4 \tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right) - 5(3x^2+1)\sqrt{x^4+5}}{50x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]), x]

[Out] (-5*(1 + 3*x^2)*Sqrt[5 + x^4] + Sqrt[5]*x^4*ArcTanh[Sqrt[1 + x^4/5]])/(50*x^4)

IntegrateAlgebraic [A] time = 0.23, size = 60, normalized size = 1.03

$$\frac{(-3x^2-1)\sqrt{x^4+5}}{10x^4} - \frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]), x]

[Out] ((-1 - 3*x^2)*Sqrt[5 + x^4])/(10*x^4) - ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])

fricas [A] time = 0.72, size = 50, normalized size = 0.86

$$\frac{\sqrt{5}x^4 \log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right) - 15x^4 - 5\sqrt{x^4+5}(3x^2+1)}{50x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2), x, algorithm="fricas")

[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4

giac [B] time = 0.22, size = 114, normalized size = 1.97

$$-\frac{1}{50}\sqrt{5}\log\left(\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{(x^2-\sqrt{x^4+5})^3 + 15(x^2-\sqrt{x^4+5})^2 + 5x^2 - 5\sqrt{x^4+5} - 75}{5\left((x^2-\sqrt{x^4+5})^2 - 5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")

[Out] $-1/50*\sqrt{5}*\log(-(x^2 + \sqrt{5}) - \sqrt{x^4 + 5})/(x^2 - \sqrt{5}) - \sqrt{x^4 + 5})) + 1/5*((x^2 - \sqrt{x^4 + 5})^3 + 15*(x^2 - \sqrt{x^4 + 5})^2 + 5*x^2 - 5*\sqrt{x^4 + 5} - 75)/((x^2 - \sqrt{x^4 + 5})^2 - 5)^2$

maple [A] time = 0.02, size = 43, normalized size = 0.74

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50} - \frac{3\sqrt{x^4+5}}{10x^2} - \frac{\sqrt{x^4+5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x)

[Out] $-1/10*(x^4+5)^{(1/2)}/x^4+1/50*5^{(1/2)}*\operatorname{arctanh}(5^{(1/2)}/(x^4+5)^{(1/2)})-3/10*(x^4+5)^{(1/2)}/x^2$

maxima [A] time = 1.12, size = 59, normalized size = 1.02

$$-\frac{1}{100} \sqrt{5} \log\left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{\sqrt{5} + \sqrt{x^4 + 5}}\right) - \frac{3\sqrt{x^4 + 5}}{10x^2} - \frac{\sqrt{x^4 + 5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] $-1/100*\sqrt{5}*\log(-(\sqrt{5}) - \sqrt{x^4 + 5})/(\sqrt{5} + \sqrt{x^4 + 5})) - 3/10*\sqrt{x^4 + 5}/x^2 - 1/10*\sqrt{x^4 + 5}/x^4$

mupad [B] time = 0.69, size = 43, normalized size = 0.74

$$\frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5} \sqrt{x^4+5}}{5}\right)}{50} - \frac{3\sqrt{x^4+5}}{10x^2} - \frac{\sqrt{x^4+5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)

[Out] $(5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*(x^4 + 5)^{(1/2)})/5))/50 - (3*(x^4 + 5)^{(1/2)})/(10*x^2) - (x^4 + 5)^{(1/2)}/(10*x^4)$

sympy [A] time = 14.33, size = 88, normalized size = 1.52

$$\frac{\sqrt{5} \left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4} + \frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4} - \frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)} - \frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)} \right)}{25} - \frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)

[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt(5*x**4 + 25)/(50*x**2)

$$3.29 \quad \int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=58

$$-\frac{45}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(3x^2+2)x^4}{2\sqrt{x^4+5}} + \frac{1}{4}(9x^2+8)\sqrt{x^4+5}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 819, 780, 215}

$$-\frac{(3x^2+2)x^4}{2\sqrt{x^4+5}} + \frac{1}{4}(9x^2+8)\sqrt{x^4+5} - \frac{45}{4} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] -(x^4*(2 + 3*x^2))/(2*sqrt[5 + x^4]) + ((8 + 9*x^2)*sqrt[5 + x^4])/4 - (45*ArcSinh[x^2/sqrt[5]])/4

Rule 215

Int[1/sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 780

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 819

Int[((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||

!ILtQ[m + 2*p + 3, 0])

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3(2+3x)}{(5+x^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10} \text{Subst} \left(\int \frac{x(20+45x)}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}(8+9x^2)\sqrt{5+x^4} - \frac{45}{4} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}(8+9x^2)\sqrt{5+x^4} - \frac{45}{4} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.88

$$\frac{3x^6 + 4x^4 + 45x^2 - 45\sqrt{x^4 + 5} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + 40}{4\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6 - 45*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(4*Sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.18, size = 51, normalized size = 0.88

$$\frac{45}{4} \log \left(\sqrt{x^4 + 5} - x^2 \right) + \frac{3x^6 + 4x^4 + 45x^2 + 40}{4\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6)/(4*sqrt[5 + x^4]) + (45*Log[-x^2 + Sqrt[5 + x^4]])/4

fricas [A] time = 0.87, size = 62, normalized size = 1.07

$$\frac{30x^4 + 45(x^4 + 5)\log(-x^2 + \sqrt{x^4 + 5}) + (3x^6 + 4x^4 + 45x^2 + 40)\sqrt{x^4 + 5} + 150}{4(x^4 + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")

[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)

giac [A] time = 0.21, size = 45, normalized size = 0.78

$$\frac{((3x^2 + 4)x^2 + 45)x^2 + 40}{4\sqrt{x^4 + 5}} + \frac{45}{4} \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")

[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.02, size = 50, normalized size = 0.86

$$\frac{3x^6}{4\sqrt{x^4 + 5}} + \frac{45x^2}{4\sqrt{x^4 + 5}} - \frac{45 \operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{x^4 + 10}{\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x)

[Out] 3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(1/2)-45/4*arcsinh(1/5*5^(1/2)*x^2)+1/(x^4+5)^(1/2)*(x^4+10)

maxima [A] time = 1.21, size = 89, normalized size = 1.53

$$\sqrt{x^4 + 5} - \frac{15\left(\frac{3(x^4+5)}{x^4} - 2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2} - \frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)} + \frac{5}{\sqrt{x^4 + 5}} - \frac{45}{8} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) + \frac{45}{8} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")

[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 1.11, size = 97, normalized size = 1.67

$$\sqrt{x^4+5} \left(\frac{3x^2}{4} + 1 \right) - \frac{45 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{\sqrt{5} (10 + \sqrt{5} 15i) \sqrt{x^4+5} 1i}{20 (-x^2 + \sqrt{5} 1i)} - \frac{\sqrt{5} (-10 + \sqrt{5} 15i) \sqrt{x^4+5} 1i}{20 (x^2 + \sqrt{5} 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)

[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)*(5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))

sympy [A] time = 14.28, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)

[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)

$$3.30 \quad \int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=45

$$3\sqrt{x^4+5} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{(3x^2+2)x^2}{2\sqrt{x^4+5}}$$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1252, 819, 641, 215}

$$-\frac{(3x^2+2)x^2}{2\sqrt{x^4+5}} + 3\sqrt{x^4+5} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !LtQ[m + 2*p + 3, 0])

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(2+3x)}{(5+x^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10} \text{Subst} \left(\int \frac{10+30x}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\frac{3x^4 - 2x^2 + 2\sqrt{x^4 + 5} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + 30}{2\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] (30 - 2*x^2 + 3*x^4 + 2*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.21, size = 44, normalized size = 0.98

$$\frac{3x^4 - 2x^2 + 30}{2\sqrt{x^4 + 5}} - \log \left(\sqrt{x^4 + 5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] (30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) - Log[-x^2 + Sqrt[5 + x^4]]

fricas [A] time = 0.75, size = 58, normalized size = 1.29

$$\frac{2x^4 + 2(x^4 + 5)\log(-x^2 + \sqrt{x^4 + 5}) - (3x^4 - 2x^2 + 30)\sqrt{x^4 + 5} + 10}{2(x^4 + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")

[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*sqrt(x^4 + 5) + 10)/(x^4 + 5)

giac [A] time = 0.23, size = 39, normalized size = 0.87

$$\frac{(3x^2 - 2)x^2 + 30}{2\sqrt{x^4 + 5}} - \log(-x^2 + \sqrt{x^4 + 5})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")

[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{x^2}{\sqrt{x^4 + 5}} + \operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{\frac{3x^4}{2} + 15}{\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x)

[Out] 3/2/(x^4+5)^(1/2)*(x^4+10)-1/(x^4+5)^(1/2)*x^2+arcsinh(1/5*5^(1/2)*x^2)

maxima [A] time = 1.41, size = 63, normalized size = 1.40

$$-\frac{x^2}{\sqrt{x^4 + 5}} + \frac{3}{2}\sqrt{x^4 + 5} + \frac{15}{2\sqrt{x^4 + 5}} + \frac{1}{2}\log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")

[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.89, size = 89, normalized size = 1.98

$$\operatorname{asinh}\left(\frac{\sqrt{5} x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{5}(-15+\sqrt{5}2i)\sqrt{x^4+5}1i}{20(-x^2+\sqrt{5}1i)} + \frac{\sqrt{5}(15+\sqrt{5}2i)\sqrt{x^4+5}1i}{20(x^2+\sqrt{5}1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)`

[Out] `asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15) * (x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))`

sympy [A] time = 12.34, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)`

[Out] `3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 15/sqrt(x**4 + 5)`

$$3.31 \quad \int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=35

$$\frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) + \frac{-3x^2 - 2}{2\sqrt{x^4 + 5}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1252, 778, 215}

$$\frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - \frac{3x^2 + 2}{2\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 778

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(2+3x)}{(5+x^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{5+x^2}} dx, x, x^2 \right) \\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.17

$$\frac{-3x^2 + 3\sqrt{x^4 + 5} \sinh^{-1} \left(\frac{x^2}{\sqrt{5}} \right) - 2}{2\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] (-2 - 3*x^2 + 3*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.19, size = 41, normalized size = 1.17

$$\frac{-3x^2 - 2}{2\sqrt{x^4 + 5}} - \frac{3}{2} \log \left(\sqrt{x^4 + 5} - x^2 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] (-2 - 3*x^2)/(2*Sqrt[5 + x^4]) - (3*Log[-x^2 + Sqrt[5 + x^4]])/2

fricas [A] time = 0.70, size = 52, normalized size = 1.49

$$\frac{3x^4 + 3(x^4 + 5) \log(-x^2 + \sqrt{x^4 + 5}) + \sqrt{x^4 + 5}(3x^2 + 2) + 15}{2(x^4 + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2), x, algorithm="fricas")

[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2 + 2) + 15)/(x^4 + 5)

giac [A] time = 0.25, size = 33, normalized size = 0.94

$$-\frac{3x^2 + 2}{2\sqrt{x^4 + 5}} - \frac{3}{2} \log\left(-x^2 + \sqrt{x^4 + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")

[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$-\frac{3x^2}{2\sqrt{x^4 + 5}} + \frac{3 \operatorname{arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{1}{\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x)

[Out] -3/2/(x^4+5)^(1/2)*x^2+3/2*arcsinh(1/5*5^(1/2)*x^2)-1/(x^4+5)^(1/2)

maxima [A] time = 1.23, size = 54, normalized size = 1.54

$$-\frac{3x^2}{2\sqrt{x^4 + 5}} - \frac{1}{\sqrt{x^4 + 5}} + \frac{3}{4} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} + 1\right) - \frac{3}{4} \log\left(\frac{\sqrt{x^4 + 5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")

[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)

mupad [B] time = 0.84, size = 82, normalized size = 2.34

$$\frac{3 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{\sqrt{5} (2 + \sqrt{5} 3i) \sqrt{x^4 + 5} 1i}{20 (-x^2 + \sqrt{5} 1i)} + \frac{\sqrt{5} (-2 + \sqrt{5} 3i) \sqrt{x^4 + 5} 1i}{20 (x^2 + \sqrt{5} 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)

[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))

sympy [A] time = 10.68, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}} + \frac{3 \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)`

[Out] `-3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)`

$$3.32 \quad \int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=20

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1248, 637}

$$-\frac{15 - 2x^2}{10\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2), x]

[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])

Rule 637

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+3x}{(5+x^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{15-2x^2}{10\sqrt{5+x^4}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (-15 + 2*x^2)/(10*sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.18, size = 20, normalized size = 1.00

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (-15 + 2*x^2)/(10*sqrt[5 + x^4])

fricas [A] time = 0.54, size = 31, normalized size = 1.55

$$\frac{2x^4 + \sqrt{x^4 + 5}(2x^2 - 15) + 10}{10(x^4 + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")

[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)

giac [A] time = 0.24, size = 16, normalized size = 0.80

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")

[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x)

[Out] $1/10*(2*x^2-15)/(x^4+5)^{(1/2)}$

maxima [A] time = 1.49, size = 22, normalized size = 1.10

$$\frac{x^2}{5\sqrt{x^4+5}} - \frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")`

[Out] $1/5*x^2/\text{sqrt}(x^4 + 5) - 3/2/\text{sqrt}(x^4 + 5)$

mupad [B] time = 0.16, size = 16, normalized size = 0.80

$$\frac{2x^2 - 15}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)`

[Out] $(2*x^2 - 15)/(10*(x^4 + 5)^{(1/2)})$

sympy [B] time = 7.86, size = 31, normalized size = 1.55

$$\frac{\sqrt{5}x^2}{5\sqrt{5x^4+25}} - \frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)`

[Out] $\text{sqrt}(5)*x**2/(5*\text{sqrt}(5*x**4 + 25)) - 3/(2*\text{sqrt}(x**4 + 5))$

$$3.33 \quad \int \frac{2+3x^2}{x(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=46

$$\frac{3x^2 + 2}{10\sqrt{x^4 + 5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 823, 12, 266, 63, 207}

$$\frac{3x^2 + 2}{10\sqrt{x^4 + 5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]

[Out] (2 + 3*x^2)/(10*sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/sqrt[5]]/(5*sqrt[5])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 823

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_.) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x(5 + x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x(5 + x^2)^{3/2}} dx, x, x^2 \right) \\
 &= \frac{2 + 3x^2}{10\sqrt{5 + x^4}} - \frac{1}{50} \text{Subst} \left(\int -\frac{10}{x\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= \frac{2 + 3x^2}{10\sqrt{5 + x^4}} + \frac{1}{5} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x^2}} dx, x, x^2 \right) \\
 &= \frac{2 + 3x^2}{10\sqrt{5 + x^4}} + \frac{1}{10} \text{Subst} \left(\int \frac{1}{x\sqrt{5 + x}} dx, x, x^4 \right) \\
 &= \frac{2 + 3x^2}{10\sqrt{5 + x^4}} + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-5 + x^2} dx, x, \sqrt{5 + x^4} \right) \\
 &= \frac{2 + 3x^2}{10\sqrt{5 + x^4}} - \frac{\tanh^{-1} \left(\frac{\sqrt{5 + x^4}}{\sqrt{5}} \right)}{5\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{1}{50} \left(\frac{5(3x^2 + 2)}{\sqrt{x^4 + 5}} - 2\sqrt{5} \tanh^{-1} \left(\frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)), x]

[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] - 2*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/50

IntegrateAlgebraic [A] time = 0.21, size = 57, normalized size = 1.24

$$\frac{3x^2 + 2}{10\sqrt{x^4 + 5}} + \frac{2 \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4 + 5}}{\sqrt{5}} \right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)), x]

[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) + (2*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/(5*Sqrt[5])

fricas [A] time = 0.54, size = 61, normalized size = 1.33

$$\frac{15x^4 + 2\sqrt{5}(x^4 + 5) \log\left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{x^2}\right) + 5\sqrt{x^4 + 5}(3x^2 + 2) + 75}{50(x^4 + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2), x, algorithm="fricas")

[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)

giac [A] time = 0.25, size = 61, normalized size = 1.33

$$\frac{1}{25} \sqrt{5} \log\left(x^2 + \sqrt{5} - \sqrt{x^4 + 5}\right) - \frac{1}{25} \sqrt{5} \log\left(-x^2 + \sqrt{5} + \sqrt{x^4 + 5}\right) + \frac{3x^2 + 2}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2), x, algorithm="giac")

[Out] $1/25*\sqrt{5}*\log(x^2 + \sqrt{5} - \sqrt{x^4 + 5}) - 1/25*\sqrt{5}*\log(-x^2 + \sqrt{5} + \sqrt{x^4 + 5}) + 1/10*(3*x^2 + 2)/\sqrt{x^4 + 5}$

maple [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{3x^2}{10\sqrt{x^4 + 5}} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4 + 5}}\right)}{25} + \frac{1}{5\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((3*x^2+2)/x/(x^4+5)^{(3/2)}, x)$

[Out] $3/10/(x^4+5)^{(1/2)}*x^2+1/5/(x^4+5)^{(1/2)}-1/25*5^{(1/2)}*\operatorname{arctanh}(5^{(1/2)}/(x^4+5)^{(1/2)})$

maxima [A] time = 1.41, size = 56, normalized size = 1.22

$$\frac{3x^2}{10\sqrt{x^4 + 5}} + \frac{1}{50}\sqrt{5} \log\left(-\frac{\sqrt{5} - \sqrt{x^4 + 5}}{\sqrt{5} + \sqrt{x^4 + 5}}\right) + \frac{1}{5\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((3*x^2+2)/x/(x^4+5)^{(3/2)}, x, \operatorname{algorithm}="maxima")$

[Out] $3/10*x^2/\sqrt{x^4 + 5} + 1/50*\sqrt{5}*\log(-(\sqrt{5} - \sqrt{x^4 + 5})/(\sqrt{5} + \sqrt{x^4 + 5})) + 1/5/\sqrt{x^4 + 5}$

mupad [B] time = 0.48, size = 40, normalized size = 0.87

$$\frac{1}{5\sqrt{x^4 + 5}} - \frac{\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{25} + \frac{3x^2}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((3*x^2 + 2)/(x*(x^4 + 5)^{(3/2)}), x)$

[Out] $1/(5*(x^4 + 5)^{(1/2)}) - (5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*(x^4 + 5)^{(1/2)})/5))/25 + (3*x^2)/(10*(x^4 + 5)^{(1/2)})$

sympy [B] time = 19.62, size = 212, normalized size = 4.61

$$\frac{2x^4 \log(x^4)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{4x^4 \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{2x^4 \log(5)}{20\sqrt{5}x^4 + 100\sqrt{5}} + \frac{3x^2}{10\sqrt{x^4 + 5}} + \frac{4\sqrt{5}\sqrt{x^4 + 5}}{20\sqrt{5}x^4 + 100\sqrt{5}} + \frac{10\log(x^4)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{20\log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{10\log(5)}{20\sqrt{5}x^4 + 100\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5))
```

$$3.34 \quad \int \frac{2+3x^2}{x^3(5+x^4)^{3/2}} dx$$

Optimal. Leaf size=65

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}} + \frac{3x^2+2}{10x^2\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{25x^2}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1252, 823, 807, 266, 63, 207}

$$\frac{3x^2+2}{10x^2\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{25x^2} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]

[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+3x}{x^2(5+x^2)^{3/2}} dx, x, x^2 \right) \\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{1}{50} \text{Subst} \left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x^2}} dx, x, x^2 \right) \\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{20} \text{Subst} \left(\int \frac{1}{x\sqrt{5+x}} dx, x, x^4 \right) \\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} + \frac{3}{10} \text{Subst} \left(\int \frac{1}{-5+x^2} dx, x, \sqrt{5+x^4} \right) \\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}} - \frac{2\sqrt{5+x^4}}{25x^2} - \frac{3 \tanh^{-1} \left(\frac{\sqrt{5+x^4}}{\sqrt{5}} \right)}{10\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.69

$$\frac{15x^2 {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{x^4}{5} + 1 \right) - 4x^4 - 10}{50x^2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)), x]

[Out] (-10 - 4*x^4 + 15*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^4/5])/(50*x^2*Sqrt[5 + x^4])

IntegrateAlgebraic [A] time = 0.22, size = 65, normalized size = 1.00

$$\frac{-4x^4 + 15x^2 - 10}{50x^2\sqrt{x^4+5}} + \frac{3 \tanh^{-1} \left(\frac{x^2}{\sqrt{5}} - \frac{\sqrt{x^4+5}}{\sqrt{5}} \right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)), x]

[Out] (-10 + 15*x^2 - 4*x^4)/(50*x^2*Sqrt[5 + x^4]) + (3*ArcTanh[x^2/Sqrt[5] - Sqrt[5 + x^4]/Sqrt[5]])/(5*Sqrt[5])

fricas [A] time = 0.63, size = 77, normalized size = 1.18

$$\frac{4x^6 - 3\sqrt{5}(x^6 + 5x^2)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) + 20x^2 + (4x^4 - 15x^2 + 10)\sqrt{x^4+5}}{50(x^6 + 5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")

[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)

giac [A] time = 0.24, size = 82, normalized size = 1.26

$$\frac{3}{50}\sqrt{5}\log\left(-\frac{x^2 + \sqrt{5} - \sqrt{x^4 + 5}}{x^2 - \sqrt{5} - \sqrt{x^4 + 5}}\right) - \frac{2x^2 - 15}{50\sqrt{x^4 + 5}} + \frac{2}{5\left((x^2 - \sqrt{x^4 + 5})^2 - 5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")

[Out] 3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2 - 5)

maple [A] time = 0.01, size = 47, normalized size = 0.72

$$-\frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50} - \frac{2x^4 + 5}{25\sqrt{x^4 + 5}x^2} + \frac{3}{10\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x)

[Out] 3/10/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/25/x^2*(2*x^4+5)/(x^4+5)^(1/2)

maxima [A] time = 1.10, size = 68, normalized size = 1.05

$$-\frac{x^2}{25\sqrt{x^4+5}} + \frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{10\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")

[Out] $-1/25*x^2/\sqrt{x^4 + 5} + 3/100*\sqrt{5}*\log(-(\sqrt{5} - \sqrt{x^4 + 5})/(\sqrt{5} + \sqrt{x^4 + 5})) + 3/10/\sqrt{x^4 + 5} - 1/25*\sqrt{x^4 + 5}/x^2$

mupad [B] time = 0.54, size = 47, normalized size = 0.72

$$\frac{3}{10\sqrt{x^4 + 5}} - \frac{3\sqrt{5} \operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{50} - \frac{2x^4 + 5}{25x^2\sqrt{x^4 + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)

[Out] $3/(10*(x^4 + 5)^{(1/2)}) - (3*5^{(1/2)}*\operatorname{atanh}((5^{(1/2)}*(x^4 + 5)^{(1/2)})/5))/50 - (2*x^4 + 5)/(25*x^2*(x^4 + 5)^{(1/2)})$

sympy [B] time = 12.98, size = 228, normalized size = 3.51

$$\frac{3x^4 \log(x^4)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{6x^4 \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{3x^4 \log(5)}{20\sqrt{5}x^4 + 100\sqrt{5}} + \frac{6\sqrt{5}\sqrt{x^4 + 5}}{20\sqrt{5}x^4 + 100\sqrt{5}} + \frac{15 \log(x^4)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{30 \log\left(\sqrt{\frac{x^4}{5} + 1} + 1\right)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{15 \log(5)}{20\sqrt{5}x^4 + 100\sqrt{5}} - \frac{2}{25\sqrt{1 + \frac{5}{x^4}}} - \frac{1}{5x^4\sqrt{1 + \frac{5}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)

[Out] $3*x**4*\log(x**4)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) - 6*x**4*\log(\sqrt{x**4/5 + 1} + 1)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) - 3*x**4*\log(5)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) + 6*\sqrt{5}*\sqrt{x**4 + 5}/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) + 15*\log(x**4)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) - 30*\log(\sqrt{x**4/5 + 1} + 1)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) - 15*\log(5)/(20*\sqrt{5}*x**4 + 100*\sqrt{5}) - 2/(25*\sqrt{1 + 5/x**4}) - 1/(5*x**4*\sqrt{1 + 5/x**4})$

$$3.35 \quad \int (fx)^m (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=269

$$\frac{(d + 10e)(fx)^{m+21}}{f^{21}(m + 21)} + \frac{5(2d + 9e)(fx)^{m+19}}{f^{19}(m + 19)} + \frac{15(3d + 8e)(fx)^{m+17}}{f^{17}(m + 17)} + \frac{30(4d + 7e)(fx)^{m+15}}{f^{15}(m + 15)} + \frac{42(5d + 6e)(fx)^{m+13}}{f^{13}(m + 13)} + \frac{42(6d + 5e)(fx)^{m+11}}{f^{11}(m + 11)} + \frac{42(5d + 6e)(fx)^{m+13}}{f^{13}(m + 13)} + \frac{30(4d + 7e)(fx)^{m+15}}{f^{15}(m + 15)} + \frac{15(3d + 8e)(fx)^{m+17}}{f^{17}(m + 17)} + \frac{5(2d + 9e)(fx)^{m+19}}{f^{19}(m + 19)} + \frac{(d + 10e)(fx)^{m+21}}{f^{21}(m + 21)} + \frac{d(fx)^{m+1}}{f(m + 1)} + \frac{e(fx)^{m+23}}{f^{23}(m + 23)}$$

Rubi [A] time = 0.16, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {28, 448}

$$\frac{(10d + e)(fx)^{m+3}}{f^3(m + 3)} + \frac{5(9d + 2e)(fx)^{m+5}}{f^5(m + 5)} + \frac{15(8d + 3e)(fx)^{m+7}}{f^7(m + 7)} + \frac{30(7d + 4e)(fx)^{m+9}}{f^9(m + 9)} + \frac{42(6d + 5e)(fx)^{m+11}}{f^{11}(m + 11)} + \frac{42(5d + 6e)(fx)^{m+13}}{f^{13}(m + 13)} + \frac{30(4d + 7e)(fx)^{m+15}}{f^{15}(m + 15)} + \frac{15(3d + 8e)(fx)^{m+17}}{f^{17}(m + 17)} + \frac{5(2d + 9e)(fx)^{m+19}}{f^{19}(m + 19)} + \frac{(d + 10e)(fx)^{m+21}}{f^{21}(m + 21)} + \frac{d(fx)^{m+1}}{f(m + 1)} + \frac{e(fx)^{m+23}}{f^{23}(m + 23)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) + (5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d + 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d + 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^19*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 + m))/(f^23*(23 + m))

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int (fx)^m (1 + x^2)^{10} (d + ex^2) dx \\
&= \int \left(d(fx)^m + \frac{(10d + e)(fx)^{2+m}}{f^2} + \frac{5(9d + 2e)(fx)^{4+m}}{f^4} + \frac{15(8d + 3e)(fx)^6}{f^6} \right. \\
&= \frac{d(fx)^{1+m}}{f(1+m)} + \frac{(10d + e)(fx)^{3+m}}{f^3(3+m)} + \frac{5(9d + 2e)(fx)^{5+m}}{f^5(5+m)} + \left. \frac{15(8d + 3e)(fx)^{7+m}}{f^7(7+m)} \right) dx
\end{aligned}$$

Mathematica [A] time = 0.56, size = 189, normalized size = 0.70

$$x(fx)^m \left(\frac{x^{20}(d+10e)}{m+21} + \frac{5x^{18}(2d+9e)}{m+19} + \frac{15x^{16}(3d+8e)}{m+17} + \frac{30x^{14}(4d+7e)}{m+15} + \frac{42x^{12}(5d+6e)}{m+13} + \frac{42x^{10}(6d+5e)}{m+11} + \frac{30x^8(7d+4e)}{m+9} + \frac{15x^6(8d+3e)}{m+7} + \frac{5x^4(9d+2e)}{m+5} + \frac{x^2(10d+e)}{m+3} + \frac{d}{m+1} + \frac{ex^{22}}{m+23} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 + m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 + m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))

IntegrateAlgebraic [F] time = 1.20, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.82, size = 1571, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 51069018*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 24372200061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 123*(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178*(d + 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 29170139

$$\begin{aligned}
&70*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2 \\
&+ 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^{21} + 5*((2* \\
&d + 9*e)*m^{11} + 125*(2*d + 9*e)*m^{10} + 6843*(2*d + 9*e)*m^9 + 215823*(2*d + \\
&9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 524676662* \\
&(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d + 9*e)*m^3 \\
&+ 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 33287804550*d + \\
&149795120475*e)*x^{19} + 15*((3*d + 8*e)*m^{11} + 127*(3*d + 8*e)*m^{10} + 7059* \\
&(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198 \\
&94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d + 8*e)*m^4 \\
&+ 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2 + 40283194455* \\
&(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^{17} + 30*((4*d + 7*e)*m^{11} \\
&+ 129*(4*d + 7*e)*m^{10} + 7283*(4*d + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4 \\
&890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m \\
&^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829 \\
&*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 147575933505 \\
&e)*x^{15} + 42*((5*d + 6*e)*m^{11} + 131*(5*d + 6*e)*m^{10} + 7515*(5*d + 6*e)*m \\
&^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*m^7 + 72748638*(5*d + 6*e) \\
&)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264053730*(5*d + 6*e)*m^4 + 1714556090 \\
&1*(5*d + 6*e)*m^3 + 41408337231*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m \\
&+ 121628516625*d + 145954219950*e)*x^{13} + 42*((6*d + 5*e)*m^{11} + 133*(6*d \\
&+ 5*e)*m^{10} + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d \\
&+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5 + 4811326 \\
&190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 48110244633*(6*d + 5*e) \\
&)*m^2 + 61333432335*(6*d + 5*e)*m + 172491350850*d + 143742792375*e)*x^{11} + \\
&30*((7*d + 4*e)*m^{11} + 135*(7*d + 4*e)*m^{10} + 8003*(7*d + 4*e)*m^9 + 273813 \\
&*(7*d + 4*e)*m^8 + 5975466*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847 \\
&550822*(7*d + 4*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4* \\
&e)*m^3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 24595988 \\
&9175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^{11} + 137*(8*d + 3*e)*m^{10} \\
&+ 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 + 6423594*(8*d + 3*e)*m^7 + \\
&95564154*(8*d + 3*e)*m^6 + 959352806*(8*d + 3*e)*m^5 + 6421988758*(8*d + 3 \\
&e)*m^4 + 27624338085*(8*d + 3*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034 \\
&286855*(8*d + 3*e)*m + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e) \\
&)*m^{11} + 139*(9*d + 2*e)*m^{10} + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m \\
&^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 1098746774*(9*d \\
&+ 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(9*d + 2*e)*m^3 + 925 \\
&02445239*(9*d + 2*e)*m^2 + 128033897103*(9*d + 2*e)*m + 569221457805*d + 12 \\
&6493657290*e)*x^5 + ((10*d + e)*m^{11} + 141*(10*d + e)*m^{10} + 8795*(10*d + e) \\
&)*m^9 + 319455*(10*d + e)*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d + \\
&e)*m^6 + 1274046710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 4463230458 \\
&1*(10*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d + e)*m \\
&+ 1054113810750*d + 105411381075*e)*x^3 + (d*m^{11} + 143*d*m^{10} + 9075*d*m^9 \\
&+ 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 1164 \\
&1582810*d*m^4 + 60936676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m + \\
&316234143225*d)*x)*(f*x)^m/(m^{12} + 144*m^{11} + 9218*m^{10} + 345840*m^9 + 843
\end{aligned}$$

9783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)

giac [B] time = 0.46, size = 3752, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] ((f*x)^m*m^11*x^23*e + 121*(f*x)^m*m^10*x^23*e + (f*x)^m*d*m^11*x^21 + 10*(f*x)^m*m^11*x^21*e + 6435*(f*x)^m*m^9*x^23*e + 123*(f*x)^m*d*m^10*x^21 + 1230*(f*x)^m*m^10*x^21*e + 197835*(f*x)^m*m^8*x^23*e + 10*(f*x)^m*d*m^11*x^19 + 6635*(f*x)^m*d*m^9*x^21 + 45*(f*x)^m*m^11*x^19*e + 66350*(f*x)^m*m^9*x^21*e + 3889578*(f*x)^m*m^7*x^23*e + 1250*(f*x)^m*d*m^10*x^19 + 206505*(f*x)^m*d*m^8*x^21 + 5625*(f*x)^m*m^10*x^19*e + 2065050*(f*x)^m*m^8*x^21*e + 51069018*(f*x)^m*m^6*x^23*e + 45*(f*x)^m*d*m^11*x^17 + 68430*(f*x)^m*d*m^9*x^19 + 4103178*(f*x)^m*d*m^7*x^21 + 120*(f*x)^m*m^11*x^17*e + 307935*(f*x)^m*m^9*x^19*e + 41031780*(f*x)^m*m^7*x^21*e + 453714470*(f*x)^m*m^5*x^23*e + 5715*(f*x)^m*d*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x^19 + 54362574*(f*x)^m*d*m^6*x^21 + 15240*(f*x)^m*m^10*x^17*e + 9712035*(f*x)^m*m^8*x^19*e + 543625740*(f*x)^m*m^6*x^21*e + 2702025590*(f*x)^m*m^4*x^23*e + 120*(f*x)^m*d*m^11*x^15 + 317655*(f*x)^m*d*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 486687830*(f*x)^m*d*m^5*x^21 + 210*(f*x)^m*m^11*x^15*e + 847080*(f*x)^m*m^9*x^17*e + 195261570*(f*x)^m*m^7*x^19*e + 4866878300*(f*x)^m*m^5*x^21*e + 10431670821*(f*x)^m*m^3*x^23*e + 15480*(f*x)^m*d*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 + 580855380*(f*x)^m*d*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^21 + 27090*(f*x)^m*m^10*x^15*e + 27100440*(f*x)^m*m^8*x^17*e + 2613849210*(f*x)^m*m^6*x^19*e + 29170139700*(f*x)^m*m^4*x^21*e + 24372200061*(f*x)^m*m^2*x^23*e + 210*(f*x)^m*d*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^17 + 5246766620*(f*x)^m*d*m^5*x^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 252*(f*x)^m*m^11*x^13*e + 1529430*(f*x)^m*m^9*x^15*e + 552066480*(f*x)^m*m^7*x^17*e + 23610449790*(f*x)^m*m^5*x^19*e + 113209660210*(f*x)^m*m^3*x^21*e + 29985521895*(f*x)^m*m*x^23*e + 27510*(f*x)^m*d*m^10*x^13 + 28391400*(f*x)^m*d*m^8*x^15 + 2804395230*(f*x)^m*d*m^6*x^17 + 31686018220*(f*x)^m*d*m^4*x^19 + 26560342503*(f*x)^m*d*m^2*x^21 + 33012*(f*x)^m*m^10*x^13*e + 49684950*(f*x)^m*m^8*x^15*e + 7478387280*(f*x)^m*m^6*x^17*e + 142587081990*(f*x)^m*m^4*x^19*e + 265603425030*(f*x)^m*m^2*x^21*e + 13749310575*(f*x)^m*x^23*e + 252*(f*x)^m*d*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13 + 586902960*(f*x)^m*d*m^7*x^15 + 25598865870*(f*x)^m*d*m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 + 32778930735*(f*x)^m*d*m*x^21 + 210*(f*x)^m*m^11*x^11*e + 1893780*(f*x)^m*m^9*x^13*e + 1027080180*(f*x)^m*m^7*x^15*e + 68263642320*(f*x)^m*m^5*x^17*e + 556867114785*(f*x)^m*m^3*x^19*e + 327789307350*(f*x)^m*m*x^21*e + 33516*(f*x)^m*d*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 +

$15058768725*(f*x)^m*d*x^{21} + 27930*(f*x)^m*m^{10}*x^{11}*e + 62532540*(f*x)^m*m^{8}*x^{13}*e + 14104952820*(f*x)^m*m^{6}*x^{15}*e + 416013088560*(f*x)^m*m^{4}*x^{17}*e + 1313053121565*(f*x)^m*m^{2}*x^{19}*e + 150587687250*(f*x)^m*x^{21}*e + 210*(f*x)^m*d*m^{11}*x^9 + 1954260*(f*x)^m*d*m^9*x^{11} + 1094918580*(f*x)^m*d*m^7*x^{13} + 74496630480*(f*x)^m*d*m^5*x^{15} + 613938233025*(f*x)^m*d*m^3*x^{17} + 361459164150*(f*x)^m*d*m*x^{19} + 120*(f*x)^m*m^{11}*x^9*e + 1628550*(f*x)^m*m^9*x^{11}*e + 1313902296*(f*x)^m*m^7*x^{13}*e + 130369103340*(f*x)^m*m^5*x^{15}*e + 1637168621400*(f*x)^m*m^3*x^{17}*e + 1626566238675*(f*x)^m*m*x^{19}*e + 28350*(f*x)^m*d*m^{10}*x^9 + 65654820*(f*x)^m*d*m^8*x^{11} + 15277213980*(f*x)^m*d*m^6*x^{13} + 459045550800*(f*x)^m*d*m^4*x^{15} + 1456578341055*(f*x)^m*d*m^2*x^{17} + 166439022750*(f*x)^m*d*x^{19} + 16200*(f*x)^m*m^{10}*x^9*e + 54712350*(f*x)^m*m^8*x^{11}*e + 18332656776*(f*x)^m*m^6*x^{13}*e + 803329713900*(f*x)^m*m^4*x^{15}*e + 3884208909480*(f*x)^m*m^2*x^{17}*e + 748975602375*(f*x)^m*x^{19}*e + 120*(f*x)^m*d*m^{11}*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^{11} + 143339613900*(f*x)^m*d*m^5*x^{13} + 1823707864920*(f*x)^m*d*m^3*x^{15} + 1812743750475*(f*x)^m*d*m*x^{17} + 45*(f*x)^m*m^{11}*x^7*e + 960360*(f*x)^m*m^9*x^9*e + 1170518580*(f*x)^m*m^7*x^{11}*e + 172007536680*(f*x)^m*m^5*x^{13}*e + 3191488763610*(f*x)^m*m^3*x^{15}*e + 4833983334600*(f*x)^m*m*x^{17}*e + 16440*(f*x)^m*d*m^{10}*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x^{11} + 895451283300*(f*x)^m*d*m^4*x^{13} + 4360457499480*(f*x)^m*d*m^2*x^{15} + 837090379125*(f*x)^m*d*x^{17} + 6165*(f*x)^m*m^{10}*x^7*e + 32857560*(f*x)^m*m^8*x^9*e + 16635451140*(f*x)^m*m^6*x^{11}*e + 1074541539960*(f*x)^m*m^4*x^{13}*e + 7630800624090*(f*x)^m*m^2*x^{15}*e + 2232241011000*(f*x)^m*x^{17}*e + 45*(f*x)^m*d*m^{11}*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 + 190744119720*(f*x)^m*d*m^5*x^{11} + 3600567789210*(f*x)^m*d*m^3*x^{13} + 5458672303560*(f*x)^m*d*m*x^{15} + 10*(f*x)^m*m^{11}*x^5*e + 371655*(f*x)^m*m^9*x^7*e + 717055920*(f*x)^m*m^7*x^9*e + 158953433100*(f*x)^m*m^5*x^{11}*e + 4320681347052*(f*x)^m*m^3*x^{13}*e + 9552676531230*(f*x)^m*m*x^{15}*e + 6255*(f*x)^m*d*m^{10}*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 1212454199880*(f*x)^m*d*m^4*x^{11} + 8695750818510*(f*x)^m*d*m^2*x^{13} + 2529873145800*(f*x)^m*d*x^{15} + 1390*(f*x)^m*m^{10}*x^5*e + 12961215*(f*x)^m*m^8*x^7*e + 10410014160*(f*x)^m*m^6*x^9*e + 1010378499900*(f*x)^m*m^4*x^{11}*e + 10434900982212*(f*x)^m*m^2*x^{13}*e + 4427278005150*(f*x)^m*x^{15}*e + 10*(f*x)^m*d*m^{11}*x^3 + 383535*(f*x)^m*d*m^9*x^5 + 770831280*(f*x)^m*d*m^7*x^7 + 177985672620*(f*x)^m*d*m^5*x^9 + 4952725167852*(f*x)^m*d*m^3*x^{11} + 10969925251950*(f*x)^m*d*m*x^{13} + (f*x)^m*m^{11}*x^3*e + 85230*(f*x)^m*m^9*x^5*e + 289061730*(f*x)^m*m^7*x^7*e + 101706098640*(f*x)^m*m^5*x^9*e + 4127270973210*(f*x)^m*m^3*x^{11}*e + 13163910302340*(f*x)^m*m*x^{13}*e + 1410*(f*x)^m*d*m^{10}*x^3 + 13645125*(f*x)^m*d*m^8*x^5 + 11467698480*(f*x)^m*d*m^6*x^7 + 1156995210420*(f*x)^m*d*m^4*x^9 + 12123781647516*(f*x)^m*d*m^2*x^{11} + 5108397698250*(f*x)^m*d*x^{13} + 141*(f*x)^m*m^{10}*x^3*e + 3032250*(f*x)^m*m^8*x^5*e + 4300386930*(f*x)^m*m^6*x^7*e + 661140120240*(f*x)^m*m^4*x^9*e + 10103151372930*(f*x)^m*m^2*x^{11}*e + 6130077237900*(f*x)^m*x^{13}*e + (f*x)^m*d*m^{11}*x + 87950*(f*x)^m*d*m^9*x^3 + 311564610*(f*x)^m*d*m^7*x^5 + 115122336720*(f*x)^m*d*m^5*x^7 + 4828477578330*(f*x)^m*d*m^3*x^9 + 15456024948420*(f*x)^m*d*m*x^{11} + 87$

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95*(f*x)^m*m^9*x^3*e + 69236580*(f*x)^m*m^7*x^5*e + 43170876270*(f*x)^m*m^5
*x^7*e + 2759130044760*(f*x)^m*m^3*x^9*e + 12880020790350*(f*x)^m*m*x^11*e
+ 143*(f*x)^m*d*m^10*x + 3194550*(f*x)^m*d*m^8*x^3 + 4765995990*(f*x)^m*d*m
^6*x^5 + 770638650960*(f*x)^m*d*m^4*x^7 + 12046833873270*(f*x)^m*d*m^2*x^9
+ 7244636735700*(f*x)^m*d*x^11 + 319455*(f*x)^m*m^8*x^3*e + 1059110220*(f*x
)^m*m^6*x^5*e + 288989494110*(f*x)^m*m^4*x^7*e + 6883905070440*(f*x)^m*m^2*
x^9*e + 6037197279750*(f*x)^m*x^11*e + 9075*(f*x)^m*d*m^9*x + 74814180*(f*x
)^m*d*m^7*x^3 + 49443604830*(f*x)^m*d*m^5*x^5 + 3314920570200*(f*x)^m*d*m^3
*x^7 + 15593181033150*(f*x)^m*d*m*x^9 + 7481418*(f*x)^m*m^7*x^3*e + 1098746
7740*(f*x)^m*m^5*x^5*e + 1243095213825*(f*x)^m*m^3*x^7*e + 8910389161800*(f
*x)^m*m*x^9*e + 336765*(f*x)^m*d*m^8*x + 1180850580*(f*x)^m*d*m^6*x^3 + 343
967603850*(f*x)^m*d*m^4*x^5 + 8511631481880*(f*x)^m*d*m^2*x^7 + 73787966752
50*(f*x)^m*d*x^9 + 118085058*(f*x)^m*m^6*x^3*e + 76437245300*(f*x)^m*m^4*x^
5*e + 3191861805705*(f*x)^m*m^2*x^7*e + 4216455243000*(f*x)^m*x^9*e + 81030
18*(f*x)^m*d*m^7*x + 12740467100*(f*x)^m*d*m^5*x^3 + 1546183653345*(f*x)^m*
d*m^3*x^5 + 11284114422600*(f*x)^m*d*m*x^7 + 1274046710*(f*x)^m*m^5*x^3*e +
343596367410*(f*x)^m*m^3*x^5*e + 4231542908475*(f*x)^m*m*x^7*e + 132426294
*(f*x)^m*d*m^6*x + 93153182700*(f*x)^m*d*m^4*x^3 + 4162610035755*(f*x)^m*d*
m^2*x^5 + 5421156741000*(f*x)^m*d*x^7 + 9315318270*(f*x)^m*m^4*x^3*e + 9250
24452390*(f*x)^m*m^2*x^5*e + 2032933777875*(f*x)^m*x^7*e + 1495875590*(f*x)
^m*d*m^5*x + 446323045810*(f*x)^m*d*m^3*x^3 + 5761525369635*(f*x)^m*d*m*x^5
+ 44632304581*(f*x)^m*m^3*x^3*e + 1280338971030*(f*x)^m*m*x^5*e + 11641582
810*(f*x)^m*d*m^4*x + 1304037152010*(f*x)^m*d*m^2*x^3 + 2846107289025*(f*x)
^m*d*x^5 + 130403715201*(f*x)^m*m^2*x^3*e + 632468286450*(f*x)^m*x^5*e + 60
936676581*(f*x)^m*d*m^3*x + 1993349776950*(f*x)^m*d*m*x^3 + 199334977695*(f
*x)^m*m*x^3*e + 203363952363*(f*x)^m*d*m^2*x + 1054113810750*(f*x)^m*d*x^3
+ 105411381075*(f*x)^m*x^3*e + 387182170935*(f*x)^m*d*m*x + 316234143225*(f
*x)^m*d*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529
312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944
*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)

```

maple [B] time = 0.03, size = 2295, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5, x)$

[Out] $(f*x)^m*(e*m^{11}*x^{22}+121*e*m^{10}*x^{22}+d*m^{11}*x^{20}+10*e*m^{11}*x^{20}+6435*e*m^9*x^{22}+123*d*m^{10}*x^{20}+1230*e*m^{10}*x^{20}+197835*e*m^8*x^{22}+10*d*m^{11}*x^{18}+6635*d*m^9*x^{20}+45*e*m^{11}*x^{18}+66350*e*m^9*x^{20}+3889578*e*m^7*x^{22}+1250*d*m^{10}*x^{18}+206505*d*m^8*x^{20}+5625*e*m^{10}*x^{18}+2065050*e*m^8*x^{20}+51069018*e*m^6*x^{22}+45*d*m^{11}*x^{16}+68430*d*m^9*x^{18}+4103178*d*m^7*x^{20}+120*e*m^{11}*x^{16}+307935*e*m^9*x^{18}+41031780*e*m^7*x^{20}+453714470*e*m^5*x^{22}+5715*d*m^{10}*x^{16}+2158230*d*m^8*x^{18}+54362574*d*m^6*x^{20}+15240*e*m^{10}*x^{16}+9712035*e*m^8*x^{18}+54$

3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+27090*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+1529430*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+265603425030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+586902960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+68263642320*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+1637168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+19962541368*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+1074541539960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+3600567789210*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+717055920*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+1212454199880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+10434900982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+10969925251950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+13645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+4300386930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379

$00 * e^x^{12} + d * m^{11} + 87950 * d * m^9 * x^2 + 311564610 * d * m^7 * x^4 + 115122336720 * d * m^5 * x^6$
 $+ 4828477578330 * d * m^3 * x^8 + 15456024948420 * d * m * x^{10} + 8795 * e * m^9 * x^2 + 69236580 * e * m^7 * x^4$
 $+ 43170876270 * e * m^5 * x^6 + 2759130044760 * e * m^3 * x^8 + 12880020790350 * e * m * x^{10} + 143 * d * m^{10} + 3194550 * d * m^8 * x^2$
 $+ 4765995990 * d * m^6 * x^4 + 770638650960 * d * m^4 * x^6 + 12046833873270 * d * m^2 * x^8 + 7244636735700 * d * x^{10} + 319455 * e * m^8 * x^2 + 1059110220 * e * m^6 * x^4$
 $+ 288989494110 * e * m^4 * x^6 + 6883905070440 * e * m^2 * x^8 + 6037197279750 * e * x^{10} + 9075 * d * m^9 + 74814180 * d * m^7 * x^2 + 49443604830 * d * m^5 * x^4 + 3314920570200 * d * m^3 * x^6$
 $+ 15593181033150 * d * m * x^8 + 7481418 * e * m^7 * x^2 + 10987467740 * e * m^5 * x^4 + 1243095213825 * e * m^3 * x^6 + 8910389161800 * e * m * x^8 + 336765 * d * m^8 + 1180850580 * d * m^6 * x^2 + 343967603850 * d * m^4 * x^4 + 8511631481880 * d * m^2 * x^6 + 7378796675250 * d * x^8 + 118085058 * e * m^6 * x^2 + 76437245300 * e * m^4 * x^4 + 3191861805705 * e * m^2 * x^6 + 4216455243000 * e * x^8 + 8103018 * d * m^7 + 12740467100 * d * m^5 * x^2 + 1546183653345 * d * m^3 * x^4 + 11284114422600 * d * m * x^6 + 1274046710 * e * m^5 * x^2 + 343596367410 * e * m^3 * x^4 + 4231542908475 * e * m * x^6 + 132426294 * d * m^6 + 93153182700 * d * m^4 * x^2 + 4162610035755 * d * m^2 * x^4 + 5421156741000 * d * x^6 + 9315318270 * e * m^4 * x^2 + 925024452390 * e * m^2 * x^4 + 2032933777875 * e * x^6 + 1495875590 * d * m^5 + 446323045810 * d * m^3 * x^2 + 5761525369635 * d * m * x^4 + 44632304581 * e * m^3 * x^2 + 1280338971030 * e * m * x^4 + 11641582810 * d * m^4 + 1304037152010 * d * m^2 * x^2 + 2846107289025 * d * x^4 + 130403715201 * e * m^2 * x^2 + 632468286450 * e * x^4 + 60936676581 * d * m^3 + 1993349776950 * d * m * x^2 + 199334977695 * e * m * x^2 + 203363952363 * d * m^2 + 1054113810750 * d * x^2 + 105411381075 * e * x^2 + 387182170935 * d * m + 316234143225 * d) * x / ((1+m) / (3+m) / (5+m) / (7+m) / (9+m) / (11+m) / (13+m) / (15+m) / (17+m) / (19+m) / (21+m) / (23+m))$

maxima [A] time = 0.89, size = 372, normalized size = 1.38

$$\frac{e^{f^m x^{23}}}{m+23} + \frac{d f^m x^{21}}{m+21} + \frac{10 e^{f^m x^{19}}}{m+19} + \frac{45 e^{f^m x^{17}}}{m+17} + \frac{120 e^{f^m x^{15}}}{m+15} + \frac{210 e^{f^m x^{13}}}{m+13} + \frac{252 e^{f^m x^{11}}}{m+11} + \frac{210 e^{f^m x^9}}{m+9} + \frac{120 e^{f^m x^7}}{m+7} + \frac{45 e^{f^m x^5}}{m+5} + \frac{10 e^{f^m x^3}}{m+3} + \frac{e^{f^m x}}{m+1} + \frac{(f x)^d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] $e^{f^m x^{23}} x^m / (m + 23) + d f^m x^{21} x^m / (m + 21) + 10 e^{f^m x^{19}} x^m / (m + 21) + 10 d f^m x^{19} x^m / (m + 19) + 45 e^{f^m x^{17}} x^m / (m + 19) + 45 d f^m x^{17} x^m / (m + 17) + 120 e^{f^m x^{15}} x^m / (m + 17) + 120 d f^m x^{15} x^m / (m + 15) + 210 e^{f^m x^{13}} x^m / (m + 15) + 210 d f^m x^{13} x^m / (m + 13) + 252 e^{f^m x^{11}} x^m / (m + 13) + 252 d f^m x^{11} x^m / (m + 11) + 210 e^{f^m x^9} x^m / (m + 11) + 210 d f^m x^9 x^m / (m + 9) + 120 e^{f^m x^7} x^m / (m + 9) + 120 d f^m x^7 x^m / (m + 7) + 45 e^{f^m x^5} x^m / (m + 7) + 45 d f^m x^5 x^m / (m + 5) + 10 e^{f^m x^3} x^m / (m + 5) + 10 d f^m x^3 x^m / (m + 3) + e^{f^m x} x^m / (m + 3) + (f x)^m (m + 1) * d / (f * (m + 1))$

mupad [B] time = 1.78, size = 1539, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)


```
[Out] (d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 1164158
2810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 9075
*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 1
40529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316
234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431670821
*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835
*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863*m + 363371
45829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 + 67166442*m^6
+ 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 21082276215))/(7
03416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131
37458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x)^m*(5*d + 6*e)*
(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 4264053730*m^4 + 68256
9590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515*m^9 + 131*m^10 +
m^11 + 24325703325))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3
+ 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439
783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (30*x^
9*(f*x)^m*(7*d + 4*e)*(74253243015*m + 57365875587*m^2 + 22992750373*m^3 +
5509501002*m^4 + 847550822*m^5 + 86750118*m^6 + 5975466*m^7 + 273813*m^8 +
8003*m^9 + 135*m^10 + m^11 + 35137127025))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
+ 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 +
316234143225) + (x^3*(f*x)^m*(10*d + e)*(199334977695*m + 130403715201*m^2
+ 44632304581*m^3 + 9315318270*m^4 + 1274046710*m^5 + 118085058*m^6 + 74814
18*m^7 + 319455*m^8 + 8795*m^9 + 141*m^10 + m^11 + 105411381075))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (5*x^19*(f*x)^m*(2*d + 9*e)*(3614591
6415*m + 29178958257*m^2 + 12374824773*m^3 + 3168601822*m^4 + 524676662*m^5
+ 58085538*m^6 + 4339146*m^7 + 215823*m^8 + 6843*m^9 + 125*m^10 + m^11 + 1
6643902275))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782
59391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^11*(f*x)
^m*(6*d + 5*e)*(61333432335*m + 48110244633*m^2 + 19653671301*m^3 + 4811326
190*m^4 + 756921110*m^5 + 79216434*m^6 + 5573898*m^7 + 260535*m^8 + 7755*m^
9 + 133*m^10 + m^11 + 28748558475))/(703416314160*m + 590546123298*m^2 + 26
4300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 14052
9312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 3162341
43225) + (15*x^7*(f*x)^m*(8*d + 3*e)*(94034286855*m + 70930262349*m^2 + 276
24338085*m^3 + 6421988758*m^4 + 959352806*m^5 + 95564154*m^6 + 6423594*m^7
+ 288027*m^8 + 8259*m^9 + 137*m^10 + m^11 + 45176306175))/(703416314160*m +
```

$$\frac{590546123298m^2 + 264300628944m^3 + 72578259391m^4 + 13137458400m^5 + 1628301884m^6 + 140529312m^7 + 8439783m^8 + 345840m^9 + 9218m^{10} + 144m^{11} + m^{12} + 316234143225) + (5x^5(fx)^m(9d + 2e)(128033897103m + 92502445239m^2 + 34359636741m^3 + 7643724530m^4 + 1098746774m^5 + 105911022m^6 + 6923658m^7 + 303225m^8 + 8523m^9 + 139m^{10} + m^{11} + 63246828645))}{(703416314160m + 590546123298m^2 + 264300628944m^3 + 72578259391m^4 + 13137458400m^5 + 1628301884m^6 + 140529312m^7 + 8439783m^8 + 345840m^9 + 9218m^{10} + 144m^{11} + m^{12} + 316234143225) + (15x^{17}(fx)^m(3d + 8e)(40283194455m + 32368407579m^2 + 13643071845m^3 + 3466775738m^4 + 568863686m^5 + 62319894m^6 + 4600554m^7 + 225837m^8 + 7059m^9 + 127m^{10} + m^{11} + 18602008425))}{(703416314160m + 590546123298m^2 + 264300628944m^3 + 72578259391m^4 + 13137458400m^5 + 1628301884m^6 + 140529312m^7 + 8439783m^8 + 345840m^9 + 9218m^{10} + 144m^{11} + m^{12} + 316234143225) + (x^{21}(fx)^m(d + 10e)(32778930735m + 26560342503m^2 + 11320966021m^3 + 2917013970m^4 + 486687830m^5 + 54362574m^6 + 4103178m^7 + 206505m^8 + 6635m^9 + 123m^{10} + m^{11} + 15058768725))}{(703416314160m + 590546123298m^2 + 264300628944m^3 + 72578259391m^4 + 13137458400m^5 + 1628301884m^6 + 140529312m^7 + 8439783m^8 + 345840m^9 + 9218m^{10} + 144m^{11} + m^{12} + 316234143225)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)

[Out] Timed out

$$3.36 \quad \int x^5 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=63

$$\frac{1}{26} (x^2 + 1)^{13} (d - 3e) - \frac{1}{24} (x^2 + 1)^{12} (2d - 3e) + \frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{28} e (x^2 + 1)^{14}$$

Rubi [A] time = 0.20, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 446, 76}

$$\frac{1}{26} (x^2 + 1)^{13} (d - 3e) - \frac{1}{24} (x^2 + 1)^{12} (2d - 3e) + \frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{28} e (x^2 + 1)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 + x^2)^13)/26 + (e*(1 + x^2)^14)/28

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 76

Int[((d_)*(x_)^(n_))*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_)*((c_) + (d_)*(x_)^(n_)]^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^5 (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int x^5 (1 + x^2)^{10} (d + ex^2) dx \\
&= \frac{1}{2} \text{Subst} \left(\int x^2 (1 + x)^{10} (d + ex) dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int ((d - e)(1 + x)^{10} + (-2d + 3e)(1 + x)^{11} + (d - 3e)(1 + x)^{12} + e(1 + x)^{13}) dx, x, x^2 \right) \\
&= \frac{1}{22} (d - e) (1 + x^2)^{11} - \frac{1}{24} (2d - 3e) (1 + x^2)^{12} + \frac{1}{26} (d - 3e) (1 + x^2)^{13} + \frac{1}{28} e (1 + x^2)^{14}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 153, normalized size = 2.43

$$\frac{1}{26}x^{26}(d+10e) + \frac{5}{24}x^{24}(2d+9e) + \frac{15}{22}x^{22}(3d+8e) + \frac{3}{2}x^{20}(4d+7e) + \frac{7}{3}x^{18}(5d+6e) + \frac{21}{8}x^{16}(6d+5e) + \frac{15}{7}x^{14}(7d+4e) + \frac{5}{4}x^{12}(8d+3e) + \frac{1}{2}x^{10}(9d+2e) + \frac{1}{8}x^8(10d+e) + \frac{dx^6}{6} + \frac{ex^{28}}{28}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.51, size = 132, normalized size = 2.10

$$\frac{1}{28}x^{28}e + \frac{5}{13}x^{26}e + \frac{1}{26}x^{26}d + \frac{15}{8}x^{24}e + \frac{5}{12}x^{24}d + \frac{60}{11}x^{22}e + \frac{45}{22}x^{22}d + \frac{21}{2}x^{20}e + 6x^{20}d + 14x^{18}e + \frac{35}{3}x^{18}d + \frac{105}{8}x^{16}e + \frac{63}{4}x^{16}d + \frac{60}{7}x^{14}e + 15x^{14}d + \frac{15}{4}x^{12}e + 10x^{12}d + x^{10}e + \frac{9}{2}x^{10}d + \frac{1}{8}x^8e + \frac{5}{4}x^8d + \frac{1}{6}x^6d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/28*x^28*e + 5/13*x^26*e + 1/26*x^26*d + 15/8*x^24*e + 5/12*x^24*d + 60/11*x^22*e + 45/22*x^22*d + 21/2*x^20*e + 6*x^20*d + 14*x^18*e + 35/3*x^18*d + 105/8*x^16*e + 63/4*x^16*d + 60/7*x^14*e + 15*x^14*d + 15/4*x^12*e + 10*x^12*d + x^10*e + 9/2*x^10*d + 1/8*x^8*e + 5/4*x^8*d + 1/6*x^6*d

giac [B] time = 0.28, size = 143, normalized size = 2.27

$$\frac{1}{28}x^{28}e + \frac{1}{26}dx^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}dx^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}dx^{22} + \frac{60}{11}x^{22}e + 6dx^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}dx^{18} + 14x^{18}e + \frac{63}{4}dx^{16} + \frac{105}{8}x^{16}e + 15dx^{14} + \frac{60}{7}x^{14}e + 10dx^{12} + \frac{15}{4}x^{12}e + \frac{9}{2}dx^{10} + x^{10}e + \frac{5}{4}dx^8 + \frac{1}{8}x^8e + \frac{1}{6}dx^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] $\frac{1}{28}x^{28}e + \frac{1}{26}d*x^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}d*x^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}d*x^{22} + \frac{60}{11}x^{22}e + 6d*x^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}d*x^{18} + 14x^{18}e + \frac{63}{4}d*x^{16} + \frac{105}{8}x^{16}e + 15d*x^{14} + \frac{60}{7}x^{14}e + 10d*x^{12} + \frac{15}{4}x^{12}e + \frac{9}{2}d*x^{10} + x^{10}e + \frac{5}{4}d*x^8 + \frac{1}{8}x^8e + \frac{1}{6}d*x^6$

maple [B] time = 0.00, size = 130, normalized size = 2.06

$$\frac{e x^{28}}{28} + \frac{(d+10e)x^{26}}{26} + \frac{(10d+45e)x^{24}}{24} + \frac{(45d+120e)x^{22}}{22} + \frac{(120d+210e)x^{20}}{20} + \frac{(210d+252e)x^{18}}{18} + \frac{(252d+210e)x^{16}}{16} + \frac{(210d+120e)x^{14}}{14} + \frac{(120d+45e)x^{12}}{12} + \frac{(45d+10e)x^{10}}{10} + \frac{(10d+e)x^8}{8} + \frac{d x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x)

[Out] $\frac{1}{28}e*x^{28} + \frac{1}{26}*(d+10*e)*x^{26} + \frac{1}{24}*(10*d+45*e)*x^{24} + \frac{1}{22}*(45*d+120*e)*x^{22} + \frac{1}{20}*(120*d+210*e)*x^{20} + \frac{1}{18}*(210*d+252*e)*x^{18} + \frac{1}{16}*(252*d+210*e)*x^{16} + \frac{1}{14}*(210*d+120*e)*x^{14} + \frac{1}{12}*(120*d+45*e)*x^{12} + \frac{1}{10}*(45*d+10*e)*x^{10} + \frac{1}{8}*(10*d+e)*x^8 + \frac{1}{6}d*x^6$

maxima [B] time = 0.59, size = 129, normalized size = 2.05

$$\frac{1}{28}e x^{28} + \frac{1}{26}(d+10e)x^{26} + \frac{5}{24}(2d+9e)x^{24} + \frac{15}{22}(3d+8e)x^{22} + \frac{3}{2}(4d+7e)x^{20} + \frac{7}{3}(5d+6e)x^{18} + \frac{21}{8}(6d+5e)x^{16} + \frac{15}{7}(7d+4e)x^{14} + \frac{5}{4}(8d+3e)x^{12} + \frac{1}{2}(9d+2e)x^{10} + \frac{1}{8}(10d+e)x^8 + \frac{1}{6}d x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] $\frac{1}{28}e*x^{28} + \frac{1}{26}*(d+10*e)*x^{26} + \frac{5}{24}*(2*d+9*e)*x^{24} + \frac{15}{22}*(3*d+8*e)*x^{22} + \frac{3}{2}*(4*d+7*e)*x^{20} + \frac{7}{3}*(5*d+6*e)*x^{18} + \frac{21}{8}*(6*d+5*e)*x^{16} + \frac{15}{7}*(7*d+4*e)*x^{14} + \frac{5}{4}*(8*d+3*e)*x^{12} + \frac{1}{2}*(9*d+2*e)*x^{10} + \frac{1}{8}*(10*d+e)*x^8 + \frac{1}{6}d*x^6$

mupad [B] time = 0.09, size = 121, normalized size = 1.92

$$\frac{e x^{28}}{28} + \left(\frac{d}{26} + \frac{5e}{13}\right)x^{26} + \left(\frac{5d}{12} + \frac{15e}{8}\right)x^{24} + \left(\frac{45d}{22} + \frac{60e}{11}\right)x^{22} + \left(6d + \frac{21e}{2}\right)x^{20} + \left(\frac{35d}{3} + 14e\right)x^{18} + \left(\frac{63d}{4} + \frac{105e}{8}\right)x^{16} + \left(15d + \frac{60e}{7}\right)x^{14} + \left(10d + \frac{15e}{4}\right)x^{12} + \left(\frac{9d}{2} + e\right)x^{10} + \left(\frac{5d}{4} + \frac{e}{8}\right)x^8 + \frac{d x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] $x^8 \cdot \left(\frac{5d}{4} + \frac{e}{8}\right) + x^{12} \cdot \left(\frac{10d}{3} + \frac{15e}{4}\right) + x^{20} \cdot \left(\frac{6d}{2} + \frac{21e}{2}\right) + x^{24} \cdot \left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{18} \cdot \left(\frac{35d}{3} + 14e\right) + x^{26} \cdot \left(\frac{d}{26} + \frac{5e}{13}\right) + x^{14} \cdot \left(\frac{15d}{7} + \frac{60e}{7}\right) + x^{22} \cdot \left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{16} \cdot \left(\frac{63d}{4} + \frac{105e}{8}\right) + \frac{d \cdot x^6}{6} + \frac{e \cdot x^{28}}{28} + x^{10} \cdot \left(\frac{9d}{2} + e\right)$

sympy [B] time = 0.10, size = 134, normalized size = 2.13

$$\frac{dx^6}{6} + \frac{ex^{28}}{28} + x^{26} \left(\frac{d}{26} + \frac{5e}{13}\right) + x^{24} \left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{22} \left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{20} \left(\frac{6d}{2} + \frac{21e}{2}\right) + x^{18} \left(\frac{35d}{3} + 14e\right) + x^{16} \left(\frac{63d}{4} + \frac{105e}{8}\right) + x^{14} \left(\frac{15d}{7} + \frac{60e}{7}\right) + x^{12} \left(\frac{10d}{3} + \frac{15e}{4}\right) + x^{10} \left(\frac{9d}{2} + e\right) + x^8 \left(\frac{5d}{4} + \frac{e}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)`

[Out] $d \cdot x^{26} / 6 + e \cdot x^{28} / 28 + x^{26} \cdot \left(\frac{d}{26} + \frac{5e}{13}\right) + x^{24} \cdot \left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{22} \cdot \left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{20} \cdot \left(\frac{6d}{2} + \frac{21e}{2}\right) + x^{18} \cdot \left(\frac{35d}{3} + 14e\right) + x^{16} \cdot \left(\frac{63d}{4} + \frac{105e}{8}\right) + x^{14} \cdot \left(\frac{15d}{7} + \frac{60e}{7}\right) + x^{12} \cdot \left(\frac{10d}{3} + \frac{15e}{4}\right) + x^{10} \cdot \left(\frac{9d}{2} + e\right) + x^8 \cdot \left(\frac{5d}{4} + \frac{e}{8}\right)$

$$3.37 \quad \int x^4 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=153

$$\frac{1}{25}x^{25}(d+10e) + \frac{5}{23}x^{23}(2d+9e) + \frac{5}{7}x^{21}(3d+8e) + \frac{30}{19}x^{19}(4d+7e) + \frac{42}{17}x^{17}(5d+6e) + \frac{14}{5}x^{15}(6d+5e) + \frac{30}{13}x^{13}(7d+4e) + \frac{15}{11}x^{11}(8d+3e) + \frac{5}{9}x^9(9d+2e) + \frac{1}{7}x^7(10d+e) + \frac{dx^5}{5} + \frac{ex^{27}}{27}$$

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{25}x^{25}(d+10e) + \frac{5}{23}x^{23}(2d+9e) + \frac{5}{7}x^{21}(3d+8e) + \frac{30}{19}x^{19}(4d+7e) + \frac{42}{17}x^{17}(5d+6e) + \frac{14}{5}x^{15}(6d+5e) + \frac{30}{13}x^{13}(7d+4e) + \frac{15}{11}x^{11}(8d+3e) + \frac{5}{9}x^9(9d+2e) + \frac{1}{7}x^7(10d+e) + \frac{dx^5}{5} + \frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^4 (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int x^4 (1 + x^2)^{10} (d + ex^2) dx \\ &= \int (dx^4 + (10d + e)x^6 + 5(9d + 2e)x^8 + 15(8d + 3e)x^{10} + 30(7d + 4e)x^{12} + \\ &= \frac{dx^5}{5} + \frac{1}{7}(10d + e)x^7 + \frac{5}{9}(9d + 2e)x^9 + \frac{15}{11}(8d + 3e)x^{11} + \frac{30}{13}(7d + 4e)x^{13} + \end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{25}x^{25}(d+10e) + \frac{5}{23}x^{23}(2d+9e) + \frac{5}{7}x^{21}(3d+8e) + \frac{30}{19}x^{19}(4d+7e) + \frac{42}{17}x^{17}(5d+6e) + \frac{14}{5}x^{15}(6d+5e) + \frac{30}{13}x^{13}(7d+4e) + \frac{15}{11}x^{11}(8d+3e) + \frac{5}{9}x^9(9d+2e) + \frac{1}{7}x^7(10d+e) + \frac{dx^5}{5} + \frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 0.87, size = 133, normalized size = 0.87

$$\frac{1}{27}x^{27}e + \frac{2}{5}x^{25}e + \frac{1}{25}x^{25}d + \frac{45}{23}x^{23}e + \frac{10}{23}x^{23}d + \frac{40}{7}x^{21}e + \frac{15}{7}x^{21}d + \frac{210}{19}x^{19}e + \frac{120}{19}x^{19}d + \frac{252}{17}x^{17}e + \frac{210}{17}x^{17}d + 14x^{15}e + \frac{84}{5}x^{15}d + \frac{120}{13}x^{13}e + \frac{210}{13}x^{13}d + \frac{45}{11}x^{11}e + \frac{120}{11}x^{11}d + \frac{10}{9}x^9e + 5x^9d + \frac{1}{7}x^7e + \frac{10}{7}x^7d + \frac{1}{5}x^5d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/27*x^27*e + 2/5*x^25*e + 1/25*x^25*d + 45/23*x^23*e + 10/23*x^23*d + 40/7*x^21*e + 15/7*x^21*d + 210/19*x^19*e + 120/19*x^19*d + 252/17*x^17*e + 210/17*x^17*d + 14*x^15*e + 84/5*x^15*d + 120/13*x^13*e + 210/13*x^13*d + 45/11*x^11*e + 120/11*x^11*d + 10/9*x^9*e + 5*x^9*d + 1/7*x^7*e + 10/7*x^7*d + 1/5*x^5*d

giac [A] time = 0.30, size = 144, normalized size = 0.94

$$\frac{1}{27}x^{27}e + \frac{1}{25}dx^{25} + \frac{2}{5}x^{25}e + \frac{10}{23}dx^{23} + \frac{45}{23}x^{23}e + \frac{15}{7}dx^{21} + \frac{40}{7}x^{21}e + \frac{120}{19}dx^{19} + \frac{210}{19}x^{19}e + \frac{210}{17}dx^{17} + \frac{252}{17}x^{17}e + \frac{84}{5}dx^{15} + 14x^{15}e + \frac{210}{13}dx^{13} + \frac{120}{13}x^{13}e + \frac{120}{11}dx^{11} + \frac{45}{11}x^{11}e + 5dx^9 + \frac{10}{9}x^9e + \frac{10}{7}dx^7 + \frac{1}{7}x^7e + \frac{1}{5}dx^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^23*e + 15/7*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e + 210/17*d*x^17 + 252

$/17*x^{17}*e + 84/5*d*x^{15} + 14*x^{15}*e + 210/13*d*x^{13} + 120/13*x^{13}*e + 120/11*d*x^{11} + 45/11*x^{11}*e + 5*d*x^9 + 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e + 1/5*d*x^5$

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e x^{27}}{27} + \frac{(d+10e)x^{25}}{25} + \frac{(10d+45e)x^{23}}{23} + \frac{(45d+120e)x^{21}}{21} + \frac{(120d+210e)x^{19}}{19} + \frac{(210d+252e)x^{17}}{17} + \frac{(252d+210e)x^{15}}{15} + \frac{(210d+120e)x^{13}}{13} + \frac{(120d+45e)x^{11}}{11} + \frac{(45d+10e)x^9}{9} + \frac{(10d+e)x^7}{7} + \frac{d x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5, x)$

[Out] $1/27*e*x^{27}+1/25*(d+10*e)*x^{25}+1/23*(10*d+45*e)*x^{23}+1/21*(45*d+120*e)*x^{21}+1/19*(120*d+210*e)*x^{19}+1/17*(210*d+252*e)*x^{17}+1/15*(252*d+210*e)*x^{15}+1/13*(210*d+120*e)*x^{13}+1/11*(120*d+45*e)*x^{11}+1/9*(45*d+10*e)*x^9+1/7*(10*d+e)*x^7+1/5*d*x^5$

maxima [A] time = 0.67, size = 129, normalized size = 0.84

$$\frac{1}{27} e x^{27} + \frac{1}{25} (d+10e)x^{25} + \frac{5}{23} (2d+9e)x^{23} + \frac{5}{7} (3d+8e)x^{21} + \frac{30}{19} (4d+7e)x^{19} + \frac{42}{17} (5d+6e)x^{17} + \frac{14}{5} (6d+5e)x^{15} + \frac{30}{13} (7d+4e)x^{13} + \frac{15}{11} (8d+3e)x^{11} + \frac{5}{9} (9d+2e)x^9 + \frac{1}{7} (10d+e)x^7 + \frac{1}{5} d x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5, x, \text{algorithm}="maxima")$

[Out] $1/27*e*x^{27} + 1/25*(d + 10*e)*x^{25} + 5/23*(2*d + 9*e)*x^{23} + 5/7*(3*d + 8*e)*x^{21} + 30/19*(4*d + 7*e)*x^{19} + 42/17*(5*d + 6*e)*x^{17} + 14/5*(6*d + 5*e)*x^{15} + 30/13*(7*d + 4*e)*x^{13} + 15/11*(8*d + 3*e)*x^{11} + 5/9*(9*d + 2*e)*x^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5$

mupad [B] time = 0.12, size = 123, normalized size = 0.80

$$\frac{e x^{27}}{27} + \left(\frac{d}{25} + \frac{2e}{5}\right) x^{25} + \left(\frac{10d}{23} + \frac{45e}{23}\right) x^{23} + \left(\frac{15d}{7} + \frac{40e}{7}\right) x^{21} + \left(\frac{120d}{19} + \frac{210e}{19}\right) x^{19} + \left(\frac{210d}{17} + \frac{252e}{17}\right) x^{17} + \left(\frac{84d}{5} + 14e\right) x^{15} + \left(\frac{210d}{13} + \frac{120e}{13}\right) x^{13} + \left(\frac{120d}{11} + \frac{45e}{11}\right) x^{11} + \left(5d + \frac{10e}{9}\right) x^9 + \left(\frac{10d}{7} + \frac{e}{7}\right) x^7 + \frac{d x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5, x)$

[Out] $x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^{25}*(d/25 + (2*e)/5) + x^{21}*((15*d)/7 + (40*e)/7) + x^{15}*((84*d)/5 + 14*e) + x^{23}*((10*d)/23 + (45*e)/23) + x^{11}*((120*d)/11 + (45*e)/11) + x^{13}*((210*d)/13 + (120*e)/13) + x^{19}*((120*d)/19 + (210*e)/19) + x^{17}*((210*d)/17 + (252*e)/17) + (d*x^5)/5 + (e*x^{27})/27$

sympy [A] time = 0.10, size = 141, normalized size = 0.92

$$\frac{d x^5}{5} + \frac{e x^{27}}{27} + x^{25} \left(\frac{d}{25} + \frac{2e}{5}\right) + x^{23} \left(\frac{10d}{23} + \frac{45e}{23}\right) + x^{21} \left(\frac{15d}{7} + \frac{40e}{7}\right) + x^{19} \left(\frac{120d}{19} + \frac{210e}{19}\right) + x^{17} \left(\frac{210d}{17} + \frac{252e}{17}\right) + x^{15} \left(\frac{84d}{5} + 14e\right) + x^{13} \left(\frac{210d}{13} + \frac{120e}{13}\right) + x^{11} \left(\frac{120d}{11} + \frac{45e}{11}\right) + x^9 \left(5d + \frac{10e}{9}\right) + x^7 \left(\frac{10d}{7} + \frac{e}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +  
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2  
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120  
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

$$3.38 \quad \int x^3 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=45

$$\frac{1}{24} (x^2 + 1)^{12} (d - 2e) - \frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{26} e (x^2 + 1)^{13}$$

Rubi [A] time = 0.12, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 446, 76}

$$\frac{1}{24} (x^2 + 1)^{12} (d - 2e) - \frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{26} e (x^2 + 1)^{13}$$

Antiderivative was successfully verified.

[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^13)/26

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 76

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int x^3 (1 + x^2)^{10} (d + ex^2) dx \\
&= \frac{1}{2} \text{Subst} \left(\int x(1+x)^{10} (d+ex) dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int ((-d+e)(1+x)^{10} + (d-2e)(1+x)^{11} + e(1+x)^{12}) dx, x, x^2 \right) \\
&= -\frac{1}{22} (d-e) (1+x^2)^{11} + \frac{1}{24} (d-2e) (1+x^2)^{12} + \frac{1}{26} e (1+x^2)^{13}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 151, normalized size = 3.36

$$\frac{1}{24}x^{24}(d+10e) + \frac{5}{22}x^{22}(2d+9e) + \frac{3}{4}x^{20}(3d+8e) + \frac{5}{3}x^{18}(4d+7e) + \frac{21}{8}x^{16}(5d+6e) + 3x^{14}(6d+5e) + \frac{5}{2}x^{12}(7d+4e) + \frac{3}{2}x^{10}(8d+3e) + \frac{5}{8}x^8(9d+2e) + \frac{1}{6}x^6(10d+e) + \frac{dx^4}{4} + \frac{ex^{26}}{26}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^10)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.48, size = 133, normalized size = 2.96

$$\frac{1}{26}x^{26}e + \frac{5}{12}x^{24}e + \frac{1}{24}x^{24}d + \frac{45}{22}x^{22}e + \frac{5}{11}x^{22}d + 6x^{20}e + \frac{9}{4}x^{20}d + \frac{35}{3}x^{18}e + \frac{20}{3}x^{18}d + \frac{63}{4}x^{16}e + \frac{105}{8}x^{16}d + 15x^{14}e + 18x^{14}d + 10x^{12}e + \frac{35}{2}x^{12}d + \frac{9}{2}x^{10}e + 12x^{10}d + \frac{5}{4}x^8e + \frac{45}{8}x^8d + \frac{1}{6}x^6e + \frac{5}{3}x^6d + \frac{1}{4}x^4d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/26*x^26*e + 5/12*x^24*e + 1/24*x^24*d + 45/22*x^22*e + 5/11*x^22*d + 6*x^20*e + 9/4*x^20*d + 35/3*x^18*e + 20/3*x^18*d + 63/4*x^16*e + 105/8*x^16*d + 15*x^14*e + 18*x^14*d + 10*x^12*e + 35/2*x^12*d + 9/2*x^10*e + 12*x^10*d + 5/4*x^8*e + 45/8*x^8*d + 1/6*x^6*e + 5/3*x^6*d + 1/4*x^4*d

giac [B] time = 0.40, size = 144, normalized size = 3.20

$$\frac{1}{26}x^{26}e + \frac{1}{24}dx^{24} + \frac{5}{12}x^{24}e + \frac{5}{11}dx^{22} + \frac{45}{22}x^{22}e + \frac{9}{4}dx^{20} + 6x^{20}e + \frac{20}{3}dx^{18} + \frac{35}{3}x^{18}e + \frac{105}{8}dx^{16} + \frac{63}{4}x^{16}e + 18dx^{14} + 15x^{14}e + \frac{35}{2}dx^{12} + 10x^{12}e + 12dx^{10} + \frac{9}{2}x^{10}e + \frac{45}{8}dx^8 + \frac{5}{4}x^8e + \frac{5}{3}dx^6 + \frac{1}{6}x^6e + \frac{1}{4}dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/26*x^{26}*e + 1/24*d*x^{24} + 5/12*x^{24}*e + 5/11*d*x^{22} + 45/22*x^{22}*e + 9/4* \\ & d*x^{20} + 6*x^{20}*e + 20/3*d*x^{18} + 35/3*x^{18}*e + 105/8*d*x^{16} + 63/4*x^{16}*e \\ & + 18*d*x^{14} + 15*x^{14}*e + 35/2*d*x^{12} + 10*x^{12}*e + 12*d*x^{10} + 9/2*x^{10}*e \\ & + 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^6 + 1/6*x^6*e + 1/4*d*x^4 \end{aligned}$$

maple [B] time = 0.00, size = 130, normalized size = 2.89

$$\frac{e x^{26}}{26} + \frac{(d+10e)x^{24}}{24} + \frac{(10d+45e)x^{22}}{22} + \frac{(45d+120e)x^{20}}{20} + \frac{(120d+210e)x^{18}}{18} + \frac{(210d+252e)x^{16}}{16} + \frac{(252d+210e)x^{14}}{14} + \frac{(210d+120e)x^{12}}{12} + \frac{(120d+45e)x^{10}}{10} + \frac{(45d+10e)x^8}{8} + \frac{(10d+e)x^6}{6} + \frac{d x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x)

$$\begin{aligned} \text{[Out]} & 1/26*e*x^{26}+1/24*(d+10*e)*x^{24}+1/22*(10*d+45*e)*x^{22}+1/20*(45*d+120*e)*x^{20} \\ & +1/18*(120*d+210*e)*x^{18}+1/16*(210*d+252*e)*x^{16}+1/14*(252*d+210*e)*x^{14}+1/ \\ & 12*(210*d+120*e)*x^{12}+1/10*(120*d+45*e)*x^{10}+1/8*(45*d+10*e)*x^8+1/6*(10*d+ \\ & e)*x^6+1/4*d*x^4 \end{aligned}$$

maxima [B] time = 0.65, size = 129, normalized size = 2.87

$$\frac{1}{26}ex^{26} + \frac{1}{24}(d+10e)x^{24} + \frac{5}{22}(2d+9e)x^{22} + \frac{3}{4}(3d+8e)x^{20} + \frac{5}{3}(4d+7e)x^{18} + \frac{21}{8}(5d+6e)x^{16} + 3(6d+5e)x^{14} + \frac{5}{2}(7d+4e)x^{12} + \frac{3}{2}(8d+3e)x^{10} + \frac{5}{8}(9d+2e)x^8 + \frac{1}{6}(10d+e)x^6 + \frac{1}{4}dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/26*e*x^{26} + 1/24*(d + 10*e)*x^{24} + 5/22*(2*d + 9*e)*x^{22} + 3/4*(3*d + 8*e) \\ &)*x^{20} + 5/3*(4*d + 7*e)*x^{18} + 21/8*(5*d + 6*e)*x^{16} + 3*(6*d + 5*e)*x^{14} \\ & + 5/2*(7*d + 4*e)*x^{12} + 3/2*(8*d + 3*e)*x^{10} + 5/8*(9*d + 2*e)*x^8 + 1/6*(\\ & 10*d + e)*x^6 + 1/4*d*x^4 \end{aligned}$$

mupad [B] time = 0.08, size = 123, normalized size = 2.73

$$\frac{e x^{26}}{26} + \left(\frac{d}{24} + \frac{5e}{12}\right)x^{24} + \left(\frac{5d}{11} + \frac{45e}{22}\right)x^{22} + \left(\frac{9d}{4} + 6e\right)x^{20} + \left(\frac{20d}{3} + \frac{35e}{3}\right)x^{18} + \left(\frac{105d}{8} + \frac{63e}{4}\right)x^{16} + (18d+15e)x^{14} + \left(\frac{35d}{2} + 10e\right)x^{12} + \left(12d + \frac{9e}{2}\right)x^{10} + \left(\frac{45d}{8} + \frac{5e}{4}\right)x^8 + \left(\frac{5d}{3} + \frac{e}{6}\right)x^6 + \frac{d x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] $x^6 \left(\frac{5d}{3} + \frac{e}{6} \right) + x^{10} (12d + \frac{9e}{2}) + x^{20} \left(\frac{9d}{4} + 6e \right) + x^{14} (18d + 15e) + x^{12} \left(\frac{35d}{2} + 10e \right) + x^{24} \left(\frac{d}{24} + \frac{5e}{12} \right) + x^8 \left(\frac{45d}{8} + \frac{5e}{4} \right) + x^{18} \left(\frac{20d}{3} + \frac{35e}{3} \right) + x^{22} \left(\frac{5d}{11} + \frac{45e}{22} \right) + x^{16} \left(\frac{105d}{8} + \frac{63e}{4} \right) + \frac{d x^4}{4} + \frac{e x^{26}}{26}$

sympy [B] time = 0.10, size = 136, normalized size = 3.02

$$\frac{dx^4}{4} + \frac{ex^{26}}{26} + x^{24} \left(\frac{d}{24} + \frac{5e}{12} \right) + x^{22} \left(\frac{5d}{11} + \frac{45e}{22} \right) + x^{20} \left(\frac{9d}{4} + 6e \right) + x^{18} \left(\frac{20d}{3} + \frac{35e}{3} \right) + x^{16} \left(\frac{105d}{8} + \frac{63e}{4} \right) + x^{14} (18d + 15e) + x^{12} \left(\frac{35d}{2} + 10e \right) + x^{10} \left(\frac{9d}{4} + 6e \right) + x^8 \left(\frac{45d}{8} + \frac{5e}{4} \right) + x^6 \left(\frac{5d}{3} + \frac{e}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)

[Out] $d x^4/4 + e x^{26}/26 + x^{24} (d/24 + 5e/12) + x^{22} (5d/11 + 45e/22) + x^{20} (9d/4 + 6e) + x^{18} (20d/3 + 35e/3) + x^{16} (105d/8 + 63e/4) + x^{14} (18d + 15e) + x^{12} (35d/2 + 10e) + x^{10} (12d + 9e/2) + x^8 (45d/8 + 5e/4) + x^6 (5d/3 + e/6)$

$$3.39 \quad \int x^2 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=153

$$\frac{1}{23}x^{23}(d+10e) + \frac{5}{21}x^{21}(2d+9e) + \frac{15}{19}x^{19}(3d+8e) + \frac{30}{17}x^{17}(4d+7e) + \frac{14}{5}x^{15}(5d+6e) + \frac{42}{13}x^{13}(6d+5e) + \frac{30}{11}x^{11}(7d+4e) + \frac{4}{1}x^9(8d+3e) + \frac{5}{7}x^7(9d+2e) + \frac{1}{5}x^5(10d+e) + \frac{dx^3}{3} + \frac{ex^{25}}{25}$$

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{23}x^{23}(d+10e) + \frac{5}{21}x^{21}(2d+9e) + \frac{15}{19}x^{19}(3d+8e) + \frac{30}{17}x^{17}(4d+7e) + \frac{14}{5}x^{15}(5d+6e) + \frac{42}{13}x^{13}(6d+5e) + \frac{30}{11}x^{11}(7d+4e) + \frac{5}{3}x^9(8d+3e) + \frac{5}{7}x^7(9d+2e) + \frac{1}{5}x^5(10d+e) + \frac{dx^3}{3} + \frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int x^2 (1 + x^2)^{10} (d + ex^2) dx \\ &= \int (dx^2 + (10d + e)x^4 + 5(9d + 2e)x^6 + 15(8d + 3e)x^8 + 30(7d + 4e)x^{10} + 4(6d + 3e)x^{12} + 2(5d + 2e)x^{14} + dx^{16}) dx \\ &= \frac{dx^3}{3} + \frac{1}{5}(10d + e)x^5 + \frac{5}{7}(9d + 2e)x^7 + \frac{5}{3}(8d + 3e)x^9 + \frac{30}{11}(7d + 4e)x^{11} + \frac{4}{13}(6d + 3e)x^{13} + \frac{2}{15}(5d + 2e)x^{15} + \frac{d}{17}x^{17} \end{aligned}$$

Mathematica [A] time = 0.02, size = 153, normalized size = 1.00

$$\frac{1}{23}x^{23}(d+10e) + \frac{5}{21}x^{21}(2d+9e) + \frac{15}{19}x^{19}(3d+8e) + \frac{30}{17}x^{17}(4d+7e) + \frac{14}{5}x^{15}(5d+6e) + \frac{42}{13}x^{13}(6d+5e) + \frac{30}{11}x^{11}(7d+4e) + \frac{5}{3}x^9(8d+3e) + \frac{5}{7}x^7(9d+2e) + \frac{1}{5}x^5(10d+e) + \frac{dx^3}{3} + \frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 0.81, size = 133, normalized size = 0.87

$$\frac{1}{25}x^{25}e + \frac{10}{23}x^{23}e + \frac{1}{23}x^{23}d + \frac{15}{7}x^{21}e + \frac{10}{21}x^{21}d + \frac{120}{19}x^{19}e + \frac{45}{19}x^{19}d + \frac{210}{17}x^{17}e + \frac{120}{17}x^{17}d + \frac{84}{5}x^{15}e + 14x^{15}d + \frac{210}{13}x^{13}e + \frac{252}{13}x^{13}d + \frac{120}{11}x^{11}e + \frac{210}{11}x^{11}d + 5x^9e + \frac{40}{3}x^9d + \frac{10}{7}x^7e + \frac{45}{7}x^7d + \frac{1}{5}x^5e + 2x^5d + \frac{1}{3}x^3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/25*x^25*e + 10/23*x^23*e + 1/23*x^23*d + 15/7*x^21*e + 10/21*x^21*d + 120/19*x^19*e + 45/19*x^19*d + 210/17*x^17*e + 120/17*x^17*d + 84/5*x^15*e + 14*x^15*d + 210/13*x^13*e + 252/13*x^13*d + 120/11*x^11*e + 210/11*x^11*d + 5*x^9*e + 40/3*x^9*d + 10/7*x^7*e + 45/7*x^7*d + 1/5*x^5*e + 2*x^5*d + 1/3*x^3*d

giac [A] time = 0.29, size = 144, normalized size = 0.94

$$\frac{1}{25}x^{25}e + \frac{1}{23}dx^{23} + \frac{10}{23}x^{23}e + \frac{10}{21}dx^{21} + \frac{15}{7}x^{21}e + \frac{45}{19}dx^{19} + \frac{120}{19}x^{19}e + \frac{120}{17}dx^{17} + \frac{210}{17}x^{17}e + 14dx^{15} + \frac{84}{5}x^{15}e + \frac{252}{13}dx^{13} + \frac{210}{13}x^{13}e + \frac{210}{11}dx^{11} + \frac{120}{11}x^{11}e + \frac{40}{3}dx^9 + 5x^9e + \frac{45}{7}dx^7 + \frac{10}{7}x^7e + 2dx^5 + \frac{1}{5}x^5e + \frac{1}{3}dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^21*e + 45/19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17*e + 14*d*x^15 + 84/

$$5*x^{15}*e + 252/13*d*x^{13} + 210/13*x^{13}*e + 210/11*d*x^{11} + 120/11*x^{11}*e + 40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 + 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*d*x^3$$

maple [A] time = 0.00, size = 130, normalized size = 0.85

$$\frac{e x^{25}}{25} + \frac{(d+10e)x^{23}}{23} + \frac{(10d+45e)x^{21}}{21} + \frac{(45d+120e)x^{19}}{19} + \frac{(120d+210e)x^{17}}{17} + \frac{(210d+252e)x^{15}}{15} + \frac{(252d+210e)x^{13}}{13} + \frac{(210d+120e)x^{11}}{11} + \frac{(120d+45e)x^9}{9} + \frac{(45d+10e)x^7}{7} + \frac{(10d+e)x^5}{5} + \frac{d x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x)

[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+120*e)*x^19+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(252*d+210*e)*x^13+1/11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/7*(45*d+10*e)*x^7+1/5*(10*d+e)*x^5+1/3*d*x^3

maxima [A] time = 0.50, size = 129, normalized size = 0.84

$$\frac{1}{25} e x^{25} + \frac{1}{23} (d+10e)x^{23} + \frac{5}{21} (2d+9e)x^{21} + \frac{15}{19} (3d+8e)x^{19} + \frac{30}{17} (4d+7e)x^{17} + \frac{14}{5} (5d+6e)x^{15} + \frac{42}{13} (6d+5e)x^{13} + \frac{30}{11} (7d+4e)x^{11} + \frac{5}{3} (8d+3e)x^9 + \frac{5}{7} (9d+2e)x^7 + \frac{1}{5} (10d+e)x^5 + \frac{1}{3} d x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3

mupad [B] time = 0.08, size = 123, normalized size = 0.80

$$\frac{e x^{25}}{25} + \left(\frac{d}{23} + \frac{10e}{23}\right) x^{23} + \left(\frac{10d}{21} + \frac{15e}{7}\right) x^{21} + \left(\frac{45d}{19} + \frac{120e}{19}\right) x^{19} + \left(\frac{120d}{17} + \frac{210e}{17}\right) x^{17} + \left(14d + \frac{84e}{5}\right) x^{15} + \left(\frac{252d}{13} + \frac{210e}{13}\right) x^{13} + \left(\frac{210d}{11} + \frac{120e}{11}\right) x^{11} + \left(\frac{40d}{3} + 5e\right) x^9 + \left(\frac{45d}{7} + \frac{10e}{7}\right) x^7 + \left(2d + \frac{e}{5}\right) x^5 + \frac{d x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7*((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5) + x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((120*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*x^25)/25

sympy [A] time = 0.10, size = 139, normalized size = 0.91

$$\frac{d x^3}{3} + \frac{e x^{25}}{25} + x^{23} \left(\frac{d}{23} + \frac{10e}{23}\right) + x^{21} \left(\frac{10d}{21} + \frac{15e}{7}\right) + x^{19} \left(\frac{45d}{19} + \frac{120e}{19}\right) + x^{17} \left(\frac{120d}{17} + \frac{210e}{17}\right) + x^{15} \left(14d + \frac{84e}{5}\right) + x^{13} \left(\frac{252d}{13} + \frac{210e}{13}\right) + x^{11} \left(\frac{210d}{11} + \frac{120e}{11}\right) + x^9 \left(\frac{40d}{3} + 5e\right) + x^7 \left(\frac{45d}{7} + \frac{10e}{7}\right) + x^5 \left(2d + \frac{e}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +  
x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8  
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(  
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

$$3.40 \quad \int x (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=29

$$\frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{24} e (x^2 + 1)^{12}$$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 444, 43}

$$\frac{1}{22} (x^2 + 1)^{11} (d - e) + \frac{1}{24} e (x^2 + 1)^{12}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_)]^(p_.)*((c_) + (d_.)*(x_)^(n_)]^(q_.
, x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]

Rubi steps

$$\begin{aligned}
\int x(d+ex^2)(1+2x^2+x^4)^5 dx &= \int x(1+x^2)^{10}(d+ex^2) dx \\
&= \frac{1}{2} \text{Subst} \left(\int (1+x)^{10}(d+ex) dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int ((d-e)(1+x)^{10} + e(1+x)^{11}) dx, x, x^2 \right) \\
&= \frac{1}{22}(d-e)(1+x^2)^{11} + \frac{1}{24}e(1+x^2)^{12}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 149, normalized size = 5.14

$$\frac{1}{22}x^{22}(d+10e) + \frac{1}{4}x^{20}(2d+9e) + \frac{5}{6}x^{18}(3d+8e) + \frac{15}{8}x^{16}(4d+7e) + 3x^{14}(5d+6e) + \frac{7}{2}x^{12}(6d+5e) + 3x^{10}(7d+4e) + \frac{15}{8}x^8(8d+3e) + \frac{5}{6}x^6(9d+2e) + \frac{1}{4}x^4(10d+e) + \frac{dx^2}{2} + \frac{ex^24}{24}$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((d + 10*e)*x^22)/22 + (e*x^24)/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(d+ex^2)(1+2x^2+x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.84, size = 133, normalized size = 4.59

$$\frac{1}{24}x^{24}e + \frac{5}{11}x^{22}e + \frac{1}{22}x^{22}d + \frac{9}{4}x^{20}e + \frac{1}{2}x^{20}d + \frac{20}{3}x^{18}e + \frac{5}{2}x^{18}d + \frac{105}{8}x^{16}e + \frac{15}{2}x^{16}d + 18x^{14}e + 15x^{14}d + \frac{35}{2}x^{12}e + 21x^{12}d + 12x^{10}e + 21x^{10}d + \frac{45}{8}x^8e + 15x^8d + \frac{5}{3}x^6e + \frac{15}{2}x^6d + \frac{1}{4}x^4e + \frac{5}{2}x^4d + \frac{1}{2}x^2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/24*x^24*e + 5/11*x^22*e + 1/22*x^22*d + 9/4*x^20*e + 1/2*x^20*d + 20/3*x^18*e + 5/2*x^18*d + 105/8*x^16*e + 15/2*x^16*d + 18*x^14*e + 15*x^14*d + 35/2*x^12*e + 21*x^12*d + 12*x^10*e + 21*x^10*d + 45/8*x^8*e + 15*x^8*d + 5/3*x^6*e + 15/2*x^6*d + 1/4*x^4*e + 5/2*x^4*d + 1/2*x^2*d

giac [B] time = 0.31, size = 144, normalized size = 4.97

$$\frac{1}{24}x^{24}e + \frac{1}{22}dx^{22} + \frac{5}{11}x^{22}e + \frac{1}{2}dx^{20} + \frac{9}{4}x^{20}e + \frac{5}{2}dx^{18} + \frac{20}{3}x^{18}e + \frac{15}{2}dx^{16} + \frac{105}{8}x^{16}e + 15dx^{14} + 18x^{14}e + 21dx^{12} + \frac{35}{2}x^{12}e + 21dx^{10} + 12x^{10}e + 15dx^8 + \frac{45}{8}x^8e + \frac{15}{2}dx^6 + \frac{5}{3}x^6e + \frac{5}{2}dx^4 + \frac{1}{4}x^4e + \frac{1}{2}dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/24*x^{24}*e + 1/22*d*x^{22} + 5/11*x^{22}*e + 1/2*d*x^{20} + 9/4*x^{20}*e + 5/2*d*x \\ & ^{18} + 20/3*x^{18}*e + 15/2*d*x^{16} + 105/8*x^{16}*e + 15*d*x^{14} + 18*x^{14}*e + 21 \\ & *d*x^{12} + 35/2*x^{12}*e + 21*d*x^{10} + 12*x^{10}*e + 15*d*x^8 + 45/8*x^8*e + 15/ \\ & 2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1/4*x^4*e + 1/2*d*x^2 \end{aligned}$$

maple [B] time = 0.00, size = 130, normalized size = 4.48

$$\frac{e x^{24}}{24} + \frac{(d+10e)x^{22}}{22} + \frac{(10d+45e)x^{20}}{20} + \frac{(45d+120e)x^{18}}{18} + \frac{(120d+210e)x^{16}}{16} + \frac{(210d+252e)x^{14}}{14} + \frac{(252d+210e)x^{12}}{12} + \frac{(210d+120e)x^{10}}{10} + \frac{(120d+45e)x^8}{8} + \frac{(45d+10e)x^6}{6} + \frac{(10d+e)x^4}{4} + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x)

$$\begin{aligned} \text{[Out]} & 1/24*e*x^{24}+1/22*(d+10*e)*x^{22}+1/20*(10*d+45*e)*x^{20}+1/18*(45*d+120*e)*x^{18} \\ & +1/16*(120*d+210*e)*x^{16}+1/14*(210*d+252*e)*x^{14}+1/12*(252*d+210*e)*x^{12}+1/ \\ & 10*(210*d+120*e)*x^{10}+1/8*(120*d+45*e)*x^8+1/6*(45*d+10*e)*x^6+1/4*(10*d+e) \\ & *x^4+1/2*d*x^2 \end{aligned}$$

maxima [B] time = 0.65, size = 129, normalized size = 4.45

$$\frac{1}{24}ex^{24} + \frac{1}{22}(d+10e)x^{22} + \frac{1}{4}(2d+9e)x^{20} + \frac{5}{6}(3d+8e)x^{18} + \frac{15}{8}(4d+7e)x^{16} + 3(5d+6e)x^{14} + \frac{7}{2}(6d+5e)x^{12} + 3(7d+4e)x^{10} + \frac{15}{8}(8d+3e)x^8 + \frac{5}{6}(9d+2e)x^6 + \frac{1}{4}(10d+e)x^4 + \frac{1}{2}dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/24*e*x^{24} + 1/22*(d + 10*e)*x^{22} + 1/4*(2*d + 9*e)*x^{20} + 5/6*(3*d + 8*e) \\ & *x^{18} + 15/8*(4*d + 7*e)*x^{16} + 3*(5*d + 6*e)*x^{14} + 7/2*(6*d + 5*e)*x^{12} + \\ & 3*(7*d + 4*e)*x^{10} + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10* \\ & d + e)*x^4 + 1/2*d*x^2 \end{aligned}$$

mupad [B] time = 0.08, size = 123, normalized size = 4.24

$$\frac{e x^{24}}{24} + \left(\frac{d}{22} + \frac{5e}{11}\right)x^{22} + \left(\frac{d}{2} + \frac{9e}{4}\right)x^{20} + \left(\frac{5d}{2} + \frac{20e}{3}\right)x^{18} + \left(\frac{15d}{2} + \frac{105e}{8}\right)x^{16} + (15d+18e)x^{14} + \left(21d + \frac{35e}{2}\right)x^{12} + (21d+12e)x^{10} + \left(15d + \frac{45e}{8}\right)x^8 + \left(\frac{15d}{2} + \frac{5e}{3}\right)x^6 + \left(\frac{5d}{2} + \frac{e}{4}\right)x^4 + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] $x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^{20}*(d/2 + (9*e)/4) + x^{10}*(21*d + 12*e) + x^{14}*(15*d + 18*e) + x^{18}*((5*d)/2 + (20*e)/3) + x^{22}*(d/22 + (5*e)/11) + x^{12}*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^{16}*((15*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^{24})/24$

sympy [B] time = 0.10, size = 133, normalized size = 4.59

$$\frac{dx^2}{2} + \frac{ex^{24}}{24} + x^{22}\left(\frac{d}{22} + \frac{5e}{11}\right) + x^{20}\left(\frac{d}{2} + \frac{9e}{4}\right) + x^{18}\left(\frac{5d}{2} + \frac{20e}{3}\right) + x^{16}\left(\frac{15d}{2} + \frac{105e}{8}\right) + x^{14}(15d + 18e) + x^{12}\left(21d + \frac{35e}{2}\right) + x^{10}(21d + 12e) + x^8\left(15d + \frac{45e}{8}\right) + x^6\left(\frac{15d}{2} + \frac{5e}{3}\right) + x^4\left(\frac{5d}{2} + \frac{e}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)

[Out] $d*x^{22}/2 + e*x^{24}/24 + x^{22}*(d/22 + 5*e/11) + x^{20}*(d/2 + 9*e/4) + x^{18}*(5*d/2 + 20*e/3) + x^{16}*(15*d/2 + 105*e/8) + x^{14}*(15*d + 18*e) + x^{12}*(21*d + 35*e/2) + x^{10}*(21*d + 12*e) + x^8*(15*d + 45*e/8) + x^6*(15*d/2 + 5*e/3) + x^4*(5*d/2 + e/4)$

$$3.41 \quad \int (d + ex^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=143

$$\frac{1}{21}x^{21}(d+10e) + \frac{5}{19}x^{19}(2d+9e) + \frac{15}{17}x^{17}(3d+8e) + 2x^{15}(4d+7e) + \frac{42}{13}x^{13}(5d+6e) + \frac{42}{11}x^{11}(6d+5e) + \frac{10}{3}x^9(7d+4e) + \frac{15}{7}x^7(8d+3e) + x^5(9d+2e) + \frac{1}{3}x^3(10d+e) + dx + \frac{ex^{23}}{23}$$

Rubi [A] time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {28, 373}

$$\frac{1}{21}x^{21}(d+10e) + \frac{5}{19}x^{19}(2d+9e) + \frac{15}{17}x^{17}(3d+8e) + 2x^{15}(4d+7e) + \frac{42}{13}x^{13}(5d+6e) + \frac{42}{11}x^{11}(6d+5e) + \frac{10}{3}x^9(7d+4e) + \frac{15}{7}x^7(8d+3e) + x^5(9d+2e) + \frac{1}{3}x^3(10d+e) + dx + \frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((d + 10*e)*x^21)/21 + (e*x^23)/23

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 373

Int[((a_.) + (b_.)*(x_)^(n_)]^(p_.)*((c_.) + (d_.)*(x_)^(n_)]^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (d + ex^2) (1 + 2x^2 + x^4)^5 dx &= \int (1 + x^2)^{10} (d + ex^2) dx \\ &= \int (d + (10d + e)x^2 + 5(9d + 2e)x^4 + 15(8d + 3e)x^6 + 30(7d + 4e)x^8 + 42(6d + 5e)x^{10} + 10(5d + 6e)x^{12} + 2(4d + 7e)x^{14} + (3d + 8e)x^{16} + (2d + 9e)x^{18} + (d + 10e)x^{20} + ex^{22}) dx \\ &= dx + \frac{1}{3}(10d + e)x^3 + (9d + 2e)x^5 + \frac{15}{7}(8d + 3e)x^7 + \frac{10}{3}(7d + 4e)x^9 + \frac{42}{11}(6d + 5e)x^{11} + \frac{42}{13}(5d + 6e)x^{13} + 2(4d + 7e)x^{15} + \frac{15}{17}(3d + 8e)x^{17} + \frac{5}{19}(2d + 9e)x^{19} + \frac{1}{21}(d + 10e)x^{21} + \frac{e}{23}x^{23} \end{aligned}$$

Mathematica [A] time = 0.02, size = 143, normalized size = 1.00

$$\frac{1}{21}x^{21}(d+10e) + \frac{5}{19}x^{19}(2d+9e) + \frac{15}{17}x^{17}(3d+8e) + 2x^{15}(4d+7e) + \frac{42}{13}x^{13}(5d+6e) + \frac{42}{11}x^{11}(6d+5e) + \frac{10}{3}x^9(7d+4e) + \frac{15}{7}x^7(8d+3e) + x^5(9d+2e) + \frac{1}{3}x^3(10d+e) + dx + \frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((d + 10*e)*x^21)/21 + (e*x^23)/23

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)(1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[(d + e*x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 0.56, size = 130, normalized size = 0.91

$$\frac{1}{23}x^{23}e + \frac{10}{21}x^{21}e + \frac{1}{21}x^{21}d + \frac{45}{19}x^{19}e + \frac{10}{19}x^{19}d + \frac{120}{17}x^{17}e + \frac{45}{17}x^{17}d + 14x^{15}e + 8x^{15}d + \frac{252}{13}x^{13}e + \frac{210}{13}x^{13}d + \frac{210}{11}x^{11}e + \frac{252}{11}x^{11}d + \frac{40}{3}x^9e + \frac{70}{3}x^9d + \frac{45}{7}x^7e + \frac{120}{7}x^7d + 2x^5e + 9x^5d + \frac{1}{3}x^3e + \frac{10}{3}x^3d + xd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/23*x^23*e + 10/21*x^21*e + 1/21*x^21*d + 45/19*x^19*e + 10/19*x^19*d + 120/17*x^17*e + 45/17*x^17*d + 14*x^15*e + 8*x^15*d + 252/13*x^13*e + 210/13*x^13*d + 210/11*x^11*e + 252/11*x^11*d + 40/3*x^9*e + 70/3*x^9*d + 45/7*x^7*e + 120/7*x^7*d + 2*x^5*e + 9*x^5*d + 1/3*x^3*e + 10/3*x^3*d + x*d

giac [A] time = 0.36, size = 141, normalized size = 0.99

$$\frac{1}{23}x^{23}e + \frac{1}{21}dx^{21} + \frac{10}{21}x^{21}e + \frac{10}{19}dx^{19} + \frac{45}{19}x^{19}e + \frac{45}{17}dx^{17} + \frac{120}{17}x^{17}e + 8dx^{15} + 14x^{15}e + \frac{210}{13}dx^{13} + \frac{252}{13}x^{13}e + \frac{252}{11}dx^{11} + \frac{210}{11}x^{11}e + \frac{70}{3}dx^9 + \frac{40}{3}x^9e + \frac{120}{7}dx^7 + \frac{45}{7}x^7e + 9dx^5 + 2x^5e + \frac{10}{3}dx^3 + \frac{1}{3}x^3e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x^19*e + 45/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 252/13*

$$x^{13}e + 252/11*d*x^{11} + 210/11*x^{11}e + 70/3*d*x^9 + 40/3*x^9e + 120/7*d*x^7 + 45/7*x^7e + 9*d*x^5 + 2*x^5e + 10/3*d*x^3 + 1/3*x^3e + d*x$$

maple [A] time = 0.00, size = 127, normalized size = 0.89

$$\frac{e x^{23}}{23} + \frac{(d+10e)x^{21}}{21} + \frac{(10d+45e)x^{19}}{19} + \frac{(45d+120e)x^{17}}{17} + \frac{(120d+210e)x^{15}}{15} + \frac{(210d+252e)x^{13}}{13} + \frac{(252d+210e)x^{11}}{11} + \frac{(210d+120e)x^9}{9} + \frac{(120d+45e)x^7}{7} + \frac{(45d+10e)x^5}{5} + \frac{(10d+e)x^3}{3} + dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x)

[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+120*e)*x^17+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(252*d+210*e)*x^11+1/9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*(45*d+10*e)*x^5+1/3*(10*d+e)*x^3+d*x

maxima [A] time = 0.58, size = 125, normalized size = 0.87

$$\frac{1}{23} e x^{23} + \frac{1}{21} (d+10e)x^{21} + \frac{5}{19} (2d+9e)x^{19} + \frac{15}{17} (3d+8e)x^{17} + 2(4d+7e)x^{15} + \frac{42}{13} (5d+6e)x^{13} + \frac{42}{11} (6d+5e)x^{11} + \frac{10}{3} (7d+4e)x^9 + \frac{15}{7} (8d+3e)x^7 + (9d+2e)x^5 + \frac{1}{3} (10d+e)x^3 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*(10*d + e)*x^3 + d*x

mupad [B] time = 0.08, size = 120, normalized size = 0.84

$$\frac{e x^{23}}{23} + \left(\frac{d}{21} + \frac{10e}{21}\right) x^{21} + \left(\frac{10d}{19} + \frac{45e}{19}\right) x^{19} + \left(\frac{45d}{17} + \frac{120e}{17}\right) x^{17} + (8d+14e)x^{15} + \left(\frac{210d}{13} + \frac{252e}{13}\right) x^{13} + \left(\frac{252d}{11} + \frac{210e}{11}\right) x^{11} + \left(\frac{70d}{3} + \frac{40e}{3}\right) x^9 + \left(\frac{120d}{7} + \frac{45e}{7}\right) x^7 + (9d+2e)x^5 + \left(\frac{10d}{3} + \frac{e}{3}\right) x^3 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 + (10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^7*((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/11 + (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23

sympy [A] time = 0.10, size = 134, normalized size = 0.94

$$dx + \frac{e x^{23}}{23} + x^{21} \left(\frac{d}{21} + \frac{10e}{21}\right) + x^{19} \left(\frac{10d}{19} + \frac{45e}{19}\right) + x^{17} \left(\frac{45d}{17} + \frac{120e}{17}\right) + x^{15} (8d+14e) + x^{13} \left(\frac{210d}{13} + \frac{252e}{13}\right) + x^{11} \left(\frac{252d}{11} + \frac{210e}{11}\right) + x^9 \left(\frac{70d}{3} + \frac{40e}{3}\right) + x^7 \left(\frac{120d}{7} + \frac{45e}{7}\right) + x^5 (9d+2e) + x^3 \left(\frac{10d}{3} + \frac{e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13) + x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

$$3.42 \quad \int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx$$

Optimal. Leaf size=93

$$\frac{dx^{20}}{20} + \frac{5dx^{18}}{9} + \frac{45dx^{16}}{16} + \frac{60dx^{14}}{7} + \frac{35dx^{12}}{2} + \frac{126dx^{10}}{5} + \frac{105dx^8}{4} + 20dx^6 + \frac{45dx^4}{4} + 5dx^2 + d \log(x) + \frac{1}{22}e(x^2+1)^{11}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {28, 446, 80, 43}

$$\frac{dx^{20}}{20} + \frac{5dx^{18}}{9} + \frac{45dx^{16}}{16} + \frac{60dx^{14}}{7} + \frac{35dx^{12}}{2} + \frac{126dx^{10}}{5} + \frac{105dx^8}{4} + 20dx^6 + \frac{45dx^4}{4} + 5dx^2 + d \log(x) + \frac{1}{22}e(x^2+1)^{11}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]

[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*(1 + x^2)^11)/22 + d*Log[x]

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(1 + 2x^2 + x^4)^5}{x} dx &= \int \frac{(1 + x^2)^{10}(d + ex^2)}{x} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{(1 + x)^{10}(d + ex)}{x} dx, x, x^2 \right) \\ &= \frac{1}{22} e (1 + x^2)^{11} + \frac{1}{2} d \text{Subst} \left(\int \frac{(1 + x)^{10}}{x} dx, x, x^2 \right) \\ &= \frac{1}{22} e (1 + x^2)^{11} + \frac{1}{2} d \text{Subst} \left(\int \left(10 + \frac{1}{x} + 45x + 120x^2 + 210x^3 + 252x^4 + 210x^5 + 105x^6 + 35x^7 + 7x^8 + x^9 \right) dx, x, x^2 \right) \\ &= 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16} + \frac{5dx^{18}}{18} + \frac{d \log(x)}{22} + \frac{e x^{22}}{22} \end{aligned}$$

Mathematica [A] time = 0.03, size = 149, normalized size = 1.60

$$\frac{1}{20} x^{20} (d + 10e) + \frac{5}{18} x^{18} (2d + 9e) + \frac{15}{16} x^{16} (3d + 8e) + \frac{15}{7} x^{14} (4d + 7e) + \frac{7}{2} x^{12} (5d + 6e) + \frac{21}{5} x^{10} (6d + 5e) + \frac{15}{4} x^8 (7d + 4e) + \frac{5}{2} x^6 (8d + 3e) + \frac{5}{4} x^4 (9d + 2e) + \frac{1}{2} x^2 (10d + e) + d \log(x) + \frac{e x^{22}}{22}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]

[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + ((d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(1 + 2x^2 + x^4)^5}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x, x]

fricas [A] time = 0.76, size = 127, normalized size = 1.37

$$\frac{1}{22}ex^{22} + \frac{1}{20}(d+10e)x^{20} + \frac{5}{18}(2d+9e)x^{18} + \frac{15}{16}(3d+8e)x^{16} + \frac{15}{7}(4d+7e)x^{14} + \frac{7}{2}(5d+6e)x^{12} + \frac{21}{5}(6d+5e)x^{10} + \frac{15}{4}(7d+4e)x^8 + \frac{5}{2}(8d+3e)x^6 + \frac{5}{4}(9d+2e)x^4 + \frac{1}{2}(10d+e)x^2 + d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")

[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1/2*(10*d + e)*x^2 + d*log(x)

giac [A] time = 0.27, size = 145, normalized size = 1.56

$$\frac{1}{22}x^{22}e + \frac{1}{20}dx^{20} + \frac{1}{2}x^{20}e + \frac{5}{9}dx^{18} + \frac{5}{2}x^{18}e + \frac{45}{16}dx^{16} + \frac{15}{2}x^{16}e + \frac{60}{7}dx^{14} + 15x^{14}e + \frac{35}{2}dx^{12} + 21x^{12}e + \frac{126}{5}dx^{10} + 21x^{10}e + \frac{105}{4}dx^8 + 15x^8e + 20dx^6 + \frac{15}{2}x^6e + \frac{45}{4}dx^4 + \frac{5}{2}x^4e + 5dx^2 + \frac{1}{2}x^2e + \frac{1}{2}d \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")

[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e + 45/16*d*x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 21*x^12*e + 126/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8*e + 20*d*x^6 + 15/2*x^6*e + 45/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1/2*x^2*e + 1/2*d*log(x^2)

maple [A] time = 0.00, size = 132, normalized size = 1.42

$$\frac{ex^{22}}{22} + \frac{dx^{20}}{20} + \frac{ex^{20}}{2} + \frac{5dx^{18}}{9} + \frac{5ex^{18}}{2} + \frac{45dx^{16}}{16} + \frac{15ex^{16}}{2} + \frac{60dx^{14}}{7} + 15ex^{14} + \frac{35dx^{12}}{2} + 21ex^{12} + \frac{126dx^{10}}{5} + 21ex^{10} + \frac{105dx^8}{4} + 15ex^8 + 20dx^6 + \frac{15ex^6}{2} + \frac{45dx^4}{4} + \frac{5ex^4}{2} + 5dx^2 + \frac{ex^2}{2} + d \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x)

[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*e*x^18+45/16*d*x^16+15/2*x^16*e+60/7*d*x^14+15*e*x^14+35/2*d*x^12+21*e*x^12+126/5*d*x^10+21*e*x^10+105/4*d*x^8+15*e*x^8+20*d*x^6+15/2*e*x^6+45/4*d*x^4+5/2*e*x^4+5*d*x^2+1/2*e*x^2+d*ln(x)

maxima [A] time = 0.48, size = 130, normalized size = 1.40

$$\frac{1}{22}ex^{22} + \frac{1}{20}(d+10e)x^{20} + \frac{5}{18}(2d+9e)x^{18} + \frac{15}{16}(3d+8e)x^{16} + \frac{15}{7}(4d+7e)x^{14} + \frac{7}{2}(5d+6e)x^{12} + \frac{21}{5}(6d+5e)x^{10} + \frac{15}{4}(7d+4e)x^8 + \frac{5}{2}(8d+3e)x^6 + \frac{5}{4}(9d+2e)x^4 + \frac{1}{2}(10d+e)x^2 + \frac{1}{2}d \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")

[Out] $1/22*e*x^{22} + 1/20*(d + 10*e)*x^{20} + 5/18*(2*d + 9*e)*x^{18} + 15/16*(3*d + 8*e)*x^{16} + 15/7*(4*d + 7*e)*x^{14} + 7/2*(5*d + 6*e)*x^{12} + 21/5*(6*d + 5*e)*x^{10} + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1/2*(10*d + e)*x^2 + 1/2*d*\log(x^2)$

mupad [B] time = 0.13, size = 121, normalized size = 1.30

$$x^2 \left(5d + \frac{e}{2}\right) + x^{18} \left(\frac{5d}{9} + \frac{5e}{2}\right) + x^6 \left(20d + \frac{15e}{2}\right) + x^{20} \left(\frac{d}{20} + \frac{e}{2}\right) + x^4 \left(\frac{45d}{4} + \frac{5e}{2}\right) + x^{12} \left(\frac{35d}{2} + 21e\right) + x^{16} \left(\frac{45d}{16} + \frac{15e}{2}\right) + x^{14} \left(\frac{60d}{7} + 15e\right) + x^8 \left(\frac{105d}{4} + 15e\right) + x^{10} \left(\frac{126d}{5} + 21e\right) + \frac{ex^{22}}{22} + d \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)`

[Out] $x^2*(5*d + e/2) + x^{18}*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^{20}*(d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^{12}*((35*d)/2 + 21*e) + x^{16}*((45*d)/16 + (15*e)/2) + x^{14}*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x^{10}*((126*d)/5 + 21*e) + (e*x^{22})/22 + d*\log(x)$

sympy [A] time = 0.33, size = 131, normalized size = 1.41

$$d \log(x) + \frac{ex^{22}}{22} + x^{20} \left(\frac{d}{20} + \frac{e}{2}\right) + x^{18} \left(\frac{5d}{9} + \frac{5e}{2}\right) + x^{16} \left(\frac{45d}{16} + \frac{15e}{2}\right) + x^{14} \left(\frac{60d}{7} + 15e\right) + x^{12} \left(\frac{35d}{2} + 21e\right) + x^{10} \left(\frac{126d}{5} + 21e\right) + x^8 \left(\frac{105d}{4} + 15e\right) + x^6 \left(20d + \frac{15e}{2}\right) + x^4 \left(\frac{45d}{4} + \frac{5e}{2}\right) + x^2 \left(5d + \frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)`

[Out] $d*\log(x) + e*x^{22}/22 + x^{20}*(d/20 + e/2) + x^{18}*(5*d/9 + 5*e/2) + x^{16}*(45*d/16 + 15*e/2) + x^{14}*(60*d/7 + 15*e) + x^{12}*(35*d/2 + 21*e) + x^{10}*(126*d/5 + 21*e) + x^8*(105*d/4 + 15*e) + x^6*(20*d + 15*e/2) + x^4*(45*d/4 + 5*e/2) + x^2*(5*d + e/2)$

$$3.43 \quad \int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$$

Optimal. Leaf size=141

$$\frac{1}{19}x^{19}(d+10e) + \frac{5}{17}x^{17}(2d+9e) + x^{15}(3d+8e) + \frac{30}{13}x^{13}(4d+7e) + \frac{42}{11}x^{11}(5d+6e) + \frac{14}{3}x^9(6d+5e) + \frac{30}{7}x^7(7d+4e) + 3x^5(8d+3e) + \frac{5}{3}x^3(9d+2e) + x(10d+e) - \frac{d}{x} + \frac{ex^{21}}{21}$$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 448}

$$\frac{1}{19}x^{19}(d+10e) + \frac{5}{17}x^{17}(2d+9e) + x^{15}(3d+8e) + \frac{30}{13}x^{13}(4d+7e) + \frac{42}{11}x^{11}(5d+6e) + \frac{14}{3}x^9(6d+5e) + \frac{30}{7}x^7(7d+4e) + 3x^5(8d+3e) + \frac{5}{3}x^3(9d+2e) + x(10d+e) - \frac{d}{x} + \frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2, x]

[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10*e)*x^19)/19 + (e*x^21)/21

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx &= \int \frac{(1+x^2)^{10}(d+ex^2)}{x^2} dx \\ &= \int \left(10d \left(1 + \frac{e}{10d} \right) + \frac{d}{x^2} + 5(9d+2e)x^2 + 15(8d+3e)x^4 + 30(7d+4e)x^6 + 42(6d+5e)x^8 + 3(2d+9e)x^{10} \right) dx \\ &= -\frac{d}{x} + (10d+e)x + \frac{5}{3}(9d+2e)x^3 + 3(8d+3e)x^5 + \frac{30}{7}(7d+4e)x^7 + \frac{14}{3}(6d+5e)x^9 + \frac{3}{11}(2d+9e)x^{11} \end{aligned}$$

Mathematica [A] time = 0.03, size = 141, normalized size = 1.00

$$\frac{1}{19}x^{19}(d+10e) + \frac{5}{17}x^{17}(2d+9e) + x^{15}(3d+8e) + \frac{30}{13}x^{13}(4d+7e) + \frac{42}{11}x^{11}(5d+6e) + \frac{14}{3}x^9(6d+5e) + \frac{30}{7}x^7(7d+4e) + 3x^5(8d+3e) + \frac{5}{3}x^3(9d+2e) + x(10d+e) - \frac{d}{x} + \frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]

[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10*e)*x^19)/19 + (e*x^21)/21

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(1 + 2x^2 + x^4)^5}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2, x]

fricas [A] time = 0.88, size = 131, normalized size = 0.93

$$\frac{46189ex^{22} + 51051(d+10e)x^{20} + 285285(2d+9e)x^{18} + 969969(3d+8e)x^{16} + 2238390(4d+7e)x^{14} + 3703518(5d+6e)x^{12} + 4526522(6d+5e)x^{10} + 4157010(7d+4e)x^8 + 2909907(8d+3e)x^6 + 1616615(9d+2e)x^4 + 969969(10d+e)x^2 - 969969d}{969969x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")

[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 + 969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*x^12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d + 3*e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x

giac [A] time = 0.36, size = 139, normalized size = 0.99

$$\frac{1}{21}x^{21}e + \frac{1}{19}dx^{19} + \frac{10}{19}x^{19}e + \frac{10}{17}dx^{17} + \frac{45}{17}x^{17}e + 3dx^{15} + 8x^{15}e + \frac{120}{13}dx^{13} + \frac{210}{13}x^{13}e + \frac{210}{11}dx^{11} + \frac{252}{11}x^{11}e + 28dx^9 + \frac{70}{3}x^9e + 30dx^7 + \frac{120}{7}x^7e + 24dx^5 + 9x^5e + 15dx^3 + \frac{10}{3}x^3e + 10dx + xe - \frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")

[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x^17*e + 3*d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 252/11*

$$x^{11}e + 28d*x^9 + 70/3*x^9e + 30*d*x^7 + 120/7*x^7*e + 24*d*x^5 + 9*x^5*e + 15*d*x^3 + 10/3*x^3e + 10*d*x + x*e - d/x$$

maple [A] time = 0.00, size = 129, normalized size = 0.91

$$\frac{e x^{21}}{21} + \frac{d x^{19}}{19} + \frac{10 e x^{19}}{19} + \frac{10 d x^{17}}{17} + \frac{45 e x^{17}}{17} + 3 d x^{15} + 8 e x^{15} + \frac{120 d x^{13}}{13} + \frac{210 e x^{13}}{13} + \frac{210 d x^{11}}{11} + \frac{252 e x^{11}}{11} + 28 d x^9 + \frac{70 e x^9}{3} + 30 d x^7 + \frac{120 e x^7}{7} + 24 d x^5 + 9 e x^5 + 15 d x^3 + \frac{10 e x^3}{3} + 10 d x + e x - \frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x)

[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*x^15*d+8*x^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*x^11*e+28*x^9*d+70/3*x^9*e+30*x^7*d+120/7*x^7*e+24*d*x^5+9*x^5*e+15*d*x^3+10/3*x^3*e+10*d*x+e*x-d/x

maxima [A] time = 0.50, size = 125, normalized size = 0.89

$$\frac{1}{21} e x^{21} + \frac{1}{19} (d + 10 e) x^{19} + \frac{5}{17} (2 d + 9 e) x^{17} + (3 d + 8 e) x^{15} + \frac{30}{13} (4 d + 7 e) x^{13} + \frac{42}{11} (5 d + 6 e) x^{11} + \frac{14}{3} (6 d + 5 e) x^9 + \frac{30}{7} (7 d + 4 e) x^7 + 3 (8 d + 3 e) x^5 + \frac{5}{3} (9 d + 2 e) x^3 + (10 d + e) x - \frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")

[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d + e)*x - d/x

mupad [B] time = 0.08, size = 119, normalized size = 0.84

$$x^{15} (3 d + 8 e) + x^3 \left(15 d + \frac{10 e}{3} \right) + x^5 (24 d + 9 e) + x^{19} \left(\frac{d}{19} + \frac{10 e}{19} \right) + x^{17} \left(\frac{10 d}{17} + \frac{45 e}{17} \right) + x^9 \left(28 d + \frac{70 e}{3} \right) + x^7 \left(30 d + \frac{120 e}{7} \right) + x^{13} \left(\frac{120 d}{13} + \frac{210 e}{13} \right) + x^{11} \left(\frac{210 d}{11} + \frac{252 e}{11} \right) + x (10 d + e) - \frac{d}{x} + \frac{e x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)

[Out] x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 + (10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(30*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (252*e)/11) + x*(10*d + e) - d/x + (e*x^21)/21

sympy [A] time = 0.32, size = 124, normalized size = 0.88

$$-\frac{d}{x} + \frac{e x^{21}}{21} + x^{19} \left(\frac{d}{19} + \frac{10 e}{19} \right) + x^{17} \left(\frac{10 d}{17} + \frac{45 e}{17} \right) + x^{15} (3 d + 8 e) + x^{13} \left(\frac{120 d}{13} + \frac{210 e}{13} \right) + x^{11} \left(\frac{210 d}{11} + \frac{252 e}{11} \right) + x^9 \left(28 d + \frac{70 e}{3} \right) + x^7 \left(30 d + \frac{120 e}{7} \right) + x^5 (24 d + 9 e) + x^3 \left(15 d + \frac{10 e}{3} \right) + x (10 d + e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*  
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)  
+ x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*  
(15*d + 10*e/3) + x*(10*d + e)
```

$$3.44 \quad \int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx$$

Optimal. Leaf size=147

$$\frac{1}{18}x^{18}(d+10e) + \frac{5}{16}x^{16}(2d+9e) + \frac{15}{14}x^{14}(3d+8e) + \frac{5}{2}x^{12}(4d+7e) + \frac{21}{5}x^{10}(5d+6e) + \frac{21}{4}x^8(6d+5e) + 5x^6(7d+4e) + \frac{15}{4}x^4$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 446, 76}

$$\frac{1}{18}x^{18}(d+10e) + \frac{5}{16}x^{16}(2d+9e) + \frac{15}{14}x^{14}(3d+8e) + \frac{5}{2}x^{12}(4d+7e) + \frac{21}{5}x^{10}(5d+6e) + \frac{21}{4}x^8(6d+5e) + 5x^6(7d+4e) + \frac{15}{4}x^4(8d+3e) + \frac{5}{2}x^2(9d+2e) + (10d+e)\log(x) - \frac{d}{2x^2} + \frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]

[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 76

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_)]^(p_.)*((c_) + (d_.)*(x_)^(n_)]^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(1 + 2x^2 + x^4)^5}{x^3} dx &= \int \frac{(1 + x^2)^{10}(d + ex^2)}{x^3} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1 + x)^{10}(d + ex)}{x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(5(9d + 2e) + \frac{d}{x^2} + \frac{10d + e}{x} + 15(8d + 3e)x + 30(7d + 4e)x^2 + 42(6d + 5e)x^3 + 15(8d + 3e)x^4 + 5(7d + 4e)x^5 + \frac{21}{4}(6d + 5e)x^6 + \frac{21}{5}(6d + 5e)x^7 + \frac{21}{5}(6d + 5e)x^8 + \frac{21}{5}(6d + 5e)x^9 + \frac{21}{5}(6d + 5e)x^{10} \right) dx, x, x^2 \right) \\
&= -\frac{d}{2x^2} + \frac{5}{2}(9d + 2e)x^2 + \frac{15}{4}(8d + 3e)x^4 + 5(7d + 4e)x^6 + \frac{21}{4}(6d + 5e)x^8 + \frac{21}{5}(6d + 5e)x^{10}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 147, normalized size = 1.00

$$\frac{1}{18}x^{18}(d + 10e) + \frac{5}{16}x^{16}(2d + 9e) + \frac{15}{14}x^{14}(3d + 8e) + \frac{5}{2}x^{12}(4d + 7e) + \frac{21}{5}x^{10}(5d + 6e) + \frac{21}{4}x^8(6d + 5e) + 5x^6(7d + 4e) + \frac{15}{4}x^4(8d + 3e) + \frac{5}{2}x^2(9d + 2e) + (10d + e)\log(x) - \frac{d}{2x^2} + \frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]

[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)(1 + 2x^2 + x^4)^5}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]

[Out] IntegrateAlgebraic[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]

fricas [A] time = 0.78, size = 133, normalized size = 0.90

$$\frac{252ex^{22} + 280(d + 10e)x^{20} + 1575(2d + 9e)x^{18} + 5400(3d + 8e)x^{16} + 12600(4d + 7e)x^{14} + 21168(5d + 6e)x^{12} + 26460(6d + 5e)x^{10} + 25200(7d + 4e)x^8 + 18900(8d + 3e)x^6 + 12600(9d + 2e)x^4 + 5040(10d + e)x^2 \log(x) - 2520d}{5040x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")

[Out] $1/5040*(252*e*x^{22} + 280*(d + 10*e)*x^{20} + 1575*(2*d + 9*e)*x^{18} + 5400*(3*d + 8*e)*x^{16} + 12600*(4*d + 7*e)*x^{14} + 21168*(5*d + 6*e)*x^{12} + 26460*(6*d + 5*e)*x^{10} + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9*d + 2*e)*x^4 + 5040*(10*d + e)*x^2*\log(x) - 2520*d)/x^2$

giac [A] time = 0.26, size = 156, normalized size = 1.06

$$\frac{1}{20}x^{20}e + \frac{1}{18}dx^{18} + \frac{5}{9}x^{18}e + \frac{5}{8}dx^{16} + \frac{45}{16}x^{16}e + \frac{45}{14}dx^{14} + \frac{60}{7}x^{14}e + 10dx^{12} + \frac{35}{2}x^{12}e + 21dx^{10} + \frac{126}{5}x^{10}e + \frac{63}{2}dx^8 + \frac{105}{4}x^8e + 35dx^6 + 20x^6e + 30dx^4 + \frac{45}{4}x^4e + \frac{45}{2}dx^2 + 5x^2e + \frac{1}{2}(10d + e)\log(x^2) - \frac{10dx^2 + x^2e + d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")`

[Out] $1/20*x^{20}*e + 1/18*d*x^{18} + 5/9*x^{18}*e + 5/8*d*x^{16} + 45/16*x^{16}*e + 45/14*d*x^{14} + 60/7*x^{14}*e + 10*d*x^{12} + 35/2*x^{12}*e + 21*d*x^{10} + 126/5*x^{10}*e + 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6*e + 30*d*x^4 + 45/4*x^4*e + 45/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e)*\log(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/x^2$

maple [A] time = 0.01, size = 131, normalized size = 0.89

$$\frac{e x^{20}}{20} + \frac{d x^{18}}{18} + \frac{5 e x^{18}}{9} + \frac{5 d x^{16}}{8} + \frac{45 e x^{16}}{16} + \frac{45 d x^{14}}{14} + \frac{60 e x^{14}}{7} + 10 d x^{12} + \frac{35 e x^{12}}{2} + 21 d x^{10} + \frac{126 e x^{10}}{5} + \frac{63 d x^8}{2} + \frac{105 e x^8}{4} + 35 d x^6 + 20 e x^6 + 30 d x^4 + \frac{45 e x^4}{4} + \frac{45 d x^2}{2} + 5 e x^2 + 10 d \ln(x) + e \ln(x) - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x)`

[Out] $1/20*e*x^{20}+1/18*d*x^{18}+5/9*e*x^{18}+5/8*d*x^{16}+45/16*e*x^{16}+45/14*d*x^{14}+60/7*e*x^{14}+10*d*x^{12}+35/2*e*x^{12}+21*d*x^{10}+126/5*e*x^{10}+63/2*d*x^8+105/4*e*x^8+35*d*x^6+20*e*x^6+30*d*x^4+45/4*e*x^4+45/2*d*x^2+5*e*x^2+10*d*\ln(x)+\ln(x)*e-1/2*d/x^2$

maxima [A] time = 0.70, size = 130, normalized size = 0.88

$$\frac{1}{20}ex^{20} + \frac{1}{18}(d+10e)x^{18} + \frac{5}{16}(2d+9e)x^{16} + \frac{15}{14}(3d+8e)x^{14} + \frac{5}{2}(4d+7e)x^{12} + \frac{21}{5}(5d+6e)x^{10} + \frac{21}{4}(6d+5e)x^8 + 5(7d+4e)x^6 + \frac{15}{4}(8d+3e)x^4 + \frac{5}{2}(9d+2e)x^2 + \frac{1}{2}(10d+e)\log(x^2) - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")`

[Out] $1/20*e*x^{20} + 1/18*(d + 10*e)*x^{18} + 5/16*(2*d + 9*e)*x^{16} + 15/14*(3*d + 8*e)*x^{14} + 5/2*(4*d + 7*e)*x^{12} + 21/5*(5*d + 6*e)*x^{10} + 21/4*(6*d + 5*e)*x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*(10*d + e)*\log(x^2) - 1/2*d/x^2$

mupad [B] time = 0.08, size = 120, normalized size = 0.82

$$x^{18} \left(\frac{d}{18} + \frac{5e}{9} \right) + x^2 \left(\frac{45d}{2} + 5e \right) + x^{12} \left(10d + \frac{35e}{2} \right) + x^6 (35d + 20e) + x^4 \left(30d + \frac{45e}{4} \right) + x^{16} \left(\frac{5d}{8} + \frac{45e}{16} \right) + x^{14} \left(\frac{45d}{14} + \frac{60e}{7} \right) + x^{10} \left(21d + \frac{126e}{5} \right) + x^8 \left(\frac{63d}{2} + \frac{105e}{4} \right) - \frac{d}{2x^2} + \frac{e x^{20}}{20} + \ln(x) (10d + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)`

[Out] $x^{18}*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^{12}*(10*d + (35*e)/2) + x^6*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^{16}*((5*d)/8 + (45*e)/16) + x^{14}*((45*d)/14 + (60*e)/7) + x^{10}*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*e)/4) - d/(2*x^2) + (e*x^{20})/20 + \log(x)*(10*d + e)$

sympy [A] time = 0.39, size = 131, normalized size = 0.89

$$-\frac{d}{2x^2} + \frac{ex^{20}}{20} + x^{18}\left(\frac{d}{18} + \frac{5e}{9}\right) + x^{16}\left(\frac{5d}{8} + \frac{45e}{16}\right) + x^{14}\left(\frac{45d}{14} + \frac{60e}{7}\right) + x^{12}\left(10d + \frac{35e}{2}\right) + x^{10}\left(21d + \frac{126e}{5}\right) + x^8\left(\frac{63d}{2} + \frac{105e}{4}\right) + x^6(35d + 20e) + x^4\left(30d + \frac{45e}{4}\right) + x^2\left(\frac{45d}{2} + 5e\right) + (10d + e)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)`

[Out] $-d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) + x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5) + x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**2*(45*d/2 + 5*e) + (10*d + e)*\log(x)$

$$3.45 \quad \int (fx)^m (1+x^2)(1+2x^2+x^4)^5 dx$$

Optimal. Leaf size=203

$$\frac{(fx)^{m+23}}{f^{23}(m+23)} + \frac{11(fx)^{m+21}}{f^{21}(m+21)} + \frac{55(fx)^{m+19}}{f^{19}(m+19)} + \frac{165(fx)^{m+17}}{f^{17}(m+17)} + \frac{330(fx)^{m+15}}{f^{15}(m+15)} + \frac{462(fx)^{m+13}}{f^{13}(m+13)} + \frac{462(fx)^{m+11}}{f^{11}(m+11)} + \frac{330(fx)^m}{f^9(m+9)}$$

Rubi [A] time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 270}

$$\frac{11(fx)^{m+3}}{f^3(m+3)} + \frac{55(fx)^{m+5}}{f^5(m+5)} + \frac{165(fx)^{m+7}}{f^7(m+7)} + \frac{330(fx)^{m+9}}{f^9(m+9)} + \frac{462(fx)^{m+11}}{f^{11}(m+11)} + \frac{462(fx)^{m+13}}{f^{13}(m+13)} + \frac{330(fx)^{m+15}}{f^{15}(m+15)} + \frac{165(fx)^{m+17}}{f^{17}(m+17)} + \frac{55(fx)^{m+19}}{f^{19}(m+19)} + \frac{11(fx)^{m+21}}{f^{21}(m+21)} + \frac{(fx)^{m+23}}{f^{23}(m+23)} + \frac{(fx)^{m+1}}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(1+x^2)*(1+2*x^2+x^4)^5,x]

[Out] (f*x)^(1+m)/(f*(1+m)) + (11*(f*x)^(3+m))/(f^3*(3+m)) + (55*(f*x)^(5+m))/(f^5*(5+m)) + (165*(f*x)^(7+m))/(f^7*(7+m)) + (330*(f*x)^(9+m))/(f^9*(9+m)) + (462*(f*x)^(11+m))/(f^11*(11+m)) + (462*(f*x)^(13+m))/(f^13*(13+m)) + (330*(f*x)^(15+m))/(f^15*(15+m)) + (165*(f*x)^(17+m))/(f^17*(17+m)) + (55*(f*x)^(19+m))/(f^19*(19+m)) + (11*(f*x)^(21+m))/(f^21*(21+m)) + (f*x)^(23+m)/(f^23*(23+m))

Rule 28

Int[(u_.)*((a_.)+(c_.)*(x_)^(n2_.))+(b_.)*(x_)^(n_)^(p_.), x_Symbol] :=> Dist[1/c^p, Int[u*(b/2+c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2-4*a*c, 0] && IntegerQ[p]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.)+(b_.)*(x_)^(n_)^(p_.), x_Symbol] :=> Int[ExpandIntegrand[(c*x)^m*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (fx)^m (1+x^2)(1+2x^2+x^4)^5 dx &= \int (fx)^m (1+x^2)^{11} dx \\ &= \int \left((fx)^m + \frac{11(fx)^{2+m}}{f^2} + \frac{55(fx)^{4+m}}{f^4} + \frac{165(fx)^{6+m}}{f^6} + \frac{330(fx)^{8+m}}{f^8} + \frac{462(fx)^{10+m}}{f^{10}} \right. \\ &= \frac{(fx)^{1+m}}{f(1+m)} + \frac{11(fx)^{3+m}}{f^3(3+m)} + \frac{55(fx)^{5+m}}{f^5(5+m)} + \frac{165(fx)^{7+m}}{f^7(7+m)} + \frac{330(fx)^{9+m}}{f^9(9+m)} + \frac{462(fx)^{11+m}}{f^{11}(11+m)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 122, normalized size = 0.60

$$x \left(\frac{x^{22}}{m+23} + \frac{11x^{20}}{m+21} + \frac{55x^{18}}{m+19} + \frac{165x^{16}}{m+17} + \frac{330x^{14}}{m+15} + \frac{462x^{12}}{m+13} + \frac{462x^{10}}{m+11} + \frac{330x^8}{m+9} + \frac{165x^6}{m+7} + \frac{55x^4}{m+5} + \frac{11x^2}{m+3} + \frac{1}{m+1} \right) (fx)^m$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (330*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(21 + m) + x^22/(23 + m))

IntegrateAlgebraic [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (fx)^m (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.82, size = 759, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 453714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 16643902275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 62319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 32368407579*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 + 462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 + 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130

$$1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^{11} + 330*(m^{11} + 135*m^{10} + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35137127025)*x^9 + 165*(m^{11} + 137*m^{10} + 8259*m^9 + 288027*m^8 + 6423594*m^7 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 70930262349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^{11} + 139*m^{10} + 8523*m^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^5 + 11*(m^{11} + 141*m^{10} + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 + 199334977695*m + 105411381075)*x^3 + (m^{11} + 143*m^{10} + 9075*m^9 + 336765*m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 60936676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)/(m^{12} + 144*m^{11} + 9218*m^{10} + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)$$

giac [B] time = 0.62, size = 1848, normalized size = 9.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 165*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 + 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 5353566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 32087153670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^13 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23 + 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 + 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^13 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 680615362515*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*

$$\begin{aligned}
& m^7 x^{13} + 204865733820 (f x)^m m^5 x^{15} + 2251106854425 (f x)^m m^3 x^{17} + \\
& 1988025402825 (f x)^m m x^{19} + 44550 (f x)^m m^{10} x^9 + 120367170 (f x)^m \\
& m^8 x^{11} + 33609870756 (f x)^m m^6 x^{13} + 1262375264700 (f x)^m m^4 x^{15} + \\
& 5340787250535 (f x)^m m^2 x^{17} + 915414625125 (f x)^m x^{19} + 165 (f x)^m m^{11} x^7 + \\
& 2640990 (f x)^m m^9 x^9 + 2575140876 (f x)^m m^7 x^{11} + 3153471505 \\
& 80 (f x)^m m^5 x^{13} + 5015196628530 (f x)^m m^3 x^{15} + 6646727085075 (f x)^m \\
& m x^{17} + 22605 (f x)^m m^{10} x^7 + 90358290 (f x)^m m^8 x^9 + 36597992508 \\
& (f x)^m m^6 x^{11} + 1969992823260 (f x)^m m^4 x^{13} + 11991258123570 (f x)^m \\
& m^2 x^{15} + 3069331390125 (f x)^m x^{17} + 55 (f x)^m m^{11} x^5 + 1362735 (f x)^m \\
& m^9 x^7 + 1971903780 (f x)^m m^7 x^9 + 349697552820 (f x)^m m^5 x^{11} + 7 \\
& 921249136262 (f x)^m m^3 x^{13} + 15011348834790 (f x)^m m x^{15} + 7645 (f x)^m \\
& m^{10} x^5 + 47524455 (f x)^m m^8 x^7 + 28627538940 (f x)^m m^6 x^9 + 22228 \\
& 32699780 (f x)^m m^4 x^{11} + 19130651800722 (f x)^m m^2 x^{13} + 6957151150950 \\
& (f x)^m x^{15} + 11 (f x)^m m^{11} x^3 + 468765 (f x)^m m^9 x^5 + 1059893010 (f \\
& x)^m m^7 x^7 + 279691771260 (f x)^m m^5 x^9 + 9079996141062 (f x)^m m^3 x^{11} + \\
& 24133835554290 (f x)^m m x^{13} + 1551 (f x)^m m^{10} x^3 + 16677375 (f x)^m \\
& m^8 x^5 + 15768085410 (f x)^m m^6 x^7 + 1818135330660 (f x)^m m^4 x^9 + \\
& 22226933020446 (f x)^m m^2 x^{11} + 11238474936150 (f x)^m x^{13} + (f x)^m m^{11} x + \\
& 96745 (f x)^m m^9 x^3 + 380801190 (f x)^m m^7 x^5 + 158293212990 (f x)^m \\
& m^5 x^7 + 7587607623090 (f x)^m m^3 x^9 + 28336045738770 (f x)^m m x^{11} + \\
& 143 (f x)^m m^{10} x + 3514005 (f x)^m m^8 x^3 + 5825106210 (f x)^m m^6 x^5 + \\
& 1059628145070 (f x)^m m^4 x^7 + 18930738943710 (f x)^m m^2 x^9 + 13281 \\
& 834015450 (f x)^m x^{11} + 9075 (f x)^m m^9 x + 82295598 (f x)^m m^7 x^3 + 60 \\
& 431072570 (f x)^m m^5 x^5 + 4558015784025 (f x)^m m^3 x^7 + 24503570194950 (f \\
& x)^m m x^9 + 336765 (f x)^m m^8 x + 1298935638 (f x)^m m^6 x^3 + 4204048 \\
& 49150 (f x)^m m^4 x^5 + 11703493287585 (f x)^m m^2 x^7 + 11595251918250 (f x)^m \\
& x^9 + 8103018 (f x)^m m^7 x + 14014513810 (f x)^m m^5 x^3 + 1889780020 \\
& 755 (f x)^m m^3 x^5 + 15515657331075 (f x)^m m x^7 + 132426294 (f x)^m m^6 x + \\
& 102468500970 (f x)^m m^4 x^3 + 5087634488145 (f x)^m m^2 x^5 + 74540905 \\
& 18875 (f x)^m x^7 + 1495875590 (f x)^m m^5 x + 490955350391 (f x)^m m^3 x^3 + \\
& 7041864340665 (f x)^m m x^5 + 11641582810 (f x)^m m^4 x + 1434440867211 (f \\
& x)^m m^2 x^3 + 3478575575475 (f x)^m x^5 + 60936676581 (f x)^m m^3 x + 2 \\
& 192684754645 (f x)^m m x^3 + 203363952363 (f x)^m m^2 x + 1159525191825 (f x)^m \\
& x^3 + 387182170935 (f x)^m m x + 316234143225 (f x)^m x / (m^{12} + 144 m^{11} + \\
& 9218 m^{10} + 345840 m^9 + 8439783 m^8 + 140529312 m^7 + 1628301884 m^6 + \\
& 13137458400 m^5 + 72578259391 m^4 + 264300628944 m^3 + 590546123298 m^2 + \\
& 703416314160 m + 316234143225)
\end{aligned}$$

maple [B] time = 0.01, size = 1121, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f x)^m (x^2+1) (x^4+2 x^2+1)^5, x)$

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(m+1)/(m+3)/(m+5)/(m+7)/(m+9)/(m+11)/(m+
13)/(m+15)/(m+17)/(m+19)/(m+21)/(m+23)
```

maxima [A] time = 0.97, size = 192, normalized size = 0.95

$$\frac{f^m x^{23} x^m}{m+23} + \frac{11 f^m x^{21} x^m}{m+21} + \frac{55 f^m x^{19} x^m}{m+19} + \frac{165 f^m x^{17} x^m}{m+17} + \frac{330 f^m x^{15} x^m}{m+15} + \frac{462 f^m x^{13} x^m}{m+13} + \frac{462 f^m x^{11} x^m}{m+11} + \frac{330 f^m x^9 x^m}{m+9} + \frac{165 f^m x^7 x^m}{m+7} + \frac{55 f^m x^5 x^m}{m+5} + \frac{11 f^m x^3 x^m}{m+3} + \frac{(f x)^{m+1}}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] $f^m x^{23} x^m / (m + 23) + 11 f^m x^{21} x^m / (m + 21) + 55 f^m x^{19} x^m / (m + 19) + 165 f^m x^{17} x^m / (m + 17) + 330 f^m x^{15} x^m / (m + 15) + 462 f^m x^{13} x^m / (m + 13) + 462 f^m x^{11} x^m / (m + 11) + 330 f^m x^9 x^m / (m + 9) + 165 f^m x^7 x^m / (m + 7) + 55 f^m x^5 x^m / (m + 5) + 11 f^m x^3 x^m / (m + 3) + (f x)^m + 1) / (f (m + 1))$

mupad [B] time = 1.25, size = 1483, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2 + 1)(f x)^m (2x^2 + x^4 + 1)^5, x)$

[Out] $(x^3 (f x)^m (2192684754645 m + 1434440867211 m^2 + 490955350391 m^3 + 102468500970 m^4 + 14014513810 m^5 + 1298935638 m^6 + 82295598 m^7 + 3514005 m^8 + 96745 m^9 + 1551 m^{10} + 11 m^{11} + 1159525191825)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225) + (x^{19} (f x)^m (1988025402825 m + 1604842704135 m^2 + 680615362515 m^3 + 174273100210 m^4 + 28857216410 m^5 + 3194704590 m^6 + 238653030 m^7 + 11870265 m^8 + 376365 m^9 + 6875 m^{10} + 55 m^{11} + 915414625125)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225) + (x^{11} (f x)^m (28336045738770 m + 22226933020446 m^2 + 9079996141062 m^3 + 2222832699780 m^4 + 349697552820 m^5 + 36597992508 m^6 + 2575140876 m^7 + 120367170 m^8 + 3582810 m^9 + 61446 m^{10} + 462 m^{11} + 13281834015450)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225) + (x^{21} (f x)^m (360568238085 m + 292163767533 m^2 + 124530626231 m^3 + 32087153670 m^4 + 5353566130 m^5 + 597988314 m^6 + 45134958 m^7 + 2271555 m^8 + 72985 m^9 + 1353 m^{10} + 11 m^{11} + 165646455975)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225) + (x^5 (f x)^m (7041864340665 m + 5087634488145 m^2 + 1889780020755 m^3 + 420404849150 m^4 + 60431072570 m^5 + 5825106210 m^6 + 380801190 m^7 + 16677375 m^8 + 468765 m^9 + 7645 m^{10} + 55 m^{11} + 3478575575475)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225) + (x^{17} (f x)^m (6646727085075 m + 5340787250535 m^2 + 2251106854425 m^3 + 572017996770 m^4 + 93862508190 m^5 + 10282782510 m^6 + 759091410 m^7 + 37263105 m^8 + 1164735 m^9 + 20955 m^{10} + 165 m^{11} + 3069331390125)) / (703416314160 m + 590546123298 m^2 + 264300628944 m^3 + 72578259391 m^4 + 13137458400 m^5 + 1628301884 m^6 + 140529312 m^7 + 8439783 m^8 + 345840 m^9 + 9218 m^{10} + 144 m^{11} + m^{12} + 316234143225)$

```

10 + 144*m^11 + m^12 + 316234143225) + (x*(f*x)^m*(387182170935*m + 2033639
52363*m^2 + 60936676581*m^3 + 11641582810*m^4 + 1495875590*m^5 + 132426294*
m^6 + 8103018*m^7 + 336765*m^8 + 9075*m^9 + 143*m^10 + m^11 + 316234143225)
)/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 +
13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^
9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^23*(f*x)^m*(2998552189
5*m + 24372200061*m^2 + 10431670821*m^3 + 2702025590*m^4 + 453714470*m^5 +
51069018*m^6 + 3889578*m^7 + 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 1374
9310575))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782593
91*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3
45840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^7*(f*x)^m*(155
15657331075*m + 11703493287585*m^2 + 4558015784025*m^3 + 1059628145070*m^4
+ 158293212990*m^5 + 15768085410*m^6 + 1059893010*m^7 + 47524455*m^8 + 1362
735*m^9 + 22605*m^10 + 165*m^11 + 7454090518875))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (x^15*(f*x)^m*(15011348834790*m + 11991258123570*m^2
+ 5015196628530*m^3 + 1262375264700*m^4 + 204865733820*m^5 + 22164925860*m
^6 + 1613983140*m^7 + 78076350*m^8 + 2403390*m^9 + 42570*m^10 + 330*m^11 +
6957151150950))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725
78259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m
^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^9*(f*x)^
m*(24503570194950*m + 18930738943710*m^2 + 7587607623090*m^3 + 181813533066
0*m^4 + 279691771260*m^5 + 28627538940*m^6 + 1971903780*m^7 + 90358290*m^8
+ 2640990*m^9 + 44550*m^10 + 330*m^11 + 11595251918250))/(703416314160*m +
590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1
628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*
m^11 + m^12 + 316234143225) + (x^13*(f*x)^m*(24133835554290*m + 19130651800
722*m^2 + 7921249136262*m^3 + 1969992823260*m^4 + 315347150580*m^5 + 336098
70756*m^6 + 2408820876*m^7 + 114642990*m^8 + 3471930*m^9 + 60522*m^10 + 462
*m^11 + 11238474936150))/(703416314160*m + 590546123298*m^2 + 264300628944*
m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 +
8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] Timed out

$$3.46 \quad \int x^5 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=34

$$\frac{1}{28} (x^2 + 1)^{14} - \frac{1}{13} (x^2 + 1)^{13} + \frac{1}{24} (x^2 + 1)^{12}$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 266, 43}

$$\frac{1}{28} (x^2 + 1)^{14} - \frac{1}{13} (x^2 + 1)^{13} + \frac{1}{24} (x^2 + 1)^{12}$$

Antiderivative was successfully verified.

[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^5 (1+x^2)(1+2x^2+x^4)^5 dx &= \int x^5 (1+x^2)^{11} dx \\
&= \frac{1}{2} \text{Subst} \left(\int x^2 (1+x)^{11} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int ((1+x)^{11} - 2(1+x)^{12} + (1+x)^{13}) dx, x, x^2 \right) \\
&= \frac{1}{24} (1+x^2)^{12} - \frac{1}{13} (1+x^2)^{13} + \frac{1}{28} (1+x^2)^{14}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 85, normalized size = 2.50

$$\frac{x^{28}}{28} + \frac{11x^{26}}{26} + \frac{55x^{24}}{24} + \frac{15x^{22}}{2} + \frac{33x^{20}}{2} + \frac{77x^{18}}{3} + \frac{231x^{16}}{8} + \frac{165x^{14}}{7} + \frac{55x^{12}}{4} + \frac{11x^{10}}{2} + \frac{11x^8}{8} + \frac{x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(1+x^2)*(1+2*x^2+x^4)^5,x]

[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 + x^28/28

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (1+x^2)(1+2x^2+x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5*(1+x^2)*(1+2*x^2+x^4)^5,x]

[Out] IntegrateAlgebraic[x^5*(1+x^2)*(1+2*x^2+x^4)^5, x]

fricas [B] time = 0.56, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^8 + \frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6

giac [B] time = 0.34, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^8 + \frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6

maple [B] time = 0.00, size = 62, normalized size = 1.82

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^8 + \frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x)

[Out] 1/28*x^28+11/26*x^26+55/24*x^24+15/2*x^22+33/2*x^20+77/3*x^18+231/8*x^16+165/7*x^14+55/4*x^12+11/2*x^10+11/8*x^8+1/6*x^6

maxima [B] time = 0.64, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^8 + \frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6

mupad [B] time = 0.06, size = 61, normalized size = 1.79

$$\frac{x^{28}}{28} + \frac{11x^{26}}{26} + \frac{55x^{24}}{24} + \frac{15x^{22}}{2} + \frac{33x^{20}}{2} + \frac{77x^{18}}{3} + \frac{231x^{16}}{8} + \frac{165x^{14}}{7} + \frac{55x^{12}}{4} + \frac{11x^{10}}{2} + \frac{11x^8}{8} + \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 + x^28/28

sympy [B] time = 0.07, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28} + \frac{11x^{26}}{26} + \frac{55x^{24}}{24} + \frac{15x^{22}}{2} + \frac{33x^{20}}{2} + \frac{77x^{18}}{3} + \frac{231x^{16}}{8} + \frac{165x^{14}}{7} + \frac{55x^{12}}{4} + \frac{11x^{10}}{2} + \frac{11x^8}{8} + \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
+ 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6

$$3.47 \quad \int x^4 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=83

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^9}{9} + \frac{11x^7}{7} + \frac{x^5}{5}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {28, 270}

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^9}{9} + \frac{11x^7}{7} + \frac{x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 + x^27/27

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^p_.], x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^4 (1 + x^2) (1 + 2x^2 + x^4)^5 dx &= \int x^4 (1 + x^2)^{11} dx \\ &= \int (x^4 + 11x^6 + 55x^8 + 165x^{10} + 330x^{12} + 462x^{14} + 462x^{16} + 330x^{18} + 165x^{20} + x^{22}) dx \\ &= \frac{x^5}{5} + \frac{11x^7}{7} + \frac{55x^9}{9} + 15x^{11} + \frac{330x^{13}}{13} + \frac{154x^{15}}{5} + \frac{462x^{17}}{17} + \frac{330x^{19}}{19} + \frac{55x^{21}}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^9}{9} + \frac{11x^7}{7} + \frac{x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 + x^27/27

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 0.76, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^9 + \frac{11}{7}x^7 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5

giac [A] time = 0.41, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^9 + \frac{11}{7}x^7 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^9 + \frac{11}{7}x^7 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x)`

[Out] `1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27`

maxima [A] time = 0.76, size = 61, normalized size = 0.73

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15x^{11} + \frac{55}{9}x^9 + \frac{11}{7}x^7 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")`

[Out] `1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5`

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^9}{9} + \frac{11x^7}{7} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)`

[Out] `x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 + x^27/27`

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^9}{9} + \frac{11x^7}{7} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)`

[Out] `x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**17/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5/5`

$$3.48 \quad \int x^3 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=23

$$\frac{1}{26} (x^2 + 1)^{13} - \frac{1}{24} (x^2 + 1)^{12}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 266, 43}

$$\frac{1}{26} (x^2 + 1)^{13} - \frac{1}{24} (x^2 + 1)^{12}$$

Antiderivative was successfully verified.

[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^3 (1+x^2) (1+2x^2+x^4)^5 dx &= \int x^3 (1+x^2)^{11} dx \\
&= \frac{1}{2} \text{Subst} \left(\int x(1+x)^{11} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int (-(1+x)^{11} + (1+x)^{12}) dx, x, x^2 \right) \\
&= -\frac{1}{24} (1+x^2)^{12} + \frac{1}{26} (1+x^2)^{13}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 83, normalized size = 3.61

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^8}{8} + \frac{11x^6}{6} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (231*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (1+x^2) (1+2x^2+x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.86, size = 61, normalized size = 2.65

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^8 + \frac{11}{6}x^6 + \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4

giac [B] time = 0.36, size = 61, normalized size = 2.65

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^8 + \frac{11}{6}x^6 + \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4

maple [B] time = 0.00, size = 62, normalized size = 2.70

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^8 + \frac{11}{6}x^6 + \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x)

[Out] 1/26*x^26+11/24*x^24+5/2*x^22+33/4*x^20+55/3*x^18+231/8*x^16+33*x^14+55/2*x^12+33/2*x^10+55/8*x^8+11/6*x^6+1/4*x^4

maxima [B] time = 0.56, size = 61, normalized size = 2.65

$$\frac{1}{26}x^{26} + \frac{11}{24}x^{24} + \frac{5}{2}x^{22} + \frac{33}{4}x^{20} + \frac{55}{3}x^{18} + \frac{231}{8}x^{16} + 33x^{14} + \frac{55}{2}x^{12} + \frac{33}{2}x^{10} + \frac{55}{8}x^8 + \frac{11}{6}x^6 + \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4

mupad [B] time = 0.06, size = 61, normalized size = 2.65

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^8}{8} + \frac{11x^6}{6} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (231*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26

sympy [B] time = 0.07, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^8}{8} + \frac{11x^6}{6} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8 + 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4

$$3.49 \quad \int x^2 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=83

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {28, 270}

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 + x^25/25

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^2 (1 + x^2) (1 + 2x^2 + x^4)^5 dx &= \int x^2 (1 + x^2)^{11} dx \\ &= \int (x^2 + 11x^4 + 55x^6 + 165x^8 + 330x^{10} + 462x^{12} + 462x^{14} + 330x^{16} + 165x^{18} + 11x^{20} + x^{22}) dx \\ &= \frac{x^3}{3} + \frac{11x^5}{5} + \frac{55x^7}{7} + \frac{55x^9}{3} + 30x^{11} + \frac{462x^{13}}{13} + \frac{154x^{15}}{5} + \frac{330x^{17}}{17} + \frac{165x^{19}}{19} \end{aligned}$$

Mathematica [A] time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 + x^25/25

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (1 + x^2) (1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 0.57, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3

giac [A] time = 0.30, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3

maple [A] time = 0.00, size = 62, normalized size = 0.75

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x)`

[Out] `1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25`

maxima [A] time = 0.88, size = 61, normalized size = 0.73

$$\frac{1}{25}x^{25} + \frac{11}{23}x^{23} + \frac{55}{21}x^{21} + \frac{165}{19}x^{19} + \frac{330}{17}x^{17} + \frac{154}{5}x^{15} + \frac{462}{13}x^{13} + 30x^{11} + \frac{55}{3}x^9 + \frac{55}{7}x^7 + \frac{11}{5}x^5 + \frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")`

[Out] `1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3`

mupad [B] time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)`

[Out] `x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 + x^25/25`

sympy [A] time = 0.07, size = 75, normalized size = 0.90

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^9}{3} + \frac{55x^7}{7} + \frac{11x^5}{5} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)`

[Out] `x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x**15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/3`

$$3.50 \quad \int x(1+x^2)(1+2x^2+x^4)^5 dx$$

Optimal. Leaf size=11

$$\frac{1}{24}(x^2+1)^{12}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {28, 261}

$$\frac{1}{24}(x^2+1)^{12}$$

Antiderivative was successfully verified.

[In] Int[x*(1+x^2)*(1+2*x^2+x^4)^5,x]

[Out] (1+x^2)^12/24

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 261

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(a + b*x^n)^(p+1)/(b*n*(p+1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n-1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x(1+x^2)(1+2x^2+x^4)^5 dx &= \int x(1+x^2)^{11} dx \\ &= \frac{1}{24}(1+x^2)^{12} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{24}(x^2+1)^{12}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (1 + x^2)^12/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(1+x^2)(1+2x^2+x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [B] time = 0.51, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^8 + \frac{55}{6}x^6 + \frac{11}{4}x^4 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2

giac [B] time = 0.31, size = 76, normalized size = 6.91

$$\frac{1}{24}(x^4 + 2x^2)^6 + \frac{1}{4}(x^4 + 2x^2)^5 + \frac{5}{8}(x^4 + 2x^2)^4 + \frac{1}{4}x^4 + \frac{5}{6}(x^4 + 2x^2)^3 + \frac{5}{8}(x^4 + 2x^2)^2 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4 + 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2

maple [B] time = 0.00, size = 62, normalized size = 5.64

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^8 + \frac{55}{6}x^6 + \frac{11}{4}x^4 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x)

[Out] 1/24*x^24+1/2*x^22+11/4*x^20+55/6*x^18+165/8*x^16+33*x^14+77/2*x^12+33*x^10+165/8*x^8+55/6*x^6+11/4*x^4+1/2*x^2

maxima [B] time = 0.78, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24} + \frac{1}{2}x^{22} + \frac{11}{4}x^{20} + \frac{55}{6}x^{18} + \frac{165}{8}x^{16} + 33x^{14} + \frac{77}{2}x^{12} + 33x^{10} + \frac{165}{8}x^8 + \frac{55}{6}x^6 + \frac{11}{4}x^4 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")

[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2

mupad [B] time = 0.06, size = 61, normalized size = 5.55

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11x^{20}}{4} + \frac{55x^{18}}{6} + \frac{165x^{16}}{8} + 33x^{14} + \frac{77x^{12}}{2} + 33x^{10} + \frac{165x^8}{8} + \frac{55x^6}{6} + \frac{11x^4}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33*x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24

sympy [B] time = 0.07, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11x^{20}}{4} + \frac{55x^{18}}{6} + \frac{165x^{16}}{8} + 33x^{14} + \frac{77x^{12}}{2} + 33x^{10} + \frac{165x^8}{8} + \frac{55x^6}{6} + \frac{11x^4}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2

$$3.51 \quad \int (1 + x^2)(1 + 2x^2 + x^4)^5 dx$$

Optimal. Leaf size=73

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^9}{3} + \frac{165x^7}{7} + 11x^5 + \frac{11x^3}{3} + x$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 194}

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^9}{3} + \frac{165x^7}{7} + 11x^5 + \frac{11x^3}{3} + x$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + x^2)(1 + 2x^2 + x^4)^5 dx &= \int (1 + x^2)^{11} dx \\ &= \int (1 + 11x^2 + 55x^4 + 165x^6 + 330x^8 + 462x^{10} + 462x^{12} + 330x^{14} + 165x^{16} + 11x^{18} + 11x^{20} + x^{22}) dx \\ &= x + \frac{11x^3}{3} + 11x^5 + \frac{165x^7}{7} + \frac{110x^9}{3} + 42x^{11} + \frac{462x^{13}}{13} + 22x^{15} + \frac{165x^{17}}{17} + \frac{55x^{19}}{19} + \frac{11x^{21}}{21} + \frac{x^{23}}{23} \end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^9}{3} + \frac{165x^7}{7} + 11x^5 + \frac{11x^3}{3} + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + x^2)(1 + 2x^2 + x^4)^5 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] IntegrateAlgebraic[(1 + x^2)*(1 + 2*x^2 + x^4)^5, x]

fricas [A] time = 1.10, size = 57, normalized size = 0.78

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^9 + \frac{165}{7}x^7 + 11x^5 + \frac{11}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")

[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 + 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x

giac [A] time = 0.27, size = 57, normalized size = 0.78

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^9 + \frac{165}{7}x^7 + 11x^5 + \frac{11}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")

[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 + 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^9 + \frac{165}{7}x^7 + 11x^5 + \frac{11}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)*(x^4+2*x^2+1)^5,x)`

[Out] $x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^{11}+462/13*x^{13}+22*x^{15}+165/17*x^{17}+55/19*x^{19}+11/21*x^{21}+1/23*x^{23}$

maxima [A] time = 0.98, size = 57, normalized size = 0.78

$$\frac{1}{23}x^{23} + \frac{11}{21}x^{21} + \frac{55}{19}x^{19} + \frac{165}{17}x^{17} + 22x^{15} + \frac{462}{13}x^{13} + 42x^{11} + \frac{110}{3}x^9 + \frac{165}{7}x^7 + 11x^5 + \frac{11}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")`

[Out] $1/23*x^{23} + 11/21*x^{21} + 55/19*x^{19} + 165/17*x^{17} + 22*x^{15} + 462/13*x^{13} + 42*x^{11} + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x$

mupad [B] time = 0.06, size = 57, normalized size = 0.78

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^9}{3} + \frac{165x^7}{7} + 11x^5 + \frac{11x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)`

[Out] $x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^{11} + (462*x^{13})/13 + 22*x^{15} + (165*x^{17})/17 + (55*x^{19})/19 + (11*x^{21})/21 + x^{23}/23$

sympy [A] time = 0.07, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^9}{3} + \frac{165x^7}{7} + 11x^5 + \frac{11x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)*(x**4+2*x**2+1)**5,x)`

[Out] $x^{**23}/23 + 11*x^{**21}/21 + 55*x^{**19}/19 + 165*x^{**17}/17 + 22*x^{**15} + 462*x^{**13}/13 + 42*x^{**11} + 110*x^{**9}/3 + 165*x^{**7}/7 + 11*x^{**5} + 11*x^{**3}/3 + x$

$$3.52 \quad \int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} dx$$

Optimal. Leaf size=80

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 266, 43}

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]

[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22 + Log[x]

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} dx &= \int \frac{(1+x^2)^{11}}{x} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1+x)^{11}}{x} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(11 + \frac{1}{x} + 55x + 165x^2 + 330x^3 + 462x^4 + 462x^5 + 330x^6 + 165x^7 \right) dx, x, x^2 \right) \\
&= \frac{11x^2}{2} + \frac{55x^4}{4} + \frac{55x^6}{2} + \frac{165x^8}{4} + \frac{231x^{10}}{5} + \frac{77x^{12}}{2} + \frac{165x^{14}}{7} + \frac{165x^{16}}{16} + \frac{55x^{18}}{18} + \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x, x]

[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22 + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x, x]

[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x, x]

fricas [A] time = 0.85, size = 58, normalized size = 0.72

$$\frac{1}{22} x^{22} + \frac{11}{20} x^{20} + \frac{55}{18} x^{18} + \frac{165}{16} x^{16} + \frac{165}{7} x^{14} + \frac{77}{2} x^{12} + \frac{231}{5} x^{10} + \frac{165}{4} x^8 + \frac{55}{2} x^6 + \frac{55}{4} x^4 + \frac{11}{2} x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x, x, algorithm="fricas")

[Out] $\frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + 231/5x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \log(x)$

giac [A] time = 0.23, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + \frac{231}{5}x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")`

[Out] $\frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + 231/5x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \frac{1}{2}\log(x^2)$

maple [A] time = 0.00, size = 59, normalized size = 0.74

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)*(x^4+2*x^2+1)^5/x,x)`

[Out] $\frac{11}{2}x^2 + \frac{55}{4}x^4 + \frac{55}{2}x^6 + \frac{165}{4}x^8 + \frac{231}{5}x^{10} + \frac{77}{2}x^{12} + \frac{165}{7}x^{14} + \frac{165}{16}x^{16} + \frac{55}{18}x^{18} + \frac{11}{20}x^{20} + \frac{1}{22}x^{22} + \ln(x)$

maxima [A] time = 0.82, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + \frac{231}{5}x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")`

[Out] $\frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + 231/5x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \frac{1}{2}\log(x^2)$

mupad [B] time = 0.06, size = 58, normalized size = 0.72

$$\ln(x) + \frac{11x^2}{2} + \frac{55x^4}{4} + \frac{55x^6}{2} + \frac{165x^8}{4} + \frac{231x^{10}}{5} + \frac{77x^{12}}{2} + \frac{165x^{14}}{7} + \frac{165x^{16}}{16} + \frac{55x^{18}}{18} + \frac{11x^{20}}{20} + \frac{x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)`

[Out] $\log(x) + (11x^2)/2 + (55x^4)/4 + (55x^6)/2 + (165x^8)/4 + (231x^{10})/5$
 $+ (77x^{12})/2 + (165x^{14})/7 + (165x^{16})/16 + (55x^{18})/18 + (11x^{20})/20$
 $+ x^{22}/22$

sympy [A] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^8}{4} + \frac{55x^6}{2} + \frac{55x^4}{4} + \frac{11x^2}{2} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)`

[Out] $x^{22}/22 + 11x^{20}/20 + 55x^{18}/18 + 165x^{16}/16 + 165x^{14}/7 + 77x^{12}$
 $/2 + 231x^{10}/5 + 165x^8/4 + 55x^6/2 + 55x^4/4 + 11x^2/2 + \log(x)$

$$3.53 \quad \int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx$$

Optimal. Leaf size=73

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^9}{3} + \frac{330x^7}{7} + 33x^5 + \frac{55x^3}{3} + 11x - \frac{1}{x}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {28, 270}

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^9}{3} + \frac{330x^7}{7} + 33x^5 + \frac{55x^3}{3} + 11x - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]

[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx &= \int \frac{(1+x^2)^{11}}{x^2} dx \\ &= \int \left(11 + \frac{1}{x^2} + 55x^2 + 165x^4 + 330x^6 + 462x^8 + 462x^{10} + 330x^{12} + 165x^{14} + 55x^{16} + 11x^{18} + x^{20} \right) dx \\ &= -\frac{1}{x} + 11x + \frac{55x^3}{3} + 33x^5 + \frac{330x^7}{7} + \frac{154x^9}{3} + 42x^{11} + \frac{330x^{13}}{13} + 11x^{15} + \frac{55x^{17}}{17} + \frac{x^{21}}{21} \end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.00

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^9}{3} + \frac{330x^7}{7} + 33x^5 + \frac{55x^3}{3} + 11x - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]

[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]

[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2, x]

fricas [A] time = 0.79, size = 62, normalized size = 0.85

$$\frac{4199x^{22} + 51051x^{20} + 285285x^{18} + 969969x^{16} + 2238390x^{14} + 3703518x^{12} + 4526522x^{10} + 4157010x^8 + 2909907x^6 + 1616615x^4 + 969969x^2 - 88179}{88179x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")

[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14 + 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 969969*x^2 - 88179)/x

giac [A] time = 0.30, size = 59, normalized size = 0.81

$$\frac{1}{21}x^{21} + \frac{11}{19}x^{19} + \frac{55}{17}x^{17} + 11x^{15} + \frac{330}{13}x^{13} + 42x^{11} + \frac{154}{3}x^9 + \frac{330}{7}x^7 + 33x^5 + \frac{55}{3}x^3 + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")

[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x

maple [A] time = 0.00, size = 60, normalized size = 0.82

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^9}{3} + \frac{330x^7}{7} + 33x^5 + \frac{55x^3}{3} + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x)`

[Out] `-1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5/17*x^17+11/19*x^19+1/21*x^21`

maxima [A] time = 0.83, size = 59, normalized size = 0.81

$$\frac{1}{21}x^{21} + \frac{11}{19}x^{19} + \frac{55}{17}x^{17} + 11x^{15} + \frac{330}{13}x^{13} + 42x^{11} + \frac{154}{3}x^9 + \frac{330}{7}x^7 + 33x^5 + \frac{55}{3}x^3 + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")`

[Out] `1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x`

mupad [B] time = 0.06, size = 59, normalized size = 0.81

$$11x - \frac{1}{x} + \frac{55x^3}{3} + 33x^5 + \frac{330x^7}{7} + \frac{154x^9}{3} + 42x^{11} + \frac{330x^{13}}{13} + 11x^{15} + \frac{55x^{17}}{17} + \frac{11x^{19}}{19} + \frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)`

[Out] `11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21`

sympy [A] time = 0.10, size = 66, normalized size = 0.90

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^9}{3} + \frac{330x^7}{7} + 33x^5 + \frac{55x^3}{3} + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)`

[Out] `x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 + 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x`

$$3.54 \quad \int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx$$

Optimal. Leaf size=80

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} - \frac{1}{2x^2} + 11 \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 266, 43}

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} - \frac{1}{2x^2} + 11 \log(x)$$

Antiderivative was successfully verified.

[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]

[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11*Log[x]

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx &= \int \frac{(1+x^2)^{11}}{x^3} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1+x)^{11}}{x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(55 + \frac{1}{x^2} + \frac{11}{x} + 165x + 330x^2 + 462x^3 + 462x^4 + 330x^5 + 165x^6 \right) dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{55x^2}{2} + \frac{165x^4}{4} + 55x^6 + \frac{231x^8}{4} + \frac{231x^{10}}{5} + \frac{55x^{12}}{2} + \frac{165x^{14}}{14} + \frac{55x^{16}}{16} + 11 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} - \frac{1}{2x^2} + 11 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]

[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]

[Out] IntegrateAlgebraic[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3, x]

fricas [A] time = 0.50, size = 64, normalized size = 0.80

$$\frac{252x^{22} + 3080x^{20} + 17325x^{18} + 59400x^{16} + 138600x^{14} + 232848x^{12} + 291060x^{10} + 277200x^8 + 207900x^6 + 138600x^4 + 55440x^2 \log(x) - 2520}{5040x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")

[Out] $1/5040*(252*x^{22} + 3080*x^{20} + 17325*x^{18} + 59400*x^{16} + 138600*x^{14} + 232848*x^{12} + 291060*x^{10} + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2 + 2520)/x^2$

giac [A] time = 0.36, size = 69, normalized size = 0.86

$$\frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^8 + 55x^6 + \frac{165}{4}x^4 + \frac{55}{2}x^2 - \frac{11x^2+1}{2x^2} + \frac{11}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")`

[Out] $1/20*x^{20} + 11/18*x^{18} + 55/16*x^{16} + 165/14*x^{14} + 55/2*x^{12} + 231/5*x^{10} + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*\log(x^2)$

maple [A] time = 0.00, size = 61, normalized size = 0.76

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} + 11\ln(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x)`

[Out] $-1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^{10}+55/2*x^{12}+165/14*x^{14}+55/16*x^{16}+11/18*x^{18}+1/20*x^{20}+11*\ln(x)$

maxima [A] time = 0.74, size = 62, normalized size = 0.78

$$\frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^8 + 55x^6 + \frac{165}{4}x^4 + \frac{55}{2}x^2 - \frac{1}{2x^2} + \frac{11}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")`

[Out] $1/20*x^{20} + 11/18*x^{18} + 55/16*x^{16} + 165/14*x^{14} + 55/2*x^{12} + 231/5*x^{10} + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*\log(x^2)$

mupad [B] time = 0.06, size = 60, normalized size = 0.75

$$11\ln(x) - \frac{1}{2x^2} + \frac{55x^2}{2} + \frac{165x^4}{4} + 55x^6 + \frac{231x^8}{4} + \frac{231x^{10}}{5} + \frac{55x^{12}}{2} + \frac{165x^{14}}{14} + \frac{55x^{16}}{16} + \frac{11x^{18}}{18} + \frac{x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)`

[Out] $11 \cdot \log(x) - 1/(2 \cdot x^2) + (55 \cdot x^2)/2 + (165 \cdot x^4)/4 + 55 \cdot x^6 + (231 \cdot x^8)/4 + (231 \cdot x^{10})/5 + (55 \cdot x^{12})/2 + (165 \cdot x^{14})/14 + (55 \cdot x^{16})/16 + (11 \cdot x^{18})/18 + x^{20}/20$

sympy [A] time = 0.11, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^8}{4} + 55x^6 + \frac{165x^4}{4} + \frac{55x^2}{2} + 11 \log(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)`

[Out] `x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**10/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2)`

$$3.55 \quad \int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=145

$$\frac{x(a+bx^2)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1250, 459, 321, 205}

$$\frac{x(a+bx^2)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(b*e*(m+n*(p+1)+1)), x] - Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(b*(m+n*(p+1)+1)), Int[(e*x)^m*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,

$n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m + n*(p + 1) + 1, 0]$

Rule 1250

$\text{Int}[(f_)*(x_)]^{(m_)}*((d_)+(e_)*(x_)^2)^{(q_)}*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_Symbol] := \text{Dist}[(a + b*x^2 + c*x^4)^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]}*(b/2 + c*x^2)^{(2*\text{FracPart}[p])}), \text{Int}[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p, q\}, x] \&\& \text{EqQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx &= \frac{(ab+b^2x^2) \int \frac{x^2(d+ex^2)}{ab+b^2x^2} dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\ &= \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-3b^2d+3abe)(ab+b^2x^2)) \int \frac{x^2}{ab+b^2x^2} dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a(-3b^2d+3abe)(ab+b^2x^2))}{3b^3\sqrt{a^2+2abx^2+b^2x^4}} \\ &= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}(bd-ae)(a+bx^2) \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.55

$$\frac{(a+bx^2) \left(\sqrt{bx}(-3ae+3bd+bx^2) + 3\sqrt{a}(ae-bd) \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 9.87, size = 81, normalized size = 0.56

$$\frac{(a + bx^2) \left(\frac{(a^{3/2}e - \sqrt{a}bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x(-3ae + 3bd + bex^2)}{3b^2} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] ((a + b*x^2)*((x*(3*b*d - 3*a*e + b*e*x^2))/(3*b^2) + ((-(Sqrt[a]*b*d) + a^(3/2)*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/b^(5/2)))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.77, size = 129, normalized size = 0.89

$$\left[\frac{2bex^3 - 3(bd - ae)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6(bd - ae)x}{6b^2}, \frac{bex^3 - 3(bd - ae)\sqrt{\frac{a}{b}} \arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3(bd - ae)x}{3b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2), x, algorithm="fricas")

[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]

giac [A] time = 0.43, size = 101, normalized size = 0.70

$$\frac{(abd\operatorname{sgn}(bx^2 + a) - a^2e\operatorname{sgn}(bx^2 + a)) \arctan\left(\frac{bx}{\sqrt{ab}}\right) + b^2x^3e\operatorname{sgn}(bx^2 + a) + 3b^2dx\operatorname{sgn}(bx^2 + a) - 3abx\operatorname{sgn}(bx^2 + a)}{\sqrt{ab}b^2} + \frac{b^2x^3e\operatorname{sgn}(bx^2 + a) + 3b^2dx\operatorname{sgn}(bx^2 + a) - 3abx\operatorname{sgn}(bx^2 + a)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2), x, algorithm="giac")

[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*e*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*b*x*e*sgn(b*x^2 + a))/b^3

maple [A] time = 0.04, size = 90, normalized size = 0.62

$$\frac{(bx^2 + a) \left(\sqrt{ab} bex^3 + 3a^2e \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3abd \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 3\sqrt{ab} aex + 3\sqrt{ab} bdx \right)}{3\sqrt{(bx^2 + a)^2} \sqrt{ab} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)`

[Out] $\frac{1}{3}(b*x^2+a)*((a*b)^(1/2)*x^3*b*e+3*\arctan(b*x/(a*b)^(1/2))*a^2*e-3*\arctan(b*x/(a*b)^(1/2))*a*b*d-3*(a*b)^(1/2)*x*a*e+3*(a*b)^(1/2)*x*b*d)/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)$

maxima [A] time = 1.52, size = 54, normalized size = 0.37

$$-\frac{(abd - a^2e) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab} b^2} + \frac{bex^3 + 3(bd - ae)x}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $-(a*b*d - a^2*e)*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*b^2) + 1/3*(b*e*x^3 + 3*(b*d - a*e)*x)/b^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 (e x^2 + d)}{\sqrt{(b x^2 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)`

[Out] `int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)`

sympy [A] time = 0.36, size = 90, normalized size = 0.62

$$x\left(-\frac{ae}{b^2} + \frac{d}{b}\right) - \frac{\sqrt{-\frac{a}{b^5}}(ae - bd) \log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}(ae - bd) \log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{ex^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)`

[Out] $x*(-a*e/b**2 + d/b) - \sqrt{-a/b**5}*(a*e - b*d)*\log(-b**2*\sqrt{-a/b**5} + x)/2 + \sqrt{-a/b**5}*(a*e - b*d)*\log(b**2*\sqrt{-a/b**5} + x)/2 + e*x**3/(3*b)$

$$3.56 \quad \int \frac{x^{(d+ex^2)}}{\sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=83

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1247, 640, 608, 31}

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p+1))/(2*c*(p+1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}} dx, x, x^2 \right) \\ &= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae) \text{Subst} \left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}} dx, x, x^2 \right)}{2b} \\ &= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{((bd-ae)(ab+b^2x^2)) \text{Subst} \left(\int \frac{1}{ab+b^2x} dx, x, x^2 \right)}{2b\sqrt{a^2+2abx^2+b^2x^4}} \\ &= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)(a+bx^2) \log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{(a+bx^2)((bd-ae)\log(a+bx^2)+bex^2)}{2b^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [B] time = 0.46, size = 208, normalized size = 2.51

$$\frac{(a\sqrt{b^2e+abe+b^2(-d)-\sqrt{b^2}bd})\log(\sqrt{a^2+2abx^2+b^2x^4}-a-\sqrt{b^2}x^2)}{4(b^2)^{3/2}} + \frac{(-a\sqrt{b^2e+abe+b^2(-d)+\sqrt{b^2}bd})\log(\sqrt{a^2+2abx^2+b^2x^4}+a-\sqrt{b^2}x^2)}{4(b^2)^{3/2}} + \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{4b^2} - \frac{ex^2}{4\sqrt{b^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] -1/4*(e*x^2)/Sqrt[b^2] + (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + ((-(b^2*d) - b*Sqrt[b^2]*d + a*b*e + a*Sqrt[b^2]*e)*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(b^2)^(3/2)) + ((-(b^2*d) + b*Sqrt[b^2]*d + a*b*e - a*Sqrt[b^2]*e)*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*(b^2)^(3/2))

fricas [A] time = 0.83, size = 29, normalized size = 0.35

$$\frac{bex^2 + (bd - ae) \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2

giac [A] time = 0.38, size = 42, normalized size = 0.51

$$\frac{1}{2} \left(\frac{x^2 e}{b} + \frac{(bd - ae) \log(|bx^2 + a|)}{b^2} \right) \operatorname{sgn}(bx^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*(x^2*e/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 55, normalized size = 0.66

$$\frac{(bx^2 + a)(-be x^2 + ae \ln(bx^2 + a) - bd \ln(bx^2 + a))}{2\sqrt{(bx^2 + a)^2} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)

[Out] -1/2*(b*x^2+a)*(-x^2*e*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(1/2)/b^2

maxima [A] time = 0.78, size = 31, normalized size = 0.37

$$\frac{ex^2}{2b} + \frac{(bd - ae) \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*e*x^2/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2

mupad [B] time = 0.88, size = 103, normalized size = 1.24

$$\frac{e\sqrt{a^2 + 2abx^2 + b^2x^4}}{2b^2} - \frac{abe \ln\left(ab + \sqrt{(bx^2 + a)^2} \sqrt{b^2 + b^2x^2}\right)}{2(b^2)^{3/2}} + \frac{b^2d \ln(b^2x^2 + ab) \operatorname{sign}(2b^2x^2 + 2ab)}{2(b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)`

[Out] `(e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))`

sympy [A] time = 0.28, size = 27, normalized size = 0.33

$$\frac{ex^2}{2b} - \frac{(ae - bd) \log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)/((b*x**2+a)**2)^(1/2), x)`

[Out] `e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)`

$$3.57 \quad \int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=97

$$\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1148, 388, 205}

$$\frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 1148

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{\sqrt{a^2 + 2abx^2 + b^2x^4}} dx &= \frac{(ab + b^2x^2) \int \frac{d+ex^2}{ab+b^2x^2} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{ex(a + bx^2)}{b\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{((-b^2d + abe)(ab + b^2x^2)) \int \frac{1}{ab+b^2x^2} dx}{b^2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{ex(a + bx^2)}{b\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(bd - ae)(a + bx^2) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2} \sqrt{a^2 + 2abx^2 + b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.71

$$-\frac{(a + bx^2) \left((ae - bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \sqrt{a} \sqrt{b} ex \right)}{\sqrt{a} b^{3/2} \sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + -(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 8.95, size = 60, normalized size = 0.62

$$\frac{(a + bx^2) \left(\frac{(bd - ae) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2}} + \frac{ex}{b} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] ((a + b*x^2)*((e*x)/b + ((b*d - a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.59, size = 98, normalized size = 1.01

$$\left[\frac{2 abex + \sqrt{-ab} (bd - ae) \log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)}{2 ab^2}, \frac{abex + \sqrt{ab} (bd - ae) \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a))/(a*b^2)]

giac [A] time = 0.26, size = 59, normalized size = 0.61

$$\frac{x e \operatorname{sgn}(b x^2 + a)}{b} + \frac{(b d \operatorname{sgn}(b x^2 + a) - a e \operatorname{sgn}(b x^2 + a)) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] x*e*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)

maple [A] time = 0.01, size = 62, normalized size = 0.64

$$\frac{(b x^2 + a) \left(-a e \arctan\left(\frac{b x}{\sqrt{a b}}\right) + b d \arctan\left(\frac{b x}{\sqrt{a b}}\right) + \sqrt{a b} e x \right)}{\sqrt{(b x^2 + a)^2} \sqrt{a b} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x)

[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(1/(a*b)^(1/2)*b*x)*a*e+arctan(1/(a*b)^(1/2)*b*x)*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)

maxima [A] time = 1.51, size = 33, normalized size = 0.34

$$\frac{e x}{b} + \frac{(b d - a e) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] e*x/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e x^2 + d}{\sqrt{(b x^2 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)`

[Out] `int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)`

sympy [A] time = 0.32, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}} (ae - bd) \log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}} (ae - bd) \log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2), x)`

[Out] `sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + e*x/b`

$$3.58 \quad \int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=92

$$\frac{d \log(x)(a+bx^2)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a+bx^2)(bd-ae) \log(a+bx^2)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1250, 446, 72}

$$\frac{d \log(x)(a+bx^2)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a+bx^2)(bd-ae) \log(a+bx^2)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]), x]

[Out] (d*(a + b*x^2)*Log[x])/(a*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a*b*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1250

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^2}{x\sqrt{a^2 + 2abx^2 + b^2x^4}} dx &= \frac{(ab + b^2x^2) \int \frac{d+ex^2}{x(ab+b^2x^2)} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
&= \frac{(ab + b^2x^2) \text{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)} dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
&= \frac{(ab + b^2x^2) \text{Subst}\left(\int \left(\frac{d}{abx} + \frac{-bd+ae}{ab(a+bx)}\right) dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
&= \frac{d(a + bx^2) \log(x)}{a\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)(a + bx^2) \log(a + bx^2)}{2ab\sqrt{a^2 + 2abx^2 + b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.59

$$\frac{(a + bx^2) \left((ae - bd) \log(a + bx^2) + 2bd \log(x) \right)}{2ab\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]), x]

[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-b*d) + a*e)*Log[a + b*x^2])/(2*a*b*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [B] time = 0.44, size = 198, normalized size = 2.15

$$\frac{(a\sqrt{b^2}e - abe - 2b\sqrt{b^2}d) \log(\sqrt{a^2 + 2abx^2 + b^2x^4} + a - \sqrt{b^2}x^2)}{4ab\sqrt{b^2}} + \frac{(2bd - ae) \log(-ab\sqrt{a^2 + 2abx^2 + b^2x^4} + a^2b + ab\sqrt{b^2}x^2)}{4ab} - \frac{e \log(\sqrt{a^2 + 2abx^2 + b^2x^4} - a - \sqrt{b^2}x^2)}{4\sqrt{b^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]), x]

[Out] -1/4*(e*Log[-a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/Sqrt[b^2] + ((-2*b*Sqrt[b^2]*d - a*b*e + a*Sqrt[b^2]*e)*Log[a - Sqrt[b^2]*x^2 + Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*a*b*Sqrt[b^2]) + ((2*b*d - a*e)*Log[a^2*b + a*b*Sqrt[b^2]*x^2 - a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]])/(4*a*b)

fricas [A] time = 0.78, size = 33, normalized size = 0.36

$$\frac{2bd \log(x) - (bd - ae) \log(bx^2 + a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)

giac [A] time = 0.38, size = 61, normalized size = 0.66

$$\frac{d \log(x^2) \operatorname{sgn}(bx^2 + a)}{2a} - \frac{(bd \operatorname{sgn}(bx^2 + a) - ae \operatorname{sgn}(bx^2 + a)) \log(|bx^2 + a|)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a*b)

maple [A] time = 0.01, size = 57, normalized size = 0.62

$$\frac{(bx^2 + a)(ae \ln(bx^2 + a) + 2bd \ln(x) - bd \ln(bx^2 + a))}{2\sqrt{(bx^2 + a)^2} ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x)

[Out] 1/2*(b*x^2+a)*(2*d*ln(x)*b+a*e*ln(b*x^2+a)-b*d*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/a/b

maxima [A] time = 0.73, size = 35, normalized size = 0.38

$$\frac{d \log(x^2)}{2a} - \frac{(bd - ae) \log(bx^2 + a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)

mupad [B] time = 0.77, size = 83, normalized size = 0.90

$$\frac{e \ln(b^2 x^2 + ab) \operatorname{sign}(2b^2 x^2 + 2ab)}{2\sqrt{b^2}} - \frac{d \ln\left(\frac{1}{x^2}\right)}{2\sqrt{a^2}} - \frac{d \ln\left(\sqrt{(bx^2 + a)^2} \sqrt{a^2} + a^2 + abx^2\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)`

[Out] $(e \cdot \log(a \cdot b + b^2 \cdot x^2) \cdot \text{sign}(2 \cdot a \cdot b + 2 \cdot b^2 \cdot x^2)) / (2 \cdot (b^2)^{(1/2)}) - (d \cdot \log(1/x^2)) / (2 \cdot (a^2)^{(1/2)}) - (d \cdot \log(((a + b \cdot x^2)^2)^{(1/2)} \cdot (a^2)^{(1/2)} + a^2 + a \cdot b \cdot x^2)) / (2 \cdot (a^2)^{(1/2)})$

sympy [A] time = 0.71, size = 26, normalized size = 0.28

$$\frac{d \log(x)}{a} + \frac{(ae - bd) \log\left(\frac{a}{b} + x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)`

[Out] $d \cdot \log(x) / a + (a \cdot e - b \cdot d) \cdot \log(a/b + x^2) / (2 \cdot a \cdot b)$

$$3.59 \quad \int \frac{d+ex^2}{x^2 \sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=101

$$-\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1250, 453, 205}

$$-\frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]

[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 1250

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[(a+b*x^2+c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2+c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d+e*x^2)^q*(b/2+c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*

a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{x^2 \sqrt{a^2 + 2abx^2 + b^2x^4}} dx &= \frac{(ab + b^2x^2) \int \frac{d+ex^2}{x^2(ab+b^2x^2)} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= -\frac{d(a + bx^2)}{ax\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{((b^2d - abe)(ab + b^2x^2)) \int \frac{1}{ab+b^2x^2} dx}{ab\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= -\frac{d(a + bx^2)}{ax\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)(a + bx^2) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2 + 2abx^2 + b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.71

$$\frac{(a + bx^2) \left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) (aex - bdx) - \sqrt{a} \sqrt{b} d \right)}{a^{3/2} \sqrt{b} x \sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]

[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + -(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 6.38, size = 63, normalized size = 0.62

$$\frac{(a + bx^2) \left(\frac{(ae-bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b}} - \frac{d}{ax} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]

[Out] ((a + b*x^2)*(-(d/(a*x)) + ((-b*d) + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.70, size = 105, normalized size = 1.04

$$\left[\frac{\sqrt{-ab}(bd - ae)x \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) - 2abd}{2a^2bx}, -\frac{\sqrt{ab}(bd - ae)x \arctan\left(\frac{\sqrt{ab}x}{a}\right) + abd}{a^2bx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) + a*b*d)/(a^2*b*x)]

giac [A] time = 0.34, size = 62, normalized size = 0.61

$$-\frac{(bd\operatorname{sgn}(bx^2 + a) - a\operatorname{sgn}(bx^2 + a)) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{d\operatorname{sgn}(bx^2 + a)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d*sgn(b*x^2 + a)/(a*x)

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$-\frac{(bx^2 + a)\left(-aex \arctan\left(\frac{bx}{\sqrt{ab}}\right) + bdx \arctan\left(\frac{bx}{\sqrt{ab}}\right) + \sqrt{ab}d\right)}{\sqrt{(bx^2 + a)^2} \sqrt{ab}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x)

[Out] -(b*x^2+a)*(-arctan(1/(a*b)^(1/2)*b*x)*x*a*e+arctan(1/(a*b)^(1/2)*b*x)*x*b*d+d*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/x/(a*b)^(1/2)

maxima [A] time = 1.22, size = 37, normalized size = 0.37

$$-\frac{(bd - ae) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] -(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e x^2 + d}{x^2 \sqrt{(b x^2 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)

[Out] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)

sympy [A] time = 0.37, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}} (ae - bd) \log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b}} (ae - bd) \log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{2} - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)

[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - d/(a*x)

$$3.60 \quad \int \frac{d+ex^2}{x^3 \sqrt{a^2+2abx^2+b^2x^4}} dx$$

Optimal. Leaf size=137

$$\frac{\log(x)(a+bx^2)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1250, 446, 77}

$$\frac{\log(x)(a+bx^2)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x^3*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]

[Out] -(d*(a + b*x^2))/(2*a*x^2*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(a + b*x^2)*Log[x])/(a^2*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^2*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1250

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*

a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^2}{x^3 \sqrt{a^2 + 2abx^2 + b^2x^4}} dx &= \frac{(ab + b^2x^2) \int \frac{d+ex^2}{x^3(ab+b^2x^2)} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= \frac{(ab + b^2x^2) \text{Subst}\left(\int \frac{d+ex}{x^2(ab+b^2x)} dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= \frac{(ab + b^2x^2) \text{Subst}\left(\int \left(\frac{d}{abx^2} + \frac{-bd+ae}{a^2bx} + \frac{bd-ae}{a^2(a+bx)}\right) dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{d(a + bx^2)}{2ax^2\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)(a + bx^2) \log(x)}{a^2\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(bd - ae)(a + bx^2) \log(x)}{2a^2\sqrt{a^2 + 2abx^2 + b^2x^4}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.51

$$\frac{(a + bx^2) \left(2x^2 \log(x)(ae - bd) + x^2(bd - ae) \log(a + bx^2) - ad \right)}{2a^2x^2\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]), x]

[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e))*x^2*Log[x] + (b*d - a*e)*x^2*Log[a + b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [B] time = 1.17, size = 713, normalized size = 5.20

$$\frac{\frac{\sqrt{b^2} \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2}}{\sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2} + \frac{d \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}{2}}{\sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{a^2 + 4abx^2 + 4b^2x^4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]), x]

[Out] (-(Sqrt[b^2]*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^2 + 4*a*b*x^2 + 4*b^2*x^4)) - d*(-(a^3*b) - 5*a^2*b^2*x^2 - 8*a*b^3*x^4 - 4*b^4*x^6))/(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-2*a^3*b*x^2 - 12*a^2*b^2*x^4 - 24*a*b^3*x^6 - 16*b^4*x^8) + Sqrt[b^2]*(2*a^4*x^2 + 14*a^3*b*x^4 + 36*a^2*b^2*x^6 + 40*a*b^3*x^8 +

$$16b^4x^{10}) + ((-\text{Sqrt}[b^2]*x^2) + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4])^2*((b*d*\text{Log}[a - \text{Sqrt}[b^2]*x^2 + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a^2) - (b*d*\text{Log}[a^3 + a^2*\text{Sqrt}[b^2]*x^2 - a^2*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a^2)))/(a^2 + 2a*b*x^2 + 2*b^2*x^4 - 2*\text{Sqrt}[b^2]*x^2*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]) + (-((b*d)/a) + (\text{Sqrt}[b^2]*e*x^2*\text{Log}[a - \text{Sqrt}[b^2]*x^2 + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a) - (e*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]*\text{Log}[a - \text{Sqrt}[b^2]*x^2 + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a) - (\text{Sqrt}[b^2]*e*x^2*\text{Log}[a^3 + a^2*\text{Sqrt}[b^2]*x^2 - a^2*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a) + (e*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]*\text{Log}[a^3 + a^2*\text{Sqrt}[b^2]*x^2 - a^2*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]])/(2*a))/(-(\text{Sqrt}[b^2]*x^2) + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4])$$

fricas [A] time = 0.71, size = 48, normalized size = 0.35

$$\frac{(bd - ae)x^2 \log(bx^2 + a) - 2(bd - ae)x^2 \log(x) - ad}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2*x^2)

giac [A] time = 0.39, size = 131, normalized size = 0.96

$$-\frac{(bd\text{sgn}(bx^2 + a) - a\text{esgn}(bx^2 + a))\log(x^2)}{2a^2} + \frac{(b^2d\text{sgn}(bx^2 + a) - ab\text{esgn}(bx^2 + a))\log(|bx^2 + a|)}{2a^2b} + \frac{bdx^2\text{sgn}(bx^2 + a) - ax^2\text{esgn}(bx^2 + a) - ad\text{sgn}(bx^2 + a)}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sgn(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d*x^2*sgn(b*x^2 + a) - a*x^2*e*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2)

maple [A] time = 0.01, size = 79, normalized size = 0.58

$$\frac{(bx^2 + a)(2aex^2 \ln(x) - aex^2 \ln(bx^2 + a) - 2bdx^2 \ln(x) + bdx^2 \ln(bx^2 + a) - ad)}{2\sqrt{(bx^2 + a)^2} a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x)

[Out] $1/2*(b*x^2+a)*(2*\ln(x)*x^2*a*e-2*\ln(x)*x^2*b*d-\ln(b*x^2+a)*x^2*a*e+\ln(b*x^2+a)*x^2*b*d-a*d)/((b*x^2+a)^2)^{(1/2)}/x^2/a^2$

maxima [A] time = 0.78, size = 48, normalized size = 0.35

$$\frac{(bd - ae) \log(bx^2 + a)}{2a^2} - \frac{(bd - ae) \log(x^2)}{2a^2} - \frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $1/2*(b*d - a*e)*\log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*\log(x^2)/a^2 - 1/2*d/(a*x^2)$

mupad [B] time = 0.81, size = 125, normalized size = 0.91

$$\frac{abd \operatorname{atanh}\left(\frac{a^2+ba x^2}{\sqrt{a^2} \sqrt{a^2+2abx^2+b^2x^4}}\right)}{2(a^2)^{3/2}} - \frac{e \ln\left(\frac{1}{x^2}\right)}{2\sqrt{a^2}} - \frac{d\sqrt{a^2+2abx^2+b^2x^4}}{2a^2x^2} - \frac{e \ln\left(\sqrt{(bx^2+a)^2} \sqrt{a^2+a^2+abx^2}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)`

[Out] $(a*b*d*\operatorname{atanh}((a^2 + a*b*x^2)/((a^2)^{(1/2)}*(a^2 + b^2*x^4 + 2*a*b*x^2)^{(1/2)})))/(2*(a^2)^{(3/2)}) - (e*\log(1/x^2))/(2*(a^2)^{(1/2)}) - (d*(a^2 + b^2*x^4 + 2*a*b*x^2)^{(1/2)})/(2*a^2*x^2) - (e*\log(((a + b*x^2)^2)^{(1/2)}*(a^2)^{(1/2)} + a^2 + a*b*x^2))/(2*(a^2)^{(1/2)})$

sympy [A] time = 0.73, size = 41, normalized size = 0.30

$$-\frac{d}{2ax^2} + \frac{(ae - bd) \log(x)}{a^2} - \frac{(ae - bd) \log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)`

[Out] $-d/(2*a*x**2) + (a*e - b*d)*\log(x)/a**2 - (a*e - b*d)*\log(a/b + x**2)/(2*a**2)$

$$3.61 \quad \int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=153

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x(bd-ae)}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1250, 455, 385, 205}

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x(bd-ae)}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p+1))/(2*b^(m/2 + 1)*(p+1)), x] + Dist[1/(2*b^(m/2 + 1)*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*x^2*Together[(b^(m/2)*x^(m-2)*(c + d*x^2) - (-a)^(m/2 -

1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1250

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (d + ex^2)}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^2)) \int \frac{x^2(d+ex^2)}{(ab+b^2x^2)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{(bd - ae)x}{4b^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(ab + b^2x^2) \int \frac{-b(bd-ae)-4b^2ex^2}{(ab+b^2x^2)^2} dx}{4b^2 \sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= \frac{(bd - 5ae)x}{8ab^2 \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)x}{4b^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{((bd + 3ae)(ab + b^2x^2)) \int \frac{1}{ab+b^2x^2} dx}{8ab^2 \sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= \frac{(bd - 5ae)x}{8ab^2 \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)x}{4b^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(bd + 3ae)(a + bx^2)}{8a^{3/2}b^{5/2} \sqrt{a^2 + 2abx^2 + b^2x^4}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 108, normalized size = 0.71

$$\frac{(a + bx^2)^2 (3ae + bd) \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) - \sqrt{a} \sqrt{b} x (3a^2e + ab(d + 5ex^2) - b^2dx^2)}{8a^{3/2}b^{5/2} (a + bx^2) \sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] (- (Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 15.92, size = 105, normalized size = 0.69

$$\frac{(a + bx^2) \left(\frac{(3ae+bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} - \frac{x(3a^2e+abd+5abex^2-b^2dx^2)}{8ab^2(a+bx^2)^2} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]

[Out] ((a + b*x^2)*(-1/8*(x*(a*b*d + 3*a^2*e - b^2*d*x^2 + 5*a*b*e*x^2))/(a*b^2*(a + b*x^2)^2) + ((b*d + 3*a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.93, size = 300, normalized size = 1.96

$$\frac{2(ab^3d - 5a^2b^2e)x^3 - ((b^3d + 3ab^2e)x^4 + a^2bd + 3a^3e + 2(ab^2d + 3a^2be)x^2)\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x + a}{bx^2 + a}\right) - 2(a^2b^2d + 3a^3be)x + (ab^3d - 5a^2b^2e)x^3 + ((b^3d + 3ab^2e)x^4 + a^2bd + 3a^3e + 2(ab^2d + 3a^2be)x^2)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right) - (a^2b^2d + 3a^3be)x}{16(a^2b^5x^4 + 2a^3b^4x^2 + a^4b^3) \cdot 8(a^2b^5x^4 + 2a^3b^4x^2 + a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 188, normalized size = 1.23

$$\frac{(-3ab^2ex^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - b^2dx^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 6a^2be x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 2ab^2dx^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 5\sqrt{ab} abe x^3 - \sqrt{ab} b^2dx^3 - 3a^3e \arctan\left(\frac{bx}{\sqrt{ab}}\right) - a^2bd \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3\sqrt{ab} a^2ex + \sqrt{ab} abdx)(bx^2 + a)}{8\sqrt{ab} \left((bx^2 + a)^2\right)^{\frac{3}{2}} ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x)`

[Out]
$$-1/8*(-3*\arctan(1/(a*b)^(1/2)*b*x)*x^4*a*b^2*e-\arctan(1/(a*b)^(1/2)*b*x)*x^4*b^3*d+5*(a*b)^(1/2)*x^3*a*b*e-(a*b)^(1/2)*x^3*b^2*d-6*\arctan(1/(a*b)^(1/2)*b*x)*x^2*a^2*b*e-2*\arctan(1/(a*b)^(1/2)*b*x)*x^2*a*b^2*d+3*(a*b)^(1/2)*x*a^2*e+(a*b)^(1/2)*x*a*b*d-3*\arctan(1/(a*b)^(1/2)*b*x)*a^3*e-\arctan(1/(a*b)^(1/2)*b*x)*a^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/a/b^2/((b*x^2+a)^2)^(3/2)$$

maxima [A] time = 1.49, size = 125, normalized size = 0.82

$$-\frac{1}{8}e\left(\frac{5bx^3+3ax}{b^4x^4+2ab^3x^2+a^2b^2}-\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}\right)+\frac{1}{8}d\left(\frac{bx^3-ax}{ab^3x^4+2a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="maxima")`

[Out]
$$-1/8*e*((5*b*x^3+3*a*x)/(b^4*x^4+2*a*b^3*x^2+a^2*b^2)-3*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*b^2))+1/8*d*((b*x^3-a*x)/(a*b^3*x^4+2*a^2*b^2*x^2+a^3*b)+\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a*b))$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 (e x^2 + d)}{(a^2 + 2 a b x^2 + b^2 x^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)`

[Out] `int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + e x^2)}{\left((a + b x^2)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2), x)`

[Out] `Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)`

$$3.62 \quad \int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=77

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1247, 640, 607}

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 607

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x + c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 640

Int[((d_) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae) \text{Subst} \left(\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}} dx, x, x^2 \right)}{2b} \\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.58

$$\frac{-ae - b(d + 2ex^2)}{4b^2(a + bx^2)\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [B] time = 0.85, size = 192, normalized size = 2.49

$$\frac{-a^3be + \sqrt{b^2}\sqrt{a^2+2abx^2+b^2x^4}(-a^2e + abd + abex^2 - b^2dx^2 - 2b^2ex^4) + a^2b^2d + ab^3ex^4 + b^4dx^4 + 2b^4ex^6}{2x^4(-2ab^5 - 2b^6x^2)\sqrt{a^2+2abx^2+b^2x^4} + 2\sqrt{b^2}x^4(2a^2b^4 + 4ab^5x^2 + 2b^6x^4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] (a^2*b^2*d - a^3*b*e + b^4*d*x^4 + a*b^3*e*x^4 + 2*b^4*e*x^6 + Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a*b*d - a^2*e - b^2*d*x^2 + a*b*e*x^2 - 2*b^2*e*x^4))/(2*x^4*(-2*a*b^5 - 2*b^6*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4] + 2*Sqrt[b^2]*x^4*(2*a^2*b^4 + 4*a*b^5*x^2 + 2*b^6*x^4))

fricas [A] time = 0.85, size = 42, normalized size = 0.55

$$-\frac{2bex^2 + bd + ae}{4(b^4x^4 + 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")

[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)

giac [A] time = 0.51, size = 40, normalized size = 0.52

$$-\frac{2bx^2e + bd + ae}{4(bx^2 + a)^2 b^2 \operatorname{sgn}(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

[Out] -1/4*(2*b*x^2*e + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))

maple [A] time = 0.01, size = 38, normalized size = 0.49

$$-\frac{(bx^2 + a)(2bex^2 + ae + bd)}{4\left((bx^2 + a)^2\right)^{\frac{3}{2}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)

[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)

maxima [A] time = 0.64, size = 65, normalized size = 0.84

$$-\frac{(2bx^2 + a)e}{4(b^4x^4 + 2ab^3x^2 + a^2b^2)} - \frac{d}{4(b^3x^4 + 2ab^2x^2 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")

[Out] -1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)

mupad [B] time = 0.18, size = 48, normalized size = 0.62

$$-\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} (2bex^2 + ae + bd)}{4b^2(bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)`

[Out] `-((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b*x^2)^3)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex^2)}{\left((a + bx^2)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2), x)`

[Out] `Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)`

$$3.63 \quad \int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=156

$$\frac{x(ae + 3bd)}{8a^2b\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{x(bd - ae)}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(a + bx^2)(ae + 3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1148, 385, 199, 205}

$$\frac{x(ae + 3bd)}{8a^2b\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{x(bd - ae)}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(a + bx^2)(ae + 3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +

p, 0])

Rule 1148

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx &= \frac{(b^2(ab + b^2x^2)) \int \frac{d+ex^2}{(ab+b^2x^2)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(bd - ae)x}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{((3bd + ae)(ab + b^2x^2)) \int \frac{1}{(ab+b^2x^2)^2} dx}{4a\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(3bd + ae)x}{8a^2b\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(bd - ae)x}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{((3bd + ae)(ab + b^2x^2)) \int \frac{1}{(ab+b^2x^2)^2} dx}{8a^2b\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(3bd + ae)x}{8a^2b\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(bd - ae)x}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(3bd + ae)(a + bx^2)}{8a^{5/2}b^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}x(a^2(-e) + ab(5d + ex^2) + 3b^2dx^2) + (a + bx^2)^2(ae + 3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}(a + bx^2)\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d + a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 15.78, size = 105, normalized size = 0.67

$$\frac{(a + bx^2) \left(\frac{(ae+3bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} - \frac{x(a^2e-5abd-abex^2-3b^2dx^2)}{8a^2b(a+bx^2)^2} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] ((a + b*x^2)*(-1/8*(x*(-5*a*b*d + a^2*e - 3*b^2*d*x^2 - a*b*e*x^2)))/(a^2*b*(a + b*x^2)^2) + ((3*b*d + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.78, size = 301, normalized size = 1.93

$$\frac{2(3ab^3d + a^2b^2e)x^3 - ((3b^3d + ab^2e)x^4 + 3a^2bd + a^3e + 2(3ab^2d + a^2be)x^2)\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) + 2(5a^2b^2d - a^3be)x(3ab^3d + a^2b^2e)x^3 + ((3b^3d + ab^2e)x^4 + 3a^2bd + a^3e + 2(3ab^2d + a^2be)x^2)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right) + (5a^2b^2d - a^3be)x}{16(a^3b^4x^4 + 2a^4b^3x^2 + a^5b^2)} \cdot \frac{1}{8(a^3b^4x^4 + 2a^4b^3x^2 + a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="fricas")

[Out] [1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b))*x - a)/(b*x^2 + a) + 2*(5*a^2*b^2*d - a^3*b*e)*x/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 186, normalized size = 1.19

$$\frac{(a b^2 e x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3b^3 d x^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 2a^2 b e x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 6a b^2 d x^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + \sqrt{ab} a b e x^3 + 3\sqrt{ab} b^2 d x^3 + a^3 e \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3a^2 b d \arctan\left(\frac{bx}{\sqrt{ab}}\right) - \sqrt{ab} a^2 e x + 5\sqrt{ab} a b d x)(b x^2 + a)}{8\sqrt{ab} \left((b x^2 + a)^2\right)^{\frac{3}{2}} a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)`

[Out] $\frac{1}{8}*(a*b^2*e*x^4*\arctan(1/(a*b)^(1/2)*b*x)+3*b^3*d*x^4*\arctan(1/(a*b)^(1/2)*b*x)+(a*b)^(1/2)*a*b*e*x^3+3*(a*b)^(1/2)*b^2*d*x^3+2*a^2*b*e*x^2*\arctan(1/(a*b)^(1/2)*b*x)+6*a*b^2*d*x^2*\arctan(1/(a*b)^(1/2)*b*x)-(a*b)^(1/2)*a^2*e*x+5*(a*b)^(1/2)*a*b*d*x+a^3*e*\arctan(1/(a*b)^(1/2)*b*x)+3*a^2*b*d*\arctan(1/(a*b)^(1/2)*b*x))*(b*x^2+a)/(a*b)^(1/2)/b/a^2/((b*x^2+a)^2)^(3/2)$

maxima [A] time = 1.67, size = 124, normalized size = 0.79

$$\frac{1}{8}d \left(\frac{3bx^3 + 5ax}{a^2b^2x^4 + 2a^3bx^2 + a^4} + \frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} \right) + \frac{1}{8}e \left(\frac{bx^3 - ax}{ab^3x^4 + 2a^2b^2x^2 + a^3b} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")`

[Out] $\frac{1}{8}*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*\arctan(b*x/\sqrt{a*b}))/(\sqrt{a*b}*a^2) + \frac{1}{8}*e*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + \arctan(b*x/\sqrt{a*b}))/(\sqrt{a*b}*a*b)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e x^2 + d}{(a^2 + 2 a b x^2 + b^2 x^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)`

[Out] `int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x^2}{((a + b x^2)^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)`

[Out] `Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)`

$$3.64 \quad \int \frac{d+ex^2}{x(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=161

$$\frac{bd - ae}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{d}{2a^2\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{d \log(x)(a + bx^2)}{a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a + bx^2) \log(a + bx^2)}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1250, 446, 77}

$$\frac{bd - ae}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{d}{2a^2\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{d \log(x)(a + bx^2)}{a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a + bx^2) \log(a + bx^2)}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]

[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1250

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int

Part[p]*(b/2 + c*x^2)^(2*FracPart[p]), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{x(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx &= \frac{(b^2(ab + b^2x^2)) \int \frac{d+ex^2}{x(ab+b^2x^2)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(b^2(ab + b^2x^2)) \text{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)^3} dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(b^2(ab + b^2x^2)) \text{Subst}\left(\int \left(\frac{d}{a^3b^3x} + \frac{-bd+ae}{ab^3(a+bx)^3} - \frac{d}{a^2b^2(a+bx)^2} - \frac{d}{a^3b^2(a+bx)}\right) dx, x, x^2\right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{d}{2a^2\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{bd - ae}{4ab(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{d(a + bx^2)}{a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.57

$$\frac{a(a^2(-e) + 3abd + 2b^2dx^2) + 4bd \log(x)(a + bx^2)^2 - 2bd(a + bx^2)^2 \log(a + bx^2)}{4a^3b(a + bx^2)\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)), x]

[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 0.89, size = 258, normalized size = 1.60

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{b^2x^2} - \sqrt{a^2+2abx^2+b^2x^4}}{a}\right)}{a^3} + \frac{a^4be - a^3b^2d + a^2b^3ex^4 + \sqrt{b^2}\sqrt{a^2 + 2abx^2 + b^2x^4}(a^3e - a^2bd - a^2bex^2 + ab^2dx^2 + 2b^3dx^4) - 3ab^4dx^4 - 2b^5dx^6}{2a^2b\sqrt{b^2x^4}(2a^2b^2 + 4ab^3x^2 + 2b^4x^4) + 2a^2bx^4(-2ab^3 - 2b^4x^2)\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)), x]

[Out] $(-(a^3 b^2 d) + a^4 b e - 3 a^2 b^3 d x^4 + a^2 b^3 e x^4 - 2 b^5 d x^6 + \text{Sqrt}[b^2] \text{Sqrt}[a^2 + 2 a b x^2 + b^2 x^4] * (-(a^2 b d) + a^3 e + a b^2 d x^2 - a^2 b e x^2 + 2 b^3 d x^4)) / (2 a^2 b x^4 * (-2 a b^3 - 2 b^4 x^2) * \text{Sqrt}[a^2 + 2 a b x^2 + b^2 x^4] + 2 a^2 b * \text{Sqrt}[b^2] * x^4 * (2 a^2 b^2 + 4 a b^3 x^2 + 2 b^4 x^4)) + (d * \text{ArcTanh}[(\text{Sqrt}[b^2] * x^2) / a - \text{Sqrt}[a^2 + 2 a b x^2 + b^2 x^4] / a]) / a^3$

fricas [A] time = 0.65, size = 119, normalized size = 0.74

$$\frac{2 a b^2 d x^2 + 3 a^2 b d - a^3 e - 2 (b^3 d x^4 + 2 a b^2 d x^2 + a^2 b d) \log(b x^2 + a) + 4 (b^3 d x^4 + 2 a b^2 d x^2 + a^2 b d) \log(x)}{4 (a^3 b^3 x^4 + 2 a^4 b^2 x^2 + a^5 b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")`

[Out] $1/4 * (2 a^2 b^2 d x^2 + 3 a^2 b^3 d - a^3 e - 2 (b^3 d x^4 + 2 a b^2 d x^2 + a^2 b^3 d) * \log(b x^2 + a) + 4 (b^3 d x^4 + 2 a b^2 d x^2 + a^2 b^3 d) * \log(x)) / (a^3 b^3 x^4 + 2 a^4 b^2 x^2 + a^5 b)$

giac [A] time = 0.53, size = 96, normalized size = 0.60

$$-\frac{d \log(|b x^2 + a|)}{2 a^3 \text{sgn}(b x^2 + a)} + \frac{d \log(|x|)}{a^3 \text{sgn}(b x^2 + a)} + \frac{2 a b^2 d x^2 + 3 a^2 b d - a^3 e}{4 (b x^2 + a)^2 a^3 b \text{sgn}(b x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

[Out] $-1/2 * d * \log(\text{abs}(b x^2 + a)) / (a^3 * \text{sgn}(b x^2 + a)) + d * \log(\text{abs}(x)) / (a^3 * \text{sgn}(b x^2 + a)) + 1/4 * (2 a^2 b^2 d x^2 + 3 a^2 b^3 d - a^3 e) / ((b x^2 + a)^2 * a^3 * b * \text{sgn}(b x^2 + a))$

maple [A] time = 0.02, size = 133, normalized size = 0.83

$$\frac{(4 b^3 d x^4 \ln(x) - 2 b^3 d x^4 \ln(b x^2 + a) + 8 a b^2 d x^2 \ln(x) - 4 a b^2 d x^2 \ln(b x^2 + a) + 2 a b^2 d x^2 + 4 a^2 b d \ln(x) - 2 a^2 b d \ln(b x^2 + a) - a^3 e + 3 a^2 b d) (b x^2 + a)}{4 ((b x^2 + a)^2)^{3/2} a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)`

[Out] $1/4 * (4 * \ln(x) * x^4 * b^3 * d - 2 * \ln(b x^2 + a) * x^4 * b^3 * d + 8 * \ln(x) * x^2 * a * b^2 * d - 4 * \ln(b x^2 + a) * x^2 * a * b^2 * d + 2 * b^2 * d * x^2 * a + 4 * \ln(x) * a^2 * b * d - 2 * \ln(b x^2 + a) * a^2 * b * d - a^3 * e + 3 * a^2 * b * d) * (b x^2 + a) / b / a^3 / ((b x^2 + a)^2)^{3/2}$

maxima [A] time = 0.89, size = 88, normalized size = 0.55

$$\frac{1}{4}d\left(\frac{2bx^2 + 3a}{a^2b^2x^4 + 2a^3bx^2 + a^4} - \frac{2\log(bx^2 + a)}{a^3} + \frac{4\log(x)}{a^3}\right) - \frac{e}{4(b^3x^4 + 2ab^2x^2 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{ex^2 + d}{x(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)

[Out] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x\left((a + bx^2)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)

$$3.65 \quad \int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=190

$$\frac{x(bd - ae)}{4a^2(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{3(a + bx^2)(5bd - ae)\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{x(7bd - 3ae)}{8a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a - bx^2)}{a^3x\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Rubi [A] time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1250, 456, 453, 205}

$$\frac{x(7bd - 3ae)}{8a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{x(bd - ae)}{4a^2(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{3(a + bx^2)(5bd - ae)\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a + bx^2)}{a^3x\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)), x]

[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2,
0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1250

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^2}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^2)) \int \frac{d+ex^2}{x^2(ab+b^2x^2)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{(bd - ae)x}{4a^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(b^2 (ab + b^2x^2)) \int \frac{\frac{4d}{ab} + \frac{3(bd-ae)x^2}{a^2b}}{x^2(ab+b^2x^2)^2} dx}{4\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{(7bd - 3ae)x}{8a^3 \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)x}{4a^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(b^2 (ab + b^2x^2)) \int \frac{d}{a^3x\sqrt{a^2 + 2abx^2 + b^2x^4}} dx}{8\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{(7bd - 3ae)x}{8a^3 \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)x}{4a^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d (a + bx^2)}{a^3x\sqrt{a^2 + 2abx^2 + b^2x^4}} \\
 &= -\frac{(7bd - 3ae)x}{8a^3 \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{(bd - ae)x}{4a^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d (a + bx^2)}{a^3x\sqrt{a^2 + 2abx^2 + b^2x^4}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.65

$$\frac{\sqrt{a} \sqrt{b} (a^2 (5ex^2 - 8d) + ab (3ex^4 - 25dx^2) - 15b^2dx^4) + 3x (a + bx^2)^2 (ae - 5bd) \tan^{-1} \left(\frac{\sqrt{b}x}{\sqrt{a}} \right)}{8a^{7/2} \sqrt{b} x (a + bx^2) \sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)), x]

[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [A] time = 19.38, size = 117, normalized size = 0.62

$$\frac{(a + bx^2) \left(\frac{3(ae - 5bd) \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{-8a^2d + 5a^2ex^2 - 25abd^2 + 3abex^4 - 15b^2dx^4}{8a^3x(a + bx^2)^2} \right)}{\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)), x]

[Out] ((a + b*x^2)*((-8*a^2*d - 25*a*b*d*x^2 + 5*a^2*e*x^2 - 15*b^2*d*x^4 + 3*a*b*e*x^4)/(8*a^3*x*(a + b*x^2)^2) + (3*(-5*b*d + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]))/Sqrt[(a + b*x^2)^2]

fricas [A] time = 0.96, size = 334, normalized size = 1.76

$$\left[\frac{16a^3bd + 6(5ab^2d - a^2b^2e)x^4 + 10(5a^2b^2d - a^3be)x^2 - 3((5b^3d - ab^2e)x^5 + 2(5ab^2d - a^2be)x^3 + (5a^2bd - a^3e)x)\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x + a}{bx + a}\right)}{16(a^4b^3x^5 + 2a^5b^2x^3 + a^6bx)}, \frac{8a^3bd + 3(5ab^2d - a^2b^2e)x^4 + 5(5a^2b^2d - a^3be)x^2 + 3((5b^3d - ab^2e)x^5 + 2(5ab^2d - a^2be)x^3 + (5a^2bd - a^3e)x)\sqrt{ab} \arctan\left(\frac{\sqrt{b}x}{a}\right)}{8(a^4b^3x^5 + 2a^5b^2x^3 + a^6bx)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="fricas")

[Out] [-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3*b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*d - a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 206, normalized size = 1.08

$$\frac{(3ab^2e^{x^5} \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 15b^3dx^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 6a^2bex^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 30ab^2dx^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 3\sqrt{ab} abex^4 - 15\sqrt{ab} b^2dx^4 + 3a^3ex \arctan\left(\frac{bx}{\sqrt{ab}}\right) - 15a^2bdx \arctan\left(\frac{bx}{\sqrt{ab}}\right) + 5\sqrt{ab} a^2ex^2 - 25\sqrt{ab} abd^2x^2 - 8\sqrt{ab} a^2d)(bx^2 + a)}{8\sqrt{ab} \left((bx^2 + a)^2\right)^{\frac{3}{2}} a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x)

[Out] $\frac{1}{8} * (3 * \arctan(1/(a*b)^{(1/2)} * b*x) * x^5 * a*b^2 * e - 15 * \arctan(1/(a*b)^{(1/2)} * b*x) * x^5 * b^3 * d + 3 * (a*b)^{(1/2)} * x^4 * a*b * e - 15 * (a*b)^{(1/2)} * x^4 * b^2 * d + 6 * \arctan(1/(a*b)^{(1/2)} * b*x) * x^3 * a^2 * b * e - 30 * \arctan(1/(a*b)^{(1/2)} * b*x) * x^3 * a * b^2 * d + 5 * (a*b)^{(1/2)} * x^2 * a^2 * e - 25 * (a*b)^{(1/2)} * x^2 * a * b * d + 3 * \arctan(1/(a*b)^{(1/2)} * b*x) * x * a^3 * e - 15 * \arctan(1/(a*b)^{(1/2)} * b*x) * x * a^2 * b * d - 8 * (a*b)^{(1/2)} * a^2 * d) * (b*x^2 + a) / (a*b)^{(1/2)} / x / a^3 / ((b*x^2 + a)^2)^{(3/2)}$

maxima [A] time = 1.40, size = 134, normalized size = 0.71

$$-\frac{1}{8} d \left(\frac{15b^2x^4 + 25abx^2 + 8a^2}{a^3b^2x^5 + 2a^4bx^3 + a^5x} + \frac{15b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab} a^3} \right) + \frac{1}{8} e \left(\frac{3bx^3 + 5ax}{a^2b^2x^4 + 2a^3bx^2 + a^4} + \frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab} a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2), x, algorithm="maxima")

[Out] $-1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x) + 15*b*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a^3)) + 1/8*e*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a^2))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{x^2} + d}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)

[Out] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x^2 \left((a + bx^2)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*(a + b*x**2)**2)**(3/2)), x)
```

$$3.66 \quad \int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal. Leaf size=223

$$\frac{bd - ae}{4a^2(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{\log(x)(a + bx^2)(3bd - ae)}{a^4\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(a + bx^2)(3bd - ae)\log(a + bx^2)}{2a^4\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{2bd}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Rubi [A] time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1250, 446, 77}

$$\frac{bd - ae}{4a^2(a + bx^2)\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{2bd - ae}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{\log(x)(a + bx^2)(3bd - ae)}{a^4\sqrt{a^2 + 2abx^2 + b^2x^4}} + \frac{(a + bx^2)(3bd - ae)\log(a + bx^2)}{2a^4\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d(a + bx^2)}{2a^3x^2\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]

[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1250

Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int

Part[p]*(b/2 + c*x^2)^(2*FracPart[p]), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{d + ex^2}{x^3 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^2)) \int \frac{d+ex^2}{x^3(ab+b^2x^2)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(b^2 (ab + b^2x^2)) \text{Subst} \left(\int \frac{d+ex}{x^2(ab+b^2x)^3} dx, x, x^2 \right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= \frac{(b^2 (ab + b^2x^2)) \text{Subst} \left(\int \left(\frac{d}{a^3b^3x^2} + \frac{-3bd+ae}{a^4b^3x} + \frac{bd-ae}{a^2b^2(a+bx)^3} + \frac{2bd-ae}{a^3b^2(a+bx)^2} + \frac{3bd-ae}{a^4b^2(a+bx)} \right) dx, x, x^2 \right)}{2\sqrt{a^2 + 2abx^2 + b^2x^4}} \\ &= -\frac{2bd - ae}{2a^3\sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{bd - ae}{4a^2 (a + bx^2) \sqrt{a^2 + 2abx^2 + b^2x^4}} - \frac{d (a + bx^2)}{2a^3x^2\sqrt{a^2 + 2abx^2 + b^2x^4}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 130, normalized size = 0.58

$$\frac{a(a^2(3ex^2 - 2d) + ab(2ex^4 - 9dx^2) - 6b^2dx^4) + 4x^2 \log(x)(a + bx^2)^2(ae - 3bd) + 2x^2(a + bx^2)^2(3bd - ae) \log(a + bx^2)}{4a^4x^2(a + bx^2)\sqrt{(a + bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]

[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2)) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Log[a + b*x^2]/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])

IntegrateAlgebraic [B] time = 6.04, size = 1473, normalized size = 6.61

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]

```
[Out] 
$$\begin{aligned} & (-a^{15}b^2d) + a^{16}b^*e + 2a^{14}b^3d*x^2 + 49a^{13}b^4d*x^4 - 3a^{14}b^3e*x^4 + 552a^{12}b^5d*x^6 - 80a^{13}b^4e*x^6 + 3784a^{11}b^6d*x^8 - 988a^{12}b^5e*x^8 + 17600a^{10}b^7d*x^{10} - 7488a^{11}b^6e*x^{10} + 58608a^9b^8d*x^{12} - 38896a^{10}b^7e*x^{12} + 143616a^8b^9d*x^{14} - 146432a^9b^8e*x^{14} + 261888a^7b^{10}d*x^{16} - 411840a^8b^9e*x^{16} + 354816a^6b^{11}d*x^{18} - 878592a^7b^{10}e*x^{18} + 352000a^5b^{12}d*x^{20} - 1427712a^6b^{11}e*x^{20} + 247808a^4b^{13}d*x^{22} - 1757184a^5b^{12}e*x^{22} + 116736a^3b^{14}d*x^{24} - 1610752a^4b^{13}e*x^{24} + 32768a^2b^{15}d*x^{26} - 1064960a^3b^{14}e*x^{26} + 4096a*b^{16}d*x^{28} - 479232a^2b^{15}e*x^{28} - 131072a*b^{16}e*x^{30} - 16384b^{17}e*x^{32} + \text{Sqrt}[b^2]*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]*(-a^{14}b*d) + a^{15}e - a^{13}b^2d*x^2 - a^{14}b^*e*x^2 - 48a^{12}b^3d*x^4 + 4a^{13}b^2e*x^4 - 504a^{11}b^4d*x^6 + 76a^{12}b^3e*x^6 - 3280a^{10}b^5d*x^8 + 912a^{11}b^4e*x^8 - 14320a^9b^6d*x^{10} + 6576a^{10}b^5e*x^{10} - 44288a^8b^7d*x^{12} + 32320a^9b^6e*x^{12} - 99328a^7b^8d*x^{14} + 114112a^8b^7e*x^{14} - 162560a^6b^9d*x^{16} + 297728a^7b^8e*x^{16} - 192256a^5b^{10}d*x^{18} + 580864a^6b^9e*x^{18} - 159744a^4b^{11}d*x^{20} + 846848a^5b^{10}e*x^{20} - 88064a^3b^{12}d*x^{22} + 910336a^4b^{11}e*x^{22} - 28672a^2b^{13}d*x^{24} + 700416a^3b^{12}e*x^{24} - 4096a*b^{14}d*x^{26} + 364544a^2b^{13}e*x^{26} + 114688a*b^{14}e*x^{28} + 16384b^{15}e*x^{30})/(2a^2b*x^4*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]*(-2a^{14}b^2 - 54a^{13}b^3*x^2 - 676a^{12}b^4*x^4 - 5200a^{11}b^5*x^6 - 27456a^{10}b^6*x^8 - 105248a^9b^7*x^{10} - 302016a^8b^8*x^{12} - 658944a^7b^9*x^{14} - 1098240a^6b^{10}x^{16} - 1391104a^5b^{11}x^{18} - 1317888a^4b^{12}x^{20} - 905216a^3b^{13}x^{22} - 425984a^2b^{14}x^{24} - 122880a*b^{15}x^{26} - 16384b^{16}x^{28}) + 2a^2b*\text{Sqrt}[b^2]*x^4*(2a^{15}b + 56a^{14}b^2*x^2 + 730a^{13}b^3*x^4 + 5876a^{12}b^4*x^6 + 32656a^{11}b^5*x^8 + 132704a^{10}b^6*x^{10} + 407264a^9b^7*x^{12} + 960960a^8b^8*x^{14} + 1757184a^7b^9*x^{16} + 2489344a^6b^{10}x^{18} + 2708992a^5b^{11}x^{20} + 2223104a^4b^{12}x^{22} + 1331200a^3b^{13}x^{24} + 548864a^2b^{14}x^{26} + 139264a*b^{15}x^{28} + 16384b^{16}x^{30})) + ((-\text{Sqrt}[b^2]*x^2) + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4])^3*((-3*b*d)/a^3 + (\text{Sqrt}[b^2]*e*x^2*\text{ArcTanh}[(-\text{Sqrt}[b^2]*x^2) + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4])/a])/a^3 - (e*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]*\text{ArcTanh}[(-\text{Sqrt}[b^2]*x^2) + \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4])/a])/a^3)/(a^4 + 4a^3b*x^2 + 12a^2b^2*x^4 + 16a*b^3*x^6 + 8b^4*x^8 - 4a^2*\text{Sqrt}[b^2]*x^2*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4] - 8a*b*\text{Sqrt}[b^2]*x^4*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4] - 8*(b^2)^(3/2)*x^6*\text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]) - (3*b*d*\text{ArcTanh}[(\text{Sqrt}[b^2]*x^2)/a - \text{Sqrt}[a^2 + 2a*b*x^2 + b^2*x^4]/a])/a^4 \end{aligned}$$

```

fricas [A] time = 0.67, size = 205, normalized size = 0.92

$$\frac{2(3ab^2d - a^2be)x^4 + 2a^3d + 3(3a^2bd - a^3e)x^2 - 2((3b^3d - ab^2e)x^6 + 2(3ab^2d - a^2be)x^4 + (3a^2bd - a^3e)x^2) \log(bx^2 + a) + 4((3b^3d - ab^2e)x^6 + 2(3ab^2d - a^2be)x^4 + (3a^2bd - a^3e)x^2) \log(x)}{4(a^4b^2x^6 + 2a^5bx^4 + a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

[Out] $-1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 - 2*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*\log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*\log(x)/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)$

giac [A] time = 0.40, size = 144, normalized size = 0.65

$$-\frac{(3bd - ae) \log(|x|)}{a^4 \operatorname{sgn}(bx^2 + a)} + \frac{(3b^2d - abe) \log(|bx^2 + a|)}{2a^4 b \operatorname{sgn}(bx^2 + a)} - \frac{2(3ab^2d - a^2be)x^4 + 2a^3d + 3(3a^2bd - a^3e)x^2}{4(bx^2 + a)^2 a^4 x^2 \operatorname{sgn}(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

[Out] $-(3*b*d - a*e)*\log(\operatorname{abs}(x))/(a^4*\operatorname{sgn}(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*\log(\operatorname{abs}(b*x^2 + a))/(a^4*b*\operatorname{sgn}(b*x^2 + a)) - 1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2)/((b*x^2 + a)^2*a^4*x^2*\operatorname{sgn}(b*x^2 + a))$

maple [A] time = 0.02, size = 249, normalized size = 1.12

$$\frac{(4a^4b^2e^2 \ln(x) - 2a^4b^2e^2 \ln(bx^2 + a) - 12b^2d^2 \ln(x) + 6b^2d^2 \ln(bx^2 + a) + 8a^2be^2 \ln(x) - 4a^2be^2 \ln(bx^2 + a) - 24ab^2d^2 \ln(x) + 12ab^2d^2 \ln(bx^2 + a) + 2a^2be^2 - 6a^2be^2 + 4a^2e^2 \ln(x) - 2a^2e^2 \ln(bx^2 + a) - 12a^2bd^2 \ln(x) + 6a^2bd^2 \ln(bx^2 + a) + 3a^2e^2 - 9a^2bd^2 - 2a^2d)(bx^2 + a)}{4(bx^2 + a)^{3/2} a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)`

[Out] $1/4*(4*\ln(x)*x^6*a*b^2*e - 12*\ln(x)*x^6*b^3*d - 2*\ln(b*x^2+a)*x^6*a*b^2*e + 6*\ln(b*x^2+a)*x^6*b^3*d + 8*\ln(x)*x^4*a^2*b*e - 24*\ln(x)*x^4*a*b^2*d - 4*\ln(b*x^2+a)*x^4*a^2*b*e + 12*\ln(b*x^2+a)*x^4*a*b^2*d + 2*x^4*a^2*b*e - 6*x^4*a*b^2*d + 4*\ln(x)*x^2*a^3*e - 12*\ln(x)*x^2*a^2*b*d - 2*\ln(b*x^2+a)*x^2*a^3*e + 6*\ln(b*x^2+a)*x^2*a^2*b*d + 3*x^2*a^3*e - 9*x^2*a^2*b*d - 2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/2)$

maxima [A] time = 0.84, size = 138, normalized size = 0.62

$$-\frac{1}{4}d\left(\frac{6b^2x^4 + 9abx^2 + 2a^2}{a^3b^2x^6 + 2a^4bx^4 + a^5x^2} - \frac{6b \log(bx^2 + a)}{a^4} + \frac{12b \log(x)}{a^4}\right) + \frac{1}{4}e\left(\frac{2bx^2 + 3a}{a^2b^2x^4 + 2a^3bx^2 + a^4} - \frac{2 \log(bx^2 + a)}{a^3} + \frac{4 \log(x)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")`

[Out] $-\frac{1}{4}d \frac{(6b^2x^4 + 9abx^2 + 2a^2)}{(a^3b^2x^6 + 2a^4bx^4 + a^5x^2)} - 6b \frac{\log(bx^2 + a)}{a^4} + 12b \frac{\log(x)}{a^4} + \frac{1}{4}e \frac{(2bx^2 + 3a)}{(a^2b^2x^4 + 2a^3bx^2 + a^4)} - 2 \frac{\log(bx^2 + a)}{a^3} + 4 \frac{\log(x)}{a^3}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{ex^2 + d}{x^3 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)`

[Out] `int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{x^3 \left((a + bx^2)^2 \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2), x)`

[Out] `Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)`

$$3.67 \quad \int (fx)^m (d + ex^2) (a^2 + 2abx^2 + b^2x^4)^{5/2} dx$$

Optimal. Leaf size=400

$$\frac{10a^2b^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+7} (ae + bd)}{f^7(m+7)(a + bx^2)} + \frac{b^5e\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+13}}{f^{13}(m+13)(a + bx^2)} + \frac{b^4\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1}}{f^{11}(m+11)(a + bx^2)}$$

Rubi [A] time = 0.24, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1250, 448}

$$\frac{a^4\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+3} (ae + 5bd)}{f^{7(m+3)(a + bx^2)}} + \frac{5a^2b\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1} (ae + 2bd)}{f^{5(m+5)(a + bx^2)}} + \frac{10a^2b^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+7} (ae + bd)}{f^{7(m+7)(a + bx^2)}} + \frac{5ab^4\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+9} (2ae + bd)}{f^{9(m+9)(a + bx^2)}} + \frac{b^4\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+11} (5ae + bd)}{f^{11(m+11)(a + bx^2)}} + \frac{a^2d\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1}}{f^{(m+1)(a + bx^2)}} + \frac{b^2e\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+13}}{f^{13(m+13)(a + bx^2)}}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2), x]

[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f*(1 + m)*(a + b*x^2)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^3*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^7*(7 + m)*(a + b*x^2)) + (5*a*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^9*(9 + m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(f^13*(13 + m)*(a + b*x^2))

Rule 448

Int[((e._)*(x_))^(m._)*((a_) + (b._)*(x_)^(n_))^(p._)*((c_) + (d._)*(x_)^(n_))^(q._), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1250

Int[((f._)*(x_))^(m._)*((d_) + (e._)*(x_)^2)^(q._)*((a_) + (b._)*(x_)^2 + (c._)*(x_)^4)^(p._), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2) (a^2 + 2abx^2 + b^2x^4)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int (fx)^m (ab + b^2x^2)^5 (d + ex^2) dx}{b^4 (ab + b^2x^2)} \\
&= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int (a^5 b^5 d (fx)^m + \frac{a^4 b^5 (5bd + ae) (fx)^{2+m}}{f^2} + \frac{5a^3 b^6 (2bd + ae) (fx)^{4+m}}{f^4}) dx}{b^4 (ab + b^2x^2)} \\
&= \frac{a^5 d (fx)^{1+m} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f(1+m)(a + bx^2)} + \frac{a^4 (5bd + ae) (fx)^{3+m} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^3(3+m)(a + bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 160, normalized size = 0.40

$$\frac{x \sqrt{(a + bx^2)^2} (fx)^m \left(\frac{a^5 d}{m+1} + \frac{a^4 x^2 (ae + 5bd)}{m+3} + \frac{5a^3 bx^4 (ae + 2bd)}{m+5} + \frac{10a^2 b^2 x^6 (ae + bd)}{m+7} + \frac{b^4 x^{10} (5ae + bd)}{m+11} + \frac{5ab^3 x^8 (2ae + bd)}{m+9} + \frac{b^5 ex^{12}}{m+13} \right)}{a + bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2), x]

[Out] (x*(f*x)^m*sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(11 + m) + (b^5*e*x^12)/(13 + m))/(a + b*x^2)

IntegrateAlgebraic [F] time = 5.12, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a^2 + 2abx^2 + b^2x^4)^{5/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2), x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2), x]

fricas [B] time = 0.86, size = 853, normalized size = 2.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2), x, algorithm="fricas")


```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*m^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285*b^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4*e)*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a^2*b^3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d + a^3*b^2*e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b^2*e)*m^5 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)*m^3 + 20335*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*((2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d + a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*b^2*d + a^4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d + 45045*a^5*e + 46*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7540*(5*a^4*b*d + a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x*(f*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

giac [B] time = 0.69, size = 2213, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((f*x)^m*b^5*m^6*x^13*e*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*m^5*x^13*e*sgn(b*x^2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*m^6*x^11*e*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*m^4*x^13*e*sgn(b*x^2 + a) + 38*(f*x)^m*b^5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*m^5*x^11*e*sgn(b*x^2 + a) + 3480*(f*x)^m*b^5*m^3*x^13*e*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*m^6*x^9*e*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*m^4*x^11*e*sgn(b*x^2 + a) + 12139*(f*x)^m*b^5*m^2*x^13*e*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 400*(f*x)^m*a^2*b^3*m^5*x^9*e*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*m^3*x^11*e*sgn(b*x^2 + a) + 19524*(f*x)^m*b^5*m*x^13*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 14039*(f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*m^6*x^7*e*sgn(b*x^2 + a) + 6130*(f*x)^m*a^2*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b^4*m^2*x^11*e*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*x^13*e*sgn(b*x^2 + a) + 420*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9
```

$$\begin{aligned} & \operatorname{sgn}(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^{11}*\operatorname{sgn}(b*x^2 + a) + 420*(f*x)^m*a^3*b^2*m^5*x^7*e*\operatorname{sgn}(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*m^3*x^9*e*\operatorname{sgn}(b*x^2 + a) + 114510*(f*x)^m*a*b^4*m*x^{11}*e*\operatorname{sgn}(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*m^6*x^5*\operatorname{sgn}(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*\operatorname{sgn}(b*x^2 + a) + 83135*(f*x)^m*a*b^4*d*m^2*x^9*\operatorname{sgn}(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^{11}*\operatorname{sgn}(b*x^2 + a) + 5*(f*x)^m*a^4*b*m^6*x^5*e*\operatorname{sgn}(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2*m^4*x^7*e*\operatorname{sgn}(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*m^2*x^9*e*\operatorname{sgn}(b*x^2 + a) + 61425*(f*x)^m*a*b^4*x^{11}*e*\operatorname{sgn}(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^5*\operatorname{sgn}(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*\operatorname{sgn}(b*x^2 + a) + 138440*(f*x)^m*a*b^4*d*m*x^9*\operatorname{sgn}(b*x^2 + a) + 220*(f*x)^m*a^4*b*m^5*x^5*e*\operatorname{sgn}(b*x^2 + a) + 52920*(f*x)^m*a^3*b^2*m^3*x^7*e*\operatorname{sgn}(b*x^2 + a) + 276880*(f*x)^m*a^2*b^3*m*x^9*e*\operatorname{sgn}(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*\operatorname{sgn}(b*x^2 + a) + 7530*(f*x)^m*a^3*b^2*d*m^4*x^5*\operatorname{sgn}(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2*x^7*\operatorname{sgn}(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*\operatorname{sgn}(b*x^2 + a) + (f*x)^m*a^5*m^6*x^3*e*\operatorname{sgn}(b*x^2 + a) + 3765*(f*x)^m*a^4*b*m^4*x^5*e*\operatorname{sgn}(b*x^2 + a) + 203350*(f*x)^m*a^3*b^2*m^2*x^7*e*\operatorname{sgn}(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*x^9*e*\operatorname{sgn}(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*\operatorname{sgn}(b*x^2 + a) + 62800*(f*x)^m*a^3*b^2*d*m^3*x^5*\operatorname{sgn}(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*\operatorname{sgn}(b*x^2 + a) + 46*(f*x)^m*a^5*m^5*x^3*e*\operatorname{sgn}(b*x^2 + a) + 31400*(f*x)^m*a^4*b*m^3*x^5*e*\operatorname{sgn}(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*m*x^7*e*\operatorname{sgn}(b*x^2 + a) + (f*x)^m*a^5*d*m^6*x*\operatorname{sgn}(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*\operatorname{sgn}(b*x^2 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*\operatorname{sgn}(b*x^2 + a) + 193050*(f*x)^m*a^2*b^3*d*x^7*\operatorname{sgn}(b*x^2 + a) + 835*(f*x)^m*a^5*m^4*x^3*e*\operatorname{sgn}(b*x^2 + a) + 129895*(f*x)^m*a^4*b*m^2*x^5*e*\operatorname{sgn}(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*x^7*e*\operatorname{sgn}(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*\operatorname{sgn}(b*x^2 + a) + 37700*(f*x)^m*a^4*b*d*m^3*x^3*\operatorname{sgn}(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*\operatorname{sgn}(b*x^2 + a) + 7540*(f*x)^m*a^5*m^3*x^3*e*\operatorname{sgn}(b*x^2 + a) + 237180*(f*x)^m*a^4*b*m*x^5*e*\operatorname{sgn}(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*\operatorname{sgn}(b*x^2 + a) + 173795*(f*x)^m*a^4*b*d*m^2*x^3*\operatorname{sgn}(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*\operatorname{sgn}(b*x^2 + a) + 34759*(f*x)^m*a^5*m^2*x^3*e*\operatorname{sgn}(b*x^2 + a) + 135135*(f*x)^m*a^4*b*x^5*e*\operatorname{sgn}(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*\operatorname{sgn}(b*x^2 + a) + 365270*(f*x)^m*a^4*b*d*m*x^3*\operatorname{sgn}(b*x^2 + a) + 73054*(f*x)^m*a^5*m*x^3*e*\operatorname{sgn}(b*x^2 + a) + 48259*(f*x)^m*a^5*d*m^2*x*\operatorname{sgn}(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*\operatorname{sgn}(b*x^2 + a) + 45045*(f*x)^m*a^5*x^3*e*\operatorname{sgn}(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*\operatorname{sgn}(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*\operatorname{sgn}(b*x^2 + a))/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135) \end{aligned}$$

maple [B] time = 0.01, size = 1099, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^{(5/2)}, x)$

[Out] $x*(b^5*e*m^6*x^{12}+36*b^5*e*m^5*x^{12}+5*a*b^4*e*m^6*x^{10}+b^5*d*m^6*x^{10}+505*b$

$$\begin{aligned} &^5e^m^4x^{12}+190a^2b^3e^m^6x^8+5a^2b^3e^m^6x^8+2775a^2b^3e^m^6x^8+555b^5d^m^4x^{10}+12139b^5e^m^2x^{12}+400a^2b^3e^m^5x^8+200a^2b^3e^m^5x^8+19700a^2b^3e^m^3x^{10}+3940b^5d^m^3x^{10}+19524b^5e^m^3x^{12}+10a^3b^2e^m^6x^6+10a^2b^3d^m^6x^6+6130a^2b^3e^m^4x^8+3065a^2b^3e^m^4x^8+70195a^2b^3e^m^2x^{10}+14039b^5d^m^2x^{10}+10395b^5e^m^2x^{12}+420a^3b^2e^m^5x^6+420a^2b^3d^m^5x^6+45280a^2b^3e^m^3x^8+22640a^2b^3e^m^3x^8+114510a^2b^3e^m^2x^{10}+22902b^5d^m^2x^{10}+5a^4b^2e^m^6x^4+10a^3b^2d^m^6x^4+6790a^3b^2e^m^4x^6+6790a^2b^3d^m^4x^6+166270a^2b^3e^m^2x^8+83135a^2b^3e^m^2x^8+61425a^2b^3e^m^2x^8+12285b^5d^m^2x^8+220a^4b^2e^m^5x^4+440a^3b^2d^m^5x^4+52920a^3b^2e^m^3x^6+52920a^2b^3d^m^3x^6+276880a^2b^3e^m^2x^8+138440a^2b^3e^m^2x^8+a^5e^m^6x^2+5a^4b^2d^m^6x^2+3765a^4b^2e^m^4x^4+7530a^3b^2d^m^4x^4+203350a^3b^2e^m^2x^6+203350a^2b^3d^m^2x^6+150150a^2b^3e^m^2x^8+75075a^2b^3e^m^2x^8+46a^5e^m^5x^2+230a^4b^2d^m^5x^2+31400a^4b^2e^m^3x^4+62800a^3b^2d^m^3x^4+349860a^3b^2e^m^2x^6+349860a^2b^3d^m^2x^6+a^5d^m^6+835a^5e^m^4x^2+4175a^4b^2d^m^4x^2+129895a^4b^2e^m^2x^4+259790a^3b^2d^m^2x^4+193050a^3b^2e^m^2x^6+193050a^2b^3d^m^2x^6+48a^5d^m^5+7540a^5e^m^3x^2+37700a^4b^2d^m^3x^2+237180a^4b^2e^m^2x^4+474360a^3b^2d^m^2x^4+925a^5d^m^4+34759a^5e^m^2x^2+173795a^4b^2d^m^2x^2+135135a^4b^2e^m^2x^4+270270a^3b^2d^m^2x^4+9120a^5d^m^3+73054a^5e^m^2x^2+365270a^4b^2d^m^2x^2+48259a^5d^m^2+45045a^5e^m^2x^2+225225a^4b^2d^m^2x^2+129072a^5d^m+135135a^5d^m)(f*x)^m*((b*x^2+a)^2)^(5/2)/(m+13)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^5 \end{aligned}$$

maxima [A] time = 0.82, size = 491, normalized size = 1.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 + 27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 + 406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 12673*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 15681*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 20409*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +

45045)*a⁵*f^m*x³)*e*x^m/(m⁶ + 48*m⁵ + 925*m⁴ + 9120*m³ + 48259*m² + 129072*m + 135135)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (ex^2 + d) (a^2 + 2abx^2 + b^2x^4)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x²)*(a² + b²*x⁴ + 2*a*b*x²)^(5/2), x)

[Out] int((f*x)^m*(d + e*x²)*(a² + b²*x⁴ + 2*a*b*x²)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) \left((a + bx^2)^2 \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2), x)

[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)

$$3.68 \quad \int (fx)^m (d + ex^2) (a^2 + 2abx^2 + b^2x^4)^{3/2} dx$$

Optimal. Leaf size=276

$$\frac{b^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+7} (3ae + bd)}{f^7(m+7)(a + bx^2)} + \frac{3ab\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+5} (ae + bd)}{f^5(m+5)(a + bx^2)} + \frac{a^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+3} (3bd)}{f^3(m+3)(a + bx^2)}$$

Rubi [A] time = 0.15, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1250, 448}

$$\frac{a^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+3} (ae + 3bd)}{f^3(m+3)(a + bx^2)} + \frac{3ab\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+5} (ae + bd)}{f^5(m+5)(a + bx^2)} + \frac{b^2\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+7} (3ae + bd)}{f^7(m+7)(a + bx^2)} + \frac{a^3d\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1}}{f^{m+1}(a + bx^2)} + \frac{b^3e\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+9}}{f^{m+9}(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2), x]

[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1250

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps


```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*e)
*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*((a
*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*
m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^
2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 +
164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^4
+ 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^m/(m^5 +
25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.52, size = 1013, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*m^4*x^7*e*sgn
(b*x^2 + a) + 86*(f*x)^m*b^3*m^2*x^9*e*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*m*x^9*e*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
104*(f*x)^m*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*m^4*x^5*e*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*x^
9*e*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^
m*b^3*d*m*x^7*sgn(b*x^2 + a) + 60*(f*x)^m*a^2*b*m^3*x^5*e*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*m*x^7*e*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^m*b^3*d*x
^7*sgn(b*x^2 + a) + (f*x)^m*a^3*m^4*x^3*e*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*
b*m^2*x^5*e*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*x^7*e*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2
+ a) + 22*(f*x)^m*a^3*m^3*x^3*e*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*m*x^5*e*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
^2*x^3*sgn(b*x^2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 164*(f*x)^
m*a^3*m^2*x^3*e*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*x^5*e*sgn(b*x^2 + a) + 2
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
+ a) + 458*(f*x)^m*a^3*m*x^3*e*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*x^3
*e*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 495, normalized size = 1.79

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^{(3/2)}, x)$

[Out] $x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b*x^2+a)^2)^{(3/2)}/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)/(b*x^2+a)^3$

maxima [A] time = 0.96, size = 243, normalized size = 0.88

$\frac{((m^3 + 9m^2 + 23m + 15)b^3 f^m x^7 + 3(m^3 + 11m^2 + 31m + 21)a b^2 f^m x^5 + 3(m^3 + 13m^2 + 47m + 35)a^2 b f^m x^3 + (m^3 + 15m^2 + 71m + 105)a^3 f^m x) d x^m + ((m^3 + 15m^2 + 71m + 105)b^3 f^m x^9 + 3(m^3 + 17m^2 + 87m + 135)a b^2 f^m x^7 + 3(m^3 + 19m^2 + 111m + 189)a^2 b f^m x^5 + (m^3 + 21m^2 + 143m + 315)a^3 f^m x^3) e x^m}{m^4 + 16m^3 + 86m^2 + 176m + 105} \cdot \frac{((m^3 + 15m^2 + 71m + 105)b^3 f^m x^9 + 3(m^3 + 17m^2 + 87m + 135)a b^2 f^m x^7 + 3(m^3 + 19m^2 + 111m + 189)a^2 b f^m x^5 + (m^3 + 21m^2 + 143m + 315)a^3 f^m x^3) e x^m}{m^4 + 24m^3 + 206m^2 + 744m + 945}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $((m^3 + 9m^2 + 23m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11m^2 + 31m + 21)*a*b^2*f^m*x^5 + 3*(m^3 + 13m^2 + 47m + 35)*a^2*b*f^m*x^3 + (m^3 + 15m^2 + 71m + 105)*a^3*f^m*x) * d*x^m / (m^4 + 16m^3 + 86m^2 + 176m + 105) + ((m^3 + 15m^2 + 71m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17m^2 + 87m + 135)*a*b^2*f^m*x^7 + 3*(m^3 + 19m^2 + 111m + 189)*a^2*b*f^m*x^5 + (m^3 + 21m^2 + 143m + 315)*a^3*f^m*x^3) * e*x^m / (m^4 + 24m^3 + 206m^2 + 744m + 945)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f x)^m (e x^2 + d) (a^2 + 2 a b x^2 + b^2 x^4)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^{(3/2)}, x)$

[Out] $\text{int}((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^{(3/2)}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (f x)^m (d + e x^2) \left((a + b x^2)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

$$3.69 \quad \int (fx)^m (d + ex^2) \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Optimal. Leaf size=153

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+3} (ae + bd)}{f^3(m+3)(a + bx^2)} + \frac{ad\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1}}{f(m+1)(a + bx^2)} + \frac{be\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+5}}{f^5(m+5)(a + bx^2)}$$

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1250, 448}

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+3} (ae + bd)}{f^3(m+3)(a + bx^2)} + \frac{ad\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+1}}{f(m+1)(a + bx^2)} + \frac{be\sqrt{a^2 + 2abx^2 + b^2x^4} (fx)^{m+5}}{f^5(m+5)(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2)) + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2))

Rule 448

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 1250

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) \sqrt{a^2 + 2abx^2 + b^2x^4} dx &= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int (fx)^m (ab + b^2x^2) (d + ex^2) dx}{ab + b^2x^2} \\ &= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \left(abd(fx)^m + \frac{b(bd+ae)(fx)^{2+m}}{f^2} + \frac{b^2e(fx)^{4+m}}{f^4} \right) dx}{ab + b^2x^2} \\ &= \frac{ad(fx)^{1+m} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f(1+m)(a + bx^2)} + \frac{(bd + ae)(fx)^{3+m} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^3(3+m)(a + bx^2)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.56

$$\frac{x \sqrt{(a + bx^2)^2} (fx)^m (a(m+5)(d(m+3) + e(m+1)x^2) + b(m+1)x^2(d(m+5) + e(m+3)x^2))}{(m+1)(m+3)(m+5)(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^2))

IntegrateAlgebraic [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

fricas [A] time = 0.73, size = 94, normalized size = 0.61

$$\frac{((bem^2 + 4bem + 3be)x^5 + ((bd + ae)m^2 + 5bd + 5ae + 6(bd + ae)m)x^3 + (adm^2 + 8adm + 15ad)x)(fx)^m}{m^3 + 9m^2 + 23m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2), x, algorithm="fricas")

[Out] $((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b*d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 + 23*m + 15)$

giac [B] time = 0.32, size = 269, normalized size = 1.76

$(f_1)^m \ln^2 x \operatorname{sgn}(bx^2 + a) + 4(f_1)^m \ln x \operatorname{sgn}(bx^2 + a) + (f_1)^m \operatorname{sgn}(bx^2 + a) + (f_1)^m \ln^2 x \operatorname{sgn}(bx^2 + a) + 3(f_1)^m \ln x \operatorname{sgn}(bx^2 + a) + 6(f_1)^m \operatorname{sgn}(bx^2 + a) + (f_1)^m \ln^2 x \operatorname{sgn}(bx^2 + a) + 5(f_1)^m \ln x \operatorname{sgn}(bx^2 + a) + 8(f_1)^m \operatorname{sgn}(bx^2 + a) + 15(f_1)^m \operatorname{sgn}(bx^2 + a)$
 $m^3 + 9m^2 + 23m + 15$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")`

[Out] $((f*x)^m*b*m^2*x^5*e*\operatorname{sgn}(b*x^2 + a) + 4*(f*x)^m*b*m*x^5*e*\operatorname{sgn}(b*x^2 + a) + (f*x)^m*b*d*m^2*x^3*\operatorname{sgn}(b*x^2 + a) + (f*x)^m*a*m^2*x^3*e*\operatorname{sgn}(b*x^2 + a) + 3*(f*x)^m*b*x^5*e*\operatorname{sgn}(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*\operatorname{sgn}(b*x^2 + a) + 6*(f*x)^m*a*m*x^3*e*\operatorname{sgn}(b*x^2 + a) + (f*x)^m*a*d*m^2*x*\operatorname{sgn}(b*x^2 + a) + 5*(f*x)^m*b*d*x^3*\operatorname{sgn}(b*x^2 + a) + 5*(f*x)^m*a*x^3*e*\operatorname{sgn}(b*x^2 + a) + 8*(f*x)^m*a*d*m*x*\operatorname{sgn}(b*x^2 + a) + 15*(f*x)^m*a*d*x*\operatorname{sgn}(b*x^2 + a))/(m^3 + 9*m^2 + 23*m + 15)$

maple [A] time = 0.01, size = 131, normalized size = 0.86

$$\frac{(be m^2 x^4 + 4bem x^4 + ae m^2 x^2 + bd m^2 x^2 + 3be x^4 + 6aem x^2 + 6bdm x^2 + ad m^2 + 5ae x^2 + 5bd x^2 + 8adm + 15ad) \sqrt{(bx^2 + a)^2} x (fx)^m}{(m + 5)(m + 3)(m + 1)(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)`

[Out] $x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)^(1/2)/(m+5)/(m+3)/(m+1)/(b*x^2+a)$

maxima [A] time = 0.81, size = 75, normalized size = 0.49

$$\frac{(bf^m(m+1)x^3 + af^m(m+3)x)dx^m}{m^2 + 4m + 3} + \frac{(bf^m(m+3)x^5 + af^m(m+5)x^3)ex^m}{m^2 + 8m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")`

[Out] $(b*f^m*(m+1)*x^3 + a*f^m*(m+3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m+3)*x^5 + a*f^m*(m+5)*x^3)*e*x^m/(m^2 + 8*m + 15)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (fx)^m (ex^2 + d) \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)`

[Out] `int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) \sqrt{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2), x)`

[Out] `Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)`

$$3.70 \quad \int x (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Optimal. Leaf size=34

$$\frac{(a^2 + 2abx^2 + b^2x^4)^{p+1}}{4b(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1247, 629}

$$\frac{(a^2 + 2abx^2 + b^2x^4)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx &= \frac{1}{2} \text{Subst} \left(\int (a + bx) (a^2 + 2abx + b^2x^2)^p dx, x, x^2 \right) \\ &= \frac{(a^2 + 2abx^2 + b^2x^4)^{1+p}}{4b(1+p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{\left((a + bx^2)^2\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] Defer[IntegrateAlgebraic][x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]

fricas [A] time = 0.71, size = 47, normalized size = 1.38

$$\frac{(b^2x^4 + 2abx^2 + a^2)(b^2x^4 + 2abx^2 + a^2)^p}{4(bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")

[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)

giac [A] time = 0.29, size = 32, normalized size = 0.94

$$\frac{(b^2x^4 + 2abx^2 + a^2)^{p+1}}{4b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")

[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))

maple [A] time = 0.00, size = 40, normalized size = 1.18

$$\frac{(bx^2 + a)^2 (b^2x^4 + 2abx^2 + a^2)^p}{4(p+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)`

[Out] `1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p`

maxima [B] time = 0.72, size = 86, normalized size = 2.53

$$\frac{(bx^2 + a)(bx^2 + a)^{2p} a}{2b(2p+1)} + \frac{(b^2(2p+1)x^4 + 2abpx^2 - a^2)(bx^2 + a)^{2p}}{4(2p^2 + 3p + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")`

[Out] `1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)`

mupad [B] time = 0.14, size = 59, normalized size = 1.74

$$(a^2 + 2abx^2 + b^2x^4)^p \left(\frac{a^2}{4b(p+1)} + \frac{ax^2}{2(p+1)} + \frac{bx^4}{4(p+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)`

[Out] `(a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (b*x^4)/(4*(p + 1)))`

sympy [A] time = 9.60, size = 165, normalized size = 4.85

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{ax^2(a^2)^p}{2} & \text{for } b = 0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for } p = -1 \\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
), Eq(p, -1)), (a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a
*b*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b), True))
```

$$3.71 \quad \int x^3 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Optimal. Leaf size=86

$$\frac{(a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{2b^2(2p + 3)} - \frac{a(a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^2(p + 1)}$$

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1249, 770, 21, 43}

$$\frac{(a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{2b^2(2p + 3)} - \frac{a(a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^2(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p)) + ((a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))

Rule 21

```
Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
 \int x^3 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx &= \frac{1}{2} \text{Subst} \left(\int x(a + bx) (a^2 + 2abx + b^2x^2)^p dx, x, x^2 \right) \\
 &= \frac{1}{2} \left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int x(a + bx) (ab + b^2x) dx, x, x^2 \right) \\
 &= \frac{\left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int x(ab + b^2x)^{1+2p} dx, x, x^2 \right)}{2b} \\
 &= \frac{\left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int \left(-\frac{a(ab+b^2x)^{1+2p}}{b} + \frac{a^2(ab+b^2x)^{1+2p}}{b} \right) dx, x, x^2 \right)}{2b} \\
 &= -\frac{a(a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^2(1 + p)} + \frac{(a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{2b^2(3 + 2p)}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.52

$$\frac{\left((a + bx^2)^2 \right)^{p+1} (2b(p + 1)x^2 - a)}{4b^2(p + 1)(2p + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^3 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
```

fricas [A] time = 0.66, size = 92, normalized size = 1.07

$$\frac{(2(b^3p + b^3)x^6 + 2a^2bpx^2 + (4ab^2p + 3ab^2)x^4 - a^3)(b^2x^4 + 2abx^2 + a^2)^p}{4(2b^2p^2 + 5b^2p + 3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")

[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)

giac [B] time = 0.43, size = 196, normalized size = 2.28

$$\frac{2(b^2x^4 + 2abx^2 + a^2)^p b^3 p x^6 + 2(b^2x^4 + 2abx^2 + a^2)^p b^3 x^6 + 4(b^2x^4 + 2abx^2 + a^2)^p ab^2 p x^4 + 3(b^2x^4 + 2abx^2 + a^2)^p ab^2 x^4 + 2(b^2x^4 + 2abx^2 + a^2)^p a^2 b p x^2 - (b^2x^4 + 2abx^2 + a^2)^p a^3}{4(2b^2p^2 + 5b^2p + 3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")

[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 - (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)

maple [A] time = 0.01, size = 62, normalized size = 0.72

$$\frac{(-2x^2pb - 2bx^2 + a)(bx^2 + a)^2(b^2x^4 + 2abx^2 + a^2)^p}{4(2p^2 + 5p + 3)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)

[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^2+5*p+3)

maxima [A] time = 0.74, size = 135, normalized size = 1.57

$$\frac{(b^2(2p+1)x^4 + 2abpx^2 - a^2)(bx^2 + a)^{2p} a}{4(2p^2 + 3p + 1)b^2} + \frac{((2p^2 + 3p + 1)b^3x^6 + (2p^2 + p)ab^2x^4 - 2a^2bpx^2 + a^3)(bx^2 + a)^{2p}}{2(4p^3 + 12p^2 + 11p + 3)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")

[Out] $\frac{1}{4}(b^2(2p+1)x^4 + 2abpx^2 - a^2)(bx^2 + a)^{(2p)a}/((2p^2 + 3p + 1)b^2) + \frac{1}{2}((2p^2 + 3p + 1)b^3x^6 + (2p^2 + p)ab^2x^4 - 2a^2bpx^2 + a^3)(bx^2 + a)^{(2p)}/((4p^3 + 12p^2 + 11p + 3)b^2)$

mupad [B] time = 0.17, size = 108, normalized size = 1.26

$$(a^2 + 2abx^2 + b^2x^4)^p \left(\frac{bx^6(p+1)}{2(2p^2 + 5p + 3)} - \frac{a^3}{4b^2(2p^2 + 5p + 3)} + \frac{ax^4(4p+3)}{4(2p^2 + 5p + 3)} + \frac{a^2px^2}{2b(2p^2 + 5p + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3(a + bx^2)(a^2 + b^2x^4 + 2abx^2)^p, x)$

[Out] $(a^2 + b^2x^4 + 2abx^2)^p \left(\frac{bx^6(p+1)}{2(5p + 2p^2 + 3)} - \frac{a^3}{4b^2(5p + 2p^2 + 3)} + \frac{ax^4(4p+3)}{4(5p + 2p^2 + 3)} + \frac{a^2px^2}{2b(5p + 2p^2 + 3)} \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^4(a^2)^p}{4} & \text{for } b = 0 \\ \int \frac{x^3(a+bx^2)}{(a+bx^2)^{\frac{3}{2}}} dx & \text{for } p = -\frac{3}{2} \\ -\frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}}+x)}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}}+x)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} + \frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} + \frac{4ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} + \frac{3ab^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} + \frac{2b^3px^6(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} + \frac{2b^3x^6(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p, x)$

[Out] $\text{Piecewise}((a*x**4*(a**2)**p/4, \text{Eq}(b, 0)), (\text{Integral}(x**3*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x), \text{Eq}(p, -3/2)), (-a*\log(-I*\text{sqrt}(a)*\text{sqrt}(1/b) + x)/(2*b**2) - a*\log(I*\text{sqrt}(a)*\text{sqrt}(1/b) + x)/(2*b**2) + x**2/(2*b), \text{Eq}(p, -1)), (-a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*a**2*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), \text{True}))$

$$3.72 \quad \int x^5 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Optimal. Leaf size=128

$$\frac{(a + bx^2)^4 (a^2 + 2abx^2 + b^2x^4)^p}{4b^3(p + 2)} - \frac{a(a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{b^3(2p + 3)} + \frac{a^2 (a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^3(p + 1)}$$

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1249, 770, 21, 43}

$$\frac{(a + bx^2)^4 (a^2 + 2abx^2 + b^2x^4)^p}{4b^3(p + 2)} - \frac{a(a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{b^3(2p + 3)} + \frac{a^2 (a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^3(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 770

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (
c_)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1249

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
 \int x^5 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx &= \frac{1}{2} \text{Subst} \left(\int x^2 (a + bx) (a^2 + 2abx + b^2x^2)^p dx, x, x^2 \right) \\
 &= \frac{1}{2} \left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int x^2 (a + bx) (ab + b^2x) (a^2 + 2abx + b^2x^2)^p dx, x, x^2 \right) \\
 &= \frac{\left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int x^2 (ab + b^2x)^{1+2p} dx, x, x^2 \right)}{2b} \\
 &= \frac{\left((b(a + bx^2))^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst} \left(\int \left(\frac{a^2(ab+b^2x)^{1+2p}}{b^2} - \frac{2a(ab+b^2x)^{1+2p}}{b} \right) dx, x, x^2 \right)}{2b} \\
 &= \frac{a^2 (a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^3(1+p)} - \frac{a (a + bx^2)^3 (a^2 + 2abx^2 + b^2x^4)^p}{b^3(3+2p)}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.53

$$\frac{\left((a + bx^2)^2 \right)^{p+1} (a^2 - 2ab(p+1)x^2 + b^2(2p^2 + 5p + 3)x^4)}{4b^3(p+1)(p+2)(2p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (((a + b*x^2)^2)^(p+1)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))

IntegrateAlgebraic [F] time = 0.42, size = 0, normalized size = 0.00

$$\int x^5 (a + bx^2) (a^2 + 2abx^2 + b^2x^4)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] Defer[IntegrateAlgebraic][x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p, x]
fricas [A] time = 0.77, size = 140, normalized size = 1.09

$$\frac{\left((2b^4p^2 + 5b^4p + 3b^4)x^8 - 2a^3bpx^2 + 4(ab^3p^2 + 2ab^3p + ab^3)x^6 + (2a^2b^2p^2 + a^2b^2p)x^4 + a^4\right)(b^2x^4 + 2abx^2 + a^2)^p}{4(2b^3p^3 + 9b^3p^2 + 13b^3p + 6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")

[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)

giac [B] time = 0.38, size = 331, normalized size = 2.59

$$\frac{2(b^2x^4 + 2abx^2 + a^2)^p b^4 p^2 x^8 + 5(b^2x^4 + 2abx^2 + a^2)^p b^4 p x^8 + 4(b^2x^4 + 2abx^2 + a^2)^p b^3 p^2 x^6 + 3(b^2x^4 + 2abx^2 + a^2)^p b^3 p x^6 + 2(b^2x^4 + 2abx^2 + a^2)^p b^2 p^2 x^4 + 4(b^2x^4 + 2abx^2 + a^2)^p b^2 p x^4 + (b^2x^4 + 2abx^2 + a^2)^p a^2 b^2 p x^4 - 2(b^2x^4 + 2abx^2 + a^2)^p a^2 b p x^2 + (b^2x^4 + 2abx^2 + a^2)^p a^3 b p x^2 + (b^2x^4 + 2abx^2 + a^2)^p a^4}{4(2b^3p^3 + 9b^3p^2 + 13b^3p + 6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")

[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)

maple [A] time = 0.01, size = 99, normalized size = 0.77

$$\frac{(bx^2 + a)^2 (2b^2p^2x^4 + 5b^2px^4 + 3b^2x^4 - 2abpx^2 - 2abx^2 + a^2) (b^2x^4 + 2abx^2 + a^2)^p}{4(2p^3 + 9p^2 + 13p + 6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)

[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)

maxima [A] time = 0.70, size = 196, normalized size = 1.53

$$\frac{((2p^2 + 3p + 1)b^3x^6 + (2p^2 + p)ab^2x^4 - 2a^2bpx^2 + a^3)(bx^2 + a)^{2p}}{2(4p^3 + 12p^2 + 11p + 3)b^3} + \frac{((4p^3 + 12p^2 + 11p + 3)b^4x^8 + 2(2p^3 + 3p^2 + p)ab^3x^6 - 3(2p^2 + p)a^2b^2x^4 + 6a^3bpx^2 - 3a^4)(bx^2 + a)^{2p}}{4(4p^4 + 20p^3 + 35p^2 + 25p + 6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")

[Out] $\frac{1}{2} * ((2 * p^2 + 3 * p + 1) * b^3 * x^6 + (2 * p^2 + p) * a * b^2 * x^4 - 2 * a^2 * b * p * x^2 + a^3) * (b * x^2 + a)^{(2 * p)} * a / ((4 * p^3 + 12 * p^2 + 11 * p + 3) * b^3) + \frac{1}{4} * ((4 * p^3 + 12 * p^2 + 11 * p + 3) * b^4 * x^8 + 2 * (2 * p^3 + 3 * p^2 + p) * a * b^3 * x^6 - 3 * (2 * p^2 + p) * a^2 * b^2 * x^4 + 6 * a^3 * b * p * x^2 - 3 * a^4) * (b * x^2 + a)^{(2 * p)} / ((4 * p^4 + 20 * p^3 + 35 * p^2 + 25 * p + 6) * b^3)$

mupad [B] time = 0.20, size = 169, normalized size = 1.32

$$(a^2 + 2abx^2 + b^2x^4)^p \left(\frac{a^4}{4b^3(2p^3 + 9p^2 + 13p + 6)} + \frac{ax^6(p+1)^2}{2p^3 + 9p^2 + 13p + 6} + \frac{bx^8(2p^2 + 5p + 3)}{4(2p^3 + 9p^2 + 13p + 6)} - \frac{a^3px^2}{2b^2(2p^3 + 9p^2 + 13p + 6)} + \frac{a^2px^4(2p+1)}{4b(2p^3 + 9p^2 + 13p + 6)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)

[Out] $(a^2 + b^2 * x^4 + 2 * a * b * x^2)^p * (a^4 / (4 * b^3 * (13 * p + 9 * p^2 + 2 * p^3 + 6)) + (a * x^6 * (p + 1)^2) / (13 * p + 9 * p^2 + 2 * p^3 + 6) + (b * x^8 * (5 * p + 2 * p^2 + 3)) / (4 * (13 * p + 9 * p^2 + 2 * p^3 + 6)) - (a^3 * p * x^2) / (2 * b^2 * (13 * p + 9 * p^2 + 2 * p^3 + 6)) + (a^2 * p * x^4 * (2 * p + 1)) / (4 * b * (13 * p + 9 * p^2 + 2 * p^3 + 6)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{aligned} & \frac{ax^6(x^2)^p}{6} && \text{for } b = 0 \\ & \frac{2x^2 \log(-i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{2x^2 \log(i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{3x^2}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{4abx^2 \log(-i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{4abx^2 \log(i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{4abx^2}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{2b^2x^4 \log(-i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} + \frac{2b^2x^4 \log(i\sqrt{a}\sqrt{\frac{x}{b}+1})}{4a^2b^3+8a^4x^2+4b^5x^4} && \text{for } p = -2 \\ & \int \frac{x^5(a+bx^2)^p}{(a+bx^2)^{\frac{3}{2}}} dx && \text{for } p = -\frac{3}{2} \\ & \frac{x^2 \log(-i\sqrt{a}\sqrt{\frac{x}{b}+1})}{2b^3} + \frac{x^2 \log(i\sqrt{a}\sqrt{\frac{x}{b}+1})}{2b^3} - \frac{ax^2}{3b} + \frac{x^4}{2b} && \text{for } p = -1 \\ & \frac{a^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} - \frac{2a^2b^2(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{2a^2b^2x^2(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{a^4b^2x^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{4ab^2x^2(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{8ab^3px^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{4ab^3x^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{2b^4x^2(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{5b^4px^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} + \frac{3b^4x^4(\sqrt{-2abx^2+1}^2)^p}{8b^3p^4+36b^5p^2+52b^7p+24b^9} && \text{otherwise} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)

[Out] Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -2)), (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b)

```

+ x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3)
- 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3
*p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b
**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + a**2*b**2
*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52
*b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b**3*p*x**6*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b
**3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b
**3*p**2 + 52*b**3*p + 24*b**3) + 2*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 5*b**4*p*x*
**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3
*p + 24*b**3) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3
+ 36*b**3*p**2 + 52*b**3*p + 24*b**3), True))

```

$$3.73 \quad \int x^3 (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=166

$$\frac{1}{4}a^3Ax^4 + \frac{1}{6}a^2x^6(aB+3Ab) + \frac{3}{14}cx^{14}(aBc + Abc + b^2B) + \frac{3}{8}ax^8(A(ac + b^2) + abB) + \frac{1}{12}x^{12}(3aAc^2 + 6abBc + 3a^2c^2)$$

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 765}

$$\frac{1}{6}a^2x^6(aB + 3Ab) + \frac{1}{4}a^3Ax^4 + \frac{1}{12}x^{12}(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{3}{14}cx^{14}(aBc + Abc + b^2B) + \frac{1}{10}x^{10}(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{3}{8}ax^8(A(ac + b^2) + abB) + \frac{1}{16}c^2x^{16}(Ac + 3bB) + \frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18

Rule 765

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int x^3 (A + Bx^2) (a + bx^2 + cx^4)^3 dx &= \frac{1}{2} \text{Subst} \left(\int x(A + Bx) (a + bx + cx^2)^3 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (a^3Ax + a^2(3Ab + aB)x^2 + 3a(abB + A(b^2 + ac)))x^3 + (3a^2b^2 + 6abBc + 3a^2c^2)x^5 + (3a^2b^2c + 6abBc^2 + 3a^2c^3)x^7 + (3a^2b^2c^2 + 6abBc^3 + 3a^2c^4)x^9 + (3a^2b^2c^3 + 6abBc^4 + 3a^2c^5)x^{11} + (3a^2b^2c^4 + 6abBc^5 + 3a^2c^6)x^{13} + (3a^2b^2c^5 + 6abBc^6 + 3a^2c^7)x^{15} + (3a^2b^2c^6 + 6abBc^7 + 3a^2c^8)x^{17} + (3a^2b^2c^7 + 6abBc^8 + 3a^2c^9)x^{19} dx, x, x^2 \right) \\ &= \frac{1}{4}a^3Ax^4 + \frac{1}{6}a^2(3Ab + aB)x^6 + \frac{3}{8}a(abB + A(b^2 + ac))x^8 + \frac{1}{10}(3a^2b^2c + 6abBc^2 + 3a^2c^3)x^{10} + \frac{1}{12}(3a^2b^2c^2 + 6abBc^3 + 3a^2c^4)x^{12} + \frac{1}{14}(3a^2b^2c^3 + 6abBc^4 + 3a^2c^5)x^{14} + \frac{1}{16}(3a^2b^2c^4 + 6abBc^5 + 3a^2c^6)x^{16} + \frac{1}{18}(3a^2b^2c^5 + 6abBc^6 + 3a^2c^7)x^{18} \end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4 + \frac{1}{6}a^2x^6(aB + 3Ab) + \frac{3}{14}cx^{14}(aBc + Abc + b^2B) + \frac{3}{5}ax^8(A(ac + b^2) + abB) + \frac{1}{12}x^{12}(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{1}{10}x^{10}(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{16}c^2x^{16}(Ac + 3bB) + \frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [A] time = 0.71, size = 193, normalized size = 1.16

$$\frac{1}{18}x^{18}c^3B + \frac{3}{16}x^{16}c^2bB + \frac{1}{16}x^{16}c^3A + \frac{3}{14}x^{14}c^2bB + \frac{3}{14}x^{14}c^2aB + \frac{3}{14}x^{14}c^2bA + \frac{1}{12}x^{12}b^3B + \frac{1}{2}x^{12}cbaB + \frac{1}{4}x^{12}c^2aA + \frac{1}{4}x^{12}c^2aA + \frac{3}{10}x^{10}b^2aB + \frac{3}{10}x^{10}ca^2B + \frac{1}{10}x^{10}b^3A + \frac{3}{5}x^{10}cbaA + \frac{3}{8}x^8ba^2B + \frac{3}{8}x^8b^2aA + \frac{3}{8}x^8ca^2A + \frac{1}{6}x^6a^3B + \frac{1}{2}x^6ba^2A + \frac{1}{4}x^4a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/18*x^18*c^3*B + 3/16*x^16*c^2*b*B + 1/16*x^16*c^3*A + 3/14*x^14*c^2*b*B + 3/14*x^14*c^2*a*B + 3/14*x^14*c^2*b*A + 1/12*x^12*b^3*B + 1/2*x^12*c*b*a*B + 1/4*x^12*c*b^2*A + 1/4*x^12*c^2*a*A + 3/10*x^10*b^2*a*B + 3/10*x^10*c*a^2*B + 1/10*x^10*b^3*A + 3/5*x^10*c*b*a*A + 3/8*x^8*b*a^2*B + 3/8*x^8*b^2*a*A + 3/8*x^8*c*a^2*A + 1/6*x^6*a^3*B + 1/2*x^6*b*a^2*A + 1/4*x^4*a^3*A

giac [A] time = 0.34, size = 193, normalized size = 1.16

$$\frac{1}{18}Bc^3x^{18} + \frac{3}{16}Bbc^2x^{16} + \frac{1}{16}Ac^3x^{16} + \frac{3}{14}Bb^2cx^{14} + \frac{3}{14}Bac^2x^{14} + \frac{3}{14}Abc^2x^{14} + \frac{1}{12}Bb^3x^{12} + \frac{1}{2}Babcx^{12} + \frac{1}{4}Ab^2cx^{12} + \frac{1}{4}Aac^2x^{12} + \frac{3}{10}Bab^2x^{10} + \frac{1}{10}Ab^3x^{10} + \frac{3}{10}Ba^2cx^{10} + \frac{3}{5}Aabcx^{10} + \frac{3}{8}Ba^2bx^8 + \frac{3}{8}Aab^2x^8 + \frac{3}{8}Aa^2cx^8 + \frac{1}{6}Ba^3x^6 + \frac{1}{2}Aa^2bx^6 + \frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 + 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12

$$+ 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^10 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4$$

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{18}}{18} + \frac{(Ac^3 + 3Bbc^2)x^{16}}{16} + \frac{(3Abc^2 + (a^2c + 2b^2c + (2ac + b^2)c)B)x^{14}}{14} + \frac{((a^2c + 2b^2c + (2ac + b^2)c)A + (4abc + (2ac + b^2)b)B)x^{12}}{12} + \frac{((4abc + (2ac + b^2)b)A + (a^2c + 2ab^2 + (2ac + b^2)a)B)x^{10}}{10} + \frac{Aa^3x^4}{4} + \frac{(3Ba^2b + (a^2c + 2ab^2 + (2ac + b^2)a)A)x^8}{8} + \frac{(3Aa^2b + Ba^3)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)

$$[Out] 1/18*B*c^3*x^18 + 1/16*(A*c^3 + 3*B*b*c^2)*x^16 + 1/14*(3*A*b*c^2 + B*(a*c^2 + 2*b^2*c + c*(2*a*c + b^2)))*x^14 + 1/12*(A*(a*c^2 + 2*b^2*c + c*(2*a*c + b^2)) + B*(4*a*b*c + b*(2*a*c + b^2)))*x^12 + 1/10*(A*(4*a*b*c + b*(2*a*c + b^2)) + B*(a*(2*a*c + b^2) + 2*b^2*a + c*a^2))*x^10 + 1/8*(A*(a*(2*a*c + b^2) + 2*b^2*a + c*a^2) + 3*B*a^2*b)*x^8 + 1/6*(3*A*a^2*b + B*a^3)*x^6 + 1/4*a^3*A*x^4$$

maxima [A] time = 0.60, size = 166, normalized size = 1.00

$$\frac{1}{18}Bc^3x^{18} + \frac{1}{16}(3Bbc^2 + Ac^3)x^{16} + \frac{3}{14}(Bb^2c + (Ba + Ab)c)x^{14} + \frac{1}{12}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^{12} + \frac{1}{10}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^{10} + \frac{3}{8}(Ba^2b + Aab^2 + Aa^2c)x^8 + \frac{1}{4}Aa^3x^4 + \frac{1}{6}(Ba^3 + 3Aa^2b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

$$[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6$$

mupad [B] time = 0.08, size = 169, normalized size = 1.02

$$x^{10} \left(\frac{3Bca^2}{10} + \frac{3Bab^2}{10} + \frac{3Acab}{5} + \frac{Ab^3}{10} \right) + x^{12} \left(\frac{Bb^3}{12} + \frac{Ab^2c}{4} + \frac{Babc}{2} + \frac{Aac^2}{4} \right) + x^6 \left(\frac{Ba^3}{6} + \frac{Aba^2}{2} \right) + x^{16} \left(\frac{Ac^3}{16} + \frac{3Bbc^2}{16} \right) + x^8 \left(\frac{3Ba^2b}{8} + \frac{3Aca^2}{8} + \frac{3Aab^2}{8} \right) + x^{14} \left(\frac{3Bb^2c}{14} + \frac{3Aab^2}{14} + \frac{3Ba^2c}{14} \right) + \frac{Aa^3x^4}{4} + \frac{Bc^3x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)

$$[Out] x^{10} * ((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^{12} * ((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6 * ((B*a^3)/6 + (A*a^2*b)/2) + x^{16} * ((A*c^3)/16 + (3*B*b*c^2)/16) + x^8 * ((3*A*a*b^2)/8 + (3*A*a^2*c)/8 + (3*B*a^2*b)/8) + x^{14} * ((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (3*B*b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18$$

sympy [A] time = 0.10, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4} + \frac{Bc^3x^{18}}{18} + x^{16} \left(\frac{Ac^3}{16} + \frac{3Bbc^2}{16} \right) + x^{14} \left(\frac{3Abc^2}{14} + \frac{3Bac^2}{14} + \frac{3Bb^2c}{14} \right) + x^{12} \left(\frac{Aac^2}{4} + \frac{Ab^2c}{4} + \frac{Babc}{2} + \frac{Bb^3}{12} \right) + x^{10} \left(\frac{3Aabc}{5} + \frac{Ab^3}{10} + \frac{3Ba^2c}{10} + \frac{3Bab^2}{10} \right) + x^8 \left(\frac{3Aa^2c}{8} + \frac{3Aab^2}{8} + \frac{3Ba^2b}{8} \right) + x^6 \left(\frac{Aa^2b}{2} + \frac{Ba^3}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)`

[Out] $A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b**2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8) + x**6*(A*a**2*b/2 + B*a**3/6)$

$$3.74 \quad \int x^2 (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=166

$$\frac{1}{3}a^3Ax^3 + \frac{1}{5}a^2x^5(aB+3Ab) + \frac{3}{13}cx^{13}(aBc + Abc + b^2B) + \frac{3}{7}ax^7(A(ac + b^2) + abB) + \frac{1}{11}x^{11}(3aAc^2 + 6abBc + 3a^2c^2)$$

Rubi [A] time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$\frac{1}{5}a^2x^5(aB+3Ab) + \frac{1}{3}a^3Ax^3 + \frac{1}{11}x^{11}(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{3}{13}cx^{13}(aBc + Abc + b^2B) + \frac{1}{9}x^9(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{3}{7}ax^7(A(ac + b^2) + abB) + \frac{1}{15}c^2x^{15}(Ac + 3bB) + \frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int x^2 (A + Bx^2) (a + bx^2 + cx^4)^3 dx &= \int (a^3Ax^2 + a^2(3Ab + aB)x^4 + 3a(abB + A(b^2 + ac))x^6 + (3aB(b^2 + ac) + a^2c^2)x^8 + (3a^2cx^2 + 6abBc + 3a^2c^2)x^{10} + (3a^3cx^4 + 6abBc^2 + 3a^2c^3)x^{12} + 3a^3cx^6 + 6abBc^3 + 3a^2c^4)x^{14} + Bc^3x^{16}) dx \\ &= \frac{1}{3}a^3Ax^3 + \frac{1}{5}a^2(3Ab + aB)x^5 + \frac{3}{7}a(abB + A(b^2 + ac))x^7 + \frac{1}{9}(3aB(b^2 + ac) + a^2c^2)x^9 + \frac{1}{11}(3a^2cx^2 + 6abBc + 3a^2c^2)x^{11} + \frac{1}{13}(3a^3cx^4 + 6abBc^2 + 3a^2c^3)x^{13} + \frac{1}{15}3a^3cx^6 + \frac{1}{17}(6abBc^3 + 3a^2c^4)x^{15} + \frac{1}{17}Bc^3x^{17} \end{aligned}$$

Mathematica [A] time = 0.05, size = 166, normalized size = 1.00

$$\frac{1}{3}a^3Ax^3 + \frac{1}{5}a^2x^5(aB+3Ab) + \frac{3}{13}cx^{13}(aBc + Abc + b^2B) + \frac{3}{7}ax^7(A(ac + b^2) + abB) + \frac{1}{11}x^{11}(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{1}{9}x^9(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{15}c^2x^{15}(Ac + 3bB) + \frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] $(a^3Ax^3)/3 + (a^2(3Ab + aB)x^5)/5 + (3a(aB + A(b^2 + ac))x^7)/7 + ((3aB(b^2 + ac) + A(b^3 + 6ab^2c))x^9)/9 + ((b^3B + 3Ab^2c + 6aBb^2c + 3aAc^2)x^{11})/11 + (3c(b^2B + Ab^2c + aBc)x^{13})/13 + (c^2(3bB + Ac)x^{15})/15 + (Bc^3x^{17})/17$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [A] time = 0.61, size = 193, normalized size = 1.16

$$\frac{1}{17}x^{17}c^3B + \frac{1}{5}x^{15}c^2bB + \frac{1}{15}x^{15}c^3A + \frac{3}{13}x^{13}c^2B + \frac{3}{13}x^{13}c^2aB + \frac{3}{13}x^{13}c^2bA + \frac{1}{11}x^{11}b^3B + \frac{6}{11}x^{11}c^2aB + \frac{3}{11}x^{11}c^2A + \frac{3}{11}x^{11}c^2aA + \frac{1}{3}x^9b^2aB + \frac{1}{3}x^9c^2B + \frac{1}{3}x^9b^3A + \frac{2}{3}x^9c^2aA + \frac{3}{7}x^7b^2B + \frac{3}{7}x^7c^2aA + \frac{3}{7}x^7c^2A + \frac{1}{5}x^5a^3B + \frac{3}{5}x^5b^2A + \frac{1}{3}x^3a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] $1/17*x^{17}*c^3*B + 1/5*x^{15}*c^2*b*B + 1/15*x^{15}*c^3*A + 3/13*x^{13}*c^2*b*B + 3/13*x^{13}*c^2*a*B + 3/13*x^{13}*c^2*b*A + 1/11*x^{11}*b^3*B + 6/11*x^{11}*c^2*b*A + 3/11*x^{11}*c^2*a*A + 1/3*x^9*b^2*a*B + 1/3*x^9*c^2*a*A + 1/9*x^9*b^3*A + 2/3*x^9*c^2*b*A + 3/7*x^7*b^2*a*B + 3/7*x^7*c^2*a*A + 3/7*x^7*c^2*a*A + 1/5*x^5*a^3*B + 3/5*x^5*b^2*a*A + 1/3*x^3*a^3*A$

giac [A] time = 0.27, size = 193, normalized size = 1.16

$$\frac{1}{17}Bc^3x^{17} + \frac{1}{5}Bbc^2x^{15} + \frac{1}{15}Ac^3x^{15} + \frac{3}{13}Bc^2cx^{13} + \frac{3}{13}Bac^2x^{13} + \frac{3}{13}Abc^2x^{13} + \frac{1}{11}Bb^3x^{11} + \frac{6}{11}Babcx^{11} + \frac{3}{11}Ab^2cx^{11} + \frac{3}{11}Aac^2x^{11} + \frac{1}{3}Bab^2x^9 + \frac{1}{3}Ab^3x^9 + \frac{1}{3}Ba^2cx^9 + \frac{2}{3}Aabcx^9 + \frac{3}{7}Ba^2bx^7 + \frac{3}{7}Aab^2x^7 + \frac{3}{7}Aa^2cx^7 + \frac{1}{5}Ba^3x^5 + \frac{3}{5}Aa^2bx^5 + \frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $1/17*B*c^3*x^{17} + 1/5*B*b*c^2*x^{15} + 1/15*A*c^3*x^{15} + 3/13*B*b^2*c*x^{13} + 3/13*B*a*c^2*x^{13} + 3/13*A*b*c^2*x^{13} + 1/11*B*b^3*x^{11} + 6/11*B*a*b*c*x^{11} + 3/11*A*b^2*c*x^{11} + 3/11*A*a*c^2*x^{11} + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9 + 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3$

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{17}}{17} + \frac{(Ac^3 + 3Bbc^2)x^{15}}{15} + \frac{(3Ab^2c^2 + (a^2 + 2b^2c + (2ac + b^2)c)B)x^{13}}{13} + \frac{((a^2 + 2b^2c + (2ac + b^2)c)A + (4abc + (2ac + b^2)b)B)x^{11}}{11} + \frac{((4abc + (2ac + b^2)b)A + (a^2c + 2ab^2 + (2ac + b^2)a)B)x^9}{9} + \frac{Aa^3x^5}{3} + \frac{(3Ba^2b + (a^2c + 2ab^2 + (2ac + b^2)a)A)x^7}{7} + \frac{(3Aa^2b + Ba^3)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3, x)$

[Out] $\frac{1}{17}B*c^3*x^{17} + \frac{1}{15}*(A*c^3 + 3*B*b*c^2)*x^{15} + \frac{1}{13}*(3*A*b*c^2 + (a*c^2 + 2*b^2*c + (2*a*c + b^2)*c)*B)*x^{13} + \frac{1}{11}*((a*c^2 + 2*b^2*c + (2*a*c + b^2)*c)*A + (4*a*b*c + (2*a*c + b^2)*b)*B)*x^{11} + \frac{1}{9}*((4*a*b*c + (2*a*c + b^2)*b)*A + (a^2*c + 2*a*b^2 + (2*a*c + b^2)*a)*B)*x^9 + \frac{1}{7}*(3*B*a^2*b + (a^2*c + 2*a*b^2 + (2*a*c + b^2)*a)*A)*x^7 + \frac{1}{5}*(3*A*a^2*b + B*a^3)*x^5 + \frac{1}{3}a^3*A*x^3$

maxima [A] time = 0.73, size = 166, normalized size = 1.00

$$\frac{1}{17}Bc^3x^{17} + \frac{1}{15}(3Bbc^2 + Ac^3)x^{15} + \frac{3}{13}(Bb^2c + (Ba + Ab)c^2)x^{13} + \frac{1}{11}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^{11} + \frac{1}{9}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^9 + \frac{3}{7}(Ba^2b + Aab^2 + Aa^2c)x^7 + \frac{1}{3}Aa^3x^3 + \frac{1}{5}(Ba^3 + 3Aa^2b)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{17}B*c^3*x^{17} + \frac{1}{15}*(3*B*b*c^2 + A*c^3)*x^{15} + \frac{3}{13}*(B*b^2*c + (B*a + A*b)*c^2)*x^{13} + \frac{1}{11}*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^{11} + \frac{1}{9}*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + \frac{3}{7}*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^7 + \frac{1}{3}A*a^3*x^3 + \frac{1}{5}*(B*a^3 + 3*A*a^2*b)*x^5$

mupad [B] time = 0.10, size = 169, normalized size = 1.02

$$x^9 \left(\frac{Bc^2}{3} + \frac{Bab^2}{3} + \frac{2Acab}{3} + \frac{Ab^3}{9} \right) + x^{11} \left(\frac{Bb^3}{11} + \frac{3Ab^2c}{11} + \frac{6Babc}{11} + \frac{3Aac^2}{11} \right) + x^5 \left(\frac{Ba^3}{5} + \frac{3Ab^2a^2}{5} \right) + x^{15} \left(\frac{Ac^3}{15} + \frac{Bb^2c^2}{5} \right) + x^7 \left(\frac{3Ba^2b}{7} + \frac{3Aca^2}{7} + \frac{3Aab^2}{7} \right) + x^{13} \left(\frac{3Bb^2c}{13} + \frac{3Aab^2}{13} + \frac{3Ba^2c}{13} \right) + \frac{Aa^3x^3}{3} + \frac{Bc^3x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x)$

[Out] $x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^{11}*((B*b^3)/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5 + (3*A*a^2*b)/5) + x^{15}*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 + (3*A*a^2*c)/7 + (3*B*a^2*b)/7) + x^{13}*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (3*B*b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^{17})/17$

sympy [A] time = 0.10, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3} + \frac{Bc^3x^{17}}{17} + x^{15} \left(\frac{Ac^3}{15} + \frac{Bbc^2}{5} \right) + x^{13} \left(\frac{3Abc^2}{13} + \frac{3Bac^2}{13} + \frac{3Bb^2c}{13} \right) + x^{11} \left(\frac{3Aac^2}{11} + \frac{3Ab^2c}{11} + \frac{6Babc}{11} + \frac{Bb^3}{11} \right) + x^9 \left(\frac{2Aabc}{3} + \frac{Ab^3}{9} + \frac{Ba^2c}{3} + \frac{Bab^2}{3} \right) + x^7 \left(\frac{3Aa^2c}{7} + \frac{3Aab^2}{7} + \frac{3Ba^2b}{7} \right) + x^5 \left(\frac{3Aa^2b}{5} + \frac{Ba^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3, x)$

[Out] $A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a**2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) + x**5*(3*A*a**2*b/5 + B*a**3/5)$

$$3.75 \quad \int x (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=166

$$\frac{1}{2}a^3Ax^2 + \frac{1}{4}a^2x^4(aB+3Ab) + \frac{1}{4}cx^{12}(aBc + Abc + b^2B) + \frac{1}{2}ax^6(A(ac + b^2) + abB) + \frac{1}{10}x^{10}(3aAc^2 + 6abBc + 3Ab^2c)$$

Rubi [A] time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1247, 631}

$$\frac{1}{4}a^2x^4(aB + 3Ab) + \frac{1}{2}a^3Ax^2 + \frac{1}{10}x^{10}(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{1}{4}cx^{12}(aBc + Abc + b^2B) + \frac{1}{8}x^8(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{2}ax^6(A(ac + b^2) + abB) + \frac{1}{14}c^2x^{14}(Ac + 3bB) + \frac{1}{16}Bc^3x^{16}$$

Antiderivative was successfully verified.

[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (A + Bx^2) (a + bx^2 + cx^4)^3 dx &= \frac{1}{2} \text{Subst} \left(\int (A + Bx) (a + bx + cx^2)^3 dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (a^3A + a^2(3Ab + aB)x + 3a(abB + A(b^2 + ac)))x^2 + (3aB(b^2 + a^2c))x^4 + (3a^2B(b^2 + a^2c) + 3a^2B^2)x^6 + 3a^2B^2cx^8 dx, x, x^2 \right) \\ &= \frac{1}{2}a^3Ax^2 + \frac{1}{4}a^2(3Ab + aB)x^4 + \frac{1}{2}a(abB + A(b^2 + ac))x^6 + \frac{1}{8}(3aB(b^2 + a^2c) + 3a^2B^2)x^8 \end{aligned}$$

Mathematica [A] time = 0.06, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2(280a^3A + 140a^2x^2(aB + 3Ab) + 140cx^{10}(aBc + Abc + b^2B) + 280ax^4(A(ac + b^2) + abB) + 56x^8(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + 70x^6(A(6abc + b^3) + 3aB(ac + b^2)) + 40c^2x^{12}(Ac + 3bB) + 35Bc^3x^{14})$$

Antiderivative was successfully verified.

[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14)/560

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(A + Bx^2)(a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [A] time = 0.60, size = 193, normalized size = 1.16

$$\frac{1}{16}x^{16}c^3B + \frac{3}{14}x^{14}c^2bB + \frac{1}{14}x^{14}c^3A + \frac{1}{4}x^{12}cb^2B + \frac{1}{4}x^{12}c^2aB + \frac{1}{4}x^{12}c^2bA + \frac{1}{10}x^{10}b^3B + \frac{3}{5}x^{10}cbaB + \frac{3}{10}x^{10}cb^2A + \frac{3}{10}x^{10}c^2aA + \frac{3}{8}x^8b^2aB + \frac{3}{8}x^8ca^2B + \frac{1}{8}x^8b^3A + \frac{3}{4}x^8cbaA + \frac{1}{2}x^6ba^2B + \frac{1}{2}x^6b^2aA + \frac{1}{2}x^6ca^2A + \frac{1}{4}x^4a^2B + \frac{3}{4}x^4ba^2A + \frac{1}{2}x^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*x^16*c^3*B + 3/14*x^14*c^2*b*B + 1/14*x^14*c^3*A + 1/4*x^12*c*b^2*B + 1/4*x^12*c^2*a*B + 1/4*x^12*c^2*b*A + 1/10*x^10*b^3*B + 3/5*x^10*c*b*a*B + 3/10*x^10*c*b^2*A + 3/10*x^10*c^2*a*A + 3/8*x^8*b^2*a*B + 3/8*x^8*c*a^2*B + 1/8*x^8*b^3*A + 3/4*x^8*c*b*a*A + 1/2*x^6*b*a^2*B + 1/2*x^6*b^2*a*A + 1/2*x^6*c*a^2*A + 1/4*x^4*a^3*B + 3/4*x^4*b*a^2*A + 1/2*x^2*a^3*A

giac [A] time = 0.29, size = 193, normalized size = 1.16

$$\frac{1}{16}Bc^3x^{16} + \frac{3}{14}Bbc^2x^{14} + \frac{1}{14}Ac^3x^{14} + \frac{1}{4}Bb^2cx^{12} + \frac{1}{4}Bac^2x^{12} + \frac{1}{4}Abc^2x^{12} + \frac{1}{10}Bb^3x^{10} + \frac{3}{5}Babcx^{10} + \frac{3}{10}Ab^2cx^{10} + \frac{3}{10}Aac^2x^{10} + \frac{3}{8}Bab^2x^8 + \frac{1}{8}Ab^3x^8 + \frac{3}{8}Ba^2cx^8 + \frac{3}{4}Aabcx^8 + \frac{1}{2}Ba^2bx^6 + \frac{1}{2}Aab^2x^6 + \frac{1}{2}Aa^2cx^6 + \frac{1}{4}Ba^3x^4 + \frac{3}{4}Aa^2bx^4 + \frac{1}{2}Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 + 1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +

$$3/10*A*b^2*c*x^{10} + 3/10*A*a*c^2*x^{10} + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2$$

maple [A] time = 0.00, size = 226, normalized size = 1.36

$$\frac{Bc^3x^{16}}{16} + \frac{(Ac^3 + 3Bbc^2)x^{14}}{14} + \frac{(3Ab^2c^2 + (a^2c + 2b^2c + (2ac + b^2)c)B)x^{12}}{12} + \frac{((a^2c + 2b^2c + (2ac + b^2)c)A + (4abc + (2ac + b^2)b)B)x^{10}}{10} + \frac{((4abc + (2ac + b^2)b)A + (a^2c + 2ab^2 + (2ac + b^2)a)B)x^8}{8} + \frac{Aa^3x^6}{2} + \frac{(3Ba^2b + (a^2c + 2ab^2 + (2ac + b^2)a)A)x^6}{6} + \frac{(3Aa^2b + Ba^3)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)

$$[Out] 1/16*B*c^3*x^{16} + 1/14*(A*c^3 + 3*B*b*c^2)*x^{14} + 1/12*(3*A*b*c^2 + (a*c^2 + 2*b^2*c + (2*a*c + b^2)*c)*B)*x^{12} + 1/10*((a*c^2 + 2*b^2*c + (2*a*c + b^2)*c)*A + (4*a*b*c + (2*a*c + b^2)*b)*B)*x^{10} + 1/8*((4*a*b*c + (2*a*c + b^2)*b)*A + (a^2*c + 2*a*b^2 + (2*a*c + b^2)*a)*B)*x^8 + 1/6*(3*B*a^2*b + (a^2*c + 2*a*b^2 + (2*a*c + b^2)*a)*A)*x^6 + 1/4*(3*A*a^2*b + B*a^3)*x^4 + 1/2*a^3*A*x^2$$

maxima [A] time = 0.77, size = 166, normalized size = 1.00

$$\frac{1}{16}Bc^3x^{16} + \frac{1}{14}(3Bbc^2 + Ac^3)x^{14} + \frac{1}{4}(Bb^2c + (Ba + Ab)c^2)x^{12} + \frac{1}{10}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^{10} + \frac{1}{8}(3Bat^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^8 + \frac{1}{2}(Ba^2b + Aab^2 + Aa^2c)x^6 + \frac{1}{2}Aa^3x^2 + \frac{1}{4}(Ba^3 + 3Aa^2b)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

$$[Out] 1/16*B*c^3*x^{16} + 1/14*(3*B*b*c^2 + A*c^3)*x^{14} + 1/4*(B*b^2*c + (B*a + A*b)*c^2)*x^{12} + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^{10} + 1/8*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4$$

mupad [B] time = 0.05, size = 169, normalized size = 1.02

$$x^8 \left(\frac{3Bc^2a^2}{8} + \frac{3Bab^2}{8} + \frac{3Acab}{4} + \frac{Ab^3}{8} \right) + x^{10} \left(\frac{Bb^3}{10} + \frac{3Ab^2c}{10} + \frac{3Babc}{5} + \frac{3Aac^2}{10} \right) + x^4 \left(\frac{Ba^3}{4} + \frac{3Aba^2}{4} \right) + x^{14} \left(\frac{Ac^3}{14} + \frac{3Bbc^2}{14} \right) + x^6 \left(\frac{Ba^2b}{2} + \frac{Ac^2a}{2} + \frac{Aab^2}{2} \right) + x^{12} \left(\frac{Bb^2c}{4} + \frac{Ab^2c}{4} + \frac{Ba^2c}{4} \right) + \frac{Aa^3x^2}{2} + \frac{Bc^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)

$$[Out] x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^{10}*((B*b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)/4 + (3*A*a^2*b)/4) + x^{14}*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/2 + (A*a^2*c)/2 + (B*a^2*b)/2) + x^{12}*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*c)/4) + (A*a^3*x^2)/2 + (B*c^3*x^{16})/16$$

sympy [A] time = 0.10, size = 199, normalized size = 1.20

$$\frac{Aa^3x^2}{2} + \frac{Bc^3x^{16}}{16} + x^{14} \left(\frac{Ac^3}{14} + \frac{3Bbc^2}{14} \right) + x^{12} \left(\frac{Abc^2}{4} + \frac{Bac^2}{4} + \frac{Bb^2c}{4} \right) + x^{10} \left(\frac{3Aac^2}{10} + \frac{3Ab^2c}{10} + \frac{3Babc}{5} + \frac{Bb^3}{10} \right) + x^8 \left(\frac{3Aabc}{4} + \frac{Ab^3}{8} + \frac{3Ba^2c}{8} + \frac{3Bab^2}{8} \right) + x^6 \left(\frac{Aa^2c}{2} + \frac{Aab^2}{2} + \frac{Ba^2b}{2} \right) + x^4 \left(\frac{3Aa^2b}{4} + \frac{Ba^3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)`

[Out] $A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A*a**2*b/4 + B*a**3/4)$

$$3.76 \quad \int (A + Bx^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=161

$$a^3 Ax + \frac{1}{3} a^2 x^3 (aB + 3Ab) + \frac{3}{11} cx^{11} (aBc + Abc + b^2 B) + \frac{3}{5} ax^5 (A(ac + b^2) + abB) + \frac{1}{9} x^9 (3aAc^2 + 6abBc + 3Ab^2c +$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$\frac{1}{3} a^2 x^3 (aB + 3Ab) + a^3 Ax + \frac{1}{9} x^9 (3aAc^2 + 6abBc + 3Ab^2c + b^3 B) + \frac{3}{11} cx^{11} (aBc + Abc + b^2 B) + \frac{1}{7} x^7 (A(6abc + b^3) + 3aB(ac + b^2)) + \frac{3}{5} ax^5 (A(ac + b^2) + abB) + \frac{1}{13} c^2 x^{13} (Ac + 3bB) + \frac{1}{15} Bc^3 x^{15}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (A + Bx^2) (a + bx^2 + cx^4)^3 dx &= \int (a^3 A + a^2(3Ab + aB)x^2 + 3a(abB + A(b^2 + ac))x^4 + (3aB(b^2 + ac) + A^2)x^6 + (3a^2Bc + 6aAb^2 + 3a^2Bc)x^8 + (3a^2Bc + 6aAb^2 + 3a^2Bc)x^{10} + (3a^2Bc + 6aAb^2 + 3a^2Bc)x^{12} + Bc^3x^{14}) dx \\ &= a^3 Ax + \frac{1}{3} a^2 (3Ab + aB)x^3 + \frac{3}{5} a (abB + A(b^2 + ac))x^5 + \frac{1}{7} (3aB(b^2 + ac) + A^2)x^7 + \frac{1}{9} (3a^2Bc + 6aAb^2 + 3a^2Bc)x^9 + \frac{1}{11} (3a^2Bc + 6aAb^2 + 3a^2Bc)x^{11} + \frac{1}{13} Bc^3x^{13} \end{aligned}$$

Mathematica [A] time = 0.05, size = 161, normalized size = 1.00

$$a^3 Ax + \frac{1}{3} a^2 x^3 (aB + 3Ab) + \frac{3}{11} cx^{11} (aBc + Abc + b^2 B) + \frac{3}{5} ax^5 (A(ac + b^2) + abB) + \frac{1}{9} x^9 (3aAc^2 + 6abBc + 3Ab^2c + b^3 B) + \frac{1}{7} x^7 (A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{13} c^2 x^{13} (Ac + 3bB) + \frac{1}{15} Bc^3 x^{15}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] $a^3 A x + (a^2 (3 A b + a B) x^3) / 3 + (3 a (a b B + A (b^2 + a c)) x^5) / 5 + ((3 a B (b^2 + a c) + A (b^3 + 6 a b c)) x^7) / 7 + ((b^3 B + 3 A b^2 c + 6 a b B c + 3 a A c^2) x^9) / 9 + (3 c (b^2 B + A b c + a B c) x^{11}) / 11 + (c^2 (3 b B + A c) x^{13}) / 13 + (B c^3 x^{15}) / 15$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Bx^2)(a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]

[Out] IntegrateAlgebraic[(A + B*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [A] time = 0.76, size = 189, normalized size = 1.17

$$\frac{1}{15}x^{15}c^3B + \frac{3}{13}x^{13}c^2bB + \frac{1}{13}x^{13}c^3A + \frac{3}{11}x^{11}ct^2B + \frac{3}{11}x^{11}c^2aB + \frac{3}{11}x^{11}c^2bA + \frac{1}{9}x^9b^3B + \frac{2}{3}x^9cbaB + \frac{1}{3}x^9ct^2A + \frac{1}{3}x^9c^2aA + \frac{3}{7}x^7b^2aB + \frac{3}{7}x^7ct^2B + \frac{1}{7}x^7b^2A + \frac{6}{7}x^7cbaA + \frac{3}{5}x^5b^2aB + \frac{3}{5}x^5b^2aA + \frac{3}{5}x^5ct^2A + \frac{1}{3}x^3a^3B + x^3ba^2A + xa^3A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] $1/15*x^{15}*c^3*B + 3/13*x^{13}*c^2*b*B + 1/13*x^{13}*c^3*A + 3/11*x^{11}*c*b^2*B + 3/11*x^{11}*c^2*a*B + 3/11*x^{11}*c^2*b*A + 1/9*x^9*b^3*B + 2/3*x^9*c*b*a*B + 1/3*x^9*c*b^2*A + 1/3*x^9*c^2*a*A + 3/7*x^7*b^2*a*B + 3/7*x^7*c*a^2*B + 1/7*x^7*b^3*A + 6/7*x^7*c*b*a*A + 3/5*x^5*b*a^2*B + 3/5*x^5*b^2*a*A + 3/5*x^5*c*a^2*A + 1/3*x^3*a^3*B + x^3*b*a^2*A + x*a^3*A$

giac [A] time = 0.26, size = 189, normalized size = 1.17

$$\frac{1}{15}Bc^3x^{15} + \frac{3}{13}Bbc^2x^{13} + \frac{1}{13}Ac^3x^{13} + \frac{3}{11}Bt^2cx^{11} + \frac{3}{11}Bat^2x^{11} + \frac{3}{11}Abc^2x^{11} + \frac{1}{9}Bb^3x^9 + \frac{2}{3}Babcx^9 + \frac{1}{3}Ab^2cx^9 + \frac{1}{3}Aac^2x^9 + \frac{3}{7}Bab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{3}{7}Ba^2cx^7 + \frac{6}{7}Aabcx^7 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{3}{5}Aa^2cx^5 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $1/15*B*c^3*x^{15} + 3/13*B*b*c^2*x^{13} + 1/13*A*c^3*x^{13} + 3/11*B*b^2*c*x^{11} + 3/11*B*a*c^2*x^{11} + 3/11*A*b*c^2*x^{11} + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 + 1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x$

maple [A] time = 0.00, size = 223, normalized size = 1.39

$$\frac{Bc^3x^{15}}{15} + \frac{(Ac^3 + 3Bb^2c^2)x^{13}}{13} + \frac{(3Abc^2 + (a^2 + 2b^2c + (2ac + b^2)c)B)x^{11}}{11} + \frac{((a^2 + 2b^2c + (2ac + b^2)c)A + (4abc + (2ac + b^2)b)B)x^9}{9} + \frac{((4abc + (2ac + b^2)b)A + (a^2c + 2ab^2 + (2ac + b^2)a)B)x^7}{7} + \frac{(3Bab^2 + (a^2c + 2ab^2 + (2ac + b^2)a)A)x^5}{5} + \frac{(3Aa^2b + B a^3)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x)`

[Out] $1/15*B*c^3*x^{15}+1/13*(A*c^3+3*B*b*c^2)*x^{13}+1/11*(3*A*b*c^2+(a*c^2+2*b^2*c+(2*a*c+b^2)*c)*B)*x^{11}+1/9*((a*c^2+2*b^2*c+(2*a*c+b^2)*c)*A+(4*a*b*c+(2*a*c+b^2)*b)*B)*x^9+1/7*((4*a*b*c+(2*a*c+b^2)*b)*A+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*B)*x^7+1/5*(3*B*a^2*b+(a^2*c+2*a*b^2+(2*a*c+b^2)*a)*A)*x^5+1/3*(3*A*a^2*b+B*a^3)*x^3+a^3*A*x$

maxima [A] time = 0.73, size = 163, normalized size = 1.01

$$\frac{1}{15}Bc^3x^{15} + \frac{1}{13}(3Bbc^2 + Ac^3)x^{13} + \frac{3}{11}(Bb^2c + (Ba + Ab)c^2)x^{11} + \frac{1}{9}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^9 + \frac{1}{7}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^7 + \frac{3}{5}(Ba^2b + Aab^2 + Aa^2c)x^5 + Aa^3x + \frac{1}{3}(Ba^3 + 3Aa^2b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")`

[Out] $1/15*B*c^3*x^{15} + 1/13*(3*B*b*c^2 + A*c^3)*x^{13} + 3/11*(B*b^2*c + (B*a + A*b)*c^2)*x^{11} + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3$

mupad [B] time = 0.05, size = 165, normalized size = 1.02

$$x^7 \left(\frac{3Bca^2}{7} + \frac{3Bab^2}{7} + \frac{6Acab}{7} + \frac{Ab^3}{7} \right) + x^9 \left(\frac{Bb^3}{9} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Aac^2}{3} \right) + x^3 \left(\frac{Ba^3}{3} + Aba^2 \right) + x^{15} \left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13} \right) + x^5 \left(\frac{3Ba^2b}{5} + \frac{3Aca^2}{5} + \frac{3Aab^2}{5} \right) + x^{11} \left(\frac{3Bb^2c}{11} + \frac{3Abc^2}{11} + \frac{3Bac^2}{11} \right) + \frac{Bc^3x^{15}}{15} + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)`

[Out] $x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*b^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^{13}*((B*a^3)/3 + A*a^2*b) + x^{15}*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)/5 + (3*B*a^2*b)/5) + x^{11}*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*c)/11) + (B*c^3*x^{15})/15 + A*a^3*x$

sympy [A] time = 0.10, size = 199, normalized size = 1.24

$$Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13} \left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13} \right) + x^{11} \left(\frac{3Abc^2}{11} + \frac{3Bac^2}{11} + \frac{3Bb^2c}{11} \right) + x^9 \left(\frac{Aac^2}{3} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Bb^3}{9} \right) + x^7 \left(\frac{6Aabc}{7} + \frac{Ab^3}{7} + \frac{3Ba^2c}{7} + \frac{3Bab^2}{7} \right) + x^5 \left(\frac{3Aa^2c}{5} + \frac{3Aab^2}{5} + \frac{3Ba^2b}{5} \right) + x^3 \left(Aa^2b + \frac{Ba^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)`

[Out] $A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 + 3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A*a**2*b + B*a**3/3)$

$$3.77 \quad \int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} dx$$

Optimal. Leaf size=162

$$a^3 A \log(x) + \frac{1}{2} a^2 x^2 (aB + 3Ab) + \frac{3}{10} cx^{10} (aBc + Abc + b^2 B) + \frac{3}{4} ax^4 (A(ac + b^2) + abB) + \frac{1}{8} x^8 (3aAc^2 + 6abBc + 3a^2 c^2)$$

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 765}

$$\frac{1}{2} a^2 x^2 (aB + 3Ab) + a^3 A \log(x) + \frac{1}{8} x^8 (3aAc^2 + 6abBc + 3a^2 c^2) + \frac{3}{10} cx^{10} (aBc + Abc + b^2 B) + \frac{1}{6} x^6 (A(6abc + b^3) + 3aB(ac + b^2)) + \frac{3}{4} ax^4 (A(ac + b^2) + abB) + \frac{1}{12} c^2 x^{12} (Ac + 3bB) + \frac{1}{14} Bc^3 x^{14}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]

[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]

Rule 765

Int[((e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x} dx = \frac{1}{2} \text{Subst} \left(\int \frac{(A + Bx)(a + bx + cx^2)^3}{x} dx, x, x^2 \right)$$

$$= \frac{1}{2} \text{Subst} \left(\int \left(a^2(3Ab + aB) + \frac{a^3A}{x} + 3a(abB + A(b^2 + ac)) \right) x + (3aB(b^2 + ac)) \right) dx, x, x^2$$

$$= \frac{1}{2} a^2(3Ab + aB)x^2 + \frac{3}{4} a(abB + A(b^2 + ac))x^4 + \frac{1}{6} (3aB(b^2 + ac) + A(b^3 + ac^2))x^6 + \frac{1}{12} c^2 x^{12} + \frac{1}{14} Bc^3 x^{14} + A \log(x)$$

Mathematica [A] time = 0.06, size = 162, normalized size = 1.00

$$a^3 A \log(x) + \frac{1}{2} a^2 x^2 (aB + 3Ab) + \frac{3}{10} c x^{10} (aBc + Abc + b^2 B) + \frac{3}{4} a x^4 (A(ac + b^2) + abB) + \frac{1}{8} x^8 (3aAc^2 + 6abBc + 3Ab^2c + b^2 B) + \frac{1}{6} x^6 (A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{12} c^2 x^{12} (Ac + 3bB) + \frac{1}{14} Bc^3 x^{14}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]

[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]

[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x, x]

fricas [A] time = 0.82, size = 164, normalized size = 1.01

$$\frac{1}{14} Bc^3 x^{14} + \frac{1}{12} (3Bbc^2 + Ac^3) x^{12} + \frac{3}{10} (Bb^2c + (Ba + Ab)c^2) x^{10} + \frac{1}{8} (Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c) x^8 + \frac{1}{6} (3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c) x^6 + \frac{3}{4} (Ba^2b + Aab^2 + Aa^2c) x^4 + Aa^3 \log(x) + \frac{1}{2} (Ba^3 + 3Aa^2b) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")

[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3

$*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 + A*a^3*\log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2$

giac [A] time = 0.34, size = 193, normalized size = 1.19

$$\frac{1}{14}Bc^3x^{14} + \frac{1}{4}Bbc^2x^{12} + \frac{1}{12}Ac^3x^{12} + \frac{3}{10}Bb^2cx^{10} + \frac{3}{10}Ba^2x^{10} + \frac{3}{10}Abc^2x^{10} + \frac{1}{8}Bb^3x^8 + \frac{3}{4}Babcx^8 + \frac{3}{8}Ab^2cx^8 + \frac{3}{8}Aac^2x^8 + \frac{1}{2}Bab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{1}{2}Ba^2cx^6 + Aabcx^6 + \frac{3}{4}Ba^2bx^4 + \frac{3}{4}Aab^2x^4 + \frac{3}{4}Aa^2cx^4 + \frac{1}{2}Ba^3x^2 + \frac{3}{2}Aa^2bx^2 + \frac{1}{2}Aa^3\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")

[Out] $1/14*B*c^3*x^{14} + 1/4*B*b*c^2*x^{12} + 1/12*A*c^3*x^{12} + 3/10*B*b^2*c*x^{10} + 3/10*B*a*c^2*x^{10} + 3/10*A*b*c^2*x^{10} + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*\log(x^2)$

maple [A] time = 0.00, size = 191, normalized size = 1.18

$$\frac{Bc^3x^{14}}{14} + \frac{Ac^3x^{12}}{12} + \frac{Bb^2cx^{12}}{4} + \frac{3Ab^2cx^{10}}{10} + \frac{3Ba^2cx^{10}}{10} + \frac{3Bb^3x^8}{8} + \frac{3Aa^2cx^8}{8} + \frac{3Babcx^8}{4} + \frac{Bb^3x^8}{8} + Aabcx^6 + \frac{Ab^3x^6}{6} + \frac{Ba^2cx^6}{2} + \frac{Ba^2bx^4}{2} + \frac{3Aa^2cx^4}{4} + \frac{3Aab^2x^4}{4} + \frac{3Ba^2bx^4}{4} + \frac{3Aa^2bx^2}{2} + \frac{Ba^3x^2}{2} + Aa^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x)

[Out] $1/14*B*c^3*x^{14} + 1/12*A*x^{12}*c^3 + 1/4*B*x^{12}*b*c^2 + 3/10*A*x^{10}*b*c^2 + 3/10*B*x^{10}*a*c^2 + 3/10*B*x^{10}*b^2*c + 3/8*A*x^8*a*c^2 + 3/8*A*x^8*b^2*c + 3/4*B*x^8*a*b*c + 1/8*B*x^8*b^3 + A*x^6*a*b*c + 1/6*A*x^6*b^3 + 1/2*B*x^6*a^2*c + 1/2*B*x^6*a*b^2 + 3/4*A*x^4*a^2*c + 3/4*A*x^4*a*b^2 + 3/4*B*x^4*a^2*b + 3/2*A*x^2*a^2*b + 1/2*B*x^2*a^3 + a^3*A*\ln(x)$

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{14}Bc^3x^{14} + \frac{1}{12}(3Bbc^2 + Ac^3)x^{12} + \frac{3}{10}(Bb^2c + (Ba + Ab)c^2)x^{10} + \frac{1}{8}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^8 + \frac{1}{6}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^6 + \frac{3}{4}(Ba^2b + Aab^2 + Aa^2c)x^4 + \frac{1}{2}Aa^3\log(x^2) + \frac{1}{2}(Ba^3 + 3Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")

[Out] $1/14*B*c^3*x^{14} + 1/12*(3*B*b*c^2 + A*c^3)*x^{12} + 3/10*(B*b^2*c + (B*a + A*b)*c^2)*x^{10} + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 + 1/2*A*a^3*\log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2$

mupad [B] time = 0.10, size = 166, normalized size = 1.02

$$x^6 \left(\frac{Bca^2}{2} + \frac{Bab^2}{2} + Acab + \frac{Ab^3}{6} \right) + x^8 \left(\frac{Bb^3}{8} + \frac{3Ab^2c}{8} + \frac{3Babc}{4} + \frac{3Aac^2}{8} \right) + x^2 \left(\frac{Ba^3}{2} + \frac{3Aba^2}{2} \right) + x^{12} \left(\frac{Ac^3}{12} + \frac{Bb^2c}{4} \right) + x^4 \left(\frac{3Ba^2b}{4} + \frac{3Aca^2}{4} + \frac{3Aab^2}{4} \right) + x^{10} \left(\frac{3Bb^2c}{10} + \frac{3Ab^2c}{10} + \frac{3Ba^2c}{10} \right) + \frac{Bc^3x^{14}}{14} + Aa^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)`

[Out] $x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (3*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^{12}*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4 + (3*B*a^2*b)/4) + x^{10}*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/10) + (B*c^3*x^{14})/14 + A*a^3*\log(x)$

sympy [A] time = 0.31, size = 199, normalized size = 1.23

$$Aa^3 \log(x) + \frac{Bc^3 x^{14}}{14} + x^{12} \left(\frac{Ac^3}{12} + \frac{Bbc^2}{4} \right) + x^{10} \left(\frac{3Abc^2}{10} + \frac{3Bac^2}{10} + \frac{3Bb^2c}{10} \right) + x^8 \left(\frac{3Aa^2c}{8} + \frac{3Ab^2c}{8} + \frac{3Babc}{4} + \frac{Bb^3}{8} \right) + x^6 \left(Aabc + \frac{Ab^3}{6} + \frac{Ba^2c}{2} + \frac{Bab^2}{2} \right) + x^4 \left(\frac{3Aa^2c}{4} + \frac{3Aab^2}{4} + \frac{3Ba^2b}{4} \right) + x^2 \left(\frac{3Aa^2b}{2} + \frac{Ba^3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)`

[Out] $A*a**3*\log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b**2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 + B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A*a**2*b/2 + B*a**3/2)$

$$3.78 \quad \int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2} dx$$

Optimal. Leaf size=156

$$-\frac{a^3 A}{x} + a^2 x(aB + 3Ab) + \frac{1}{3} cx^9 (aBc + Abc + b^2 B) + ax^3 (A(ac + b^2) + abB) + \frac{1}{7} x^7 (3aAc^2 + 6abBc + 3Ab^2c + b^3 B)$$

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$a^2 x(aB + 3Ab) - \frac{a^3 A}{x} + \frac{1}{7} x^7 (3aAc^2 + 6abBc + 3Ab^2c + b^3 B) + \frac{1}{3} cx^9 (aBc + Abc + b^2 B) + \frac{1}{5} x^5 (A(6abc + b^3) + 3aB(ac + b^2)) + ax^3 (A(ac + b^2) + abB) + \frac{1}{11} c^2 x^{11} (Ac + 3bB) + \frac{1}{13} Bc^3 x^{13}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2, x]

[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13

Rule 1261

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x^2} dx &= \int \left(a^2(3Ab + aB) + \frac{a^3 A}{x^2} + 3a(abB + A(b^2 + ac)) \right) x^2 + (3aB(b^2 + ac) + A) x^4 \\ &= -\frac{a^3 A}{x} + a^2(3Ab + aB)x + a(abB + A(b^2 + ac))x^3 + \frac{1}{5}(3aB(b^2 + ac) + A)x^5 \end{aligned}$$

Mathematica [A] time = 0.08, size = 156, normalized size = 1.00

$$-\frac{a^3 A}{x} + a^2 x(aB + 3Ab) + \frac{1}{3} cx^9 (aBc + Abc + b^2 B) + ax^3 (A(ac + b^2) + abB) + \frac{1}{7} x^7 (3aAc^2 + 6abBc + 3Ab^2c + b^3 B) + \frac{1}{5} x^5 (A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{11} c^2 x^{11} (Ac + 3bB) + \frac{1}{13} Bc^3 x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]

[Out] $-\frac{(a^3 A)}{x} + a^2(3A^2 b + a^2 B)x + a^2(a^2 b^2 B + A^2(b^2 + a^2 c))x^3 + ((3a^2 B(b^2 + a^2 c) + A^2(b^3 + 6a^2 b^2 c))x^5)/5 + ((b^3 B + 3A^2 b^2 c + 6a^2 b^2 B c + 3a^2 A^2 c^2)x^7)/7 + (c(b^2 B + A^2 b^2 c + a^2 B^2 c)x^9)/3 + (c^2(3b^2 B + A^2 c)x^{11})/11 + (B^2 c^3 x^{13})/13$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]

[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2, x]

fricas [A] time = 0.96, size = 168, normalized size = 1.08

$\frac{1155 B^3 c^{14} + 1365 (3 B b c^2 + A c^3) x^{12} + 5005 (B^2 c + (B a + A b) c^2) x^{10} + 2145 (B b^3 + 3 A a c^2 + 3 (2 B a b + A b^2) c) x^8 + 3003 (3 B a b^2 + A b^3 + 3 (B a^2 + 2 A a b) c) x^6 + 15015 (B a^2 b + A a^2 c) x^4 - 15015 A a^3 + 15015 (B a^3 + 3 A a^2 b) x^2}{15015 x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")

[Out] $\frac{1}{15015} (1155 B^3 c^3 x^{14} + 1365 (3 B^2 b c^2 + A c^3) x^{12} + 5005 (B^2 b^2 c + (B^2 a + A^2 b) c^2) x^{10} + 2145 (B^2 b^3 + 3 A^2 a c^2 + 3 (2 B^2 a b + A^2 b^2) c) x^8 + 3003 (3 B^2 a b^2 + A^2 b^3 + 3 (B^2 a^2 + 2 A^2 a b) c) x^6 + 15015 (B^2 a^2 b + A^2 a b^2 + A^2 a^2 c) x^4 - 15015 A^2 a^3 + 15015 (B^2 a^3 + 3 A^2 a^2 b) x^2) / x$

giac [A] time = 0.31, size = 185, normalized size = 1.19

$\frac{\frac{1}{13} B^3 c^{13} + \frac{3}{11} B b c^2 x^{11} + \frac{1}{11} A c^3 x^{11} + \frac{1}{3} B b^2 c x^9 + \frac{1}{3} B a c^2 x^9 + \frac{1}{3} A b c^2 x^9 + \frac{1}{7} B b^2 x^7 + \frac{6}{7} B a b c x^7 + \frac{3}{7} A b^2 c x^7 + \frac{3}{5} A a c^2 x^7 + \frac{3}{5} B a^2 x^5 + \frac{1}{5} A b^3 x^5 + \frac{3}{5} B a^2 c x^5 + \frac{6}{5} A a b c x^5 + B a^2 b x^3 + A a^2 c x^3 + B a^3 x + 3 A a^2 b x - \frac{A a^3}{x}}{15015 x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")

[Out] $\frac{1}{13} B^3 c^3 x^{13} + \frac{3}{11} B^2 b c^2 x^{11} + \frac{1}{11} A^2 c^3 x^{11} + \frac{1}{3} B^2 b^2 c x^9 + \frac{1}{3} B^2 a c^2 x^9 + \frac{1}{3} A^2 b c^2 x^9 + \frac{1}{7} B^2 b^3 x^7 + \frac{6}{7} B^2 a b c x^7 + \frac{3}{7} A^2 b^2 c x^7 + \frac{3}{5} B^2 a b^2 x^5 + \frac{1}{5} A^2 b^3 x^5 + \frac{3}{5} B^2 a^2 c x^5 + \frac{6}{5} A^2 a b c x^5 + B^2 a^2 b x^3 + A^2 a b^2 x^3 + A^2 a^2 c x^3 + B^2 a^3 x + 3 A^2 a^2 b x - A^2 a^3 / x$

maple [A] time = 0.00, size = 186, normalized size = 1.19

$\frac{B^3 c^{13} + \frac{A c^3 x^{11}}{11} + \frac{3 B b c^2 x^{11}}{11} + \frac{A b c^2 x^9}{3} + \frac{B a c^2 x^9}{3} + \frac{B b^2 c x^9}{3} + \frac{3 A a c^2 x^7}{7} + \frac{3 A b^2 c x^7}{7} + \frac{6 B a b c x^7}{7} + \frac{B b^3 x^7}{7} + \frac{6 A a b c x^5}{5} + \frac{A b^3 x^5}{5} + \frac{3 B a^2 c x^5}{5} + \frac{3 B a b^2 x^5}{5} + A a^2 c x^3 + A a b^2 x^3 + B a^2 b x^3 + 3 A a^2 b x + B a^3 x - \frac{A a^3}{x}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x)`

[Out] $1/13*B*c^3*x^{13}+1/11*A*x^{11}*c^3+3/11*B*x^{11}*b*c^2+1/3*A*x^9*b*c^2+1/3*B*x^9*a*c^2+1/3*B*x^9*b^2*c+3/7*A*x^7*a*c^2+3/7*A*x^7*b^2*c+6/7*B*x^7*a*b*c+1/7*B*x^7*b^3+6/5*A*x^5*a*b*c+1/5*A*x^5*b^3+3/5*B*x^5*a^2*c+3/5*B*x^5*a*b^2+A*x^3*a^2*c+A*x^3*a*b^2+B*x^3*a^2*b+3*A*a^2*b*x+B*a^3*x-a^3*A/x$

maxima [A] time = 0.71, size = 162, normalized size = 1.04

$$\frac{1}{13}Bc^3x^{13} + \frac{1}{11}(3Bbc^2 + Ac^3)x^{11} + \frac{1}{3}(Bb^2c + (Ba + Ab)c^2)x^9 + \frac{1}{7}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^7 + \frac{1}{5}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^5 + (Ba^2b + Aab^2 + Aa^2c)x^3 - \frac{Aa^3}{x} + (Ba^3 + 3Aa^2b)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")`

[Out] $1/13*B*c^3*x^{13} + 1/11*(3*B*b*c^2 + A*c^3)*x^{11} + 1/3*(B*b^2*c + (B*a + A*b)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x$

mupad [B] time = 0.05, size = 163, normalized size = 1.04

$$x^5 \left(\frac{3Bca^2}{5} + \frac{3Bal^2}{5} + \frac{6Acab}{5} + \frac{Ab^3}{5} \right) + x^7 \left(\frac{Bb^3}{7} + \frac{3Ab^2c}{7} + \frac{6Babc}{7} + \frac{3Aac^2}{7} \right) + x(Ba^3 + 3Aba^2) + x^{11} \left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11} \right) + x^3 (Ba^2b + Aca^2 + Aab^2) + x^9 \left(\frac{Bb^2c}{3} + \frac{Abc^2}{3} + \frac{Ba^2c}{3} \right) - \frac{Aa^3}{x} + \frac{Bc^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)`

[Out] $x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*b^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a^2*b) + x^{11}*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^2*b) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*x^{13})/13$

sympy [A] time = 0.31, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x} + \frac{Bc^3x^{13}}{13} + x^{11} \left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11} \right) + x^9 \left(\frac{Abc^2}{3} + \frac{Ba^2c}{3} + \frac{Bb^2c}{3} \right) + x^7 \left(\frac{3Aac^2}{7} + \frac{3Ab^2c}{7} + \frac{6Babc}{7} + \frac{Bb^3}{7} \right) + x^5 \left(\frac{6Aabc}{5} + \frac{Ab^3}{5} + \frac{3Ba^2c}{5} + \frac{3Bal^2}{5} \right) + x^3 (Aa^2c + Aab^2 + Ba^2b) + x(3Aa^2b + Ba^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)`

[Out] $-A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)$

$$3.79 \quad \int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} dx$$

Optimal. Leaf size=162

$$-\frac{a^3A}{2x^2} + a^2 \log(x)(aB+3Ab) + \frac{3}{8}cx^8(aBc + Abc + b^2B) + \frac{3}{2}ax^2(A(ac + b^2) + abB) + \frac{1}{6}x^6(3aAc^2 + 6abBc + 3Ab^2c)$$

Rubi [A] time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 765}

$$a^2 \log(x)(aB + 3Ab) - \frac{a^3A}{2x^2} + \frac{1}{6}x^6(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{3}{8}cx^8(aBc + Abc + b^2B) + \frac{1}{4}x^4(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{3}{2}ax^2(A(ac + b^2) + abB) + \frac{1}{10}c^2x^{10}(Ac + 3bB) + \frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]

[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + (((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]

Rule 765

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x^3} dx = \frac{1}{2} \text{Subst} \left(\int \frac{(A + Bx)(a + bx + cx^2)^3}{x^2} dx, x, x^2 \right)$$

$$= \frac{1}{2} \text{Subst} \left(\int \left(3a(abB + A(b^2 + ac)) + \frac{a^3A}{x^2} + \frac{a^2(3Ab + aB)}{x} + (3aB(b^2 + ac) + A(b^3 + 6abc))x \right) dx, x, x^2 \right)$$

$$= -\frac{a^3A}{2x^2} + \frac{3}{2}a(abB + A(b^2 + ac))x^2 + \frac{1}{4}(3aB(b^2 + ac) + A(b^3 + 6abc))x^4$$

Mathematica [A] time = 0.07, size = 162, normalized size = 1.00

$$-\frac{a^3A}{2x^2} + a^2 \log(x)(aB + 3Ab) + \frac{3}{8}cx^8(aBc + Abc + b^2B) + \frac{3}{2}ax^2(A(ac + b^2) + abB) + \frac{1}{6}x^6(3aAc^2 + 6abBc + 3Ab^2c + b^3B) + \frac{1}{4}x^4(A(6abc + b^3) + 3aB(ac + b^2)) + \frac{1}{10}c^2x^{10}(Ac + 3bB) + \frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]

[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx^2)(a + bx^2 + cx^4)^3}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]

[Out] IntegrateAlgebraic[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3, x]

fricas [A] time = 0.84, size = 170, normalized size = 1.05

$$\frac{10Bc^3x^{14} + 12(3Bbc^2 + Ac^3)x^{12} + 45(Bb^2c + (Ba + Ab)c^2)x^{10} + 20(Bb^3 + 3Auc^2 + 3(2Bab + Ab^2)c)x^8 + 30(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^6 + 180(Ba^2b + Aab^2 + Aa^2c)x^4 - 60Aa^3 + 120(Ba^3 + 3Aa^2b)x^2 \log(x)}{120x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")

[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*

$B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*\log(x))/x^2$

giac [A] time = 0.40, size = 212, normalized size = 1.31

$$\frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Bac^2x^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + Babcx^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{2}Aac^2x^6 + \frac{3}{4}Bab^2x^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{4}Ba^2cx^4 + \frac{3}{2}Aabcx^4 + \frac{3}{2}Ba^2bx^2 + \frac{3}{2}Aa^2cx^2 + \frac{1}{2}(Ba^3 + 3Aa^2b)\log(x^2) - \frac{Ba^3x^2 + 3Aa^2bx^2 + Aa^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")

[Out] $1/12*B*c^3*x^{12} + 3/10*B*b*c^2*x^{10} + 1/10*A*c^3*x^{10} + 3/8*B*b^2*c*x^8 + 3/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*\log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/x^2$

maple [A] time = 0.01, size = 190, normalized size = 1.17

$$\frac{Bc^3x^{12}}{12} + \frac{Ac^3x^{10}}{10} + \frac{3Bbc^2x^{10}}{10} + \frac{3Abc^2x^8}{8} + \frac{3Ba^2cx^8}{8} + \frac{3Bb^2cx^8}{8} + \frac{Aa^2x^6}{2} + \frac{Ab^2cx^6}{2} + Babcx^6 + \frac{Bb^3x^6}{6} + \frac{3Aabcx^4}{2} + \frac{Ab^3x^4}{4} + \frac{3Ba^2cx^4}{4} + \frac{3Ba^2bx^2}{4} + \frac{3Aa^2cx^2}{2} + \frac{3Aa^2bx^2}{2} + \frac{3Ba^2b^2x^2}{2} + 3Aa^2b\ln(x) + Ba^3\ln(x) - \frac{Aa^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x)

[Out] $1/12*B*c^3*x^{12} + 1/10*A*x^{10}*c^3 + 3/10*B*x^{10}*b*c^2 + 3/8*A*x^8*b*c^2 + 3/8*B*x^8*a*c^2 + 3/8*B*x^8*b^2*c + 1/2*A*x^6*a*c^2 + 1/2*A*x^6*b^2*c + B*x^6*a*b*c + 1/6*B*x^6*b^3 + 3/2*A*x^4*a*b*c + 1/4*A*x^4*b^3 + 3/4*B*x^4*a^2*c + 3/4*B*x^4*a*b^2 + 3/2*A*x^2*a^2*c + 3/2*A*x^2*a*b^2 + 3/2*B*x^2*a^2*b + 3*A*\ln(x)*a^2*b + B*\ln(x)*a^3 - 1/2*a^3*A/x^2$

maxima [A] time = 0.79, size = 167, normalized size = 1.03

$$\frac{1}{12}Bc^3x^{12} + \frac{1}{10}(3Bbc^2 + Ac^3)x^{10} + \frac{3}{8}(Bb^2c + (Ba + Ab)c^2)x^8 + \frac{1}{6}(Bb^3 + 3Aac^2 + 3(2Bab + Ab^2)c)x^6 + \frac{1}{4}(3Bab^2 + Ab^3 + 3(Ba^2 + 2Aab)c)x^4 + \frac{3}{2}(Ba^2b + Aab^2 + Aa^2c)x^2 - \frac{Aa^3}{2x^2} + \frac{1}{2}(Ba^3 + 3Aa^2b)\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")

[Out] $1/12*B*c^3*x^{12} + 1/10*(3*B*b*c^2 + A*c^3)*x^{10} + 3/8*(B*b^2*c + (B*a + A*b)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*\log(x^2)$

mupad [B] time = 0.06, size = 166, normalized size = 1.02

$$x^4 \left(\frac{3Bc^2}{4} + \frac{3Bab^2}{4} + \frac{3Acab}{2} + \frac{Ab^3}{4} \right) + x^6 \left(\frac{Bb^3}{6} + \frac{Ab^2c}{2} + Babc + \frac{Aac^2}{2} \right) + x^{10} \left(\frac{Ac^3}{10} + \frac{3Bbc^2}{10} \right) + \ln(x) (Ba^3 + 3Aba^2) + x^2 \left(\frac{3Ba^2b}{2} + \frac{3Aca^2}{2} + \frac{3Aab^2}{2} \right) + x^8 \left(\frac{3Bb^2c}{8} + \frac{3Abc^2}{8} + \frac{3Ba^2c}{8} \right) - \frac{Aa^3}{2x^2} + \frac{Bc^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)`

[Out] $x^4 \left(\frac{(A*b^3)}{4} + \frac{(3*B*a*b^2)}{4} + \frac{(3*B*a^2*c)}{4} + \frac{(3*A*a*b*c)}{2} \right) + x^6 \left(\frac{(B*b^3)}{6} + \frac{(A*a*c^2)}{2} + \frac{(A*b^2*c)}{2} + B*a*b*c \right) + x^{10} \left(\frac{(A*c^3)}{10} + \frac{(3*B*b*c^2)}{10} \right) + \log(x) * (B*a^3 + 3*A*a^2*b) + x^2 * \left(\frac{(3*A*a*b^2)}{2} + \frac{(3*A*a^2*c)}{2} + \frac{(3*B*a^2*b)}{2} \right) + x^8 * \left(\frac{(3*A*b*c^2)}{8} + \frac{(3*B*a*c^2)}{8} + \frac{(3*B*b^2*c)}{8} \right) - \frac{(A*a^3)}{(2*x^2)} + \frac{(B*c^3*x^{12})}{12}$

sympy [A] time = 0.40, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2} + \frac{Bc^3x^{12}}{12} + a^2(3Ab + Ba)\log(x) + x^{10}\left(\frac{Ac^3}{10} + \frac{3Bb^2c}{10}\right) + x^8\left(\frac{3Abc^2}{8} + \frac{3Bac^2}{8} + \frac{3Bb^2c}{8}\right) + x^6\left(\frac{Aac^2}{2} + \frac{Ab^2c}{2} + Bab + \frac{Bb^3}{6}\right) + x^4\left(\frac{3Aabc}{2} + \frac{Ab^3}{4} + \frac{3Ba^2c}{4} + \frac{3Bab^2}{4}\right) + x^2\left(\frac{3Aa^2c}{2} + \frac{3Aab^2}{2} + \frac{3Ba^2b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)`

[Out] $-A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*\log(x) + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8) + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2 + A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2/2 + 3*B*a**2*b/2)$

$$3.80 \quad \int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=133

$$\frac{(-aBc - Abc + b^2B) \log(a + bx^2 + cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{x^2(bB - Ac)}{2c^2} + \frac{Bx^4}{4c}$$

Rubi [A] time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(-aBc - Abc + b^2B) \log(a + bx^2 + cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{x^2(bB - Ac)}{2c^2} + \frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

Rule 800

$\text{Int}[((d_) + (e_)*(x_))^{(m_)}*((f_) + (g_)*(x_))]/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1251

$\text{Int}[(x_)^{(m_)}*((d_) + (e_)*(x_)^2)^{(q_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}], x_Symbol] \text{:>} \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q, x\} \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{x^5(A + Bx^2)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(A + Bx)}{a + bx + cx^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{bB - Ac}{c^2} + \frac{Bx}{c} + \frac{a(bB - Ac) + (b^2B - Abc - aBc)x}{c^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\ &= -\frac{(bB - Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{\text{Subst} \left(\int \frac{a(bB - Ac) + (b^2B - Abc - aBc)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2} \\ &= -\frac{(bB - Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{(b^2B - Abc - aBc) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} - \frac{(b^3B - Ab^2c - 3abBc + 2aAc^2)}{4c^3} \\ &= -\frac{(bB - Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{(b^2B - Abc - aBc) \log(a + bx^2 + cx^4)}{4c^3} + \frac{(b^3B - Ab^2c - 3abBc + 2aAc^2)}{4c^3} \\ &= -\frac{(bB - Ac)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{(b^3B - Ab^2c - 3abBc + 2aAc^2) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2B - Abc)}{4c^3} \end{aligned}$$

Mathematica [A] time = 0.06, size = 126, normalized size = 0.95

$$\frac{(-aBc - Abc + b^2B) \log(a + bx^2 + cx^4) + \frac{2(-2aAc^2 + 3abBc + Ab^2c + b^3(-B)) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + 2cx^2(Ac - bB) + Bc^2x^4}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c - 2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4]/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

fricas [A] time = 1.59, size = 421, normalized size = 3.17

$$\frac{(b^2c^2 - 4Bbc^2 + 2Aa^2c^2 - 4Bb^2c^2 + 2Aa^2c^2 - 4Bb^2c^2) \sqrt{-b^2 + 4ac} \log\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + (b^2c^2 - 4Bbc^2 + 2Aa^2c^2 - 4Bb^2c^2 + 2Aa^2c^2 - 4Bb^2c^2) \sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + (b^2c^2 - 4Bbc^2 + 2Aa^2c^2 - 4Bb^2c^2 + 2Aa^2c^2 - 4Bb^2c^2) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]

giac [A] time = 1.87, size = 126, normalized size = 0.95

$$\frac{Bcx^4 - 2Bbx^2 + 2Acx^2}{4c^2} + \frac{(Bb^2 - Bac - Abc) \log(cx^4 + bx^2 + a)}{4c^3} - \frac{(Bb^3 - 3Babc - Ab^2c + 2Aac^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] $1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*\log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*c^3)$

maple [B] time = 0.01, size = 261, normalized size = 1.96

$$\frac{Bx^4}{4c} - \frac{Aa \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{Ab^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} + \frac{Ax^2}{2c} + \frac{3Bab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} - \frac{Bb^3 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^3} - \frac{Bbx^2}{2c^2} - \frac{Ab \ln(cx^4 + bx^2 + a)}{4c^2} - \frac{Ba \ln(cx^4 + bx^2 + a)}{4c^2} + \frac{Bb^2 \ln(cx^4 + bx^2 + a)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x)`

[Out] $1/4*B*x^4/c + 1/2/c*A*x^2 - 1/2/c^2*B*x^2*b - 1/4/c^2*\ln(c*x^4 + b*x^2 + a)*A*b - 1/4/c^2*\ln(c*x^4 + b*x^2 + a)*a*B + 1/4/c^3*\ln(c*x^4 + b*x^2 + a)*b^2*B - 1/c/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*a*A + 3/2/c^2/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*a*b*B + 1/2/c^2/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*A*b^2 - 1/2/c^3/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^{(1/2)})*b^3*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.46, size = 1343, normalized size = 10.10



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)`

[Out] $x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (\log(a + b*x^2 + c*x^4)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) + (\operatorname{atan}((2*c^4*(4*a*c - b^2)*(x^2*(((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)))/(8*c^3*(4*a*c - b^2)^{(1/2)}) - (b*(B*b^3 + 2*$

$$\begin{aligned}
& A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^{(1/2)}*(16*a*c^4 - 4*b^2*c^3))/a - \\
& (b*(((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2*A*B*b^4*c - A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 + 4*A*B*a*b^2*c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^{(1/2)})) + (((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^{(1/2)}) - (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(c*(4*a*c - b^2)^{(1/2)}*(16*a*c^4 - 4*b^2*c^3))/a - (b*(((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*a*b^4 + B^2*a^3*c^2 + A^2*a*b^2*c^2 - 2*B^2*a^2*b^2*c - 2*A*B*a*b^3*c + 2*A*B*a^2*b*c^2)/c^4 + (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^{(1/2)})))/(B^2*b^6 + 4*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 12*A*B*a^2*b*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^3*(4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [B] time = 43.89, size = 620, normalized size = 4.66

$$\frac{B^4}{c^4} + \frac{B^3}{c^3} \left(\frac{A}{4c} \right) \left(\frac{\sqrt{-4ac + b^2} (-2a^2c^2 + 2b^2c + 3Bab - B^2)}{4^2(4ac - b^2)} \right) \log \left(\frac{A^2c + 2B^2c^2 - Bb^2 + Bc^2}{4^2(4ac - b^2)} \right) + \frac{2B^2c^2 \left(\frac{\sqrt{-4ac + b^2} (-2a^2c^2 + 2b^2c + 3Bab - B^2)}{4^2(4ac - b^2)} \right) - 2B^2c^2 \left(\frac{\sqrt{-4ac + b^2} (-2a^2c^2 + 2b^2c + 3Bab - B^2)}{4^2(4ac - b^2)} \right)}{4^2(4ac - b^2)} \left(\frac{\sqrt{-4ac + b^2} (-2a^2c^2 + 2b^2c + 3Bab - B^2)}{4^2(4ac - b^2)} \right) \log \left(\frac{A^2c + 2B^2c^2 - Bb^2 + Bc^2}{4^2(4ac - b^2)} \right) + \frac{A^2c + 2B^2c^2 - Bb^2 + Bc^2}{4^2(4ac - b^2)} \left(\frac{\sqrt{-4ac + b^2} (-2a^2c^2 + 2b^2c + 3Bab - B^2)}{4^2(4ac - b^2)} \right) \log \left(\frac{A^2c + 2B^2c^2 - Bb^2 + Bc^2}{4^2(4ac - b^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a), x)

[Out] $B*x**4/(4*c) + x**2*(A/(2*c) - B*b/(2*c**2)) + (-sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(-sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3))/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3) - 2*b**2*c**2*(-sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3) + (sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3))/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3) - 2*b**2*c**2*(sqrt(-4*a*c + b**2))*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b$

$$\frac{(A^2c + B^2a^2c - B^2b^2)/(4c^3)}{(-2A^2ac^2 + A^2b^2c + 3B^2ab^2c - B^2b^2c^3)}$$

$$3.81 \quad \int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=97

$$\frac{(-2aBc - Abc + b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{(bB - Ac) \log(a + bx^2 + cx^4)}{4c^2} + \frac{Bx^2}{2c}}{2c^2\sqrt{b^2 - 4ac}}$$

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 773, 634, 618, 206, 628}

$$\frac{(-2aBc - Abc + b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{(bB - Ac) \log(a + bx^2 + cx^4)}{4c^2} + \frac{Bx^2}{2c}}{2c^2\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]

[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 773

$\text{Int}[(((d_.) + (e_.)*(x_.))*((f_.) + (g_.)*(x_.)))/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{:>} \text{Simp}[(e*g*x)/c, x] + \text{Dist}[1/c, \text{Int}[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1251

$\text{Int}[(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(q_.)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \text{:>} \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m-1)/2)}*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{x^3 (A + Bx^2)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Bx)}{a + bx + cx^2} dx, x, x^2 \right) \\ &= \frac{Bx^2}{2c} + \frac{\text{Subst} \left(\int \frac{-aB + (-bB + Ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c} \\ &= \frac{Bx^2}{2c} - \frac{(bB - Ac) \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} + \frac{(b^2B - Abc - 2aBc) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} \\ &= \frac{Bx^2}{2c} - \frac{(bB - Ac) \log(a + bx^2 + cx^4)}{4c^2} - \frac{(b^2B - Abc - 2aBc) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + cx^2 \right)}{2c^2} \\ &= \frac{Bx^2}{2c} - \frac{(b^2B - Abc - 2aBc) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c^2 \sqrt{b^2-4ac}} - \frac{(bB - Ac) \log(a + bx^2 + cx^4)}{4c^2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$\frac{2(-2aBc - Abc + b^2B) \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right) + (Ac - bB) \log(a + bx^2 + cx^4) + 2Bcx^2}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (- (b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.77, size = 312, normalized size = 3.22

$$\frac{2(Bb^2c - 4Bac^2)x^2 - (Bb^2 - (2Ba + Ab)c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{c^2x^4 + bx^2 + a}\right) - (Bb^2 + 4Aac^2 - (4Bab + Ab^2)c) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)} + \frac{2(Bb^2c - 4Bac^2)x^2 - 2(Bb^2 - (2Ba + Ab)c)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) - (Bb^2 + 4Aac^2 - (4Bab + Ab^2)c) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c))/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 1.81, size = 91, normalized size = 0.94

$$\frac{Bx^2}{2c} - \frac{(Bb - Ac) \log(cx^4 + bx^2 + a)}{4c^2} + \frac{(Bb^2 - 2Bac - Abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$\frac{Ab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} - \frac{Ba \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{Bb^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} + \frac{Bx^2}{2c} + \frac{A \ln(cx^4+bx^2+a)}{4c} - \frac{Bb \ln(cx^4+bx^2+a)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a), x)

[Out] $\frac{1}{2}Bx^2/c + \frac{1}{4}c \ln(cx^4+bx^2+a) \cdot A - \frac{1}{4}c^2 \ln(cx^4+bx^2+a) \cdot b \cdot B - \frac{1}{c} \left(4ac - b^2\right)^{1/2} \arctan\left(\frac{2cx^2+b}{\left(4ac - b^2\right)^{1/2}}\right) \cdot a \cdot B - \frac{1}{2}c \left(4ac - b^2\right)^{1/2} \arctan\left(\frac{2cx^2+b}{\left(4ac - b^2\right)^{1/2}}\right) \cdot A \cdot b + \frac{1}{2}c^2 \left(4ac - b^2\right)^{1/2} \arctan\left(\frac{2cx^2+b}{\left(4ac - b^2\right)^{1/2}}\right) \cdot b^2 \cdot B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.65, size = 979, normalized size = 10.09

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4), x)

[Out] $\frac{Bx^2}{2c} + \frac{\log(a + bx^2 + cx^4) \cdot (2Bb^3 + 8Aac^2 - 2Ab^2c - 8Babc)}{2(16ac^3 - 4b^2c^2)} - \frac{\operatorname{atan}\left(\frac{2c^2(4ac - b^2)}{\left(4ac - b^2\right)^{1/2}}\right) \cdot \left(\frac{8Aac^3 - 8Babc^2}{c^2} - \frac{8a^2c^2(2Bb^3 + 8Aac^2 - 2Ab^2c - 8Babc)}{(16ac^3 - 4b^2c^2)} \cdot (Abc - Bb^2 + 2Bac)\right)}{(8c^2(4ac - b^2)^{1/2})} - \frac{a(Abc - Bb^2 + 2Bac) \cdot (2Bb^3 + 8Aac^2 - 2Ab^2c - 8Babc)}{\left(4ac - b^2\right)^{1/2} \cdot (16ac^3 - 4b^2c^2)} + x^2 \cdot \left(\frac{(6Abc^3 - 6Bb^2c^2 + 4Bac^3)/c^2 - (4b^2c^2(2Bb^3 + 8Aac^2 - 2Ab^2c - 8Babc))/(16ac^3 - 4b^2c^2)}{(8c^2(4ac - b^2)^{1/2})} - \frac{b(Abc - Bb^2 + 2Bac) \cdot (2Bb^3 + 8Aac^2 - 2Ab^2c - 8Babc)}{2(4ac - b^2)^{1/2} \cdot (16ac^3 - 4b^2c^2)}\right)$

2)))/a + (b*(((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (B^2*b^3 + A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A*b*c - B*b^2 + 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*(((8*A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2 - 2*A*B*a*b*c)/c^2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 4*A*B*a*b*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)^(1/2)))

sympy [B] time = 10.55, size = 434, normalized size = 4.47

$$\frac{B^2}{2c} + \left(\frac{-Ac + Bb}{4c^2} - \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right) \log \left(x^2 + \frac{2Aac - Bbb - 8ac^2 \left(-\frac{Ac + Bb}{4c^2} - \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right) + 2b^2c \left(-\frac{Ac + Bb}{4c^2} - \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right)}{Abc + 2Bac - Bb^2} \right) + \left(\frac{-Ac + Bb}{4c^2} + \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right) \log \left(x^2 + \frac{2Aac - Bbb - 8ac^2 \left(-\frac{Ac + Bb}{4c^2} + \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right) + 2b^2c \left(-\frac{Ac + Bb}{4c^2} + \frac{\sqrt{-4ac + b^2} (Abc + 2Bac - Bb^2)}{4c^2(4ac - b^2)} \right)}{Abc + 2Bac - Bb^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a), x)

[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*c - B*b**2) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*c - B*b**2))

$$3.82 \quad \int \frac{x(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=71

$$\frac{(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B \log(a + bx^2 + cx^4)}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1247, 634, 618, 206, 628}

$$\frac{(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B \log(a + bx^2 + cx^4)}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1247

$\text{Int}[(x_*)*((d_*) + (e_*)*(x_*)^2)^(q_*)*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^(p_*), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{x(A + Bx^2)}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{a + bx + cx^2} dx, x, x^2 \right) \\ &= \frac{B \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c} + \frac{(-bB + 2Ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4c} \\ &= \frac{B \log(a + bx^2 + cx^4)}{4c} - \frac{(-bB + 2Ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c} \\ &= \frac{(bB - 2Ac) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c\sqrt{b^2-4ac}} + \frac{B \log(a + bx^2 + cx^4)}{4c} \end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 1.00

$$\frac{B \log(a + bx^2 + cx^4) - \frac{2(bB-2Ac) \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.83, size = 219, normalized size = 3.08

$$\left[\frac{(Bb - 2Ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (Bb^2 - 4Bac) \log(cx^4 + bx^2 + a)}{4(b^2c - 4ac^2)}, \frac{2(Bb - 2Ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (Bb^2 - 4Bac) \log(cx^4 + bx^2 + a)}{4(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]

giac [A] time = 1.71, size = 67, normalized size = 0.94

$$\frac{B \log(cx^4 + bx^2 + a)}{4c} - \frac{(Bb - 2Ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)

maple [A] time = 0.00, size = 98, normalized size = 1.38

$$\frac{A \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} - \frac{Bb \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}c} + \frac{B \ln(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a), x)

[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b/c

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo re details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.50, size = 606, normalized size = 8.54

$$\frac{\operatorname{atan}\left(\frac{\frac{\frac{2(Ac-B)\left(8Bc-4A^2\sqrt{4b^2-8Bac}\right)}{8c\sqrt{4ac-b^2}}+\frac{4\left(2B^2-8Bac\right)\left(2Ac-Bb\right)}{2\left(16ac^2-4b^2c\right)\sqrt{4ac-b^2}}}{2\left(4ac-b^2\right)}+\frac{\frac{2B^2-8Bac}{2\left(16ac^2-4b^2c\right)}+\frac{4\left(2B^2-8Bac\right)\left(2Ac-Bb\right)}{2\left(16ac^2-4b^2c\right)\sqrt{4ac-b^2}}}{2c\sqrt{4ac-b^2}}+\frac{\frac{8Bc^2\left(2B^2-8Bac\right)}{8c\sqrt{4ac-b^2}}+\frac{4\left(2B^2-8Bac\right)\left(2Ac-Bb\right)}{2\left(16ac^2-4b^2c\right)\sqrt{4ac-b^2}}}{2c\sqrt{4ac-b^2}}\right)}{4A^2b^2-4ABbc+B^2b^2}}{\frac{\ln\left(cx^4+bx^2+a\right)\left(2Bb^2-8Bac\right)}{2\left(16ac^2-4b^2c\right)}-\frac{2c\sqrt{4ac-b^2}}{4A^2b^2-4ABbc+B^2b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

[Out] $-\left(\log\left(a+b x^2+c x^4\right)\left(2 B b^2-8 B a c\right)\right) / \left(2\left(16 a^2 c^2-4 b^2 c\right)\right)-\left(\operatorname{atan}\left(\frac{2\left(4 a c-b^2\right) x^2\left(\left(\left(2 A c-B b\right)\left(6 B b^2 c-4 A^2 c^2+\left(4 b^2 c^2\left(2 B b^2-8 B a c\right)\right)\right)\right)\right)}{\left(16 a^2 c^2-4 b^2 c\right)}\right) / \left(8 c\left(4 a c-b^2\right)^{1 / 2}\right)+\left(b^2 c\left(2 B b^2-8 B a c\right)\left(2 A c-B b\right)\right) / \left(2\left(16 a^2 c^2-4 b^2 c\right)\left(4 a c-b^2\right)^{1 / 2}\right)\right) / a+\left(b\left(B^2 b-A B c-\left(b\left(2 A c-B b\right)\right)^2\right)\right) / \left(2\left(4 a c-b^2\right)\right)+\left(\left(2 B b^2-8 B a c\right)\left(6 B b^2 c-4 A^2 c^2+\left(4 b^2 c^2\left(2 B b^2-8 B a c\right)\right)\right)\right) / \left(16 a^2 c^2-4 b^2 c\right)\right) / \left(2\left(16 a^2 c^2-4 b^2 c\right)\right)\right) / \left(2 a\left(4 a c-b^2\right)^{1 / 2}\right)+\left(\left(8 B a c+\left(8 a^2 c^2\left(2 B b^2-8 B a c\right)\right)\right) / \left(16 a^2 c^2-4 b^2 c\right)\right)\left(2 A c-B b\right)\right) / \left(8 c\left(4 a c-b^2\right)^{1 / 2}\right)+\left(a c\left(2 B b^2-8 B a c\right)\left(2 A c-B b\right)\right) / \left(\left(16 a^2 c^2-4 b^2 c\right)\left(4 a c-b^2\right)^{1 / 2}\right)\right) / a+\left(b\left(B^2 a+\left(\left(2 B b^2-8 B a c\right)\left(8 B a c+\left(8 a^2 c^2\left(2 B b^2-8 B a c\right)\right)\right)\right)\right) / \left(16 a^2 c^2-4 b^2 c\right)\right)\right) / \left(2\left(16 a^2 c^2-4 b^2 c\right)\right)-\left(a\left(2 A c-B b\right)^2\right) / \left(4 a c-b^2\right)\right) / \left(2 a\left(4 a c-b^2\right)^{1 / 2}\right)\right) / \left(4 A^2 c^2+B^2 b^2-4 A B b c\right)\left(2 A c-B b\right) / \left(2 c\left(4 a c-b^2\right)^{1 / 2}\right)$

sympy [B] time = 3.59, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right)+\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

[Out] $\left(\frac{B}{4c}-\left(-2Ac+Bb\right)\sqrt{-4ac+b^2}\right) / \left(4c\left(4ac-b^2\right)\right)\log\left(x^2+\left(-Ab+2Ba-8ac\left(\frac{B}{4c}-\left(-2Ac+Bb\right)\sqrt{-4ac+b^2}\right)+2b^2\left(\frac{B}{4c}-\left(-2Ac+Bb\right)\sqrt{-4ac+b^2}\right)\right) / \left(-2Ac+Bb\right)\right)$

$$\begin{aligned}
& c(4ac - b^2)) + 2b^2(B/(4c) - (-2Ac + Bb)\sqrt{-4ac + b^2}/(\\
& 4c(4ac - b^2))) / (-2Ac + Bb) + (B/(4c) + (-2Ac + Bb)\sqrt{-4a \\
& c + b^2}/(4c(4ac - b^2))) \log(x^2 + (-Ab + 2Ba - 8ac(B/(4c) \\
& + (-2Ac + Bb)\sqrt{-4ac + b^2}/(4c(4ac - b^2))) + 2b^2(B/(4c \\
&) + (-2Ac + Bb)\sqrt{-4ac + b^2}/(4c(4ac - b^2)))) / (-2Ac + Bb \\
&))
\end{aligned}$$

$$3.83 \quad \int \frac{A+Bx^2}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=78

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{A \log(a + bx^2 + cx^4)}{4a} + \frac{A \log(x)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{A \log(a + bx^2 + cx^4)}{4a} + \frac{A \log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]

[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 800

$\text{Int}[(((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1251

$\text{Int}[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] \text{:>} \text{Dist}[1/2, \text{Subst}[\text{Int}[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx + cx^2)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{A}{ax} + \frac{-Ab + aB - Acx}{a(a + bx + cx^2)} \right) dx, x, x^2 \right) \\ &= \frac{A \log(x)}{a} + \frac{\text{Subst} \left(\int \frac{-Ab + aB - Acx}{a + bx + cx^2} dx, x, x^2 \right)}{2a} \\ &= \frac{A \log(x)}{a} - \frac{A \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a} + \frac{(-Ab + 2aB) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4a} \\ &= \frac{A \log(x)}{a} - \frac{A \log(a + bx^2 + cx^4)}{4a} - \frac{(-Ab + 2aB) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2a} \\ &= \frac{(Ab - 2aB) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2a\sqrt{b^2 - 4ac}} + \frac{A \log(x)}{a} - \frac{A \log(a + bx^2 + cx^4)}{4a} \end{aligned}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 1.64

$$\frac{-\left(A\left(\sqrt{b^2 - 4ac} + b\right) - 2aB\right) \log\left(-\sqrt{b^2 - 4ac} + b + 2cx^2\right) + \left(A\left(b - \sqrt{b^2 - 4ac}\right) - 2aB\right) \log\left(\sqrt{b^2 - 4ac} + b + 2cx^2\right) + 4A \log(x) \sqrt{b^2 - 4ac}}{4a\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 1.14, size = 249, normalized size = 3.19

$$\frac{(2Ba - Ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) + (Ab^2 - 4Aac) \log(cx^4 + bx^2 + a) - 4(Ab^2 - 4Aac) \log(x)}{4(ab^2 - 4a^2c)}, \frac{2(2Ba - Ab)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (Ab^2 - 4Aac) \log(cx^4 + bx^2 + a) - 4(Ab^2 - 4Aac) \log(x)}{4(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c))/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 1.61, size = 78, normalized size = 1.00

$$-\frac{A \log(cx^4 + bx^2 + a)}{4a} + \frac{A \log(x^2)}{2a} + \frac{(2Ba - Ab) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)

maple [A] time = 0.01, size = 105, normalized size = 1.35

$$-\frac{Ab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}a} + \frac{B \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{A \ln(x)}{a} - \frac{A \ln(cx^4 + bx^2 + a)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x)

[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 4.48, size = 2424, normalized size = 31.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)

[Out] (A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b))/(4*a) + B^3*c^2*x^2*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2))))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b))/(4*a) + B^3*c^2*x^2*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (atan((2*(4*a*c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2*c^2 + ((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2

$$\begin{aligned}
& *c^2*(2*A*b^2 - 8*A*a*c)/(4*a*b^2 - 16*a^2*c))/(2*(4*a*b^2 - 16*a^2*c)) + \\
& B^2*a*c^2 - 4*A*B*b*c^2)/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b - 2*B*a)*((A*b \\
& - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4 \\
& *a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*a* \\
& c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2)))/(4*a*(4*a* \\
& c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a)^2)/(8*a*(4*a*b \\
& ^2 - 16*a^2*c)*(4*a*c - b^2)))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 4*A*B*a \\
& *b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)) - (16*a^3*x^2*((3*A* \\
& b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*((2*A*b^2 - 8*A*a*c)*(B^2*b*c^2 + 5*A \\
& *B*c^3 - ((2*A*b^2 - 8*A*a*c)*((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^ \\
& 3)))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(\\
& 4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2 - 16*a^2*c)) - B^3*c^2 + ((A*b - 2*B*a)* \\
& ((A*b - 2*B*a)*((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^ \\
& 2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2) \\
& ^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8* \\
& a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2)))/(4*a*(4*a*c - b^2)^(1/2)) + (\\
& (2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^2)/(32*a^2*(4*a \\
& *b^2 - 16*a^2*c)*(4*a*c - b^2)))/(8*a^3*c^2*(6*A^2*b^2 - B^2*a^2 - 25*A^2* \\
& a*c + A*B*a*b)) + (((12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^3)/(64*a^3*(4* \\
& a*c - b^2)^(3/2)) - ((2*A*b^2 - 8*A*a*c)*((A*b - 2*B*a)*((2*A*b^2 - 8*A*a \\
& *c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10* \\
& A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12 \\
& *b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^ \\
& 2)^(1/2)))/(2*(4*a*b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(B^2*b*c^2 + 5*A*B*c^ \\
& 3 - ((2*A*b^2 - 8*A*a*c)*((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(\\
& 2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(4*a*b \\
& ^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)))*(3*A*b^4 + 10*A*a^2*c^2 - B*a* \\
& b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*A^2*b^ \\
& 2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b))*(4*a*c - b^2)^(3/2))/(A^2*b^2*c^2 + 4 \\
& *B^2*a^2*c^2 - 4*A*B*a*b*c^2) + (2*(4*a*c - b^2)*((2*A*b^2 - 8*A*a*c)*((A \\
& *b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/ \\
& (4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A* \\
& a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2)))/(2*(4*a* \\
& b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B* \\
& a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a* \\
& b^2 - 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(4*a*(4*a*c - b^2)^(1/2)) - (b \\
& ^2*c^2*(A*b - 2*B*a)^3)/(16*a^2*(4*a*c - b^2)^(3/2)))*(3*A*b^4 + 10*A*a^2*c \\
& ^2 - B*a*b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c \\
& ^2 - 4*A*B*a*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b))*(A*b - 2 \\
& *B*a))/(2*a*(4*a*c - b^2)^(1/2))
\end{aligned}$$

`sympy [F(-1)]` time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

$$3.84 \quad \int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=112

$$-\frac{(-2aAc - abB + Ab^2) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{(Ab - aB) \log(a + bx^2 + cx^4)}{4a^2} - \frac{\log(x)(Ab - aB)}{a^2} - \frac{A}{2ax^2}$$

Rubi [A] time = 0.25, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 800, 634, 618, 206, 628}

$$-\frac{(-2aAc - abB + Ab^2) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{(Ab - aB) \log(a + bx^2 + cx^4)}{4a^2} - \frac{\log(x)(Ab - aB)}{a^2} - \frac{A}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 1251

`Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^2}{x^3(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^2(a + bx + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{A}{ax^2} + \frac{-Ab + aB}{a^2x} + \frac{-abB + A(b^2 - ac) + (Ab - aB)cx}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{A}{2ax^2} - \frac{(Ab - aB) \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{-abB + A(b^2 - ac) + (Ab - aB)cx}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2} \\
 &= -\frac{A}{2ax^2} - \frac{(Ab - aB) \log(x)}{a^2} + \frac{(Ab - aB) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2} + \frac{(-abB + A(b^2 - 2ac)) \log(x)}{4a^2} \\
 &= -\frac{A}{2ax^2} - \frac{(Ab - aB) \log(x)}{a^2} + \frac{(Ab - aB) \log(a + bx^2 + cx^4)}{4a^2} - \frac{(-abB + A(b^2 - 2ac)) \log(x)}{4a^2} \\
 &= -\frac{A}{2ax^2} + \frac{(abB - A(b^2 - 2ac)) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2 \sqrt{b^2 - 4ac}} - \frac{(Ab - aB) \log(x)}{a^2} + \frac{(Ab - aB) \log(a + bx^2 + cx^4)}{4a^2}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 186, normalized size = 1.66

$$\frac{\left(A(b\sqrt{b^2 - 4ac} - 2ac + b^2) - aB(\sqrt{b^2 - 4ac} + b) \right) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{\sqrt{b^2 - 4ac}} + \frac{\left(A(b\sqrt{b^2 - 4ac} + 2ac - b^2) + aB(b - \sqrt{b^2 - 4ac}) \right) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{\sqrt{b^2 - 4ac}} + 4 \log(x)(aB - Ab) - \frac{2aA}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] $\left(\frac{-2*a*A}{x^2} + 4*(-(A*b) + a*B)*\text{Log}[x] + \left(\frac{-(a*B*(b + \text{Sqrt}[b^2 - 4*a*c])}{+ A*(b^2 - 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c])}\right)*\text{Log}[b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2]/\text{Sqrt}[b^2 - 4*a*c] + \left(\frac{(a*B*(b - \text{Sqrt}[b^2 - 4*a*c])}{+ A*(-b^2 + 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c])}\right)*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2]/\text{Sqrt}[b^2 - 4*a*c]\right)/(4*a^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 1.08, size = 385, normalized size = 3.44

$$\left[\frac{(Bab - Ab^2 + 2Aac)\sqrt{b^2 - 4ac} \log\left(\frac{b^2 + 2bx^2 + cx^4 + \sqrt{b^2 - 4ac}x}{cx^4 + bx^2 + a}\right) - 2Aab^2 + 8Aa^2c - (Bab^2 - Ab^3 - 4(Ba^2 - Aab)c)x^2 \log(cx^4 + bx^2 + a) + 4(Bab^2 - Ab^3 - 4(Ba^2 - Aab)c)x^2 \log(x)}{4(a^2b^2 - 4a^2c)^2} \right] - 2Aab^2 + 8Aa^2c - (Bab^2 - Ab^3 - 4(Ba^2 - Aab)c)x^2 \log(cx^4 + bx^2 + a) + 4(Bab^2 - Ab^3 - 4(Ba^2 - Aab)c)x^2 \log(x)}{4(a^2b^2 - 4a^2c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] $\left[\frac{1}{4} * \left((B*a*b - A*b^2 + 2*A*a*c) * \text{sqrt}(b^2 - 4*a*c) * x^2 * \log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b) * \text{sqrt}(b^2 - 4*a*c)) / (c*x^4 + b*x^2 + a)) - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c) * x^2 * \log(c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c) * x^2 * \log(x) \right) / ((a^2*b^2 - 4*a^3*c) * x^2), \frac{1}{4} * (2*(B*a*b - A*b^2 + 2*A*a*c) * \text{sqrt}(-b^2 + 4*a*c) * x^2 * \arctan(-(2*c*x^2 + b) * \text{sqrt}(-b^2 + 4*a*c) / (b^2 - 4*a*c)) - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c) * x^2 * \log(c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c) * x^2 * \log(x)) / ((a^2*b^2 - 4*a^3*c) * x^2) \right]$

giac [A] time = 1.87, size = 124, normalized size = 1.11

$$-\frac{(Ba - Ab) \log(cx^4 + bx^2 + a)}{4a^2} + \frac{(Ba - Ab) \log(x^2)}{2a^2} - \frac{(Bab - Ab^2 + 2Aac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a^2} - \frac{Bax^2 - Abx^2 + Aa}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-1/4*(B*a - A*b)*\log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*\log(x^2)/a^2 - 1/2*(B*a*b - A*b^2 + 2*A*a*c)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)$

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$-\frac{Ac \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} a} + \frac{A b^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2} a^2} - \frac{Bb \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2} a} - \frac{Ab \ln(x)}{a^2} + \frac{Ab \ln(cx^4 + bx^2 + a)}{4a^2} + \frac{B \ln(x)}{a} - \frac{B \ln(cx^4 + bx^2 + a)}{4a} - \frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^2+A)/x^3/(c*x^4+b*x^2+a), x)$

[Out] $-1/2*A/a/x^2 - 1/a^2*\ln(x)*A*b + 1/a*\ln(x)*B + 1/4/a^2*\ln(c*x^4+b*x^2+a)*A*b - 1/4/a*\ln(c*x^4+b*x^2+a)*B - 1/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*c + 1/2/a^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b^2 - 1/2/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x^2+A)/x^3/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 4.85, size = 3729, normalized size = 33.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)), x)$

[Out] $-A/(2*a*x^2) - (\log(x)*(A*b - B*a))/a^2 - (\log(((A^3*c^5*x^2)/a^3 - (((4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)}))/a^2)*(B*a - A*b + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)}))/4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2*(B*a - A*b + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)})/(4*a^2) + (A^2*c^4*(A*b - B*a))/a^3*(((4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*$

$$\begin{aligned}
& ((A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)}) \\
& /a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)}) \\
& /((4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2) \\
& *(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^{(1/2)}) \\
& /((4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b - B*a))/a^3) \\
& *(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/((2*(16*a^3*c - 4*a^2*b^2)) \\
& - (\operatorname{atan}((16*a^6*x^2*(((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) - (A^3*c^5)/a^3 + (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c - b^2)^{(1/2})*(16*a^3*c - 4*a^2*b^2)))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b)^2*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))*((3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*B*a^2*b*c))/(8*a^3*c^2*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) + (((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c - b^2)^{(1/2})*(16*a^3*c - 4*a^2*b^2)))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*b)^3)/(64*a^9*(4*a*c - b^2)^{(3/2)}))*((6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(16*a^3*c^2*(4*a*c - b^2)^{(1/2})*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(4*a*c - b^2)^{(3/2)})/(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a^2*b*c^3) + (a^3*(4*a*c - b^2)*((((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*((2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^{(1/2})*(16*a^3*c - 4*a^2*b^2)))*((2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((
\end{aligned}$$

$$\begin{aligned}
& 4A^3a^3b^3c^3 - 4A^2a^2b^3c^2 + 4B^3a^3b^2c^2)/a^3 + (2ab^2c^2(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/((16a^3c - 4a^2b^2)(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/(2(16a^3c - 4a^2b^2))(2A^2ac - A^2b^2 + B^2ab^2)/(4a^2(4ac - b^2)^{1/2}) - (b^2c^2(2A^2ac - A^2b^2 + B^2ab^2)^3)/(16a^5(4ac - b^2)^{3/2})(6A^5b^5 - 20B^3a^3c^2 - 6B^2a^2b^4 - 30A^2a^2b^3c + 26A^2a^2b^2c^2 + 28B^2a^2b^2c^2)/(c^2(4A^2a^2c^4 + A^2b^4c^2 + B^2a^2b^2c^2 - 4A^2a^2b^2c^3 - 2AB^2a^2b^3c^2 + 4AB^2a^2b^2c^3)(25B^2a^3c - 6A^2b^4 + A^2a^2c^2 - 6B^2a^2b^2 + 12AB^2a^2b^3 + 24A^2a^2b^2c - 49AB^2a^2b^2c)) - (2a^3(4ac - b^2)^{3/2}((A^3b^4c^4 - A^2B^2a^3c^4)/a^3 - ((A^2a^2c^4 - 4A^2a^2b^2c^3 + 4AB^2a^2b^2c^3)/a^3 + (((4A^3b^3c^3 - 4A^2a^2b^3c^2 + 4B^3a^3b^2c^2)/a^3 + (2ab^2c^2(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/((16a^3c - 4a^2b^2)(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/(2(16a^3c - 4a^2b^2)))(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/(2(16a^3c - 4a^2b^2)) + (((((4A^3b^3c^3 - 4A^2a^2b^3c^2 + 4B^3a^3b^2c^2)/a^3 + (2ab^2c^2(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/((16a^3c - 4a^2b^2)(2A^2ac - A^2b^2 + B^2ab^2))/(4a^2(4ac - b^2)^{1/2}) + (b^2c^2(2A^2ac - A^2b^2 + B^2ab^2)(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/(2a(4ac - b^2)^{1/2}(16a^3c - 4a^2b^2)))(2A^2ac - A^2b^2 + B^2ab^2))/(4a^2(4ac - b^2)^{1/2}) + (b^2c^2(2A^2ac - A^2b^2 + B^2ab^2)^2(2A^3b^3 - 2B^2a^2b^2 + 8B^2a^2c - 8A^2ab^2c))/(8a^3(4ac - b^2)(16a^3c - 4a^2b^2)))(3A^4b^4 + A^2a^2c^2 - 3B^2a^2b^3 - 9A^2a^2b^2c + 8B^2a^2b^2c))/(c^2(4A^2a^2c^4 + A^2b^4c^2 + B^2a^2b^2c^2 - 4A^2a^2b^2c^3 - 2AB^2a^2b^3c^2 + 4AB^2a^2b^2c^3)(25B^2a^3c - 6A^2b^4 + A^2a^2c^2 - 6B^2a^2b^2 + 12AB^2a^2b^3 + 24A^2a^2b^2c - 49AB^2a^2b^2c)))(2A^2ac - A^2b^2 + B^2ab^2)/(2a^2(4ac - b^2)^{1/2})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.85 \quad \int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=261

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}} - aBc - Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}} - aBc - Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.49, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1279, 1166, 205}

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}} - aBc - Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}} - aBc - Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right) - \frac{x(bB - Ac)}{c^2} + \frac{Bx^3}{3c}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +

1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{x^4 (A + Bx^2)}{a + bx^2 + cx^4} dx &= \frac{Bx^3}{3c} - \frac{\int \frac{x^2(3aB+3(bB-Ac)x^2)}{a+bx^2+cx^4} dx}{3c} \\ &= -\frac{(bB - Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\int \frac{3a(bB-Ac)+3(b^2B-Abc-aBc)x^2}{a+bx^2+cx^4} dx}{3c^2} \\ &= -\frac{(bB - Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B - Abc - aBc - \frac{b^3B - Ab^2c - 3abBc + 2aAc^2}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c^2} \\ &= -\frac{(bB - Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B - Abc - aBc - \frac{b^3B - Ab^2c - 3abBc + 2aAc^2}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.40, size = 327, normalized size = 1.25

$$\frac{(-Abc\sqrt{b^2 - 4ac} - 2aAc^2 + b^2B\sqrt{b^2 - 4ac} - aBc\sqrt{b^2 - 4ac} + 3abBc + Ab^2c + b^3(-B)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) + (-Abc\sqrt{b^2 - 4ac} + 2aAc^2 + b^2B\sqrt{b^2 - 4ac} - aBc\sqrt{b^2 - 4ac} - 3abBc - Ab^2c + b^3B) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right) + \frac{x(Ac - bB)}{c^2} + \frac{Bx^3}{3c}}{\sqrt{2}c^{5/2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (((- (b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (((- (b^3*B) + A*b^2*c + 3*a*b*B*c - 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (((b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{a + bx^2 + cx^4} dx$$

$$\begin{aligned}
& a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3* \\
& a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2 \\
& *b^3 - 3*A^2*B^2*a*b^4)*c)*x - \text{sqrt}(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^ \\
& 2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 \\
& + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2 \\
& *B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a \\
& *b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*\text{sqrt}((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2 \\
& *a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B \\
& ^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^ \\
& 2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a* \\
& b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a \\
& *c^11)))*\text{sqrt}(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A* \\
& B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6))* \\
& \text{sqrt}((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c \\
& ^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b \\
& ^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B* \\
& b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4 \\
& *a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3* \\
& \text{sqrt}(1/2)*c^2*\text{sqrt}(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + \\
& 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a* \\
& c^6))*\text{sqrt}((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b \\
& ^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + \\
& A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A \\
& ^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(\\
& 3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))* \\
& \log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^ \\
& 2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 \\
& - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + \text{sqrt}(1/2)*(B^3*b \\
& ^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4* \\
& B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2* \\
& b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (\\
& B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*\text{sqrt}((B^4*b^ \\
& 8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^ \\
& 4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2* \\
& (3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (\\
& 11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A \\
& *B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*\text{sqrt}(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a* \\
& b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b \\
& ^4)*c - (b^2*c^5 - 4*a*c^6))*\text{sqrt}((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + \\
& 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2* \\
& b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + \\
& 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6 \\
& *A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)) \\
&)/(b^2*c^5 - 4*a*c^6))) - 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(B^2*b^5 - (4*A*B*a^2 + 3*A \\
& ^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*
\end{aligned}$$

$$\begin{aligned}
& A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*\sqrt{(B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^{10} - 4*a*c^{11})))/(b^2*c^5 - 4*a*c^6))*\log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x - \sqrt{1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*\sqrt{(B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^{10} - 4*a*c^{11})))*\sqrt{-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*\sqrt{(B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^{10} - 4*a*c^{11})))/(b^2*c^5 - 4*a*c^6)) - 6*(B*b - A*c)*x)/c^2
\end{aligned}$$

giac [B] time = 3.79, size = 4391, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^5*c + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^4*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^3*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^6 + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}*a^2*b^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c}
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(b^2 - 4ac) * c) * a * b^3 * c^2 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^4 * c^2 + 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * c^3 + 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^3 + 5 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^3 - 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * c^4 - 2 * (b^2 - 4ac) * b^4 * c^2 + 10 * (b^2 - 4ac) * a * b^2 * c^3 - 8 * (b^2 - 4ac) * a^2 * c^4) * B * c^2 + 2 * (\text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 * c^3 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * c^4 - 2 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c^4 + 2 * a * b^4 * c^4 + 16 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * c^5 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^5 + \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^5 - 16 * a^2 * b^2 * c^5 - 4 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * c^6 + 32 * a^3 * c^6 - 2 * (b^2 - 4ac) * a * b^2 * c^4 + 8 * (b^2 - 4ac) * a^2 * c^5) * A * \text{abs}(c) - 2 * (\text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^5 * c^2 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^3 * c^3 - 2 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 * c^3 + 2 * a * b^5 * c^3 + 16 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b * c^4 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * c^4 + \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c^4 - 16 * a^2 * b^3 * c^4 - 4 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^5 + 32 * a^3 * b * c^5 - 2 * (b^2 - 4ac) * a * b^3 * c^3 + 8 * (b^2 - 4ac) * a^2 * b * c^4) * B * \text{abs}(c) - (2 * b^5 * c^5 - 12 * a * b^3 * c^6 + 16 * a^2 * b * c^7 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^5 * c^3 + 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c^4 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^4 * c^4 - 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^5 - 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^5 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^3 * c^5 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b * c^6 - 2 * (b^2 - 4ac) * b^3 * c^5 + 4 * (b^2 - 4ac) * a * b * c^6) * A + (2 * b^6 * c^4 - 14 * a * b^4 * c^5 + 24 * a^2 * b^2 * c^6 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^6 * c^2 + 7 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 * c^3 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^5 * c^3 - 12 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * c^4 - 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c^4 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^4 * c^4 + 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^5 - 2 * (b^2 - 4ac) * b^4 * c^4 + 6 * (b^2 - 4ac) * a * b^2 * c^5) * B) * \arctan(2 * \text{sqrt}(1/2) * x / \text{sqrt}((b * c^3 + \text{sqrt}(b^2 * c^6 - 4 * a * c^7)) / c^4)) / ((a * b^4 * c^4 - 8 * a^2 * b^2 * c^5 - 2 * a * b^3 * c^5 + 16 * a^3 * c^6 + 8 * a^2 * b * c^6 + a * b^2 * c^6 - 4 * a^2 * c^7) * c^2) + 1/8 * ((2 * b^5 * c^3 - 16 * a * b^3 * c^4 + 32 * a^2 * b * c^5 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * b^5 * c + 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c^2 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * b^4 * c^2 - 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^3 - 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * b^3 * c^3 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a * b * c^4 - 2 * (b^2 - 4ac) * b^3 * c^3 + 8 * (b
\end{aligned}$$

$$\begin{aligned}
& ^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 3 \\
& 2*a^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6 + \\
& 9*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^4 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c^2 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^3 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^4 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*B)*arctan(2*\sqrt{1/2}*x/\sqrt{(b*c^3 - \sqrt{b^2*c^6 - 4*a*c^7})/c^4}))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(B*c^2*x^3 - 3*B*b*c*x + 3*A*c^2*x)/c^3
\end{aligned}$$

$$\begin{aligned}
& b^4c - 4A^2ab^2c^3 + 10ABa^3b^3c^2 - 10ABa^2b^3c^3)/c^3)*(-(B^2 \\
& *b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2ABb^6c + 25B^2 \\
& a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2c^2*(-(4ac \\
& - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 + 12A^2 \\
& a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - 36A^2 \\
& B^2a^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^4c^2 - \\
& 2ABb^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 4ABa^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} \\
&)/(8*(16a^2c^7 + b^4c^5 - 8ab^2c^6)))^{(1/2)}*i)/(((16Aa^2c^5 - \\
& 4Aab^2c^4 + 4Bab^3c^3 - 16Ba^2b^3c^4)/c^3 - (2x*(4b^3c^5 - 16 \\
& ab^3c^6))*(-(B^2b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2A \\
& ABb^6c + 25B^2a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2 \\
& c^2*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 \\
& + 12A^2a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - \\
& 36ABa^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} + 16 \\
& ABa^4c^2 - 2ABb^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 4ABa^2b^2c^2*(-(4a \\
& ac - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8ab^2c^6)))^{(1/2)}/c^3)* \\
& (-(B^2b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2ABb^6c + \\
& 25B^2a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2c^2*(- \\
& (4ac - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 + \\
& 12A^2a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - \\
& 36ABa^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^4c^2 \\
& - 2ABb^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 4ABa^2b^2c^2*(-(4ac - b^2) \\
& ^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8ab^2c^6)))^{(1/2)} - (2x*(B^2b^6 \\
& + 2A^2a^2c^4 + A^2b^4c^2 - 2B^2a^3c^3 - 2ABb^5c + 9B^2a^2b^2 \\
& c^2 - 6B^2a^2b^4c - 4A^2a^2b^2c^3 + 10ABa^3b^3c^2 - 10ABa^2b^3c^3))/c^3)* \\
& (-(B^2b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2AB \\
& B^6c + 25B^2a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2 \\
& c^2*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 \\
& + 12A^2a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - \\
& 36ABa^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} + 16 \\
& ABa^4c^2 - 2ABb^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 4ABa^2b^2c^2*(-(4a \\
& ac - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8ab^2c^6)))^{(1/2)} + (((16 \\
& Aa^2c^5 - 4Aab^2c^4 + 4Bab^3c^3 - 16Ba^2b^3c^4)/c^3 + (2x*(4b^3c^5 - \\
& 16ab^3c^6))*(-(B^2b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2A \\
& ABb^6c + 25B^2a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2 \\
& c^2*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 \\
& + 12A^2a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - \\
& 36ABa^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} + 16 \\
& ABa^4c^2 - 2ABb^3c^3*(-(4ac - b^2)^3)^{(1/2)} + 4ABa^2b^2c^2*(-(4a \\
& ac - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8ab^2c^6)))^{(1/2)}/c^3)* \\
& (-(B^2b^7 + A^2b^5c^2 + B^2b^4*(-(4ac - b^2)^3)^{(1/2)} - 2 \\
& ABb^6c + 25B^2a^2b^3c^2 + A^2b^2c^2*(-(4ac - b^2)^3)^{(1/2)} + B^2a^2 \\
& c^2*(-(4ac - b^2)^3)^{(1/2)} + 16ABa^3c^4 - 9B^2a^5b^5c - 7A^2a^3b^3c^3 \\
& + 12A^2a^2b^3c^4 - A^2a^3c^3*(-(4ac - b^2)^3)^{(1/2)} - 20B^2a^3b^3c^3 - \\
& 36ABa^2b^2c^3 - 3B^2a^2b^2c^3*(-(4ac - b^2)^3)^{(1/2)} +
\end{aligned}$$

$$\begin{aligned}
& 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + (2 \\
& *x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9 \\
& *B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10* \\
& A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3*a^2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*2i - \operatorname{atan}((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6))*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6))*(-
\end{aligned}$$

$$\begin{aligned}
& (B^2b^7 + A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - 2ABb^6c + 2 \\
& 5B^2a^2b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B^2a^2c^2(-4 \\
& ac - b^2)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^2ab^3c^3 + 1 \\
& 2A^2a^2b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^2a^3b^3c^3 - 3 \\
& 6ABa^2b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} + 16ABaab^4c \\
& ^2 + 2ABb^3c(-4ac - b^2)^3)^{1/2} - 4ABaab^2c(-4ac - b^2)^3 \\
&)^{1/2}) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}) / c^3 * (- (B^2b^7 + \\
& A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - 2ABb^6c + 25B^2a^2b^2 \\
& b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B^2a^2c^2(-4ac - b^2 \\
&)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^2ab^3c^3 + 12A^2a^2b^2 \\
& b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^2a^3b^3c^3 - 36ABa^2b^2 \\
& b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} + 16ABaab^4c^2 + 2AB \\
& *b^3c(-4ac - b^2)^3)^{1/2} - 4ABaab^2c(-4ac - b^2)^3)^{1/2}) / (\\
& 8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (2x*(B^2b^6 + 2A^2a^2c^4 + \\
& A^2b^4c^2 - 2B^2a^3c^3 - 2ABb^5c + 9B^2a^2b^2c^2 - 6B^2 \\
& *ab^4c - 4A^2ab^2c^3 + 10ABaab^3c^2 - 10ABa^2b^3c^3)) / c^3 * (- (\\
& B^2b^7 + A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - 2ABb^6c + 25 \\
& *B^2a^2b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B^2a^2c^2(-4ac \\
& - b^2)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^2ab^3c^3 + 12 \\
& *A^2a^2b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^2a^3b^3c^3 - 36 \\
& *ABa^2b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} + 16ABaab^4c^2 \\
& + 2ABb^3c(-4ac - b^2)^3)^{1/2} - 4ABaab^2c(-4ac - b^2)^3 \\
&)^{1/2}) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * i) / (((16Aa^2c^ \\
& 5 - 4Aab^2c^4 + 4Bab^3c^3 - 16Ba^2b^3c^4) / c^3 - (2x*(4b^3c^5 - \\
& 16ab^3c^6) * (- (B^2b^7 + A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - \\
& 2ABb^6c + 25B^2a^2b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B \\
& ^2a^2c^2(-4ac - b^2)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^ \\
& 2ab^3c^3 + 12A^2a^2b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^ \\
& 2a^3b^3c^3 - 36ABa^2b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} + \\
& 16ABaab^4c^2 + 2ABb^3c(-4ac - b^2)^3)^{1/2} - 4ABaab^2c(- \\
& (4ac - b^2)^3)^{1/2}) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}) / c^ \\
& 3 * (- (B^2b^7 + A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - 2ABb^6c \\
& c + 25B^2a^2b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B^2a^2c^2 \\
& *(-4ac - b^2)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^2ab^3c^ \\
& 3 + 12A^2a^2b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^2a^3b^3c^ \\
& 3 - 36ABa^2b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} + 16ABaab \\
& b^4c^2 + 2ABb^3c(-4ac - b^2)^3)^{1/2} - 4ABaab^2c(-4ac - b \\
& ^2)^3)^{1/2}) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} - (2x*(B^2b \\
& ^6 + 2A^2a^2c^4 + A^2b^4c^2 - 2B^2a^3c^3 - 2ABb^5c + 9B^2a^2b^2 \\
& b^2c^2 - 6B^2ab^4c - 4A^2ab^2c^3 + 10ABaab^3c^2 - 10ABa^2b \\
& *c^3)) / c^3 * (- (B^2b^7 + A^2b^5c^2 - B^2b^4(-4ac - b^2)^3)^{1/2} - 2 \\
& *ABb^6c + 25B^2a^2b^3c^2 - A^2b^2c^2(-4ac - b^2)^3)^{1/2} - B^ \\
& 2a^2c^2(-4ac - b^2)^3)^{1/2} + 16ABa^3c^4 - 9B^2ab^5c - 7A^2 \\
& *ab^3c^3 + 12A^2a^2b^3c^4 + A^2aac^3(-4ac - b^2)^3)^{1/2} - 20B^2 \\
& *a^3b^3c^3 - 36ABa^2b^2c^3 + 3B^2aab^2c(-4ac - b^2)^3)^{1/2} +
\end{aligned}$$

$$\begin{aligned}
& 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + ((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6))*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)} + (2*(B^3*a^4*c - B^3*a^3*b^2 + A*B^2*a^2*b^3 + A^2*B*a^3*c^2 + A^3*a^2*b*c^2 - 2*A^2*B*a^2*b^2*c))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a*b^4*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*2i + (B*x^3)/(3*c)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.86 \quad \int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=208

$$\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}} - Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}} - Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{Bx}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.53, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1279, 1166, 205}

$$\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}} - Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}} - Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{Bx}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +

1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{x^2 (A + Bx^2)}{a + bx^2 + cx^4} dx &= \frac{Bx}{c} - \frac{\int \frac{aB + (bB - Ac)x^2}{a + bx^2 + cx^4} dx}{c} \\ &= \frac{Bx}{c} - \frac{\left(bB - Ac - \frac{b^2B - Abc - 2aBc}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} - \frac{\left(bB - Ac + \frac{b^2B - Abc - 2aBc}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2c} \\ &= \frac{Bx}{c} - \frac{\left(bB - Ac - \frac{b^2B - Abc - 2aBc}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(bB - Ac + \frac{b^2B - Abc - 2aBc}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 251, normalized size = 1.21

$$\frac{\left(-Ac\sqrt{b^2 - 4ac} + bB\sqrt{b^2 - 4ac} + 2aBc + Abc + b^2(-B)\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\left(-Ac\sqrt{b^2 - 4ac} + bB\sqrt{b^2 - 4ac} - 2aBc - Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2 - 4ac} + b}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{\sqrt{b^2 - 4ac} + b}} + \frac{Bx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4), x]

[Out] (B*x)/c - (((-b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4), x]
```

fricas [B] time = 1.30, size = 2632, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)))
```

$$\begin{aligned} & / (b^2c^6 - 4ac^7)) * \sqrt{-(B^2b^3 + (4ABa + A^2b)c^2 - (3B^2ab + 2ABb^2)c - (b^2c^3 - 4ac^4) * \sqrt{(B^4b^4 + A^4c^4 - 2(A^2B^2a + 2A^3Bb)c^3 + (B^4a^2 + 4AB^3ab + 6A^2B^2b^2)c^2 - 2(B^4ab^2 + 2AB^3b^3)c) / (b^2c^6 - 4ac^7))} / (b^2c^3 - 4ac^4)) - \sqrt{(1/2) * c * \sqrt{-(B^2b^3 + (4ABa + A^2b)c^2 - (3B^2ab + 2ABb^2)c - (b^2c^3 - 4ac^4) * \sqrt{(B^4b^4 + A^4c^4 - 2(A^2B^2a + 2A^3Bb)c^3 + (B^4a^2 + 4AB^3ab + 6A^2B^2b^2)c^2 - 2(B^4ab^2 + 2AB^3b^3)c) / (b^2c^6 - 4ac^7))} / (b^2c^3 - 4ac^4)) * \log(2 * (B^4ab^2 - AB^3b^3 - 3A^3Bb^2c^2 + A^4c^3 - (B^4a^2 + AB^3ab - 3A^2B^2b^2)c) * x - \sqrt{(1/2) * (B^3b^4 - 4A^2B^2ac^3 + (4B^3a^2 + 8AB^2ab + A^2Bb^2)c^2 - (5B^3ab^2 + 2AB^2b^3)c + (Bb^3c^3 + 8A^2ac^5 - 2(2B^2ab + Ab^2)c^4) * \sqrt{(B^4b^4 + A^4c^4 - 2(A^2B^2a + 2A^3Bb)c^3 + (B^4a^2 + 4AB^3ab + 6A^2B^2b^2)c^2 - 2(B^4ab^2 + 2AB^3b^3)c) / (b^2c^6 - 4ac^7))} * \sqrt{-(B^2b^3 + (4ABa + A^2b)c^2 - (3B^2ab + 2ABb^2)c - (b^2c^3 - 4ac^4) * \sqrt{(B^4b^4 + A^4c^4 - 2(A^2B^2a + 2A^3Bb)c^3 + (B^4a^2 + 4AB^3ab + 6A^2B^2b^2)c^2 - 2(B^4ab^2 + 2AB^3b^3)c) / (b^2c^6 - 4ac^7))} / (b^2c^3 - 4ac^4))} + 2Bx) / c} \end{aligned}$$

giac [B] time = 3.50, size = 3179, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $Bx/c + 1/8 * ((2b^4c^3 - 16ab^2c^4 + 32a^2c^5 - \sqrt{2}) * \sqrt{b^2 - 4ac}) * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^4c + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b^2c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^3c^2 - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a^2c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b * c^3 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^2c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * c^4 - 2 * (b^2 - 4ac) * b^2c^3 + 8 * (b^2 - 4ac) * a * c^4) * A * c^2 - (2b^5c^2 - 16ab^3c^3 + 32a^2b^2c^4 - \sqrt{2}) * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^5 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b^3c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^4c - 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a^2 * b * c^2 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b^2c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * b^3c^2 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b * c^3 - 2 * (b^2 - 4ac) * b^3c^2 + 8 * (b^2 - 4ac) * a * b * c^3) * B * c^2 - 2 * (\sqrt{2}) * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b^4c^2 - 8 * \sqrt{2} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a^2 * b^2c^3 - 2 * \sqrt{2} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a * b^3c^3 + 2 * a * b^4c^3 + 16 * \sqrt{2} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a^3c^4 + 8 * \sqrt{2} * \sqrt{bc - \sqrt{b^2 - 4ac}} * a^2 * b * c^4 + \sqrt{2}) * \sqrt{bc - \sqrt{b^2 - 4ac}}$

$$\begin{aligned}
& a^2c^5 + 32a^3c^5 - 2(b^2 - 4ac)a^2c^3 + 8(b^2 - 4ac)a^2c^4)B\text{abs}(c) - (2b^4c^5 - 8a^2b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac})*b^4c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^2c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^2c^5 - 2(b^2 - 4ac)b^2c^5)A + (2b^5c^4 - 12a^2b^3c^5 + 16a^2b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^5c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^3c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^4c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^4 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^3c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^5 - 2(b^2 - 4ac)*b^3c^4 + 4(b^2 - 4ac)a^2b^2c^5)B) \arctan(2\sqrt{1/2}x/\sqrt{(b^2c^2 - 4ac^3)})/c^2)/((a^2b^4c^3 - 8a^2b^2c^4 - 2a^2b^3c^4 + 16a^3c^5 + 8a^2b^2c^5 + a^2b^2c^5 - 4a^2c^6)*c^2) - 1/8*((2b^4c^3 - 16a^2b^2c^4 + 32a^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^4c + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^3c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2c^4 - 2(b^2 - 4ac)*b^2c^3 + 8(b^2 - 4ac)a^2c^4)A^2 - (2b^5c^2 - 16a^2b^3c^3 + 32a^2b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^5 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^3c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^4c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^3c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^3 - 2(b^2 - 4ac)*b^3c^2 + 8(b^2 - 4ac)a^2b^2c^3)B^2 + 2(\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^4c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^3 - 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^3c^3 - 2a^2b^4c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^3c^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^4 + \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^4 + 16a^2b^2c^4 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2c^5 - 32a^3c^5 + 2(b^2 - 4ac)a^2b^2c^3 - 8(b^2 - 4ac)a^2c^4)B\text{abs}(c) - (2b^4c^5 - 8a^2b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^4c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*a^2b^2c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^2c^5 - 2(b^2 - 4ac)b^2c^5)A + (2b^5c^4 - 12a^2b^3c^5 + 16a^2b^2c^6 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*b^5c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac})*
\end{aligned}$$

$$\begin{aligned}
& (4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2 \\
& + 12AB^2a^2bc^2/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} - (2x(B^2b^4 - 2A^2ac^3 + A^2b^2c^2 + 2B^2a^2c^2 - 2AB^2b^3c - 4B^2ab^2c + 6AB^2a^2bc^2))/c \\
& *(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} \\
& + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (((16B^2a^2c^3 - 4B^2ab^2c^2)/c + (2x(4b^3c^3 - 16ab^2c^4))*(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}))/c \\
& *(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} \\
& + (2x(B^2b^4 - 2A^2ac^3 + A^2b^2c^2 + 2B^2a^2c^2 - 2AB^2b^3c - 4B^2ab^2c + 6AB^2a^2bc^2))/c *(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} \\
& + (2x(B^2b^4 - 2A^2ac^3 + A^2b^2c^2 + 2B^2a^2c^2 - 2AB^2b^3c - 4B^2ab^2c + 6AB^2a^2bc^2))/c *(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} \\
& - (2(A^3ac^2 - B^3a^2b + AB^2a^2b^2 + AB^2a^2c - 2A^2B^2a^2bc))/c *(-B^2b^5 + A^2b^3c^2 - A^2c^2(-4ac - b^2)^{3/2} - B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c + B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 + 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} \\
& *i - \operatorname{atan}\left(\frac{((16B^2a^2c^3 - 4B^2ab^2c^2)/c - (2x(4b^3c^3 - 16ab^2c^4))*(-B^2b^5 + A^2b^3c^2 + A^2c^2(-4ac - b^2)^{3/2} + B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c - B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 - 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}}{(2x(4b^3c^3 - 16ab^2c^4))*(-B^2b^5 + A^2b^3c^2 + A^2c^2(-4ac - b^2)^{3/2} + B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c - B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 - 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}}\right) \\
& *i - \left(\frac{(16B^2a^2c^3 - 4B^2ab^2c^2)/c + (2x(4b^3c^3 - 16ab^2c^4))*(-B^2b^5 + A^2b^3c^2 + A^2c^2(-4ac - b^2)^{3/2} + B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c - B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 - 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}}{(2x(4b^3c^3 - 16ab^2c^4))*(-B^2b^5 + A^2b^3c^2 + A^2c^2(-4ac - b^2)^{3/2} + B^2b^2(-4ac - b^2)^{3/2} - 2AB^2b^4c - 16AB^2a^2c^3 - 4A^2ab^2c^3 - 7B^2a^2b^3c - B^2a^2c^2(-4ac - b^2)^{3/2} + 12B^2a^2bc^2 - 2AB^2bc^2(-4ac - b^2)^{3/2} + 12AB^2a^2bc^2)/(8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}}\right) *i
\end{aligned}$$

$$\begin{aligned}
& (1/2) + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4 \\
& *A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*B^2*a^2 \\
& *b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2)/(8*(16*a^2 \\
& *c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c \\
& - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B* \\
& a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (2*x*(B^2*b^4 \\
& - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + \\
& 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4* \\
& A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*B^2*a^2 \\
& *b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2)/(8*(16*a^2* \\
& c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*ii)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/ \\
& c - (2*x*(4*b^3*c^3 - 16*a*b*c^4))*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c - 16*A* \\
& B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2 \\
&)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}/c)*(-(B^2*b^5 + A^2*b^3 \\
& *c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} - (\\
& 2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4* \\
& B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c - 16*A*B \\
& *a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2 \\
&)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (((16*B*a^2*c^3 - 4*B*a \\
& *b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4))*(-(B^2*b^5 + A^2*b^3*c^2 + A^2* \\
& c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4 \\
& *c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A* \\
& B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}/c)*(-(B^2*b^5 \\
& + A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - \\
& B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) \\
& ^{(1/2)} + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B* \\
& b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 + A^2*c \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4* \\
& c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B \\
& *a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} - (2*(A^3*a*c^2 \\
& - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c)*(-(B^2*b^5 +
\end{aligned}$$

$$A^2*b^3*c^2 + A^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c - B^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*B^2*a^2*b*c^2 - 2*A*B*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)*2i}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.87 \quad \int \frac{A+Bx^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=172

$$\frac{\left(B - \frac{bB-2Ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}} + B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1166, 205}

$$\frac{\left(B - \frac{bB-2Ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}} + B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4), x]

[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4} dx = \frac{1}{2} \left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx + \frac{1}{2} \left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}} dx$$

$$= \frac{\left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] time = 0.09, size = 173, normalized size = 1.01

$$\frac{\left(B\sqrt{b^2 - 4ac} + 2Ac - bB \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(B\sqrt{b^2 - 4ac} - 2Ac + bB \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac + b}} \right)}{\sqrt{b^2 - 4ac + b}}$$

$$\frac{\hspace{10em}}{\sqrt{2} \sqrt{c} \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4), x]

[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4), x]

fricas [B] time = 1.33, size = 1569, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2

$$\begin{aligned}
& *c - 4*a^2*c^2)) * \log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + \sqrt{1/2}*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c) * \sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)}) * \sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))) - 1/2*\sqrt{1/2}*\sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))*\log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - \sqrt{1/2}*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c) * \sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)}) * \sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))) + 1/2*\sqrt{1/2}*\sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))*\log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + \sqrt{1/2}*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c) * \sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)}) * \sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))) - 1/2*\sqrt{1/2}*\sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2))*\log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - \sqrt{1/2}*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c) * \sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)}) * \sqrt{-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3))})/(a*b^2*c - 4*a^2*c^2)))
\end{aligned}$$

giac [B] time = 2.42, size = 1400, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $1/4*((\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 2*b^4*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)$

$$\begin{aligned}
& - 4*a*c)*c)*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c) \\
& *b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& + \sqrt{b^2 - 4*a*c})*a*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& (b^2 - 4*a*c)*c)*a^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*a*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c) \\
& *a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b + \sqrt{b^2 - 4*a*c})/c})/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 \\
& + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c)) + 1/4*((\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*b^4 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*b^3*c + 2*b^4*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& (b^2 - 4*a*c)*c)*a^2*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 \\
& + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 \\
& + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b - \sqrt{b^2 - 4*a*c})/c})/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c))
\end{aligned}$$

maple [B] time = 0.02, size = 328, normalized size = 1.91

$$\frac{\sqrt{2} A c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} A c \arctan\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} B b \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} B b \arctan\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} B \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} B \arctan\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(c*x^4+b*x^2+a), x)

[Out]
$$\begin{aligned}
& -c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A-1/2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B+1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+1/2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B+1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^2 + A}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.00, size = 4109, normalized size = 23.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(a + b*x^2 + c*x^4),x)

[Out] - atan(((((-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + (((-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b*c^3)*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((((-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b

$$\begin{aligned}
& \wedge 2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * \\
& c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) - ((-(B^2 * a * b^3 + B^2 \\
& * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c - A^2 * c * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + \\
& 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * c) / (8 * (16 * a^3 * \\
& c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * (4 * A * b^2 * c^2 + x * (8 * b^3 * c^2 - 32 * a * b \\
& * c^3)) * (- (B^2 * a * b^3 + B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c - A^2 * c * (- (\\
& 4 * a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * \\
& A * B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) - 16 * A * a * c^3 \\
&) + x * (4 * A^2 * c^3 - 4 * B^2 * a * c^2 + 2 * B^2 * b^2 * c - 4 * A * B * b * c^2)) * (- (B^2 * a * b^3 + \\
& B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c - A^2 * c * (- (4 * a * c - b^2) \wedge 3) \wedge (1/2) \\
&) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * c) / (8 * (16 * \\
& a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) + 2 * A^2 * B * c^2 + 2 * B^3 * a * c - 2 * A * \\
& B^2 * b * c)) * (- (B^2 * a * b^3 + B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c - A^2 * c * \\
& (- (4 * a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c \\
& - 4 * A * B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * 2i - \text{atan} \\
& (\frac{((-(B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (-4 * \\
& a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * \\
& B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * (x * (8 * b^3 * c^2 \\
& - 32 * a * b * c^3)) * (- (B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A \\
& ^2 * c * (- (4 * a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * \\
& b * c - 4 * A * B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) - 4 * \\
& A * b^2 * c^2 + 16 * A * a * c^3) + x * (4 * A^2 * c^3 - 4 * B^2 * a * c^2 + 2 * B^2 * b^2 * c - 4 * A * B * \\
& b * c^2)) * (- (B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (\\
& - (4 * a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - \\
& 4 * A * B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * 1i + ((-(B \\
& ^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (- (4 * a * c - b^ \\
& 2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * \\
& c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * (4 * A * b^2 * c^2 + x * (8 * b^ \\
& 3 * c^2 - 32 * a * b * c^3)) * (- (B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * \\
& c + A^2 * c * (- (4 * a * c - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^ \\
& 2 * a^2 * b * c - 4 * A * B * a * b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) \\
&) - 16 * A * a * c^3) + x * (4 * A^2 * c^3 - 4 * B^2 * a * c^2 + 2 * B^2 * b^2 * c - 4 * A * B * b * c^2)) * \\
& (- (B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (- (4 * a * c \\
& - b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * \\
& b^2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * 1i) / (((-(B^2 * a * b^3 \\
& - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (- (4 * a * c - b^2) \wedge 3) \wedge (1 \\
& / 2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * c) / (8 * (1 \\
& 6 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) * (x * (8 * b^3 * c^2 - 32 * a * b * c^3)) * (- \\
& (B^2 * a * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (- (4 * a * c - \\
& b^2) \wedge 3) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^ \\
& 2 * c) / (8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) - 4 * A * b^2 * c^2 + 16 * A \\
& * a * c^3) + x * (4 * A^2 * c^3 - 4 * B^2 * a * c^2 + 2 * B^2 * b^2 * c - 4 * A * B * b * c^2)) * (- (B^2 * a \\
& * b^3 - B^2 * a * (-4 * a * c - b^2) \wedge 3) \wedge (1/2) + A^2 * b^3 * c + A^2 * c * (- (4 * a * c - b^2) \wedge 3 \\
&) \wedge (1/2) + 16 * A * B * a^2 * c^2 - 4 * A^2 * a * b * c^2 - 4 * B^2 * a^2 * b * c - 4 * A * B * a * b^2 * c) / (\\
& 8 * (16 * a^3 * c^3 - 8 * a^2 * b^2 * c^2 + a * b^4 * c)) \wedge (1/2) - ((-(B^2 * a * b^3 - B^2 * a * (-
\end{aligned}$$

$$\begin{aligned}
& (4ac - b^2)^3)^{1/2} + A^2b^3c + A^2c(-4ac - b^2)^3)^{1/2} + 16A^2 \\
& B^2a^2c^2 - 4A^2ab^2c - 4B^2a^2b^2c - 4AB^2ab^2c)/(8(16a^3c^3 - \\
& 8a^2b^2c^2 + ab^4c))^{1/2} * (4A^2b^2c^2 + x(8b^3c^2 - 32ab^2c^3) \\
& * (-B^2ab^3 - B^2a(-4ac - b^2)^3)^{1/2} + A^2b^3c + A^2c(-4ac - \\
& b^2)^3)^{1/2} + 16A^2B^2a^2c^2 - 4A^2ab^2c - 4B^2a^2b^2c - 4AB^2a \\
& b^2c)/(8(16a^3c^3 - 8a^2b^2c^2 + ab^4c))^{1/2} - 16A^2ac^3 + x \\
& * (4A^2c^3 - 4B^2a^2c^2 + 2B^2b^2c - 4AB^2b^2c^2) * (-B^2ab^3 - B^2a \\
& a(-4ac - b^2)^3)^{1/2} + A^2b^3c + A^2c(-4ac - b^2)^3)^{1/2} + 1 \\
& 6A^2B^2a^2c^2 - 4A^2ab^2c - 4B^2a^2b^2c - 4AB^2ab^2c)/(8(16a^3c^3 \\
& - 8a^2b^2c^2 + ab^4c))^{1/2} + 2A^2B^2c^2 + 2B^3a^2c - 2AB^2b^2 \\
& * c) * (-B^2ab^3 - B^2a(-4ac - b^2)^3)^{1/2} + A^2b^3c + A^2c(-4 \\
& ac - b^2)^3)^{1/2} + 16A^2B^2a^2c^2 - 4A^2ab^2c - 4B^2a^2b^2c - 4A \\
& B^2ab^2c)/(8(16a^3c^3 - 8a^2b^2c^2 + ab^4c))^{1/2} * 2i
\end{aligned}$$

sympy [A] time = 16.96, size = 314, normalized size = 1.83

$$\text{RootSum}\left(t^4(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c) + t^2(-16A^2ab^2c + 4A^2b^3c + 64AB^2a^2c^2 - 16AB^2b^2c - 16B^2a^2b^2c + 4B^2ab^3) + A^4c^2 - 2A^3Bbc + 2A^2B^2ac + A^2B^2b^2 - 2AB^3ab + B^4a^2, t \mapsto t \log\left(x + \frac{-32t^3Aa^2bc^2 + 8t^3Aab^3c + 64t^3Ba^2b^2c - 16t^3Bb^2b^2c - 4t^3Aa^2c^2 + 2t^3A^3b^2c - 6t^3A^2Babc + 12t^3AB^2a^2c - 2t^3B^3a^2b}{-A^4c^2 + A^3Bbc - AB^3ab + B^4a^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a), x)

[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-16*A**2*a*b*c**2 + 4*A**2*b**3*c + 64*A*B*a**2*c**2 - 16*A*B*a*b**2*c - 16*B**2*a**2*b*c + 4*B**2*a*b**3) + A**4*c**2 - 2*A**3*B*b*c + 2*A**2*B**2*a*c + A**2*B**2*b**2 - 2*A*B**3*a*b + B**4*a**2, Lambda(_t, _t*log(x + (-32*_t**3*A*a**2*b*c**2 + 8*_t**3*A*a*b**3*c + 64*_t**3*B*a**3*c**2 - 16*_t**3*B*a**2*b**2*c - 4*_t*A**3*a*c**2 + 2*_t*A**3*b**2*c - 6*_t*A**2*B*a*b*c + 12*_t*A*B**2*a**2*c - 2*_t*B**3*a**2*b)/(-A**4*c**2 + A**3*B*b*c - A*B**3*a*b + B**4*a**2))))

$$3.88 \quad \int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=189

$$-\frac{\sqrt{c} \left(\frac{Ab-2aB}{\sqrt{b^2-4ac}} + A \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab-2aB}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{A}{ax}$$

Rubi [A] time = 0.40, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c} \left(\frac{Ab-2aB}{\sqrt{b^2-4ac}} + A \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab-2aB}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{A}{ax}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1281

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)

)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^2(a + bx^2 + cx^4)} dx &= -\frac{A}{ax} - \frac{\int \frac{Ab - aB + Acx^2}{a + bx^2 + cx^4} dx}{a} \\ &= -\frac{A}{ax} - \frac{\left(c \left(A - \frac{Ab - 2aB}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2a} - \frac{\left(c \left(A + \frac{Ab - 2aB}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2a} \\ &= -\frac{A}{ax} - \frac{\sqrt{c} \left(A + \frac{Ab - 2aB}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}a\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(A - \frac{Ab - 2aB}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}a\sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.29, size = 206, normalized size = 1.09

$$\frac{\sqrt{2}\sqrt{c} \left(A \left(\sqrt{b^2 - 4ac} + b\right) - 2aB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c} \left(A \left(\sqrt{b^2 - 4ac} - b\right) + 2aB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2 - 4ac} + b}\right)}{\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} + b} + \frac{2A}{x}$$

2a

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] -1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

$$\begin{aligned}
& - A^3 a^2 b) c^2 - (4 B^3 a^4 - 12 A B^2 a^3 b + 13 A^2 B a^2 b^2 - 5 A^3 a b^3) c + (B a^4 b^3 - A a^3 b^4 - 8 A a^5 c^2 - 2(2 B a^5 b - 3 A a^4 b^2) c) \sqrt{(B^4 a^4 - 4 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - 4 A^3 B a b^3 + A^4 b^4 + A^4 a^2 c^2 - 2(A^2 B^2 a^3 - 2 A^3 B a^2 b + A^4 a b^2) c) / (a^6 b^2 - 4 a^7 c)} \\
& \sqrt{-(B^2 a^2 b - 2 A B a b^2 + A^2 b^3 + (4 A B a^2 - 3 A^2 a b) c - (a^3 b^2 - 4 a^4 c) \sqrt{(B^4 a^4 - 4 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - 4 A^3 B a b^3 + A^4 b^4 + A^4 a^2 c^2 - 2(A^2 B^2 a^3 - 2 A^3 B a^2 b + A^4 a b^2) c) / (a^6 b^2 - 4 a^7 c)}}) / (a^3 b^2 - 4 a^4 c)) - \sqrt{1/2} \\
& a x \sqrt{-(B^2 a^2 b - 2 A B a b^2 + A^2 b^3 + (4 A B a^2 - 3 A^2 a b) c - (a^3 b^2 - 4 a^4 c) \sqrt{(B^4 a^4 - 4 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - 4 A^3 B a b^3 + A^4 b^4 + A^4 a^2 c^2 - 2(A^2 B^2 a^3 - 2 A^3 B a^2 b + A^4 a b^2) c) / (a^6 b^2 - 4 a^7 c)}}) / (a^3 b^2 - 4 a^4 c)} \\
& \log(2(A^4 a c^3 + (A^3 B a b - A^4 b^2) c^2 - (B^4 a^3 - 3 A B^3 a^2 b + 3 A^2 B^2 a b^2 - A^3 B b^3) c) x - \sqrt{1/2} (B^3 a^3 b^2 - 3 A B^2 a^2 b^3 + 3 A^2 B a b^4 - A^3 b^5 + 4(A^2 B a^3 - A^3 a^2 b) c^2 - (4 B^3 a^4 - 12 A B^2 a^3 b + 13 A^2 B a^2 b^2 - 5 A^3 a b^3) c + (B a^4 b^3 - A a^3 b^4 - 8 A a^5 c^2 - 2(2 B a^5 b - 3 A a^4 b^2) c) \sqrt{(B^4 a^4 - 4 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - 4 A^3 B a b^3 + A^4 b^4 + A^4 a^2 c^2 - 2(A^2 B^2 a^3 - 2 A^3 B a^2 b + A^4 a b^2) c) / (a^6 b^2 - 4 a^7 c)}}) \sqrt{-(B^2 a^2 b - 2 A B a b^2 + A^2 b^3 + (4 A B a^2 - 3 A^2 a b) c - (a^3 b^2 - 4 a^4 c) \sqrt{(B^4 a^4 - 4 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - 4 A^3 B a b^3 + A^4 b^4 + A^4 a^2 c^2 - 2(A^2 B^2 a^3 - 2 A^3 B a^2 b + A^4 a b^2) c) / (a^6 b^2 - 4 a^7 c)}}) / (a^3 b^2 - 4 a^4 c)) - 2 A) / (a x)
\end{aligned}$$

giac [B] time = 3.57, size = 2805, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-A/(a x) - 1/8((2 b^4 c^2 - 16 a b^2 c^3 + 32 a^2 c^4 - \sqrt{2} \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} c) b^4 + 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a b^2 c + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) b^3 c - 16 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a^2 c^2 - 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a b c^2 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) b^2 c^2 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a c^3 - 2(b^2 - 4 a c) b^2 c^2 + 8(b^2 - 4 a c) a c^3) A a^2 + 2(\sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a b^5 - 8 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a^2 b^3 c - 2 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a b^4 c - 2 a b^5 c + 16 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a^3 b c^2 + 8 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a^2 b^2 c^2 + \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a b^3 c^2 + 16 a^2 b^3 c^2 - 4 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c) a^2 b c^3 - 32 a^3 b c^3 + 2(b^2 - 4 a c) a b^3 c - 8(b^2 - 4 a c)$

$$\begin{aligned}
& c) * a^2 * b * c^2) * A * \text{abs}(a) - 2 * (\text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^4 \\
& - 8 * \text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^2 * c - 2 * \text{sqrt}(2) * \text{sqrt}(b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^3 * c - 2 * a^2 * b^4 * c + 16 * \text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^4 * c^2 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b \\
& * c^2 + \text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^2 * c^2 + 16 * a^3 * b^2 * c^2 \\
& - 4 * \text{sqrt}(2) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * c^3 - 32 * a^4 * c^3 + 2 * (b^2 \\
& - 4 * a * c) * a^2 * b^2 * c - 8 * (b^2 - 4 * a * c) * a^3 * c^2) * B * \text{abs}(a) + (2 * a^2 * b^4 * c^2 - 8 \\
& * a^3 * b^2 * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^4 \\
& + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^2 * \\
& * c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^3 * c \\
& - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^2 * c^2 - 2 \\
& * (b^2 - 4 * a * c) * a^2 * b^2 * c^2) * A - 2 * (2 * a^3 * b^3 * c^2 - 8 * a^4 * b * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^3 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^4 * b * c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^2 * c - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b * c^2 - 2 * (b^2 - 4 * a * c) * a^3 * b * c^2) * B) * \arctan(2 * \text{sqrt}(1/2) * x / \text{sqrt}((a * b + \text{sqrt}(a^2 * b^2 - 4 * a^3 * c)) / (a * c))) / ((a^3 * b^4 - 8 * a^4 * b^2 * c - 2 * a^3 * b^3 * c + 16 * a^5 * c^2 + 8 * a^4 * b * c^2 + a^3 * b^2 * c^2 - 4 * a^4 * c^3) * \text{abs}(a) * \text{abs}(c)) + 1/8 * ((2 * b^4 * c^2 - 16 * a * b^2 * c^3 + 32 * a^2 * c^4 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * b^4 + 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b^2 * c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * b^3 * c - 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * c^2 - 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b * c^2 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * b^2 * c^2 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * c^3 - 2 * (b^2 - 4 * a * c) * b^2 * c^2 + 8 * (b^2 - 4 * a * c) * a * c^3) * A * a^2 - 2 * (\text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b^5 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^3 * c - 2 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b^4 * c + 2 * a * b^5 * c + 16 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b * c^2 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^2 * c^2 + \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b^3 * c^2 - 16 * a^2 * b^3 * c^2 - 4 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b * c^3 + 32 * a^3 * b * c^3 - 2 * (b^2 - 4 * a * c) * a * b^3 * c + 8 * (b^2 - 4 * a * c) * a^2 * b * c^2) * A * \text{abs}(a) + 2 * (\text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^4 - 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^2 * c - 2 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^3 * c + 2 * a^2 * b^4 * c + 16 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^4 * c^2 + 8 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b * c^2 + \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^2 * c^2 - 16 * a^3 * b^2 * c^2 - 4 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * c^3 + 32 * a^4 * c^3 - 2 * (b^2 - 4 * a * c) * a^2 * b^2 * c + 8 * (b^2 - 4 * a * c) * a^3 * c^2) * B * \text{abs}(a) + (2 * a^2 * b^4 * c^2 - 8 * a^3 * b^2 * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^4 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^2 * c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^3 * c - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^2 * b^2 * c^2 - 2 * (b^2 - 4 * a * c) * a^2 * b^2 * c^2) * A - 2 * (2 * a^3 * b^3 * c^2 - 8 * a^4 * b * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c)) * c) * a^3 * b^3 + 4 * s
\end{aligned}$$

$\text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 b^2 c + 2 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 b^2 c - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 b^2 c^2 - 2(b^2 - 4ac) \cdot a^3 b^2 c^2 \cdot B \cdot \arctan\left(\frac{2 \sqrt{1/2} \cdot x / \sqrt{(a^2 b^2 - 4a^3 c)}}{(a^3 b^4 - 8a^4 b^2 c - 2a^3 b^3 c + 16a^5 c^2 + 8a^4 b^2 c^2 + a^3 b^2 c^2 - 4a^4 c^3)} \cdot \frac{a}{c}\right) \cdot \frac{a}{c}$

maple [B] time = 0.03, size = 353, normalized size = 1.87

$$\frac{\sqrt{2} A b c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}a} + \frac{\sqrt{2} A b c \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}a} - \frac{\sqrt{2} B c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} B c \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} A c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c}a} - \frac{\sqrt{2} A c \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c}a} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^2+A)/x^2/(c*x^4+b*x^2+a), x)$

[Out] $-A/a/x + 1/2/a*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*A + 1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*A*b-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*B - 1/2/a*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*A + 1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*A*b-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}* \operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})c)^{(1/2)}*c*x)*B$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((B*x^2+A)/x^2/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] $-\text{integrate}((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)$

mupad [B] time = 1.35, size = 6335, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)), x)$

[Out] $-\operatorname{atan}\left(\frac{(x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A^2*a^2*b^2*c^2}{(x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A^2*a^2*b^2*c^2}\right)$

$$\begin{aligned}
& *c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2 \\
& *c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(16*B*a^6*c^3 + x*(32*a \\
& ^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 \\
& - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A \\
& ^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a \\
& ^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c \\
& ^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - \\
& 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a \\
& ^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b \\
& ^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 - \\
& 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7 \\
& *A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2 \\
& *b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 \\
& + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*2i - \operatorname{atan}(((x*(4*A^2*a^4*c^4 - 4*B^2*a \\
& ^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 - \\
& A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A* \\
& B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(x*(32*a^6 \\
& *b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 \\
& - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2 \\
& *a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3 \\
& *b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - \\
& 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16 \\
& *A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3 \\
& *b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b \\
& ^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*1i + (x*(4*A^2*a^4*c^4 \\
& - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2 \\
& *a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} \\
& *(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 \\
& - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A \\
& *B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} - 16*A*a^5 \\
& *b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 - A \\
& ^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B \\
& *a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} -
\end{aligned}$$

$$\begin{aligned}
& 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12 \\
& *A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*1i)/((x*(4* \\
& A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-A^2 \\
& *b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2* \\
& c))^{(1/2)}*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 - A^2 \\
& *b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a* \\
& b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^3)^{(1/2)} - 4 \\
& *B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^{(1/2)} + 12*A \\
& *B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} - 16*B*a^6*c^ \\
& 3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a \\
& ^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2)^3)^ \\
& (1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} - \\
& (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + \\
& (-A^2*b^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a \\
& *c*(-(4*a*c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a \\
& ^4*b^2*c))^{(1/2)}*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b \\
& ^5 + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) \\
&))^{(1/2)} - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 \\
& + B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(\\
& 1/2)} + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 - 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + \\
& B^2*a^2*b^3 - A^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c + A^2*a*c*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 + 2*A*B*a*b*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1 \\
& /2)}*2i - A/(a*x)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.89 \quad \int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=271

$$\frac{\sqrt{c} \left(aB \left(\sqrt{b^2 - 4ac} + b \right) - A \left(b\sqrt{b^2 - 4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{c} \left(aB \left(b - \sqrt{b^2 - 4ac} \right) - A \left(-b\sqrt{b^2 - 4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} + \sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 0.65, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1281, 1166, 205}

$$\frac{\sqrt{c} \left(aB \left(\sqrt{b^2 - 4ac} + b \right) - A \left(b\sqrt{b^2 - 4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{c} \left(aB \left(b - \sqrt{b^2 - 4ac} \right) - A \left(-b\sqrt{b^2 - 4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) + \frac{Ab - aB}{a^2 x} - \frac{A}{3ax^3}}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} + \sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]

[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1281

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)

)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^4(a + bx^2 + cx^4)} dx &= -\frac{A}{3ax^3} - \frac{\int \frac{3(Ab - aB) + 3Acx^2}{x^2(a + bx^2 + cx^4)} dx}{3a} \\ &= -\frac{A}{3ax^3} + \frac{Ab - aB}{a^2x} + \frac{\int \frac{3(Ab^2 - abB - aAc) + 3(Ab - aB)cx^2}{a + bx^2 + cx^4} dx}{3a^2} \\ &= -\frac{A}{3ax^3} + \frac{Ab - aB}{a^2x} + \frac{\left(c \left(aB \left(b - \sqrt{b^2 - 4ac} \right) - A \left(b^2 - 2ac - b\sqrt{b^2 - 4ac} \right) \right) \right) \int \frac{b + \frac{1}{2}}{2a^2\sqrt{b^2 - 4ac}}}{2a^2\sqrt{b^2 - 4ac}} \\ &= -\frac{A}{3ax^3} + \frac{Ab - aB}{a^2x} - \frac{\sqrt{c} \left(aB \left(b + \sqrt{b^2 - 4ac} \right) - A \left(b^2 - 2ac + b\sqrt{b^2 - 4ac} \right) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.32, size = 267, normalized size = 0.99

$$\frac{3\sqrt{2}\sqrt{c} \left(aB \left(\sqrt{b^2 - 4ac} + b \right) - A \left(b\sqrt{b^2 - 4ac} - 2ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{3\sqrt{2}\sqrt{c} \left(A \left(b\sqrt{b^2 - 4ac} + 2ac - b^2 \right) + aB \left(b - \sqrt{b^2 - 4ac} \right) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{b^2 - 4ac} \sqrt{\sqrt{b^2 - 4ac} + b}} + \frac{6Ab - 6aB}{x} - \frac{2aA}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)), x]

[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*sqrt[2]*sqrt[c]*(a*B*(b + sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c + b*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]]])/(sqrt[b^2 - 4*a*c]*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*sqrt[c]*(a*B*(b - sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]]])/(sqrt[b^2 - 4*a*c]*sqrt[b + sqrt[b^2 - 4*a*c]]))/(6*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)), x]

fricas [B] time = 3.55, size = 5442, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\frac{1}{6} \left(3 \sqrt{\frac{1}{2}} a^2 x^3 \sqrt{-(B^2 a^2 b^3 - 2 A B a^2 b^4 + A^2 b^5 - (4 A B a^3 - 5 A^2 a^2 b) c^2 - (3 B^2 a^3 b - 8 A B a^2 b^2 + 5 A^2 a b^3) c + (a^5 b^2 - 4 a^6 c) \sqrt{(B^4 a^4 b^4 - 4 A B^3 a^3 b^5 + 6 A^2 B^2 a^2 b^6 - 4 A^3 B a^2 b^7 + A^4 b^8 + A^4 a^4 c^4 - 2 (A^2 B^2 a^5 - 4 A^3 B a^4 b + 3 A^4 a^3 b^2) c^3 + (B^4 a^6 - 8 A B^3 a^5 b + 24 A^2 B^2 a^4 b^2 - 28 A^3 B a^3 b^3 + 11 A^4 a^2 b^4) c^2 - 2 (B^4 a^5 b^2 - 6 A B^3 a^4 b^3 + 12 A^2 B^2 a^3 b^4 - 10 A^3 B a^2 b^5 + 3 A^4 a b^6) c)} / (a^{10} b^2 - 4 a^{11} c)} \right) / (a^5 b^2 - 4 a^6 c) \log(2 (A^4 a^2 c^5 + 3 (A^3 B a^2 b - A^4 a b^2) c^4 - (B^4 a^4 - 5 A B^3 a^3 b + 6 A^2 B^2 a^2 b^2 - A^3 B a^2 b^3 - A^4 b^4) c^3 + (B^4 a^3 b^2 - 3 A B^3 a^2 b^3 + 3 A^2 B^2 a b^4 - A^3 B b^5) c^2) * x + \sqrt{\frac{1}{2}} (B^3 a^3 b^5 - 3 A B^2 a^2 b^6 + 3 A^2 B a^2 b^7 - A^3 b^8 - 4 A^3 a^4 c^4 + (4 A B^2 a^5 - 20 A^2 B a^4 b + 17 A^3 a^3 b^2) c^3 + (4 B^3 a^5 b - 25 A B^2 a^4 b^2 + 41 A^2 B a^3 b^3 - 20 A^3 a^2 b^4) c^2 - (5 B^3 a^4 b^3 - 18 A B^2 a^3 b^4 + 21 A^2 B a^2 b^5 - 8 A^3 a b^6) c - (B a^6 b^4 - A a^5 b^5 + 4 (2 B a^8 - 3 A a^7 b) c^2 - (6 B a^7 b^2 - 7 A a^6 b^3) c) \sqrt{(B^4 a^4 b^4 - 4 A B^3 a^3 b^5 + 6 A^2 B^2 a^2 b^6 - 4 A^3 B a^2 b^7 + A^4 b^8 + A^4 a^4 c^4 - 2 (A^2 B^2 a^5 - 4 A^3 B a^4 b + 3 A^4 a^3 b^2) c^3 + (B^4 a^6 - 8 A B^3 a^5 b + 24 A^2 B^2 a^4 b^2 - 28 A^3 B a^3 b^3 + 11 A^4 a^2 b^4) c^2 - 2 (B^4 a^5 b^2 - 6 A B^3 a^4 b^3 + 12 A^2 B^2 a^3 b^4 - 10 A^3 B a^2 b^5 + 3 A^4 a b^6) c)} / (a^{10} b^2 - 4 a^{11} c)} \right) \sqrt{-(B^2 a^2 b^3 - 2 A B a^2 b^4 + A^2 b^5 - (4 A B a^3 - 5 A^2 a^2 b) c^2 - (3 B^2 a^3 b - 8 A B a^2 b^2 + 5 A^2 a b^3) c + (a^5 b^2 - 4 a^6 c) \sqrt{(B^4 a^4 b^4 - 4 A B^3 a^3 b^5 + 6 A^2 B^2 a^2 b^6 - 4 A^3 B a^2 b^7 + A^4 b^8 + A^4 a^4 c^4 - 2 (A^2 B^2 a^5 - 4 A^3 B a^4 b + 3 A^4 a^3 b^2) c^3 + (B^4 a^6 - 8 A B^3 a^5 b + 24 A^2 B^2 a^4 b^2 - 28 A^3 B a^3 b^3 + 11 A^4 a^2 b^4) c^2 - 2 (B^4 a^5 b^2 - 6 A B^3 a^4 b^3 + 12 A^2 B^2 a^3 b^4 - 10 A^3 B a^2 b^5 + 3 A^4 a b^6) c)} / (a^{10} b^2 - 4 a^{11} c)} \right) / (a^5 b^2 - 4 a^6 c) \right) - 3 \sqrt{\frac{1}{2}} a^2 x^3 \sqrt{-(B^2 a^2 b^3 - 2 A B a^2 b^4 + A^2 b^5 - (4 A B a^3 - 5 A^2 a^2 b) c^2 - (3 B^2 a^3 b - 8 A B a^2 b^2 + 5 A^2 a b^3) c + (a^5 b^2 - 4 a^6 c) \sqrt{(B^4 a^4 b^4 - 4 A B^3 a^3 b^5 + 6 A^2 B^2 a^2 b^6 - 4 A^3 B a^2 b^7 + A^4 b^8 + A^4 a^4 c^4 - 2 (A^2 B^2 a^5 - 4 A^3 B a^4 b + 3 A^4 a^3 b^2) c^3 + (B^4 a^6 - 8 A B^3 a^5 b + 24 A^2 B^2 a^4 b^2 - 28 A^3 B a^3 b^3 + 11 A^4 a^2 b^4) c^2 - 2 (B^4 a^5 b^2 - 6 A B^3 a^4 b^3 + 12 A^2 B^2 a^3 b^4 - 10 A^3 B a^2 b^5 + 3 A^4 a b^6) c)} / (a^{10} b^2 - 4 a^{11} c)} \right) / (a^5 b^2 - 4 a^6 c) \right)$$

$$\begin{aligned}
& 2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))/(a^5*b^2 - 4*a^6*c))*\log(2* \\
& (A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + \\
& 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2 \\
& 2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - \sqrt{1/2}*(B^3*a^3*b^5 - 3*A* \\
& B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A \\
& ^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2 \\
& *B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A \\
& ^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a \\
& ^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*\sqrt{((B^4*a^4*b^4 - 4*A*B^3*a^3* \\
& b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2 \\
& 2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24* \\
& A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - \\
& 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/ \\
& (a^{10}*b^2 - 4*a^{11}*c)))*\sqrt{-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B \\
& *a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (\\
& a^5*b^2 - 4*a^6*c)*\sqrt{((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 \\
& - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + \\
& 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3 \\
& *B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2 \\
& 2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))/ \\
& (a^5*b^2 - 4*a^6*c)) + 3*\sqrt{1/2}*a^2*x^3*\sqrt{-(B^2*a^2*b^3 - 2*A*B*a*b^4 \\
& + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 \\
& + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*\sqrt{((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 \\
& + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 \\
& - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2 \\
& 2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A \\
& *B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10} \\
& 0*b^2 - 4*a^{11}*c)))/(a^5*b^2 - 4*a^6*c))*\log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2* \\
& b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a \\
& *b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3 \\
& 3*B*b^5)*c^2)*x + \sqrt{1/2}*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 \\
& - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2) \\
& *c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4) \\
& *c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)* \\
& c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7 \\
& *A*a^6*b^3)*c)*\sqrt{((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4* \\
& A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4 \\
& 4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a \\
& ^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2 \\
& 2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))*\sqrt{ \\
& -(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3 \\
& *B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*\sqrt{((B^4 \\
& *a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + \\
& A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 \\
& - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)
\end{aligned}$$

$$\begin{aligned}
& *c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c))/(a^5*b^2 - 4*a^6*c)) - 3*\text{sqrt}(1/2)*a^2*x^3*\text{sqrt}(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c))*\text{sqrt}((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))/(a^5*b^2 - 4*a^6*c))*\log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - \text{sqrt}(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*\text{sqrt}((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))*\text{sqrt}(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c))*\text{sqrt}((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^{10}*b^2 - 4*a^{11}*c)))/(a^5*b^2 - 4*a^6*c)) - 6*(B*a - A*b)*x^2 - 2*A*a)/(a^2*x^3)
\end{aligned}$$

giac [B] time = 3.35, size = 2870, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{4} * ((\text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * b^6 - 9 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a * b^4 * c - 2 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * b^5 * c - 2 * b^6 * c + 24 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a^2 * b^2 * c^2 + 10 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a * b^3 * c^2 + \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * b^4 * c^2 + 18 * a * b^4 * c^2 + 2 * b^5 * c^2 - 16 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a^3 * c^3 - 8 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a^2 * b * c^3 - 5 * \text{sqrt}(2) * \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) * a * b^2 * c^3 - 48 * a^2 * b^2 * c^3$

$$\begin{aligned}
& c^3 - 14ab^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}c)a^2c^4 + 32 \\
& a^3c^4 + 24a^2b^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2c^4 + 7\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2b^3c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2b^4c^2 - 12\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2b^2c^2 - 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} \\
& a^2b^3c^2 + 3\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac}a^2b^3c^3 + 2(b^2 - 4ac) \\
& b^4c^3 - 10(b^2 - 4ac)a^2b^2c^2 - 2(b^2 - 4ac) \\
& b^3c^2 + 8(b^2 - 4ac)a^2c^3 + 6(b^2 - 4ac) \\
& a^2b^3c^3)A - (\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^5 - 8\sqrt{2} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^3 - 2\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^4c^2 - 2a^2b^5c^2 + 16\sqrt{2} \\
& \sqrt{b^2 - 4ac}c)a^3b^2c^2 + 8\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 + \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^2 + 16a^2b^3c^2 + 2a^2b^4c^2 - 4\sqrt{2} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^3 - 32a^3b^2c^3 - 12a^2b^2c^3 + 16a^3c^4 - \sqrt{2} \\
& \sqrt{b^2 - 4ac}c)a^2b^4 + 6\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^3c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2c^3 + 2(b^2 - 4ac)a^2b^3c^2 - 8(b^2 - 4ac) \\
& a^2b^2c^2 - 2(b^2 - 4ac)a^2b^2c^2 + 4(b^2 - 4ac) \\
& a^2c^3)B) \arctan(2\sqrt{1/2}x / \sqrt{(a^2b + \sqrt{a^4b^2 - 4a^5c}) / (a^2c)}) / ((a^3b^4 - 8a^4b^2c - 2a^3b^3c + 16a^5c^2 + 8a^4b^2c^2 + a^3b^2c^2 - 4a^4c^3) \operatorname{abs}(c)) \\
& + 1/4((\sqrt{2}\sqrt{b^2 - 4ac}c)b^6 - 9\sqrt{2}\sqrt{b^2 - 4ac}c) \\
& a^2b^4c^2 - 2\sqrt{2}\sqrt{b^2 - 4ac}c)b^5c^2 + 2b^6c^2 + 24\sqrt{2}\sqrt{b^2 - 4ac}c) \\
& a^2b^2c^2 + 10\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^3c^2 + \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)b^4c^2 - 18a^2b^4c^2 - 2b^5c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^3c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^3 + 48a^2b^2c^3 + 14a^2b^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2c^4 - 32a^3c^4 - 24a^2b^3c^4 + \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)b^5 - 7\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^2 - 2\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)b^4c^2 + 12\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^2c^2 + \sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)b^3c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^3 - 2(b^2 - 4ac) \\
& b^4c^3 + 10(b^2 - 4ac)a^2b^2c^2 + 2(b^2 - 4ac) \\
& b^3c^2 - 8(b^2 - 4ac)a^2c^3 - 6(b^2 - 4ac) \\
& a^2b^3c^3)A - (\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^5 - 8\sqrt{2} \\
& \sqrt{b^2 - 4ac}c)a^2b^3c^3 - 2\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{b^2 - 4ac}c)a^2b^4c^2 + 2a^2b^5c^2 + 16\sqrt{2}
\end{aligned}$$

```

sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2
- 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 +
2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c
- 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)
) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)

```

maple [B] time = 0.03, size = 611, normalized size = 2.25

$$\frac{\sqrt{A^2 \operatorname{arctanh}\left(\frac{d \cdot c}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}}{\sqrt{4ac+d^2} \sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{A^2 \operatorname{arctan}\left(\frac{d \cdot c}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{A^2 \operatorname{arctanh}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{4ac+d^2} \sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{A^2 \operatorname{arctan}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctanh}\left(\frac{d \cdot c}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctan}\left(\frac{d \cdot c}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctanh}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{4ac+d^2} \sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctan}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctanh}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{\sqrt{Bc \operatorname{arctan}\left(\frac{d \cdot c}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}}{\sqrt{(b+\sqrt{4ac+d^2})}} \cdot \frac{Ab}{\sqrt{4ac+d^2}} \cdot \frac{B}{\sqrt{4ac+d^2}} \cdot \frac{A}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a), x)

```

[Out] -1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*A*b^2+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+1/a*c
^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*A*b^2-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B+1/2/a*c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*B

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(Ba - Ab)cx^2 + Bab - Ab^2 + Aac}{cx^4 + bx^2 + a} dx - \frac{3(Ba - Ab)x^2 + Aa}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)

mupad [B] time = 2.19, size = 10101, normalized size = 37.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)

[Out]
$$-\frac{A}{3a} - \frac{x^2(Ab - B^2)}{a^2} \frac{1}{x^3} - \operatorname{atan}\left(\frac{(-(A^2b^7 + B^2a^2b^5 + A^2b^4(-4ac - b^2)^3)^{1/2} - 2ABab^6 + 25A^2a^2b^3c^2 + A^2a^2c^2(-4ac - b^2)^3)^{1/2} + B^2a^2b^2(-4ac - b^2)^3)^{1/2} + 16ABa^4c^3 - 9A^2ab^5c - 20A^2a^3bc^3 - 7B^2a^3b^3c + 12B^2a^4bc^2 - B^2a^3c(-4ac - b^2)^3)^{1/2} - 36ABa^3b^2c^2 - 3A^2ab^2c(-4ac - b^2)^3)^{1/2} - 2ABab^3(-4ac - b^2)^3)^{1/2} + 16ABa^2b^4c + 4ABa^2b^2c(-4ac - b^2)^3)^{1/2}}{(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2}} \cdot (16Aa^{10}c^4 + x(32a^{11}bc^3 - 8a^{10}b^3c^2) \cdot (-(A^2b^7 + B^2a^2b^5 + A^2b^4(-4ac - b^2)^3)^{1/2} - 2ABab^6 + 25A^2a^2b^3c^2 + A^2a^2c^2(-4ac - b^2)^3)^{1/2} + B^2a^2b^2(-4ac - b^2)^3)^{1/2} + 16ABa^4c^3 - 9A^2ab^5c - 20A^2a^3bc^3 - 7B^2a^3b^3c + 12B^2a^4bc^2 - B^2a^3c(-4ac - b^2)^3)^{1/2} - 36ABa^3b^2c^2 - 3A^2ab^2c(-4ac - b^2)^3)^{1/2} - 2ABab^3(-4ac - b^2)^3)^{1/2} + 16ABa^2b^4c + 4ABa^2b^2c(-4ac - b^2)^3)^{1/2}}{(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2}} + 16Ba^{10}bc^3 + 4Aa^8b^4c^2 - 20Aa^9b^2c^3 - 4Ba^9b^3c^2 - x(4A^2a^8c^5 - 4B^2a^9c^4 + 2A^2a^6b^4c^3 - 8A^2a^7b^2c^4 + 2B^2a^8b^2c^3 - 4ABa^7b^3c^3 + 12ABa^8bc^4) \cdot (-(A^2b^7 + B^2a^2b^5 + A^2b^4(-4ac - b^2)^3)^{1/2} - 2ABab^6 + 25A^2a^2b^3c^2 + A^2a^2c^2(-4ac - b^2)^3)^{1/2} + B^2a^2b^2(-4ac - b^2)^3)^{1/2} + 16ABa^4c^3 - 9A^2ab^5c - 20A^2a^3bc^3 - 7B^2a^3b^3c + 12B^2a^4bc^2 - B^2a^3c(-4ac - b^2)^3)^{1/2} - 36ABa^3b^2c^2 - 3A^2ab^2c(-4ac - b^2)^3)^{1/2} - 2ABab^3(-4ac - b^2)^3)^{1/2} + 16ABa^2b^4c + 4ABa^2b^2c(-4ac - b^2)^3)^{1/2}}{(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2}} \cdot i - \left(\frac{(-(A^2b^7 + B^2a^2b^5 + A^2b^4(-4ac - b^2)^3)^{1/2} - 2ABab^6 + 25A^2a^2b^3c^2 + A^2a^2c^2(-4ac - b^2)^3)^{1/2} + B^2a^2b^2(-4ac - b^2)^3)^{1/2} + 16ABa^4c^3 - 9A^2ab^5c - 20A^2a^3bc^3 - 7B^2a^3b^3c + 12B^2a^4bc^2 - B^2a^3c(-4ac - b^2)^3)^{1/2} - 36ABa^3b^2c^2 - 3A^2ab^2c(-4ac - b^2)^3)^{1/2} - 2ABab^3(-4ac - b^2)^3)^{1/2} + 16ABa^2b^4c + 4ABa^2b^2c(-4ac - b^2)^3)^{1/2}}{(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2}} \cdot (16Aa^{10}c^4 - x(32a^{11}bc^3 - 8a^{10}b^3c^2) \cdot (-(A^2b^7 + B^2a^2b^5 + A^2b^4(-4ac - b^2)^3)^{1/2} - 2ABab^6 + 25A^2a^2b^3c^2 + A^2a^2c^2(-4ac - b^2)^3)^{1/2} + B^2a^2b^2(-4ac - b^2)^3)^{1/2} + 16ABa^4c^3 - 9A^2ab^5c - 20A^2a^3bc^3 - 7B^2a^3b^3c + 12B^2a^4bc^2 - B^2a^3c(-4ac - b^2)^3)^{1/2} - 36ABa^3b^2c^2 - 3A^2ab^2c(-4ac - b^2)^3)^{1/2} - 2ABab^3(-4ac - b^2)^3)^{1/2} + 16ABa^2b^4c + 4ABa^2b^2c(-4ac - b^2)^3)^{1/2}}{(8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2}}\right)$$

$$\begin{aligned}
& c^2) * (- (A^2 * b^7 + B^2 * a^2 * b^5 + A^2 * b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * \\
& b^6 + 25 * A^2 * a^2 * b^3 * c^2 + A^2 * a^2 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + B^2 * a^2 * b \\
& ^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^4 * c^3 - 9 * A^2 * a * b^5 * c - 20 * A^2 * a^3 * b \\
& * c^3 - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 * a^4 * b * c^2 - B^2 * a^3 * c * (- (4 * a * c - b^2)^3)^{(1 \\
& / 2)} - 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a \\
& * b^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^2 * b^4 * c + 4 * A * B * a^2 * b * c * (- (4 * a * c - \\
& b^2)^3)^{(1/2)}) / (8 * (a^5 * b^4 + 16 * a^7 * c^2 - 8 * a^6 * b^2 * c))^{(1/2)} + 16 * B * a^{10} \\
& * b * c^3 + 4 * A * a^8 * b^4 * c^2 - 20 * A * a^9 * b^2 * c^3 - 4 * B * a^9 * b^3 * c^2) + x * (4 * A^2 * a \\
& ^8 * c^5 - 4 * B^2 * a^9 * c^4 + 2 * A^2 * a^6 * b^4 * c^3 - 8 * A^2 * a^7 * b^2 * c^4 + 2 * B^2 * a^8 * \\
& b^2 * c^3 - 4 * A * B * a^7 * b^3 * c^3 + 12 * A * B * a^8 * b * c^4) * (- (A^2 * b^7 + B^2 * a^2 * b^5 + \\
& A^2 * b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^6 + 25 * A^2 * a^2 * b^3 * c^2 + A^2 * \\
& a^2 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + B^2 * a^2 * b^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 1 \\
& 6 * A * B * a^4 * c^3 - 9 * A^2 * a * b^5 * c - 20 * A^2 * a^3 * b * c^3 - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 \\
& * a^4 * b * c^2 - B^2 * a^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 \\
& * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + \\
& 16 * A * B * a^2 * b^4 * c + 4 * A * B * a^2 * b * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (a^5 * b^4 + 1 \\
& 6 * a^7 * c^2 - 8 * a^6 * b^2 * c))^{(1/2)} * i) / (((- (A^2 * b^7 + B^2 * a^2 * b^5 + A^2 * b^4 * (- \\
& (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^6 + 25 * A^2 * a^2 * b^3 * c^2 + A^2 * a^2 * c^2 * (- \\
& (4 * a * c - b^2)^3)^{(1/2)} + B^2 * a^2 * b^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^4 * \\
& c^3 - 9 * A^2 * a * b^5 * c - 20 * A^2 * a^3 * b * c^3 - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 * a^4 * b * c^2 \\
& - B^2 * a^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 * a * b^2 * c * \\
& (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^2 * \\
& b^4 * c + 4 * A * B * a^2 * b * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (a^5 * b^4 + 16 * a^7 * c^2 \\
& - 8 * a^6 * b^2 * c))^{(1/2)} * (16 * A * a^{10} * c^4 + x * (32 * a^{11} * b * c^3 - 8 * a^{10} * b^3 * c^2) * \\
& (- (A^2 * b^7 + B^2 * a^2 * b^5 + A^2 * b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^6 + \\
& 25 * A^2 * a^2 * b^3 * c^2 + A^2 * a^2 * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + B^2 * a^2 * b^2 * (- \\
& (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^4 * c^3 - 9 * A^2 * a * b^5 * c - 20 * A^2 * a^3 * b * c^3 \\
& - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 * a^4 * b * c^2 - B^2 * a^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - \\
& 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^3 * \\
& (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^2 * b^4 * c + 4 * A * B * a^2 * b * c * (- (4 * a * c - b^2) \\
& ^3)^{(1/2)}) / (8 * (a^5 * b^4 + 16 * a^7 * c^2 - 8 * a^6 * b^2 * c))^{(1/2)} + 16 * B * a^{10} * b * c^ \\
& 3 + 4 * A * a^8 * b^4 * c^2 - 20 * A * a^9 * b^2 * c^3 - 4 * B * a^9 * b^3 * c^2) - x * (4 * A^2 * a^8 * c^ \\
& 5 - 4 * B^2 * a^9 * c^4 + 2 * A^2 * a^6 * b^4 * c^3 - 8 * A^2 * a^7 * b^2 * c^4 + 2 * B^2 * a^8 * b^2 * c \\
& ^3 - 4 * A * B * a^7 * b^3 * c^3 + 12 * A * B * a^8 * b * c^4) * (- (A^2 * b^7 + B^2 * a^2 * b^5 + A^2 * \\
& b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^6 + 25 * A^2 * a^2 * b^3 * c^2 + A^2 * a^2 * c \\
& ^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + B^2 * a^2 * b^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B \\
& * a^4 * c^3 - 9 * A^2 * a * b^5 * c - 20 * A^2 * a^3 * b * c^3 - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 * a^4 * \\
& b * c^2 - B^2 * a^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 * a * b \\
& ^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 2 * A * B * a * b^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A \\
& * B * a^2 * b^4 * c + 4 * A * B * a^2 * b * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (a^5 * b^4 + 16 * a^7 \\
& * c^2 - 8 * a^6 * b^2 * c))^{(1/2)} + ((- (A^2 * b^7 + B^2 * a^2 * b^5 + A^2 * b^4 * (- (4 * a * c \\
& - b^2)^3)^{(1/2)} - 2 * A * B * a * b^6 + 25 * A^2 * a^2 * b^3 * c^2 + A^2 * a^2 * c^2 * (- (4 * a * c \\
& - b^2)^3)^{(1/2)} + B^2 * a^2 * b^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 16 * A * B * a^4 * c^3 - 9 * \\
& A^2 * a * b^5 * c - 20 * A^2 * a^3 * b * c^3 - 7 * B^2 * a^3 * b^3 * c + 12 * B^2 * a^4 * b * c^2 - B^2 * a \\
& ^3 * c * (- (4 * a * c - b^2)^3)^{(1/2)} - 36 * A * B * a^3 * b^2 * c^2 - 3 * A^2 * a * b^2 * c * (- (4 * a * c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c \\
& + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6* \\
& b^2*c))^{(1/2)}*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b \\
& ^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2* \\
& a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2* \\
& a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B* \\
& a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^3*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2} \\
&))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} + 16*B*a^10*b*c^3 + 4*A* \\
& a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^ \\
& 2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A \\
& *B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4 \\
& a*c - b^2)^3)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 \\
& - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - \\
& B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b \\
& ^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8 \\
& *a^6*b^2*c))^{(1/2)} + 2*B^3*a^8*c^4 + 2*A^2*B*a^7*c^5 - 2*A^3*a^6*b*c^5 - 4 \\
& *A*B^2*a^7*b*c^4 + 2*A^2*B*a^6*b^2*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^ \\
& 4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c \\
& ^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2* \\
& c*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B* \\
& a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^ \\
& 2 - 8*a^6*b^2*c))^{(1/2)}*2i - \operatorname{atan}(\left(\left(\left(-A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^ \\
& 3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + \\
& B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2* \\
& b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - \\
& 8*a^6*b^2*c))^{(1/2)}*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(- \\
& (A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 2 \\
& 5*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - \\
& 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 3 \\
& 6*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3 \\
&)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c))^{(1/2)} + 16*B*a^10*b*c^3 \\
& + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^5 \\
& - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 \\
& - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^
\end{aligned}$$

$$\begin{aligned}
& 4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a \\
& ^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b* \\
& c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2 \\
& *c*(-(4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B \\
& *a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c \\
& ^2 - 8*a^6*b^2*c)))^{(1/2)}*1i - (((-A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9 \\
& *A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2* \\
& a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c \\
& - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6 \\
& *b^2*c)))^{(1/2)}*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2))*(-(A^2*b \\
& ^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2 \\
& *a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2 \\
& *a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B \\
& *a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^{(1/2)} + 16*B*a^10*b*c^3 + 4*A \\
& *a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B \\
& ^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4* \\
& A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^ \\
& 3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + \\
& B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2* \\
& b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - \\
& 8*a^6*b^2*c)))^{(1/2)}*1i)/(((- (A^2*b^7 + B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9*A^2* \\
& a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3*b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c - 4* \\
& A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2* \\
& c)))^{(1/2)}*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2))*(-(A^2*b^7 + \\
& B^2*a^2*b^5 - A^2*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 2*A*B*a*b^6 + 25*A^2*a^2* \\
& b^3*c^2 - A^2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3* \\
& b^3*c + 12*B^2*a^4*b*c^2 + B^2*a^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 36*A*B*a^3* \\
& b^2*c^2 + 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 2*A*B*a*b^3*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 16*A*B*a^2*b^4*c - 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(\\
& 8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^{(1/2)} + 16*B*a^10*b*c^3 + 4*A*a^8*
\end{aligned}$$

$$\begin{aligned}
& b^4c^2 - 20Aa^9b^2c^3 - 4B^2a^9b^3c^2) - x(4A^2a^8c^5 - 4B^2a^8 \\
& 9c^4 + 2A^2a^6b^4c^3 - 8A^2a^7b^2c^4 + 2B^2a^8b^2c^3 - 4A^2B^2a^7b^3c^3 + 12A^2B^2a^8b^3c^4) * (- (A^2b^7 + B^2a^2b^5 - A^2b^4 * (- (4ac - b^2)^3)^{1/2} - 2A^2B^2a^2b^6 + 25A^2a^2b^3c^2 - A^2a^2c^2 * (- (4ac - b^2)^3)^{1/2} - B^2a^2b^2 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^4c^3 - 9A^2a^2b^5c - 20A^2a^3b^3c^3 - 7B^2a^3b^3c + 12B^2a^4b^3c^2 + B^2a^3c * (- (4ac - b^2)^3)^{1/2} - 36A^2B^2a^3b^2c^2 + 3A^2a^2b^2c * (- (4ac - b^2)^3)^{1/2} + 2A^2B^2a^2b^3 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^2b^4c - 4A^2B^2a^2b^2c * (- (4ac - b^2)^3)^{1/2}) / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2} + ((- (A^2b^7 + B^2a^2b^5 - A^2b^4 * (- (4ac - b^2)^3)^{1/2} - 2A^2B^2a^2b^6 + 25A^2a^2b^3c^2 - A^2a^2c^2 * (- (4ac - b^2)^3)^{1/2} - B^2a^2b^2 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^4c^3 - 9A^2a^2b^5c - 20A^2a^3b^3c^3 - 7B^2a^3b^3c + 12B^2a^4b^3c^2 + B^2a^3c * (- (4ac - b^2)^3)^{1/2} - 36A^2B^2a^3b^2c^2 + 3A^2a^2b^2c * (- (4ac - b^2)^3)^{1/2} + 2A^2B^2a^2b^3 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^2b^4c - 4A^2B^2a^2b^2c * (- (4ac - b^2)^3)^{1/2}) / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2} * (16Aa^10c^4 - x(32a^11b^3c^3 - 8a^10b^3c^2) * (- (A^2b^7 + B^2a^2b^5 - A^2b^4 * (- (4ac - b^2)^3)^{1/2} - 2A^2B^2a^2b^6 + 25A^2a^2b^3c^2 - A^2a^2c^2 * (- (4ac - b^2)^3)^{1/2} - B^2a^2b^2 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^4c^3 - 9A^2a^2b^5c - 20A^2a^3b^3c^3 - 7B^2a^3b^3c + 12B^2a^4b^3c^2 + B^2a^3c * (- (4ac - b^2)^3)^{1/2} - 36A^2B^2a^3b^2c^2 + 3A^2a^2b^2c * (- (4ac - b^2)^3)^{1/2} + 2A^2B^2a^2b^3 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^2b^4c - 4A^2B^2a^2b^2c * (- (4ac - b^2)^3)^{1/2}) / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2} + 16B^2a^10b^3c^3 + 4A^2a^8b^4c^2 - 20A^2a^9b^2c^3 - 4B^2a^9b^3c^2) + x(4A^2a^8c^5 - 4B^2a^8c^4 + 2A^2a^6b^4c^3 - 8A^2a^7b^2c^4 + 2B^2a^8b^2c^3 - 4A^2B^2a^7b^3c^3 + 12A^2B^2a^8b^3c^4) * (- (A^2b^7 + B^2a^2b^5 - A^2b^4 * (- (4ac - b^2)^3)^{1/2} - 2A^2B^2a^2b^6 + 25A^2a^2b^3c^2 - A^2a^2c^2 * (- (4ac - b^2)^3)^{1/2} - B^2a^2b^2 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^4c^3 - 9A^2a^2b^5c - 20A^2a^3b^3c^3 - 7B^2a^3b^3c + 12B^2a^4b^3c^2 + B^2a^3c * (- (4ac - b^2)^3)^{1/2} - 36A^2B^2a^3b^2c^2 + 3A^2a^2b^2c * (- (4ac - b^2)^3)^{1/2} + 2A^2B^2a^2b^3 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^2b^4c - 4A^2B^2a^2b^2c * (- (4ac - b^2)^3)^{1/2}) / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2} + 2B^3a^8c^4 + 2A^2B^2a^7c^5 - 2A^3a^6b^3c^5 - 4A^2B^2a^7b^3c^4 + 2A^2B^2a^6b^2c^4) * (- (A^2b^7 + B^2a^2b^5 - A^2b^4 * (- (4ac - b^2)^3)^{1/2} - 2A^2B^2a^2b^6 + 25A^2a^2b^3c^2 - A^2a^2c^2 * (- (4ac - b^2)^3)^{1/2} - B^2a^2b^2 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^4c^3 - 9A^2a^2b^5c - 20A^2a^3b^3c^3 - 7B^2a^3b^3c + 12B^2a^4b^3c^2 + B^2a^3c * (- (4ac - b^2)^3)^{1/2} - 36A^2B^2a^3b^2c^2 + 3A^2a^2b^2c * (- (4ac - b^2)^3)^{1/2} + 2A^2B^2a^2b^3 * (- (4ac - b^2)^3)^{1/2} + 16A^2B^2a^2b^4c - 4A^2B^2a^2b^2c * (- (4ac - b^2)^3)^{1/2}) / (8(a^5b^4 + 16a^7c^2 - 8a^6b^2c))^{1/2} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.90 \quad \int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=212

$$\frac{(12a^2Bc^2 + 6aAbc^2 - 12ab^2Bc - Ab^3c + 2b^4B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + x^2(-6aBc - Abc + 2b^2B) - x^4(x^2(-2aBc - 2Abc + b^2B) + a(bB - 2Ac))}{2c^3(b^2 - 4ac)^{3/2}} + \frac{x^2(-6aBc - Abc + 2b^2B)}{2c^2(b^2 - 4ac)} - \frac{x^4(x^2(-2aBc - 2Abc + b^2B) + a(bB - 2Ac))}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2bB - Ac) \log(a + bx^2 + cx^4)}{4c^3}$$

Rubi [A] time = 0.38, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$\frac{(12a^2Bc^2 + 6aAbc^2 - 12ab^2Bc - Ab^3c + 2b^4B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + x^2(-6aBc - Abc + 2b^2B) - x^4(x^2(-2aBc - 2Abc + b^2B) + a(bB - 2Ac))}{2c^3(b^2 - 4ac)^{3/2}} + \frac{x^2(-6aBc - Abc + 2b^2B)}{2c^2(b^2 - 4ac)} - \frac{x^4(x^2(-2aBc - 2Abc + b^2B) + a(bB - 2Ac))}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(2bB - Ac) \log(a + bx^2 + cx^4)}{4c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4)) + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^4(a(bB-2Ac) + (b^2B - Abc - 2aBc)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst} \left(\int \frac{x(2a(bB-2Ac) + (2b^2B - Abc - 6aBc)x)}{a+bx+cx^2} \right)}{2c(b^2-4ac)} \\
&= \frac{(2b^2B - Abc - 6aBc)x^2}{2c^2(b^2-4ac)} - \frac{x^4(a(bB-2Ac) + (b^2B - Abc - 2aBc)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst} \left(\int \frac{-a}{a+bx+cx^2} \right)}{2c(b^2-4ac)} \\
&= \frac{(2b^2B - Abc - 6aBc)x^2}{2c^2(b^2-4ac)} - \frac{x^4(a(bB-2Ac) + (b^2B - Abc - 2aBc)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(2bB - Ac)S}{2c(b^2-4ac)} \\
&= \frac{(2b^2B - Abc - 6aBc)x^2}{2c^2(b^2-4ac)} - \frac{x^4(a(bB-2Ac) + (b^2B - Abc - 2aBc)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(2bB - Ac)l}{2c(b^2-4ac)} \\
&= \frac{(2b^2B - Abc - 6aBc)x^2}{2c^2(b^2-4ac)} - \frac{x^4(a(bB-2Ac) + (b^2B - Abc - 2aBc)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{(2b^4B - Ab^3)}{2c^2(b^2-4ac)}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 208, normalized size = 0.98

$$\frac{-\frac{2(a^2c(2c(A+Bx^2)-3bB)+ab(-bc(A+4Bx^2)+3Ac^2x^2+b^2B)+b^3x^2(bB-Ac))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + (Ac-2bB) \log(a+bx^2+cx^4) + 2Bcx^2}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c))*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/(-b^2 + 4*a*c)^(3/2) + (-2*b*B + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 0.78, size = 1323, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b \\ & + A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 + 7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(\\ & 2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^2 \\ & - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2 \\ & *(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(\\ & 4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log \\ & (c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2), \\ & -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b \\ & + A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 + 7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6 \\ & *(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)* \\ & x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 \\ & - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 \\ & - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 \\ & + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)] \end{aligned}$$

giac [A] time = 1.65, size = 239, normalized size = 1.13

$$\frac{Bx^2}{2c^2} + \frac{(2Bb^4 - 12Bab^2c - Ab^3c + 12Ba^2c^2 + 6Aabc^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) + 2Bb^3x^4 - 8Babcx^4 - Ab^2cx^4 + 4Aac^2x^4 + Ab^3x^2 - 4Ba^2cx^2 - 2Aabcx^2 - 2Ba^2b + Aab^2}{2(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}} + \frac{2Bb^3x^4 - 8Babcx^4 - Ab^2cx^4 + 4Aac^2x^4 + Ab^3x^2 - 4Ba^2cx^2 - 2Aabcx^2 - 2Ba^2b + Aab^2}{4(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} - \frac{(2Bb - Ac) \log(cx^4 + bx^2 + a)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

maple [B] time = 0.02, size = 689, normalized size = 3.25



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
[Out] 1/2*B*x^2/c^2+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*A*b-1/2/c^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x^2*A*b^3+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*B-2/c^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*B+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x^2*b^4*B+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*A-1/2/c^2/(c*x^4+b*x^2+a)*
a/(4*a*c-b^2)*A*b^2-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*B+1/2/c^3/(c*
x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*B+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*A-1/4/
c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)
*a*b*B+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*B-3/c/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*B+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*B*a*b^2+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^3*A-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.83, size = 2282, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x)$

[Out]
$$\begin{aligned} & ((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(\\ & B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c - b \\ & ^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (\log(a + b*x^2 + c* \\ & x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - \\ & 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 - \\ & 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (\text{atan}(((8*a*c^5*(4*a*c - b^ \\ & 2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12 \\ & *B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c \\ & ^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A \\ & *a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4 \\ & *a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5 \\ &)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(8*c^3 \\ & *(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - \\ & A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c \\ & - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192 \\ & *B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c \\ & ^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(\\ & (4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 - 5*A^2*a*b*c^3 - 20 \\ & *B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a*c^5 - b^2*c^4) + (\\ & ((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4 \\ &)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 \\ & - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^ \\ & 2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 \\ & + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - \\ & 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B \\ & *a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5 \\ &)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b \\ & *c^2 - 12*B*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4)))/(2*a*(4 \\ & *a*c - b^2)^(3/2)) + (((8*A*a*c^4 - 16*B*a*b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 \\ & + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b* \\ & c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(256*a^3*c^6 - 4*b^6*c^3 + 48* \\ & a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c \\ & ^2 - 12*B*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*B*b^4 + 12*B*a^2*c^ \\ & 2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^ \\ & 6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + \\ & 192*B*a^2*b^3*c^2))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a* \\ & b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(((8*A*a*c^4 - 16*B*a* \\ & b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + \\ & 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2)))/(\\ & (256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*B*b^7 + 128* \end{aligned}$$

$$\frac{A^3c^4 - 2Ab^6c - 48B^2a^2b^3c^2 + 24A^2ab^4c^2 - 256B^2a^3b^3c^3 - 96A^2a^2b^2c^3 + 192B^2a^2b^3c^2)}{(2(256a^3c^6 - 4b^6c^3 + 48a^2b^4c^4 - 192a^2b^2c^5)) - (A^2ac^2 + 4B^2ab^2 - 4AB^2abc)/c^4 + (a(2B^2b^4 + 12B^2a^2c^2 - Ab^3c + 6A^2ab^2c^2 - 12B^2ab^2c)^2)/(c^4(4ac - b^2)^3)))/(2a(4ac - b^2)^{3/2})/(4B^2b^8 + A^2b^6c^2 + 144B^2a^4c^4 - 4AB^2b^7c + 36A^2a^2b^2c^4 + 192B^2a^2b^4c^2 - 288B^2a^3b^2c^3 - 48B^2ab^6c - 12A^2ab^4c^3 - 168AB^2a^2b^3c^3 + 48AB^2ab^5c^2 + 144AB^2a^3b^4c^4)*(2B^2b^4 + 12B^2a^2c^2 - Ab^3c + 6A^2ab^2c^2 - 12B^2ab^2c))/(2c^3(4ac - b^2)^{3/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.91 \quad \int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=147

$$\frac{x^2 \left(x^2 (-2aBc - Abc + b^2B) + a(bB - 2Ac) \right)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(4aAc^2 - 6abBc + b^3B) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{B \log(a + bx^2 + cx^4)}{4c^2}$$

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 818, 634, 618, 206, 628}

$$\frac{(4aAc^2 - 6abBc + b^3B) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c^2(b^2 - 4ac)^{3/2}} - \frac{x^2 \left(x^2 (-2aBc - Abc + b^2B) + a(bB - 2Ac) \right)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{B \log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))] + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^2 (a(bB - 2Ac) + (b^2B - Abc - 2aBc) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{\text{Subst} \left(\int \frac{a(bB - 2Ac) + B(b^2 - 4ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c (b^2 - 4ac)} \\
&= -\frac{x^2 (a(bB - 2Ac) + (b^2B - Abc - 2aBc) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{B \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^2} - \frac{(b^3B - 6abBc + 4a^2c^2)}{4c^2} \\
&= -\frac{x^2 (a(bB - 2Ac) + (b^2B - Abc - 2aBc) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{B \log(a + bx^2 + cx^4)}{4c^2} + \frac{(b^3B - 6abBc + 4a^2c^2)}{4c^2} \\
&= -\frac{x^2 (a(bB - 2Ac) + (b^2B - Abc - 2aBc) x^2)}{2c (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{(b^3B - 6abBc + 4a^2c^2) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2c^2 (b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 160, normalized size = 1.09

$$\frac{-\frac{2(2a^2Bc + a(bc(A + 3Bx^2) - 2Ac^2x^2 + b^2(-B)) + b^2x^2(Ac - bB))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2(4aAc^2 - 6abBc + b^3B) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}} + B \log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c))*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + B*Log[a + b*x^2 + c*x^4]/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 1.01, size = 849, normalized size = 5.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]

giac [A] time = 1.62, size = 194, normalized size = 1.32

$$-\frac{(Bb^3 - 6Babc + 4Aac^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(b^2c^2 - 4ac^3)\sqrt{-b^2+4ac}} + \frac{B \log(cx^4 + bx^2 + a)}{4c^2} - \frac{Bb^2cx^4 - 4Bac^2x^4 - Bb^3x^2 + 2Babcx^2 + 2Ab^2cx^2 - 4Aac^2x^2 - Bab^2 + 2Aabc}{4(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3))

maple [B] time = 0.01, size = 286, normalized size = 1.95

$$\frac{2Aa \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} - \frac{3Bab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}c} + \frac{Bb^3 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(4ac-b^2)^{\frac{3}{2}}c^2} + \frac{Ba \ln(cx^4 + bx^2 + a)}{(4ac-b^2)c} - \frac{Bb^2 \ln(cx^4 + bx^2 + a)}{4(4ac-b^2)c^2} + \frac{\frac{(2aAc^2 - Ab^2c - 3abBc + b^3B)x^2}{(4ac-b^2)c^2} + \frac{(Abc + 2aBc - b^2B)a}{(4ac-b^2)c^2}}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)$

[Out] $\frac{1}{2}*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)/c*\ln(c*x^4+b*x^2+a)*a*B-1/4/(4*a*c-b^2)/c^2*\ln(c*x^4+b*x^2+a)*b^2*B+2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*a*A-3/(4*a*c-b^2)^{(3/2)}/c*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*a*b*B+1/2/(4*a*c-b^2)^{(3/2)}/c^2*arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.22, size = 1527, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)$

[Out] $-\frac{((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) - (a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (\log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (\text{atan}(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*(((8*B*a + 8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)))/(8*c^2*(4*a*c - b^2)^{(3/2)}) + (a*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*a*c - b^2)^{(3/2)}*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*(((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(8*c^2*(4*a*c - b^2)^{(3/2)}) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(16*c^2*(4*a$

$$\begin{aligned}
& *c - b^2)^{(3/2)} * (4*a*c^3 - b^2*c^2) * (256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 \\
& - 192*a^2*b^2*c^4)) / (a*(4*a*c - b^2)) + (b*((B^2*b^3 + 2*A*B*a*c^2 - 5*B^2 \\
& *a*b*c) / (4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3) / (\\
& 4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5) * (2*B*b^6 - 128*B*a^3*c^3 - 2 \\
& 4*B*a*b^4*c + 96*B*a^2*b^2*c^2)) / (2*(4*a*c^3 - b^2*c^2) * (256*a^3*c^5 - 4*b^6 \\
& *c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))) * (2*B*b^6 - 128*B*a^3*c^3 - 24*B*a \\
& *b^4*c + 96*B*a^2*b^2*c^2)) / (2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 19 \\
& 2*a^2*b^2*c^4)) - (((b^3*c^4) / 2 - 2*a*b*c^5) * (B*b^3 + 4*A*a*c^2 - 6*B*a*b*c \\
&)^2) / (c^4 * (4*a*c - b^2)^3 * (4*a*c^3 - b^2*c^2))) / (2*a*(4*a*c - b^2)^{(3/2})) \\
& + (b*((8*B*a + (8*a*c^2 * (2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2 \\
& *b^2*c^2)) / (256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) * (2* \\
& B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2)) / (2*(256*a^3*c^5 - \\
& 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (B^2*a) / c^2 - (a*(B*b^3 + 4 \\
& *A*a*c^2 - 6*B*a*b*c)^2) / (c^2 * (4*a*c - b^2)^3)) / (2*a*(4*a*c - b^2)^{(3/2})) \\
&) / (B^2*b^6 + 16*A^2*a^2*c^4 + 36*B^2*a^2*b^2*c^2 - 12*B^2*a*b^4*c + 8*A*B*a \\
& *b^3*c^2 - 48*A*B*a^2*b*c^3) * (B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)) / (2*c^2 * (4*a* \\
& c - b^2)^{(3/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.92 \quad \int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=107

$$-\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} - \frac{x^2(-2aBc - Abc + b^2B) + a(bB - 2Ac)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1251, 777, 618, 206}

$$\frac{x^2(-2aBc - Abc + b^2B) + a(bB - 2Ac)}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 777

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x

+ c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(Ab - 2aB) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(Ab - 2aB) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx \right)}{b^2 - 4ac} \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{2c(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(Ab - 2aB) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 1.04

$$\frac{-2ac(A + Bx^2) + abB + bx^2(bB - Ac)}{2c(4ac - b^2)(a + bx^2 + cx^4)} - \frac{(Ab - 2aB) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 0.80, size = 538, normalized size = 5.03

$$\frac{Bb^3 + 8Aa^2c^2 + (8b^4 + 4(2Ba^2 + Ab^2)c^2 - (6Ba^2 + Ab^2)c^2 - (2Ba - Ab)c^2 + (2Ba^2 - Ab^2)c^2)\sqrt{b^2 - 4ac} \log\left(\frac{2cx^2 + b}{\sqrt{b^2 - 4ac}}\right) - 2(2Bc^2 + Ab^2)c}{2(b^2c - 8a^2b^2c + 16a^3c^2 + (b^2c - 8a^2b^2c + 16a^3c^2)^2)} - \frac{Bb^3 + 8Aa^2c^2 + (8b^4 + 4(2Ba^2 + Ab^2)c^2 - (6Ba^2 + Ab^2)c^2 - (2Ba - Ab)c^2 + (2Ba^2 - Ab^2)c^2)\sqrt{b^2 - 4ac} \arctan\left(\frac{2cx^2 + b}{\sqrt{b^2 - 4ac}}\right) - 2(2Bc^2 + Ab^2)c}{2(b^2c - 8a^2b^2c + 16a^3c^2 + (b^2c - 8a^2b^2c + 16a^3c^2)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]

giac [A] time = 1.72, size = 120, normalized size = 1.12

$$\frac{(2Ba - Ab) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{Bb^2x^2 - 2Bacx^2 - Abcx^2 + Bab - 2Aac}{2(cx^4 + bx^2 + a)(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))

maple [A] time = 0.01, size = 158, normalized size = 1.48

$$-\frac{Ab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{2Ba \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{(Abc+2aBc-b^2B)x^2}{(4ac-b^2)c} - \frac{(2Ac-bB)a}{(4ac-b^2)c}}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)`

[Out] `1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.32, size = 283, normalized size = 2.64

$$\frac{\frac{x^2(-Bb^2+Ac b+2Bac)}{2c(4ac-b^2)} + \frac{a(2Ac-Bb)}{2c(4ac-b^2)}}{cx^4 + bx^2 + a} \operatorname{atan}\left(\frac{(4ac-b^2)^4 \left(x^2 \left(\frac{(Ab-2Ba)(Abc^2-2Bac^2)}{a(4ac-b^2)^{7/2}} + \frac{(2b^3c^2-8abc^3)(Ab-2Ba)^2(b^3-4abc)}{2a(4ac-b^2)^{13/2}} \right) - \frac{2c^2(Ab-2Ba)^2(b^3-4abc)}{(4ac-b^2)^{11/2}} \right)}{2A^2b^2c^2-8ABab^2c^2+8B^2a^2c^2}}{(4ac-b^2)^{3/2}}\right) (Ab-2Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)`

[Out] `-((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*((A*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2)))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b^2*c^2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2)*(A*b - 2*B*a)/(4*a*c - b^2)^(3/2)`

sympy [B] time = 5.38, size = 394, normalized size = 3.68

$$\frac{\sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba) \log\left(x^2 + \frac{-A^2+2Bab-16a^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)+8a^2c \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)+i^4 \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)}}{-2Ab+4Bac}\right)}{2} + \frac{\sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba) \log\left(x^2 + \frac{-A^2+2Bab+16a^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)-8a^2c \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)+i^4 \sqrt{\frac{1}{(4ac-b^2)}} (-Ab+2Ba)}}{-2Ab+4Bac}\right)}{2} + \frac{-2Aac+Ba^2+a^2(-Ab-2Bac+B^2)}{8a^2c^2-2a^2c+x^2(8ac^3-2b^2c^2)+x^2(8ac^3-2b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] $-\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a)*\log(x**2 + (-A*b**2 + 2*B*a*b - 16*a**2*c**2*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a) + 8*a*b**2*c*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a) - b**4*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a)))/(-2*A*b*c + 4*B*a*c))/2 + \sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a)*\log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a) - 8*a*b**2*c*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a) + b**4*\sqrt{-1/(4*a*c - b**2)**3}*(-A*b + 2*B*a)))/(-2*A*b*c + 4*B*a*c))/2 + (-2*A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))$

$$3.93 \quad \int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=94

$$\frac{(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} - \frac{-2aB - (x^2(bB - 2Ac)) + Ab}{2(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1247, 638, 618, 206}

$$-\frac{-2aB + x^2(-(bB - 2Ac)) + Ab}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(A + Bx^2)}{(a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\ &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(bB - 2Ac) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\ &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(bB - 2Ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\ &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(bB - 2Ac) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 101, normalized size = 1.07

$$\frac{\frac{2(bB - 2Ac) \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + \frac{B(2a + bx^2) - A(b + 2cx^2)}{a + bx^2 + cx^4}}{2(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 0.69, size = 474, normalized size = 5.04

$$\frac{2Bab^2 - Ab^3 + (Bb^3 + 8Aac^2 - 2(2Bab + Ab^2))c^2 + ((Bbc - 2Ac^2)x^4 + Bab - 2Aac + (Bb^2 - 2Abc)x^2)\sqrt{b^2 - 4ac} \log\left(\frac{2x^2 + 2bx + b^2 - 2ac - (2x^2 + b)\sqrt{b^2 - 4ac}}{a^2 + 4ac}\right) - 4(2Bb^2 - Ab^3 + (Bb^3 + 8Aac^2 - 2(2Bab + Ab^2))c^2 - 2((Bbc - 2Ac^2)x^4 + Bab - 2Aac + (Bb^2 - 2Abc)x^2)\sqrt{b^2 - 4ac} \arctan\left(\frac{(2x^2 + b)\sqrt{b^2 - 4ac}}{a^2 + 4ac}\right) - 4(2Bb^2 - Ab^3)c}{2(ab^4 - 8a^2b^2c + 16a^2c^2 + (b^2c - 8ab^2c + 16a^2c^2)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]

giac [A] time = 1.37, size = 102, normalized size = 1.09

$$\frac{(Bb - 2Ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{Bbx^2 - 2Acx^2 + 2Ba - Ab}{2(cx^4 + bx^2 + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))

maple [A] time = 0.01, size = 127, normalized size = 1.35

$$\frac{2Ac \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{\frac{3}{2}}} - \frac{Bb \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{\frac{3}{2}}} + \frac{Ab - 2Ba + (2Ac - bB)x^2}{2(4ac - b^2)(cx^4 + bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)`

[Out] $\frac{1}{2} * ((2 * A * c - B * b) * x^2 + A * b - 2 * a * B) / (4 * a * c - b^2) / (c * x^4 + b * x^2 + a) + 2 / (4 * a * c - b^2)^{(3/2)} * \arctan((2 * c * x^2 + b) / (4 * a * c - b^2)^{(1/2)}) * A * c - 1 / (4 * a * c - b^2)^{(3/2)} * \arctan((2 * c * x^2 + b) / (4 * a * c - b^2)^{(1/2)}) * b * B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.30, size = 264, normalized size = 2.81

$$\frac{\frac{Ab-2Ba}{2(4ac-b^2)} + \frac{x^2(2Ac-Bb)}{2(4ac-b^2)}}{cx^4 + bx^2 + a} + \frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{(2Ac-Bb)(2Ac^3-Bb^2)}{a(4ac-b^2)^{7/2}} + \frac{(2b^3c^2-8abc^3)(2Ac-Bb)^2(b^3-4abc)}{2a(4ac-b^2)^{13/2}}\right) - \frac{2c^2(2Ac-Bb)^2(b^3-4abc)}{(4ac-b^2)^{11/2}}\right)}{8A^2c^4-8ABbc^3+2B^2b^2c^2}\right)(2Ac-Bb)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)`

[Out] $\frac{(A * b - 2 * B * a) / (2 * (4 * a * c - b^2)) + (x^2 * (2 * A * c - B * b)) / (2 * (4 * a * c - b^2))}{a + b * x^2 + c * x^4} + \frac{\operatorname{atan}\left(\frac{(x^2 * ((2 * A * c - B * b) * (2 * A * c^3 - B * b * c^2)) / (a * (4 * a * c - b^2)^{(7/2)}) + ((2 * b^3 * c^2 - 8 * a * b * c^3) * (2 * A * c - B * b)^2 * (b^3 - 4 * a * b * c)) / (2 * a * (4 * a * c - b^2)^{(13/2)}) - (2 * c^2 * (2 * A * c - B * b)^2 * (b^3 - 4 * a * b * c)) / (4 * a * c - b^2)^{(11/2)) * (4 * a * c - b^2)^4}{(8 * A^2 * c^4 + 2 * B^2 * b^2 * c^2 - 8 * A * B * b * c^3) * (2 * A * c - B * b)}\right)}{(4 * a * c - b^2)^{(3/2)}}$

sympy [B] time = 3.36, size = 374, normalized size = 3.98

$$\frac{\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)\log\left(x^2 + \frac{-2Abc+8b^2-16a^2\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)+6ab^2\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)-14\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)}{-4a^2+2Bbc}\right)}{2} - \sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)\log\left(x^2 + \frac{-2Abc+8b^2+16a^2\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)-6ab^2\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)+14\sqrt{\frac{1}{(4ac-b^2)}}(-2Ac+Bb)}{-4a^2+2Bbc}\right)}{2} + \frac{Ab-2Ba+x^2(2Ac-Bb)}{8a^2c-2ab^2+x^2(8a^2-2b^2c)+x^2(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)`

[Out] $\sqrt{-1/(4 * a * c - b * b^2) * 3} * (-2 * A * c + B * b) * \log(x^2 + (-2 * A * b * c + B * b^2 - 16 * a * b^2 * c^2) * \sqrt{-1/(4 * a * c - b * b^2) * 3} * (-2 * A * c + B * b) + 8 * a * b^2 * c * \sqrt{-1/$

$$\begin{aligned}
& (4ac - b^2)^3(-2Ac + Bb) - b^4\sqrt{-1/(4ac - b^2)^3}(-2Ac + Bb) \\
& + Bb)/(-4A^2c + 2B^2b^2c)/2 - \sqrt{-1/(4ac - b^2)^3}(-2Ac + Bb) \\
& \log(x^2 + (-2Ab^2c + B^2b^2 + 16a^2c^2)\sqrt{-1/(4ac - b^2)^3}(-2Ac + Bb) \\
& - 8ab^2c)\sqrt{-1/(4ac - b^2)^3}(-2Ac + Bb) + b^4\sqrt{-1/(4ac - b^2)^3}(-2Ac + Bb) \\
& + Bb)/(-4A^2c + 2B^2b^2c)/2 + (Ab - 2Ba + x^2(2Ac - Bb))/(8a^2c - 2ab^2 + x^4(8a^2c - 2b^2c) \\
& + x^2(8abc - 2b^3))
\end{aligned}$$

$$3.94 \quad \int \frac{A+Bx^2}{x(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=150

$$\frac{(4a^2Bc + A(b^3 - 6abc)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{A \log(a + bx^2 + cx^4)}{4a^2} + \frac{A \log(x)}{a^2} - \frac{-A(b^2 - 2ac) - (cx^2(Ab - 2aB))}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^2(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.33, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{(4a^2Bc + A(b^3 - 6abc)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{A \log(a + bx^2 + cx^4)}{4a^2} + \frac{A \log(x)}{a^2} - \frac{-A(b^2 - 2ac) + cx^2(-Ab - 2aB) + abB}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^2(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-A(b^2 - 4ac) - (Ab - 2aB)cx}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{A(-b^2 + 4ac)}{ax} + \frac{2a^2Bc + A(b^3 - 5abc) + Ac(b^2 - 4ac)}{a(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A \log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{2a^2Bc + A(b^3 - 5abc) + Ac(b^2 - 4ac)}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A \log(x)}{a^2} - \frac{A \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{A \log(x)}{a^2} - \frac{A \log(a + bx^2 + cx^4)}{4a^2} + \frac{(4a^2Bc)}{4a^2} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(4a^2Bc + A(b^3 - 6abc)) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2(b^2 - 4ac)^{3/2}} +
\end{aligned}$$

Mathematica [A] time = 0.33, size = 243, normalized size = 1.62

$$\frac{\frac{(4a^2Bc + A(b^2\sqrt{b^2 - 4ac} - 4ac\sqrt{b^2 - 4ac} - 6abc + b^3)) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(4a^2Bc + A(-b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac} - 6abc + b^3)) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2a(aB(b + 2cx^2) - A(-2ac + b^2 + bcx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + 4A \log(x)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*sqrt[b^2 - 4*a*c] - 4*a*c*sqrt[b^2 - 4*a*c]))*Log[b - sqrt[b^2 - 4*a*c] + 2*c*x^2])/((b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*sqrt[b^2 - 4*a*c] + 4*a*c*sqrt[b^2 - 4*a*c]))*Log[b + sqrt[b^2 - 4*a*c] + 2*c*x^2])/((b^2 - 4*a*c)^(3/2)))/(4*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 1.82, size = 1014, normalized size = 6.76

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - \\ & (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A \\ & *a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3 \\ & *A*a^2*b)*c)*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (\\ & 2*c*x^2 + b)*\sqrt{b^2 - 4*a*c}))/ (c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a \\ & ^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c \\ & ^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*\log(c \\ & *x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - \\ & 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)* \\ & x^2)*\log(x))/ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c \\ & ^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2 \\ & *B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B \\ & *a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)* \\ & c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2 \\ & *b)*c)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4 \\ & *a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^ \\ & 3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c \\ & + 16*A*a^2*b*c^2)*x^2)*\log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c \\ & + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8 \\ & *A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*\log(x))/ (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c \\ & ^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c \\ & + 16*a^4*b*c^2)*x^2)] \end{aligned}$$

giac [A] time = 1.71, size = 201, normalized size = 1.34

$$\frac{(Ab^3 + 4Ba^2c - 6Aabc) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) - A \log(cx^4 + bx^2 + a) + \frac{A \log(x^2)}{2a^2} + \frac{Ab^2cx^4 - 4Aac^2x^4 + Ab^3x^2 - 4Ba^2cx^2 - 2Aabcx^2 - 2Ba^2b + 3Aab^2 - 8Aa^2c}{4(cx^4 + bx^2 + a)(a^2b^2 - 4a^3c)}}{2(a^2b^2 - 4a^3c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c}) / ((a^2*b^2 - 4*a^3*c)*\sqrt{-b^2 + 4*a*c}) - 1/4*A*\log(c*x^4 + b*x^2 + a) / a^2 + 1/2*A*\log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c) / ((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))$$

maple [B] time = 0.02, size = 361, normalized size = 2.41

$$\frac{A b c x^2}{2(c x^4 + b x^2 + a)(4 a c - b^2) a} + \frac{B c x^2}{(c x^4 + b x^2 + a)(4 a c - b^2)} - \frac{3 A b c \arctan\left(\frac{2 c x^2 + b}{\sqrt{4 a c - b^2}}\right)}{(4 a c - b^2)^2 a} + \frac{A b^3 \arctan\left(\frac{2 c x^2 + b}{\sqrt{4 a c - b^2}}\right)}{2(4 a c - b^2)^2 a^2} + \frac{2 B c \arctan\left(\frac{2 c x^2 + b}{\sqrt{4 a c - b^2}}\right)}{(4 a c - b^2)^2} - \frac{A b^2}{2(c x^4 + b x^2 + a)(4 a c - b^2) a} - \frac{A c \ln(c x^4 + b x^2 + a)}{(4 a c - b^2) a} + \frac{A b^2 \ln(c x^4 + b x^2 + a)}{4(4 a c - b^2) a^2} + \frac{A c}{(c x^4 + b x^2 + a)(4 a c - b^2)} + \frac{B b}{2(c x^4 + b x^2 + a)(4 a c - b^2)} + \frac{A \ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x)

[Out]
$$A*\ln(x)/a^2 - 1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b + 1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*B + 1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c - 1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^2 + 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*B - 1/a/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)*A + 1/4/a^2/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*A*b^2 - 3/a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b*c + 1/2/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b^3 + 2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*B*c$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 7.88, size = 7119, normalized size = 47.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)

[Out]
$$((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*\log(x))/a^2 - (\log(((A + a^2*(-($$

$$\begin{aligned}
& A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)}*(((A + a^2*(\\
& -(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*((4*b*c^2* \\
& (A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(-(A*b \\
& ^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})*(a*b + 3*b^2*x^ \\
& 2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c - 10*A*a* \\
& b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c \\
& - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c \\
& - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a \\
&)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c - b^2)^2)) \\
& *(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)} \\
&))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1 \\
& /2)})*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(\\
& A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^{(1/2)})* \\
& (a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B* \\
& a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A* \\
& b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2* \\
& B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x \\
& ^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4 \\
& *a*c - b^2)^2))*((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2 \\
&))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((\\
& x^2*(((A^3*b^3*c^5 - 8*B^3*a^3*c^5 + 12*A*B^2*a^2*b*c^5 - 6*A^2*B*a*b^2*c^ \\
& 5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((44*A^2*a^2*b \\
& ^3*c^5 - 4*B^2*a^3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3 \\
& *b*c^6 + 16*B^2*a^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b \\
& ^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A \\
& *a^5*b*c^6 - 2*A*a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B \\
& a^3*b^6*c^3 + 168*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 \\
& - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c \\
& + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1 \\
& 056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4* \\
& c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c \\
& ^2))))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^ \\
& 2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^ \\
& 3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48* \\
& a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A* \\
& a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168 \\
& *B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + \\
& 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c \\
& ^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - \\
& 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^ \\
& 2))*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4* \\
& B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^{(3/2)}) - ((A*b^3 + 4*B*a^2*c - 6 \\
& *A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560 \\
& *a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6 \\
& *b^3*c^5))/(8*a^2*(4*a*c - b^2)^{(3/2)}*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
\end{aligned}$$

$$\begin{aligned}
& + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2) \\
&))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 \\
& + 4*B*a^2*c - 6*A*a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a \\
& ^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 \\
& ^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - \\
& 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 1 \\
& 92*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b^3*c + 33*A*a^2*b*c^2 + \\
& 2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4 \\
& *B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12* \\
& A*B*a^3*b*c^2)) + (((((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*a^2*b^7*c^3 \\
& + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168*B*a^4*b^4*c \\
& ^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c \\
& ^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^ \\
& 7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^ \\
& 3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^ \\
& 6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6* \\
& A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2 \\
& *A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + \\
& 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(\\
& 8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2 \\
& *c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 \\
& - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^ \\
& 5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((44*A^2*a^2*b^3*c^5 - 4*B^2*a^ \\
& 3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3*b*c^6 + 16*B^2*a^ \\
& 4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - \\
& 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A* \\
& a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168 \\
& *B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + \\
& 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c \\
& ^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - \\
& 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^ \\
& 2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - \\
& 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c \\
& ^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4* \\
& a^2*(4*a*c - b^2)^(3/2)) + ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3*(2560*a^7*b*c \\
& ^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5 \\
&))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^ \\
& 5*b^2*c^2)))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + 2*B*a^2*b^3*c - 6*B*a \\
& ^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*A^2*a^3*c \\
& ^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A \\
& *B*a^2*b^3*c - 12*A*B*a^3*b*c^2))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a \\
& ^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^ \\
& 2*c^2*(4*a*c - b^2)^(9/2)))/(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2* \\
& c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4) - (((((((4*A \\
& *a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a
\end{aligned}$$

$$\begin{aligned}
&^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5 \\
&*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a \\
&^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c \\
&^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4* \\
&a^2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^ \\
&4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c \\
&+ 96*A*a^2*b^2*c^2))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a \\
&^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A \\
&*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 25 \\
&6*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*B^2*a^4*c^4 + 17*A^2*a^ \\
&2*b^2*c^4 - 4*A^2*a*b^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^ \\
&4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A \\
&a^3*b^4*c^3 + 80*A*a^4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8 \\
&a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - \\
&128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 \\
&- 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) \\
&)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^ \\
&6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6* \\
&A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + \\
&64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^3)/(64*a^6*(4*a*c - b^2)^(9 \\
&/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) \\
&- 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 76 \\
&8*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*A*b^6 - 40*A*a^3*c^3 - 27*A*a*b^4*c + \\
&2*B*a^2*b^3*c - 6*B*a^3*b*c^2 + 69*A*a^2*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2) \\
&)^7/2)*(A^2*b^6*c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c \\
&^3 + 8*A*B*a^2*b^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4 \\
&*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12* \\
&A*B*a^3*b*c^2) + ((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - \\
&b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b \\
&^2)^(9/2))*((A^3*b^2*c^4 + 4*A*B^2*a^2*c^4 - 4*A^2*B*a*b*c^4)/(a^3*b^4 + 16 \\
&*a^5*c^2 - 8*a^4*b^2*c) + (((4*B^2*a^4*c^4 + 17*A^2*a^2*b^2*c^4 - 4*A^2*a*b \\
&^4*c^3 + 8*A*B*a^2*b^3*c^3 - 36*A*B*a^3*b*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^ \\
&4*b^2*c) + (((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^ \\
&4*b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4 \\
&*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A \\
&*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^ \\
&2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^ \\
&3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48* \\
&a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96 \\
&*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c \\
&^2)) - ((((((4*A*a^2*b^6*c^2 - 32*B*a^5*b*c^4 - 36*A*a^3*b^4*c^3 + 80*A*a^4* \\
&b^2*c^4 + 8*B*a^4*b^3*c^3)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b \\
&^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a \\
&*b^4*c + 96*A*a^2*b^2*c^2))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2* \\
&b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c -
\end{aligned}$$

$$\begin{aligned}
& 6*A*a*b*c))/((4*a^2*(4*a*c - b^2)^{(3/2)}) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 \\
& + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 \\
& - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)))/(8*a^2*(4*a*c - b^2)^{(3/2)}*(a^3*b^4 + 1 \\
& 6*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4* \\
& b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/((4*a^2*(4*a*c - b^2)^{(3/2)}) - (\\
& (4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(A*b^3 + 4*B*a^2*c - 6*A* \\
& a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)))/(32*a \\
& ^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^ \\
& 5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b \\
& ^3*c + 33*A*a^2*b*c^2 + 2*B*a^2*b^2*c))/((8*a^3*c^2*(4*a*c - b^2)^3*(A^2*b^6 \\
& *c^2 + 16*B^2*a^4*c^4 + 36*A^2*a^2*b^2*c^4 - 12*A^2*a*b^4*c^3 + 8*A*B*a^2*b \\
& ^3*c^3 - 48*A*B*a^3*b*c^4)*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4*B^2*a^4*c^2 - 2 \\
& 91*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*A*B*a^3*b*c^2))) \\
& *(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/((2*a^2*(4*a*c - b^2)^{(3/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.95 \quad \int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=223

$$\frac{(2Ab - aB) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2Ab - aB)}{a^3} - \frac{-6aAc - abB + 2Ab^2}{2a^2x^2(b^2 - 4ac)} + \frac{(abB(b^2 - 6ac) - 2A(6a^2c^2 - 6ab^2))}{2a^3(b^2 - 4ac)}$$

Rubi [A] time = 0.42, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{(abB(b^2 - 6ac) - 2A(6a^2c^2 - 6ab^2c + b^4)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{-6aAc - abB + 2Ab^2}{2a^2x^2(b^2 - 4ac)} + \frac{(2Ab - aB) \log(a + bx^2 + cx^4)}{4a^3} - \frac{\log(x)(2Ab - aB)}{a^3} - \frac{-A(b^2 - 2ac) + cx^2(-Ab - 2aB) + abB}{2ax^2(b^2 - 4ac)(a + bx^2 + cx^4)}}{2a^3(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

[Out] $-(2A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^{(3/2)}) - ((2*A*b - a*B)*Log[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^2 (a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-2Ab^2 + abB + 6aAc - 2(Ab - 2aB)cx}{x^2(a + bx + cx^2)} dx, x, \right)}{2a(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{-2Ab^2 + abB + 6aAc}{ax^2} + \frac{(-2Ab + aB)(-b^2 + 2ac)}{a^2x} \right) dx, x, \right)}{2a(b^2 - 4ac)} \\
&= -\frac{2Ab^2 - abB - 6aAc}{2a^2(b^2 - 4ac)x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{(2Ab - aB) \log(x)}{a^3} \\
&= -\frac{2Ab^2 - abB - 6aAc}{2a^2(b^2 - 4ac)x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{(2Ab - aB) \log(x)}{a^3} \\
&= -\frac{2Ab^2 - abB - 6aAc}{2a^2(b^2 - 4ac)x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{(2Ab - aB) \log(x)}{a^3} \\
&= -\frac{2Ab^2 - abB - 6aAc}{2a^2(b^2 - 4ac)x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} + \frac{(abB(b^2 - 6ac) - 2a^2(2Ab - aB) \log(x))}{4a^3}
\end{aligned}$$

Mathematica [A] time = 0.56, size = 379, normalized size = 1.70

$$\frac{(2A(6a^2b^2 - 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^2\sqrt{b^2 - 4ac} + b^4) + aB(-b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac} + 6abc - b^3)) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2) + (2A(-6a^2b^2 + 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^2\sqrt{b^2 - 4ac} - b^4) + aB(-b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac} - 6abc + b^3)) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{2a(A(-3ab^2c - 2a^2b^2 + b^2cx^2) + aB(2ac - b^2 - 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + 4 \log(x)(aB - 2Ab) - \frac{2a^2}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*sqrt[b^2 - 4*a*c] + 4*a*c*sqrt[b^2 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*sqrt[b^2 - 4*a*c] - 4*a*b*c*sqrt[b^2 - 4*a*c]))*Log[b - sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*sqrt[b^2 - 4*a*c] + 4*a*c*sqrt[b^2 - 4*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*sqrt[b^2 - 4*a*c] - 4*a*b*c

$c\sqrt{b^2 - 4ac}) \cdot \text{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2]/(b^2 - 4ac)^{(3/2)}/(4a^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 3.86, size = 1635, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2 \\ & *B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4 \\ & - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)* \\ & c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4) \\ & *c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c \\ &)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c \\ &)*x^2)*\text{sqrt}(b^2 - 4*a*c)*\text{log}((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^ \\ & 2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c \\ & ^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 \\ & - 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x \\ & ^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2 \\ & *A*a^2*b^3)*c)*x^2)*\text{log}(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 \\ & - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - \\ & 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 \\ & + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A \\ & *a^2*b^3)*c)*x^2)*\text{log}(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (\\ & a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6 \\ & *c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3 \\ & *c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2 \\ & *(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5* \\ & A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b \\ & ^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - \\ & 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2 \\ & *A*a^2*b^2)*c)*x^2)*\text{sqrt}(-b^2 + 4*a*c)*\text{arctan}(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4* \\ & a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b \end{aligned}$$

$$\begin{aligned} &^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2 \\ & *A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 \\ & + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*\log(c*x \\ & ^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3 \\ &)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A \\ & *a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 + \\ & 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*\log(x))/ \\ & (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16* \\ & a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2) \end{aligned}$$

giac [A] time = 1.65, size = 250, normalized size = 1.12

$$\frac{(Bab^3 - 2Ab^4 - 6Ba^2bc + 12Aab^2c - 12Aa^2c^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) + Babcx^4 - 2Ab^2cx^4 + 6Aac^2x^4 + Bab^2x^2 - 2Ab^3x^2 - 2Ba^2cx^2 + 7Aabcx^2 - Aab^2 + 4Aa^2c}{2(a^3b^2 - 4a^4c)\sqrt{-b^2+4ac}} + \frac{Babcx^4 - 2Ab^2cx^4 + 6Aac^2x^4 + Bab^2x^2 - 2Ab^3x^2 - 2Ba^2cx^2 + 7Aabcx^2 - Aab^2 + 4Aa^2c}{2(cx^6 + bx^4 + ax^2)(a^2b^2 - 4a^3c)} - \frac{(Ba - 2Ab)\log(cx^4 + bx^2 + a)}{4a^3} + \frac{(Ba - 2Ab)\log(x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $-1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*\arctan$
 $((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^3*b^2 - 4*a^4*c)*\sqrt{-b^2 + 4*a*c})$
 $+ 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3$
 $*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4$
 $+ a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*\log(c*x^4 + b*x^2 + a)/a$
 $^3 + 1/2*(B*a - 2*A*b)*\log(x^2)/a^3$

maple [B] time = 0.02, size = 622, normalized size = 2.79

$$\frac{A^2c^2}{(c^2x^4+b^2x^2+a^2)^2} - \frac{2Ab^2c}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{Bbc}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{6A^2c^2\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{6A^2c^2\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{A^2c^2\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{Bbc\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{B^2c^2\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{2Ab^2c}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{Bbc}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{2Ab^2c}{(4ac-b^2)^2} + \frac{A^2c^2\ln\left(\frac{c^2x^4+b^2x^2+a^2}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)}\right)}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{Bbc\ln\left(\frac{c^2x^4+b^2x^2+a^2}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)}\right)}{2(c^2x^4+b^2x^2+a^2)(4ac-b^2)} + \frac{Bc}{(4ac-b^2)^2} + \frac{2Ab^2c}{(4ac-b^2)^2} + \frac{Bbc}{(4ac-b^2)^2} + \frac{2Ab^2c}{(4ac-b^2)^2} + \frac{Bbc}{(4ac-b^2)^2} + \frac{2Ab^2c}{(4ac-b^2)^2} + \frac{Bbc}{(4ac-b^2)^2} + \frac{2Ab^2c}{(4ac-b^2)^2} + \frac{Bbc}{(4ac-b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x)

[Out] $-1/2*A/a^2/x^2 - 2/a^3*\ln(x)*A*b + 1/a^2*\ln(x)*B - 1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c$
 $-b^2)*x^2*A + 1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b^2 - 1/2/a/(c*x^4+b$
 $x^2+a)*c/(4*a*c-b^2)*x^2*b*B - 3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b*c + 1/2/a^$
 $2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^3 + 1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c - 1/2/a$
 $/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*b^2 + 2/a^2/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)*A$
 $b - 1/2/a^3/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*A*b^3 - 1/a/(4*a*c-b^2)*c*\ln(c*x^4+b$
 $x^2+a)*B + 1/4/a^2/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^2*B - 6/a/(4*a*c-b^2)^(3/2)*$
 $\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^2 + 6/a^2/(4*a*c-b^2)^(3/2)*\arctan$
 $((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2*c - 1/a^3/(4*a*c-b^2)^(3/2)*\arctan((2*c$
 $x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4 - 3/a/(4*a*c-b^2)^(3/2)*\arctan((2*c*x^2+b)/(4$
 $*a*c-b^2)^(1/2))*b*B*c + 1/2/a^2/(4*a*c-b^2)^(3/2)*\arctan((2*c*x^2+b)/(4*a*c-$
 $b^2)^(1/2))*B*b^3$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 9.09, size = 10034, normalized size = 45.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)

[Out]
$$\begin{aligned} & \left(\log\left(\frac{(c^4(2Ab - Ba)(6A^2ac - 2Ab^2 + B^2ab)^2)}{(a^6(4ac - b^2)^2)} - \frac{((B^2a - 2Ab + a^3(-2Ab^4 + 12A^2ac^2 - B^2ab^3 - 12A^2ab^2c + 6B^2a^2bc))^2}{(a^6(4ac - b^2)^3)}\right)^{1/2} \right) \left(\frac{(b^2c(B^2a - 2Ab + a^3(-2Ab^4 + 12A^2ac^2 - B^2ab^3 - 12A^2ab^2c + 6B^2a^2bc))^2}{(a^6(4ac - b^2)^3)}\right)^{1/2} \right) \left(\frac{(ab + 3b^2x^2 - 10acx^2)}{a^3} + \frac{4b^2c^2(2Ab^4 + 6A^2ac^2 - B^2ab^3 - 10A^2ab^2c + 5B^2a^2bc)}{(a^2(4ac - b^2))} + \frac{(2c^3x^2(2Ab^4 - 60A^2ac^2 - B^2ab^3 + 4A^2ab^2c + 10B^2a^2bc))}{(a^2(4ac - b^2))} \right) \left(\frac{1}{4a^3} + \frac{(c^3(36A^2a^3c^3 - 16A^2b^6 - 4B^2a^2b^4 + 16AB^2ab^5 - 216A^2a^2b^2c^2 + 116A^2ab^4c + 17B^2a^3b^2c - 92AB^2a^2b^3c + 108AB^2a^3bc^2))}{(a^4(4ac - b^2)^2)} - \frac{(2c^4x^2(12A^2b^5 + 3B^2a^2b^3 - 12AB^2ab^4 - 60AB^2a^3c^2 - 82A^2ab^3c - 10B^2a^3bc + 138A^2a^2bc^2 + 61AB^2a^2b^2c))}{(a^4(4ac - b^2)^2)} \right) \left(\frac{(B^2a - 2Ab + a^3(-2Ab^4 + 12A^2ac^2 - B^2ab^3 - 12A^2ab^2c + 6B^2a^2bc))^2}{(a^6(4ac - b^2)^3)}\right)^{1/2} \right) \left(\frac{1}{4a^3} + \frac{(c^5x^2(6A^2ac - 2Ab^2 + B^2ab)^3)}{(a^6(4ac - b^2)^3)} \right) \left(\frac{(c^4(2Ab - Ba)(6A^2ac - 2Ab^2 + B^2ab)^2)}{(a^6(4ac - b^2)^2)} - \frac{((2Ab - Ba + a^3(-2Ab^4 + 12A^2ac^2 - B^2ab^3 - 12A^2ab^2c + 6B^2a^2bc))^2}{(a^6(4ac - b^2)^3)}\right)^{1/2} \right) \left(\frac{(4b^2c^2(2Ab^4 + 6A^2ac^2 - B^2ab^3 - 10A^2ab^2c + 5B^2a^2bc))}{(a^2(4ac - b^2))} - \frac{(b^2c^2(2Ab - Ba + a^3(-2Ab^4 + 12A^2ac^2 - B^2ab^3 - 12A^2ab^2c + 6B^2a^2bc))^2}{(a^6(4ac - b^2)^3)}\right)^{1/2} \right) \left(\frac{(ab + 3b^2x^2 - 10acx^2)}{a^3} + \frac{(2c^3x^2(2Ab^4 - 60A^2ac^2 - B^2ab^3 + 4A^2ab^2c + 10B^2a^2bc))}{(a^2(4ac - b^2))} \right) \left(\frac{1}{4a^3} - \frac{(c^3(36A^2a^3c^3 - 16A^2b^6 - 4B^2a^2b^4 + 16AB^2ab^5 - 216A^2a^2b^2c^2 + 116A^2ab^4c + 17B^2a^3b^2c - 92AB^2a^2b^3c + 108AB^2a^3bc^2))}{(a^4(4ac - b^2)^2)} + \frac{(2c^4x^2(12A^2b^5 + 3B^2a^2b^3 - 12AB^2ab^4 - 60AB^2a^3c^2 - 82A^2ab^3c + 10B^2a^3bc^2))}{(a^4(4ac - b^2)^2)} \right) \end{aligned}$$

$$\begin{aligned}
& *c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c)) / (a^4*(4*a*c - \\
& b^2)^2) * (2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^ \\
& 2*c + 6*B*a^2*b*c)^2 / (a^6*(4*a*c - b^2)^3))^{(1/2)}) / (4*a^3) + (c^5*x^2*(6*A \\
& *a*c - 2*A*b^2 + B*a*b)^3 / (a^6*(4*a*c - b^2)^3)) * (4*A*b^7 + 128*B*a^4*c^3 \\
& - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2* \\
& b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 1 \\
& 92*a^5*b^2*c^2)) - (\log(x)*(2*A*b - B*a)) / a^3 - (A/(2*a) - (x^2*(2*A*b^3 - \\
& B*a*b^2 + 2*B*a^2*c - 7*A*a*b*c)) / (2*a^2*(4*a*c - b^2))) + (c*x^4*(6*A*a*c - \\
& 2*A*b^2 + B*a*b)) / (2*a^2*(4*a*c - b^2))) / (a*x^2 + b*x^4 + c*x^6) + (\operatorname{atan}((\\
& x^2*(((216*A^3*a^3*c^8 - 8*A^3*b^6*c^5 - 216*A^3*a^2*b^2*c^7 + B^3*a^3*b^3 \\
& *c^5 + 72*A^3*a*b^4*c^6 + 12*A^2*B*a*b^5*c^5 + 108*A^2*B*a^3*b*c^7 - 6*A*B^ \\
& 2*a^2*b^4*c^5 + 18*A*B^2*a^3*b^2*c^6 - 72*A^2*B*a^2*b^3*c^6) / (a^6*b^6 - 64* \\
& a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((24*A^2*a^2*b^7*c^4 - 260*A^2* \\
& a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5 \\
& + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6* \\
& c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6) / (a^6*b^6 - 64*a^9*c^3 - 12 \\
& *a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^ \\
& 4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B \\
& *a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5) / (a^6*b^6 - 64*a^9*c^3 \\
& - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184 \\
& *a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (4*A*b^7 + 128*B*a^4*c^ \\
& 3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2 \\
& *b^3*c^2 - 96*B*a^3*b^2*c^2)) / (2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48* \\
& a^8*b^2*c^2) * (4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (\\
& 4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B \\
& *a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(4*a^3*b^6 - 256*a^6 \\
& *c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b \\
& ^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - \\
& 96*B*a^3*b^2*c^2) / (2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2 \\
& *c^2)) - ((((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5 \\
& *b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B* \\
& a^6*b^5*c^4 + 192*B*a^7*b^3*c^5) / (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48* \\
& a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056* \\
& a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A \\
& *a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3* \\
& b^2*c^2)) / (2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3* \\
& b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (2*A*b^4 + 12*A*a^2*c \\
& ^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)) / (4*a^3*(4*a*c - b^2)^{(3/2)}) - (\\
& (2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26 \\
& 88*a^9*b^3*c^5) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2* \\
& b*c) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 \\
& + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2)) / (8*a^3*(4*a*c - b \\
& ^2)^{(3/2)} * (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3*b^6 \\
& - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (2*A*b^4 + 12*A*a^2*c^2 \\
& - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)) / (4*a^3*(4*a*c - b^2)^{(3/2)}) + ((25
\end{aligned}$$

$$\begin{aligned}
& 60*a^{10}*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c) \\
&)^2 * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (32*a^6*(4*a*c - b \\
& ^2)^3 * (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (6*A*a^3*c^3 - 6*A*b^6 + 3*B \\
& *a*b^5 + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2) / (8*a^3*c^2*(4*a*c - b^2)^3 * (36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) + (((((((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5) / (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c) / (4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (8*a^3*(4*a*c - b^2)^(3/2) * (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5 + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6) / (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5) / (a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) * (4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2) / (2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2))) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c) / (4*a^3*(4*a*c - b^2)^(3/2)) + ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5) * (2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c
\end{aligned}$$

$$\begin{aligned}
& + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^{(9/2)}*(a^6*b^6 - 64*a^9*c^3 - 12*a \\
& ^7*b^4*c + 48*a^8*b^2*c^2))*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 69 \\
& 12*A*a*b^5*c - 12544*A*a^3*b*c^3 + 3456*B*a^2*b^4*c + 18432*A*a^2*b^3*c^2 - \\
& 8832*B*a^3*b^2*c^2))/(1024*a^3*c^2*(4*a*c - b^2)^{(7/2)}*(36*A^2*a^4*c^4 - 2 \\
& 4*A^2*b^8 - 6*B^2*a^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b \\
& ^4*c^2 + 1528*A^2*a^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72* \\
& B^2*a^3*b^4*c + 1158*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c \\
& ^3)))*(16*a^9*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^12*c^3*(4*a*c - b^2)^{(9/2)} - \\
& 192*a^10*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^11*b^2*c^2*(4*a*c - b^2)^{(9/2)}) \\
&)/(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4*c^4 - 288*A^2*a^3*b^2* \\
& c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a^4*b^2*c^4 - 48*A^2*a* \\
& b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - 4*A*B*a*b^7*c^2 + 144* \\
& A*B*a^4*b*c^5) + (((((((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 \\
& - 184*A*a^6*b^3*c^4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^ \\
& 4)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 \\
& + 64*a^9*b^2*c^4)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256 \\
& *A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(\\
& a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c \\
& + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6* \\
& B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^{(3/2)}) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 \\
& + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^ \\
& 2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^ \\
& 3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(8*a^3*(4*a*c - \\
& b^2)^{(3/2)}*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - \\
& 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 4 \\
& 8*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a \\
& ^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) \\
& - (((36*A^2*a^5*c^6 - 16*A^2*a^2*b^6*c^3 + 116*A^2*a^3*b^4*c^4 - 216*A^2*a \\
& ^4*b^2*c^5 - 4*B^2*a^4*b^4*c^3 + 17*B^2*a^5*b^2*c^4 + 16*A*B*a^3*b^5*c^3 - \\
& 92*A*B*a^4*b^3*c^4 + 108*A*B*a^5*b*c^5)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) \\
&) - (((96*A*a^7*b*c^5 - 8*A*a^4*b^7*c^2 + 72*A*a^5*b^5*c^3 - 184*A*a^6*b^3*c^ \\
& 4 + 4*B*a^5*b^6*c^2 - 36*B*a^6*b^4*c^3 + 80*B*a^7*b^2*c^4)/(a^6*b^4 + 16* \\
& a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4) \\
& *(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24 \\
& *B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^4 + 16*a^8* \\
& c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^ \\
& 2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 \\
& + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 2 \\
& 56*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B* \\
& a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^{(3/2)}) + ((4*a^7* \\
& b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^ \\
& 3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^3)/(64*a^9*(4*a*c - b^2)^{(9/2)}*(a^6*b^4 + 1 \\
& 6*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^12*c^3* \\
& (4*a*c - b^2)^{(9/2)} - 192*a^10*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^11*b^2*c^2 \\
& *(4*a*c - b^2)^{(9/2)})*(768*A*b^7 + 5120*B*a^4*c^3 - 384*B*a*b^6 - 6912*A*a*
\end{aligned}$$

$$\begin{aligned}
& b^5c - 12544Aa^3b^2c^3 + 3456B^2a^2b^4c + 18432Aa^2b^3c^2 - 8832B \\
& *a^3b^2c^2) / (1024a^3c^2(4a^2c - b^2)^{(7/2)} * (144A^2a^4c^6 + 4A^2b \\
& ^8c^2 + 192A^2a^2b^4c^4 - 288A^2a^3b^2c^5 + B^2a^2b^6c^2 - 12B \\
& ^2a^3b^4c^3 + 36B^2a^4b^2c^4 - 48A^2a^2b^6c^3 + 48A^2B^2a^2b^5c^3 \\
& - 168A^2B^2a^3b^3c^4 - 4A^2B^2a^2b^7c^2 + 144A^2B^2a^4b^2c^5) * (36A^2a^4c \\
& ^4 - 24A^2b^8 - 6B^2a^2b^6 + 400B^2a^5c^3 + 24A^2B^2a^2b^7 - 1152A^2 \\
& *a^2b^4c^2 + 1528A^2a^3b^2c^3 - 291B^2a^4b^2c^2 + 288A^2a^2b^6c \\
& + 72B^2a^3b^4c + 1158A^2B^2a^3b^3c^2 - 288A^2B^2a^2b^5c - 1564A^2B^2a \\
& ^4b^2c^3)) + ((16a^9b^6(4a^2c - b^2)^{(9/2)} - 1024a^12c^3(4a^2c - b^2) \\
& ^{(9/2)} - 192a^10b^4c(4a^2c - b^2)^{(9/2)} + 768a^11b^2c^2(4a^2c - b^2) \\
&)^{(9/2)}) * ((B^3a^3b^2c^4 - 8A^3b^5c^4 + 36A^2B^2a^3c^6 + 48A^3a^2b^ \\
& 3c^5 - 72A^3a^2b^2c^6 + 12A^2B^2a^3b^2c^5 + 12A^2B^2a^2b^4c^4 - 6A^2B^ \\
& 2a^2b^3c^4 - 48A^2B^2a^2b^2c^5) / (a^6b^4 + 16a^8c^2 - 8a^7b^2c) \\
& - (((36A^2a^5c^6 - 16A^2a^2b^6c^3 + 116A^2a^3b^4c^4 - 216A^2a^ \\
& 4b^2c^5 - 4B^2a^4b^4c^3 + 17B^2a^5b^2c^4 + 16A^2B^2a^3b^5c^3 - 9 \\
& 2A^2B^2a^4b^3c^4 + 108A^2B^2a^5b^2c^5) / (a^6b^4 + 16a^8c^2 - 8a^7b^2c) \\
& - (((96A^2a^7b^2c^5 - 8A^2a^4b^7c^2 + 72A^2a^5b^5c^3 - 184A^2a^6b^3c \\
& ^4 + 4B^2a^5b^6c^2 - 36B^2a^6b^4c^3 + 80B^2a^7b^2c^4) / (a^6b^4 + 16a \\
& ^8c^2 - 8a^7b^2c) - ((4a^7b^6c^2 - 32a^8b^4c^3 + 64a^9b^2c^4) * \\
& (4A^2b^7 + 128B^2a^4c^3 - 2B^2a^2b^6 - 48A^2a^2b^5c - 256A^2a^3b^2c^3 + 24 \\
& B^2a^2b^4c + 192A^2a^2b^3c^2 - 96B^2a^3b^2c^2)) / (2(a^6b^4 + 16a^8c \\
& ^2 - 8a^7b^2c) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^5b^2c^2) \\
&)) * (4A^2b^7 + 128B^2a^4c^3 - 2B^2a^2b^6 - 48A^2a^2b^5c - 256A^2a^3b^2c^3 + \\
& 24B^2a^2b^4c + 192A^2a^2b^3c^2 - 96B^2a^3b^2c^2)) / (2(4a^3b^6 - 25 \\
& 6a^6c^3 - 48a^4b^4c + 192a^5b^2c^2)) * (4A^2b^7 + 128B^2a^4c^3 - 2 \\
& B^2a^2b^6 - 48A^2a^2b^5c - 256A^2a^3b^2c^3 + 24B^2a^2b^4c + 192A^2a^2b^3c \\
& ^2 - 96B^2a^3b^2c^2)) / (2(4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 192a^ \\
& 5b^2c^2)) - ((((((96A^2a^7b^2c^5 - 8A^2a^4b^7c^2 + 72A^2a^5b^5c^3 - 18 \\
& 4A^2a^6b^3c^4 + 4B^2a^5b^6c^2 - 36B^2a^6b^4c^3 + 80B^2a^7b^2c^4) / (a \\
& ^6b^4 + 16a^8c^2 - 8a^7b^2c) - ((4a^7b^6c^2 - 32a^8b^4c^3 + 64a \\
& ^9b^2c^4) * (4A^2b^7 + 128B^2a^4c^3 - 2B^2a^2b^6 - 48A^2a^2b^5c - 256A^2a^ \\
& 3b^2c^3 + 24B^2a^2b^4c + 192A^2a^2b^3c^2 - 96B^2a^3b^2c^2)) / (2(a^6b \\
& ^4 + 16a^8c^2 - 8a^7b^2c) * (4a^3b^6 - 256a^6c^3 - 48a^4b^4c + 19 \\
& 2a^5b^2c^2))) * (2A^2b^4 + 12A^2a^2c^2 - B^2a^2b^3 - 12A^2a^2b^2c + 6B^2a^2 \\
& *b^2c)) / (4a^3(4a^2c - b^2)^{(3/2)}) - ((4a^7b^6c^2 - 32a^8b^4c^3 + 64a \\
& ^9b^2c^4) * (2A^2b^4 + 12A^2a^2c^2 - B^2a^2b^3 - 12A^2a^2b^2c + 6B^2a^2b^2c \\
&) * (4A^2b^7 + 128B^2a^4c^3 - 2B^2a^2b^6 - 48A^2a^2b^5c - 256A^2a^3b^2c^3 + 2 \\
& 4B^2a^2b^4c + 192A^2a^2b^3c^2 - 96B^2a^3b^2c^2)) / (8a^3(4a^2c - b^2) \\
& ^{(3/2)} * (a^6b^4 + 16a^8c^2 - 8a^7b^2c) * (4a^3b^6 - 256a^6c^3 - 48a \\
& ^4b^4c + 192a^5b^2c^2)) * (2A^2b^4 + 12A^2a^2c^2 - B^2a^2b^3 - 12A^2a^2b^ \\
& 2c + 6B^2a^2b^2c)) / (4a^3(4a^2c - b^2)^{(3/2)}) + ((4a^7b^6c^2 - 32a^8b \\
& ^4c^3 + 64a^9b^2c^4) * (2A^2b^4 + 12A^2a^2c^2 - B^2a^2b^3 - 12A^2a^2b^2c \\
& + 6B^2a^2b^2c))^2 * (4A^2b^7 + 128B^2a^4c^3 - 2B^2a^2b^6 - 48A^2a^2b^5c - 256 \\
& A^2a^3b^2c^3 + 24B^2a^2b^4c + 192A^2a^2b^3c^2 - 96B^2a^3b^2c^2)) / (32a \\
& ^6(4a^2c - b^2)^3 * (a^6b^4 + 16a^8c^2 - 8a^7b^2c) * (4a^3b^6 - 256a^
\end{aligned}$$

$$\begin{aligned}
& (6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) * (6*A*a^3*c^3 - 6*A*b^6 + 3*B*a*b^5 \\
& + 42*A*a*b^4*c - 21*B*a^2*b^3*c + 33*B*a^3*b*c^2 - 72*A*a^2*b^2*c^2)) / (8* \\
& a^3*c^2*(4*a*c - b^2)^3*(144*A^2*a^4*c^6 + 4*A^2*b^8*c^2 + 192*A^2*a^2*b^4* \\
& c^4 - 288*A^2*a^3*b^2*c^5 + B^2*a^2*b^6*c^2 - 12*B^2*a^3*b^4*c^3 + 36*B^2*a \\
& ^4*b^2*c^4 - 48*A^2*a*b^6*c^3 + 48*A*B*a^2*b^5*c^3 - 168*A*B*a^3*b^3*c^4 - \\
& 4*A*B*a*b^7*c^2 + 144*A*B*a^4*b*c^5)*(36*A^2*a^4*c^4 - 24*A^2*b^8 - 6*B^2*a \\
& ^2*b^6 + 400*B^2*a^5*c^3 + 24*A*B*a*b^7 - 1152*A^2*a^2*b^4*c^2 + 1528*A^2*a \\
& ^3*b^2*c^3 - 291*B^2*a^4*b^2*c^2 + 288*A^2*a*b^6*c + 72*B^2*a^3*b^4*c + 115 \\
& 8*A*B*a^3*b^3*c^2 - 288*A*B*a^2*b^5*c - 1564*A*B*a^4*b*c^3)) * (2*A*b^4 + 12 \\
& *A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)) / (2*a^3*(4*a*c - b^2)^(3 \\
& /2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.96 \quad \int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=425

$$\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}} + 6aAc^2 - 13abBc - Ab^2c + 3b^3B \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) \left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}} \right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 3.67, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}} + 6aAc^2 - 13abBc - Ab^2c + 3b^3B \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}} + 6aAc^2 - 13abBc - Ab^2c + 3b^3B \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b+\sqrt{b^2-4ac}}} \right) + \frac{x(-10aBc - Abc + 3b^2B)}{2c^2(b^2-4ac)} - \frac{x^5(-2aB + x^2(-bB - 2Ac)) + Ab}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(bB - 2Ac)}{2c(b^2-4ac)}}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 - (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2} dx &= -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^4(5(Ab-2aB)-3(bB-2Ac)x^2)}{a+bx^2+cx^4} dx}{2(b^2-4ac)} \\
&= -\frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^2(-9a(bB-2Ac)-3(3b^2B-Abc-10aBc)x^2)}{a+bx^2+cx^4}}{6c(b^2-4ac)} \\
&= \frac{(3b^2B-Abc-10aBc)x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{-3a(3b^2-Abc-10aBc)}{a+bx^2+cx^4}}{6c(b^2-4ac)} \\
&= \frac{(3b^2B-Abc-10aBc)x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(3b^3B-Abc-10aBc)}{6c(b^2-4ac)} \\
&= \frac{(3b^2B-Abc-10aBc)x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(3b^3B-Abc-10aBc)}{6c(b^2-4ac)}
\end{aligned}$$

Mathematica [A] time = 1.20, size = 455, normalized size = 1.07

$$\frac{2\sqrt{c}(-2a^2bc + (-b(A+3Bb^2) + 2A^2a^2 + b^2B) + b^2(10B - Ab))}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\sqrt{2}(2a^2(3A\sqrt{b^2 - 4ac} - 10ab) + b^2(19ab - A\sqrt{b^2 - 4ac}) - ab(13B\sqrt{b^2 - 4ac} + 8A) + b^3(3B\sqrt{b^2 - 4ac} + A) - 3b^4)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) - \frac{\sqrt{2}(2a^2(3A\sqrt{b^2 - 4ac} + 10ab) - b^2(A\sqrt{b^2 - 4ac} + 19ab) - ab(8A - 13B\sqrt{b^2 - 4ac}) + b^3(3B\sqrt{b^2 - 4ac} - A) + 3b^4)}{(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right) + 4B\sqrt{c}x$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B + 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*c^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 8.56, size = 7252, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B*a*b + A*b^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6

$$\begin{aligned}
& * (425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c) / (b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) / (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) * \log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c) * x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6) * sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c) / (b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) * sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8) * sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c) / (b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) / (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) * \log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2* \\
& *B^2*a*b^6)*c)*x - 1/2*\sqrt{1/2}*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3 \\
& *b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2 \\
& *b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 1 \\
& 7*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 \\
& + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27 \\
& *(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4 \\
& *b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b \\
& ^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*\sqrt{((81*B^4*b^8 + 81*A^4*a^2 \\
& *c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 \\
& + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 \\
& - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5) \\
& *c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B \\
& ^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64 \\
& *a^3*c^13)))*\sqrt{-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2 \\
& *a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 \\
& + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + \\
& 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2 \\
& *B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3 \\
& *b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3 \\
& *b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B \\
& ^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B \\
& ^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^ \\
& 6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) - \sqrt{1/2}*(a*b^2*c^ \\
& 2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\sqrt{-(\\
& 9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2 \\
& *b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3* \\
& (35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6 \\
& 4*a^3*c^8)*\sqrt{((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3* \\
& B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a \\
& ^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^ \\
& 2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B \\
& ^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^1 \\
& 0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^ \\
& 6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + \\
& 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2 \\
& 500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c \\
& ^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B \\
& *a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x \\
& + 1/2*\sqrt{1/2}*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500 \\
& *B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (113 \\
& 60*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 \\
& - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 \\
& + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + \\
& A*B^2*b^9)*c + (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 2) * c^9 - 192 * (5 * B * a^3 * b^3 + A * a^2 * b^4) * c^8 + 24 * (14 * B * a^2 * b^5 + A * a * b^6) * c^7 \\
& - (52 * B * a * b^7 + A * b^8) * c^6) * \text{sqrt}((81 * B^4 * b^8 + 81 * A^4 * a^2 * c^6 - 18 * (25 * A^2 * B^2 * a^3 \\
& + 44 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (625 * B^4 * a^4 + 2200 * A * B^3 * a^3 * b + 2904 * A^2 * B^2 * a^2 * b^2 \\
& + 196 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 - 6 * (425 * B^4 * a^3 * b^2 + 798 * A * B^3 * a^2 * b^3 + 132 * A^2 * B^2 * a * b^4 \\
& + 2 * A^3 * B * b^5) * c^3 + 27 * (113 * B^4 * a^2 * b^4 + 52 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (17 * B^4 * a * b^6 \\
& + 2 * A * B^3 * b^7) * c) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) * \text{sqrt} \\
& (- (9 * B^2 * b^7 + 60 * (4 * A * B * a^3 + A^2 * a^2 * b) * c^4 - 15 * (28 * B^2 * a^3 * b + 20 * A * B * a^2 * b^2 \\
& + A^2 * a * b^3) * c^3 + (385 * B^2 * a^2 * b^3 + 80 * A * B * a * b^4 + A^2 * b^5) * c^2 - 3 * (35 * B^2 * a * b^5 + 2 * A * B * b^6) * c \\
& - (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8) * \text{sqrt}((81 * B^4 * b^8 + 81 * A^4 * a^2 * c^6 - 18 * (25 * A^2 * B^2 * a^3 \\
& + 44 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (625 * B^4 * a^4 + 2200 * A * B^3 * a^3 * b + 2904 * A^2 * B^2 * a^2 * b^2 \\
& + 196 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 - 6 * (425 * B^4 * a^3 * b^2 + 798 * A * B^3 * a^2 * b^3 + 132 * A^2 * B^2 * a * b^4 \\
& + 2 * A^3 * B * b^5) * c^3 + 27 * (113 * B^4 * a^2 * b^4 + 52 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (17 * B^4 * a * b^6 \\
& + 2 * A * B^3 * b^7) * c) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) / (b^6 * c^5 - 12 * a * b^4 * c^6 \\
& + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8))) + \text{sqrt}(1/2) * (a * b^2 * c^2 - 4 * a^2 * c^3 + (b^2 * c^3 - 4 * a * c^4) * x^4 + (b^3 * c^2 - 4 * a * b * c^3) * x^2) * \text{sqrt}(- (9 * B^2 * b^7 + 60 * \\
& (4 * A * B * a^3 + A^2 * a^2 * b) * c^4 - 15 * (28 * B^2 * a^3 * b + 20 * A * B * a^2 * b^2 + A^2 * a * b^3) * c^3 + (385 * B^2 * a^2 * b^3 + 80 * A * B * a * b^4 \\
& + A^2 * b^5) * c^2 - 3 * (35 * B^2 * a * b^5 + 2 * A * B * b^6) * c - (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8) * \text{sqrt} \\
& ((81 * B^4 * b^8 + 81 * A^4 * a^2 * c^6 - 18 * (25 * A^2 * B^2 * a^3 + 44 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (625 * B^4 * a^4 + 2200 * A * B^3 * a^3 * b \\
& + 2904 * A^2 * B^2 * a^2 * b^2 + 196 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 - 6 * (425 * B^4 * a^3 * b^2 + 798 * A * B^3 * a^2 * b^3 + 132 * A^2 * B^2 * a * b^4 \\
& + 2 * A^3 * B * b^5) * c^3 + 27 * (113 * B^4 * a^2 * b^4 + 52 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (17 * B^4 * a * b^6 + 2 * A * B^3 * b^7) * c) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} \\
& + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) / (b^6 * c^5 - 12 * a * b^4 * c^6 + 48 * a^2 * b^2 * c^7 - 64 * a^3 * c^8)) * \log((189 * B^4 * a^2 * b^6 - 135 * A * B^3 * a * b^7 + 324 * A^4 * a^3 * c^5 \\
& - 81 * (28 * A^3 * B * a^3 * b + A^4 * a^2 * b^2) * c^4 - (2500 * B^4 * a^5 + 2500 * A * B^3 * a^4 * b - 5016 * A^2 * B^2 * a^3 * b^2 - 647 * A^3 * B * a^2 * b^3 - 5 * A^4 * a * b^4) * c^3 + 9 * (625 * B^4 * a^4 * b^2 \\
& - 303 * A * B^3 * a^3 * b^3 - 186 * A^2 * B^2 * a^2 * b^4 - 5 * A^3 * B * a * b^5) * c^2 - 27 * (73 * B^4 * a^3 * b^4 - 49 * A * B^3 * a^2 * b^5 - 5 * A^2 * B^2 * a * b^6) * c) * x - 1/2 * \text{sqrt}(1/2) \\
& * (27 * B^3 * b^{10} + 144 * (10 * A^2 * B * a^4 + A^3 * a^3 * b) * c^6 - 8 * (500 * B^3 * a^5 + 930 * A * B^2 * a^4 * b + 252 * A^2 * B * a^3 * b^2 + 11 * A^3 * a^2 * b^3) * c^5 + (11360 * B^3 * a^4 * b^2 + 7608 * A * B^2 * a^3 * b^3 + 882 * A^2 * B * a^2 * b^4 + 17 * A^3 * a * b^5) * c^4 - (8818 * B^3 * a^3 * b^4 + 2841 * A * B^2 * a^2 * b^5 + 153 * A^2 * B * a * b^6 + A^3 * b^7) * c^3 + 9 * (329 * B^3 * a^2 * b^6 + 51 * A * B^2 * a * b^7 + A^2 * B * b^8) * c^2 - 27 * (17 * B^3 * a * b^8 + A * B^2 * b^9) * c + (3 * B * b^9 * c^5 - 768 * A * a^4 * c^{10} + 128 * (8 * B * a^4 * b + 5 * A * a^3 * b^2) * c^9 - 192 * (5 * B * a^3 * b^3 + A * a^2 * b^4) * c^8 + 24 * (14 * B * a^2 * b^5 + A * a * b^6) * c^7 - (52 * B * a * b^7 + A * b^8) * c^6) * \text{sqrt}((81 * B^4 * b^8 + 81 * A^4 * a^2 * c^6 - 18 * (25 * A^2 * B^2 * a^3 + 44 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (625 * B^4 * a^4 + 2200 * A * B^3 * a^3 * b + 2904 * A^2 * B^2 * a^2 * b^2 + 196 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 - 6 * (425 * B^4 * a^3 * b^2 + 798 * A * B^3 * a^2 * b^3 + 132 * A^2 * B^2 * a * b^4 + 2 * A^3 * B * b^5) * c^3 + 27 * (113 * B^4 * a^2 * b^4 + 52 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (17 * B^4 * a * b^6 + 2 * A * B^3 * b^7) * c) / (b^6 * c^{10} - 12 * a * b^4 * c^{11} + 48 * a^2 * b^2 * c^{12} - 64 * a^3 * c^{13})) * \text{sqrt}(- (9 * B^2 * b^7 +
\end{aligned}$$

$$60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\text{sqrt}((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) + 2*(3*B*a*b^2 - (10*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)$$

giac [B] time = 6.49, size = 5681, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B*a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^4*c + 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^2 - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*c^3 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^2*c^3 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4*(b^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^5 + 25*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^4*c - 52*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^3*c^2 + 13*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6)*A*\text{abs}(-b^2*c^2 + 4*a*c^3) - 2*(3*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*s$

$$\begin{aligned}
& \text{qrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^4 * c^4 - 6 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^5 * c^4 + 6 * a * b^6 * c^4 + 128 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b^2 * c^5 + 44 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^3 * c^5 + 3 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^4 * c^5 - 68 * a^2 * b^4 * c^5 - 160 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^4 * c^6 - 80 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b * c^6 - 22 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^2 * c^6 + 256 * a^3 * b^2 * c^6 + 40 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * c^7 - 320 * a^4 * c^7 - 6 * (b^2 - 4 * a * c) * a * b^4 * c^4 + 44 * (b^2 - 4 * a * c) * a^2 * b^2 * c^5 - 80 * (b^2 - 4 * a * c) * a^3 * c^6) * B * \text{abs}(-b^2 * c^2 + 4 * a * c^3) - (2 * b^8 * c^7 - 32 * a * b^6 * c^8 + 160 * a^2 * b^4 * c^9 - 256 * a^3 * b^2 * c^{10} - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^8 * c^5 + 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^6 * c^6 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^7 * c^6 - 80 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^4 * c^7 - 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^5 * c^7 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^6 * c^7 + 128 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b^2 * c^8 + 64 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^3 * c^8 + 12 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^4 * c^8 - 32 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^2 * c^9 - 2 * (b^2 - 4 * a * c) * b^6 * c^7 + 24 * (b^2 - 4 * a * c) * a * b^4 * c^8 - 64 * (b^2 - 4 * a * c) * a^2 * b^2 * c^9) * A + (6 * b^9 * c^6 - 86 * a * b^7 * c^7 + 440 * a^2 * b^5 * c^8 - 928 * a^3 * b^3 * c^9 + 640 * a^4 * b * c^{10} - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^9 * c^4 + 43 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^7 * c^5 + 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^8 * c^5 - 220 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^5 * c^6 - 62 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^6 * c^6 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^7 * c^6 + 464 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b^3 * c^7 + 192 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^4 * c^7 + 31 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^5 * c^7 - 320 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^4 * b * c^8 - 160 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b^2 * c^8 - 96 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * b^3 * c^8 + 80 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^3 * b * c^9 - 6 * (b^2 - 4 * a * c) * b^7 * c^6 + 62 * (b^2 - 4 * a * c) * a * b^5 * c^7 - 192 * (b^2 - 4 * a * c) * a^2 * b^3 * c^8 + 160 * (b^2 - 4 * a * c) * a^3 * b * c^9) * B) * \arctan(2 * \text{sqrt}(1/2) * x / \text{sqrt}((b^3 * c^2 - 4 * a * b * c^3 + \text{sqrt}((b^3 * c^2 - 4 * a * b * c^3)^2 - 4 * (a * b^2 * c^2 - 4 * a^2 * c^3) * (b^2 * c^3 - 4 * a * c^4))) / (b^2 * c^3 - 4 * a * c^4))) / ((a * b^6 * c^5 - 12 * a^2 * b^4 * c^6 - 2 * a * b^5 * c^6 + 48 * a^3 * b^2 * c^7 + 16 * a^2 * b^3 * c^7 + a * b^4 * c^7 - 64 * a^4 * c^8 - 32 * a^3 * b * c^8 - 8 * a^2 * b^2 * c^8 + 16 * a^3 * c^9) * \text{abs}(-b^2 * c^2 + 4 * a * c^3) * \text{abs}(c)) - 1/16 * ((2 * b^4 * c^3 - 20 * a * b^2 * c^4 + 48 * a^2 * c^5 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c) * c) * b^4 * c + 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^2 * c^2 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c) * c) * b^3 * c^2 - 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * c^3 - 12 * \text{sqrt}(2) *
\end{aligned}$$

$$\begin{aligned}
& \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^2 c^3 + 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 c^4 - 2(b^2 - 4ac) b^2 c^3 + 12(b^2 - 4ac) a^2 c^4 \\
& (b^2 c^2 - 4ac^3)^2 A - (6b^5 c^2 - 50a^2 b^3 c^3 + 104a^2 b^2 c^4 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c}) b^5 + 25\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^3 \\
& + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^4 c^3 - 52\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^2 - 26\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^2 - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^2 \\
& - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^3 c^2 + 13\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^3 - 6(b^2 - 4ac) b^3 c^2 + 26(b^2 - 4ac) a^2 b^2 c^3 (b^2 c^2 - 4ac^3)^2 B - 2(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^5 c^4 - 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^5 - 2\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^5 - 2a^2 b^5 c^5 + 16\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^6 + 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^6 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^6 + 16a^2 b^3 c^6 - 4\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^7 - 32a^3 b^2 c^7 + 2(b^2 - 4ac) a^2 b^3 c^5 - 8(b^2 - 4ac) a^2 b^2 c^6) A \operatorname{abs}(-b^2 c^2 + 4ac^3) + 2(3\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^6 c^3 - 34\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^4 - 6\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^5 c^4 - 6a^2 b^6 c^4 + 128\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^5 + 44\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^5 + 3\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^5 + 68a^2 b^4 c^5 - 160\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^4 c^6 - 80\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^6 - 22\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^6 - 256a^3 b^2 c^6 + 40\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 c^7 + 320a^4 c^7 + 6(b^2 - 4ac) a^2 b^4 c^4 - 44(b^2 - 4ac) a^2 b^2 c^5 + 80(b^2 - 4ac) a^3 c^6) B \operatorname{abs}(-b^2 c^2 + 4ac^3) - (2b^8 c^7 - 32a^2 b^6 c^8 + 160a^2 b^4 c^9 - 256a^3 b^2 c^{10} - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^8 c^5 + 16\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^6 c^6 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^7 c^6 - 80\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^7 - 24\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^5 c^7 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^6 c^7 + 128\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^8 + 64\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^8 + 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^8 - 32\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^9 - 2(b^2 - 4ac) b^6 c^7 + 24(b^2 - 4ac) a^2 b^4 c^8 - 64(b^2 - 4ac) a^2 b^2 c^9) A + (6b^9 c^6 - 86a^2 b^7 c^7 + 440a^2 b^5 c^8 - 928a^3 b^3 c^9 + 640a^4 b^2 c^{10} - 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^9 c^4 + 43\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^7 c^5 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} b^8 c^5 - 220\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^5 c^7
\end{aligned}$$

$$\begin{aligned} & (1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*A*b+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*A*b+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3+13/4/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*B+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*B-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3-13/4/c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*B+B/c^2*x \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * ((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 + A*a*b)*c)*x) / (a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + B*x/c^2 - 1/2 * \operatorname{integrate}((3*B*a*b^2 + (3*B*b^3 + 6*A*a*c^2 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c) / (c*x^4 + b*x^2 + a), x) / (b^2*c^2 - 4*a*c^3)$

mupad [B] time = 4.36, size = 16604, normalized size = 39.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out] $\frac{(B*x)/c^2 - \operatorname{atan}((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6) / (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^{11}*c^2 - 9*B^2*b^{13} + 6*A*B*b^{12}*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + 213*B^2*a*b^{11}*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*$

$$\begin{aligned}
& a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^{10}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)} / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * (16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) * \\
& (9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^{11}*c^2 - 9*B^2*b^{13} + 6*A*B*b^{12}*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + 213*B^2*a*b^{11}*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^{10}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4)) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) * ((9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^{11}*c^2 - 9*B^2*b^{13} + 6*A*B*b^{12}*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + 213*B^2*a*b^{11}*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^{10}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * ii - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6) / (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^{11}*c^2 - 9*B^2*b^{13} + 6*A*B*b^{12}*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + 213*B^2*a*b^{11}*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^{10}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6) / (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^{11}*c^2 - 9*B^2*b^{13} + 6*A*B*b^{12}*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + 213*B^2*a*b^{11}*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^{10}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * i
\end{aligned}$$

$$\begin{aligned}
& \left(-6144a^5b^2c^{10} \right)^{1/2} \cdot \left(16b^7c^5 - 192a^5b^5c^6 - 1024a^3b^3c^8 + 768a^2b^3c^7 \right) / \left(2 \cdot \left(16a^2c^5 + b^4c^3 - 8ab^2c^4 \right) \right) \cdot \left((9B^2b^4 \right. \\
& \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - A^2b^{11}c^2 - 9B^2b^{13} + 6ABb^{12}c - 288A^2a^2b^7c^4 + 1504A^2a^3b^5c^5 - 3840A^2a^4b^3c^6 - 2077B^2a^2b^9 \\
& \cdot c^2 + 10656B^2a^3b^7c^3 - 30240B^2a^4b^5c^4 + 44800B^2a^5b^3c^5 + A^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 25B^2a^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} \\
& + 15360ABa^6c^7 + 213B^2a^2b^{11}c + 27A^2a^2b^9c^3 + 3840A^2a^5b^3c^7 - 9A^2a^3c^3 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 26880B^2a^6b^3c^6 \\
& + 1548ABa^2b^8c^3 - 8064ABa^3b^6c^4 + 22400ABa^4b^4c^5 - 30720ABa^5b^2c^6 - 51B^2a^2b^2c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 152ABa^2b^{10} \\
& \cdot c^2 - 6ABb^3c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 44ABa^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} \left. \right) / \left(32 \cdot \left(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 \right. \right. \\
& \left. \left. - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10} \right) \right)^{1/2} + \left(x \cdot \left(9B^2b^8 - 72A^2a^3c^5 + A^2b^6c^2 + 200B^2a^4c^4 - 6ABb^7c \right. \right. \\
& \left. \left. + 74A^2a^2b^2c^4 + 481B^2a^2b^4c^2 - 718B^2a^3b^2c^3 - 114B^2a^2b^6c - 16A^2a^2b^4c^3 - 374ABa^2b^3c^3 + 86ABa^2b^5c^2 + 472 \right. \right. \\
& \left. \left. \cdot ABa^3b^3c^4 \right) \right) / \left(2 \cdot \left(16a^2c^5 + b^4c^3 - 8ab^2c^4 \right) \right) \cdot \left((9B^2b^4 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - A^2b^{11}c^2 - 9B^2b^{13} + 6ABb^{12}c - 288A^2a^2b^7c^4 \right. \right. \\
& \left. \left. + 1504A^2a^3b^5c^5 - 3840A^2a^4b^3c^6 - 2077B^2a^2b^9c^2 + 10656B^2a^3b^7c^3 - 30240B^2a^4b^5c^4 + 44800B^2a^5b^3c^5 + A^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 25B^2a^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} \right. \right. \\
& \left. \left. + 15360ABa^6c^7 + 213B^2a^2b^{11}c + 27A^2a^2b^9c^3 + 3840A^2a^5b^3c^7 - 9A^2a^3c^3 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 26880B^2a^6b^3c^6 + 1548ABa^2b^8c^3 - 8064ABa^3b^6c^4 + 22400ABa^4b^4c^5 - 30720 \right. \right. \\
& \left. \left. \cdot ABa^5b^2c^6 - 51B^2a^2b^2c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 152ABa^2b^{10}c^2 - 6ABb^3c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 44ABa^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} \right) \right) / \left(32 \cdot \left(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 \right. \right. \\
& \left. \left. - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10} \right) \right)^{1/2} \cdot i) / \left(\left(\left(10240B^2a^5c^7 - 16A^2a^2b^7c^4 + 1024A^2a^4b^3c^7 + 48B^2a^2b^8c^3 + 192 \right. \right. \right. \\
& \left. \left. \cdot A^2a^2b^5c^5 - 768A^2a^3b^3c^6 - 736B^2a^2b^6c^4 + 4224B^2a^3b^4c^5 - 10752B^2a^4b^2c^6 \right) / \left(8 \cdot \left(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5 \right) \right) \right. \\
& \left. - \left(x \cdot \left(9B^2b^4 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - A^2b^{11}c^2 - 9B^2b^{13} + 6ABb^{12}c - 288A^2a^2b^7c^4 + 1504A^2a^3b^5c^5 - 3840A^2a^4b^3c^6 - 2077B^2a^2b^9c^2 + 10656B^2a^3b^7c^3 - 30240B^2a^4b^5c^4 + 44800B^2a^5b^3c^5 + A^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 25B^2a^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 15360ABa^6c^7 + 213B^2a^2b^{11}c + 27A^2a^2b^9c^3 + 3840A^2a^5b^3c^7 - 9A^2a^3c^3 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 26880B^2a^6b^3c^6 + 1548ABa^2b^8c^3 - 8064ABa^3b^6c^4 + 22400ABa^4b^4c^5 - 30720ABa^5b^2c^6 - 51B^2a^2b^2c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - 152ABa^2b^{10}c^2 - 6ABb^3c \cdot \left(-(4ac - b^2)^9 \right)^{1/2} + 44ABa^2b^2c^2 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} \right) \right) / \left(32 \cdot \left(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10} \right) \right)^{1/2} \cdot \left(16b^7c^5 - 192a^5b^5c^6 - 1024a^3b^3c^8 + 768a^2b^3c^7 \right) / \left(2 \cdot \left(16a^2c^5 + b^4c^3 - 8ab^2c^4 \right) \right) \cdot \left((9B^2b^4 \cdot \left(-(4ac - b^2)^9 \right)^{1/2} - A^2b^{11}c^2 - 9B^2b^{13} + 6ABb^{12}c - 288A^2a^2b^7c^4 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 \\
& + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A \\
& ^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5 \\
& *b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548* \\
& A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B* \\
& a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^10*c^2 \\
& - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& (1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 12 \\
& 80*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10))^{(1/2)} - (x*(9*B^2* \\
& b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6*A*B*b^7*c + 74*A^2 \\
& *a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 - 114*B^2*a*b^6*c \\
& - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b \\
& *c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))*((9*B^2*b^4*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 \\
& + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 106 \\
& 56*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^ \\
& 2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^ \\
& 7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a \\
& ^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b \\
& ^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^10*c^2 - 6*A \\
& *B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^ \\
& 3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10))^{(1/2)} - (216*A^3*a^4*c^ \\
& 4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*c^3 + 45*A*B^2*a^2* \\
& b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a^5*b*c^2 - 402*A*B^ \\
& 2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 + 762*A*B^2*a^4*b^2* \\
& c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a \\
& ^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48* \\
& B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4 \\
& 224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^ \\
& 4*c^4 - 48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^ \\
& 11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5 \\
& *c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 \\
& - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 15360*A*B*a^6*c^7 + \\
& 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A \\
& *B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b \\
& ^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(4096*a^6*c^1 \\
& 1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^ \\
& 4*b^4*c^9 - 6144*a^5*b^2*c^10))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a \\
& ^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))*((9*
\end{aligned}$$

$$\begin{aligned}
& B^2 b^4 (- (4ac - b^2)^9)^{1/2} - A^2 b^{11} c^2 - 9B^2 b^{13} + 6A^2 B b^{12} c \\
& - 288A^2 a^2 b^7 c^4 + 1504A^2 a^3 b^5 c^5 - 3840A^2 a^4 b^3 c^6 - 2077B^2 a^2 b^9 c^2 + 10656B^2 a^3 b^7 c^3 - 30240B^2 a^4 b^5 c^4 + 44800B^2 \\
& a^5 b^3 c^5 + A^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} + 25B^2 a^2 c^2 (- (4ac - b^2)^9)^{1/2} + 15360A^2 B a^6 c^7 + 213B^2 a^2 b^{11} c + 27A^2 a^2 b^9 c^3 \\
& + 3840A^2 a^5 b^3 c^7 - 9A^2 a^2 c^3 (- (4ac - b^2)^9)^{1/2} - 26880B^2 a^6 b^3 c^6 + 1548A^2 B a^2 b^8 c^3 - 8064A^2 B a^3 b^6 c^4 + 22400A^2 B a^4 b^4 c^5 \\
& - 30720A^2 B a^5 b^2 c^6 - 51B^2 a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} - 152A^2 B a^2 b^{10} c^2 - 6A^2 B b^3 c^2 (- (4ac - b^2)^9)^{1/2} + 44A^2 B a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} \\
& / (32(4096a^6 c^{11} + b^{12} c^5 - 24a^2 b^{10} c^6 + 240a^2 b^8 c^7 - 1280a^3 b^6 c^8 + 3840a^4 b^4 c^9 - 6144a^5 b^2 c^{10}))^{1/2} + (x(9B^2 b^8 - 72A^2 a^3 c^5 + A^2 b^6 c^2 + 200B^2 a^4 c^4 - 6A^2 \\
& B b^7 c + 74A^2 a^2 b^2 c^4 + 481B^2 a^2 b^4 c^2 - 718B^2 a^3 b^2 c^3 - 114B^2 a^2 b^6 c - 16A^2 a^2 b^4 c^3 - 374A^2 B a^2 b^3 c^3 + 86A^2 B a^2 b^5 c^2 + 472A^2 B a^3 b^4 c^4)) / (2(16a^2 c^5 + b^4 c^3 - 8a^2 b^2 c^4)) * ((9B^2 b^4 (- (4ac - b^2)^9)^{1/2} - A^2 b^{11} c^2 - 9B^2 b^{13} + 6A^2 B b^{12} c - 28 \\
& 8A^2 a^2 b^7 c^4 + 1504A^2 a^3 b^5 c^5 - 3840A^2 a^4 b^3 c^6 - 2077B^2 a^2 b^9 c^2 + 10656B^2 a^3 b^7 c^3 - 30240B^2 a^4 b^5 c^4 + 44800B^2 a^5 b^3 c^5 + A^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} + 25B^2 a^2 c^2 (- (4ac - b^2)^9)^{1/2} + 15360A^2 B a^6 c^7 + 213B^2 a^2 b^{11} c + 27A^2 a^2 b^9 c^3 + \\
& 3840A^2 a^5 b^3 c^7 - 9A^2 a^2 c^3 (- (4ac - b^2)^9)^{1/2} - 26880B^2 a^6 b^3 c^6 + 1548A^2 B a^2 b^8 c^3 - 8064A^2 B a^3 b^6 c^4 + 22400A^2 B a^4 b^4 c^5 - 30720A^2 B a^5 b^2 c^6 - 51B^2 a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} - 152A^2 B \\
& a^2 b^{10} c^2 - 6A^2 B b^3 c^2 (- (4ac - b^2)^9)^{1/2} + 44A^2 B a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} / (32(4096a^6 c^{11} + b^{12} c^5 - 24a^2 b^{10} c^6 + 240a^2 b^8 c^7 - 1280a^3 b^6 c^8 + 3840a^4 b^4 c^9 - 6144a^5 b^2 c^{10}))^{1/2}) * ((9B^2 b^4 (- (4ac - b^2)^9)^{1/2} - A^2 b^{11} c^2 - 9B^2 b^{13} + 6A^2 B b^{12} c - 288A^2 a^2 b^7 c^4 + 1504A^2 a^3 b^5 c^5 - 3840A^2 a^4 b^3 c^6 - 2077B^2 a^2 b^9 c^2 + 10656B^2 a^3 b^7 c^3 - 30240B^2 a^4 b^5 c^4 + 44 \\
& 800B^2 a^5 b^3 c^5 + A^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} + 25B^2 a^2 c^2 (- (4ac - b^2)^9)^{1/2} + 15360A^2 B a^6 c^7 + 213B^2 a^2 b^{11} c + 27A^2 a^2 b^9 c^3 + 3840A^2 a^5 b^3 c^7 - 9A^2 a^2 c^3 (- (4ac - b^2)^9)^{1/2} - 26880B^2 a^6 b^3 c^6 + 1548A^2 B a^2 b^8 c^3 - 8064A^2 B a^3 b^6 c^4 + 22400A^2 B a^4 b^4 c^5 - 30720A^2 B a^5 b^2 c^6 - 51B^2 a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} - 152A^2 B a^2 b^{10} c^2 - 6A^2 B b^3 c^2 (- (4ac - b^2)^9)^{1/2} + 44A^2 B a^2 b^2 c^2 (- (4ac - b^2)^9)^{1/2} / (32(4096a^6 c^{11} + b^{12} c^5 - 24a^2 b^{10} c^6 + 240a^2 b^8 c^7 - 1280a^3 b^6 c^8 + 3840a^4 b^4 c^9 - 6144a^5 b^2 c^{10}))^{1/2}) * 2i - ((x^3 (B^2 b^3 + 2A^2 a^2 c^2 - A^2 b^2 c - 3B^2 a^2 b^3 c)) / (2(4ac - b^2)) - (x(2B^2 a^2 c - B^2 a^2 b^2 + A^2 a^2 b^3 c)) / (2(4ac - b^2))) / (a^2 c^2 + c^3 x^4 + b^2 c^2 x^2) - atan((((10240B^2 a^5 c^7 - 16A^2 a^2 b^7 c^4 + 1024A^2 a^4 b^3 c^7 + 48B^2 a^2 b^8 c^3 + 192A^2 a^2 b^5 c^5 - 768A^2 a^3 b^3 c^6 - 736B^2 a^2 b^6 c^4 + 4224B^2 a^3 b^4 c^5 - 10752B^2 a^4 b^2 c^6) / (8(64a^3 c^6 - b^6 c^3 + 12a^2 b^4 c^4 - 48a^2 b^2 c^5)) - (x(- (9B^2 b^{13} + A^2 b^{11} c^2 + 9B^2 b^4 (- (4ac - b^2)^9)^{1/2} - 6A^2 B b^{12} c + 288A^2 a^2 b^7 c^4 - 1504A^2 a^3 b^5 c^5 + 3840A^2 a^4 b^3 c^6 + 2077B^2 a^2 b^9 c^2 - 10656B^2
\end{aligned}$$

$$\begin{aligned}
& 2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2 \\
& *(-4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-4*a*c - b^2)^9)^{(1/2)} - 15360 \\
& *A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9 \\
& *A^2*a*c^3*(-4*a*c - b^2)^9)^{(1/2)} + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8 \\
& *c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 \\
& - 51*B^2*a*b^2*c*(-4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a*b^10*c^2 - 6*A*B*b^3 \\
& *c*(-4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-4*a*c - b^2)^9)^{(1/2))}/(32 \\
& *(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6 \\
& *c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5 \\
& *c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2 \\
& *c^4)))*(-9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-4*a*c - b^2)^9)^{(1/2)} \\
& - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4 \\
& *b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5 \\
& *c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-4*a*c - b^2)^9)^{(1/2)} + 25*B^2 \\
& *a^2*c^2*(-4*a*c - b^2)^9)^{(1/2)} - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - \\
& 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-4*a*c - b^2)^9)^{(1/2)} \\
& + 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22 \\
& 400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-4*a*c - b^2 \\
&)^9)^{(1/2)} + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-4*a*c - b^2)^9)^{(1/2)} + 44 \\
& *A*B*a*b*c^2*(-4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a \\
& *b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5 \\
& *b^2*c^10)))^{(1/2)} - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2 \\
& *a^4*c^4 - 6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2 \\
& *a^3*b^2*c^3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + \\
& 86*A*B*a*b^5*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4 \\
&)))*(-9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-4*a*c - b^2)^9)^{(1/2)} - 6 \\
& *A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3 \\
& *c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 \\
& - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2 \\
& *c^2*(-4*a*c - b^2)^9)^{(1/2)} - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27* \\
& A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-4*a*c - b^2)^9)^{(1/2)} + \\
& 26880*B^2*a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400* \\
& A*B*a^4*b^4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-4*a*c - b^2)^9) \\
& ^{(1/2)} + 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-4*a*c - b^2)^9)^{(1/2)} + 44*A*B \\
& *a*b*c^2*(-4*a*c - b^2)^9)^{(1/2))}/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^1 \\
& 0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2 \\
& *c^10)))^{(1/2)}*i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 \\
& + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 \\
& + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6))/(8*(64*a^3*c^6 - b^6*c^3 + 1 \\
& 2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-9*B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4 \\
& *(-4*a*c - b^2)^9)^{(1/2)} - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^4 - 1504*A^2* \\
& a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7 \\
& *c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-4*a \\
& *c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-4*a*c - b^2)^9)^{(1/2)} - 15360*A*B*a^6 \\
& *c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*
\end{aligned}$$

$$\begin{aligned}
& c^3 * (- (4 * a * c - b^2)^9)^{(1/2)} + 26880 * B^2 * a^6 * b * c^6 - 1548 * A * B * a^2 * b^8 * c^3 + \\
& 8064 * A * B * a^3 * b^6 * c^4 - 22400 * A * B * a^4 * b^4 * c^5 + 30720 * A * B * a^5 * b^2 * c^6 - 51 * \\
& B^2 * a * b^2 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a * b^{10} * c^2 - 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 44 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} / (32 * (4096 * a^6 * c^{11} + b^{12} * c^5 - 24 * a * b^{10} * c^6 + 240 * a^2 * b^8 * c^7 - 1280 * a^3 * b^6 * c^8 + 3840 * a^4 * b^4 * c^9 - 6144 * a^5 * b^2 * c^{10}))^{(1/2)} * (16 * b^7 * c^5 - 192 * a * b^5 * c^6 - 1024 * a^3 * b * c^8 + 768 * a^2 * b^3 * c^7) / (2 * (16 * a^2 * c^5 + b^4 * c^3 - 8 * a * b^2 * c^4)) * (- (9 * B^2 * b^{13} + A^2 * b^{11} * c^2 + 9 * B^2 * b^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * b^{12} * c + 288 * A^2 * a^2 * b^7 * c^4 - 1504 * A^2 * a^3 * b^5 * c^5 + 3840 * A^2 * a^4 * b^3 * c^6 + 2077 * B^2 * a^2 * b^9 * c^2 - 10656 * B^2 * a^3 * b^7 * c^3 + 30240 * B^2 * a^4 * b^5 * c^4 - 44800 * B^2 * a^5 * b^3 * c^5 + A^2 * b^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 25 * B^2 * a^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - 15360 * A * B * a^6 * c^7 - 213 * B^2 * a * b^{11} * c - 27 * A^2 * a * b^9 * c^3 - 3840 * A^2 * a^5 * b * c^7 - 9 * A^2 * a * c^3 * (- (4 * a * c - b^2)^9)^{(1/2)} + 26880 * B^2 * a^6 * b * c^6 - 1548 * A * B * a^2 * b^8 * c^3 + 8064 * A * B * a^3 * b^6 * c^4 - 22400 * A * B * a^4 * b^4 * c^5 + 30720 * A * B * a^5 * b^2 * c^6 - 51 * B^2 * a * b^2 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a * b^{10} * c^2 - 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 44 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} / (32 * (4096 * a^6 * c^{11} + b^{12} * c^5 - 24 * a * b^{10} * c^6 + 240 * a^2 * b^8 * c^7 - 1280 * a^3 * b^6 * c^8 + 3840 * a^4 * b^4 * c^9 - 6144 * a^5 * b^2 * c^{10}))^{(1/2)} + (x * (9 * B^2 * b^8 - 72 * A^2 * a^3 * c^5 + A^2 * b^6 * c^2 + 200 * B^2 * a^4 * c^4 - 6 * A * B * b^7 * c + 74 * A^2 * a^2 * b^2 * c^4 + 481 * B^2 * a^2 * b^4 * c^2 - 718 * B^2 * a^3 * b^2 * c^3 - 114 * B^2 * a * b^6 * c - 16 * A^2 * a * b^4 * c^3 - 374 * A * B * a^2 * b^3 * c^3 + 86 * A * B * a * b^5 * c^2 + 472 * A * B * a^3 * b * c^4)) / (2 * (16 * a^2 * c^5 + b^4 * c^3 - 8 * a * b^2 * c^4)) * (- (9 * B^2 * b^{13} + A^2 * b^{11} * c^2 + 9 * B^2 * b^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * b^{12} * c + 288 * A^2 * a^2 * b^7 * c^4 - 1504 * A^2 * a^3 * b^5 * c^5 + 3840 * A^2 * a^4 * b^3 * c^6 + 2077 * B^2 * a^2 * b^9 * c^2 - 10656 * B^2 * a^3 * b^7 * c^3 + 30240 * B^2 * a^4 * b^5 * c^4 - 44800 * B^2 * a^5 * b^3 * c^5 + A^2 * b^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 25 * B^2 * a^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - 15360 * A * B * a^6 * c^7 - 213 * B^2 * a * b^{11} * c - 27 * A^2 * a * b^9 * c^3 - 3840 * A^2 * a^5 * b * c^7 - 9 * A^2 * a * c^3 * (- (4 * a * c - b^2)^9)^{(1/2)} + 26880 * B^2 * a^6 * b * c^6 - 1548 * A * B * a^2 * b^8 * c^3 + 8064 * A * B * a^3 * b^6 * c^4 - 22400 * A * B * a^4 * b^4 * c^5 + 30720 * A * B * a^5 * b^2 * c^6 - 51 * B^2 * a * b^2 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a * b^{10} * c^2 - 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 44 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} / (32 * (4096 * a^6 * c^{11} + b^{12} * c^5 - 24 * a * b^{10} * c^6 + 240 * a^2 * b^8 * c^7 - 1280 * a^3 * b^6 * c^8 + 3840 * a^4 * b^4 * c^9 - 6144 * a^5 * b^2 * c^{10}))^{(1/2)} * ii) / (((10240 * B * a^5 * c^7 - 16 * A * a * b^7 * c^4 + 1024 * A * a^4 * b * c^7 + 48 * B * a * b^8 * c^3 + 192 * A * a^2 * b^5 * c^5 - 768 * A * a^3 * b^3 * c^6 - 736 * B * a^2 * b^6 * c^4 + 4224 * B * a^3 * b^4 * c^5 - 10752 * B * a^4 * b^2 * c^6) / (8 * (64 * a^3 * c^6 - b^6 * c^3 + 12 * a * b^4 * c^4 - 48 * a^2 * b^2 * c^5)) - (x * (- (9 * B^2 * b^{13} + A^2 * b^{11} * c^2 + 9 * B^2 * b^4 * (- (4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * b^{12} * c + 288 * A^2 * a^2 * b^7 * c^4 - 1504 * A^2 * a^3 * b^5 * c^5 + 3840 * A^2 * a^4 * b^3 * c^6 + 2077 * B^2 * a^2 * b^9 * c^2 - 10656 * B^2 * a^3 * b^7 * c^3 + 30240 * B^2 * a^4 * b^5 * c^4 - 44800 * B^2 * a^5 * b^3 * c^5 + A^2 * b^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} + 25 * B^2 * a^2 * c^2 * (- (4 * a * c - b^2)^9)^{(1/2)} - 15360 * A * B * a^6 * c^7 - 213 * B^2 * a * b^{11} * c - 27 * A^2 * a * b^9 * c^3 - 3840 * A^2 * a^5 * b * c^7 - 9 * A^2 * a * c^3 * (- (4 * a * c - b^2)^9)^{(1/2)} + 26880 * B^2 * a^6 * b * c^6 - 1548 * A * B * a^2 * b^8 * c^3 + 8064 * A * B * a^3 * b^6 * c^4 - 22400 * A * B * a^4 * b^4 * c^5 + 30720 * A * B * a^5 * b^2 * c^6 - 51 * B^2 * a * b^2 * c * (- (4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a * b^{10} * c^2 - 6 * A * B * b^3 * c * (- (4 * a * c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^1 \\
& 1 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^ \\
& 4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2)}*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a \\
& ^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9 \\
& *B^2*b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*b^12*c \\
& c + 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 207 \\
& 7*B^2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B \\
& ^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c \\
& c^3 - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*B^2 \\
& *a^6*b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^ \\
& 4*c^5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1 \\
& 52*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 24 \\
& 0*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(\\
& (1/2)} - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 - 6* \\
& A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^3 \\
& - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5*c \\
& ^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*B^2 \\
& *b^13 + A^2*b^11*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*b^12*c + \\
& 288*A^2*a^2*b^7*c^4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^ \\
& 2*a^2*b^9*c^2 - 10656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a \\
& ^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 \\
& - 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*B^2*a^6 \\
& *b*c^6 - 1548*A*B*a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^ \\
& 5 + 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A \\
& *B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 44*A*B*a*b*c^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^ \\
& 2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^{(1/2 \\
&)} - (216*A^3*a^4*c^4 - 63*B^3*a^3*b^5 + 5*A^3*a^2*b^4*c^2 - 66*A^3*a^3*b^2*c \\
& ^3 + 45*A*B^2*a^2*b^6 + 600*A*B^2*a^5*c^3 + 573*B^3*a^4*b^3*c - 1300*B^3*a \\
& ^5*b*c^2 - 402*A*B^2*a^3*b^4*c - 30*A^2*B*a^2*b^5*c - 924*A^2*B*a^4*b*c^3 + \\
& 762*A*B^2*a^4*b^2*c^2 + 339*A^2*B*a^3*b^3*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + \\
& 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 102 \\
& 4*A*a^4*b*c^7 + 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 73 \\
& 6*B*a^2*b^6*c^4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 \\
& - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*B^2*b^13 + A^2*b^11*c \\
& ^2 + 9*B^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*b^12*c + 288*A^2*a^2*b^7*c^ \\
& 4 - 1504*A^2*a^3*b^5*c^5 + 3840*A^2*a^4*b^3*c^6 + 2077*B^2*a^2*b^9*c^2 - 10 \\
& 656*B^2*a^3*b^7*c^3 + 30240*B^2*a^4*b^5*c^4 - 44800*B^2*a^5*b^3*c^5 + A^2*b \\
& ^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 15360*A*B*a^6*c^7 - 213*B^2*a*b^11*c - 27*A^2*a*b^9*c^3 - 3840*A^2*a^5*b*c \\
& ^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*B^2*a^6*b*c^6 - 1548*A*B \\
& a^2*b^8*c^3 + 8064*A*B*a^3*b^6*c^4 - 22400*A*B*a^4*b^4*c^5 + 30720*A*B*a^5*
\end{aligned}$$

$$\begin{aligned}
& b^2c^6 - 51B^2a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 152ABa^2b^{10}c^2 - 6* \\
& ABb^3c^2(-4ac - b^2)^9)^{(1/2)} + 44ABa^2b^2c^2(-4ac - b^2)^9)^{(1/2)} \\
&)/(32(4096a^6c^{11} + b^{12}c^5 - 24a^2b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 \\
& + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)}*(16b^7c^5 - 19 \\
& 2a^2b^5c^6 - 1024a^3b^3c^8 + 768a^2b^3c^7)/(2(16a^2c^5 + b^4c^3 - \\
& 8a^2b^2c^4))*(-(9B^2b^{13} + A^2b^{11}c^2 + 9B^2b^4*(-4ac - b^2)^9) \\
& ^{(1/2)} - 6ABb^{12}c + 288A^2a^2b^7c^4 - 1504A^2a^3b^5c^5 + 3840A^2a^4b^3c^6 \\
& + 2077B^2a^2b^9c^2 - 10656B^2a^3b^7c^3 + 30240B^2a^4b^5c^4 - 44800B^2a^5b^3c^5 \\
& + A^2b^2c^2*(-4ac - b^2)^9)^{(1/2)} + 25B^2a^2c^2*(-4ac - b^2)^9)^{(1/2)} - 15360ABa^6c^7 \\
& - 213B^2a^2b^{11}c - 27A^2a^2b^9c^3 - 3840A^2a^5b^3c^7 - 9A^2a^3c^3*(-4ac - b^2)^9) \\
& ^{(1/2)} + 26880B^2a^6b^3c^6 - 1548ABa^2b^8c^3 + 8064ABa^3b^6c^4 - 22400ABa^4b^4c^5 \\
& + 30720ABa^5b^2c^6 - 51B^2a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 152ABa^2b^{10}c^2 \\
& - 6ABb^3c^2(-4ac - b^2)^9)^{(1/2)} + 44ABa^2b^2c^2(-4ac - b^2)^9)^{(1/2)}/(32(4096a^6c^{11} \\
& + b^{12}c^5 - 24a^2b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})) \\
&)^{(1/2)} + (x(9B^2b^8 - 72A^2a^3c^5 + A^2b^6c^2 + 200B^2a^4c^4 - 6ABb^7c + 74A^2a^2b^2c^4 \\
& + 481B^2a^2b^4c^2 - 718B^2a^3b^2c^3 - 114B^2a^2b^6c - 16A^2a^2b^4c^3 - 374ABa^2b^3c^3 \\
& + 86ABa^2b^5c^2 + 472ABa^3b^3c^4))/(2(16a^2c^5 + b^4c^3 - 8a^2b^2c^4))*(-(9B^2b^{13} \\
& + A^2b^{11}c^2 + 9B^2b^4*(-4ac - b^2)^9)^{(1/2)} - 6ABb^{12}c + 288A^2a^2b^7c^4 \\
& - 1504A^2a^3b^5c^5 + 3840A^2a^4b^3c^6 + 2077B^2a^2b^9c^2 - 10656B^2a^3b^7c^3 + 30240B^2a^4b^5c^4 \\
& - 44800B^2a^5b^3c^5 + A^2b^2c^2*(-4ac - b^2)^9)^{(1/2)} + 25B^2a^2c^2*(-4ac - b^2)^9) \\
& ^{(1/2)} - 15360ABa^6c^7 - 213B^2a^2b^{11}c - 27A^2a^2b^9c^3 - 3840A^2a^5b^3c^7 - 9A^2a^3c^3 \\
& (-4ac - b^2)^9)^{(1/2)} + 26880B^2a^6b^3c^6 - 1548ABa^2b^8c^3 + 8064ABa^3b^6c^4 - 22400ABa^4b^4c^5 \\
& + 30720ABa^5b^2c^6 - 51B^2a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 152ABa^2b^{10}c^2 - 6ABb^3c^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 44ABa^2b^2c^2(-4ac - b^2)^9)^{(1/2)}/(32(4096a^6c^{11} + b^{12}c^5 - 24 \\
& a^2b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)})) \\
&)^{(1/2)}*(-(9B^2b^{13} + A^2b^{11}c^2 + 9B^2b^4*(-4ac - b^2)^9)^{(1/2)} - 6ABb^{12}c + 288A^2a^2b^7c^4 \\
& - 1504A^2a^3b^5c^5 + 3840A^2a^4b^3c^6 + 2077B^2a^2b^9c^2 - 10656B^2a^3b^7c^3 + 30240B^2a^4b^5c^4 \\
& - 44800B^2a^5b^3c^5 + A^2b^2c^2*(-4ac - b^2)^9)^{(1/2)} + 25B^2a^2c^2*(-4ac - b^2)^9) \\
& ^{(1/2)} - 15360ABa^6c^7 - 213B^2a^2b^{11}c - 27A^2a^2b^9c^3 - 3840A^2a^5b^3c^7 - 9A^2a^3c^3 \\
& (-4ac - b^2)^9)^{(1/2)} + 26880B^2a^6b^3c^6 - 1548ABa^2b^8c^3 + 8064ABa^3b^6c^4 - 22400ABa^4b^4c^5 \\
& + 30720ABa^5b^2c^6 - 51B^2a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 152ABa^2b^{10}c^2 - 6ABb^3c^2 \\
& (-4ac - b^2)^9)^{(1/2)} + 44ABa^2b^2c^2(-4ac - b^2)^9)^{(1/2)}/(32(4096a^6c^{11} + b^{12}c^5 - 24 \\
& a^2b^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.97 \quad \int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=336

$$\frac{x^3(-2aB - (x^2(bB - 2Ac)) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(bB - 2Ac)}{2c(b^2 - 4ac)} + \frac{\left(-\frac{4aAc^2 - 8abBc + Ab^2c + b^3B}{\sqrt{b^2 - 4ac}} - 6aBc + Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.72, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{4aAc^2 - 8abBc + Ab^2c + b^3B}{\sqrt{b^2 - 4ac}} - 6aBc + Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(\frac{4aAc^2 - 8abBc + Ab^2c + b^3B}{\sqrt{b^2 - 4ac}} - 6aBc + Abc + b^2B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{\sqrt{b^2 - 4ac} + b}} - \frac{x^3(-2aB + x^2(-bB - 2Ac)) + Ab}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{x(bB - 2Ac)}{2c(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c - (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx &= -\frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{x^2(3(Ab - 2aB) + (-bB + 2Ac)x^2)}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\ &= -\frac{(bB - 2Ac)x}{2c(b^2 - 4ac)} - \frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-a(bB - 2Ac) + (-b^2B - Abc + 6aBc)x^2}{a + bx^2 + cx^4} dx}{2c(b^2 - 4ac)} \\ &= -\frac{(bB - 2Ac)x}{2c(b^2 - 4ac)} - \frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2B + Abc - 6aBc - \frac{b^3B + Ab^2c - 8abB}{\sqrt{b^2 - 4ac}}\right)}{4c(b^2 - 4ac)} \\ &= -\frac{(bB - 2Ac)x}{2c(b^2 - 4ac)} - \frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2B + Abc - 6aBc - \frac{b^3B + Ab^2c - 8abB}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.85, size = 362, normalized size = 1.08

$$\frac{2\sqrt{c}(2acx(A+Bx^2) - abBx + bx^3(Ac - bB))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b^2(B\sqrt{b^2 - 4ac} - Ac) + b(A\sqrt{b^2 - 4ac} + 8aB) - 2ac(3B\sqrt{b^2 - 4ac} + 2Ac) + b^3(-B)) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}(b^2(B\sqrt{b^2 - 4ac} + Ac) + b(Ac\sqrt{b^2 - 4ac} - 8aBc) + 2a(2Ac - 3B\sqrt{b^2 - 4ac}) + b^3B) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c))*x^3 + 2*a*c*x*(A + B*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[b^2 - 4*a*c])) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*c^(3/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 2.82, size = 4658, normalized size = 13.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c))*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2

$$\begin{aligned}
& - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))) * \\
& \text{sqrt}(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 \\
& + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + \\
& 48*a^2*b^2*c^5 - 64*a^3*c^6))*\text{sqrt}((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2* \\
& A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4 \\
& *a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3* \\
& c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - \text{sqrt}(1/2) \\
& *((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)* \\
& \text{sqrt}(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 \\
& + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + \\
& 48*a^2*b^2*c^5 - 64*a^3*c^6))*\text{sqrt}((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2* \\
& A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4 \\
& *a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3* \\
& c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*\log(-(5*B^4* \\
& a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108 \\
& *B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4* \\
& a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*\text{sqrt}(1/2)*(B^3*b^7 - 1 \\
& 7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b \\
& ^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^ \\
& 3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256* \\
& (3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2 \\
& *b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*\text{sqrt}((B^4*b^4 + A^4*c^4 - \\
& 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2* \\
& b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^ \\
& 2*b^2*c^8 - 64*a^3*c^9))) * \text{sqrt}(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (\\
& 60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + \\
& (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*\text{sqrt}((B^4*b^4 + A^4 \\
& *c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A \\
& ^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 \\
& + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - \\
& 64*a^3*c^6))) + \text{sqrt}(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + \\
& (b^3*c - 4*a*b*c^2)*x^2)*\text{sqrt}(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (\\
& 60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - \\
& (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*\text{sqrt}((B^4*b^4 + A^4 \\
& *c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A \\
& ^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 \\
& + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - \\
& 64*a^3*c^6))*\log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b \\
& - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3 \\
& *A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x + \\
& 1/2*\text{sqrt}(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 \\
& - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12 \\
& *A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^ \\
& 5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A \\
& a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
\end{aligned}$$

$$\begin{aligned} & \sqrt{(B^4b^4 + A^4c^4 - 2(9A^2B^2a - 2A^3Bb)c^3 + 3(27B^4a^2 - 12AB^3ab + 2A^2B^2b^2)c^2 - 2(9B^4ab^2 - 2AB^3b^3)c)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} \sqrt{-(B^2b^5 - 12(4ABa^2 - A^2ab)c^3 + (60B^2a^2b - 12ABab^2 + A^2b^3)c^2 - (15B^2ab^3 - 2ABb^4)c - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} \sqrt{(B^4b^4 + A^4c^4 - 2(9A^2B^2a - 2A^3Bb)c^3 + 3(27B^4a^2 - 12AB^3ab + 2A^2B^2b^2)c^2 - 2(9B^4ab^2 - 2AB^3b^3)c)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6)) - \sqrt{1/2} * ((b^2c^2 - 4ac^3) * x^4 + ab^2c - 4a^2c^2 + (b^3c - 4ab^2c) * x^2) * \sqrt{-(B^2b^5 - 12(4ABa^2 - A^2ab)c^3 + (60B^2a^2b - 12ABab^2 + A^2b^3)c^2 - (15B^2ab^3 - 2ABb^4)c - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} \sqrt{(B^4b^4 + A^4c^4 - 2(9A^2B^2a - 2A^3Bb)c^3 + 3(27B^4a^2 - 12AB^3ab + 2A^2B^2b^2)c^2 - 2(9B^4ab^2 - 2AB^3b^3)c)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6)) * \log(-(5B^4ab^4 - 3AB^3b^5 - 4A^4ac^4 + (20A^3Bab - 3A^4b^2)c^3 + 3(108B^4a^3 - 108AB^3a^2b + 28A^2B^2ab^2 - 3A^3Bb^3)c^2 - (81B^4a^2b^2 - 65AB^3ab^3 + 9A^2B^2b^4)c) * x - 1/2 * \sqrt{1/2} * (B^3b^7 - 17B^3ab^5c - 32A^3a^2c^5 + 16(18AB^2a^3 - 3A^2Bba^2b + A^3ab^2)c^4 - 2(72B^3a^3b + 72AB^2a^2b^2 - 12A^2Bba^2b^3 + A^3b^4)c^3 + (88B^3a^2b^3 + 18AB^2ab^4 - 3A^2Bb^5)c^2 + (Bb^8c^3 + 256(3Ba^4 - Aa^3b)c^7 - 64(10Ba^3b^2 - 3Aa^2b^3)c^6 + 48(4Ba^2b^4 - Aab^5)c^5 - 4(6Bab^6 - Ab^7)c^4) * \sqrt{(B^4b^4 + A^4c^4 - 2(9A^2B^2a - 2A^3Bb)c^3 + 3(27B^4a^2 - 12AB^3ab + 2A^2B^2b^2)c^2 - 2(9B^4ab^2 - 2AB^3b^3)c)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} * \sqrt{-(B^2b^5 - 12(4ABa^2 - A^2ab)c^3 + (60B^2a^2b - 12ABab^2 + A^2b^3)c^2 - (15B^2ab^3 - 2ABb^4)c - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} \sqrt{(B^4b^4 + A^4c^4 - 2(9A^2B^2a - 2A^3Bb)c^3 + 3(27B^4a^2 - 12AB^3ab + 2A^2B^2b^2)c^2 - 2(9B^4ab^2 - 2AB^3b^3)c)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6)) + 2(Bab - 2Aac) * x / ((b^2c^2 - 4ac^3) * x^4 + ab^2c - 4a^2c^2 + (b^3c - 4ab^2c) * x^2) \end{aligned}$$

giac [B] time = 6.29, size = 4538, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(Bb^2x^3 - 2Bacx^3 - Ab^2cx^3 + B^2abx - 2A^2acx) / ((cx^4 + bx^2 + a)(b^2c - 4ac^2)) + 1/16*((2b^3c^3 - 8ab^2c^4 - \sqrt{2}) * \sqrt{(b^2 - 4ac)} * \sqrt{bc + \sqrt{(b^2 - 4ac)} * c}) * b^3c + 4 * \sqrt{2}) * \sqrt{(b^2 -$$

$$\begin{aligned}
& 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{(b*c + \sqrt{b^2 - 4*a*c}*c)} \\
&)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{2} \\
&)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c \\
& + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*a^2*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B \\
& - 4*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*\text{abs}(b^2*c - 4*a*c^2) + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^3 - 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*\text{abs}(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}) \\
&)*b^7*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c^5 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^6 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^8*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^3 - 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^4 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6*c^4 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^5 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^5 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac) a^2 b^2 c^6) B) \arctan(2\sqrt{1/2} x / \sqrt{(b^3 c - 4a b c^2 + \\
& \sqrt{(b^3 c - 4a b c^2)^2 - 4(a b^2 c - 4a^2 c^2)(b^2 c^2 - 4a c^3)})} \\
& / (b^2 c^2 - 4a c^3)) / ((a b^6 c^3 - 12a^2 b^4 c^4 - 2a b^5 c^4 + 48a^3 b \\
& b^2 c^5 + 16a^2 b^3 c^5 + a b^4 c^5 - 64a^4 c^6 - 32a^3 b c^6 - 8a^2 b^ \\
& 2 c^6 + 16a^3 c^7) \operatorname{abs}(b^2 c - 4a c^2) \operatorname{abs}(c)) - 1/16 * ((2b^3 c^3 - 8a b \\
& c^4 - \sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^3 c + 4 \\
& \sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b c^2 + 2\sqrt{2} \sqrt{ \\
& 2) \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^2 c^2 - \sqrt{2} \sqrt{ \\
& (b^2 - 4a c) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b c^3 - 2(b^2 - 4a c) b c^3 \\
&) (b^2 c - 4a c^2)^2 A + (2b^4 c^2 - 20a b^2 c^3 + 48a^2 c^4 - \sqrt{2} * \\
& \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^4 + 10\sqrt{2} \sqrt{b^2 \\
& - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b^2 c + 2\sqrt{2} \sqrt{b^2 - 4 \\
& a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^3 c - 24\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{ \\
& b c - \sqrt{b^2 - 4a c}} c) a^2 c^2 - 12\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{ \\
& b c - \sqrt{b^2 - 4a c}} c) a b c^2 - \sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - s \\
& \sqrt{b^2 - 4a c}} c) b^2 c^2 + 6\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b \\
& ^2 - 4a c}} c) a c^3 - 2(b^2 - 4a c) b^2 c^2 + 12(b^2 - 4a c) a c^3) (b \\
& ^2 c - 4a c^2)^2 B + 4(\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b^4 c^3 \\
& - 8\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a^2 b^2 c^4 - 2\sqrt{2} \sqrt{b c \\
& - \sqrt{b^2 - 4a c}} c) a b^3 c^4 + 2a b^4 c^4 + 16\sqrt{2} \sqrt{b c - s \\
& \sqrt{b^2 - 4a c}} c) a^3 c^5 + 8\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a^2 * \\
& b c^5 + \sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b^2 c^5 - 16a^2 b^2 c^5 \\
& - 4\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a^2 c^6 + 32a^3 c^6 - 2(b^2 - \\
& 4a c) a b^2 c^4 + 8(b^2 - 4a c) a^2 c^5) A \operatorname{abs}(b^2 c - 4a c^2) - 2(\sqrt{ \\
& 2) \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b^5 c^2 - 8\sqrt{2} \sqrt{b c - \sqrt{ \\
& b^2 - 4a c}} c) a^2 b^3 c^3 - 2\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a * \\
& b^4 c^3 + 2a b^5 c^3 + 16\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a^3 b c^4 \\
& + 8\sqrt{2} \sqrt{b c - \sqrt{b^2 - 4a c}} c) a^2 b^2 c^4 + \sqrt{2} \sqrt{b c \\
& - \sqrt{b^2 - 4a c}} c) a b^3 c^4 - 16a^2 b^3 c^4 - 4\sqrt{2} \sqrt{b c - \\
& \sqrt{b^2 - 4a c}} c) a^2 b c^5 + 32a^3 b c^5 - 2(b^2 - 4a c) a b^3 c^3 + \\
& 8(b^2 - 4a c) a^2 b c^4) B \operatorname{abs}(b^2 c - 4a c^2) - (2b^7 c^5 - 8a b^5 c \\
& ^6 - 32a^2 b^3 c^7 + 128a^3 b c^8 - \sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \\
& \sqrt{b^2 - 4a c}} c) b^7 c^3 + 4\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{ \\
& b^2 - 4a c}} c) a b^5 c^4 + 2\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 \\
& - 4a c}} c) b^6 c^4 + 16\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4 \\
& a c}} c) a^2 b^3 c^5 - \sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a * \\
& c}} c) b^5 c^5 - 64\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) \\
&) a^3 b c^6 - 32\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) * \\
& a^2 b^2 c^6 + 16\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) * \\
& a^2 b c^7 - 2(b^2 - 4a c) b^5 c^5 + 32(b^2 - 4a c) a^2 b c^7) A - (2b^ \\
& 8 c^4 - 32a b^6 c^5 + 160a^2 b^4 c^6 - 256a^3 b^2 c^7 - \sqrt{2} \sqrt{b^2 \\
& - 4a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^8 c^2 + 16\sqrt{2} \sqrt{b^2 - 4 \\
& a c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) a b^6 c^3 + 2\sqrt{2} \sqrt{b^2 - 4a * \\
& c}) \sqrt{b c - \sqrt{b^2 - 4a c}} c) b^7 c^3 - 80\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{ \\
& b c - \sqrt{b^2 - 4a c}} c) a^2 b^4 c^4 - 24\sqrt{2} \sqrt{b^2 - 4a c}) \sqrt{ \\
\end{aligned}$$

```

qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*
c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((
b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b
^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*
a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a
^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c))

```

maple [B] time = 0.04, size = 1030, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2, x)$

[Out]
$$\begin{aligned} & (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*A*c-B*b)/(4*a*c-b^2)*a \\ & /c*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b-1/(4*a*c-b^2) \\ & *c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*b^2-3/2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*B+1/4/(4*a*c-b^2)/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *b^3*B-1/4/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*b^2+3/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*B-1/4/(4*a*c-b^2)/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *A*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x) \\ & *b^3*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out]
$$-1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)$$

mupad [B] time = 5.18, size = 12396, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out]
$$-((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a*b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - \operatorname{atan}\left(\frac{(2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{1/2} + B^2*b^2*(-(4*a*c - b^2)^9)^{1/2} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{1/2} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{1/2} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))}{(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))}*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{1/2} + B^2*b^2*(-(4*a*c - b^2)^9)^{1/2} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{1/2} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{1/2} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))\right)^{1/2} - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(- (B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{1/2} + B^2*b^2*(-(4*a*c - b^2)^9)^{1/2} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3$$

$$\begin{aligned} & 3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)} * i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} * (16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)) * (- (B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)) * (- (B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} * i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} * i) \end{aligned}$$

$$\begin{aligned}
& 8*c^2)/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 128 \\
& 0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)}*(16*b^7*c^3 - \\
& 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - \\
& 8*a*b^2*c^2)))*(-(B^2*b^{11} + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^{10}*c - 96*A^2*a^2*b^5*c^4 + \\
& 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2 \\
& *a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - \\
& 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
& (1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 24 \\
& 0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} \\
& - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b \\
& ^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3 \\
& *c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^1 \\
& 1 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 2*A*B*b^{10}*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288* \\
& B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^ \\
& 5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b \\
& *c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 153 \\
& 6*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/ \\
& (32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b \\
& ^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} - (3*A*B^2*a*b^5 - 21 \\
& 6*B^3*a^4*c^2 - 5*B^3*a^2*b^4 - 24*A^2*B*a^3*c^3 + 3*A^3*a*b^3*c^2 + 4*A^3* \\
& a^2*b*c^3 + 66*B^3*a^3*b^2*c - 51*A*B^2*a^2*b^3*c + 204*A*B^2*a^3*b*c^2 - 4 \\
& 2*A^2*B*a^2*b^2*c^2 + 6*A^2*B*a*b^4*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^ \\
& 2 + 48*a^2*b^2*c^3)) + (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 \\
& - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5 \\
& *c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^ \\
& 2*c^3)) + (x*(-(B^2*b^{11} + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^{10}*c - 96*A^2*a^2*b^5*c^4 + 512 \\
& *A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^ \\
& 4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9 \\
&)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12 \\
& 8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
& (1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a \\
& ^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{(1/2)} \\
& *(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c \\
& + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^{11} + A^2*b^9*c^2 + A^2*c^2*(-(4*a* \\
& c - b^2)^9)^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*b^{10}*c - 96*A^ \\
& 2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^ \\
& 5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a* \\
& c*(-(4*a*c - b^2)^9)^{(1/2)} - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A \\
& *B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24 \\
& *a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
\end{aligned}$$

$$\begin{aligned}
& a^5 b^2 c^8) \Big)^{1/2} + (x(B^2 b^6 + 8A^2 a^2 c^4 + A^2 b^4 c^2 - 72B^2 a^3 c^3 + 2A^2 B b^5 c + 74B^2 a^2 b^2 c^2 - 16B^2 a b^4 c + 2A^2 a^2 b^2 c^3 - 14A^2 B a^3 c^2 - 8A^2 B a^2 b^2 c^3) / (2(b^4 c + 16a^2 c^3 - 8a b^2 c^2))) * (- (B^2 b^{11} + A^2 b^9 c^2 + A^2 c^2 * (- (4a^2 c - b^2)^9)^{1/2} + B^2 b^2 * (- (4a^2 c - b^2)^9)^{1/2} + 2A^2 B b^{10} c - 96A^2 a^2 b^5 c^4 + 512A^2 a^3 b^3 c^5 + 288B^2 a^2 b^7 c^2 - 1504B^2 a^3 b^5 c^3 + 3840B^2 a^4 b^3 c^4 + 3072A^2 B a^5 c^6 - 27B^2 a^2 b^9 c - 9B^2 a^2 c * (- (4a^2 c - b^2)^9)^{1/2} - 768A^2 a^4 b^2 c^6 - 3840B^2 a^5 b^2 c^5 + 192A^2 B a^2 b^6 c^3 - 128A^2 B a^3 b^4 c^4 - 1536A^2 B a^4 b^2 c^5 + 2A^2 B^2 b^2 c * (- (4a^2 c - b^2)^9)^{1/2} - 36A^2 B a^2 b^8 c^2) / (32(4096a^6 c^9 + b^{12} c^3 - 24a^2 b^{10} c^4 + 240a^2 b^8 c^5 - 1280a^3 b^6 c^6 + 3840a^4 b^4 c^7 - 6144a^5 b^2 c^8)))^{1/2}) * (- (B^2 b^{11} + A^2 b^9 c^2 + A^2 c^2 * (- (4a^2 c - b^2)^9)^{1/2} + B^2 b^2 * (- (4a^2 c - b^2)^9)^{1/2} + 2A^2 B b^{10} c - 96A^2 a^2 b^5 c^4 + 512A^2 a^3 b^3 c^5 + 288B^2 a^2 b^7 c^2 - 1504B^2 a^3 b^5 c^3 + 3840B^2 a^4 b^3 c^4 + 3072A^2 B a^5 c^6 - 27B^2 a^2 b^9 c - 9B^2 a^2 c * (- (4a^2 c - b^2)^9)^{1/2} - 768A^2 a^4 b^2 c^6 - 3840B^2 a^5 b^2 c^5 + 192A^2 B a^2 b^6 c^3 - 128A^2 B a^3 b^4 c^4 - 1536A^2 B a^4 b^2 c^5 + 2A^2 B^2 b^2 c * (- (4a^2 c - b^2)^9)^{1/2} - 36A^2 B a^2 b^8 c^2) / (32(4096a^6 c^9 + b^{12} c^3 - 24a^2 b^{10} c^4 + 240a^2 b^8 c^5 - 1280a^3 b^6 c^6 + 3840a^4 b^4 c^7 - 6144a^5 b^2 c^8)))^{1/2} * 2i - \operatorname{atan}(\frac{2048A^2 a^4 c^6 - 32A^2 a^2 b^6 c^3 + 16B^2 a^2 b^7 c^2 - 1024B^2 a^4 b^2 c^5 + 384A^2 a^2 b^4 c^4 - 1536A^2 a^3 b^2 c^5 - 192B^2 a^2 b^5 c^3 + 768B^2 a^3 b^3 c^4}{(8(b^6 c - 64a^3 c^4 - 12a^2 b^4 c^2 + 48a^2 b^2 c^3)) - (x((A^2 c^2 * (- (4a^2 c - b^2)^9)^{1/2} - A^2 b^9 c^2 - B^2 b^{11} + B^2 b^2 * (- (4a^2 c - b^2)^9)^{1/2} - 2A^2 B b^{10} c + 96A^2 a^2 b^5 c^4 - 512A^2 a^3 b^3 c^5 - 288B^2 a^2 b^7 c^2 + 1504B^2 a^3 b^5 c^3 - 3840B^2 a^4 b^3 c^4 - 3072A^2 B a^5 c^6 + 27B^2 a^2 b^9 c - 9B^2 a^2 c * (- (4a^2 c - b^2)^9)^{1/2} + 768A^2 a^4 b^2 c^6 + 3840B^2 a^5 b^2 c^5 - 192A^2 B a^2 b^6 c^3 + 128A^2 B a^3 b^4 c^4 + 1536A^2 B a^4 b^2 c^5 + 2A^2 B^2 b^2 c * (- (4a^2 c - b^2)^9)^{1/2} + 36A^2 B a^2 b^8 c^2) / (32(4096a^6 c^9 + b^{12} c^3 - 24a^2 b^{10} c^4 + 240a^2 b^8 c^5 - 1280a^3 b^6 c^6 + 3840a^4 b^4 c^7 - 6144a^5 b^2 c^8)))^{1/2} * (16b^7 c^3 - 192a^2 b^5 c^4 - 1024a^3 b^2 c^6 + 768a^2 b^3 c^5) / (2(b^4 c + 16a^2 c^3 - 8a^2 b^2 c^2)) * ((A^2 c^2 * (- (4a^2 c - b^2)^9)^{1/2} - A^2 b^9 c^2 - B^2 b^{11} + B^2 b^2 * (- (4a^2 c - b^2)^9)^{1/2} - 2A^2 B b^{10} c + 96A^2 a^2 b^5 c^4 - 512A^2 a^3 b^3 c^5 - 288B^2 a^2 b^7 c^2 + 1504B^2 a^3 b^5 c^3 - 3840B^2 a^4 b^3 c^4 - 3072A^2 B a^5 c^6 + 27B^2 a^2 b^9 c - 9B^2 a^2 c * (- (4a^2 c - b^2)^9)^{1/2} + 768A^2 a^4 b^2 c^6 + 3840B^2 a^5 b^2 c^5 - 192A^2 B a^2 b^6 c^3 + 128A^2 B a^3 b^4 c^4 + 1536A^2 B a^4 b^2 c^5 + 2A^2 B^2 b^2 c * (- (4a^2 c - b^2)^9)^{1/2} + 36A^2 B a^2 b^8 c^2) / (32(4096a^6 c^9 + b^{12} c^3 - 24a^2 b^{10} c^4 + 240a^2 b^8 c^5 - 1280a^3 b^6 c^6 + 3840a^4 b^4 c^7 - 6144a^5 b^2 c^8)))^{1/2} - (x(B^2 b^6 + 8A^2 a^2 c^4 + A^2 b^4 c^2 - 72B^2 a^3 c^3 + 2A^2 B b^5 c + 74B^2 a^2 b^2 c^2 - 16B^2 a b^4 c + 2A^2 a^2 b^2 c^3 - 14A^2 B a^3 c^2 - 8A^2 B a^2 b^2 c^3) / (2(b^4 c + 16a^2 c^3 - 8a^2 b^2 c^2))) * ((A^2 c^2 * (- (4a^2 c - b^2)^9)^{1/2} - A^2 b^9 c^2 - B^2 b^{11} + B^2 b^2 * (- (4a^2 c - b^2)^9)^{1/2} - 2A^2 B b^{10} c + 96A^2 a^2 b^5 c^4 - 512A^2 a^3 b^3 c^5 - 288B^2 a^2 b^7 c^2 + 1504B^2 a^3 b^5 c^3 - 3840B^2 a^4 b^3 c^4 - 3072A^2 B a^5 c^6 + 27B^2
\end{aligned}$$

$$\begin{aligned}
& 2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*1i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)} + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^11 + B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^10*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*1i)
\end{aligned}$$

$$\begin{aligned}
& \left(8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8 \right)^{1/2} \cdot (16b^7c^3 - 192ab^5c^4 - 1024a^3b^3c^6 + 768a^2b^3c^5) / (2(b^4c + 16a^2c^3 - 8ab^2c^2)) \cdot ((A^2c^2(-4ac - b^2)^9)^{1/2} - A^2b^9c^2 - B^2b^{11} + B^2b^2(-4ac - b^2)^9)^{1/2} - 2ABb^{10}c + 96A^2a^2b^5c^4 - 512A^2a^3b^3c^5 - 288B^2a^2b^7c^2 + 1504B^2a^3b^5c^3 - 3840B^2a^4b^3c^4 - 3072AABa^5c^6 + 27B^2aab^9c - 9B^2aac(-4ac - b^2)^9)^{1/2} + 768A^2a^4b^6c^6 + 3840B^2a^5b^5c^5 - 192AABa^2b^6c^3 + 128AABa^3b^4c^4 + 1536AABa^4b^2c^5 + 2AABb^8c^2 \cdot (-4ac - b^2)^9)^{1/2} + 36AABaab^8c^2) / (32(4096a^6c^9 + b^{12}c^3 - 24aab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{1/2} - (x(B^2b^6 + 8A^2a^2c^4 + A^2b^4c^2 - 72B^2a^3c^3 + 2ABb^5c + 74B^2a^2b^2c^2 - 16B^2aab^4c + 2A^2aab^2c^3 - 14AABa^3c^2 - 8AABa^2b^3c^3)) / (2(b^4c + 16a^2c^3 - 8ab^2c^2)) \cdot ((A^2c^2(-4ac - b^2)^9)^{1/2} - A^2b^9c^2 - B^2b^{11} + B^2b^2(-4ac - b^2)^9)^{1/2} - 2ABb^{10}c + 96A^2a^2b^5c^4 - 512A^2a^3b^3c^5 - 288B^2a^2b^7c^2 + 1504B^2a^3b^5c^3 - 3840B^2a^4b^3c^4 - 3072AABa^5c^6 + 27B^2aab^9c - 9B^2aac(-4ac - b^2)^9)^{1/2} + 768A^2a^4b^6c^6 + 3840B^2a^5b^5c^5 - 192AABa^2b^6c^3 + 128AABa^3b^4c^4 + 1536AABa^4b^2c^5 + 2AABb^8c^2 \cdot (-4ac - b^2)^9)^{1/2} + 36AABaab^8c^2) / (32(4096a^6c^9 + b^{12}c^3 - 24aab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{1/2} - (3AB^2aab^5 - 216B^3a^4c^2 - 5B^3a^2b^4 - 24A^2Ba^3c^3 + 3A^3aab^3c^2 + 4A^3a^2b^3c^3 + 66B^3a^3b^2c - 51AB^2a^2b^3c + 204AB^2a^3b^2c^2 - 42A^2Ba^2b^2c^2 + 6A^2Baab^4c) / (4(b^6c - 64a^3c^4 - 12aab^4c^2 + 48a^2b^2c^3)) + (((2048Aa^4c^6 - 32Aaab^6c^3 + 16B^2aab^7c^2 - 1024B^2a^4b^3c^5 + 384Aa^2b^4c^4 - 1536Aa^3b^2c^5 - 192B^2a^2b^5c^3 + 768B^2a^3b^3c^4) / (8(b^6c - 64a^3c^4 - 12aab^4c^2 + 48a^2b^2c^3)) + (x((A^2c^2(-4ac - b^2)^9)^{1/2} - A^2b^9c^2 - B^2b^{11} + B^2b^2(-4ac - b^2)^9)^{1/2} - 2ABb^{10}c + 96A^2a^2b^5c^4 - 512A^2a^3b^3c^5 - 288B^2a^2b^7c^2 + 1504B^2a^3b^5c^3 - 3840B^2a^4b^3c^4 - 3072AABa^5c^6 + 27B^2aab^9c - 9B^2aac(-4ac - b^2)^9)^{1/2} + 768A^2a^4b^6c^6 + 3840B^2a^5b^5c^5 - 192AABa^2b^6c^3 + 128AABa^3b^4c^4 + 1536AABa^4b^2c^5 + 2AABb^8c^2 \cdot (-4ac - b^2)^9)^{1/2} + 36AABaab^8c^2) / (32(4096a^6c^9 + b^{12}c^3 - 24aab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{1/2} \cdot (16b^7c^3 - 192ab^5c^4 - 1024a^3b^3c^6 + 768a^2b^3c^5) / (2(b^4c + 16a^2c^3 - 8ab^2c^2)) \cdot ((A^2c^2(-4ac - b^2)^9)^{1/2} - A^2b^9c^2 - B^2b^{11} + B^2b^2(-4ac - b^2)^9)^{1/2} - 2ABb^{10}c + 96A^2a^2b^5c^4 - 512A^2a^3b^3c^5 - 288B^2a^2b^7c^2 + 1504B^2a^3b^5c^3 - 3840B^2a^4b^3c^4 - 3072AABa^5c^6 + 27B^2aab^9c - 9B^2aac(-4ac - b^2)^9)^{1/2} + 768A^2a^4b^6c^6 + 3840B^2a^5b^5c^5 - 192AABa^2b^6c^3 + 128AABa^3b^4c^4 + 1536AABa^4b^2c^5 + 2AABb^8c^2 \cdot (-4ac - b^2)^9)^{1/2} + 36AABaab^8c^2) / (32(4096a^6c^9 + b^{12}c^3 - 24aab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{1/2} + (x(B^2b^6 + 8A^2a^2c^4 + A^2b^4c^2 -
\end{aligned}$$

$$\begin{aligned}
& 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2* \\
& a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3) / (2*(b^4*c + 16*a^2*c^3 - 8 \\
& *a*b^2*c^2)) * ((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^{11} + \\
& B^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^{10}*c + 96*A^2*a^2*b^5*c^4 - 512 \\
& *A^2*a^3*b^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4 \\
& *b^3*c^4 - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 12 \\
& 8*A*B*a^3*b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} + 36*A*B*a*b^8*c^2) / (32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a \\
& ^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)} \\
&)) * ((A^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - A^2*b^9*c^2 - B^2*b^{11} + B^2*b^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*b^{10}*c + 96*A^2*a^2*b^5*c^4 - 512*A^2*a^3*b \\
& ^3*c^5 - 288*B^2*a^2*b^7*c^2 + 1504*B^2*a^3*b^5*c^3 - 3840*B^2*a^4*b^3*c^4 \\
& - 3072*A*B*a^5*c^6 + 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 768*A^2*a^4*b*c^6 + 3840*B^2*a^5*b*c^5 - 192*A*B*a^2*b^6*c^3 + 128*A*B*a^3* \\
& b^4*c^4 + 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A* \\
& B*a*b^8*c^2) / (32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 \\
& - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.98 \quad \int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=276

$$\frac{x(-2aB - (x^2(bB - 2Ac)) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(-\frac{4aBc - 4Abc + b^2B}{\sqrt{b^2 - 4ac}} - 2Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(\frac{4aBc - 4Abc + b^2B}{\sqrt{b^2 - 4ac}} - 2Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 0.55, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1275, 1166, 205}

$$\frac{x(-2aB + x^2(-(bB - 2Ac)) + Ab)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(-\frac{4aBc - 4Abc + b^2B}{\sqrt{b^2 - 4ac}} - 2Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(\frac{4aBc - 4Abc + b^2B}{\sqrt{b^2 - 4ac}} - 2Ac + bB\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] $-(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c))/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]/(2*\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2 - 4*a*c)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*a*B*c))/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]]/(2*\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2 - 4*a*c)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{\int \frac{Ab - 2aB + (bB - 2Ac)x^2}{a + bx^2 + cx^4} dx}{2 (b^2 - 4ac)} \\ &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{\left(bB - 2Ac - \frac{b^2B - 4Abc + 4aBc}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{4 (b^2 - 4ac)} + \dots \\ &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{2 (b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{\left(bB - 2Ac - \frac{b^2B - 4Abc + 4aBc}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} \sqrt{c} (b^2 - 4ac) \sqrt{b - \sqrt{b^2 - 4ac}}} + \dots \end{aligned}$$

Mathematica [A] time = 0.66, size = 298, normalized size = 1.08

$$\frac{1}{4} \left(\frac{2x (B (2a + bx^2) - A (b + 2cx^2))}{(b^2 - 4ac) (a + bx^2 + cx^4)} + \frac{\sqrt{2} (-2Ac\sqrt{b^2 - 4ac} + bB\sqrt{b^2 - 4ac} - 4aBc + 4Abc + b^2(-B)) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} (-2Ac\sqrt{b^2 - 4ac} + bB\sqrt{b^2 - 4ac} + 4aBc - 4Abc + b^2B) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{c} (b^2 - 4ac)^{3/2} \sqrt{\sqrt{b^2 - 4ac} + b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (A + Bx^2)}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 1.92, size = 3467, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{4} * (2 * (B * b - 2 * A * c) * x^3 - \sqrt{1/2} * ((b^2 * c - 4 * a * c^2) * x^4 + a * b^2 - 4 * a^2 * c + (b^3 - 4 * a * b * c) * x^2) * \sqrt{-(B^2 * a * b^3 - 4 * (4 * A * B * a^2 - 3 * A^2 * a * b) * c^2 + (12 * B^2 * a^2 * b - 12 * A * B * a * b^2 + A^2 * b^3) * c + (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 4 * 8 * a^3 * b^2 * c^3 - 64 * a^4 * c^4) * \sqrt{(B^4 * a^2 - 2 * A^2 * B^2 * a * c + A^4 * c^2) / (a^2 * b^6 * c^2 - 12 * a^3 * b^4 * c^3 + 48 * a^4 * b^2 * c^4 - 64 * a^5 * c^5))}) / (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 48 * a^3 * b^2 * c^3 - 64 * a^4 * c^4) * \log(-(3 * B^4 * a^2 * b^2 - A * B^3 * a * b^3 - 4 * A^4 * a * c^3 + 3 * (4 * A^3 * B * a * b - A^4 * b^2) * c^2 + (4 * B^4 * a^3 - 12 * A * B^3 * a^2 * b + A^3 * B * b^3) * c) * x + 1/2 * \sqrt{1/2} * (2 * B^3 * a^2 * b^4 - A * B^2 * a * b^5 - 16 * (2 * A^2 * B * a^3 - A^3 * a^2 * b) * c^3 + 8 * (4 * B^3 * a^4 - 2 * A * B^2 * a^3 * b + 2 * A^2 * B * a^2 * b^2 - A^3 * a * b^3) * c^2 - (16 * B^3 * a^3 * b^2 - 8 * A * B^2 * a^2 * b^3 + 2 * A^2 * B * a * b^4 - A^3 * b^5) * c + (192 * B * a^4 * b^3 * c^3 + 256 * A * a^5 * c^5 - 128 * (2 * B * a^5 * b + A * a^4 * b^2) * c^4 - 8 * (6 * B * a^3 * b^5 - A * a^2 * b^6) * c^2 + (4 * B * a^2 * b^7 - A * a * b^8) * c) * \sqrt{(B^4 * a^2 - 2 * A^2 * B^2 * a * c + A^4 * c^2) / (a^2 * b^6 * c^2 - 12 * a^3 * b^4 * c^3 + 48 * a^4 * b^2 * c^4 - 64 * a^5 * c^5))} * \sqrt{-(B^2 * a * b^3 - 4 * (4 * A * B * a^2 - 3 * A^2 * a * b) * c^2 + (12 * B^2 * a^2 * b - 12 * A * B * a * b^2 + A^2 * b^3) * c + (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 48 * a^3 * b^2 * c^3 - 64 * a^4 * c^4) * \sqrt{(B^4 * a^2 - 2 * A^2 * B^2 * a * c + A^4 * c^2) / (a^2 * b^6 * c^2 - 12 * a^3 * b^4 * c^3 + 48 * a^4 * b^2 * c^4 - 64 * a^5 * c^5))}) / (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 48 * a^3 * b^2 * c^3 - 64 * a^4 * c^4)) + \sqrt{1/2} * ((b^2 * c - 4 * a * c^2) * x^4 + a * b^2 - 4 * a^2 * c + (b^3 - 4 * a * b * c) * x^2) * \sqrt{-(B^2 * a * b^3 - 4 * (4 * A * B * a^2 - 3 * A^2 * a * b) * c^2 + (12 * B^2 * a^2 * b - 12 * A * B * a * b^2 + A^2 * b^3) * c + (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 48 * a^3 * b^2 * c^3 - 64 * a^4 * c^4) * \sqrt{(B^4 * a^2 - 2 * A^2 * B^2 * a * c + A^4 * c^2) / (a^2 * b^6 * c^2 - 12 * a^3 * b^4 * c^3 + 48 * a^4 * b^2 * c^4 - 64 * a^5 * c^5))}) / (a * b^6 * c - 12 * a^2 * b^4 * c^2 + 48 * a^3 * b^2 * c^3 - 64 * a^4 * c^4) * \log(-(3 * B^4 * a^2 * b^2 - A * B^3 * a * b^3 - 4 * A^4 * a * c^3 + 3 * (4 * A^3 * B * a * b - A^4 * b^2) * c^2 + (4 * B^4 * a^3 - 12 * A * B^3 * a^2 * b + A^3 * B * b^3) * c) * x - 1/2 * \sqrt{1/2} * (2 * B^3 * a^2 * b^4 - A * B^2 * a * b^5 - 16 * (2 * A^2 * B * a^3 - A^3 * a^2 * b) * c^3 + 8 * (4 * B^3 * a^4 - 2 * A * B^2 * a^3 * b + 2 * A^2 * B * a^2 * b^2 - A^3 * a * b^3) * c^2 - (16 * B^3 * a^3 * b^2 - 8 * A * B^2 * a^2 * b^3 + 2 * A^2 * B * a * b^4 -$$

$$\begin{aligned}
& A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*\sqrt{-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))} - \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*\log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*\sqrt{1/2}*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*\sqrt{-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))} + \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*\log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*\sqrt{1/2}*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*\sqrt{-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*\sqrt{(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))} + 2*(2*B*a - A*b)*x)/(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
\end{aligned}$$

giac [B] time = 4.78, size = 3776, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*
```


$$- 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))$$

maple [B] time = 0.03, size = 733, normalized size = 2.66

$$\frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{(ac-b)\sqrt{4ac-b^2}\sqrt{a+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{(ac-b)\sqrt{4ac-b^2}\sqrt{b+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{(ac-b)\sqrt{4ac-b^2}\sqrt{a+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{(ac-b)\sqrt{4ac-b^2}\sqrt{b+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{a(ac-b)\sqrt{4ac-b^2}\sqrt{b+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{a(ac-b)\sqrt{4ac-b^2}\sqrt{a+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{2(ac-b)\sqrt{a+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{2(ac-b)\sqrt{b+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{a(ac-b)\sqrt{b+\sqrt{4ac-b^2}}} - \frac{\sqrt{2} \operatorname{Arctanh}\left(\frac{a}{\sqrt{4ac-b^2}}\right)}{a(ac-b)\sqrt{a+\sqrt{4ac-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)

[Out] $\frac{1}{2}*(2*A*c-B*b)/(4*a*c-b^2)*x^3 + \frac{1}{2}*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a) - \frac{1}{2}/(4*a*c-b^2)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A + \frac{1}{4}/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b + \frac{1}{4}/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B - \frac{1}{4}/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*B - \frac{1}{4}/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*B + \frac{1}{2}/(4*a*c-b^2)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A + \frac{1}{4}/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b - \frac{1}{4}/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B - \frac{1}{4}/(4*a*c-b^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*B - \frac{1}{4}/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*B$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Bb - 2Ac)x^3 + (2Ba - Ab)x}{2((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2)} - \frac{\int \frac{(Bb-2Ac)x^2 - 2Ba + Ab}{cx^4 + bx^2 + a} dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - \frac{1}{2}*\operatorname{integrate}(-((B*b - 2*A*c)*x^2 - 2*B*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

mupad [B] time = 4.41, size = 9444, normalized size = 34.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2, x)$

[Out]
$$\begin{aligned} & ((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2))) / (a + b*x^2 + c*x^4) - \text{atan}\left(\frac{(16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)}{8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)}\right) \\ & - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^{1/2}) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^{1/2} - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c) / (32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c))^{1/2} * (16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^{1/2}) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^{1/2} - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c) / (32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c))^{1/2} - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^{1/2}) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^{1/2} - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c) / (32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c))^{1/2} * i - \left(\frac{(16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)}{8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)}\right) + (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^{1/2}) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^{1/2} - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c) / (32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c))^{1/2} * (16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^{1/2}) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^{1/2} - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c) / (32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c))^{1/2} \end{aligned}$$

$$\begin{aligned}
& b^8 c^3 - 1280 a^4 b^6 c^4 + 3840 a^5 b^4 c^5 - 6144 a^6 b^2 c^6 + a b^{12} c \\
& \left. \right)^{(1/2)} + \left((16 A b^7 c^2 + 2048 B a^4 c^5 - 192 A a b^5 c^3 - 1024 A a^3 \right. \\
& * b c^5 - 32 B a b^6 c^2 + 768 A a^2 b^3 c^4 + 384 B a^2 b^4 c^3 - 1536 B a^3 \\
& * b^2 c^4) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + (x (-B^2 \\
& * a b^9 + B^2 a (-4 a c - b^2)^9)^{(1/2)} + A^2 b^9 c - A^2 c (-4 a c - b^2) \\
& ^9)^{(1/2)} - 96 A^2 a^2 b^5 c^3 + 512 A^2 a^3 b^3 c^4 - 96 B^2 a^3 b^5 c^2 + \\
& 512 B^2 a^4 b^3 c^3 + 1024 A B a^5 c^5 - 768 A^2 a^4 b c^5 - 768 B^2 a^5 b \\
& * c^4 + 128 A B a^2 b^6 c^2 - 384 A B a^3 b^4 c^3 - 12 A B a b^8 c) / (32 (409 \\
& 6 a^7 c^7 - 24 a^2 b^{10} c^2 + 240 a^3 b^8 c^3 - 1280 a^4 b^6 c^4 + 3840 a^5 \\
& * b^4 c^5 - 6144 a^6 b^2 c^6 + a b^{12} c)) \left. \right)^{(1/2)} * (16 b^7 c^2 - 192 a b^5 c^3 \\
& - 1024 a^3 b c^5 + 768 a^2 b^3 c^4) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * (\\
& - (B^2 a b^9 + B^2 a (-4 a c - b^2)^9)^{(1/2)} + A^2 b^9 c - A^2 c (-4 a c - \\
& b^2)^9)^{(1/2)} - 96 A^2 a^2 b^5 c^3 + 512 A^2 a^3 b^3 c^4 - 96 B^2 a^3 b^5 c^2 \\
& + 512 B^2 a^4 b^3 c^3 + 1024 A B a^5 c^5 - 768 A^2 a^4 b c^5 - 768 B^2 a^5 b \\
& a^5 b c^4 + 128 A B a^2 b^6 c^2 - 384 A B a^3 b^4 c^3 - 12 A B a b^8 c) / (32 \\
& * (4096 a^7 c^7 - 24 a^2 b^{10} c^2 + 240 a^3 b^8 c^3 - 1280 a^4 b^6 c^4 + 384 \\
& 0 a^5 b^4 c^5 - 6144 a^6 b^2 c^6 + a b^{12} c)) \left. \right)^{(1/2)} + (x (B^2 b^4 c - 8 A^2 \\
& a^2 c^4 + 10 A^2 b^2 c^3 + 8 B^2 a^2 c^3 - 6 A B b^3 c^2 + 2 B^2 a b^2 c^2 \\
& - 8 A B a b c^3) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c))) * (- (B^2 a b^9 + B^2 a \\
& (-4 a c - b^2)^9)^{(1/2)} + A^2 b^9 c - A^2 c (-4 a c - b^2)^9)^{(1/2)} - 96 A^2 \\
& a^2 b^5 c^3 + 512 A^2 a^3 b^3 c^4 - 96 B^2 a^3 b^5 c^2 + 512 B^2 a^4 b^3 \\
& c^3 + 1024 A B a^5 c^5 - 768 A^2 a^4 b c^5 - 768 B^2 a^5 b c^4 + 128 A B a^2 \\
& b^6 c^2 - 384 A B a^3 b^4 c^3 - 12 A B a b^8 c) / (32 (4096 a^7 c^7 - 24 a^2 \\
& b^{10} c^2 + 240 a^3 b^8 c^3 - 1280 a^4 b^6 c^4 + 3840 a^5 b^4 c^5 - 6144 \\
& a^6 b^2 c^6 + a b^{12} c)) \left. \right)^{(1/2)} * (- (B^2 a b^9 + B^2 a (-4 a c - b^2)^9)^{(1/2)} \\
& + A^2 b^9 c - A^2 c (-4 a c - b^2)^9)^{(1/2)} - 96 A^2 a^2 b^5 c^3 + 5 \\
& 12 A^2 a^3 b^3 c^4 - 96 B^2 a^3 b^5 c^2 + 512 B^2 a^4 b^3 c^3 + 1024 A B a^5 \\
& c^5 - 768 A^2 a^4 b c^5 - 768 B^2 a^5 b c^4 + 128 A B a^2 b^6 c^2 - 384 A \\
& * B a^3 b^4 c^3 - 12 A B a b^8 c) / (32 (4096 a^7 c^7 - 24 a^2 b^{10} c^2 + 240 a^3 \\
& b^8 c^3 - 1280 a^4 b^6 c^4 + 3840 a^5 b^4 c^5 - 6144 a^6 b^2 c^6 + a b^{12} \\
& c)) \left. \right)^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.99 \quad \int \frac{A+Bx^2}{(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=293

$$\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c} \left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}} - 2aB + Ab \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \left(-\frac{12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}} - 2aB + Ab \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.85, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1178, 1166, 205}

$$\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c} \left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}} - 2aB + Ab \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \left(-\frac{12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}} - 2aB + Ab \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b^2-4ac+b}} \right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b^2-4ac+b}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]

[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{(a + bx^2 + cx^4)^2} dx &= \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{-Ab^2 - abB + 6aAc - (Ab - 2aB)cx^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)} \\ &= \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(c\left(Ab - 2aB - \frac{Ab^2 + 4abB - 12aAc}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}}}{4a(b^2 - 4ac)} \\ &= \frac{x(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}\left(2aB\left(2b - \sqrt{b^2 - 4ac}\right) + A\left(b^2 - 12ac + 2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b}\right)\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.79, size = 304, normalized size = 1.04

$$\frac{2x(A(-2ac + b^2 + bcx^2) - ab(b + 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2 - 4ac} - 12ac + b^2\right) - 2aB\left(\sqrt{b^2 - 4ac} - 2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2 - 4ac} + 12ac - b^2\right) - 2aB\left(\sqrt{b^2 - 4ac} + 2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}}$$

4a

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2, x]

[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) + A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2 + c*x^4)^2, x]

fricas [B] time = 3.72, size = 4885, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$-1/4*(2*(2*B*a - A*b)*c*x^3 - \sqrt{1/2}*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)}*\sqrt{((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)}*\log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x + 1/2*\sqrt{1/2}*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9*5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*\sqrt{((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)}*\sqrt{((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)) + \sqrt{1/2}*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*($$

$$\begin{aligned}
& 4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48 \\
& *a^5*b^2*c^2 - 64*a^6*c^3)*\sqrt{(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 \\
& + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2 \\
& *b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)) \\
&)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log((324*A^4*a^2* \\
& c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84 \\
& *A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B \\
& ^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*\sqrt{1/2)*(B^3*a^3*b^5 \\
& + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2 \\
& *a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4 \\
& *b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2* \\
& a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144 \\
& *A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^ \\
& 3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*\sqrt{(B^4*a^4 + 4*A*B^3*a^3*b + 6 \\
& *A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a \\
& ^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
& - 64*a^9*c^3)))*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 \\
& - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^ \\
& 3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\sqrt{(B^4*a^4 + 4*A*B^3 \\
& *a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18* \\
& (A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^ \\
& 8*b^2*c^2 - 64*a^9*c^3)))/\sqrt{1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^ \\
& 3 - 4*a^2*b*c)*x^2)*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B* \\
& a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - \\
& (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\sqrt{(B^4*a^4 + 4*A* \\
& B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - \\
& 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4 \\
& 8*a^8*b^2*c^2 - 64*a^9*c^3)))/\sqrt{1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^ \\
& 3 - 4*a^2*b*c)*x^2)*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B* \\
& a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - \\
& (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\sqrt{(B^4*a^4 + 4*A* \\
& B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - \\
& 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^ \\
& 8*b^2*c^2 - 64*a^9*c^3)))*\log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B \\
& ^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)* \\
& c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x \\
& + 1/2*\sqrt{1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + \\
& 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2 \\
& *(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 \\
& - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c + \\
& (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + \\
& 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*\sqrt{(\\
& B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81 \\
& *A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 1 \\
& 2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*\sqrt{-(B^2*a^2*b^3 + 2*A*B*a*b \\
& ^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^ \\
& 2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6* \\
& c^3))*\sqrt{(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^ \\
& 4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
\end{aligned}$$

$$\begin{aligned} & \left(\frac{6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3}{(a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} + \sqrt{\frac{1}{2}} \left((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2 \right) \sqrt{-(B^2a^2b^3 + 2ABa^2b^4 + A^2b^5 - 12(4ABa^3 - 5A^2a^2b)c^2 + 3(4B^2a^3b - 4ABa^2b^2 - 5A^2ab^3)c - (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} \right) \sqrt{(B^4a^4 + 4AB^3a^3b + 6A^2B^2a^2b^2 + 4A^3Bab^3 + A^4b^4 + 81A^4a^2c^2 - 18(A^2B^2a^3 + 2A^3Bab^2 + A^4ab^2)c)} \right) / (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3) / (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3) * \log\left(\frac{(324A^4a^2c^4 - 81(4A^3Bab^2 + A^4ab^2)c^3 - (4B^4a^4 - 20AB^3a^3b - 84A^2B^2a^2b^2 - 65A^3Bab^3 - 5A^4b^4)c^2 - 3(B^4a^3b^2 + 3AB^3a^2b^3 + 3A^2B^2ab^4 + A^3Bb^5)c)x - 1/2\sqrt{1/2}(B^3a^3b^5 + 3AB^2a^2b^6 + 3A^2Bab^7 + A^3b^8 + 864A^3a^4c^4 - 48(2AB^2a^5 + 7A^2Bab^4)b + 14A^3a^3b^2)c^3 + 2(8B^3a^5b + 48AB^2a^4b^2 + 108A^2Bab^3b^3 + 95A^3a^2b^4)c^2 - (8B^3a^4b^3 + 30AB^2a^3b^4 + 45A^2Bab^2b^5 + 23A^3ab^6)c + (Ba^4b^8 + Aa^3b^9 + 144Aa^5b^5c^2 - 256(Ba^8 - 2Aa^7b)c^4 + 64(2Ba^7b^2 - 7Aa^6b^3)c^3 - 4(2Ba^5b^6 + 5Aa^4b^7)c)}{\sqrt{(B^4a^4 + 4AB^3a^3b + 6A^2B^2a^2b^2 + 4A^3Bab^3 + A^4b^4 + 81A^4a^2c^2 - 18(A^2B^2a^3 + 2A^3Bab^2 + A^4ab^2)c)} / (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)}\right) \sqrt{-(B^2a^2b^3 + 2ABa^2b^4 + A^2b^5 - 12(4ABa^3 - 5A^2a^2b)c^2 + 3(4B^2a^3b - 4ABa^2b^2 - 5A^2ab^3)c - (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3)} \sqrt{(B^4a^4 + 4AB^3a^3b + 6A^2B^2a^2b^2 + 4A^3Bab^3 + A^4b^4 + 81A^4a^2c^2 - 18(A^2B^2a^3 + 2A^3Bab^2 + A^4ab^2)c)} / (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)} \right) / (a^3b^6 - 12a^4b^4c + 48a^5b^2c^2 - 64a^6c^3) + 2(Bab - Ab^2 + 2Aac)x / ((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2) \end{aligned}$$

giac [B] time = 5.96, size = 4426, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$-1/2*(2B*ac*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2A*a*c*x) / ((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*a*b$$

$$\begin{aligned}
& *c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^2 - 2*(b \\
& ^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c})*a*b^6 - 14*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c - 2 \\
& *\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c - 2*a*b^6*c + 64*\sqrt{2}*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^2 + 20*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c})*a^2*b^3*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^2 \\
& + 28*a^2*b^4*c^2 - 96*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*c^3 - 4 \\
& 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^3 - 10*\sqrt{2}*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*\sqrt{2}*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c}*c})*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20* \\
& (b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c \\
&) + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5 - 8*\sqrt{2}*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}* \\
& c})*a^2*b^4*c - 2*a^2*b^5*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4 \\
& *b*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^2 + \sqrt{2}*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^7 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^6*c - 112*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b^3*c^2 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^4*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^2 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^5*b*c^3 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^3 - 48*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^6 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^5*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^5*b^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)*arctan(2*\sqrt{1/2}*x/\sqrt{((a*b^3 - 4*a^2*b*c + \sqrt{((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2))})/(a*b^2*c - 4*a^2*c^2))})/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c -
\end{aligned}$$

$$\begin{aligned}
& \sqrt{b^2 - 4ac} \cdot c \cdot b^2 \cdot c - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& - 4ac \cdot c \cdot b \cdot c^2 - 2 \cdot (b^2 - 4ac) \cdot b \cdot c^2 \cdot (ab^2 - 4a^2c)^2 \cdot A - 2 \cdot (2ab \\
& ^2c^2 - 8a^2c^3 - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c) \cdot a \cdot b^2 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \\
& \cdot c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a \cdot b \cdot c - \sqrt{2} \\
& \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a \cdot c^2 - 2 \cdot (b^2 - 4 \\
& ac) \cdot a \cdot c^2 \cdot (ab^2 - 4a^2c)^2 \cdot B - 2 \cdot (\sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c) \cdot a \cdot b^6 - 14 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \cdot b^4 \cdot c - 2 \cdot \sqrt{2} \\
&) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a \cdot b^5 \cdot c + 2 \cdot a \cdot b^6 \cdot c + 64 \cdot \sqrt{2} \cdot \sqrt{bc \\
& - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^2 \cdot c^2 + 20 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^2 \cdot b^3 \cdot c^2 + \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a \cdot b^4 \cdot c^2 - 28 \cdot \\
& a^2 \cdot b^4 \cdot c^2 - 96 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot c^3 - 48 \cdot \sqrt{2} \\
& \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b \cdot c^3 - 10 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^2 \cdot b^2 \cdot c^3 + 128 \cdot a^3 \cdot b^2 \cdot c^3 + 24 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^3 \cdot c^4 - 192 \cdot a^4 \cdot c^4 - 2 \cdot (b^2 - 4ac) \cdot a \cdot b^4 \cdot c + 20 \cdot (b^2 - \\
& 4ac) \cdot a^2 \cdot b^2 \cdot c^2 - 48 \cdot (b^2 - 4ac) \cdot a^3 \cdot c^3 \cdot A \cdot \text{abs}(ab^2 - 4a^2c) - 2 \cdot \\
& (\sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c) \cdot a^2 \cdot b^5 - 8 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^3 \cdot b^3 \cdot c - 2 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \cdot \\
& b^4 \cdot c + 2 \cdot a^2 \cdot b^5 \cdot c + 16 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot b \cdot c^2 \\
& + 8 \cdot \sqrt{2} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^2 \cdot c^2 + \sqrt{2} \cdot \sqrt{bc \\
& - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \cdot b^3 \cdot c^2 - 16 \cdot a^3 \cdot b^3 \cdot c^2 - 4 \cdot \sqrt{2} \cdot \sqrt{bc - \\
& \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b \cdot c^3 + 32 \cdot a^4 \cdot b \cdot c^3 - 2 \cdot (b^2 - 4ac) \cdot a^2 \cdot b^3 \cdot c + \\
& 8 \cdot (b^2 - 4ac) \cdot a^3 \cdot b \cdot c^2 \cdot B \cdot \text{abs}(ab^2 - 4a^2c) + (2 \cdot a^2 \cdot b^7 \cdot c^2 - 40 \cdot a^ \\
& 3 \cdot b^5 \cdot c^3 + 224 \cdot a^4 \cdot b^3 \cdot c^4 - 384 \cdot a^5 \cdot b \cdot c^5 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc \\
& - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \cdot b^7 + 20 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^3 \cdot b^5 \cdot c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^2 \cdot b^6 \cdot c \\
& - 112 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot b^3 \cdot c^2 - 32 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \\
& \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^4 \cdot c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^2 \cdot b^5 \cdot c^2 + 192 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^5 \cdot b \cdot c^3 \\
& + 96 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot b^2 \cdot c^3 + 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \\
& \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^3 \cdot c^3 - 48 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^4 \cdot b \cdot c^4 - 2 \cdot (b^2 - 4ac) \cdot a^2 \cdot b^5 \cdot c^2 + 32 \cdot (b^2 - 4ac) \\
& \cdot a^3 \cdot b^3 \cdot c^3 - 96 \cdot (b^2 - 4ac) \cdot a^4 \cdot b \cdot c^4 \cdot A + 4 \cdot (2 \cdot a^3 \cdot b^6 \cdot c^2 - 16 \cdot a^4 \cdot b^ \\
& 4 \cdot c^3 + 32 \cdot a^5 \cdot b^2 \cdot c^4 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^3 \cdot b^6 + 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot b^4 \cdot c \\
& + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^5 \cdot c - 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \\
& \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^5 \cdot b^2 \cdot c^2 - 8 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \\
& \cdot c \cdot a^4 \cdot b^3 \cdot c^2 - \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^3 \cdot b^ \\
& 4 \cdot c^2 + 4 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac}) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot c \cdot a^4 \cdot b^2 \\
& \cdot c^3 - 2 \cdot (b^2 - 4ac) \cdot a^3 \cdot b^4 \cdot c^2 + 8 \cdot (b^2 - 4ac) \cdot a^4 \cdot b^2 \cdot c^3 \cdot B) \cdot \arctan \\
& (2 \cdot \sqrt{1/2} \cdot x / \sqrt{(ab^3 - 4a^2 \cdot b \cdot c - \sqrt{(ab^3 - 4a^2 \cdot b \cdot c)^2 - 4 \cdot (a^ \\
& 2 \cdot b^2 - 4a^3 \cdot c) \cdot (ab^2 \cdot c - 4a^2 \cdot c^2)})) / (ab^2 \cdot c - 4a^2 \cdot c^2)) / ((a^3 \cdot b^6
\end{aligned}$$

$$- 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c))$$

maple [B] time = 0.11, size = 1761, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x)`

[Out] $\frac{1}{4} \frac{1}{(4ac-b^2)^{1/2}} \frac{(-4ac+b^2)^{1/2}}{ax} \frac{1}{(x^2+1/2(-4ac+b^2))^{1/2}} \frac{1}{c+1/2b/c} \frac{1}{c} A - \frac{1}{4} \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{ax} \frac{1}{(x^2+1/2(-4ac+b^2))^{1/2}} \frac{1}{c+1/2b/c} \frac{1}{c} A + \frac{1}{2} \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{ax} \frac{1}{(x^2+1/2(-4ac+b^2))^{1/2}} \frac{1}{c+1/2b/c} \frac{1}{c} B - 12c^3 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + a - 8c^2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + b^2 + 3/4c \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + b - 3/4c \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + b^3 + 2c^2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) B + 3/2c \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) B + b^2 + 4c^2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) B + 3c \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \arctan(2^{1/2} \frac{1}{((b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) B + b^3 - 1/4 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{ax} \frac{1}{(x^2+1/2b/c-1/2(-4ac+b^2))^{1/2}} \frac{1}{c} A - 1/4 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{ax} \frac{1}{(x^2+1/2b/c-1/2(-4ac+b^2))^{1/2}} \frac{1}{c} A + b + 1/2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{ax} \frac{1}{(x^2+1/2b/c-1/2(-4ac+b^2))^{1/2}} \frac{1}{c} B - 12c^3 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \operatorname{arctanh}(2^{1/2} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + a - 8c^2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \operatorname{arctanh}(2^{1/2} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + b^2 + 3/4c \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{(-4ac+b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \operatorname{arctanh}(2^{1/2} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) A + b^3 - 2c^2 \frac{1}{(4ac-b^2)^{1/2}} \frac{1}{a} \frac{1}{(4ac+3b^2)^{1/2}} \frac{1}{2^{1/2}} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \operatorname{arctanh}(2^{1/2} \frac{1}{((-b+(-4ac+b^2))^{1/2})^{1/2}} \frac{1}{c^{1/2}} \frac{1}{cx}) B - 3/2c$

$$\frac{c/(4ac-b^2)/(4ac+3b^2)*2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*cx)*B*b^2+4c^2/(4ac-b^2)/(-4ac+b^2)^{1/2}*a/(4ac+3b^2)*2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*cx)*b*B+3c/(4ac-b^2)/(-4ac+b^2)^{1/2}/(4ac+3b^2)*2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})*c)^{1/2}*cx)*B*b^3}{}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(2Ba - Ab)cx^3 + (Bab - Ab^2 + 2Aac)x}{2((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)} + \frac{-\int \frac{(2Ba - Ab)cx^2 - Bab - Ab^2 + 6Aac}{cx^4 + bx^2 + a} dx}{2(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $-1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*\operatorname{integrate}(-((2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)$

mupad [B] time = 4.84, size = 12349, normalized size = 42.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)

[Out] $\operatorname{atan}\left(\frac{(6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^{11} + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{1/2} + B^2*a^2*(-(4*a*c - b^2)^9)^{1/2} + 2*A*B*a*b^{10} + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{1/2} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{1/2} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))}{(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))}*(-(A^2*b^{11} + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{1/2} + B^2*a^2*(-(4*a*c - b^2)^9)^{1/2} + 2*A*B*a*b^{10} + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{1/2} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4}$

$$\begin{aligned}
& + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4 \\
& 096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7 \\
& *b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8 \\
& *B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40 \\
& *A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^{11} + B^ \\
& 2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} + 2*A*B*a*b^{10} + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2 \\
& *a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 \\
& - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 \\
& - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B* \\
& a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a \\
& ^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 \\
& + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*1i - (((6144*A*a^5*c^6 + 16 \\
& *A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - \\
& 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c \\
& ^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(A^2* \\
& b^{11} + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 2*A*B*a*b^{10} + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + \\
& 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B \\
& *a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a \\
& ^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - \\
& 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8* \\
& c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^ \\
& 6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*(1024*a^5*b*c^5 - \\
& 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c \\
& ^2 - 8*a^3*b^2*c)))*(-(A^2*b^{11} + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^ \\
& (1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^{10} + 288*A^2*a^2*b^7*c \\
& ^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512 \\
& *B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c \\
& ^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c \\
& + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5) \\
&))^{(1/2)} - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^ \\
& 2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 \\
& + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^{11} + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^{10} + 288*A^ \\
& 2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^ \\
& 5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A* \\
& B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24* \\
& a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a \\
& ^8*b^2*c^5))^{(1/2)}*1i)/((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b* \\
& c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^
\end{aligned}$$

$$\begin{aligned}
& 2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 \\
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2 \\
& *(-4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^10 \\
& + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B \\
& ^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - \\
& 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 \\
& + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B \\
& *a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 \\
& - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 \\
& - 6144*a^8*b^2*c^5))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5 \\
& *c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b \\
& ^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3 \\
& *b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B \\
& *a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^ \\
& 5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1 \\
& 536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c \\
&)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6 \\
& *b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (x*(72*A^2*a^2*c^ \\
& 5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2 \\
& *A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) \\
&)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3 \\
& *b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 \\
& + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4* \\
& b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A \\
& B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 \\
& - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (((614 \\
& 4*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920* \\
& A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + \\
& 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^ \\
& 2)) + (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2 \\
& *a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A \\
& ^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^ \\
& 3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A \\
& B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b \\
& ^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*(1 \\
& 024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^ \\
& 2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^10 + 2 \\
& 88*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a \\
& ^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^
\end{aligned}$$

$$\begin{aligned}
& 2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 1 \\
& 92*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 \\
& - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6 \\
& 144*a^8*b^2*c^5))^{(1/2)} - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 \\
& + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4)) \\
& / (2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) * (-(A^2*b^11 + B^2*a^2*b^9 + \\
& A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B \\
& *a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 \\
& - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c \\
& - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 \\
& + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A \\
& *B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 40 \\
& 96*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7* \\
& b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (5*A^3*b^3*c^4 + 8*B^3*a^3*c^4 + 6*B^ \\
& 3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 - 3*A^2*B*b^4*c^3 + 3*A*B \\
& ^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2*c^4)/(4*(a^2*b^6 - 64*a^ \\
& 5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) * (-(A^2*b^11 + B^2*a^2*b^9 + A^2*b \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} + 2*A*B*a*b^ \\
& 10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96 \\
& *B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c \\
& - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^ \\
& 4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A \\
& *B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^ \\
& 9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^ \\
& 4 - 6144*a^8*b^2*c^5))^{(1/2)} * 2i + \operatorname{atan}((((6144*A*a^5*c^6 + 16*A*a*b^8*c^ \\
& 2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4* \\
& b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2 \\
& *b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2*b^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^ \\
& 4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 2 \\
& 7*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 7 \\
& 68*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5 \\
& *b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3* \\
& b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + \\
& 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} * (1024*a^5*b*c^5 - 16*a^2*b^7*c^ \\
& 2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^ \\
& ^2*c)) * ((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2 \\
& *a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3* \\
& c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B* \\
& a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 3 \\
& 6*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8
\end{aligned}$$

$$\begin{aligned}
& *c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (x \\
& *(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^ \\
& 2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^ \\
& 11 + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 \\
& + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^ \\
& 2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 \\
& + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2 \\
& 40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(\\
& 1/2)}*1i - (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^ \\
& 2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 19 \\
& 2*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c \\
& + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 \\
& - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2* \\
& b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 \\
& - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3* \\
& b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^ \\
& 10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2 \\
& *c^5))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4* \\
& b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2 \\
& *A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3 \\
& *c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2 \\
& *a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^ \\
& 2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2* \\
& c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 \\
& + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840* \\
& a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 \\
& - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - \\
& 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A \\
& ^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^ \\
& 5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^ \\
& 5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A* \\
& B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32* \\
& (a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c \\
& ^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*1i)/((((6144*A*a^5*c^6 + \\
& 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 \\
& - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3 \\
& *c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((A^2
\end{aligned}$$

$$\begin{aligned}
& *b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^{11} + B^2*a^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 2*A*B*a*b^{10} - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 \\
& - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A* \\
& B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2* \\
& a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + \\
& 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8 \\
& *c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a \\
& ^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*(1024*a^5*b*c^5 - \\
& 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)/(2*(a^2*b^4 + 16*a^4*c \\
& ^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2 \\
& *b^{11} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^{10} - 288*A^2*a^2*b^7*c \\
& ^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512 \\
& *B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - \\
& b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c \\
& ^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c \\
& + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5) \\
&))^{(1/2)} + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^ \\
& 2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4)/(2*(a^2*b^4 \\
& + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2 \\
& *b^9 - A^2*b^{11} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^{10} - 288*A^2 \\
& *a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5 \\
& *c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B \\
& *a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a \\
& ^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^ \\
& 8*b^2*c^5))^{(1/2)} + (((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 \\
& - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^ \\
& 7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 1 \\
& 2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((A^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^ \\
& 2*a^2*b^9 - A^2*b^{11} + B^2*a^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2*A*B*a*b^{10} - 28 \\
& 8*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^ \\
& 4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2 \\
& *a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 19 \\
& 2*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b* \\
& (- (4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32*(a^3*b^{12} + 4096*a^9*c^6 - \\
& 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61 \\
& 44*a^8*b^2*c^5))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 \\
& - 768*a^4*b^3*c^4)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A^2*b^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^{11} + B^2*a^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} - 2*A*B*a*b^{10} - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 3840*A \\
& ^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^6*c^ \\
& 5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5*b*c^ \\
& 5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 1536*A*
\end{aligned}$$

$$\begin{aligned}
& B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/(32* \\
& (a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c \\
& ^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} - (x*(72*A^2*a^2*c^5 + A^ \\
& 2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a \\
& *b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((A \\
& ^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^ \\
& 3 - 3840*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072* \\
& A*B*a^6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^ \\
& 2*a^5*b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 \\
& + 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b \\
& ^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280 \\
& *a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)} + (5*A^3*b^3*c^ \\
& 4 + 8*B^3*a^3*c^4 + 6*B^3*a^2*b^2*c^3 - 36*A^3*a*b*c^5 + 72*A^2*B*a^2*c^5 - \\
& 3*A^2*B*b^4*c^3 + 3*A*B^2*a*b^3*c^3 - 60*A*B^2*a^2*b*c^4 + 18*A^2*B*a*b^2* \\
& c^4)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))*((A^2*b^2 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^9 - A^2*b^11 + B^2*a^2*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} - 2*A*B*a*b^10 - 288*A^2*a^2*b^7*c^2 + 1504*A^2*a^3*b^5*c^3 - 38 \\
& 40*A^2*a^4*b^3*c^4 + 96*B^2*a^4*b^5*c^2 - 512*B^2*a^5*b^3*c^3 - 3072*A*B*a^ \\
& 6*c^5 + 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3840*A^2*a^5* \\
& b*c^5 + 768*B^2*a^6*b*c^4 - 192*A*B*a^3*b^6*c^2 + 128*A*B*a^4*b^4*c^3 + 153 \\
& 6*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^{(1/2)} + 36*A*B*a^2*b^8*c)/ \\
& (32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b \\
& ^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5))^{(1/2)}*2i + ((x*(2*A*a*c - A \\
& *b^2 + B*a*b))/(2*a*(4*a*c - b^2)) - (c*x^3*(A*b - 2*B*a))/(2*a*(4*a*c - b^ \\
& 2)))/(a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.100 \quad \int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=389

$$\frac{-10aAc - abB + 3Ab^2}{2a^2x(b^2 - 4ac)} + \frac{\sqrt{c} \left(aB \left(b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) - A \left(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.22, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1277, 1281, 1166, 205}

$$\frac{-10aAc - abB + 3Ab^2}{2a^2x(b^2 - 4ac)} + \frac{\sqrt{c} \left(aB \left(b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) - A \left(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(\frac{aB(b^2 - 12ac) - A(3b^3 - 16abc)}{\sqrt{b^2 - 4ac}} - 10aAc - abB + 3Ab^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{2\sqrt{2}a^2(b^2 - 4ac) \sqrt{\sqrt{b^2 - 4ac} + b}} - \frac{A(b^2 - 2ac) + c^2(-Ab - 2aB) + abB}{2ax(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] $-(3A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^{3/2}*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 - 12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^2(a + bx^2 + cx^4)^2} dx &= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\int \frac{-3Ab^2 + abB + 10aAc - 3(Ab - 2aB)cx^2}{x^2(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\ &= -\frac{3Ab^2 - abB - 10aAc}{2a^2(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{\int \frac{aB(b^2 - 6ac) - A(3b^3 - 3ab^2 - 3a^2b - 3a^3)}{x^2(a + bx^2 + cx^4)} dx}{2a} \\ &= -\frac{3Ab^2 - abB - 10aAc}{2a^2(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{c \left(aB(b^2 - 12ac) - A(3b^3 - 3ab^2 - 3a^2b - 3a^3) \right)}{2a} \\ &= -\frac{3Ab^2 - abB - 10aAc}{2a^2(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(aB(b^2 - 12ac) - A(3b^3 - 3ab^2 - 3a^2b - 3a^3) \right)}{2a} \end{aligned}$$

Mathematica [A] time = 1.04, size = 382, normalized size = 0.98

$$\frac{2x(ab(-2ac + b^2 + bcx^2) - A(-3abc - 2ac^2 + b^3 + b^2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(A(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} + 16abc - 3b^3) + ab(b\sqrt{b^2 - 4ac} - 12ac + b^2) \right) \tan^{-1} \left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(A(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3) + ab(b\sqrt{b^2 - 4ac} + 12ac - b^2) \right) \tan^{-1} \left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}} - \frac{4A}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out]
$$\left(\frac{-4A}{x} + \frac{(2x(aB(b^2 - 2ac + bcx^2) - A(b^3 - 3ab^2c + b^2cx^2 - 2ac^2x^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}(aB(b^2 - 12ac + b\sqrt{b^2 - 4ac}) + A(-3b^3 + 16ab^2c - 3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac}))}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right] + \frac{\sqrt{2}\sqrt{c}(aB(-b^2 + 12ac + b\sqrt{b^2 - 4ac}) + A(3b^3 - 16ab^2c - 3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac}))}{\sqrt{b + \sqrt{b^2 - 4ac}}} \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right] \right) / (4a^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2(a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 9.54, size = 7583, normalized size = 19.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left(2(10Aac^2 + (Bab - 3Ab^2)c)x^4 - 4Aab^2 + 16Aa^2c + 2(Bab^2 - 3Ab^3 - (2Ba^2 - 11Aab)c)x^2 - \sqrt{1/2}((a^2b^2c - 4a^3c^2)x^5 + (a^2b^3 - 4a^3bc)x^3 + (a^3b^2 - 4a^4c)x) \sqrt{-(B^2a^2b^5 - 6ABab^6 + 9A^2b^7 + 60(4ABa^4 - 7A^2a^3b)c^3 + 5(12B^2a^4b - 60ABa^3b^2 + 77A^2a^2b^3)c^2 - 5(3B^2a^3b^3 - 16ABa^2b^4 + 21A^2ab^5)c + (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)} \right) \sqrt{(B^4a^4b^4 - 12AB^3a^3b^5 + 54A^2B^2a^2b^6 - 108A^3Bab^7 + 81A^4b^8 + 625A^4a^4c^4 - 50(9A^2B^2a^5 - 44A^3Bab^4 + 51A^4a^3b^2)c^3 + 3(27B^4a^6 - 264AB^3a^5b + 968A^2B^2a^4b^2 - 1596A^3Bab^3b^3 + 1017A^4a^2b^4)c^2 - 2(9B^4a^5b^2 - 98AB^3a^4b^3 + 396A^2B^2a^3b^4 - 702A^3Bab^2b^5 + 459A^4aab^6)c} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) \log((2500A^4a^3c^6 + 625(4A^3Bab^3 - 9A^4a^2b^2)c^5 - 3(108B^4a^5 - 756AB^3a^4b +$$

$$\begin{aligned}
& 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b
\end{aligned}$$

$$\begin{aligned}
&^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414* \\
&A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4* \\
&b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^ \\
&2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^1 \\
&0 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 \\
&- 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - \\
&11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 \\
&- 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44* \\
&A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A \\
&^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5* \\
&b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4* \\
&a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))*sqrt \\
&(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 \\
&+ 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^ \\
&3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2 \\
&*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^ \\
&6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44 \\
&*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968* \\
&A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5 \\
&*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4 \\
&*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^ \\
&5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2 \\
&*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq \\
&rt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c \\
&^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b \\
&b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b \\
&^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2* \\
&b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - \\
&44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 96 \\
&8*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a \\
&^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A \\
&^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(\\
&a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^ \\
&6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^ \\
&4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B \\
&^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - \\
&189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27 \\
&*A^3*B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2* \\
&B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3* \\
&a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B \\
&^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)* \\
&c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a \\
&^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486 \\
&*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10 \\
&*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^
\end{aligned}$$

$$\begin{aligned}
& 5 - 49A^4a^7b^6)c^2 - 5(4B^4a^7b^7 - 11A^4a^6b^8)c) \sqrt{(B^4a^4b^4 - 12A^3B^3a^3b^5 + 54A^2B^2a^2b^6 - 108A^3B^3a^4b^7 + 81A^4b^8 + 625A^4a^4c^4 - 50(9A^2B^2a^5 - 44A^3B^3a^4b + 51A^4a^3b^2)c^3 + 3(27B^4a^6 - 264A^3B^3a^5b + 968A^2B^2a^4b^2 - 1596A^3B^3a^3b^3 + 1017A^4a^2b^4)c^2 - 2(9B^4a^5b^2 - 98A^3B^3a^4b^3 + 396A^2B^2a^3b^4 - 702A^3B^3a^2b^5 + 459A^4a^2b^6)c)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) \sqrt{-(B^2a^2b^5 - 6A^3B^3a^4b^6 + 9A^2b^7 + 60(4A^2B^2a^4b - 7A^2a^3b)c^3 + 5(12B^2a^4b - 60A^3B^3a^3b^2 + 77A^2a^2b^3)c^2 - 5(3B^2a^3b^3 - 16A^3B^3a^2b^4 + 21A^2a^2b^5)c - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3))} \sqrt{(B^4a^4b^4 - 12A^3B^3a^3b^5 + 54A^2B^2a^2b^6 - 108A^3B^3a^4b^7 + 81A^4b^8 + 625A^4a^4c^4 - 50(9A^2B^2a^5 - 44A^3B^3a^4b + 51A^4a^3b^2)c^3 + 3(27B^4a^6 - 264A^3B^3a^5b + 968A^2B^2a^4b^2 - 1596A^3B^3a^3b^3 + 1017A^4a^2b^4)c^2 - 2(9B^4a^5b^2 - 98A^3B^3a^4b^3 + 396A^2B^2a^3b^4 - 702A^3B^3a^2b^5 + 459A^4a^2b^6)c)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) + \sqrt{1/2}((a^2b^2c - 4a^3c^2)x^5 + (a^2b^3 - 4a^3b^2c)x^3 + (a^3b^2 - 4a^4c)x) \sqrt{-(B^2a^2b^5 - 6A^3B^3a^4b^6 + 9A^2b^7 + 60(4A^2B^2a^4b - 7A^2a^3b)c^3 + 5(12B^2a^4b - 60A^3B^3a^3b^2 + 77A^2a^2b^3)c^2 - 5(3B^2a^3b^3 - 16A^3B^3a^2b^4 + 21A^2a^2b^5)c - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3))} \sqrt{(B^4a^4b^4 - 12A^3B^3a^3b^5 + 54A^2B^2a^2b^6 - 108A^3B^3a^4b^7 + 81A^4b^8 + 625A^4a^4c^4 - 50(9A^2B^2a^5 - 44A^3B^3a^4b + 51A^4a^3b^2)c^3 + 3(27B^4a^6 - 264A^3B^3a^5b + 968A^2B^2a^4b^2 - 1596A^3B^3a^3b^3 + 1017A^4a^2b^4)c^2 - 2(9B^4a^5b^2 - 98A^3B^3a^4b^3 + 396A^2B^2a^3b^4 - 702A^3B^3a^2b^5 + 459A^4a^2b^6)c)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) * \log((2500A^4a^3c^6 + 625(4A^3B^3a^3b - 9A^4a^2b^2)c^5 - 3(108B^4a^5 - 756A^3B^3a^4b + 1672A^2B^2a^3b^2 - 909A^3B^3a^2b^3 - 657A^4a^2b^4)c^4 + (81B^4a^4b^2 - 647A^3B^3a^3b^3 + 1674A^2B^2a^2b^4 - 1323A^3B^3a^2b^5 - 189A^4b^6)c^3 - 5(B^4a^3b^4 - 9A^3B^3a^2b^5 + 27A^2B^2a^2b^6 - 27A^3B^3b^7)c^2) * x - 1/2 \sqrt{1/2} (B^3a^3b^8 - 9A^3B^2a^2b^9 + 27A^2B^3a^2b^10 - 27A^3b^11 - 400(6A^2B^3a^6 - 13A^3a^5b)c^5 + 8(108B^3a^7 - 762A^2B^2a^6b + 1956A^2B^3a^5b^2 - 1801A^3a^4b^3)c^4 - (672B^3a^6b^2 - 4968A^2B^2a^5b^3 + 12414A^2B^3a^4b^4 - 10549A^3a^3b^5)c^3 + 5(38B^3a^5b^4 - 297A^2B^2a^4b^5 + 771A^2B^3a^3b^6 - 666A^3a^2b^7)c^2 - (23B^3a^4b^6 - 192A^2B^2a^3b^7 + 531A^2B^3a^2b^8 - 486A^3a^2b^9)c + (B^3a^6b^9 - 3A^2a^5b^10 + 1280A^3a^10c^5 + 128(4B^3a^10b - 17A^2a^9b^2)c^4 - 448(B^3a^9b^3 - 3A^2a^8b^4)c^3 + 8(18B^3a^8b^5 - 49A^2a^7b^6)c^2 - 5(4B^3a^7b^7 - 11A^2a^6b^8)c) \sqrt{(B^4a^4b^4 - 12A^3B^3a^3b^5 + 54A^2B^2a^2b^6 - 108A^3B^3a^4b^7 + 81A^4b^8 + 625A^4a^4c^4 - 50(9A^2B^2a^5 - 44A^3B^3a^4b + 51A^4a^3b^2)c^3 + 3(27B^4a^6 - 264A^3B^3a^5b + 968A^2B^2a^4b^2 - 1596A^3B^3a^3b^3 + 1017A^4a^2b^4)c^2 - 2(9B^4a^5b^2 - 98A^3B^3a^4b^3 + 396A^2B^2a^3b^4 - 702A^3B^3a^2b^5 + 459
\end{aligned}$$

$$\frac{(A^4 a^2 b^6 c) \sqrt{-(B^2 a^2 b^5 - 6 A B a^2 b^6 + 9 A^2 b^7 + 60(4 A B a^4 - 7 A^2 a^3 b) c^3 + 5(12 B^2 a^4 b - 60 A B a^3 b^2 + 77 A^2 a^2 b^3) c^2 - 5(3 B^2 a^3 b^3 - 16 A B a^2 b^4 + 21 A^2 a b^5) c - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3) \sqrt{(B^4 a^4 b^4 - 12 A B^3 a^3 b^5 + 54 A^2 B^2 a^2 b^6 - 108 A^3 B a b^7 + 81 A^4 b^8 + 625 A^4 a^4 c^4 - 50(9 A^2 B^2 a^5 - 44 A^3 B a^4 b + 51 A^4 a^3 b^2) c^3 + 3(27 B^4 a^6 - 264 A B^3 a^5 b + 968 A^2 B^2 a^4 b^2 - 1596 A^3 B a^3 b^3 + 1017 A^4 a^2 b^4) c^2 - 2(9 B^4 a^5 b^2 - 98 A B^3 a^4 b^3 + 396 A^2 B^2 a^3 b^4 - 702 A^3 B a^2 b^5 + 459 A^4 a b^6) c}}{(a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)) \sqrt{(a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))}}{(a^2 b^2 c - 4 a^3 c^2) x^5 + (a^2 b^3 - 4 a^3 b c) x^3 + (a^3 b^2 - 4 a^4 c) x}$$

giac [B] time = 6.16, size = 5408, normalized size = 13.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] $\frac{1}{2} \frac{(B a^2 b^2 c x^4 - 3 A a b^2 c x^4 + 10 A^2 a^2 c^2 x^4 + B a^2 b^2 x^2 - 3 A a b^3 x^2 - 2 B a^2 c x^2 + 11 A^2 a b^2 c x^2 - 2 A^2 a b^2 + 8 A^2 a^2 c)}{(c x^5 + b x^3 + a x) (a^2 b^2 - 4 a^3 c)} - \frac{1}{16} \frac{((6 b^4 c^2 - 44 a b^2 c^3 + 80 a^2 c^4 - 3 \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^4 + 22 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a b^2 c + 6 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} b^3 c - 40 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^2 c^2 - 20 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a b c^2 - 3 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} b^2 c^2 + 10 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a c^3 - 6 (b^2 - 4 a c) b^2 c^2 + 20 (b^2 - 4 a c) a c^3) (a^2 b^2 - 4 a^3 c)^2 A - (2 a b^3 c^2 - 8 a^2 b c^3 - \sqrt{2} \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} a b^3 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^2 b c + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a b^2 c - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} a b c^2 - 2 (b^2 - 4 a c) a b c^2) (a^2 b^2 - 4 a^3 c)^2 B + 2 (3 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^2 b^7 - 37 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^3 b^5 c - 6 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^2 b^6 c - 6 a^2 b^7 c + 152 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^4 b^3 c^2 + 50 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^3 b^4 c^2 + 3 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^2 b^5 c^2 + 74 a^3 b^5 c^2 - 208 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^5 b c^3 - 104 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^4 b^2 c^3 - 25 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^3 b^3 c^3 - 304 a^4 b^3 c^3 + 52 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} a^4 b c^4 + 416 a^5 b c^4 + 6 (b^2 - 4 a c) a^2 b^5 c - 50 (b^2 - 4 a c) a^3 b^3 c^2 + 104 (b^2 - 4 a c) a^4 b c^3) A \operatorname{abs}(a^2 b^2 -$

$$\begin{aligned}
& 4*a^3*c) - 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^6 - 14*\text{sqrt}(2)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^4*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c \\
&))*c)*a^5*b^2*c^2 + 20*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^3*c^2 + \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96* \\
& \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*c^3 - 48*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqr} \\
& \text{t}(b^2 - 4*a*c))*c)*a^5*b*c^3 - 10*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^ \\
& 4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^ \\
& 5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2* \\
& c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*B*\text{abs}(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 - \\
& 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^8 + 40*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^6*c + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^7*c - 176*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^4*c^2 - 56*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^5*c^2 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^6*c^2 + 256*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^2*c^3 + 128*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^3*c^3 + 28*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^4*c^3 - 64*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + \\
& 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b \\
& ^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - \text{sqrt}(2)*\text{sqrt}(b^ \\
& 2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^7 + 20*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^5*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a \\
& *c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^6*c - 112*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^3*c^2 - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a* \\
& c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^4*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^5*c^2 + 192*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& *\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^8*b*c^3 + 96*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b^2*c^3 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^3*c^3 - 48*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32 \\
& *(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*\text{arctan}(2*\text{sqrt}(1 \\
& /2)*x/\text{sqrt}((a^2*b^3 - 4*a^3*b*c + \text{sqrt}((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 \\
& - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - \\
& 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 \\
& - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*\text{abs}(a^2*b^2 - 4* \\
& a^3*c)*\text{abs}(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*\text{sqrt}(2)*s \\
& \text{qrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^4 + 22*\text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a \\
& *c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c - 40*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^2 - 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b \\
& *c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^2 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c))*c)*b^2*c^2 + 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}
\end{aligned}$$

$$\begin{aligned}
& (b^2 - 4ac)c^3 - 6(b^2 - 4ac)b^2c^2 + 20(b^2 - 4ac)ac^3) * \\
& (a^2b^2 - 4a^3c)^2A - (2ab^3c^2 - 8a^2b^3c^3 - \sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * ab^3 + 4\sqrt{2}\sqrt{b^2 - 4ac} * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^2b^3c + 2\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& ab^2c - \sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc - \sqrt{b^2 - 4ac}}) * ab^2c^2 - 2(b^2 - 4ac) * \\
& ab^2c^2) * (a^2b^2 - 4a^3c)^2 * B - 2(3\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^2b^7 - 37\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * a^3b^5c - 6\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^2b^6c + 6a^2b^7c + 152\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^3c^2 + 50\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * a^3b^4c^2 + 3\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^2b^5c^2 - 74a^3b^5c^2 - 208\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^5b^3c^3 - 104\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^2c^3 - 25\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^3b^3c^3 + 304a^4b^3c^3 + 52\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^3c^4 - 416a^5b^3c^4 - 6(b^2 - 4ac) * a^2b^5c + 50(b^2 - 4ac) * \\
& a^3b^3c^2 - 104(b^2 - 4ac) * a^4b^3c^3) * A * \text{abs}(a^2b^2 - 4a^3c) + 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^3b^6 - 14\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^4c - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^3b^5c + 2a^3b^6c + 64\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^5b^2c^2 + 20\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^3c^2 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^3b^4c^2 - 28a^4b^4c^2 - 96\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^6c^3 - 48\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^5b^3c^3 - 10\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^4b^2c^3 + 128a^5b^2c^3 + 24\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}) * \\
& a^5c^4 - 192a^6c^4 - 2(b^2 - 4ac) * a^3b^4c + 20(b^2 - 4ac) * \\
& a^4b^2c^2 - 48(b^2 - 4ac) * a^5c^3) * B * \text{abs}(a^2b^2 - 4a^3c) + (6a^4b^8c^2 - 80a^5b^6c^3 + 352a^6b^4c^4 - 512a^7b^2c^5 - 3\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^4b^8 + 40\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^6c + 6\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^4b^7c - 176\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^6b^4c^2 - 56\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^5c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^4b^6c^2 + 256\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^7b^2c^3 + 128\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^6b^3c^3 + 28\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^4c^3 - 64\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^6b^2c^4 - 6(b^2 - 4ac) * a^4b^6c^2 + 56(b^2 - 4ac) * \\
& a^5b^4c^3 - 128(b^2 - 4ac) * a^6b^2c^4) * A - (2a^5b^7c^2 - 40a^6b^5c^3 + 224a^7b^3c^4 - 384a^8b^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^7 + 20\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^6b^5c + 2\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^6c - 112\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^7b^3c^2 - 32\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^6b^4c^2 - \sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) * a^5b^5c^2 + 192\sqrt{2}\sqrt{b^2 - 4ac}) * \\
& \sqrt{bc - \sqrt{b^2 - 4ac}}) *
\end{aligned}$$

$$t(b^2 - 4ac)c)a^8b^3c^3 + 96\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^7b^2c^3 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^6b^3c^3 - 48\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^7b^4c^2 - 2(b^2 - 4ac)a^5b^5c^2 + 32(b^2 - 4ac)a^6b^3c^3 - 96(b^2 - 4ac)a^7b^4c^2)B)\arctan(2\sqrt{1/2}x/\sqrt{(a^2b^3 - 4a^3b^2c - \sqrt{(a^2b^3 - 4a^3b^2c)^2 - 4(a^3b^2 - 4a^4c)(a^2b^2c - 4a^3c^2)})/(a^2b^2c - 4a^3c^2)})/((a^5b^6 - 12a^6b^4c - 2a^5b^5c + 48a^7b^2c^2 + 16a^6b^3c^2 + a^5b^4c^2 - 64a^8c^3 - 32a^7b^3c^3 - 8a^6b^2c^3 + 16a^7c^4)\text{abs}(a^2b^2 - 4a^3c)\text{abs}(c))$$

maple [B] time = 0.04, size = 1252, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((Bx^2+A)/x^2/(cx^4+bx^2+a)^2, x)$

[Out] $-A/a^2/x - 1/a/(cx^4+bx^2+a)c^2/(4ac-b^2)x^3 + 1/2/a^2/(cx^4+bx^2+a)c/(4ac-b^2)x^3 + A/b^2 - 1/2/a/(cx^4+bx^2+a)c/(4ac-b^2)x^3 + B - 3/2/a/(cx^4+bx^2+a)/(4ac-b^2)x + A/b^2c + 1/2/a^2/(cx^4+bx^2+a)/(4ac-b^2)x + B^3 + 1/(cx^4+bx^2+a)/(4ac-b^2)x + B^2c - 1/2/a/(cx^4+bx^2+a)/(4ac-b^2)x + B^2 + 5/2/ac^2/(4ac-b^2)^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A - 3/4/a^2c/(4ac-b^2)^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A^2 + 4/ac^2/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A^2 - 3/4/a^2c/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A^2 + 3/4/ac/(4ac-b^2)2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + B - 3c^2/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2}\arctanh(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + B^2 - 5/2/ac^2/(4ac-b^2)2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A + 3/4/a^2c/(4ac-b^2)2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A^2 + 4/ac^2/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + A^2 - 3/4/a^2c/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + B + 1/4/ac/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + B - 3c^2/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x) + B + 1/4/ac/(4ac-b^2)/(-4ac+b^2)^{1/2}2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2}\arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2})c^2x)$

$$((b+(-4*a*c+b^2)^{(1/2)})^*(c)^{(1/2)*c*x})*B*b^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * ((10 * A * a * c^2 + (B * a * b - 3 * A * b^2) * c) * x^4 - 2 * A * a * b^2 + 8 * A * a^2 * c + (B * a * b^2 - 3 * A * b^3 - (2 * B * a^2 - 11 * A * a * b) * c) * x^2) / ((a^2 * b^2 * c - 4 * a^3 * c^2) * x^5 + (a^2 * b^3 - 4 * a^3 * b * c) * x^3 + (a^3 * b^2 - 4 * a^4 * c) * x) + \frac{1}{2} * \text{integrate}((B * a * b^2 - 3 * A * b^3 + (10 * A * a * c^2 + (B * a * b - 3 * A * b^2) * c) * x^2 - (6 * B * a^2 - 13 * A * a * b) * c) / (c * x^4 + b * x^2 + a), x) / (a^2 * b^2 - 4 * a^3 * c)$

mupad [B] time = 5.38, size = 17591, normalized size = 45.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)

[Out] $-(A/a - (x^2 * (3 * A * b^3 - B * a * b^2 + 2 * B * a^2 * c - 11 * A * a * b * c)) / (2 * a^2 * (4 * a * c - b^2)) + (c * x^4 * (10 * A * a * c - 3 * A * b^2 + B * a * b)) / (2 * a^2 * (4 * a * c - b^2))) / (a * x + b * x^3 + c * x^5) - \text{atan}(\frac{((-(9 * A^2 * b^{13} + B^2 * a^2 * b^{11} + 9 * A^2 * b^4 * (-(4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * a * b^{12} + 2077 * A^2 * a^2 * b^9 * c^2 - 10656 * A^2 * a^3 * b^7 * c^3 + 30240 * A^2 * a^4 * b^5 * c^4 - 44800 * A^2 * a^5 * b^3 * c^5 + 25 * A^2 * a^2 * c^2 * (-(4 * a * c - b^2)^9)^{(1/2)} + B^2 * a^2 * b^2 * (-(4 * a * c - b^2)^9)^{(1/2)} + 288 * B^2 * a^4 * b^7 * c^2 - 1504 * B^2 * a^5 * b^5 * c^3 + 3840 * B^2 * a^6 * b^3 * c^4 - 15360 * A * B * a^7 * c^6 - 213 * A^2 * a * b^{11} * c + 26880 * A^2 * a^6 * b * c^6 - 27 * B^2 * a^3 * b^9 * c - 3840 * B^2 * a^7 * b * c^5 - 9 * B^2 * a^3 * c * (-(4 * a * c - b^2)^9)^{(1/2)} - 1548 * A * B * a^3 * b^8 * c^2 + 8064 * A * B * a^4 * b^6 * c^3 - 22400 * A * B * a^5 * b^4 * c^4 + 30720 * A * B * a^6 * b^2 * c^5 - 51 * A^2 * a * b^2 * c * (-(4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * a * b^3 * (-(4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a^2 * b^{10} * c + 44 * A * B * a^2 * b * c * (-(4 * a * c - b^2)^9)^{(1/2)}) / (32 * (a^5 * b^{12} + 4096 * a^{11} * c^6 - 24 * a^6 * b^{10} * c + 240 * a^7 * b^8 * c^2 - 1280 * a^8 * b^6 * c^3 + 3840 * a^9 * b^4 * c^4 - 6144 * a^{10} * b^2 * c^5)))^{(1/2)} * (x * (-(9 * A^2 * b^{13} + B^2 * a^2 * b^{11} + 9 * A^2 * b^4 * (-(4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * a * b^{12} + 2077 * A^2 * a^2 * b^9 * c^2 - 10656 * A^2 * a^3 * b^7 * c^3 + 30240 * A^2 * a^4 * b^5 * c^4 - 44800 * A^2 * a^5 * b^3 * c^5 + 25 * A^2 * a^2 * c^2 * (-(4 * a * c - b^2)^9)^{(1/2)} + B^2 * a^2 * b^2 * (-(4 * a * c - b^2)^9)^{(1/2)} + 288 * B^2 * a^4 * b^7 * c^2 - 1504 * B^2 * a^5 * b^5 * c^3 + 3840 * B^2 * a^6 * b^3 * c^4 - 15360 * A * B * a^7 * c^6 - 213 * A^2 * a * b^{11} * c + 26880 * A^2 * a^6 * b * c^6 - 27 * B^2 * a^3 * b^9 * c - 3840 * B^2 * a^7 * b * c^5 - 9 * B^2 * a^3 * c * (-(4 * a * c - b^2)^9)^{(1/2)} - 1548 * A * B * a^3 * b^8 * c^2 + 8064 * A * B * a^4 * b^6 * c^3 - 22400 * A * B * a^5 * b^4 * c^4 + 30720 * A * B * a^6 * b^2 * c^5 - 51 * A^2 * a * b^2 * c * (-(4 * a * c - b^2)^9)^{(1/2)} - 6 * A * B * a * b^3 * (-(4 * a * c - b^2)^9)^{(1/2)} + 152 * A * B * a^2 * b^{10} * c + 44 * A * B * a^2 * b * c * (-(4 * a * c - b^2)^9)^{(1/2)}) / (32 * (a^5 * b^{12} + 4096 * a^{11} * c^6 - 24 * a^6 * b^{10} * c + 240 * a^7 * b^8 * c^2 - 1280 * a^8 * b^6 * c^3$

$$\begin{aligned}
& + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)}(1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - 393216B^8a^{15}c^8 + 851968A^8a^{14}b^8c^8 + 192A^8a^8b^{13}c^2 - 4672A^8a^9b^{11}c^3 + 47360A^8a^{10}b^9c^4 - 256000A^8a^{11}b^7c^5 + 778240A^8a^{12}b^5c^6 - 1261568A^8a^{13}b^3c^7 - 64B^8a^9b^{12}c^2 + 1664B^8a^{10}b^{10}c^3 - 17920B^8a^{11}b^8c^4 + 102400B^8a^{12}b^6c^5 - 327680B^8a^{13}b^4c^6 + 557056B^8a^{14}b^2c^7) + x(204800A^2a^{12}c^9 - 73728B^2a^{13}c^8 + 144A^2a^6b^{12}c^3 - 3264A^2a^7b^{10}c^4 + 30112A^2a^8b^8c^5 - 143360A^2a^9b^6c^6 + 365568A^2a^{10}b^4c^7 - 458752A^2a^{11}b^2c^8 + 16B^2a^8b^{10}c^3 - 416B^2a^9b^8c^4 + 4608B^2a^{10}b^6c^5 - 25600B^2a^{11}b^4c^6 + 69632B^2a^{12}b^2c^7 - 96A^2B^2a^7b^{11}c^3 + 2336A^2B^2a^8b^9c^4 - 22528A^2B^2a^9b^7c^5 + 107520A^2B^2a^{10}b^5c^6 - 253952A^2B^2a^{11}b^3c^7 + 237568A^2B^2a^{12}b^1c^8) \\
&) * (- (9A^2b^{13} + B^2a^2b^{11} + 9A^2b^4 * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^{12} + 2077A^2a^2b^9c^2 - 10656A^2a^3b^7c^3 + 30240A^2a^4b^5c^4 - 44800A^2a^5b^3c^5 + 25A^2a^2c^2 * (- (4ac - b^2)^9)^{(1/2)} + B^2a^2b^2 * (- (4ac - b^2)^9)^{(1/2)} + 288B^2a^4b^7c^2 - 1504B^2a^5b^5c^3 + 3840B^2a^6b^3c^4 - 15360A^2B^2a^7c^6 - 213A^2a^2b^{11}c + 26880A^2a^6b^3c^6 - 27B^2a^3b^9c - 3840B^2a^7b^5c^5 - 9B^2a^3c * (- (4ac - b^2)^9)^{(1/2)} - 1548A^2B^2a^3b^8c^2 + 8064A^2B^2a^4b^6c^3 - 22400A^2B^2a^5b^4c^4 + 30720A^2B^2a^6b^2c^5 - 51A^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^3 * (- (4ac - b^2)^9)^{(1/2)} + 152A^2B^2a^2b^{10}c + 44A^2B^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * i + ((- (9A^2b^{13} + B^2a^2b^{11} + 9A^2b^4 * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^{12} + 2077A^2a^2b^9c^2 - 10656A^2a^3b^7c^3 + 30240A^2a^4b^5c^4 - 44800A^2a^5b^3c^5 + 25A^2a^2c^2 * (- (4ac - b^2)^9)^{(1/2)} + B^2a^2b^2 * (- (4ac - b^2)^9)^{(1/2)} + 288B^2a^4b^7c^2 - 1504B^2a^5b^5c^3 + 3840B^2a^6b^3c^4 - 15360A^2B^2a^7c^6 - 213A^2a^2b^{11}c + 26880A^2a^6b^3c^6 - 27B^2a^3b^9c - 3840B^2a^7b^5c^5 - 9B^2a^3c * (- (4ac - b^2)^9)^{(1/2)} - 1548A^2B^2a^3b^8c^2 + 8064A^2B^2a^4b^6c^3 - 22400A^2B^2a^5b^4c^4 + 30720A^2B^2a^6b^2c^5 - 51A^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^3 * (- (4ac - b^2)^9)^{(1/2)} + 152A^2B^2a^2b^{10}c + 44A^2B^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (x * (- (9A^2b^{13} + B^2a^2b^{11} + 9A^2b^4 * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^{12} + 2077A^2a^2b^9c^2 - 10656A^2a^3b^7c^3 + 30240A^2a^4b^5c^4 - 44800A^2a^5b^3c^5 + 25A^2a^2c^2 * (- (4ac - b^2)^9)^{(1/2)} + B^2a^2b^2 * (- (4ac - b^2)^9)^{(1/2)} + 288B^2a^4b^7c^2 - 1504B^2a^5b^5c^3 + 3840B^2a^6b^3c^4 - 15360A^2B^2a^7c^6 - 213A^2a^2b^{11}c + 26880A^2a^6b^3c^6 - 27B^2a^3b^9c - 3840B^2a^7b^5c^5 - 9B^2a^3c * (- (4ac - b^2)^9)^{(1/2)} - 1548A^2B^2a^3b^8c^2 + 8064A^2B^2a^4b^6c^3 - 22400A^2B^2a^5b^4c^4 + 30720A^2B^2a^6b^2c^5 - 51A^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)} - 6A^2B^2a^2b^3 * (- (4ac - b^2)^9)^{(1/2)} + 152A^2B^2a^2b^{10}c + 44A^2B^2a^2b^2c * (- (4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12} +
\end{aligned}$$

$$\begin{aligned}
& 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840* \\
& a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^1 \\
& 3*c^2 - 6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983 \\
& 040*a^{14}*b^5*c^6 - 1572864*a^{15}*b^3*c^7) + 393216*B*a^{15}*c^8 - 851968*A*a^1 \\
& 4*b*c^8 - 192*A*a^8*b^{13}*c^2 + 4672*A*a^9*b^{11}*c^3 - 47360*A*a^{10}*b^9*c^4 + \\
& 256000*A*a^{11}*b^7*c^5 - 778240*A*a^{12}*b^5*c^6 + 1261568*A*a^{13}*b^3*c^7 + 6 \\
& 4*B*a^9*b^{12}*c^2 - 1664*B*a^{10}*b^{10}*c^3 + 17920*B*a^{11}*b^8*c^4 - 102400*B*a \\
& ^{12}*b^6*c^5 + 327680*B*a^{13}*b^4*c^6 - 557056*B*a^{14}*b^2*c^7) + x*(204800*A^ \\
& 2*a^{12}*c^9 - 73728*B^2*a^{13}*c^8 + 144*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}* \\
& c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4* \\
& c^7 - 458752*A^2*a^{11}*b^2*c^8 + 16*B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + \\
& 4608*B^2*a^{10}*b^6*c^5 - 25600*B^2*a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - \\
& 96*A*B*a^7*b^{11}*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520 \\
& *A*B*a^{10}*b^5*c^6 - 253952*A*B*a^{11}*b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*(-(9* \\
& A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^{12} \\
& + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 4 \\
& 4800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3 \\
& 840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c + 26880*A^2*a^6* \\
& b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4 \\
& *c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6* \\
& A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(- \\
& (4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240 \\
& *a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(\\
& 1/2)}*ii)/((((-(9*A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 6*A*B*a*b^{12} + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2 \\
& *a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2 \\
& *a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c \\
& + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 2 \\
& 2400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 4 \\
& 4*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24* \\
& a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a \\
& ^{10}*b^2*c^5))^{(1/2)}*(x*(-(9*A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 6*A*B*a*b^{12} + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^ \\
& 3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c \\
& ^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213* \\
& A^2*a*b^{11}*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 \\
& - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^ \\
& 4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a
\end{aligned}$$

$$\begin{aligned}
&^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a \\
&^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4 \\
&*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - \\
&6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14} \\
&4*b^5*c^6 - 1572864*a^{15}*b^3*c^7) - 393216*B*a^{15}*c^8 + 851968*A*a^{14}*b*c^8 \\
&+ 192*A*a^8*b^{13}*c^2 - 4672*A*a^9*b^{11}*c^3 + 47360*A*a^{10}*b^9*c^4 - 256000 \\
&*A*a^{11}*b^7*c^5 + 778240*A*a^{12}*b^5*c^6 - 1261568*A*a^{13}*b^3*c^7 - 64*B*a^9 \\
&*b^{12}*c^2 + 1664*B*a^{10}*b^{10}*c^3 - 17920*B*a^{11}*b^8*c^4 + 102400*B*a^{12}*b^6 \\
&*c^5 - 327680*B*a^{13}*b^4*c^6 + 557056*B*a^{14}*b^2*c^7) + x*(204800*A^2*a^{12} \\
&c^9 - 73728*B^2*a^{13}*c^8 + 144*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}*c^4 + 3 \\
&0112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4*c^7 - 4 \\
&58752*A^2*a^{11}*b^2*c^8 + 16*B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B \\
&^2*a^{10}*b^6*c^5 - 25600*B^2*a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - 96*A*B \\
&a^7*b^{11}*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^ \\
&10*b^5*c^6 - 253952*A*B*a^{11}*b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*(-(9*A^2*b^1 \\
&3 + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^{12} + 2077 \\
&*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^ \\
&2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4* \\
&a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2 \\
&*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c + 26880*A^2*a^6*b*c^6 - \\
&27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/ \\
&2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + \\
&30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b \\
&^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c \\
&- b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^ \\
&8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)} - \\
&((-9*A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B \\
&a*b^{12} + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c \\
&^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2 \\
&a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c \\
&^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c + 26880*A^ \\
&2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c \\
&- b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a \\
&^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 44*A*B*a^2* \\
&b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c \\
&+ 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^ \\
&5))^{(1/2)}*(x*(-(9*A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(\\
&1/2)} - 6*A*B*a*b^{12} + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240 \\
&A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9) \\
&^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504 \\
&B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11} \\
&*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^ \\
&3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 \\
&- 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c \\
& + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - \\
& 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - 6144*a^{11} \\
& *b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14}*b^5*c^6 \\
& - 1572864*a^{15}*b^3*c^7) + 393216*B*a^{15}*c^8 - 851968*A*a^{14}*b*c^8 - 192*A*a \\
& ^8*b^{13}*c^2 + 4672*A*a^9*b^{11}*c^3 - 47360*A*a^{10}*b^9*c^4 + 256000*A*a^{11}*b^7 \\
& *c^5 - 778240*A*a^{12}*b^5*c^6 + 1261568*A*a^{13}*b^3*c^7 + 64*B*a^9*b^{12}*c^2 \\
& - 1664*B*a^{10}*b^{10}*c^3 + 17920*B*a^{11}*b^8*c^4 - 102400*B*a^{12}*b^6*c^5 + 327 \\
& 680*B*a^{13}*b^4*c^6 - 557056*B*a^{14}*b^2*c^7) + x*(204800*A^2*a^{12}*c^9 - 7372 \\
& 8*B^2*a^{13}*c^8 + 144*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}*c^4 + 30112*A^2*a \\
& ^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4*c^7 - 458752*A^2* \\
& a^{11}*b^2*c^8 + 16*B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^{10}*b^6 \\
& *c^5 - 25600*B^2*a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - 96*A*B*a^7*b^{11}*c \\
& ^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^{10}*b^5*c^6 \\
& - 253952*A*B*a^{11}*b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*(-(9*A^2*b^{13} + B^2*a^2 \\
& *b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^{12} + 2077*A^2*a^2*b^9 \\
& *c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)} + 128000*A^3*a^{10}*c^9 + 504*A^3*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^{10}*b^3*c^6 + 46080*A*B^2*a^{11}*c^8 + 4608*B^3*a^{11}*b*c^7 - 84480*A^2*B*a^{10}*b*c^8 + 240*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 52992*A*B^2*a^{10}*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 - 33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7))*(-(9*A^2*b^{13} + B^2*a^2*b^{11} + 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^{12} + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^{11}*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} + 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*2i - atan((((9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A
\end{aligned}$$

$$\begin{aligned}
&^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 302 \\
&40*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2) \\
&^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 15 \\
&04*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b \\
&^{11}*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2 \\
&*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c \\
&^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a* \\
&c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10} \\
&*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 \\
&- 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - \\
&6144*a^{10}*b^2*c^5))^{(1/2)}*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^ \\
&2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b \\
&^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(\\
&4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4* \\
&b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + \\
&213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b \\
&*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A \\
&*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b \\
&^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152* \\
&A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4 \\
&096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^ \\
&9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}* \\
&c^2 - 6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 98304 \\
&0*a^{14}*b^5*c^6 - 1572864*a^{15}*b^3*c^7) - 393216*B*a^{15}*c^8 + 851968*A*a^{14}* \\
&b*c^8 + 192*A*a^8*b^{13}*c^2 - 4672*A*a^9*b^{11}*c^3 + 47360*A*a^{10}*b^9*c^4 - 2 \\
&56000*A*a^{11}*b^7*c^5 + 778240*A*a^{12}*b^5*c^6 - 1261568*A*a^{13}*b^3*c^7 - 64* \\
&B*a^9*b^{12}*c^2 + 1664*B*a^{10}*b^{10}*c^3 - 17920*B*a^{11}*b^8*c^4 + 102400*B*a^{1 \\
&2}*b^6*c^5 - 327680*B*a^{13}*b^4*c^6 + 557056*B*a^{14}*b^2*c^7) + x*(204800*A^2* \\
&a^{12}*c^9 - 73728*B^2*a^{13}*c^8 + 144*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}*c^ \\
&4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4*c^ \\
&7 - 458752*A^2*a^{11}*b^2*c^8 + 16*B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + 4 \\
&608*B^2*a^{10}*b^6*c^5 - 25600*B^2*a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - 96 \\
&*A*B*a^7*b^{11}*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A \\
&*B*a^{10}*b^5*c^6 - 253952*A*B*a^{11}*b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*((9*A^2 \\
&*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - \\
&2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 4480 \\
&0*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(- \\
&-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840 \\
&*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c \\
&^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9) \\
&^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^ \\
&4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B \\
&*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4* \\
&a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^ \\
&7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
&) * i + \left((9A^2b^4(-4ac - b^2)^9)^{1/2} - B^2a^2b^{11} - 9A^2b^{13} + 6ABa^2b^{12} - 2077A^2a^2b^9c^2 + 10656A^2a^3b^7c^3 - 30240A^2a^4b^5c^4 + 44800A^2a^5b^3c^5 + 25A^2a^2c^2(-4ac - b^2)^9)^{1/2} \right. \\
& + B^2a^2b^2(-4ac - b^2)^9)^{1/2} - 288B^2a^4b^7c^2 + 1504B^2a^5b^5c^3 - 3840B^2a^6b^3c^4 + 15360ABa^7c^6 + 213A^2a^2b^{11}c - 26880A^2a^6b^3c^6 + 27B^2a^3b^9c + 3840B^2a^7b^5c^5 - 9B^2a^3c(-4ac - b^2)^9)^{1/2} \\
& + 1548ABa^3b^8c^2 - 8064ABa^4b^6c^3 + 22400ABa^5b^4c^4 - 30720ABa^6b^2c^5 - 51A^2a^2b^2c(-4ac - b^2)^9)^{1/2} - 6ABa^2b^3(-4ac - b^2)^9)^{1/2} - 152ABa^2b^{10}c + 44ABa^2b^2c(-4ac - b^2)^9)^{1/2} \\
& \left. \right) / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{1/2} \\
& * (x((9A^2b^4(-4ac - b^2)^9)^{1/2} - B^2a^2b^{11} - 9A^2b^{13} + 6ABa^2b^{12} - 2077A^2a^2b^9c^2 + 10656A^2a^3b^7c^3 - 30240A^2a^4b^5c^4 + 44800A^2a^5b^3c^5 + 25A^2a^2c^2(-4ac - b^2)^9)^{1/2} \\
& + B^2a^2b^2(-4ac - b^2)^9)^{1/2} - 288B^2a^4b^7c^2 + 1504B^2a^5b^5c^3 - 3840B^2a^6b^3c^4 + 15360ABa^7c^6 + 213A^2a^2b^{11}c - 26880A^2a^6b^3c^6 + 27B^2a^3b^9c + 3840B^2a^7b^5c^5 - 9B^2a^3c(-4ac - b^2)^9)^{1/2} \\
& + 1548ABa^3b^8c^2 - 8064ABa^4b^6c^3 + 22400ABa^5b^4c^4 - 30720ABa^6b^2c^5 - 51A^2a^2b^2c(-4ac - b^2)^9)^{1/2} - 6ABa^2b^3(-4ac - b^2)^9)^{1/2} - 152ABa^2b^{10}c + 44ABa^2b^2c(-4ac - b^2)^9)^{1/2} \\
& \left. \right) / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{1/2} \\
& * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) + 393216Ba^{15}c^8 - 851968Aa^{14}b^8c^8 - 192Aa^8b^{13}c^2 + 4672Aa^9b^{11}c^3 - 47360Aa^{10}b^9c^4 + 256000Aa^{11}b^7c^5 - 778240Aa^{12}b^5c^6 + 1261568Aa^{13}b^3c^7 + 64Ba^9b^{12}c^2 - 1664Ba^{10}b^{10}c^3 + 17920Ba^{11}b^8c^4 - 102400Ba^{12}b^6c^5 + 327680Ba^{13}b^4c^6 - 557056Ba^{14}b^2c^7) + x(204800A^2a^{12}c^9 - 73728B^2a^{13}c^8 + 144A^2a^6b^{12}c^3 - 3264A^2a^7b^{10}c^4 + 30112A^2a^8b^8c^5 - 143360A^2a^9b^6c^6 + 365568A^2a^{10}b^4c^7 - 458752A^2a^{11}b^2c^8 + 16B^2a^8b^{10}c^3 - 416B^2a^9b^8c^4 + 4608B^2a^{10}b^6c^5 - 25600B^2a^{11}b^4c^6 + 69632B^2a^{12}b^2c^7 - 96ABa^7b^{11}c^3 + 2336ABa^8b^9c^4 - 22528ABa^9b^7c^5 + 107520ABa^{10}b^5c^6 - 253952ABa^{11}b^3c^7 + 237568ABa^{12}b^1c^8) * ((9A^2b^4(-4ac - b^2)^9)^{1/2} - B^2a^2b^{11} - 9A^2b^{13} + 6ABa^2b^{12} - 2077A^2a^2b^9c^2 + 10656A^2a^3b^7c^3 - 30240A^2a^4b^5c^4 + 44800A^2a^5b^3c^5 + 25A^2a^2c^2(-4ac - b^2)^9)^{1/2} + B^2a^2b^2(-4ac - b^2)^9)^{1/2} - 288B^2a^4b^7c^2 + 1504B^2a^5b^5c^3 - 3840B^2a^6b^3c^4 + 15360ABa^7c^6 + 213A^2a^2b^{11}c - 26880A^2a^6b^3c^6 + 27B^2a^3b^9c + 3840B^2a^7b^5c^5 - 9B^2a^3c(-4ac - b^2)^9)^{1/2} + 1548ABa^3b^8c^2 - 8064ABa^4b^6c^3 + 22400ABa^5b^4c^4 - 30720ABa^6b^2c^5 - 51A^2a^2b^2c(-4ac - b^2)^9)^{1/2} - 6ABa^2b^3(-4ac - b^2)^9)^{1/2} - 152ABa^2b^{10}c + 44ABa^2b^2c(-4ac - b^2)^9)^{1/2} \\
& \left. \right) / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2
\end{aligned}$$

$$\begin{aligned}
& - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)*1i)/((((\\
& 9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^ \\
& 12 - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + \\
& 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2* \\
& b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - \\
& 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^ \\
& 6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b \\
& ^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c* \\
& (- (4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 2 \\
& 40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))) \\
& ^{(1/2)}*(x*((9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} \\
& + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a \\
& ^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a \\
& ^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - \\
& 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 224 \\
& 00*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44* \\
& A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^ \\
& 6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1 \\
& 0*b^2*c^5)))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - 6144*a^{11}*b^{11} \\
& *c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14}*b^5*c^6 - 157 \\
& 2864*a^{15}*b^3*c^7) - 393216*B*a^{15}*c^8 + 851968*A*a^{14}*b*c^8 + 192*A*a^8*b^ \\
& 13*c^2 - 4672*A*a^9*b^{11}*c^3 + 47360*A*a^{10}*b^9*c^4 - 256000*A*a^{11}*b^7*c^5 \\
& + 778240*A*a^{12}*b^5*c^6 - 1261568*A*a^{13}*b^3*c^7 - 64*B*a^9*b^{12}*c^2 + 166 \\
& 4*B*a^{10}*b^{10}*c^3 - 17920*B*a^{11}*b^8*c^4 + 102400*B*a^{12}*b^6*c^5 - 327680*B \\
& *a^{13}*b^4*c^6 + 557056*B*a^{14}*b^2*c^7) + x*(204800*A^2*a^{12}*c^9 - 73728*B^2 \\
& *a^{13}*c^8 + 144*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}*c^4 + 30112*A^2*a^8*b^ \\
& 8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4*c^7 - 458752*A^2*a^{11} \\
& b^2*c^8 + 16*B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^{10}*b^6*c^5 \\
& - 25600*B^2*a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - 96*A*B*a^7*b^{11}*c^3 + \\
& 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^{10}*b^5*c^6 - 25 \\
& 3952*A*B*a^{11}*b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 \\
& + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + \\
& 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 1 \\
& 5360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9* \\
& c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^ \\
& 3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^ \\
& 2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})
\end{aligned}$$

$$\begin{aligned}
& / (32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8 \\
& *b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)} - (((9*A^2*b^4*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2 \\
& *a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^ \\
& 5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6 \\
& *b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c^6 + 27* \\
& B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 3072 \\
& 0*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)}) / (32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^ \\
& 2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(x*((9 \\
& *A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{1 \\
& 2} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + \\
& 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - \\
& 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6 \\
& *b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^ \\
& 4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6 \\
& *A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(\\
& -(4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 24 \\
& 0*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(\\
& 1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - 6144*a^{11}*b^{11}*c^3 + 61440* \\
& a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14}*b^5*c^6 - 1572864*a^{15}*b^3 \\
& *c^7) + 393216*B*a^{15}*c^8 - 851968*A*a^{14}*b*c^8 - 192*A*a^8*b^{13}*c^2 + 4672 \\
& *A*a^9*b^{11}*c^3 - 47360*A*a^{10}*b^9*c^4 + 256000*A*a^{11}*b^7*c^5 - 778240*A*a \\
& ^{12}*b^5*c^6 + 1261568*A*a^{13}*b^3*c^7 + 64*B*a^9*b^{12}*c^2 - 1664*B*a^{10}*b^{10} \\
& *c^3 + 17920*B*a^{11}*b^8*c^4 - 102400*B*a^{12}*b^6*c^5 + 327680*B*a^{13}*b^4*c^6 \\
& - 557056*B*a^{14}*b^2*c^7) + x*(204800*A^2*a^{12}*c^9 - 73728*B^2*a^{13}*c^8 + 1 \\
& 44*A^2*a^6*b^{12}*c^3 - 3264*A^2*a^7*b^{10}*c^4 + 30112*A^2*a^8*b^8*c^5 - 14336 \\
& 0*A^2*a^9*b^6*c^6 + 365568*A^2*a^{10}*b^4*c^7 - 458752*A^2*a^{11}*b^2*c^8 + 16* \\
& B^2*a^8*b^{10}*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^{10}*b^6*c^5 - 25600*B^2* \\
& a^{11}*b^4*c^6 + 69632*B^2*a^{12}*b^2*c^7 - 96*A*B*a^7*b^{11}*c^3 + 2336*A*B*a^8* \\
& b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^{10}*b^5*c^6 - 253952*A*B*a^{11} \\
& *b^3*c^7 + 237568*A*B*a^{12}*b*c^8))*((9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B \\
& ^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2* \\
& a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2 \\
& *a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7* \\
& c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2* \\
& a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8 \\
& 064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^ \\
& 2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} -
\end{aligned}$$

$$\begin{aligned}
& 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)} + 128000*A^3*a^{10}*c^9 + 504*A^3*a^6*b^8*c^5 - 8112*A^3*a^7*b^6*c^6 + 48704*A^3*a^8*b^4*c^7 - 129280*A^3*a^9*b^2*c^8 - 40*B^3*a^8*b^7*c^4 + 608*B^3*a^9*b^5*c^5 - 2944*B^3*a^{10}*b^3*c^6 + 46080*A*B^2*a^{11}*c^8 + 4608*B^3*a^{11}*b*c^7 - 84480*A^2*B*a^{10}*b*c^8 + 240*A*B^2*a^7*b^8*c^4 - 3792*A*B^2*a^8*b^6*c^5 + 21696*A*B^2*a^9*b^4*c^6 - 52992*A*B^2*a^{10}*b^2*c^7 - 360*A^2*B*a^6*b^9*c^4 + 5736*A^2*B*a^7*b^7*c^5 - 33888*A^2*B*a^8*b^5*c^6 + 87936*A^2*B*a^9*b^3*c^7)*((9*A^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} - B^2*a^2*b^{11} - 9*A^2*b^{13} + 6*A*B*a*b^{12} - 2077*A^2*a^2*b^9*c^2 + 10656*A^2*a^3*b^7*c^3 - 30240*A^2*a^4*b^5*c^4 + 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 288*B^2*a^4*b^7*c^2 + 1504*B^2*a^5*b^5*c^3 - 3840*B^2*a^6*b^3*c^4 + 15360*A*B*a^7*c^6 + 213*A^2*a*b^{11}*c - 26880*A^2*a^6*b*c^6 + 27*B^2*a^3*b^9*c + 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 1548*A*B*a^3*b^8*c^2 - 8064*A*B*a^4*b^6*c^3 + 22400*A*B*a^5*b^4*c^4 - 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^{(1/2)} - 152*A*B*a^2*b^{10}*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.101 \quad \int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2} dx$$

Optimal. Leaf size=522

$$\frac{aB(3b^2 - 10ac) - A(5b^3 - 19abc)}{2a^3x(b^2 - 4ac)} - \frac{-14aAc - 3abB + 5Ab^2}{6a^2x^3(b^2 - 4ac)} - \frac{\sqrt{c} \left(aB(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc) \right)}{6a^2x^3(b^2 - 4ac)}$$

Rubi [A] time = 1.36, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1277, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(aB(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} - 16abc) - A(5b^3 - 19abc) \right) \tan^{-1} \left(\frac{\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{c} \left(aB(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3) - A(2ab^2 - 5b^3\sqrt{b^2 - 4ac} - 29ab^2c + 19abc\sqrt{b^2 - 4ac} + 5b^4) \right) \tan^{-1} \left(\frac{\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) - \frac{14aAc - 3abB + 5Ab^2}{6a^2x^3(b^2 - 4ac)} - \frac{aB(3b^2 - 10ac) - A(5b^3 - 19abc)}{2a^3x(b^2 - 4ac)} - \frac{A(b^2 - 2ac) + c^2c(Ab - 2ab) + abB}{2a^2x^3(b^2 - 4ac)(c + b^2 + cx^2)}}{2\sqrt{c} \left(b^2 - 4ac \right)^2 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x]

[Out] $-(5A*b^2 - 3a*b*B - 14a*A*c)/(6a^2*(b^2 - 4a*c)*x^3) - (a*B*(3b^2 - 10a*c) - A*(5b^3 - 19a*b*c))/(2a^3*(b^2 - 4a*c)*x) - (a*b*B - A*(b^2 - 2a*c) - (A*b - 2a*B)*c*x^2)/(2a*(b^2 - 4a*c)*x^3*(a + b*x^2 + c*x^4)) - (\text{Sqrt}[c]*(a*B*(3b^3 - 16a*b*c + 3b^2*\text{Sqrt}[b^2 - 4a*c] - 10a*c*\text{Sqrt}[b^2 - 4a*c]) - A*(5b^4 - 29a*b^2*c + 28a^2*c^2 + 5b^3*\text{Sqrt}[b^2 - 4a*c] - 19a*b*c*\text{Sqrt}[b^2 - 4a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4a*c)^(3/2)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4a*c]]) + (\text{Sqrt}[c]*(a*B*(3b^3 - 16a*b*c - 3b^2*\text{Sqrt}[b^2 - 4a*c] + 10a*c*\text{Sqrt}[b^2 - 4a*c]) - A*(5b^4 - 29a*b^2*c + 28a^2*c^2 - 5b^3*\text{Sqrt}[b^2 - 4a*c] + 19a*b*c*\text{Sqrt}[b^2 - 4a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4a*c)^(3/2)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne

$Q[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 1277

$\text{Int}[(f \cdot x)^m \cdot (d + e \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^p, x_Symbol] \rightarrow -\text{Simp}[(f \cdot x)^{m+1} \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1} \cdot (d \cdot (b^2 - 2 \cdot a \cdot c) - a \cdot b \cdot e + (b \cdot d - 2 \cdot a \cdot e) \cdot c \cdot x^2)] / (2 \cdot a \cdot f \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)) + \text{Dist}[1 / (2 \cdot a \cdot (p + 1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[(f \cdot x)^m \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1} \cdot \text{Simp}[d \cdot (b^2 \cdot (m + 2 \cdot (p + 1) + 1) - 2 \cdot a \cdot c \cdot (m + 4 \cdot (p + 1) + 1)) - a \cdot b \cdot e \cdot (m + 1) + c \cdot (m + 2 \cdot (2 \cdot p + 3) + 1) \cdot (b \cdot d - 2 \cdot a \cdot e) \cdot x^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1281

$\text{Int}[(f \cdot x)^m \cdot (d + e \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^p, x_Symbol] \rightarrow \text{Simp}[(d \cdot (f \cdot x)^{m+1} \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1}) / (a \cdot f \cdot (m + 1)), x] + \text{Dist}[1 / (a \cdot f^2 \cdot (m + 1)), \text{Int}[(f \cdot x)^{m+2} \cdot (a + b \cdot x^2 + c \cdot x^4)^p \cdot \text{Simp}[a \cdot e \cdot (m + 1) - b \cdot d \cdot (m + 2 \cdot p + 3) - c \cdot d \cdot (m + 4 \cdot p + 5) \cdot x^2, x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^4 (a + bx^2 + cx^4)^2} dx &= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} - \frac{\int \frac{-5Ab^2 + 3abB + 14aAc - 5(Ab - 2aB)cx^2}{x^4(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\
&= -\frac{5Ab^2 - 3abB - 14aAc}{6a^2(b^2 - 4ac)x^3} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} + \frac{\int \frac{-3(5Ab^3 - 3ab^2B - 19a^2Ac)}{x^4(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\
&= -\frac{5Ab^2 - 3abB - 14aAc}{6a^2(b^2 - 4ac)x^3} - \frac{aB(3b^2 - 10ac) - A(5b^3 - 19abc)}{2a^3(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac)}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} \\
&= -\frac{5Ab^2 - 3abB - 14aAc}{6a^2(b^2 - 4ac)x^3} - \frac{aB(3b^2 - 10ac) - A(5b^3 - 19abc)}{2a^3(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac)}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} \\
&= -\frac{5Ab^2 - 3abB - 14aAc}{6a^2(b^2 - 4ac)x^3} - \frac{aB(3b^2 - 10ac) - A(5b^3 - 19abc)}{2a^3(b^2 - 4ac)x} - \frac{abB - A(b^2 - 2ac)}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)}
\end{aligned}$$

Mathematica [A] time = 1.20, size = 487, normalized size = 0.93

$$\frac{6(A(2a^2 - 4ab^2 - 3ab^2c + 9a^2c^2) + ab(5ab^2 + 2a^2c^2 - b^2 - b^2c^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{3\sqrt{2}\sqrt{c}\left(A(28a^2 - 28ab^2 - 18abc\sqrt{b^2 - 4ac} + 5b^3\sqrt{b^2 - 4ac} + 5b^4) + ab(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} + 16abc - 3b^3)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(A(28a^2 - 28ab^2c + 19abc\sqrt{b^2 - 4ac} - 5b^3\sqrt{b^2 - 4ac} + 5b^4) + ab(3b^2\sqrt{b^2 - 4ac} - 10ac\sqrt{b^2 - 4ac} + 16abc - 3b^3)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{24Ab - 12aB}{x} - \frac{4aA}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x]

[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*sqrt[2]*sqrt[c]*(a*B*(-3*b^3 + 16*a*b*c - 3*b^2*sqrt[b^2 - 4*a*c] + 10*a*c*sqrt[b^2 - 4*a*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (3*sqrt[2]*sqrt[c]*(a*B*(-3*b^3 + 16*a*b*c + 3*b^2*sqrt[b^2 - 4*a*c] - 10*a*c*sqrt[b^2 - 4*a*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*sqrt[b^2 - 4*a*c] + 19*a*b*c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]]))/(12*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4 (a + bx^2 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x]

fricas [B] time = 22.20, size = 10190, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 4*A*a^2*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6

$$\begin{aligned}
& - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*
\end{aligned}$$

$$\begin{aligned}
& A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^{10} \\
&)*c)/(a^{14}*b^6 - 12*a^{15}*b^4*c + 48*a^{16}*b^2*c^2 - 64*a^{17}*c^3))/(a^7*b^6 \\
& - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^{10}*c^3))*\log((9604*A^4*a^4*c^8 + 720 \\
& 3*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + \\
& 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (562 \\
& 5*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a \\
& ^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5 \\
& 560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b \\
& ^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x - 1/2*sq \\
& rt(1/2)*(27*B^3*a^3*b^{11} - 135*A*B^2*a^2*b^{12} + 225*A^2*B*a*b^{13} - 125*A^3* \\
& b^{14} + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^ \\
& 6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 \\
& - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 2247 \\
& 05*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B \\
& ^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b \\
& ^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^{10})*c^2 - \\
& (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^{10} + 4260*A^2*B*a^2*b^{11} - 2425*A^3*a*b \\
& ^{12})*c - (3*B*a^8*b^{10} - 5*A*a^7*b^{11} - 256*(5*B*a^{13} - 13*A*a^{12}*b)*c^5 + \\
& 64*(34*B*a^{12}*b^2 - 73*A*a^{11}*b^3)*c^4 - 112*(12*B*a^{11}*b^4 - 23*A*a^{10}*b^5 \\
&)*c^3 + 28*(14*B*a^{10}*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^ \\
& 9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^{10} - 15 \\
& 00*A^3*B*a*b^{11} + 625*A^4*b^{12} + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 18 \\
& 6*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51 \\
& 894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12 \\
& 75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B* \\
& a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 \\
& + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2 \\
& *(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B* \\
& a^2*b^9 + 4125*A^4*a*b^{10})*c)/(a^{14}*b^6 - 12*a^{15}*b^4*c + 48*a^{16}*b^2*c^2 - \\
& 64*a^{17}*c^3))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A \\
& *B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3* \\
& b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(\\
& 15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c \\
& + 48*a^9*b^2*c^2 - 64*a^{10}*c^3))*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + \\
& 1350*A^2*B^2*a^2*b^{10} - 1500*A^3*B*a*b^{11} + 625*A^4*b^{12} + 2401*A^4*a^6*c^ \\
& 6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4* \\
& a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 766 \\
& 86*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2 \\
& *B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a \\
& ^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + \\
& 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^ \\
& 2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^{10})*c)/(a^{14}*b^6 - 12*a^{1 \\
& 5}*b^4*c + 48*a^{16}*b^2*c^2 - 64*a^{17}*c^3))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9 \\
& *b^2*c^2 - 64*a^{10}*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3 \\
& *b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*
\end{aligned}$$

$$\begin{aligned}
& A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5 \\
& *b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3 \\
& *b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a \\
& *b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*\sqrt{(81* \\
& B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 \\
& + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + \\
& 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6 \\
& *b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 \\
& - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815 \\
& *A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^ \\
& 2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b \\
& ^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125* \\
& A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)) \\
& / (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*\log((9604*A^4*a^4 \\
& *c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B \\
& ^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)* \\
& c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 201 \\
& 90*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a \\
& ^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27 \\
& *B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)* \\
& x + 1/2*\sqrt{1/2}*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 \\
& - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + \\
& 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2* \\
& B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6* \\
& b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 \\
& - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330 \\
& *B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^ \\
& 10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 24 \\
& 25*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12 \\
& *b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23* \\
& A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 9 \\
& 4*A*a^8*b^9)*c)*\sqrt{(81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2 \\
& *b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^ \\
& 2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3* \\
& a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c \\
& ^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77 \\
& 424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B \\
& ^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^ \\
& 8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9 \\
& 550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16 \\
& *b^2*c^2 - 64*a^17*c^3))*\sqrt{-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 \\
& - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 2 \\
& 3*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5) \\
& *c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12 \\
& *a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*\sqrt{(81*B^4*a^4*b^8 - 540*A*B^3
\end{aligned}$$

$$\begin{aligned}
& a^3b^9 + 1350A^2B^2a^2b^{10} - 1500A^3B^2a^2b^{11} + 625A^4b^{12} + 2401A^4a^6c^6 - 98(25A^2B^2a^7 - 186A^3B^2a^6b + 246A^4a^5b^2)c^5 + \\
& (625B^4a^8 - 9300A^2B^3a^7b + 51894A^2B^2a^6b^2 - 109544A^3B^2a^5b^3 + 76686A^4a^4b^4)c^4 - 2(1275B^4a^7b^2 - 14086A^2B^3a^6b^3 + \\
& 51336A^2B^2a^5b^4 - 77424A^3B^2a^4b^5 + 41815A^4a^3b^6)c^3 + 3(1017B^4a^6b^4 - 7872A^2B^3a^5b^5 + 22508A^2B^2a^4b^6 - 28260A^3B^2a^3b^7 + 13175A^4a^2b^8)c^2 - 2(459B^4a^5b^6 - 3186A^2B^3a^4b^7 + 8280A^2B^2a^3b^8 - 9550A^3B^2a^2b^9 + 4125A^4a^2b^{10})c) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) / (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) + 3\sqrt{1/2}((a^3b^2c - 4a^4c^2)x^7 + (a^3b^3 - 4a^4bc)x^5 + (a^4b^2 - 4a^5c)x^3)\sqrt{-(9B^2a^2b^7 - 30A^2B^2a^2b^8 + 25A^2b^9 - 140(4A^2B^2a^5 - 9A^2a^4b)c^4 - 105(4B^2a^5b - 20A^2B^2a^4b^2 + 23A^2a^3b^3)c^3 + 7(55B^2a^4b^3 - 210A^2B^2a^3b^4 + 198A^2a^2b^5)c^2 - 7(15B^2a^3b^5 - 52A^2B^2a^2b^6 + 45A^2a^2b^7)c - (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3)\sqrt{(81B^4a^4b^8 - 540A^2B^3a^3b^9 + 1350A^2B^2a^2b^{10} - 1500A^3B^2a^2b^{11} + 625A^4b^{12} + 2401A^4a^6c^6 - 98(25A^2B^2a^7 - 186A^3B^2a^6b + 246A^4a^5b^2)c^5 + (625B^4a^8 - 9300A^2B^3a^7b + 51894A^2B^2a^6b^2 - 109544A^3B^2a^5b^3 + 76686A^4a^4b^4)c^4 - 2(1275B^4a^7b^2 - 14086A^2B^3a^6b^3 + 51336A^2B^2a^5b^4 - 77424A^3B^2a^4b^5 + 41815A^4a^3b^6)c^3 + 3(1017B^4a^6b^4 - 7872A^2B^3a^5b^5 + 22508A^2B^2a^4b^6 - 28260A^3B^2a^3b^7 + 13175A^4a^2b^8)c^2 - 2(459B^4a^5b^6 - 3186A^2B^3a^4b^7 + 8280A^2B^2a^3b^8 - 9550A^3B^2a^2b^9 + 4125A^4a^2b^{10})c) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) / (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) * \log((9604A^4a^4c^8 + 7203(4A^3B^2a^4b - 7A^4a^3b^2)c^7 - (2500B^4a^6 - 22500A^2B^3a^5b + 43524A^2B^2a^4b^2 + 4343A^3B^2a^3b^3 - 43410A^4a^2b^4)c^6 + (5625B^4a^5b^2 - 31137A^2B^3a^4b^3 + 52821A^2B^2a^3b^4 - 20190A^3B^2a^2b^5 - 12325A^4a^2b^6)c^5 - 3(657B^4a^4b^4 - 3351A^2B^3a^3b^5 + 5560A^2B^2a^2b^6 - 2775A^3B^2a^2b^7 - 375A^4b^8)c^4 + 7(27B^4a^3b^6 - 135A^2B^3a^2b^7 + 225A^2B^2a^2b^8 - 125A^3B^2a^2b^9)c^3)x - 1/2\sqrt{1/2}(27B^3a^3b^{11} - 135A^2B^2a^2b^{12} + 225A^2B^2a^2b^{13} - 125A^3b^{14} + 10976A^3a^7c^7 - 112(50A^2B^2a^8 - 463A^2B^2a^7b + 709A^3a^6b^2)c^6 - 2(2600B^3a^8b - 31256A^2B^2a^7b^2 + 96044A^2B^2a^6b^3 - 86495A^3a^5b^4)c^5 + (14408B^3a^7b^3 - 101006A^2B^2a^6b^4 + 224705A^2B^2a^5b^5 - 160932A^3a^4b^6)c^4 - 7(1507B^3a^6b^5 - 8820A^2B^2a^5b^6 + 16991A^2B^2a^4b^7 - 10797A^3a^3b^8)c^3 + (3330B^3a^5b^7 - 17889A^2B^2a^4b^8 + 31929A^2B^2a^3b^9 - 18940A^3a^2b^{10})c^2 - (486B^3a^4b^9 - 2493A^2B^2a^3b^{10} + 4260A^2B^2a^2b^{11} - 2425A^3a^2b^{12})c + (3B^2a^8b^{10} - 5A^2a^7b^{11} - 256(5B^2a^{13} - 13A^2a^{12}b)c^5 + 64(34B^2a^{12}b^2 - 73A^2a^{11}b^3)c^4 - 112(12B^2a^{11}b^4 - 23A^2a^{10}b^5)c^3 + 28(14B^2a^{10}b^6 - 25A^2a^9b^7)c^2 - (55B^2a^9b^8 - 94A^2a^8b^9)c)\sqrt{(81B^4a^4b^8 - 540A^2B^3a^3b^9 + 1350A^2B^2a^2b^{10} - 1500A^3B^2a^2b^{11} + 625A^4b^{12} + 2401A^4a^6c^6 - 98(25A^2B^2a^7 - 186A^3B^2a^6b + 246A^4a^5b^2)c^5 + (625B^4a^8 - 9
\end{aligned}$$

$$\begin{aligned}
& 300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4* \\
& a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^ \\
& 5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 \\
& - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175* \\
& A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^ \\
& 3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c \\
& + 48*a^16*b^2*c^2 - 64*a^17*c^3))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 2 \\
& 5*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^ \\
& 4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^ \\
& 2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^ \\
& 7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*sqrt((81*B^4*a^4*b^8 - \\
& 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^ \\
& 12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5* \\
& b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544 \\
& *A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3 \\
& *a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6) \\
& *c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 2 \\
& 8260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B \\
& ^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c \\
&))/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 1 \\
& 2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)))/((a^3*b^2*c - 4*a^4*c^2)*x^7 \\
& + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
\end{aligned}$$

giac [B] time = 8.15, size = 6327, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*a \\
& *b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b^2 \\
& - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a \\
& ^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 \\
& + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10 \\
& *sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(\\
& 2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2) \\
& *sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sq \\
& rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b \\
& ^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3* \\
& c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44 \\
& *a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 \\
& - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a \\
& *c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c) \\
& *c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
\end{aligned}$$

$$\begin{aligned}
& a^3c^2 - 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2* \\
& b^2c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2* \\
& b^2c^2 + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^3 - \\
& 6(b^2 - 4ac)a^2b^2c^2 + 20(b^2 - 4ac)a^2c^3)(a^3b^2 - 4a^4c)^ \\
& 2B + 2(5\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^8 - 64\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^6c \\
& - 10\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^7c - 10a^3b^8c + 286\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^4c^2 \\
& + 88\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^5c^2 + 5\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^6c^2 \\
& + 128a^4b^6c^2 - 496\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^2c^3 - 220\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^3c^3 \\
& - 44\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^4c^3 - 572a^5b^4c^3 + 224\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7c^4 \\
& + 112\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^2c^4 + 110\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^2c^4 \\
& + 992a^6b^2c^4 - 56\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6c^5 - 448a^7c^5 + 10(b^2 - 4ac)a^3b^6c \\
& - 88(b^2 - 4ac)a^4b^4c^2 + 220(b^2 - 4ac)a^5b^2c^3 - 112(b^2 - 4ac)a^6c^4)A\sqrt{a^3b^2 - 4a^4c} \\
& - 2(3\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^7 - 37\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^5c \\
& - 6\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^6c - 6a^4b^7c + 152\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^3c^2 \\
& + 50\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^4c^2 + 3\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^5c^2 \\
& + 74a^5b^5c^2 - 208\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^2c^3 - 104\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^2c^3 \\
& - 25\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^3c^3 - 304a^6b^3c^3 + 52\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^2c^4 \\
& + 416a^7b^2c^4 + 6(b^2 - 4ac)a^4b^5c - 50(b^2 - 4ac)a^5b^3c^2 + 104(b^2 - 4ac)a^6b^2c^3)B\sqrt{a^3b^2 - 4a^4c} \\
& + (10a^6b^9c^2 - 138a^7b^7c^3 + 680a^8b^5c^4 - 1376a^9b^3c^5 + 896a^10b^2c^6 - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^9 \\
& + 69\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^7c + 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^8c \\
& - 340\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^8b^5c^2 - 98\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^6c^2 \\
& - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^7c^2 + 688\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^9b^3c^3 \\
& + 288\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^8b^4c^3 + 49\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^5c^3 \\
& - 448\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^10b^2c^4 - 224\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^9b^2c^4 \\
& - 144\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^8b^3c^4 + 112\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^9b^2c^5 \\
& - 10(b^2 - 4ac)a^6b^7c^2 + 98(b^2 - 4ac)a^7b^5c^3 - 288(b^2 - 4ac)a^8b^3c^4 + 224(b^2 - 4ac)a^9b^2c^5)A \\
& - (6a^7b^8c^2 - 80a^8b^6c^3 + 352a^9b^4c^4 - 512a^10b^2c^5 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^8 \\
& + 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^8 + 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^8 \\
& + 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^8 + 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^8
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(b^2 - 4ac) * c * a^8 * b^6 * c + 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b^7 * c - 176 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^9 * b^4 * c^2 - 56 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^8 * b^5 * c^2 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^7 * b^6 * c^2 + 256 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^{10} * b^2 * c^3 + 128 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^9 * b^3 * c^3 + 28 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^8 * b^4 * c^3 - 64 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4ac) * c) * a^9 * b^2 * c^4 - 6 * (b^2 - 4ac) * a^7 * b^6 * c^2 + 56 * (b^2 - 4ac) * a^8 * b^4 * c^3 - 128 * (b^2 - 4ac) * a^9 * b^2 * c^4) * B * \arctan(2 * \text{sqrt}(1/2) * x / \text{sqrt}((a^3 * b^3 - 4a^4 * b * c + \text{sqrt}((a^3 * b^3 - 4a^4 * b * c)^2 - 4 * (a^4 * b^2 - 4a^5 * c) * (a^3 * b^2 * c - 4a^4 * c^2)))) / (a^3 * b^2 * c - 4a^4 * c^2)) / ((a^7 * b^6 - 12a^8 * b^4 * c - 2a^7 * b^5 * c + 48a^9 * b^2 * c^2 + 16a^8 * b^3 * c^2 + a^7 * b^4 * c^2 - 64a^{10} * c^3 - 32a^9 * b * c^3 - 8a^8 * b^2 * c^3 + 16a^9 * c^4) * \text{abs}(a^3 * b^2 - 4a^4 * c) * \text{abs}(c)) - 1/16 * ((10 * b^5 * c^2 - 78 * a * b^3 * c^3 + 152 * a^2 * b * c^4 - 5 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^5 + 39 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c + 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^4 * c - 76 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^2 - 38 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^2 - 5 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * b^3 * c^2 + 19 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b * c^3 - 10 * (b^2 - 4ac) * b^3 * c^2 + 38 * (b^2 - 4ac) * a * b * c^3) * (a^3 * b^2 - 4a^4 * c)^2 * A - (6 * a * b^4 * c^2 - 44 * a^2 * b^2 * c^3 + 80 * a^3 * c^4 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 + 22 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b^2 * c + 6 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 * c - 40 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * c^2 - 20 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * b * c^2 - 3 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 * c^2 + 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^2 * c^3 - 6 * (b^2 - 4ac) * a * b^2 * c^2 + 20 * (b^2 - 4ac) * a^2 * c^3) * (a^3 * b^2 - 4a^4 * c)^2 * B - 2 * (5 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^8 - 64 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^6 * c - 10 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^7 * c + 10 * a^3 * b^8 * c + 286 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^4 * c^2 + 88 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^5 * c^2 + 5 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^3 * b^6 * c^2 - 128 * a^4 * b^6 * c^2 - 496 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^6 * b^2 * c^3 - 220 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^3 * c^3 - 44 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^4 * c^3 + 572 * a^5 * b^4 * c^3 + 224 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^7 * c^4 + 112 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^6 * b * c^4 + 110 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^2 * c^4 - 992 * a^6 * b^2 * c^4 - 56 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^6 * c^5 + 448 * a^7 * c^5 - 10 * (b^2 - 4ac) * a^3 * b^6 * c + 88 * (b^2 - 4ac) * a^4 * b^4 * c^2 - 220 * (b^2 - 4ac) * a^5 * b^2 * c^3 + 112 * (b^2 - 4ac) * a^6 * c^4) * A * \text{abs}(a^3 * b^2 - 4a^4 * c) + 2 * (3 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^4 * b^7 - 37 * \text{sqrt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4ac) * c) * a^5 * b^5 * c - 6
\end{aligned}$$

```

*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c))*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a
^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^2*c^3 - 25*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4
*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c
^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b
^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c))*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c))*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c))*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c))*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c))*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c))*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b
^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)
*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^1
0*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7
*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^8*b^6
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b^7*c
- 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^9*b^4*c^2
- 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^8*b^5*c^2
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^7*b^6*c^2
+ 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^10*b^2*c
^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^9*b^3*c
^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^8*b^4*c
^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^9*b^2*c
^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2
- 4*a*c)*a^9*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - s
qrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)
))/((a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a
^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a
^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/3*(3*B*a*x^2 -
6*A*b*x^2 + A*a)/(a^3*x^3)

```

maple [B] time = 0.05, size = 1653, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2, x)$

[Out] $\frac{1}{2} \frac{1}{a^2} \frac{1}{(c*x^4+b*x^2+a)*c} \frac{1}{(4*a*c-b^2)*x^3*B*b^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^2*c-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*B*c+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A*b-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*A*b^3+5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(h(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B-5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B+2/a^3/x*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A-3/4/a^2*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^2+19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+3/4/a^2*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^2+5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3-5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3-19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b-1/3*A/a^2/x^3-1/a^2/x*B-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^3+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^4+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^4-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*B-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*c^2+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^3-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^4$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{6}*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2*b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - \frac{1}{2}*\text{integrate}((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)$

mupad [B] time = 5.70, size = 21554, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2), x)

[Out] $-\text{atan}\left(\frac{((-25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 11692*8*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})}{32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)}\right)*(917504*A*a^{19}*c^9 + x*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})}$

$$\begin{aligned}
& 4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2 \\
& *b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240 \\
& *a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))) \\
& ^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440 \\
& *a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^ \\
& 3*c^7) + 851968*B*a^{19}*b*c^8 - 320*A*a^{12}*b^{14}*c^2 + 7936*A*a^{13}*b^{12}*c^3 - \\
& 82816*A*a^{14}*b^{10}*c^4 + 468480*A*a^{15}*b^8*c^5 - 1536000*A*a^{16}*b^6*c^6 + 2 \\
& 867200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672 \\
& *B*a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B* \\
& a^{17}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) - x*(401408*A^2*a^{16}*c^{10} - 204800*B \\
& ^2*a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^1 \\
& 1*b^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A \\
& ^2*a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B \\
& ^2*a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 36556 \\
& 8*B^2*a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 1110 \\
& 4*A*B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1 \\
& 469440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9 \\
&))*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2* \\
& a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3* \\
& c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 207 \\
& 7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B \\
& ^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^ \\
& 7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2 \\
& *a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^ \\
& 10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^ \\
& 4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 \\
& - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - \\
& 6144*a^{12}*b^2*c^5)))^{(1/2)}*1i - ((-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2* \\
& b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 3576 \\
& 7*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 21504 \\
& 0*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + \\
& 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - \\
& 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B* \\
& a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b \\
& ^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b \\
& ^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})
\end{aligned}$$

$$\begin{aligned}
& /((32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(917504*A*a^{19}*c^9 - x*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) + 851968*B*a^{19}*b*c^8 - 320*A*a^{12}*b^{14}*c^2 + 7936*A*a^{13}*b^{12}*c^3 - 82816*A*a^{14}*b^{10}*c^4 + 468480*A*a^{15}*b^8*c^5 - 1536000*A*a^{16}*b^6*c^6 + 2867200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672*B*a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B*a^{17}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) + x*(401408*A^2*a^{16}*c^{10} - 204800*B^2*a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^{11}*b^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A^2*a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B^2*a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 365568*B^2*a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 11104*A*B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1469440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9))*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*1i)/(((-(25*A^2*b^{15} +
\end{aligned}$$

$$\begin{aligned}
& 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 636 \\
& 6*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 21974 \\
& 4*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - \\
& 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25 \\
& *B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}* \\
& c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^ \\
& 2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B* \\
& a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B* \\
& a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a \\
& ^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2* \\
& (-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 2 \\
& 40*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5) \\
&))^{(1/2)}*(917504*A*a^{19}*c^9 + x*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^ \\
& 6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767* \\
& A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040* \\
& A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30 \\
& 240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 2 \\
& 13*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^ \\
& 5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4 \\
& *c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 3 \\
& 0*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3 \\
& *c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})/(\\
& 32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10} \\
& b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)}*(1048576*a^{21}*b*c^ \\
& 8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^ \\
& 18*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) + 851968*B*a^{19}*b \\
& c^8 - 320*A*a^{12}*b^{14}*c^2 + 7936*A*a^{13}*b^{12}*c^3 - 82816*A*a^{14}*b^{10}*c^4 + \\
& 468480*A*a^{15}*b^8*c^5 - 1536000*A*a^{16}*b^6*c^6 + 2867200*A*a^{17}*b^4*c^7 - 2 \\
& 719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672*B*a^{14}*b^{11}*c^3 + 47360* \\
& B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B*a^{17}*b^5*c^6 - 1261568*B* \\
& a^{18}*b^3*c^7) - x*(401408*A^2*a^{16}*c^{10} - 204800*B^2*a^{17}*c^9 - 400*A^2*a^9 \\
& *b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^{11}*b^{10}*c^5 + 488096*A^2*a \\
& ^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A^2*a^{14}*b^4*c^8 - 1871872 \\
& *A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B^2*a^{12}*b^{10}*c^4 - 30112* \\
& B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 365568*B^2*a^{15}*b^4*c^7 + 4587 \\
& 52*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 11104*A*B*a^{11}*b^{11}*c^4 + 105 \\
& 824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1469440*A*B*a^{14}*b^5*c^7 - \\
& 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9))*(-(25*A^2*b^{15} + 9*B^2 \\
& *a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2* \\
& a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*
\end{aligned}$$

$$\begin{aligned}
& a^5 b^5 c^5 + 215040 A^2 a^6 b^3 c^6 + 49 A^2 a^3 c^3 (-4ac - b^2)^9)^{(1/2)} - 9 B^2 a^2 b^4 (-4ac - b^2)^9)^{(1/2)} + 2077 B^2 a^4 b^9 c^2 - 10656 \\
& * B^2 a^5 b^7 c^3 + 30240 B^2 a^6 b^5 c^4 - 44800 B^2 a^7 b^3 c^5 - 25 B^2 a^4 c^2 (-4ac - b^2)^9)^{(1/2)} + 35840 A B a^8 c^7 - 615 A^2 a b^{13} c - 80 \\
& 640 A^2 a^7 b c^7 - 213 B^2 a^3 b^{11} c + 26880 B^2 a^8 b c^6 - 246 A^2 a^2 b^2 c^2 (-4ac - b^2)^9)^{(1/2)} - 7278 A B a^3 b^{10} c^2 + 39132 A B a^4 b^8 c^3 \\
& - 119616 A B a^5 b^6 c^4 + 201600 A B a^6 b^4 c^5 - 161280 A B a^7 b^2 c^6 + 165 A^2 a b^4 c (-4ac - b^2)^9)^{(1/2)} + 51 B^2 a^3 b^2 c (-4ac - b^2)^9)^{(1/2)} \\
& + 30 A B a b^5 (-4ac - b^2)^9)^{(1/2)} + 724 A B a^2 b^{12} c - 184 A B a^2 b^3 c (-4ac - b^2)^9)^{(1/2)} + 186 A B a^3 b c^2 (-4ac - b^2)^9)^{(1/2)} \\
& / (32 (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c + 240 a^9 b^8 c^2 - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5)))^{(1/2)} \\
& + ((-25 A^2 b^{15} + 9 B^2 a^2 b^{13} - 25 A^2 b^6 (-4ac - b^2)^9)^{(1/2)} - 30 A B a b^{14} + 6366 A^2 a^2 b^{11} c^2 - 35767 A^2 a^3 b^9 c^3 + 116928 A^2 a^4 b^7 c^4 \\
& - 219744 A^2 a^5 b^5 c^5 + 215040 A^2 a^6 b^3 c^6 + 49 A^2 a^3 c^3 (-4ac - b^2)^9)^{(1/2)} - 9 B^2 a^2 b^4 (-4ac - b^2)^9)^{(1/2)} + 2077 B^2 a^4 b^9 c^2 \\
& - 10656 B^2 a^5 b^7 c^3 + 30240 B^2 a^6 b^5 c^4 - 44800 B^2 a^7 b^3 c^5 - 25 B^2 a^4 c^2 (-4ac - b^2)^9)^{(1/2)} + 35840 A B a^8 c^7 - 615 A^2 a b^{13} c - 80640 A^2 a^7 b c^7 \\
& - 213 B^2 a^3 b^{11} c + 26880 B^2 a^8 b c^6 - 246 A^2 a^2 b^2 c^2 (-4ac - b^2)^9)^{(1/2)} - 7278 A B a^3 b^{10} c^2 + 39132 A B a^4 b^8 c^3 - 119616 A B a^5 b^6 c^4 \\
& + 201600 A B a^6 b^4 c^5 - 161280 A B a^7 b^2 c^6 + 165 A^2 a b^4 c (-4ac - b^2)^9)^{(1/2)} + 51 B^2 a^3 b^2 c (-4ac - b^2)^9)^{(1/2)} + 30 A B a b^5 (-4ac - b^2)^9)^{(1/2)} \\
& + 724 A B a^2 b^{12} c - 184 A B a^2 b^3 c (-4ac - b^2)^9)^{(1/2)} + 186 A B a^3 b c^2 (-4ac - b^2)^9)^{(1/2)} / (32 (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c + 240 a^9 b^8 c^2 \\
& - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5)))^{(1/2)} * (917504 A a^{19} c^9 - x (-25 A^2 b^{15} + 9 B^2 a^2 b^{13} - 25 A^2 b^6 (-4ac - b^2)^9)^{(1/2)} \\
& - 30 A B a b^{14} + 6366 A^2 a^2 b^{11} c^2 - 35767 A^2 a^3 b^9 c^3 + 116928 A^2 a^4 b^7 c^4 - 219744 A^2 a^5 b^5 c^5 + 215040 A^2 a^6 b^3 c^6 + 49 A^2 a^3 c^3 (-4ac - b^2)^9)^{(1/2)} \\
& - 9 B^2 a^2 b^4 (-4ac - b^2)^9)^{(1/2)} + 2077 B^2 a^4 b^9 c^2 - 10656 B^2 a^5 b^7 c^3 + 30240 B^2 a^6 b^5 c^4 - 44800 B^2 a^7 b^3 c^5 - 25 B^2 a^4 c^2 (-4ac - b^2)^9)^{(1/2)} \\
& + 35840 A B a^8 c^7 - 615 A^2 a b^{13} c - 80640 A^2 a^7 b c^7 - 213 B^2 a^3 b^{11} c + 26880 B^2 a^8 b c^6 - 246 A^2 a^2 b^2 c^2 (-4ac - b^2)^9)^{(1/2)} - 7278 A B a^3 b^{10} c^2 \\
& + 39132 A B a^4 b^8 c^3 - 119616 A B a^5 b^6 c^4 + 201600 A B a^6 b^4 c^5 - 161280 A B a^7 b^2 c^6 + 165 A^2 a b^4 c (-4ac - b^2)^9)^{(1/2)} + 51 B^2 a^3 b^2 c (-4ac - b^2)^9)^{(1/2)} \\
& + 30 A B a b^5 (-4ac - b^2)^9)^{(1/2)} + 724 A B a^2 b^{12} c - 184 A B a^2 b^3 c (-4ac - b^2)^9)^{(1/2)} + 186 A B a^3 b c^2 (-4ac - b^2)^9)^{(1/2)} / (32 (a^7 b^{12} + 4096 a^{13} c^6 - 24 a^8 b^{10} c + 240 a^9 b^8 c^2 \\
& - 1280 a^{10} b^6 c^3 + 3840 a^{11} b^4 c^4 - 6144 a^{12} b^2 c^5)))^{(1/2)} * (1048576 a^{21} b c^8 + 256 a^{15} b^{13} c^2 - 6144 a^{16} b^{11} c^3 + 61440 a^{17} b^9 c^4 - 327680 a^{18} b^7 c^5 \\
& + 983040 a^{19} b^5 c^6 - 1572864 a^{20} b^3 c^7) + 851968 B a^{19} b c^8 - 320 A a^{12} b^{14} c^2 + 7936 A a^{13} b^{12} c^3 - 82816 A a^{14} b^{10} c^4 + 468480 A a^{15} b^8 c^5 - 1536000 A a^{16} b^6 c^6 + 2867
\end{aligned}$$

$$\begin{aligned}
& 200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672*B* \\
& a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B*a^{17}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) + x*(401408*A^2*a^{16}*c^{10} - 204800*B^2* \\
& a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^{11}*b^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A^2* \\
& a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B^2* \\
& a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 365568*B^2* \\
& a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 11104*A \\
& *B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1469 \\
& 440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9))* \\
& (- (25*A^2*b^{15} + 9*B^2*a^2*b^{13} - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30* \\
& A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4 \\
& *b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B \\
& ^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2* \\
& a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - \\
& 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8* \\
& b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}* \\
& c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - \\
& 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51 \\
& *B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 724*A*B*a^2*b^{12}*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 18 \\
& 6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 2 \\
& 4*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 61 \\
& 44*a^{12}*b^2*c^5)))^{(1/2)} + 128000*B^3*a^{15}*c^9 - 1800*A^3*a^9*b^9*c^6 + 290 \\
& 80*A^3*a^{10}*b^7*c^7 - 176032*A^3*a^{11}*b^5*c^8 + 473216*A^3*a^{12}*b^3*c^9 + 5 \\
& 04*B^3*a^{11}*b^8*c^5 - 8112*B^3*a^{12}*b^6*c^6 + 48704*B^3*a^{13}*b^4*c^7 - 1292 \\
& 80*B^3*a^{14}*b^2*c^8 + 250880*A^2*B*a^{14}*c^{10} - 476672*A^3*a^{13}*b*c^{10} - 442 \\
& 880*A*B^2*a^{14}*b*c^9 - 1680*A*B^2*a^{10}*b^9*c^5 + 27176*A*B^2*a^{11}*b^7*c^6 - \\
& 164448*A*B^2*a^{12}*b^5*c^7 + 441216*A*B^2*a^{13}*b^3*c^8 + 1400*A^2*B*a^9*b^1 \\
& 0*c^5 - 21680*A^2*B*a^{10}*b^8*c^6 + 121648*A^2*B*a^{11}*b^6*c^7 - 275264*A^2*B \\
& *a^{12}*b^4*c^8 + 121088*A^2*B*a^{13}*b^2*c^9))*(- (25*A^2*b^{15} + 9*B^2*a^2*b^{13} \\
& - 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}* \\
& c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 \\
& + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 9*B \\
& ^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5* \\
& b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7* \\
& b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(- \\
& -(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 1 \\
& 19616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 1 \\
& 65*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^ \\
& 9)^{(1/2)} + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c - 184 \\
& *A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2) \\
& ^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2
\end{aligned}$$

$$\begin{aligned}
& - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)}*2i - a \\
& \tan((((-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A \\
& ^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a \\
& ^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + \\
& 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4480 \\
& 0*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8 \\
& *c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880* \\
& B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3 \\
& *b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6 \\
& *b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2 \\
&)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c \\
& ^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 \\
& - 6144*a^{12}*b^2*c^5))^{(1/2)}*(917504*A*a^{19}*c^9 + x*(-(25*A^2*b^{15} + 9*B^ \\
& 2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2 \\
& *a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2 \\
& *a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 1065 \\
& 6*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2* \\
& a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 8 \\
& 0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2 \\
& *b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b \\
& ^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b \\
& ^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a \\
& *c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^ \\
& ^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^ \\
& 9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1 \\
& /2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^ \\
& ^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c \\
& ^7) + 851968*B*a^{19}*b*c^8 - 320*A*a^{12}*b^{14}*c^2 + 7936*A*a^{13}*b^{12}*c^3 - 82 \\
& 816*A*a^{14}*b^{10}*c^4 + 468480*A*a^{15}*b^8*c^5 - 1536000*A*a^{16}*b^6*c^6 + 2867 \\
& 200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672*B* \\
& a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B*a^{1 \\
& 7}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) - x*(401408*A^2*a^{16}*c^{10} - 204800*B^2* \\
& a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^{11}*b \\
& ^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A^2* \\
& a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B^2* \\
& a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 365568*B \\
& ^2*a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 11104*A \\
& *B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1469 \\
& 440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9))* \\
& (-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*
\end{aligned}$$

$$\begin{aligned}
& A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4* \\
& *b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(- (4*a*c - b^2)^9)^{(1/2)} + 2077*B \\
& ^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2* \\
& a^7*b^3*c^5 + 25*B^2*a^4*c^2*(- (4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - \\
& 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^ \\
& 8*b*c^6 + 246*A^2*a^2*b^2*c^2*(- (4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}* \\
& c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c \\
& ^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(- (4*a*c - b^2)^9)^{(1/2)} - 51 \\
& *B^2*a^3*b^2*c*(- (4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(- (4*a*c - b^2)^9)^{(\\
& 1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(- (4*a*c - b^2)^9)^{(1/2)} - 18 \\
& 6*A*B*a^3*b*c^2*(- (4*a*c - b^2)^9)^{(1/2))}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 2 \\
& 4*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 61 \\
& 44*a^{12}*b^2*c^5)))^{(1/2)}*i - ((-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6 \\
& *(- (4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A \\
& ^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A \\
& ^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(- (4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(- \\
& (4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 302 \\
& 40*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(- (4*a*c - b^2) \\
& ^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 21 \\
& 3*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(- (4*a*c - b^2 \\
&)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5 \\
& *b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4* \\
& c*(- (4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(- (4*a*c - b^2)^9)^{(1/2)} - 30 \\
& *A*B*a*b^5*(- (4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3* \\
& c*(- (4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(- (4*a*c - b^2)^9)^{(1/2))}/(3 \\
& 2*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b \\
& ^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(917504*A*a^{19}*c^9 \\
& - x*(- (25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(- (4*a*c - b^2)^9)^{(1/2)} - \\
& 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2 \\
& *a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3 \\
& *c^3*(- (4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(- (4*a*c - b^2)^9)^{(1/2)} + 20 \\
& 77*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800* \\
& B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(- (4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c \\
& ^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^ \\
& 2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(- (4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b \\
& ^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b \\
& ^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(- (4*a*c - b^2)^9)^{(1/2)} \\
& - 51*B^2*a^3*b^2*c*(- (4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(- (4*a*c - b^2)^ \\
& 9)^{(1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(- (4*a*c - b^2)^9)^{(1/2)} \\
& - 186*A*B*a^3*b*c^2*(- (4*a*c - b^2)^9)^{(1/2))}/(32*(a^7*b^{12} + 4096*a^{13}*c^6 \\
& - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 \\
& - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144 \\
& *a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5 \\
& *c^6 - 1572864*a^{20}*b^3*c^7) + 851968*B*a^{19}*b*c^8 - 320*A*a^{12}*b^{14}*c^2 +
\end{aligned}$$

$$\begin{aligned}
& 7936*A*a^{13}*b^{12}*c^3 - 82816*A*a^{14}*b^{10}*c^4 + 468480*A*a^{15}*b^8*c^5 - 1536 \\
& 000*A*a^{16}*b^6*c^6 + 2867200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192* \\
& B*a^{13}*b^{13}*c^2 - 4672*B*a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}* \\
& b^7*c^5 + 778240*B*a^{17}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) + x*(401408*A^{2*} \\
& a^{16}*c^{10} - 204800*B^2*a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}* \\
& c^4 - 92816*A^2*a^{11}*b^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}* \\
& b^6*c^7 + 2401280*A^2*a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}* \\
& b^{12}*c^3 + 3264*B^2*a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}* \\
& b^6*c^6 - 365568*B^2*a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}* \\
& b^{13}*c^3 - 11104*A*B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}* \\
& b^7*c^6 + 1469440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}* \\
& b*c^9))*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{1/2} - 30*A*B*a*b^{14} \\
& + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 \\
& + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} + 9*B^2*a^2*b^4*(-(4*a*c \\
& - b^2)^9)^{1/2} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 \\
& - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{1/2} + 35840*A*B*a^8*c^7 \\
& - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 \\
& + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 \\
& - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c* \\
& (- (4*a*c - b^2)^9)^{1/2} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{1/2} - 30*A*B*a*b^5* \\
& (- (4*a*c - b^2)^9)^{1/2} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{1/2} \\
& - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{1/2})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c \\
& + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{1/2} * i) / (((- (25*A^2*b^{15} \\
& + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{1/2} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}* \\
& c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 \\
& - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{1/2} + 2077*B^2*a^4*b^9*c^2 \\
& - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c \\
& - b^2)^9)^{1/2} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c \\
& + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 7278*A*B*a^3*b^{10}*c^2 \\
& + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 \\
& - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{1/2} - 30*A*B*a*b^5* \\
& (- (4*a*c - b^2)^9)^{1/2} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{1/2} - 186*A*B*a^3*b*c^2*(-(4 \\
& a*c - b^2)^9)^{1/2})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}* \\
& b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{1/2} * (917504*A*a^{19}*c^9 + x*(-(25*A^2*b^{15} \\
& + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{1/2} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 \\
& - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 \\
& - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{1/2} + 2077*B^2*a^4*b^9*c^2 \\
& - 10656*B^2*a^5*b^7*c^3 + 30240
\end{aligned}$$

$$\begin{aligned}
& *B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) + 851968*B*a^{19}*b*c^8 - 320*A*a^{12}*b^{14}*c^2 + 7936*A*a^{13}*b^{12}*c^3 - 82816*A*a^{14}*b^{10}*c^4 + 468480*A*a^{15}*b^8*c^5 - 1536000*A*a^{16}*b^6*c^6 + 2867200*A*a^{17}*b^4*c^7 - 2719744*A*a^{18}*b^2*c^8 + 192*B*a^{13}*b^{13}*c^2 - 4672*B*a^{14}*b^{11}*c^3 + 47360*B*a^{15}*b^9*c^4 - 256000*B*a^{16}*b^7*c^5 + 778240*B*a^{17}*b^5*c^6 - 1261568*B*a^{18}*b^3*c^7) - x*(401408*A^2*a^{16}*c^{10} - 204800*B^2*a^{17}*c^9 - 400*A^2*a^9*b^{14}*c^3 + 9440*A^2*a^{10}*b^{12}*c^4 - 92816*A^2*a^{11}*b^{10}*c^5 + 488096*A^2*a^{12}*b^8*c^6 - 1458688*A^2*a^{13}*b^6*c^7 + 2401280*A^2*a^{14}*b^4*c^8 - 1871872*A^2*a^{15}*b^2*c^9 - 144*B^2*a^{11}*b^{12}*c^3 + 3264*B^2*a^{12}*b^{10}*c^4 - 30112*B^2*a^{13}*b^8*c^5 + 143360*B^2*a^{14}*b^6*c^6 - 365568*B^2*a^{15}*b^4*c^7 + 458752*B^2*a^{16}*b^2*c^8 + 480*A*B*a^{10}*b^{13}*c^3 - 11104*A*B*a^{11}*b^{11}*c^4 + 105824*A*B*a^{12}*b^9*c^5 - 530432*A*B*a^{13}*b^7*c^6 + 1469440*A*B*a^{14}*b^5*c^7 - 2121728*A*B*a^{15}*b^3*c^8 + 1236992*A*B*a^{16}*b*c^9))*(-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^{10}*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^{12}*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)} + ((-(25*A^2*b^{15} + 9*B^2*a^2*b^{13} + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^{14} + 6366*A^2*a^2*b^{11}*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^{13}*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^{11}*c + 26880*B^2
\end{aligned}$$

$$\begin{aligned}
& *a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)}*(917504*A*a^19*c^9 - x*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)}*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 - 82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) + x*(401408*A^2*a^16*c^10 - 204800*B^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^11*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 365568*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 11104*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5
\end{aligned}$$

$$\begin{aligned}
& - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)} + 128000*B^3*a^15*c^9 - 1800*A^3*a^9*b^9*c^6 + 29080*A^3*a^10*b^7*c^7 - 176032*A^3*a^11*b^5*c^8 + 473216*A^3*a^12*b^3*c^9 + 504*B^3*a^11*b^8*c^5 - 8112*B^3*a^12*b^6*c^6 + 48704*B^3*a^13*b^4*c^7 - 129280*B^3*a^14*b^2*c^8 + 250880*A^2*B*a^14*c^10 - 476672*A^3*a^13*b*c^10 - 442880*A*B^2*a^14*b*c^9 - 1680*A*B^2*a^10*b^9*c^5 + 27176*A*B^2*a^11*b^7*c^6 - 164448*A*B^2*a^12*b^5*c^7 + 441216*A*B^2*a^13*b^3*c^8 + 1400*A^2*B*a^9*b^10*c^5 - 21680*A^2*B*a^10*b^8*c^6 + 121648*A^2*B*a^11*b^6*c^7 - 275264*A^2*B*a^12*b^4*c^8 + 121088*A^2*B*a^13*b^2*c^9))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 + 25*A^2*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 - 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} + 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 + 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 + 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 - 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} - 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)} - 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^{(1/2)} + 724*A*B*a^2*b^12*c + 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^{(1/2)} - 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5))^{(1/2)}*2i - (A/(3*a) - (x^2*(5*A*b - 3*B*a))/(3*a^2) + (x^4*(15*A*b^4 + 14*A*a^2*c^2 - 9*B*a*b^3 - 62*A*a*b^2*c + 33*B*a^2*b*c))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*A*b^3 - 3*B*a*b^2 + 10*B*a^2*c - 19*A*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*x^3 + b*x^5 + c*x^7)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

$$3.102 \quad \int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=365

$$\frac{x^4 \left(x^2 \left(20a^2Bc^2 + 10aAbc^2 - 20ab^2Bc - Ab^3c + 3b^4B \right) + a \left(16aAc^2 - 18abBc - Ab^2c + 3b^3B \right) \right) x^2 \left(30a^2Bc^2 - 20a^2Bc^2 \right)}{4c^2 \left(b^2 - 4ac \right)^2 \left(a + bx^2 + cx^4 \right)} + \dots$$

Rubi [A] time = 1.45, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$\frac{x^4 \left(x^2 \left(20a^2Bc^2 + 10aAbc^2 - 20ab^2Bc - Ab^3c + 3b^4B \right) + a \left(16aAc^2 - 18abBc - Ab^2c + 3b^3B \right) \right) x^2 \left(30a^2Bc^2 - 20a^2Bc^2 \right)}{4c^2 \left(b^2 - 4ac \right)^2 \left(a + bx^2 + cx^4 \right)} + \dots$$

Antiderivative was successfully verified.

[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c - 18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 773

Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x]/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 818

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4)) + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^5 (A + Bx)}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\
&= -\frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{\text{Subst} \left(\int \frac{x^3 (4a(bB - 2Ac) + (3b^2B - Abc - 10aBc)x)}{(a + bx + cx^2)^2} dx, x, x^2 \right)}{4c (b^2 - 4ac)} \\
&= -\frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^4 (a (3b^3B - Ab^2c - 18abBc + 16aAc^2))}{4c^2 (b^2 - 4ac)} \\
&= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2) x^2}{2c^3 (b^2 - 4ac)^2} - \frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)} \\
&= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2) x^2}{2c^3 (b^2 - 4ac)^2} - \frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)} \\
&= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2) x^2}{2c^3 (b^2 - 4ac)^2} - \frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)} \\
&= \frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2) x^2}{2c^3 (b^2 - 4ac)^2} - \frac{x^8 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 435, normalized size = 1.19

$$\frac{x^2 \sqrt{2(A+Bx^2-3B)} \sqrt{x(-b(4A+9Bx^2)+5Ac^2+9B^2)} + \sqrt{b(A+6Bx^2)-5Ac^2+9B^2} \sqrt{c(Ac-3B)} - \frac{2(6b^2Bc^2+30a^2Abc^2-90a^2B^2c^2-30a^2B^2c^2-30a^2B^2c^2-30a^2B^2c^2) \sqrt{\frac{b+2cx^2}{4ac-3B^2}}}{(4ac-3B^2)^2} + \frac{4c^2 \sqrt{b(A+6Bx^2)-5Ac^2+9B^2} \sqrt{2(13A+38Bx^2)+2c^2(25Ac^2-39B)}}{(b^2-4ac)^2 \sqrt{b+2cx^2}} + c(Ac-3B) \log(a+bx^2+cx^4) + 2Bc^2x^2}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6*1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B + 25*A*c*x^2))/(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4) + (b^5*(-(b*B) + A*c)*x^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/

$((b^2 - 4ac)(a + bx^2 + cx^4)^2) - (2c(-3b^6B + Ab^5c + 30ab^4Bc - 10aAb^3c^2 - 90a^2b^2Bc^2 + 30a^2Ab^3c^3 + 60a^3Bc^3) \operatorname{ArcTan}[(b + 2cx^2)/\sqrt{-b^2 + 4ac}]) / (-b^2 + 4ac)^{5/2} + c(-3bB + Ac) \operatorname{Log}[a + bx^2 + cx^4] / (4c^5)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 1.41, size = 3196, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] $[1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^{10} - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4ac)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4ac))/(c*x^4 + b*x^2 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3$

$$\begin{aligned}
& - (36B^2a^2b^6 + A^2b^7)c^2)x^6 + (3B^2b^9 + 128A^2a^4c^5 - 32(12B^2a^4b \\
& + A^2a^3b^2)c^4 + 24(4B^2a^3b^3 - A^2a^2b^4)c^3 + 2(36B^2a^2b^5 + 5A^2a^2b^6)c^2 - (30B^2a^2b^7 + A^2b^8)c \\
&)x^4 - 48(4B^2a^5b + A^2a^4b^2)c^3 + 12(12B^2a^4b^3 + A^2a^3b^4)c^2 + 2(3B^2a^2b^8 + 64A^2a^4b^2c^4 - 48(\\
& 4B^2a^4b^2 + A^2a^3b^3)c^3 + 12(12B^2a^3b^4 + A^2a^2b^5)c^2 - (36B^2a^2b^6 + A^2a^2b^7)c)x^2 - (36B^2a^3b^5 + A^2a^2b^6)c \\
&)\log(cx^4 + bx^2 + a)/(a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7 + (b^6c^6 - 12a^2b^4c^7 + 48a^2b^2c^8 - 64a^3c^9)x^8 \\
& + 2(b^7c^5 - 12a^2b^5c^6 + 48a^2b^3c^7 - 64a^3b^2c^8)x^6 + (b^8c^4 - 10a^2b^6c^5 + 24a^2b^4c^6 + 32a^3b^2c^7 - 128a^4c^8)x^4 \\
& + 2(a^2b^7c^4 - 12a^2b^5c^5 + 48a^3b^3c^6 - 64a^4b^2c^7)x^2), 1/4(2(B^2b^6c^3 - 12B^2a^2b^4c^4 + 48B^2a^2b^2c^5 - 64B^2a^3c^6)x^10 \\
& - 5B^2a^2b^7 - 96A^2a^5c^4 + 4(B^2b^7c^2 - 12B^2a^2b^5c^3 + 48B^2a^2b^3c^4 - 64B^2a^3b^2c^5)x^8 - 2(2B^2b^8c + 100(2B^2a^4 + A^2a^3b)c^5 \\
& - (254B^2a^3b^2 + 85A^2a^2b^3)c^4 + (123B^2a^2b^4 + 23A^2a^2b^5)c^3 - 2(13B^2a^2b^6 + A^2b^7)c^2)x^6 - (5B^2b^9 + 128A^2a^4c^5 \\
& + 4(22B^2a^4b + 3A^2a^3b^2)c^4 - (314B^2a^3b^3 + 87A^2a^2b^4)c^3 + (225B^2a^2b^5 + 31A^2a^2b^6)c^2 - (58B^2a^2b^7 + 3A^2b^8)c \\
&)x^4 + 4(58B^2a^5b + 27A^2a^4b^2)c^3 - (202B^2a^4b^3 + 33A^2a^3b^4)c^2 - 2(5B^2a^2b^8 + 4(30B^2a^5 + 31A^2a^4b)c^4 \\
& - (346B^2a^4b^2 + 119A^2a^3b^3)c^3 + (235B^2a^3b^4 + 34A^2a^2b^5)c^2 - (59B^2a^2b^6 + 3A^2a^2b^7)c)x^2 - 2(3B^2a^2b^6 + (3B^2b^6c^2 - 30(2B^2a^3 + A^2a^2b)c^5 \\
& + 10(9B^2a^2b^2 + A^2a^2b^3)c^4 - (30B^2a^2b^4 + A^2b^5)c^3)x^8 + 2(3B^2b^7c - 30(2B^2a^3b + A^2a^2b^2)c^4 + 10(9B^2a^2b^3 + A^2a^2b^4)c^3 \\
& - (30B^2a^2b^5 + A^2b^6)c^2)x^6 + (3B^2b^8 - 60(2B^2a^4 + A^2a^3b)c^4 + 10(12B^2a^3b^2 - A^2a^2b^3)c^3 + 2(15B^2a^2b^4 + 4A^2a^2b^5)c^2 \\
& - (24B^2a^2b^6 + A^2b^7)c)x^4 - 30(2B^2a^5 + A^2a^4b)c^3 + 10(9B^2a^4b^2 + A^2a^3b^3)c^2 + 2(3B^2a^2b^7 - 30(2B^2a^4b + A^2a^3b^2)c^3 \\
& + 10(9B^2a^3b^3 + A^2a^2b^4)c^2 - (30B^2a^2b^5 + A^2a^2b^6)c)x^2 - (30B^2a^3b^4 + A^2a^2b^5)c)\sqrt{-b^2 + 4ac}\arctan(-(2cx^2 + b)\sqrt{-b^2 + 4ac})/(b^2 - 4ac) \\
&) + (56B^2a^3b^5 + 3A^2a^2b^6)c - (3B^2a^2b^7 + 64A^2a^5c^4 + (3B^2b^7c^2 + 64A^2a^3c^6 - 48(4B^2a^3b + A^2a^2b^2)c^5 + 12(12B^2a^2b^3 + A^2a^2b^4)c^4 \\
& - (36B^2a^2b^5 + A^2b^6)c^3)x^8 + 2(3B^2b^8c + 64A^2a^3b^2c^5 - 48(4B^2a^3b^2 + A^2a^2b^3)c^4 + 12(12B^2a^2b^4 + A^2a^2b^5)c^3 \\
& - (36B^2a^2b^6 + A^2b^7)c^2)x^6 + (3B^2b^9 + 128A^2a^4c^5 - 32(12B^2a^4b + A^2a^3b^2)c^4 + 24(4B^2a^3b^3 - A^2a^2b^4)c^3 + 2(36B^2a^2b^5 + 5A^2a^2b^6)c^2 \\
& - (30B^2a^2b^7 + A^2b^8)c)x^4 - 48(4B^2a^5b + A^2a^4b^2)c^3 + 12(12B^2a^4b^3 + A^2a^3b^4)c^2 + 2(3B^2a^2b^8 + 64A^2a^4b^2c^4 - 48(\\
& 4B^2a^4b^2 + A^2a^3b^3)c^3 + 12(12B^2a^3b^4 + A^2a^2b^5)c^2 - (36B^2a^2b^6 + A^2a^2b^7)c)x^2 - (36B^2a^3b^5 + A^2a^2b^6)c \\
&)\log(cx^4 + bx^2 + a)/(a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7 + (b^6c^6 - 12a^2b^4c^7 + 48a^2b^2c^8 - 64a^3c^9)x^8 \\
& + 2(b^7c^5 - 12a^2b^5c^6 + 48a^2b^3c^7 - 64a^3b^2c^8)x^6 + (b^8c^4 - 10a^2b^6c^5 + 24a^2b^4c^6 + 32a^3b^2c^7 - 128a^4c^8)x^4 \\
& + 2(a^2b^7c^4 - 12a^2b^5c^5 + 48a^3b^3c^6 - 64a^4b^2c^7)x^2)]
\end{aligned}$$

giac [A] time = 5.88, size = 598, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

maple [B] time = 0.03, size = 2054, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
[Out] 1/2*B*x^2/c^3+6/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a*b^3-1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^5*A-2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a*
b^2-12/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a^2*b+1/c^2/(c*x^
4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^5-5/4/c^4/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^7+7/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2*B-21/4/c^2/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+
b^4)*A*b^2-29/2/c^2/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b-5/
4/c^4/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^5+25/2/(c*x^4+b*
x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a^2*b+3/2/c^4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6*B-30/c/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*a^3*B-3/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^
6+3/4/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^6+3/4/c^3/(c
*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4+9/c^3/(c*x^4+b*x^2+a)^
2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3+3/2/c^3/(c*x^4+b*x^2+a)^2*a/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^2*A*b^5-11/c^2/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b
```


$$\begin{aligned} & \frac{2c+b^4}{c} x^2 A b^3 + \frac{11}{4} \frac{1}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 \\ & A a^2 b^2 - \frac{19}{4} \frac{1}{c^2} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 A a b^4 - \frac{21}{2} \frac{1}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 B a^3 b - \frac{41}{4} \frac{1}{c^2} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 B a^2 b^3 + \frac{31}{2} \frac{1}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^2 A b - \frac{71}{2} \frac{1}{c^2} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^2 B b^2 - \frac{5}{2} \frac{1}{c^4} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^2 B b^6 - \frac{15}{2} \frac{1}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^6 A a b^3 + \frac{12}{c^2} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^6 B a b^4 - \frac{15}{c} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \frac{1}{(4 a c - b^2)^{1/2}} \arctan\left(\frac{2 c x^2 + b}{(4 a c - b^2)^{1/2}}\right) A a^2 b + \frac{5}{c^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \frac{1}{(4 a c - b^2)^{1/2}} \arctan\left(\frac{2 c x^2 + b}{(4 a c - b^2)^{1/2}}\right) A a b^3 + \frac{45}{c^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \frac{1}{(4 a c - b^2)^{1/2}} \arctan\left(\frac{2 c x^2 + b}{(4 a c - b^2)^{1/2}}\right) B a^2 b^2 - \frac{15}{c^3} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \frac{1}{(4 a c - b^2)^{1/2}} \arctan\left(\frac{2 c x^2 + b}{(4 a c - b^2)^{1/2}}\right) B a b^4 + \frac{19}{c^3} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^2 B b^4 + \frac{17}{2} \frac{1}{c^3} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 B a b^5 - \frac{51}{2} \frac{1}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^6 B a^2 b^2 + \frac{9}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^6 B a^3 + \frac{8}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} x^4 A a^3 + \frac{6}{c} \frac{1}{(c x^4 + b x^2 + a)^2} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} A + \frac{1}{4} \frac{1}{c^3} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \ln(c x^4 + b x^2 + a) A b^4 + \frac{4}{c} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \ln(c x^4 + b x^2 + a) A a^2 - \frac{3}{4} \frac{1}{c^4} \frac{1}{(16 a^2 c^2 - 8 a b^2 c + b^4)} \ln(c x^4 + b x^2 + a) B b^5 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(B*x²+A)/(c*x⁴+b*x²+a)³,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b²>0)', see `assume?` for more details)Is 4*a*c-b² positive or negative?

mupad [B] time = 4.66, size = 4501, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x¹¹*(A + B*x²))/(a + b*x² + c*x⁴)³,x)

[Out]
$$\begin{aligned} & \left(\frac{x^6 (18 B a^3 c^3 - 3 B b^6 + 2 A b^5 c + 24 B a b^4 c - 15 A a b^3 c^2 + 25 A a^2 b c^3 - 51 B a^2 b^2 c^2)}{(2 (b^4 + 16 a^2 c^2 - 8 a b^2 c))} + \right. \\ & \left. \frac{a (24 A a^3 c^3 - 5 B a b^5 + 3 A a b^4 c + 36 B a^2 b^3 c - 58 B a^3 b c^2 - 21 A a^2 b^2 c^2)}{(4 c (b^4 + 16 a^2 c^2 - 8 a b^2 c))} + \right. \\ & \left. \frac{x^2 (14 B a^4 c^3 - 5 B a b^6 + 3 A a b^5 c + 31 A a^3 b c^3 + 38 B a^2 b^4 c - 22 A a^2} \right. \end{aligned}$$

$$\begin{aligned}
& *b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(\\
& 5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 + 42*B*a \\
& ^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8 \\
& *a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^6 + 2* \\
& a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (\log(((a*(A*c - 3*B*b))^2)/c^6 - (((8*a*(A* \\
& c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B \\
& *b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2* \\
& b^2*c^2))^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a^3*c^3 - 9*B* \\
& b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - 186*B*a^ \\
& 2*b^2*c^2))/(c^2*(4*a*c - b^2)^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B \\
& *b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2* \\
& b^2*c^2))^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*B*b)*(30* \\
& B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*A*a^2*b*c \\
& ^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c - 3*B*b))^2)/c^6 + (\\
& ((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + \\
& 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2))^2/(c^8*(\\
& 4*a*c - b^2)^5))^(1/2)))/c^2 + (8*a*(A*c - 3*B*b))/c^2 + (2*x^2*(60*B*a^3*c \\
& ^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - \\
& 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(3*B*b - A*c + c^4*(-(60*B*a^3* \\
& c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - \\
& 90*B*a^2*b^2*c^2))^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3* \\
& B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23* \\
& A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((6*B*b^11 + 2048*A \\
& *a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - \\
& 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^ \\
& 7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10 \\
& *c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8 \\
&)) - (\operatorname{atan}(((32*a^2*c^8*(4*a*c - b^2)^5 + 2*b^4*c^6*(4*a*c - b^2)^5 - 16*a* \\
& b^2*c^7*(4*a*c - b^2)^5)*(x^2*(((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c \\
& ^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6 \\
&))/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a \\
& ^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a \\
& *b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560 \\
& *A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^ \\
& 4)))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80 \\
& *a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(60*B \\
& *a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c \\
& ^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^8 - 64*a*b^ \\
& 3*c^9 + 128*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - \\
& 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5* \\
& c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320* \\
& A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 \\
& - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(16*c^4*(4*a*c - b^2)^(5/2)*(1 \\
& 6*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^ \\
& 5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^2) + (b((((6A^5b^5c^5 + 120B^3a^3c^7 - 18B^6b^6c^4 - 52A^2a^2b^3c^6 \\
& + 124A^2a^2b^3c^7 + 156B^2a^2b^4c^5 - 372B^2a^2b^2c^6)/(16a^2c^8 + b^4c^6 - 8a^2b^2c^7) - ((8b^5c^8 - 64a^2b^3c^9 + 128a^2b^2c^10)*(6B^6b^11 \\
& + 2048A^5a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960 \\
& *B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(2*(16a^2c^8 + b^4c^6 - 8a^2b^2c^7)*(4096a^5c^9 - 4b^10c^4 + 80a^2b^8c^5 - 640a^2 \\
& *b^6c^6 + 2560a^3b^4c^7 - 5120a^4b^2c^8)))*(6B^6b^11 + 2048A^5a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2 \\
& a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(2*(4096a^5c^9 - 4b^10c^4 + 80a^2b^8c^5 - 640a^2b^6c^6 + 2560a^3b^4c^7 - 5120a^4b^2c^8)) - (9 \\
& *B^2b^7 + A^2b^5c^2 - 6A^2B^2b^6c + 207B^2a^2b^3c^2 + 30A^2B^2a^3c^4 - 81B^2a^2b^5c - 9A^2a^2b^3c^3 + 23A^2a^2b^2c^4 - 90B^2a^3b^2c^3 - 138A^2B^2a^2b^2c^3 + 54A^2B^2a^2b^4c^2)/(16a^2c^8 + b^4c^6 - 8a^2b^2c^7) + (((b^5c^8)/2 - 4a^2b^3c^9 + 8a^2b^2c^10)*(60B^2a^3c^3 - 3B^2b^6 + A^2b^5c + 30B^2a^2b^4c - 10A^2a^2b^3c^2 + 30A^2a^2b^2c^2)^2)/(c^8(4a^2c - b^2)^5(16a^2c^8 + b^4c^6 - 8a^2b^2c^7)))/(2a^2(4a^2c - b^2)^5) + (((8A^2a^2c^5 - 24B^2a^2b^2c^4)/c^6 - (8a^2c^2*(6B^2b^11 + 2048A^2a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(4096a^5c^9 - 4b^10c^4 + 80a^2b^8c^5 - 640a^2b^6c^6 + 2560a^3b^4c^7 - 5120a^4b^2c^8))*(60B^2a^3c^3 - 3B^2b^6 + A^2b^5c + 30B^2a^2b^4c - 10A^2a^2b^3c^2 + 30A^2a^2b^2c^2))/(8c^4(4a^2c - b^2)^5) - (a*(60B^2a^3c^3 - 3B^2b^6 + A^2b^5c + 30B^2a^2b^4c - 10A^2a^2b^3c^2 + 30A^2a^2b^2c^2)^3 - 90B^2a^2b^2c^2)*(6B^2b^11 + 2048A^2a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(c^2(4a^2c - b^2)^5(16a^2c^8 + b^4c^6 - 8a^2b^2c^7)))/(a*(4a^2c - b^2)^2) + (b((((8A^2a^2c^5 - 24B^2a^2b^2c^4)/c^6 - (8a^2c^2*(6B^2b^11 + 2048A^2a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(4096a^5c^9 - 4b^10c^4 + 80a^2b^8c^5 - 640a^2b^6c^6 + 2560a^3b^4c^7 - 5120a^4b^2c^8))*(6B^2b^11 + 2048A^2a^5c^6 - 2A^2b^10c - 120B^2a^2b^9c + 40A^2a^2b^8c^2 - 6144B^2a^5b^5c^5 - 320A^2a^2b^6c^3 + 1280A^2a^3b^4c^4 - 2560A^2a^4b^2c^5 + 960B^2a^2b^7c^2 - 3840B^2a^3b^5c^3 + 7680B^2a^4b^3c^4))/(2*(4096a^5c^9 - 4b^10c^4 + 80a^2b^8c^5 - 640a^2b^6c^6 + 2560a^3b^4c^7 - 5120a^4b^2c^8)) - (A^2a^2c^2 + 9B^2a^2b^2 - 6A^2B^2a^2b^2c)/c^6 + (a*(60B^2a^3c^3 - 3B^2b^6 + A^2b^5c + 30B^2a^2b^4c - 10A^2a^2b^3c^2 + 30A^2a^2b^2c^2)^2)/(c^6(4a^2c - b^2)^5)))/(2a^2(4a^2c - b^2)^5)))/(9B^2b^12 + A^2b^10c^2 + 3600B^2a^6c^6 - 6A^2B^2b^11c + 160A^2a^2b^6c^4 - 600A^2a^3b^4c^5 + 900A^2a^4b^2c^6 + 1440B^2a^2a^
\end{aligned}$$

$$\begin{aligned} & 2*b^8*c^2 - 5760*B^2*a^3*b^6*c^3 + 11700*B^2*a^4*b^4*c^4 - 10800*B^2*a^5*b^2*c^5 - 180*B^2*a*b^{10}*c - 20*A^2*a*b^8*c^3 - 960*A*B*a^2*b^7*c^3 + 3720*A*B*a^3*b^5*c^4 - 6600*A*B*a^4*b^3*c^5 + 120*A*B*a*b^9*c^2 + 3600*A*B*a^5*b*c^6) * (60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2) / (2*c^4*(4*a*c - b^2)^(5/2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.103 \quad \int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=254

$$\frac{(-12a^2Ac^3 + 30a^2bBc^2 - 10ab^3Bc + b^5B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) x^2 (x^2(16a^2Bc^2 + 6aAbc^2 - 15ab^2Bc + 2b^4B) + 2a)}{2c^3(b^2 - 4ac)^{5/2} - 4c^2(b^2 - 4ac)^2(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.40, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 818, 634, 618, 206, 628}

$$-\frac{x^2(16a^2Bc^2 + 6aAbc^2 - 15ab^2Bc + 2b^4B) + 2a(6aAc^2 - 7abBc + b^3B)}{4c^2(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(-12a^2Ac^3 + 30a^2bBc^2 - 10ab^3Bc + b^5B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3(b^2 - 4ac)^{5/2}} - \frac{x^6(x^2(-2aBc - Abc + b^2B) + a(bB - 2Ac))}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{B \log(a + bx^2 + cx^4)}{4c^3}$$

Antiderivative was successfully verified.

[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] $-(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*A*c^3)*\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^{(5/2)}) + (B*\text{Log}[a + b*x^2 + c*x^4])/(4*c^3)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4)) + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4 (A + Bx)}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{\text{Subst} \left(\int \frac{x^2 (3a(bB - 2Ac) + 2B(b^2 - 4ac)x)}{(a + bx + cx^2)^2} dx, x \right)}{4c (b^2 - 4ac)} \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^2 (2a (b^3B - 7abBc + 6aAc^2) + (2b^4B - 4a^2c^2))}{4c^2 (b^2 - 4ac)^2} \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^2 (2a (b^3B - 7abBc + 6aAc^2) + (2b^4B - 4a^2c^2))}{4c^2 (b^2 - 4ac)^2} \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^2 (2a (b^3B - 7abBc + 6aAc^2) + (2b^4B - 4a^2c^2))}{4c^2 (b^2 - 4ac)^2} \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^2 (2a (b^3B - 7abBc + 6aAc^2) + (2b^4B - 4a^2c^2))}{4c^2 (b^2 - 4ac)^2} \\
&= -\frac{x^6 (a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2)}{4c (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^2 (2a (b^3B - 7abBc + 6aAc^2) + (2b^4B - 4a^2c^2))}{4c^2 (b^2 - 4ac)^2}
\end{aligned}$$

Mathematica [A] time = 0.48, size = 354, normalized size = 1.39

$$\frac{-\frac{2(-12a^2A^2+30a^2Bc^2-10ab^3Bc+ab^3B)}{(4ac-b^2)^2} \tan^{-1}\left(\frac{bx+2c}{\sqrt{4ac-b^2}}\right) + \frac{2a^2bc^2(11A+25Bc^2)+4a^2c^2(8ab-5Ac^2)+b^4(11bB-2Ac^2)-2ab^2c^2(4A+15Bc^2)+ab^2c^2(16Ac^2-39ab)+b^4(A+4Bc^2)+b^4(-B)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^2Bc^2+a^2c^2(b(3A+5Bc^2)-2Ac^2-4b^2B)+ab^2(-b(A+5Bc^2)+4Ac^2+b^2B)+b^4c^2(bB-Ac)}{(b^2-4ac)^2(a+bx^2+cx^4)} + Bc \log(a + bx^2 + cx^4)}{4c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 1.16, size = 2167, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12

$$\begin{aligned}
& *B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 \\
& + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A \\
& *a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^ \\
& 3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3 \\
& 0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c \\
& ^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a \\
& ^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c \\
& + 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x \\
& ^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + \\
& (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 \\
& - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B \\
& *a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c \\
& + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - \\
& 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 \\
& + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^ \\
& 5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5 \\
& *c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^ \\
& 2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c \\
& ^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]
\end{aligned}$$

giac [A] time = 6.31, size = 466, normalized size = 1.83

$$\frac{(8b^7 - 10Ba^6c^2 + 30B^2a^5c^2 - 12A^2b^4c^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 8 \log(c^2x^4 + bx^2 + a)}{2(b^2c^3 - 8a^2b^2c^4 + 16a^2c^5) \sqrt{-b^2 + 4ac}} + \frac{3Bb^7c^4 - 24Ba^6b^2c^4 + 48B^2a^5c^4 - 2Bb^7c^4 + 12Ba^6b^2c^4 + 4A^2b^4c^4 - 4B^2b^4c^4 - 32A^2b^4c^4 + 40A^2b^4c^4 - 3Bb^7c^4 + 20Ba^6b^2c^4 + 2A^2b^4c^4 - 23B^2b^4c^4 - 16A^2b^4c^4 + 32B^2b^4c^4 - 4A^2b^4c^4 - 6Ba^6c^4 + 40B^2b^4c^4 + 4A^2b^4c^4 - 28B^2b^4c^4 - 40A^2b^4c^4 - 24A^2b^4c^4 - 3Bb^7c^4 + 18Ba^6b^2c^4 + 2A^2b^4c^4 - 20A^2b^4c^4}{8(b^7c^3 - 8a^2b^6c^4 + 16a^2c^7)(c^2 + b^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $-1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*\sqrt{-b^2 + 4*a*c}) + 1/4*B*\log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)$

maple [B] time = 0.03, size = 723, normalized size = 2.85

$$\frac{6A^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 12B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 8B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 8B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 4B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 4B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 4B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 4B^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(b^2c^3 - 8a^2b^2c^4 + 16a^2c^5) \sqrt{-b^2 + 4ac}} + \frac{3Bb^7c^4 - 24Ba^6b^2c^4 + 48B^2a^5c^4 - 2Bb^7c^4 + 12Ba^6b^2c^4 + 4A^2b^4c^4 - 4B^2b^4c^4 - 32A^2b^4c^4 + 40A^2b^4c^4 - 3Bb^7c^4 + 20Ba^6b^2c^4 + 2A^2b^4c^4 - 23B^2b^4c^4 - 16A^2b^4c^4 + 32B^2b^4c^4 - 4A^2b^4c^4 - 6Ba^6c^4 + 40B^2b^4c^4 + 4A^2b^4c^4 - 28B^2b^4c^4 - 40A^2b^4c^4 - 24A^2b^4c^4 - 3Bb^7c^4 + 18Ba^6b^2c^4 + 2A^2b^4c^4 - 20A^2b^4c^4}{4(b^2c^3 - 8a^2b^2c^4 + 16a^2c^5) \sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5))/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2*B-2
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2*B+1/4/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4*B+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a^2-15/c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*
b*B+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*B*a*b^3-1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.15, size = 3062, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2*
A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 25*B
*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2 + 3*
B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*B*a^2*
b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log
(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2
*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*(-(B*b^5 -
12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/
2)))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B
```

$$\begin{aligned}
& a^2 b c^2) / (c(4 a^2 c - b^2)^2)) / (4 c^3) + (B x^2 (B b^5 - 6 A a^2 c^3 - \\
& 9 B a b^3 c + 23 B a^2 b c^2)) / (c^4 (4 a^2 c - b^2)^2)) * ((B^2 a) / c^4 - ((B - \\
& c^3 * (- (B b^5 - 12 A a^2 c^3 - 10 B a b^3 c + 30 B a^2 b c^2))^2 / (c^6 (4 a^2 c \\
& - b^2)^5))^{(1/2)}) * ((8 B a) / c - (2 * (B - c^3 * (- (B b^5 - 12 A a^2 c^3 - 10 B a \\
& b^3 c + 30 B a^2 b c^2))^2 / (c^6 (4 a^2 c - b^2)^5))^{(1/2)}) * (2 a + b x^2)) / c + \\
& (2 x^2 * (3 B b^5 - 12 A a^2 c^3 - 26 B a b^3 c + 62 B a^2 b c^2)) / (c * (4 a^2 c \\
& - b^2)^2)) / (4 c^3) + (B x^2 * (B b^5 - 6 A a^2 c^3 - 9 B a b^3 c + 23 B a^2 \\
& b c^2)) / (c^4 (4 a^2 c - b^2)^2)) * (2 B b^{10} - 2048 B a^5 c^5 - 40 B a b^8 c \\
& + 320 B a^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4)) / (2 * (4096 a^5 \\
& c^8 - 4 b^{10} c^3 + 80 a b^8 c^4 - 640 a^2 b^6 c^5 + 2560 a^3 b^4 c^6 - 51 \\
& 20 a^4 b^2 c^7)) + (\operatorname{atan}(((32 a^2 c^6 (4 a^2 c - b^2)^5 + 2 b^4 c^4 (4 a^2 c - \\
& b^2)^5 - 16 a b^2 c^5 (4 a^2 c - b^2)^5) * (x^2 * (((((24 A a^2 c^6 - 6 B b^5 c^3 \\
& + 52 B a b^3 c^4 - 124 B a^2 b c^5) / (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5) - \\
& ((8 b^5 c^6 - 64 a b^3 c^7 + 128 a^2 b c^8) * (2 B b^{10} - 2048 B a^5 c^5 - 4 \\
& 0 B a b^8 c + 320 B a^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4)) \\
& / (2 * (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5) * (4096 a^5 c^8 - 4 b^{10} c^3 + 80 a b^8 \\
& c^4 - 640 a^2 b^6 c^5 + 2560 a^3 b^4 c^6 - 5120 a^4 b^2 c^7))) * (B b^5 - \\
& 12 A a^2 c^3 - 10 B a b^3 c + 30 B a^2 b c^2)) / (8 c^3 (4 a^2 c - b^2)^{(5/2)}) \\
& - ((8 b^5 c^6 - 64 a b^3 c^7 + 128 a^2 b c^8) * (B b^5 - 12 A a^2 c^3 - 10 B \\
& a b^3 c + 30 B a^2 b c^2) * (2 B b^{10} - 2048 B a^5 c^5 - 40 B a b^8 c + 320 B \\
& a^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4)) / (16 c^3 (4 a^2 c - \\
& b^2)^{(5/2)} * (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5) * (4096 a^5 c^8 - 4 b^{10} c^3 \\
& + 80 a b^8 c^4 - 640 a^2 b^6 c^5 + 2560 a^3 b^4 c^6 - 5120 a^4 b^2 c^7)))) / (\\
& a (4 a^2 c - b^2)^2) - (b * (((((24 A a^2 c^6 - 6 B b^5 c^3 + 52 B a b^3 c^4 - 1 \\
& 24 B a^2 b c^5) / (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5) - ((8 b^5 c^6 - 64 a b \\
& ^3 c^7 + 128 a^2 b c^8) * (2 B b^{10} - 2048 B a^5 c^5 - 40 B a b^8 c + 320 B a \\
& ^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4)) / (2 * (16 a^2 c^6 + b^4 \\
& c^4 - 8 a b^2 c^5) * (4096 a^5 c^8 - 4 b^{10} c^3 + 80 a b^8 c^4 - 640 a^2 b^6 \\
& c^5 + 2560 a^3 b^4 c^6 - 5120 a^4 b^2 c^7))) * (2 B b^{10} - 2048 B a^5 c^5 - \\
& 40 B a b^8 c + 320 B a^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4) \\
&) / (2 * (4096 a^5 c^8 - 4 b^{10} c^3 + 80 a b^8 c^4 - 640 a^2 b^6 c^5 + 2560 a^3 \\
& b^4 c^6 - 5120 a^4 b^2 c^7)) - (B^2 b^5 - 6 A B a^2 c^3 - 9 B^2 a b^3 c + \\
& 23 B^2 a^2 b c^2) / (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5) + (((b^5 c^6) / 2 - 4 \\
& a b^3 c^7 + 8 a^2 b c^8) * (B b^5 - 12 A a^2 c^3 - 10 B a b^3 c + 30 B a^2 b \\
& c^2)^2) / (c^6 (4 a^2 c - b^2)^5 * (16 a^2 c^6 + b^4 c^4 - 8 a b^2 c^5)))) / (2 a * (\\
& 4 a^2 c - b^2)^{(5/2)}) - (((((8 B a) / c + (8 a c^2 * (2 B b^{10} - 2048 B a^5 c^5 - \\
& 40 B a b^8 c + 320 B a^2 b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4 \\
&)) / (4096 a^5 c^8 - 4 b^{10} c^3 + 80 a b^8 c^4 - 640 a^2 b^6 c^5 + 2560 a^3 b \\
& ^4 c^6 - 5120 a^4 b^2 c^7)) * (B b^5 - 12 A a^2 c^3 - 10 B a b^3 c + 30 B a^2 \\
& b c^2)) / (8 c^3 (4 a^2 c - b^2)^{(5/2)}) + (a * (B b^5 - 12 A a^2 c^3 - 10 B a b^3 \\
& c + 30 B a^2 b c^2) * (2 B b^{10} - 2048 B a^5 c^5 - 40 B a b^8 c + 320 B a^2 \\
& b^6 c^2 - 1280 B a^3 b^4 c^3 + 2560 B a^4 b^2 c^4)) / (c * (4 a^2 c - b^2)^{(5/2)} \\
& * (4096 a^5 c^8 - 4 b^{10} c^3 + 80 a b^8 c^4 - 640 a^2 b^6 c^5 + 2560 a^3 b^4 \\
& c^6 - 5120 a^4 b^2 c^7))) / (a * (4 a^2 c - b^2)^2) + (b * ((B^2 a) / c^4 + (((8 B a \\
&) / c + (8 a c^2 * (2 B b^{10} - 2048 B a^5 c^5 - 40 B a b^8 c + 320 B a^2 b^6 c^
\end{aligned}$$

$$\frac{(2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4)/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4)/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*a*c - b^2)^(5/2)))/(B^2*b^10 + 144*A^2*a^4*c^6 + 160*B^2*a^2*b^6*c^2 - 600*B^2*a^3*b^4*c^3 + 900*B^2*a^4*b^2*c^4 - 20*B^2*a*b^8*c - 24*A*B*a^2*b^5*c^3 + 240*A*B*a^3*b^3*c^4 - 720*A*B*a^4*b*c^5))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(2*c^3*(4*a*c - b^2)^(5/2))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.104 \quad \int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=146

$$\frac{3a(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}} + \frac{3x^2(2a + bx^2)(Ab - 2aB)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{x^6(-2aB - (x^2(bB - 2Ac)) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2}$$

Rubi [A] time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 804, 722, 618, 206}

$$-\frac{x^6(-2aB + x^2(-(bB - 2Ac)) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3x^2(2a + bx^2)(Ab - 2aB)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{3a(Ab - 2aB) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 722

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &

& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]

Rule 804

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^7 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3 (A + Bx)}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\ &= -\frac{x^6 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{(3(Ab - 2aB)) \text{Subst} \left(\int \frac{x^2}{(a + bx + cx^2)^2} dx, x, x^2 \right)}{4 (b^2 - 4ac)} \\ &= -\frac{x^6 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2 (2a + bx^2)}{4 (b^2 - 4ac)^2 (a + bx^2 + cx^4)} - \frac{(3a(Ab - 2aB)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2 (b^2 - 4ac)} \\ &= -\frac{x^6 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2 (2a + bx^2)}{4 (b^2 - 4ac)^2 (a + bx^2 + cx^4)} + \frac{(3a(Ab - 2aB)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{(b^2 - 4ac)} \\ &= -\frac{x^6 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{3(Ab - 2aB)x^2 (2a + bx^2)}{4 (b^2 - 4ac)^2 (a + bx^2 + cx^4)} + \frac{3a(Ab - 2aB) \tanh^{-1} \left(\frac{bx + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)} \end{aligned}$$

Mathematica [A] time = 0.27, size = 261, normalized size = 1.79

$$\frac{1}{4} \left(\frac{a^2 c (2c(A + Bx^2) - 3bB) + ab(-bc(A + 4Bx^2) + 3Ac^2x^2 + b^2B) + b^3x^2(bB - Ac) - 4a^2c^3(4A + 5Bx^2) + ab^2c^2(5A + 16Bx^2) + 2abc^2(11aB - 3Acx^2) - 8ab^3Bc - b^4c(A + 2Bx^2) + b^5B}{c^3(4ac - b^2)(a + bx^2 + cx^4)^2} + \frac{-4a^2c^3(4A + 5Bx^2) + ab^2c^2(5A + 16Bx^2) + 2abc^2(11aB - 3Acx^2) - 8ab^3Bc - b^4c(A + 2Bx^2) + b^5B}{c^3(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{12a(Ab - 2aB) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 0.95, size = 1378, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2

$$\begin{aligned}
 & *c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - \\
 & 64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4 \\
 & 4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 \\
 & + (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 \\
 & - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)* \\
 & c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3) \\
 & *c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2* \\
 & (2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^4 + (2*(2*B \\
 & *a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b \\
 &)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4 \\
 & *a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a \\
 & ^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3 \\
 & *c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6 \\
 & + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)* \\
 & x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
 \end{aligned}$$

giac [B] time = 6.46, size = 318, normalized size = 2.18

$$\frac{3(2Ba^2 - Aab) \arctan\left(\frac{2c^2 + b}{\sqrt{4ac - b^2}}\right) - 2Bb^4c^6 - 16Bab^2c^2x^6 + 20Ba^2c^3x^6 + 6Aabc^3x^6 + Bb^5x^4 - 8Bab^3cx^4 + Ab^4cx^4 - 2Ba^2bc^2x^4 + Aab^2c^2x^4 + 16Aa^2c^3x^4 + 2Bab^4x^2 - 20Ba^2b^2cx^2 + 2Aab^3cx^2 + 12Ba^3c^2x^2 + 10Aa^2bc^2x^2 + Ba^2b^3 - 10Ba^2bc + Aa^2b^2c + 8Aa^2c^2}{(b^4 - 8a^2b^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} - \frac{2(14B^2a^3b^3 - A^2a^2b^4)c}{4(b^4c^2 - 8ab^2c^3 + 16a^2c^4)(cx^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

$$\begin{aligned}
 & [Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2 \\
 & *c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2 \\
 & *x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A \\
 & *b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a \\
 & *b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a \\
 & ^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4 \\
 & c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
 \end{aligned}$$

maple [B] time = 0.02, size = 398, normalized size = 2.73

$$\frac{3Aab \arctan\left(\frac{2c^2 + b}{\sqrt{4ac - b^2}}\right) + 6B a^2 \arctan\left(\frac{2c^2 + b}{\sqrt{4ac - b^2}}\right) - \frac{(3aAb^2c^2 + 10a^2B^2c^2 - 8a^2b^2Bc + b^4B)c^6}{(16a^2c^2 - 8a^2b^2c + 16a^2c^2)c} - \frac{(16Aa^2c^3 + Aa^2b^2c^2 + Ab^4c - 2B^2a^2b^2c^2 - 8Ba^2b^3c + Bb^5)c^4}{2(16a^2c^2 - 8a^2b^2c + 16a^2c^2)^2} - \frac{(5aAb^2c^2 + Ab^3c + 6a^2B^2c^2 - 10a^2b^2Bc + b^4B)c^2}{(16a^2c^2 - 8a^2b^2c + 16a^2c^2)^2} - \frac{(8aA^2c^2 + A^2b^2c - 10aAb^2c + b^3B)c^2}{2(16a^2c^2 - 8a^2b^2c + 16a^2c^2)^2}}{(16a^2c^2 - 8a^2b^2c + 16a^2c^2)\sqrt{4ac - b^2}} + \frac{6B a^2 \arctan\left(\frac{2c^2 + b}{\sqrt{4ac - b^2}}\right)}{(16a^2c^2 - 8a^2b^2c + 16a^2c^2)\sqrt{4ac - b^2}} + \frac{2(c^4 + bx^2 + a)^2}{2(c^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)

$$\begin{aligned}
 & [Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+ \\
 & b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B \\
 & b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2- \\
 & 10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c \\
 & ^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
 \end{aligned}$$


```
[Out] -3*a*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*
B*a**2*b - 192*a**4*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 144*a*
*3*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 36*a**2*b**4*c*sqr
t(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5
)*(-A*b + 2*B*a))/(-6*A*a*b*c + 12*B*a**2*c))/2 + 3*a*sqrt(-1/(4*a*c - b**2
)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*B*a**2*b + 192*a**4*c**3*s
qrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 144*a**3*b**2*c**2*sqrt(-1/(4*a*
c - b**2)**5)*(-A*b + 2*B*a) + 36*a**2*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-
A*b + 2*B*a) - 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a))/(-6*A*a*
b*c + 12*B*a**2*c))/2 + (-8*A*a**3*c**2 - A*a**2*b**2*c + 10*B*a**3*b*c - B
*a**2*b**3 + x**6*(-6*A*a*b*c**3 - 20*B*a**2*c**3 + 16*B*a*b**2*c**2 - 2*B*
b**4*c) + x**4*(-16*A*a**2*c**3 - A*a*b**2*c**2 - A*b**4*c + 2*B*a**2*b*c**
2 + 8*B*a*b**3*c - B*b**5) + x**2*(-10*A*a**2*b*c**2 - 2*A*a*b**3*c - 12*B*
a**3*c**2 + 20*B*a**2*b**2*c - 2*B*a*b**4))/ (64*a**4*c**4 - 32*a**3*b**2*c*
*3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2*c**5 + 4*b**4*c**4)
+ x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3) + x**4*(128*a**3*c*
*5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*c**4 - 64*a**2*b**3*c
**3 + 8*a*b**5*c**2))
```

$$3.105 \quad \int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=185

$$\frac{(3abB - A(2ac + b^2)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) x^4(-2aB - (x^2(bB - 2Ac)) + Ab) a(8aBc - 6Abc + b^2B) + x^2(4aA^2 - 4abB + 4a^2c)}{(b^2 - 4ac)^{5/2} 4(b^2 - 4ac)(a + bx^2 + cx^4)^2 4c(b^2 - 4ac)^2 (a + bx^2 + cx^4)}$$

Rubi [A] time = 0.26, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 820, 777, 618, 206}

$$\frac{x^2(4aAc^2 + 2abBc - 4Ab^2c + b^3B) + a(8aBc - 6Abc + b^2B)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{x^4(-2aB + x^2(-(bB - 2Ac)) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(3abB - A(2ac + b^2)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c + 4*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 777

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4

*a*c, 0] && LtQ[p, -1]

Rule 820

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\
 &= -\frac{x^4 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{\text{Subst} \left(\int \frac{x(-2(Ab - 2aB) - (bB - 2Ac)x)}{(a + bx + cx^2)^2} dx, x, x^2 \right)}{4 (b^2 - 4ac)} \\
 &= -\frac{x^4 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{a (b^2 B - 6Abc + 8aBc) + (b^3 B - 4Ab^2 c + 2abBc + 4a^2 c)}{4c (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\
 &= -\frac{x^4 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{a (b^2 B - 6Abc + 8aBc) + (b^3 B - 4Ab^2 c + 2abBc + 4a^2 c)}{4c (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\
 &= -\frac{x^4 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{a (b^2 B - 6Abc + 8aBc) + (b^3 B - 4Ab^2 c + 2abBc + 4a^2 c)}{4c (b^2 - 4ac)^2 (a + bx^2 + cx^4)}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 233, normalized size = 1.26

$$\frac{1}{4} \left(\frac{2a^2Bc + a(bc(A + 3Bx^2) - 2Ac^2x^2 + b^2(-B)) + b^2x^2(Ac - bB)}{c^2(4ac - b^2)(a + bx^2 + cx^4)^2} + \frac{4(A(2ac + b^2) - 3abB) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{5/2}} + \frac{b^2c(5aB + 2Acx^2) + 2abc^2(A - 3Bx^2) + 4ac^2(Acx^2 - 4aB) + Ab^3c + b^4(-B)}{c^2(b^2 - 4ac)^2(a + bx^2 + cx^4)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out]
$$\frac{(-b^4B + Ab^3c + 2a*b*c^2*(A - 3B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2) + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^{(5/2)}/4$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 0.95, size = 1369, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*c) * log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c)) / (c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c / (a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - \end{aligned}$$

$$64a^3b^5c^5)x^6 + (b^8c - 10a^2b^6c^2 + 24a^2b^4c^3 + 32a^3b^2c^4 - 128a^4c^5)x^4 + 2(a^2b^7c - 12a^2b^5c^2 + 48a^3b^3c^3 - 64a^4b^2c^4)x^2, -1/4(B^2a^2b^4 + 2(8A^2a^2c^4 - 2(6B^2a^2b - A^2a^2b^2)c^3 + (3B^2a^2b^3 - A^2b^4)c^2)x^6 + (B^2b^6 - 8(8B^2a^3 - 3A^2a^2b)c^3 + 6(2B^2a^2b^2 + A^2a^2b^3)c^2 - 3(B^2a^2b^4 + A^2b^5)c)x^4 - 8(4B^2a^4 - 3A^2a^3b)c^2 + 2(B^2a^2b^5 - 8A^2a^3c^3 - 2(10B^2a^3b - 11A^2a^2b^2)c^2 + (B^2a^2b^3 - 5A^2a^2b^4)c)x^2 + 4((2A^2a^2c^4 - (3B^2a^2b - A^2b^2)c^3)x^8 + 2(2A^2a^2b^2c^3 - (3B^2a^2b^2 - A^2b^3)c^2)x^6 + 2A^2a^3c^2 + (4A^2a^2c^3 - 2(3B^2a^2b - 2A^2a^2b^2)c^2 - (3B^2a^2b^3 - A^2b^4)c)x^4 + 2(2A^2a^2b^2c^2 - (3B^2a^2b^2 - A^2a^2b^3)c)x^2 - (3B^2a^3b - A^2a^2b^2)c) \sqrt{-b^2 + 4ac} \arctan(-(2cx^2 + b) \sqrt{-b^2 + 4ac}) / (b^2 - 4ac)) + 2(2B^2a^3b^2 - 3A^2a^2b^3)c) / (a^2b^6c - 12a^3b^4c^2 + 48a^4b^2c^3 - 64a^5c^4 + (b^6c^3 - 12a^2b^4c^4 + 48a^2b^2c^5 - 64a^3c^6)x^8 + 2(b^7c^2 - 12a^2b^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)x^6 + (b^8c - 10a^2b^6c^2 + 24a^2b^4c^3 + 32a^3b^2c^4 - 128a^4c^5)x^4 + 2(a^2b^7c - 12a^2b^5c^2 + 48a^3b^3c^3 - 64a^4b^2c^4)x^2)]$$

giac [A] time = 6.59, size = 268, normalized size = 1.45

$$\frac{(3Bab - Ab^2 - 2Aac) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) - 6Babc^2x^6 - 2Ab^2c^2x^6 - 4Aac^3x^6 + Bb^4x^4 + Bab^2cx^4 - 3Ab^3cx^4 + 16Ba^2c^2x^4 - 6Aabc^2x^4 + 2Bab^3x^2 + 10Ba^2bcx^2 - 10Aab^2cx^2 + 4Aa^2c^2x^2 + Ba^2b^2 + 8Ba^3c - 6Aa^2bc}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} - \frac{6Babc^2x^6 - 2Ab^2c^2x^6 - 4Aac^3x^6 + Bb^4x^4 + Bab^2cx^4 - 3Ab^3cx^4 + 16Ba^2c^2x^4 - 6Aabc^2x^4 + 2Bab^3x^2 + 10Ba^2bcx^2 - 10Aab^2cx^2 + 4Aa^2c^2x^2 + Ba^2b^2 + 8Ba^3c - 6Aa^2bc}{4(b^4c - 8ab^2c^2 + 16a^2c^3)(cx^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $-(3B^2a^2b - A^2b^2 - 2A^2a^2c) \arctan((2cx^2 + b) / \sqrt{-b^2 + 4ac}) / ((b^4 - 8a^2b^2c + 16a^2c^2) \sqrt{-b^2 + 4ac}) - 1/4(6B^2a^2b^2c^2x^6 - 2A^2a^2b^2c^2x^6 - 4A^2a^2c^3x^6 + B^2b^4x^4 + B^2a^2b^2c^2x^4 - 3A^2b^3c^2x^4 + 16B^2a^2c^2x^4 - 6A^2a^2b^2c^2x^4 + 2B^2a^2b^3x^2 + 10B^2a^2b^2c^2x^2 - 10A^2a^2b^2c^2x^2 + 4A^2a^2c^2x^2 + B^2a^2b^2 + 8B^2a^3c - 6A^2a^2b^2c) / ((b^4c - 8a^2b^2c^2 + 16a^2c^3)(cx^4 + bx^2 + a)^2)$

maple [B] time = 0.02, size = 411, normalized size = 2.22

$$\frac{2Aac \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{A^2b^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} - \frac{3Bab \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{(2aAc + Ab^2 - 3abB)c^2x^6}{16a^2c^2 - 8ab^2c + b^4} + \frac{(6aAb^2 + 3A^2b^3c - 16a^2B^2c^2 - a^2b^2Bc - b^4B)x^4}{2(16a^2c^2 - 8ab^2c + b^4)c} - \frac{(2aA^2c^2 - 5A^2b^2c + 5abBc + b^3B)x^2}{(16a^2c^2 - 8ab^2c + b^4)c} + \frac{(6Abc - 8aBc - b^2B)a^2}{2(16a^2c^2 - 8ab^2c + b^4)c} + \frac{6A^2a^2c^2x^2 + B^2a^2b^2 + 8B^2a^3c - 6A^2a^2b^2c}{2(cx^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)

[Out] $1/2(c(2A^2a^2c + A^2b^2 - 3B^2a^2b) / (16a^2c^2 - 8a^2b^2c + b^4)x^6 + 1/2(6A^2a^2b^2c^2 + 3A^2b^3c - 16B^2a^2c^2 - B^2a^2b^2c - B^2b^4) / c / (16a^2c^2 - 8a^2b^2c + b^4)x^4 - a/c(2A^2a^2c^2 - 5A^2b^2c + 5B^2a^2b^2c + B^2b^3) / (16a^2c^2 - 8a^2b^2c + b^4)x^2 + 1/2a^2(6A^2b^2c - 8B^2a^2c - B^2b^2) / c / (16a^2c^2 - 8a^2b^2c + b^4) / (cx^4 + bx^2 + a)^2 + 2 / (16a^2c^2 - 8a^2b^2c + b^4) / (4a^2c - b^2)^{(1/2)} \arctan((2cx^2 + b) / (4a$

[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] $\sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) \log(x^2 + (-2Aab - Ab^3 + 3Bab^2 - 64a^3c^3 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) + 48a^2b^2c^2 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) - 12ab^4c \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) + b^6 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab)))/(-4Aa^2c^2 - 2Ab^2c + 6Bab^2c))/2 - \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) \log(x^2 + (-2Aab - Ab^3 + 3Bab^2 + 64a^3c^3 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) - 48a^2b^2c^2 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) + 12ab^4c \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab) - b^6 \sqrt{-1/(4ac - b^2)^5}(-2Aac - Ab^2 + 3Bab)))/(-4Aa^2c^2 - 2Ab^2c + 6Bab^2c))/2 + (6Aa^2b^2c - 8Ba^3c - Ba^2b^2 + x^6(4Aa^3c^3 + 2Ab^2c^2 - 6Bab^2c^2) + x^4(6Aab^2c^2 + 3Ab^3c - 16Ba^2c^2 - Bab^2c - Bb^4) + x^2(-4Aa^2c^2 + 10Aab^2c - 10Ba^2b^2c - 2Bab^3))/ (64a^4c^3 - 32a^3b^2c^2 + 4a^2b^4c + x^8(64a^2c^5 - 32ab^2c^4 + 4b^4c^3) + x^6(128a^2b^2c^4 - 64ab^3c^3 + 8b^5c^2) + x^4(128a^3c^4 - 24ab^4c^2 + 4b^6c) + x^2(128a^3b^2c^3 - 64a^2b^3c^2 + 8ab^5c))$

$$3.106 \quad \int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=170

$$\frac{(2aBc - 3Abc + b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}} + \frac{(b + 2cx^2)(2aBc - 3Abc + b^2B)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{x^2(-2aBc - Abc + b^2B) + a(bB - 2Ac)}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2}$$

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 777, 614, 618, 206}

$$\frac{(b + 2cx^2)(2aBc - 3Abc + b^2B)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{x^2(-2aBc - Abc + b^2B) + a(bB - 2Ac)}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{(2aBc - 3Abc + b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 777

Int[((d_.) + (e_.)*(x_.))*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Bx)}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{(b^2B - 3Abc + 2aBc) \text{Subst} \left(\int \frac{1}{(a + bx + cx^2)^2} dx \right)}{4c(b^2 - 4ac)} \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(b^2B - 3Abc + 2aBc)(b + 2cx^2)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(b^2B - 3Abc + 2aBc)}{4c(b^2 - 4ac)} \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(b^2B - 3Abc + 2aBc)(b + 2cx^2)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{(b^2B - 3Abc + 2aBc)}{4c(b^2 - 4ac)} \\ &= -\frac{a(bB - 2Ac) + (b^2B - Abc - 2aBc)x^2}{4c(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(b^2B - 3Abc + 2aBc)(b + 2cx^2)}{4c(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{(b^2B - 3Abc + 2aBc)}{4c(b^2 - 4ac)} \end{aligned}$$

Mathematica [A] time = 0.20, size = 172, normalized size = 1.01

$$\frac{1}{4} \left(\frac{4(2aBc - 3Abc + b^2B) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{(b+2cx^2)(2aBc - 3Abc + b^2B)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{-2ac(A+Bx^2) + abB + bx^2(bB - Ac)}{c(4ac-b^2)(a+bx^2+cx^4)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 0.93, size = 1226, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x

$$\begin{aligned} & \text{^2}), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b))*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2) \\ & *x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3 \\ & *A*a*b^2))*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2 \\ & *B*a^3 + 5*A*a^2*b))*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 + \\ & (2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2 \\ & *b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b))*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + \\ & 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*\text{sqrt} \\ & (-b^2 + 4*a*c)*\arctan(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4 \\ & *(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3 \\ & *c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7 \\ & *c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c \\ & + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5 \\ & *c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)] \end{aligned}$$

giac [A] time = 6.06, size = 228, normalized size = 1.34

$$\frac{(Bb^2 + 2Bac - 3Abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) + 2Bb^2cx^6 + 4Bac^2x^6 - 6Abc^2x^6 + 3Bb^3x^4 + 6Babcx^4 - 9Ab^2cx^4 + 10Bab^2x^2 - 2Ab^3x^2 - 4Ba^2cx^2 - 10Aabcx^2 + 6Ba^2b - Aab^2 - 8Aa^2c}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} + \frac{4(cx^4 + bx^2 + a)^2(b^4 - 8ab^2c + 16a^2c^2)}{4(cx^4 + bx^2 + a)^2(b^4 - 8ab^2c + 16a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))

maple [B] time = 0.02, size = 379, normalized size = 2.23

$$\frac{3Abc \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + 2Bac \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + Bb^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + \frac{(3Abc - 2aBc - b^2B)cx^6}{16a^2c^2 - 8ab^2c + b^4} - \frac{3(3Abc - 2aBc - b^2B)b^4x^4}{2(16a^2c^2 - 8ab^2c + b^4)} - \frac{(5Aabc + Ab^3 + 2a^2Bc - 5Ba^2b^2)x^2}{16a^2c^2 - 8ab^2c + b^4} - \frac{(8aAc + Ab^2 - 6abB)a}{2(16a^2c^2 - 8ab^2c + b^4)}}{2(cx^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)

[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.66, size = 587, normalized size = 3.45

$$\operatorname{atan}\left(\frac{\frac{\frac{c^2\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)+\frac{c^2\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)^2\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)}{2(4ac-b^2)^{5/2}\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)}+\frac{21c^2\left(\frac{B^2-3Ac+2Ba}{4ac-b^2}\right)^2}{(4ac-b^2)^{5/2}}}{18a^2b^2c^2-24ABb^2c^2-12AB^2b^2c^2+8B^2b^2c^2+8B^2a^2c^2+2B^2b^2c^2}\right)}{(4ac-b^2)^{5/2}}\left(Bb^2-3Ac+2Ba\right)-\frac{-6B^2b^2Ac^2+18a^2b^2}{4(16a^2c^2-8a^2b^2+c^4)}+\frac{c^2(2Bc^2-5Ba^2+5Ac+Ab^2)}{2(16a^2c^2-8a^2b^2+c^4)}-\frac{31c^4(Bb^2-3Ac+2Ba)}{4(16a^2c^2-8a^2b^2+c^4)}-\frac{c^4(Bb^2-3Ac+2Ba)}{2(16a^2c^2-8a^2b^2+c^4)}}{x^4(b^2+2ac)+a^2+c^2x^2+2abx^2+2bcx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out] (atan(((x^2*((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a*c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 - 3*A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b*c + 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c^4 + 2*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4)*(B*b^2 - 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c - 6*B*a^2*c*b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 + 2*B*a^2*c + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b^2 - 3*A*b*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^2 - 3*A*b*c + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)

sympy [B] time = 21.37, size = 789, normalized size = 4.64

$$\frac{\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}+\frac{c^2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}+\frac{c^2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}+\frac{c^2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}{2\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}\sqrt{\frac{c^2(-5Ab+2Ba+BF)}{(4ac-b^2)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5))*(-3*A

$$\begin{aligned}
& *b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**5)*(-3 \\
& *A*b*c + 2*B*a*c + B*b**2) - 12*a*b**4*c*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b \\
& *c + 2*B*a*c + B*b**2) + b**6*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a* \\
& c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + \sqrt{-1/(4*a*c - \\
& b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*\log(x**2 + (-3*A*b**2*c + 2*B*a*b*c \\
& + B*b**3 + 64*a**3*c**3*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B \\
& *b**2) - 48*a**2*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + \\
& B*b**2) + 12*a*b**4*c*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b \\
& **2) - b**6*\sqrt{-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A \\
& *b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2* \\
& b + x**6*(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B* \\
& a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 10*B*a*b**2 \\
&))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a \\
& *b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5 \\
& *c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - \\
& 64*a**2*b**3*c + 8*a*b**5))
\end{aligned}$$

$$3.107 \quad \int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=139

$$\frac{3c(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}} - \frac{3(b + 2cx^2)(bB - 2Ac)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{-2aB - (x^2(bB - 2Ac)) + Ab}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2}$$

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1247, 638, 614, 618, 206}

$$-\frac{3(b + 2cx^2)(bB - 2Ac)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{-2aB + x^2(-(bB - 2Ac)) + Ab}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3c(bB - 2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1247

Int[(x_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{(a + bx + cx^2)^3} dx, x, x^2 \right) \\
 &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{(3(bB - 2Ac)) \text{Subst} \left(\int \frac{1}{(a + bx + cx^2)^2} dx, x, x^2 \right)}{4(b^2 - 4ac)} \\
 &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{(3c(bB - 2Ac)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
 &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(3c(bB - 2Ac)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{(b^2 - 4ac)} \\
 &= -\frac{Ab - 2aB - (bB - 2Ac)x^2}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{3(bB - 2Ac)(b + 2cx^2)}{4(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{3c(bB - 2Ac) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 142, normalized size = 1.02

$$\frac{-\frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{(b^2-4ac)(B(2a+bx^2)-A(b+2cx^2))}{(a+bx^2+cx^4)^2} - \frac{3(b+2cx^2)(bB-2Ac)}{a+bx^2+cx^4}}{4(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 0.69, size = 1109, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*

$$(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]$$

giac [A] time = 5.57, size = 208, normalized size = 1.50

$$\frac{3(Bbc - 2Ac^2)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) - \frac{6Bbc^2x^6 - 12Ac^3x^6 + 9Bb^2cx^4 - 18Abc^2x^4 + 2Bb^3x^2 + 10Babcx^2 - 4Ab^2cx^2 - 20Aac^2x^2 + Bab^2 + Ab^3 + 8Ba^2c - 10Aabc}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}}}{4(cx^4 + bx^2 + a)^2(b^4 - 8ab^2c + 16a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $-3*(B*b*c - 2*A*c^2)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-b^2 + 4*a*c}) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))$

maple [A] time = 0.01, size = 262, normalized size = 1.88

$$\frac{3Ac^2x^2}{(4ac-b^2)^2(cx^4+bx^2+a)} - \frac{3Bbcx^2}{2(4ac-b^2)^2(cx^4+bx^2+a)} + \frac{6Ac^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} - \frac{3Bbc\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{3Abc}{2(4ac-b^2)^2(cx^4+bx^2+a)} - \frac{3Bb^2}{4(4ac-b^2)^2(cx^4+bx^2+a)} + \frac{Ab-2Ba+(2Ac-bB)x^2}{4(4ac-b^2)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)

[Out] $1/4*(A*b-2*B*a+(2*A*c-B*b)*x^2)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*c^2*x^2*A-3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*c*x^2*b*B+3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b*A*c-3/4/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b^2*B+6/(4*a*c-b^2)^{(5/2)}*c^2*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A-3/(4*a*c-b^2)^{(5/2)}*c*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.59, size = 517, normalized size = 3.72

$$3 \operatorname{catan} \left(\frac{\left(\frac{x^2 \left(\frac{3+2(A-c)(6A^4-3Bb^2)}{4(4ac-b^2)^2} - \frac{9x^2(2A-c)(Bb^2(3x^2+4-16a^2+2b^2))}{2a(4ac-b^2)^2} \right)}{72A^2c^6-72ABb^2c^4+18B^2b^2c^4} \right) + \frac{18A^4(2A-c)(Bb^2)}{4(4ac-b^2)^2} \right)}{(4ac-b^2)^{5/2}} \right) (2Ac-Bb) - \frac{8Bc^2+Ba^2-10Acab+Ab^3}{4(16a^2c^2-8a^2b^2+c^4)} - \frac{9x^4(2Ab^2-B^2c)}{4(16a^2c^2-8a^2b^2+c^4)} + \frac{x^2(B^2-2A^2c^2+5Babc-10Aac^2)}{2(16a^2c^2-8a^2b^2+c^4)} - \frac{3c^2x^6(2Ac-Bb)}{2(16a^2c^2-8a^2b^2+c^4)} \\ x^4 (b^2+2ac) + a^2 + c^2x^8 + 2abx^2 + 2bcx^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out] (3*c*atan(((x^2*((3*c*(2*A*c - B*b))*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/((72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c - b^2)^(5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/((x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6))

sympy [B] time = 12.40, size = 661, normalized size = 4.76

$$\sqrt{\frac{x^2}{(4ac-b^2)^2}} \left(\frac{x^2 \left(\frac{3+2(A-c)(6A^4-3Bb^2)}{4(4ac-b^2)^2} - \frac{9x^2(2A-c)(Bb^2(3x^2+4-16a^2+2b^2))}{2a(4ac-b^2)^2} \right)}{72A^2c^6-72ABb^2c^4+18B^2b^2c^4} \right) + \frac{18A^4(2A-c)(Bb^2)}{4(4ac-b^2)^2} \right) (2Ac-Bb) - \frac{8Bc^2+Ba^2-10Acab+Ab^3}{4(16a^2c^2-8a^2b^2+c^4)} - \frac{9x^4(2Ab^2-B^2c)}{4(16a^2c^2-8a^2b^2+c^4)} + \frac{x^2(B^2-2A^2c^2+5Babc-10Aac^2)}{2(16a^2c^2-8a^2b^2+c^4)} - \frac{3c^2x^6(2Ac-Bb)}{2(16a^2c^2-8a^2b^2+c^4)} \\ x^4 (b^2+2ac) + a^2 + c^2x^8 + 2abx^2 + 2bcx^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2 + x**6*(1

$$\begin{aligned} & 2A^3c - 6B^2bc^2) + x^4(18Ab^2c^2 - 9B^2b^2c) + x^2(20A^2ac^2 \\ & + 4A^2b^2c - 10B^2abc - 2B^2b^3)/(64a^4c^2 - 32a^3b^2c + 4 \\ & a^2b^4 + x^8(64a^2c^4 - 32ab^2c^3 + 4b^4c^2) + x^6(128 \\ & a^2b^3c^3 - 64ab^3c^2 + 8b^5c) + x^4(128a^3c^3 - 24ab^4 \\ & c + 4b^6) + x^2(128a^3b^2c^2 - 64a^2b^3c + 8ab^5) \end{aligned}$$

$$3.108 \quad \int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=252

$$\frac{A \log(a+bx^2+cx^4)}{4a^3} + \frac{A \log(x)}{a^3} + \frac{2cx^2(6a^2Bc + A(b^3 - 7abc)) + A(16a^2c^2 - 15ab^2c + 2b^4) + 6a^2bBc}{4a^2(b^2 - 4ac)^2(a+bx^2+cx^4)} \quad (12a)$$

Rubi [A] time = 0.54, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{2cx^2(6a^2Bc + A(b^3 - 7abc)) + A(16a^2c^2 - 15ab^2c + 2b^4) + 6a^2bBc}{4a^2(b^2 - 4ac)^2(a+bx^2+cx^4)} - \frac{(12a^3Bc^2 - A(30a^2bc^2 - 10ab^3c + b^5)) \tanh^{-1}\left(\frac{bx+2cx^2}{\sqrt{b^2-4ac}}\right) - \frac{A \log(a+bx^2+cx^4)}{4a^3} + \frac{A \log(x)}{a^3} - \frac{-A(b^2-2ac) + cx^2(-Ab-2aB) + abB}{4a(b^2-4ac)(a+bx^2+cx^4)^2}}{2a^3(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x]

[Out] $-(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*c*(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^{(5/2)}) + (A*\text{Log}[x])/a^3 - (A*\text{Log}[a + b*x^2 + c*x^4])/(4*a^3)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x(a + bx + cx^2)^3} dx, x, x^2 \right) \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{\text{Subst} \left(\int \frac{-2A(b^2 - 4ac) - 3(Ab - 2aB)cx}{x(a + bx + cx^2)^2} dx, x, x^2 \right)}{4a(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{6a^2bBc + A(2b^4 - 15ab^2c + 16a^2c^2) + 2c}{4a^2(b^2 - 4ac)^2(a + bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.69, size = 396, normalized size = 1.57

$$\frac{\frac{a^2(A(-2ac + b^2 + bcx^2) - aB(b + 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{a(2a^2c(8Ac + 3bB + 6Bc^2) - aAb(15b + 14cx^2) + 2AB^2)(b + cx^2)}{(b^2 - 4ac)^2(a + bx^2 + cx^4)} - \frac{\left(A(16a^2c^2\sqrt{b^2 - 4ac} + 30a^2b^2c^2 - 10ab^3c - 8ab^2c\sqrt{b^2 - 4ac} + b^4\sqrt{b^2 - 4ac} + b^5) - 12a^2Bc^2 \right) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{(12a^2Bc^2 + A(16a^2c^2\sqrt{b^2 - 4ac} - 30a^2b^2c^2 + 10ab^3c - 8ab^2c\sqrt{b^2 - 4ac} + b^4\sqrt{b^2 - 4ac} + b^5)) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}}}{4a^3} + 4A \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x]

[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2))

$$+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*sqrt[b^2 - 4*a*c] - 8*a*b^2*c*sqrt[b^2 - 4*a*c] + 16*a^2*c^2*sqrt[b^2 - 4*a*c]))*Log[b - sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*sqrt[b^2 - 4*a*c] - 8*a*b^2*c*sqrt[b^2 - 4*a*c] + 16*a^2*c^2*sqrt[b^2 - 4*a*c]))*Log[b + sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x]

fricas [B] time = 7.45, size = 2494, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*

$$\begin{aligned}
& A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A \\
& *a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*\log(x))/(\\
& a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4 \\
& *b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 \\
& + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4 \\
& *c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48 \\
& *a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5 \\
& *c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a \\
& ^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b \\
& ^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4 \\
& *b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a \\
& ^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b \\
& ^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2* \\
& (A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - \\
& 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3) \\
& *c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(\\
& 2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b) \\
& *\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a \\
& ^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12 \\
& *A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b \\
& ^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2 \\
& 4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A \\
& *a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*\log(c*x^4 + b*x^2 + a) \\
& - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6 \\
& *c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - \\
& 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a \\
& b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b \\
& ^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*\log(x))/(a^5 \\
& b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^ \\
& 4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + \\
& 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c \\
& ^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6 \\
& *b^3*c^2 - 64*a^7*b*c^3)*x^2)]
\end{aligned}$$

giac [A] time = 6.44, size = 421, normalized size = 1.67

$$\frac{(A^5 - 10 A a b^2 c - 12 B a^2 c^2 + 30 A a^2 b c^2) \arctan\left(\frac{2 c x^2 + b}{\sqrt{-b^2 + 4 a c}}\right) - \frac{A \log(x^2 + a)}{4 a^3} - \frac{A \log(x^2)}{2 a^3} - \frac{3 A a^2 c^2 - 24 A a b^2 c^2 + 48 A a^2 c^2 + 6 A b^2 c^2 - 44 A a b^2 c^2 + 24 B a^2 c^2 + 68 A a^2 b c^2 + 3 A b^2 c^2 - 10 A a b^2 c^2 + 36 B a^2 b c^2 - 58 A a^2 b^2 c^2 + 128 A a^2 c^2 + 10 A a b^2 c^2 + 8 B a^2 b c^2 - 72 A a^2 b^2 c^2 + 40 B a^2 c^2 + 92 A a^2 b c^2 - 2 B a^2 c^2 + 9 A a b^2 c^2 + 20 B a^2 b c^2 - 66 A a^2 b^2 c^2 + 96 A a^2 c^2}{8 (c^2 b^3 - 8 a^2 b^2 c + 16 a^2 c^2) (c^2 + b^2 + a)}}{2 (a^5 b^6 - 8 a^6 b^4 c + 16 a^7 b^2 c^2 + 4 a^8 c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $-1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*\sqrt{-b^2 + 4*a*c}) - 1/4*A*\log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*\log(x^2)/a^3 + 1/8*(3*A*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A$

$$*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*x^2 + a)^2)$$

maple [B] time = 0.03, size = 1161, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x)

[Out] $\frac{9/2}{(c*x^4+b*x^2+a)^2*c^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*B-1/2} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b*c^2+1} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2*c+1} + \frac{1/2}{a^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^5+5*a} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*c^2+5} + \frac{1/2}{a} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*b*B*c+2} + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^2-1/2} + \frac{1/2}{a^3} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)} + \frac{1/2}{(4*a*c-b^2)^{1/2}} + \frac{1/2}{(4*a*c-b^2)^{1/2}} * \arctan\left(\frac{2*c*x^2+b}{(4*a*c-b^2)^{1/2}}\right) + \frac{1/2}{a^3-1/4} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3-7/2} + \frac{1/2}{a} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b+1/2} + \frac{1/2}{(c*x^4+b*x^2+a)^2*c^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^3-29/4} + \frac{1/2}{a} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^2+1} + \frac{1/2}{(c*x^4+b*x^2+a)^2*c} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^4-3/a} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3*c-15/a} + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)} + \frac{1/2}{(4*a*c-b^2)^{1/2}} + \frac{1/2}{(4*a*c-b^2)^{1/2}} * \arctan\left(\frac{2*c*x^2+b}{(4*a*c-b^2)^{1/2}}\right) + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)} + \frac{1/2}{(4*a*c-b^2)^{1/2}} * \arctan\left(\frac{2*c*x^2+b}{(4*a*c-b^2)^{1/2}}\right) + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^3*c-21/4} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^2*c+6/a} + \frac{1/2}{(c*x^4+b*x^2+a)^2} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*A*c^2+3/4} + \frac{1/2}{a} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A-1/4} + \frac{1/2}{a^3} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*b^4+6} + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)} + \frac{1/2}{(4*a*c-b^2)^{1/2}} * \arctan\left(\frac{2*c*x^2+b}{(4*a*c-b^2)^{1/2}}\right) + \frac{1/2}{(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2+3} + \frac{1/2}{(c*x^4+b*x^2+a)^2*c^3} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B+4} + \frac{1/2}{(c*x^4+b*x^2+a)^2*c^3} + \frac{16*a^2*c^2-8*a*b^2*c+b^4}{(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is $4*a*c-b^2$ positive or negative?

mupad [B] time = 11.57, size = 11674, normalized size = 46.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3), x)$

[Out]
$$\begin{aligned} & ((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^4 \\ & + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c - A* \\ & a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(\\ & 16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^4 + \\ & 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(2*a^2 \\ & *(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a* \\ & b*x^2 + 2*b*c*x^6) + (A*\log(x))/a^3 - (\log(((c^5*x^2*(A*b^3 + 6*B*a^2*c - 7 \\ & *A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10 \\ & *A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*A^2*b \\ & ^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A^2*a* \\ & b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(a^4*(\\ & 4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a \\ & ^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^3*c^2 \\ & - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^2) + \\ & (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4*a*c \\ & - b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a \\ & ^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a \\ & ^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4*c^3 \\ & - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*b^2*c \\ & ^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^3 + 6 \\ & *B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((c^5*x^2*(A*b^3 + 6*B*a^2* \\ & c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 \\ & - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4* \\ & A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A \\ & ^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(\\ & a^4*(4*a*c - b^2)^4) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3 \\ & 0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^ \\ & 3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^ \\ & 2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4 \\ & *a*c - b^2)^2) + (b*c^2*(A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3 \\ & 0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^ \\ & 2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4 \\ & *c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3* \\ & b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^ \\ & 3 + 6*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((2*A*b^10 - 2048*A*a^ \\ & \end{aligned}$$

$$\begin{aligned}
& 5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4* \\
& b^2*c^4)/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - \\
& 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (\operatorname{atan}((x^2*(((30720*B*a^11*c^9 \\
& + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9 \\
& *c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B* \\
& a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7 \\
& - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^ \\
& 8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2 \\
& *A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^ \\
& 4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7* \\
& b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227 \\
& 328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80 \\
& *a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^ \\
& 12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3 \\
& 840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3* \\
& c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 - \\
& 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + \\
& 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13* \\
& b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9* \\
& b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/ \\
& (8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640* \\
& a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^ \\
& 6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 \\
& - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^ \\
& 2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096 \\
& *a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2 \\
& *c^4)) - (((6*A^2*a^2*b^11*c^4 - 137*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6 \\
& - 5256*A^2*a^5*b^5*c^7 + 11024*A^2*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288* \\
& B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9 - 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8 \\
& + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A \\
& *B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 \\
& - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - (((30720*B*a \\
& ^11*c^9 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A* \\
& a^6*b^9*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 \\
& - 48*B*a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9* \\
& b^4*c^7 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + \\
& 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) \\
&) - ((2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A \\
& *a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 3 \\
& 28*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^ \\
& 6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c \\
& ^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)* \\
& (a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6* \\
& c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - \\
& 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4
\end{aligned}$$

$$\begin{aligned}
&)) / (2 * (4 * a^3 * b^{10} - 4096 * a^8 * c^5 - 80 * a^4 * b^8 * c + 640 * a^5 * b^6 * c^2 - 2560 * a^6 * b^4 * c^3 + 5120 * a^7 * b^2 * c^4)) * (A * b^5 - 12 * B * a^3 * c^2 - 10 * A * a * b^3 * c + 30 * A * a^2 * b * c^2) / (4 * a^3 * (4 * a * c - b^2)^{(5/2)}) + ((A * b^5 - 12 * B * a^3 * c^2 - 10 * A * a * b^3 * c + 30 * A * a^2 * b * c^2)^3 * (163840 * a^{13} * b * c^9 - 12 * a^6 * b^{15} * c^2 + 328 * a^7 * b^{13} * c^3 - 3840 * a^8 * b^{11} * c^4 + 24960 * a^9 * b^9 * c^5 - 97280 * a^{10} * b^7 * c^6 + 227328 * a^{11} * b^5 * c^7 - 294912 * a^{12} * b^3 * c^8)) / (64 * a^9 * (4 * a * c - b^2)^{(15/2)} * (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5)) * (3 * A * b^8 + 160 * A * a^4 * c^4 - 39 * A * a * b^6 * c + 18 * B * a^4 * b * c^3 + 180 * A * a^2 * b^4 * c^2 - 325 * A * a^3 * b^2 * c^3 - 6 * B * a^3 * b^3 * c^2) / (8 * a^3 * c^2 * (4 * a * c - b^2)^{(13/2)} * (6 * A^2 * b^{10} - 6400 * A^2 * a^5 * c^5 - 36 * B^2 * a^6 * c^4 + 960 * A^2 * a^2 * b^6 * c^2 - 3850 * A^2 * a^3 * b^4 * c^3 + 7775 * A^2 * a^4 * b^2 * c^4 - 120 * A^2 * a * b^8 * c + 6 * A * B * a^3 * b^5 * c^2 - 60 * A * B * a^4 * b^3 * c^3 + 180 * A * B * a^5 * b * c^4)) + (((A^3 * b^9 * c^5 + 216 * B^3 * a^6 * c^8 + 147 * A^3 * a^2 * b^5 * c^7 - 343 * A^3 * a^3 * b^3 * c^8 - 21 * A^3 * a * b^7 * c^6 - 756 * A * B^2 * a^5 * b * c^8 + 108 * A * B^2 * a^4 * b^3 * c^7 + 18 * A^2 * B * a^2 * b^6 * c^6 - 252 * A^2 * B * a^3 * b^4 * c^7 + 882 * A^2 * B * a^4 * b^2 * c^8) / (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5) - (((6 * A^2 * a^2 * b^{11} * c^4 - 137 * A^2 * a^3 * b^9 * c^5 + 1217 * A^2 * a^4 * b^7 * c^6 - 5256 * A^2 * a^5 * b^5 * c^7 + 11024 * A^2 * a^6 * b^3 * c^8 + 36 * B^2 * a^6 * b^5 * c^6 - 288 * B^2 * a^7 * b^3 * c^7 + 7680 * A * B * a^8 * c^9 - 8960 * A^2 * a^7 * b * c^9 + 576 * B^2 * a^8 * b * c^8 + 42 * A * B * a^4 * b^8 * c^5 - 660 * A * B * a^5 * b^6 * c^6 + 3744 * A * B * a^6 * b^4 * c^7 - 9024 * A * B * a^7 * b^2 * c^8) / (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5) - (((30720 * B * a^{11} * c^9 + 5120 * A * a^{10} * b * c^9 + 2 * A * a^4 * b^{13} * c^3 - 36 * A * a^5 * b^{11} * c^4 + 276 * A * a^6 * b^9 * c^5 - 1216 * A * a^7 * b^7 * c^6 + 3456 * A * a^8 * b^5 * c^7 - 6144 * A * a^9 * b^3 * c^8 - 48 * B * a^6 * b^{10} * c^4 + 888 * B * a^7 * b^8 * c^5 - 6528 * B * a^8 * b^6 * c^6 + 23808 * B * a^9 * b^4 * c^7 - 43008 * B * a^{10} * b^2 * c^8) / (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5) - ((2 * A * b^{10} - 2048 * A * a^5 * c^5 - 40 * A * a * b^8 * c + 320 * A * a^2 * b^6 * c^2 - 1280 * A * a^3 * b^4 * c^3 + 2560 * A * a^4 * b^2 * c^4) * (163840 * a^{13} * b * c^9 - 12 * a^6 * b^{15} * c^2 + 328 * a^7 * b^{13} * c^3 - 3840 * a^8 * b^{11} * c^4 + 24960 * a^9 * b^9 * c^5 - 97280 * a^{10} * b^7 * c^6 + 227328 * a^{11} * b^5 * c^7 - 294912 * a^{12} * b^3 * c^8)) / (2 * (4 * a^3 * b^{10} - 4096 * a^8 * c^5 - 80 * a^4 * b^8 * c + 640 * a^5 * b^6 * c^2 - 2560 * a^6 * b^4 * c^3 + 5120 * a^7 * b^2 * c^4)) * (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5)) * (2 * A * b^{10} - 2048 * A * a^5 * c^5 - 40 * A * a * b^8 * c + 320 * A * a^2 * b^6 * c^2 - 1280 * A * a^3 * b^4 * c^3 + 2560 * A * a^4 * b^2 * c^4) / (2 * (4 * a^3 * b^{10} - 4096 * a^8 * c^5 - 80 * a^4 * b^8 * c + 640 * a^5 * b^6 * c^2 - 2560 * a^6 * b^4 * c^3 + 5120 * a^7 * b^2 * c^4)) * (2 * A * b^{10} - 2048 * A * a^5 * c^5 - 40 * A * a * b^8 * c + 320 * A * a^2 * b^6 * c^2 - 1280 * A * a^3 * b^4 * c^3 + 2560 * A * a^4 * b^2 * c^4) / (2 * (4 * a^3 * b^{10} - 4096 * a^8 * c^5 - 80 * a^4 * b^8 * c + 640 * a^5 * b^6 * c^2 - 2560 * a^6 * b^4 * c^3 + 5120 * a^7 * b^2 * c^4)) - (((((30720 * B * a^{11} * c^9 + 5120 * A * a^{10} * b * c^9 + 2 * A * a^4 * b^{13} * c^3 - 36 * A * a^5 * b^{11} * c^4 + 276 * A * a^6 * b^9 * c^5 - 1216 * A * a^7 * b^7 * c^6 + 3456 * A * a^8 * b^5 * c^7 - 6144 * A * a^9 * b^3 * c^8 - 48 * B * a^6 * b^{10} * c^4 + 888 * B * a^7 * b^8 * c^5 - 6528 * B * a^8 * b^6 * c^6 + 23808 * B * a^9 * b^4 * c^7 - 43008 * B * a^{10} * b^2 * c^8) / (a^6 * b^{12} + 4096 * a^{12} * c^6 - 24 * a^7 * b^{10} * c + 240 * a^8 * b^8 * c^2 - 1280 * a^9 * b^6 * c^3 + 3840 * a^{10} * b^4 * c^4 - 6144 * a^{11} * b^2 * c^5) -
\end{aligned}$$

$$\begin{aligned}
& ((2*A*b^{10} - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^{13}*b*c^9 - 12*a^6*b^{15}*c^2 + 328*a^7*b^{13}*c^3 - 3840*a^8*b^{11}*c^4 + 24960*a^9*b^9*c^5 - 97280*a^{10}*b^7*c^6 + 227328*a^{11}*b^5*c^7 - 294912*a^{12}*b^3*c^8))/(2*(4*a^3*b^{10} - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^{(5/2)}) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^{10} - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^{13}*b*c^9 - 12*a^6*b^{15}*c^2 + 328*a^7*b^{13}*c^3 - 3840*a^8*b^{11}*c^4 + 24960*a^9*b^9*c^5 - 97280*a^{10}*b^7*c^6 + 227328*a^{11}*b^5*c^7 - 294912*a^{12}*b^3*c^8))/(8*a^3*(4*a*c - b^2)^{(5/2)}*(4*a^3*b^{10} - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^{(5/2)}) + ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2*(2*A*b^{10} - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^{13}*b*c^9 - 12*a^6*b^{15}*c^2 + 328*a^7*b^{13}*c^3 - 3840*a^8*b^{11}*c^4 + 24960*a^9*b^9*c^5 - 97280*a^{10}*b^7*c^6 + 227328*a^{11}*b^5*c^7 - 294912*a^{12}*b^3*c^8))/(32*a^6*(4*a*c - b^2)^5*(4*a^3*b^{10} - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(3*A*b^7 + 6*B*a^4*c^3 - 33*A*a*b^5*c - 135*A*a^3*b*c^3 + 120*A*a^2*b^3*c^2 - 6*B*a^3*b^2*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(6*A^2*b^{10} - 6400*A^2*a^5*c^5 - 36*B^2*a^6*c^4 + 960*A^2*a^2*b^6*c^2 - 3850*A^2*a^3*b^4*c^3 + 7775*A^2*a^4*b^2*c^4 - 120*A^2*a*b^8*c + 6*A*B*a^3*b^5*c^2 - 60*A*B*a^4*b^3*c^3 + 180*A*B*a^5*b*c^4)))*(16*a^9*b^{12}*(4*a*c - b^2)^{(15/2)} + 65536*a^{15}*c^6*(4*a*c - b^2)^{(15/2)} - 384*a^{10}*b^{10}*c*(4*a*c - b^2)^{(15/2)} + 3840*a^{11}*b^8*c^2*(4*a*c - b^2)^{(15/2)} - 20480*a^{12}*b^6*c^3*(4*a*c - b^2)^{(15/2)} + 61440*a^{13}*b^4*c^4*(4*a*c - b^2)^{(15/2)} - 98304*a^{14}*b^2*c^5*(4*a*c - b^2)^{(15/2)))/(A^2*b^{10}*c^2 + 144*B^2*a^6*c^6 + 160*A^2*a^2*b^6*c^4 - 600*A^2*a^3*b^4*c^5 + 900*A^2*a^4*b^2*c^6 - 20*A^2*a*b^8*c^3 - 24*A*B*a^3*b^5*c^4 + 240*A*B*a^4*b^3*c^5 - 720*A*B*a^5*b*c^6) - (((((((384*B*a^9*b*c^6 - 4*A*a^4*b^{10}*c^2 + 68*A*a^5*b^8*c^3 - 444*A*a^6*b^6*c^4 + 1312*A*a^7*b^4*c^5 - 1472*A*a^8*b^2*c^6 + 24*B*a^7*b^5*c^4 - 192*B*a^8*b^3*c^5)/(a^6*b^8 + 256*a^{10}*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - ((4*a^7*b^{10}*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^{10}*b^4*c^5 + 1024*a^{11}*b^2*c^6)*(2*A*b^{10} - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(a^6*b^8 + 256*a^{10}*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^{10} - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^{(5/2)}) - ((A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(4*a^7*b^{10}*c^2 - 64*a
\end{aligned}$$

$$\begin{aligned}
& ^8b^8c^3 + 384a^9b^6c^4 - 1024a^{10}b^4c^5 + 1024a^{11}b^2c^6) \cdot (2Ab^{10} - 2048A^5c^5 - 40A^2b^8c + 320A^2b^6c^2 - 1280A^3b^4c^3 + 2560A^4b^2c^4) / (8a^3(4ac - b^2)^{(5/2)} \cdot (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) \cdot (4a^3b^{10} - 4096a^8c^5 - 80a^4b^8c + 640a^5b^6c^2 - 2560a^6b^4c^3 + 5120a^7b^2c^4)) \cdot (2Ab^{10} - 2048A^5c^5 - 40A^2b^8c + 320A^2b^6c^2 - 1280A^3b^4c^3 + 2560A^4b^2c^4) / (2(4a^3b^{10} - 4096a^8c^5 - 80a^4b^8c + 640a^5b^6c^2 - 2560a^6b^4c^3 + 5120a^7b^2c^4)) - (((36B^2a^7c^6 - 4A^2a^2b^8c^3 + 61A^2a^3b^6c^4 - 302A^2a^4b^4c^5 + 497A^2a^5b^2c^6 - 24ABa^4b^5c^4 + 204ABa^5b^3c^5 - 468ABa^6b^2c^6) / (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) - (((384B^2a^9b^2c^6 - 4A^2a^4b^10c^2 + 68A^5b^8c^3 - 444A^6b^6c^4 + 1312A^7b^4c^5 - 1472A^8b^2c^6 + 24B^2a^7b^5c^4 - 192B^2a^8b^3c^5) / (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) - ((4a^7b^{10}c^2 - 64a^8b^8c^3 + 384a^9b^6c^4 - 1024a^{10}b^4c^5 + 1024a^{11}b^2c^6) \cdot (2Ab^{10} - 2048A^5c^5 - 40A^2b^8c + 320A^2b^6c^2 - 1280A^3b^4c^3 + 2560A^4b^2c^4)) / (2(a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) \cdot (4a^3b^{10} - 4096a^8c^5 - 80a^4b^8c + 640a^5b^6c^2 - 2560a^6b^4c^3 + 5120a^7b^2c^4))) \cdot (2Ab^{10} - 2048A^5c^5 - 40A^2b^8c + 320A^2b^6c^2 - 1280A^3b^4c^3 + 2560A^4b^2c^4)) / (2(4a^3b^{10} - 4096a^8c^5 - 80a^4b^8c + 640a^5b^6c^2 - 2560a^6b^4c^3 + 5120a^7b^2c^4))) \cdot (Ab^5 - 12B^2a^3c^2 - 10A^2b^3c + 30A^2b^2c^2)) / (4a^3(4ac - b^2)^{(5/2)}) + ((Ab^5 - 12B^2a^3c^2 - 10A^2b^3c + 30A^2b^2c^2)^3 \cdot (4a^7b^{10}c^2 - 64a^8b^8c^3 + 384a^9b^6c^4 - 1024a^{10}b^4c^5 + 1024a^{11}b^2c^6)) / (64a^9(4ac - b^2)^{(15/2)} \cdot (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3)) \cdot (16a^9b^{12}(4ac - b^2)^{(15/2)} + 65536a^{15}c^6(4ac - b^2)^{(15/2)} - 384a^{10}b^{10}c(4ac - b^2)^{(15/2)} + 3840a^{11}b^8c^2(4ac - b^2)^{(15/2)} - 20480a^{12}b^6c^3(4ac - b^2)^{(15/2)} + 61440a^{13}b^4c^4(4ac - b^2)^{(15/2)} - 98304a^{14}b^2c^5(4ac - b^2)^{(15/2)}) \cdot (3Ab^8 + 160A^4c^4 - 39A^2b^6c + 18B^2a^4b^3c^3 + 180A^2a^2b^4c^2 - 325A^3b^2c^3 - 6B^2a^3b^3c^2)) / (8a^3c^2(4ac - b^2)^{(13/2)} \cdot (A^2b^{10}c^2 + 144B^2a^6c^6 + 160A^2a^2b^6c^4 - 600A^2a^3b^4c^5 + 900A^2a^4b^2c^6 - 20A^2a^5b^8c^3 - 24ABa^3b^5c^4 + 240ABa^4b^3c^5 - 720ABa^5b^2c^6) \cdot (6A^2b^{10} - 6400A^2a^5c^5 - 36B^2a^6c^4 + 960A^2a^2b^6c^2 - 3850A^2a^3b^4c^3 + 7775A^2a^4b^2c^4 - 120A^2a^5b^8c + 6ABa^3b^5c^2 - 60ABa^4b^3c^3 + 180ABa^5b^2c^4)) + (((A^3b^6c^4 + 49A^3a^2b^2c^6 + 36AB^2a^4c^6 - 14A^3a^2b^4c^5 - 84A^2B^2a^3b^2c^6 + 12A^2B^2a^2b^3c^5) / (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) + (((36B^2a^7c^6 - 4A^2a^2b^8c^3 + 61A^2a^3b^6c^4 - 302A^2a^4b^4c^5 + 497A^2a^5b^2c^6 - 24ABa^4b^5c^4 + 204ABa^5b^3c^5 - 468ABa^6b^2c^6) / (a^6b^8 + 256a^{10}c^4 - 16a^7b^6c + 96a^8b^4c^2 - 256a^9b^2c^3) - (((384B^2a^9b^2c^6 - 4A^2a^4b^10c^2 + 68A^5b^8c^3 - 444A^6b^6c^4 + 1312A^7b^4c^5 - 1472A^8b^2c^6 + 24B^2
\end{aligned}$$

$$\begin{aligned}
& a^7 b^5 c^4 - 192 B a^8 b^3 c^5) / (a^6 b^8 + 256 a^{10} c^4 - 16 a^7 b^6 c + 96 a^8 b^4 c^2 - 256 a^9 b^2 c^3) - ((4 a^7 b^{10} c^2 - 64 a^8 b^8 c^3 + 384 a^9 b^6 c^4 - 1024 a^{10} b^4 c^5 + 1024 a^{11} b^2 c^6) * (2 A b^{10} - 2048 A a^5 c^5 - 40 A a b^8 c + 320 A a^2 b^6 c^2 - 1280 A a^3 b^4 c^3 + 2560 A a^4 b^2 c^4)) / (2 (a^6 b^8 + 256 a^{10} c^4 - 16 a^7 b^6 c + 96 a^8 b^4 c^2 - 256 a^9 b^2 c^3) * (4 a^3 b^{10} - 4096 a^8 c^5 - 80 a^4 b^8 c + 640 a^5 b^6 c^2 - 2560 a^6 b^4 c^3 + 5120 a^7 b^2 c^4)) * (2 A b^{10} - 2048 A a^5 c^5 - 40 A a b^8 c + 320 A a^2 b^6 c^2 - 1280 A a^3 b^4 c^3 + 2560 A a^4 b^2 c^4)) / (2 (4 a^3 b^{10} - 4096 a^8 c^5 - 80 a^4 b^8 c + 640 a^5 b^6 c^2 - 2560 a^6 b^4 c^3 + 5120 a^7 b^2 c^4)) * (2 A b^{10} - 2048 A a^5 c^5 - 40 A a b^8 c + 320 A a^2 b^6 c^2 - 1280 A a^3 b^4 c^3 + 2560 A a^4 b^2 c^4)) / (2 (4 a^3 b^{10} - 4096 a^8 c^5 - 80 a^4 b^8 c + 640 a^5 b^6 c^2 - 2560 a^6 b^4 c^3 + 5120 a^7 b^2 c^4)) * (A b^5 - 12 B a^3 c^2 - 10 A a b^3 c + 30 A a^2 b c^2)) / (4 a^3 (4 a c - b^2)^{(5/2)}) - ((A b^5 - 12 B a^3 c^2 - 10 A a b^3 c + 30 A a^2 b c^2) * (4 a^7 b^{10} c^2 - 64 a^8 b^8 c^3 + 384 a^9 b^6 c^4 - 1024 a^{10} b^4 c^5 + 1024 a^{11} b^2 c^6) * (2 A b^{10} - 2048 A a^5 c^5 - 40 A a b^8 c + 320 A a^2 b^6 c^2 - 1280 A a^3 b^4 c^3 + 2560 A a^4 b^2 c^4)) / (8 a^3 (4 a c - b^2)^{(5/2)}) * (a^6 b^8 + 256 a^{10} c^4 - 16 a^7 b^6 c + 96 a^8 b^4 c^2 - 256 a^9 b^2 c^3) * (4 a^3 b^{10} - 4096 a^8 c^5 - 80 a^4 b^8 c + 640 a^5 b^6 c^2 - 2560 a^6 b^4 c^3 + 5120 a^7 b^2 c^4)) * (A b^5 - 12 B a^3 c^2 - 10 A a b^3 c + 30 A a^2 b c^2)) / (4 a^3 (4 a c - b^2)^{(5/2)}) - ((A b^5 - 12 B a^3 c^2 - 10 A a b^3 c + 30 A a^2 b c^2)^2 * (4 a^7 b^{10} c^2 - 64 a^8 b^8 c^3 + 384 a^9 b^6 c^4 - 1024 a^{10} b^4 c^5 + 1024 a^{11} b^2 c^6) * (2 A b^{10} - 2048 A a^5 c^5 - 40 A a b^8 c + 320 A a^2 b^6 c^2 - 1280 A a^3 b^4 c^3 + 2560 A a^4 b^2 c^4)) / (32 a^6 (4 a c - b^2)^5 * (a^6 b^8 + 256 a^{10} c^4 - 16 a^7 b^6 c + 96 a^8 b^4 c^2 - 256 a^9 b^2 c^3) * (4 a^3 b^{10} - 4096 a^8 c^5 - 80 a^4 b^8 c + 640 a^5 b^6 c^2 - 2560 a^6 b^4 c^3 + 5120 a^7 b^2 c^4)) * (3 A b^7 + 6 B a^4 c^3 - 33 A a b^5 c - 135 A a^3 b c^3 + 120 A a^2 b^3 c^2 - 6 B a^3 b^2 c^2) * (16 a^9 b^{12} (4 a c - b^2)^{(15/2)} + 65536 a^{15} c^6 (4 a c - b^2)^{(15/2)}) - 384 a^{10} b^{10} c (4 a c - b^2)^{(15/2)} + 3840 a^{11} b^8 c^2 (4 a c - b^2)^{(15/2)} - 20480 a^{12} b^6 c^3 (4 a c - b^2)^{(15/2)} + 61440 a^{13} b^4 c^4 (4 a c - b^2)^{(15/2)} - 98304 a^{14} b^2 c^5 (4 a c - b^2)^{(15/2))} / (8 a^3 c^2 (4 a c - b^2)^6 * (A^2 b^{10} c^2 + 144 B^2 a^6 c^6 + 160 A^2 a^2 b^6 c^4 - 600 A^2 a^3 b^4 c^5 + 900 A^2 a^4 b^2 c^6 - 20 A^2 a b^8 c^3 - 24 A B a^3 b^5 c^4 + 240 A B a^4 b^3 c^5 - 720 A B a^5 b c^6) * (6 A^2 b^{10} - 6400 A^2 a^5 c^5 - 36 B^2 a^6 c^4 + 960 A^2 a^2 b^6 c^2 - 3850 A^2 a^3 b^4 c^3 + 7775 A^2 a^4 b^2 c^4 - 120 A^2 a b^8 c + 6 A B a^3 b^5 c^2 - 60 A B a^4 b^3 c^3 + 180 A
\end{aligned}$$

$$\frac{(B a^5 b c^4) (A b^5 - 12 B a^3 c^2 - 10 A a b^3 c + 30 A a^2 b c^2)}{2 a^3 (4 a c - b^2)^{5/2}}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.109 \quad \int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=363

$$\frac{(3Ab - aB) \log(a + bx^2 + cx^4)}{4a^4} - \frac{\log(x)(3Ab - aB)}{a^4} - \frac{-A(20a^2c^2 - 20ab^2c + 3b^4) + cx^2(aB(b^2 - 16ac) - 3A(b^3 - 4a^2x^2(b^2 - 4ac)^2(a + bx^2 + cx^4))}{4a^2x^2(b^2 - 4ac)^2(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.77, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{abB(b^2 - 7ac) - 3A(10a^2c^2 - 7ab^2c + b^4)}{2a^3b^2(b^2 - 4ac)^2} - \frac{-A(20a^2c^2 - 20ab^2c + 3b^4) + cx^2(aB(b^2 - 16ac) - 3A(b^3 - 6abc)) + abB(b^2 - 10ac)}{4a^2b^2(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{(abB(30a^2c^2 - 10ab^2c + b^4) - 3A(30a^2b^2c^2 - 20a^3c^3 - 10ab^4c + b^5)) \tanh^{-1}\left(\frac{bx^2}{\sqrt{b^2 - 4ac}}\right)}{2a^4(b^2 - 4ac)^{3/2}} + \frac{(3Ab - aB) \log(a + bx^2 + cx^4)}{4a^4} - \frac{\log(x)(3Ab - aB)}{a^4} - \frac{-A(b^2 - 2ac) + cx^2(-aB - 2aB) + abB}{4a^2x^2(b^2 - 4ac)^2(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3), x]

[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c)))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c + 30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 822

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^m*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^2 (a + bx + cx^2)^3} dx, x, x^2 \right) \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} - \frac{\text{Subst} \left(\int \frac{-3Ab^2 + abB + 10aAc - 4(Ab - 2aB)cx}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right)}{4a(b^2 - 4ac)} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} - \frac{abB(b^2 - 10ac) - A(3b^4 - 20ab^2c + 20a^2c^2)}{4a^2(b^2 - 4ac)^2 x^2} \\
&= -\frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} - \frac{abB(b^2 - 10ac) - A(3b^4 - 20ab^2c + 20a^2c^2)}{4a^2(b^2 - 4ac)^2 x^2} \\
&= \frac{abB(b^2 - 7ac) - 3A(b^4 - 7ab^2c + 10a^2c^2)}{2a^3(b^2 - 4ac)^2 x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} \\
&= \frac{abB(b^2 - 7ac) - 3A(b^4 - 7ab^2c + 10a^2c^2)}{2a^3(b^2 - 4ac)^2 x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} \\
&= \frac{abB(b^2 - 7ac) - 3A(b^4 - 7ab^2c + 10a^2c^2)}{2a^3(b^2 - 4ac)^2 x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2} \\
&= \frac{abB(b^2 - 7ac) - 3A(b^4 - 7ab^2c + 10a^2c^2)}{2a^3(b^2 - 4ac)^2 x^2} - \frac{abB - A(b^2 - 2ac) - (Ab - 2aB)cx^2}{4a(b^2 - 4ac)x^2(a + bx^2 + cx^4)^2}
\end{aligned}$$

Mathematica [A] time = 1.50, size = 642, normalized size = 1.77

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3), x]

[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*

$$\frac{b^4 - 15ab^2c + 16a^2c^2 + 2b^3cx^2 - 14ab^2c^2x^2 - A(4b^5 - 29ab^3c + 46a^2b^2c^2 + 4b^4cx^2 - 26ab^2c^2x^2 + 28a^2c^3x^2)}{((b^2 - 4ac)^2(a + bx^2 + cx^4)) + 4(-3Ab + aB)\text{Log}[x] + ((-aB(b^5 - 10ab^3c + 30a^2b^2c^2 + b^4\sqrt{b^2 - 4ac}) - 8ab^2c\sqrt{b^2 - 4ac} + 16a^2c^2\sqrt{b^2 - 4ac})) + 3A(b^6 - 10ab^4c + 30a^2b^2c^2 - 20a^3c^3 + b^5\sqrt{b^2 - 4ac}) - 8ab^3c\sqrt{b^2 - 4ac} + 16a^2b^2c^2\sqrt{b^2 - 4ac})\text{Log}[b - \sqrt{b^2 - 4ac} + 2cx^2]}{(b^2 - 4ac)^{5/2} + ((aB(b^5 - 10ab^3c + 30a^2b^2c^2 - b^4\sqrt{b^2 - 4ac}) + 8ab^2c\sqrt{b^2 - 4ac} - 16a^2c^2\sqrt{b^2 - 4ac}) + 3A(-b^6 + 10ab^4c - 30a^2b^2c^2 + 20a^3c^3 + b^5\sqrt{b^2 - 4ac}) - 8ab^3c\sqrt{b^2 - 4ac} + 16a^2b^2c^2\sqrt{b^2 - 4ac})\text{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2)]/(b^2 - 4ac)^{5/2}}/(4a^4)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3), x]

[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3), x]

fricas [B] time = 14.44, size = 3956, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2* \\ & (120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^{10} + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3 \end{aligned}$$

$$\begin{aligned}
& *b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B* \\
& a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^4 + 2*b*c*x^2 \\
& + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((\\
& 64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b \\
& ^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^{10} + 2*(64*(B*a^4*b - 3*A* \\
& a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6 \\
&)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A* \\
& a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5 \\
&)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(\\
& B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b \\
& ^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b \\
&)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)* \\
& x^2)*\log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^ \\
& 2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7) \\
&)*c^2)*x^{10} + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^ \\
& 4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a \\
& *b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3) \\
&)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 \\
& - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^ \\
& 3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3 \\
& *A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 \\
& - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*\log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c \\
& ^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^{10} + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48 \\
& *a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 \\
& + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b \\
& ^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64* \\
& a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128* \\
& A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^ \\
& 3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - \\
& 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3* \\
& A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^ \\
& 8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A \\
& *a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^ \\
& 2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^ \\
& 2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b \\
& - 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c \\
& ^2)*x^{10} + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2 \\
& *b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1 \\
& 20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4) \\
&)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A* \\
& a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5) \\
&)*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b \\
& ^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-(\\
& 2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c \\
& ^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
\end{aligned}$$

$$\begin{aligned}
& *a*b^6 - 3*A*b^7)*c^2)*x^{10} + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3 \\
& *b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b \\
& ^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b \\
& ^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - \\
& 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c \\
& ^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 \\
& - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - \\
& 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*\log(c*x^4 + b*x^2 + \\
& a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 1 \\
& 2*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^{10} + 2*(64*(B*a^ \\
& 4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - \\
& 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B* \\
& a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3 \\
& *A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a \\
& b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 1 \\
& 2*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - \\
& 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^ \\
& 3*b^5)*c)*x^2)*\log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64 \\
& *a^7*c^5)*x^{10} + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b \\
& c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128* \\
& a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x \\
& ^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
\end{aligned}$$

giac [A] time = 6.37, size = 648, normalized size = 1.79

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 - \\
& 90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\\
& (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*\sqrt{-b^2 + 4*a*c}) + 1/8*(3*B*a*b^4*c \\
& ^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B \\
& *a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44* \\
& B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^ \\
& 2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c \\
& *x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12 \\
& 8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 - \\
& 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3* \\
& b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2* \\
& c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2 \\
& *c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*\log(c*x^4 + b*x \\
& ^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*\log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A \\
& *a)/(a^4*x^2)
\end{aligned}$$

maple [B] time = 0.04, size = 1862, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3, x)$

[Out]
$$\frac{6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^4*c-3/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^3*c+13/2/a^2/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^2-1/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^4-7/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b*B+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^3-37/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b+55/4/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^3+45/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b^2*c^2-15/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b^4*c-15/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*B*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*B*b^3*c-2/a^3/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^5-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^2+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^4-7/2/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^2*c^2-3/a^4*\ln(x)*A*b+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^4+3/4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*\ln(c*x^4+b*x^2+a)*A*b^5-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*\ln(c*x^4+b*x^2+a)*B-1/4/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*\ln(c*x^4+b*x^2+a)*B*b^4+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*c^3-29/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^2*c-1/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b*c^2-7/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^6+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^5+9/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^3*c+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*\ln(c*x^4+b*x^2+a)*A*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*\ln(c*x^4+b*x^2+a)*A*b^3+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*\ln(c*x^4+b*x^2+a)*B*b^2-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*c^3-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*B*b^5+3/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*A*b^6-5/4/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^5+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2-1/2*A/a^3/x^2+1/a^3*\ln(x)*B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 15.91, size = 16265, normalized size = 44.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b
*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^
6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2
*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3
- 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A
*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3 - 3*A
*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*A*a^2*b
^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^3 - 3*A*
b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*c^2))/(a^
3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4)))/(4*
a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A*B*a*b^9 -
3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c^4 - 302*B
^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a^3*b^6*c +
1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c + 2340*A*B
*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9 + 6*B^2*a^2*b^7 -
36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409*B^2*a^4*b
^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4 - 89*B^2*a
^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^4*b^2*c^3
+ 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A*b - B*a)*
(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2)/(a^9*(4*
a*c - b^2)^4))*((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c +
7*B*a^2*b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*A*b - B*a + a^4*(-(60*A*a^3*c^
3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90
*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((3*A*b - B*a + a^4*(-(60*
A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*
c^2 - 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3
*c^3 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 -
69*A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*
```

$$\begin{aligned}
& c^3 - 3A^*b^6 + 6A^*a^*b^4*c - 2B^*a^2*b^3*c + 10B^*a^3*b*c^2 + 90A^*a^2*b^2 \\
& *c^2)) / (a^3*(4*a*c - b^2)^2) - (b*c^2*(3A^*b - B^*a + a^4*(-(60A^*a^3*c^3 - \\
& 3A^*b^6 + B^*a*b^5 + 30A^*a*b^4*c - 10B^*a^2*b^3*c + 30B^*a^3*b*c^2 - 90A^*a^2*b^2*c^2)^2 / (a^8*(4*a*c - b^2)^5))^{(1/2)})*(a*b + 3*b^2*x^2 - 10*a*c*x^2)) \\
& / a^4)) / (4*a^4) - (c^3*(900A^2*a^5*c^5 - 36A^2*b^10 - 4B^2*a^2*b^8 + 24A^ \\
& *B^*a*b^9 - 3078A^2*a^2*b^6*c^2 + 7533A^2*a^3*b^4*c^3 - 7020A^2*a^4*b^2*c^ \\
& ^4 - 302B^2*a^4*b^4*c^2 + 497B^2*a^5*b^2*c^3 + 549A^2*a*b^8*c + 61B^2*a^ \\
& ^3*b^6*c + 1932A^*B^*a^3*b^5*c^2 - 4002A^*B^*a^4*b^3*c^3 - 366A^*B^*a^2*b^7*c \\
& + 2340A^*B^*a^5*b*c^4)) / (a^6*(4*a*c - b^2)^4) + (c^4*x^2*(54A^2*b^9 + 6B^2 \\
& *a^2*b^7 - 36A^*B^*a*b^8 + 4311A^2*a^2*b^5*c^2 - 9900A^2*a^3*b^3*c^3 + 409 \\
& *B^2*a^4*b^3*c^2 - 2400A^*B^*a^5*c^4 - 801A^2*a*b^7*c + 8100A^2*a^4*b*c^4 \\
& - 89B^2*a^3*b^5*c - 560B^2*a^5*b*c^3 - 2664A^*B^*a^3*b^4*c^2 + 4980A^*B^*a^ \\
& 4*b^2*c^3 + 534A^*B^*a^2*b^6*c)) / (a^6*(4*a*c - b^2)^4)) / (4*a^4) + (c^4*(3A^ \\
& *b - B^*a)*(3A^*b^4 + 30A^*a^2*c^2 - B^*a*b^3 - 21A^*a*b^2*c + 7B^*a^2*b*c)^2 \\
&) / (a^9*(4*a*c - b^2)^4)) * (6A^*b^11 + 2048B^*a^6*c^5 - 2B^*a*b^10 - 120A^*a \\
& *b^9*c - 6144A^*a^5*b*c^5 + 40B^*a^2*b^8*c + 960A^*a^2*b^7*c^2 - 3840A^*a^3 \\
& *b^5*c^3 + 7680A^*a^4*b^3*c^4 - 320B^*a^3*b^6*c^2 + 1280B^*a^4*b^4*c^3 - 25 \\
& 60B^*a^5*b^2*c^4)) / (2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b \\
& ^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (log(x)*(3A^*b - B^*a)) / a^4 \\
& - (A / (2*a) + (x^2*(9A^*b^5 - 24B^*a^3*c^2 - 3B^*a*b^4 - 68A^*a*b^3*c + 122 \\
& *A^*a^2*b*c^2 + 21B^*a^2*b^2*c)) / (4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x \\
& ^6*(16B^*a^3*c^3 - 12A^*b^5*c + 4B^*a*b^4*c + 87A^*a*b^3*c^2 - 138A^*a^2*b* \\
& c^3 - 29B^*a^2*b^2*c^2)) / (4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(3A^ \\
& *b^6 + 50A^*a^3*c^3 - B^*a*b^5 - 18A^*a*b^4*c + 6B^*a^2*b^3*c + B^*a^3*b*c^2 \\
& + 7A^*a^2*b^2*c^2)) / (2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^8*(3A^ \\
& b^4 + 30A^*a^2*c^2 - B^*a*b^3 - 21A^*a*b^2*c + 7B^*a^2*b*c)) / (2*a^3*(b^4 + 1 \\
& 6*a^2*c^2 - 8*a*b^2*c))) / (x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^ \\
& 4 + 2*b*c*x^8) - (atan((x^2*((((((((((((((((153600A^*a^13*c^10 - 5120B^*a^13*b*c^9 \\
& + 6A^*a^6*b^14*c^3 - 108A^*a^7*b^12*c^4 + 588A^*a^8*b^10*c^5 + 792A^*a^9*b^ \\
& 8*c^6 - 22272A^*a^10*b^6*c^7 + 100608A^*a^11*b^4*c^8 - 199680A^*a^12*b^2*c^ \\
& 9 - 2B^*a^7*b^13*c^3 + 36B^*a^8*b^11*c^4 - 276B^*a^9*b^9*c^5 + 1216B^*a^10* \\
& b^7*c^6 - 3456B^*a^11*b^5*c^7 + 6144B^*a^12*b^3*c^8)) / (a^9*b^12 + 4096*a^15* \\
& c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4 \\
& *c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1 \\
& 0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + \\
& 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)) * (6A^*b^11 + 2048B^*a^6*c^5 - 2* \\
& B^*a*b^10 - 120A^*a*b^9*c - 6144A^*a^5*b*c^5 + 40B^*a^2*b^8*c + 960A^*a^2*b^ \\
& 7*c^2 - 3840A^*a^3*b^5*c^3 + 7680A^*a^4*b^3*c^4 - 320B^*a^3*b^6*c^2 + 1280* \\
& B^*a^4*b^4*c^3 - 2560B^*a^5*b^2*c^4)) / (2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5 \\
& *b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) * (a^9*b^12 + \\
& 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 38 \\
& 40*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)) * (60A^*a^3*c^3 - 3A^*b^6 + B^*a*b^5 + \\
& 30A^*a*b^4*c - 10B^*a^2*b^3*c + 30B^*a^3*b*c^2 - 90A^*a^2*b^2*c^2)) / (4*a^4* \\
& (4*a*c - b^2)^{(5/2)}) - ((60A^*a^3*c^3 - 3A^*b^6 + B^*a*b^5 + 30A^*a*b^4*c - \\
& 10B^*a^2*b^3*c + 30B^*a^3*b*c^2 - 90A^*a^2*b^2*c^2) * (163840*a^16*b*c^9 - 12
\end{aligned}$$

$$\begin{aligned}
& a^9 b^{15} c^2 + 328 a^{10} b^{13} c^3 - 3840 a^{11} b^{11} c^4 + 24960 a^{12} b^9 c^5 \\
& - 97280 a^{13} b^7 c^6 + 227328 a^{14} b^5 c^7 - 294912 a^{15} b^3 c^8) * (6 A^* b^{11} \\
& + 2048 B^* a^6 c^5 - 2 B^* a^* b^{10} - 120 A^* a^* b^9 c - 6144 A^* a^5 b^* c^5 + 40 B^* a^ \\
& ^2 b^8 c + 960 A^* a^2 b^7 c^2 - 3840 A^* a^3 b^5 c^3 + 7680 A^* a^4 b^3 c^4 - 32 \\
& 0 B^* a^3 b^6 c^2 + 1280 B^* a^4 b^4 c^3 - 2560 B^* a^5 b^2 c^4) / (8 a^4 (4 a^* c - \\
& b^2)^{(5/2)} * (4 a^4 b^{10} - 4096 a^9 c^5 - 80 a^5 b^8 c + 640 a^6 b^6 c^2 - 2 \\
& 560 a^7 b^4 c^3 + 5120 a^8 b^2 c^4) * (a^9 b^{12} + 4096 a^{15} c^6 - 24 a^{10} b^{10} c \\
& + 240 a^{11} b^8 c^2 - 1280 a^{12} b^6 c^3 + 3840 a^{13} b^4 c^4 - 6144 a^{14} b^2 c^5)) * (6 A^* b^{11} + 2048 B^* a^6 c^5 - 2 B^* a^* b^{10} - 120 A^* a^* b^9 c - 6144 A^* \\
& a^5 b^* c^5 + 40 B^* a^2 b^8 c + 960 A^* a^2 b^7 c^2 - 3840 A^* a^3 b^5 c^3 + 7680 \\
& * A^* a^4 b^3 c^4 - 320 B^* a^3 b^6 c^2 + 1280 B^* a^4 b^4 c^3 - 2560 B^* a^5 b^2 c^4) / (2 * (4 a^4 b^{10} - 4096 a^9 c^5 - 80 a^5 b^8 c + 640 a^6 b^6 c^2 - 2560 a^ \\
& ^7 b^4 c^3 + 5120 a^8 b^2 c^4)) - (((54 A^2 a^3 b^{13} c^4 - 1233 A^2 a^4 b^{11} \\
& c^5 + 11583 A^2 a^5 b^9 c^6 - 57204 A^2 a^6 b^7 c^7 + 156276 A^2 a^7 b^5 c^8 - 223200 A^2 a^8 b^3 c^9 + 6 B^2 a^5 b^{11} c^4 - 137 B^2 a^6 b^9 c^5 + 1 \\
& 217 B^2 a^7 b^7 c^6 - 5256 B^2 a^8 b^5 c^7 + 11024 B^2 a^9 b^3 c^8 - 38400 A^* B^* a^{10} c^{10} + 129600 A^2 a^9 b^* c^{10} - 8960 B^2 a^{10} b^* c^9 - 36 A^* B^* a^4 b^ \\
& ^{12} c^4 + 822 A^* B^* a^5 b^{10} c^5 - 7512 A^* B^* a^6 b^8 c^6 + 34836 A^* B^* a^7 b^6 c^7 - 84864 A^* B^* a^8 b^4 c^8 + 98880 A^* B^* a^9 b^2 c^9) / (a^9 b^{12} + 4096 a^{15} c^6 \\
& - 24 a^{10} b^{10} c + 240 a^{11} b^8 c^2 - 1280 a^{12} b^6 c^3 + 3840 a^{13} b^4 c^4 - 6144 a^{14} b^2 c^5) - (((153600 A^* a^{13} c^{10} - 5120 B^* a^{13} b^* c^9 + 6 A^* a^ \\
& ^6 b^{14} c^3 - 108 A^* a^7 b^{12} c^4 + 588 A^* a^8 b^{10} c^5 + 792 A^* a^9 b^8 c^6 - 22272 A^* a^{10} b^6 c^7 + 100608 A^* a^{11} b^4 c^8 - 199680 A^* a^{12} b^2 c^9 - 2 B^* \\
& a^7 b^{13} c^3 + 36 B^* a^8 b^{11} c^4 - 276 B^* a^9 b^9 c^5 + 1216 B^* a^{10} b^7 c^6 \\
& - 3456 B^* a^{11} b^5 c^7 + 6144 B^* a^{12} b^3 c^8) / (a^9 b^{12} + 4096 a^{15} c^6 - 2 \\
& 4 a^{10} b^{10} c + 240 a^{11} b^8 c^2 - 1280 a^{12} b^6 c^3 + 3840 a^{13} b^4 c^4 - \\
& 6144 a^{14} b^2 c^5) - ((163840 a^{16} b^* c^9 - 12 a^9 b^{15} c^2 + 328 a^{10} b^{13} c^3 \\
& - 3840 a^{11} b^{11} c^4 + 24960 a^{12} b^9 c^5 - 97280 a^{13} b^7 c^6 + 227328 \\
& a^{14} b^5 c^7 - 294912 a^{15} b^3 c^8) * (6 A^* b^{11} + 2048 B^* a^6 c^5 - 2 B^* a^* b^{10} \\
& - 120 A^* a^* b^9 c - 6144 A^* a^5 b^* c^5 + 40 B^* a^2 b^8 c + 960 A^* a^2 b^7 c^2 - \\
& 3840 A^* a^3 b^5 c^3 + 7680 A^* a^4 b^3 c^4 - 320 B^* a^3 b^6 c^2 + 1280 B^* a^4 b^4 c^ \\
& ^3 - 2560 B^* a^5 b^2 c^4) / (2 * (4 a^4 b^{10} - 4096 a^9 c^5 - 80 a^5 b^8 c + 640 \\
& a^6 b^6 c^2 - 2560 a^7 b^4 c^3 + 5120 a^8 b^2 c^4) * (a^9 b^{12} + 4096 a^{15} c^6 - 24 a^{10} b^{10} c + 240 a^{11} b^8 c^2 - 1280 a^{12} b^6 c^3 + 3840 a^{13} \\
& b^4 c^4 - 6144 a^{14} b^2 c^5)) * (6 A^* b^{11} + 2048 B^* a^6 c^5 - 2 B^* a^* b^{10} - 1 \\
& 20 A^* a^* b^9 c - 6144 A^* a^5 b^* c^5 + 40 B^* a^2 b^8 c + 960 A^* a^2 b^7 c^2 - 3840 \\
& * A^* a^3 b^5 c^3 + 7680 A^* a^4 b^3 c^4 - 320 B^* a^3 b^6 c^2 + 1280 B^* a^4 b^4 c^ \\
& ^3 - 2560 B^* a^5 b^2 c^4) / (2 * (4 a^4 b^{10} - 4096 a^9 c^5 - 80 a^5 b^8 c + 640 \\
& a^6 b^6 c^2 - 2560 a^7 b^4 c^3 + 5120 a^8 b^2 c^4)) * (60 A^* a^3 c^3 - 3 A^* b^ \\
& ^6 + B^* a^* b^5 + 30 A^* a^* b^4 c - 10 B^* a^2 b^3 c + 30 B^* a^3 b^* c^2 - 90 A^* a^2 b^ \\
& ^2 c^2) / (4 a^4 (4 a^* c - b^2)^{(5/2)}) + ((60 A^* a^3 c^3 - 3 A^* b^6 + B^* a^* b^5 + \\
& 30 A^* a^* b^4 c - 10 B^* a^2 b^3 c + 30 B^* a^3 b^* c^2 - 90 A^* a^2 b^2 c^2)^3 * (16384 \\
& 0 a^{16} b^* c^9 - 12 a^9 b^{15} c^2 + 328 a^{10} b^{13} c^3 - 3840 a^{11} b^{11} c^4 + 2 \\
& 4960 a^{12} b^9 c^5 - 97280 a^{13} b^7 c^6 + 227328 a^{14} b^5 c^7 - 294912 a^{15} b^3 c^8) / (64 a^{12} (4 a^* c - b^2)^{(15/2)} * (a^9 b^{12} + 4096 a^{15} c^6 - 24 a^{10}
\end{aligned}$$

$$\begin{aligned} & *b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (9A^8b^9 - 160B^8a^5c^4 - 3B^8a^8b - 117A^8a^8b^7c + 570A^8 \\ & *a^4b^6c^4 + 39B^8a^2b^6c + 540A^8a^2b^5c^2 - 1005A^8a^3b^3c^3 - 180B^8a^3b^4c^2 + 325B^8a^4b^2c^3)) / (8*(4a^8c - b^2)^{(13/2)} * (900A^8a^2a^9c^8 \\ & + 6400B^8a^10c^7 - 54A^8a^2a^3b^12c^2 - 960a^6b^6c^4*(B^2a - 36A^2c) + 120a^5b^8c^3*(B^2a - 72A^2c) - 6a^4b^10c^2*(B^2a - 180A^2 \\ & *c) - 25a^8b^2c^6*(311B^2a - 2196A^2c) + 25a^7b^4c^5*(154B^2a - 2763A^2c) + 36A^8B^8a^4b^11c^2 - 720A^8B^8a^5b^9c^3 + 5760A^8B^8a^6b^7 \\ & *c^4 - 23070A^8B^8a^7b^5c^5 + 46350A^8B^8a^8b^3c^6 - 37500A^8B^8a^9b^5c^7)) - (((27000A^8a^3a^6c^11 + 27A^8a^3b^12c^5 + 4779A^8a^3a^2b^8c^7 - 20601 \\ & *A^8a^3a^3b^6c^8 + 47790A^8a^3a^4b^4c^9 - 56700A^8a^3a^5b^2c^10 - B^3a^3 \\ & *b^9c^5 + 21B^3a^4b^7c^6 - 147B^3a^5b^5c^7 + 343B^3a^6b^3c^8 - 567A^3a^8b^10c^6 - 27A^2B^8a^8b^11c^5 + 18900A^2B^8a^6b^8c^10 + 9A^8B^8 \\ & *a^2b^10c^5 - 189A^8B^2a^3b^8c^6 + 1413A^8B^2a^4b^6c^7 - 4347A^8B^2 \\ & *a^5b^4c^8 + 4410A^8B^2a^6b^2c^9 + 567A^2B^8a^2b^9c^6 - 4509A^2B^8 \\ & *a^3b^7c^7 + 16821A^2B^8a^4b^5c^8 - 29160A^2B^8a^5b^3c^9)) / (a^9b^12 \\ & + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) - (((54A^8a^2a^3b^13c^4 - 1233A^2a^4 \\ & *b^11c^5 + 11583A^2a^5b^9c^6 - 57204A^2a^6b^7c^7 + 156276A^2a^7 \\ & *b^5c^8 - 223200A^2a^8b^3c^9 + 6B^2a^5b^11c^4 - 137B^2a^6b^9c^5 \\ & + 1217B^2a^7b^7c^6 - 5256B^2a^8b^5c^7 + 11024B^2a^9b^3c^8 - 38400A^8B^8a^10c^10 + 129600A^2a^9b^8c^10 - 8960B^2a^10b^8c^9 - 36A^8B^8 \\ & *a^4b^12c^4 + 822A^8B^8a^5b^10c^5 - 7512A^8B^8a^6b^8c^6 + 34836A^8B^8a^7 \\ & *b^6c^7 - 84864A^8B^8a^8b^4c^8 + 98880A^8B^8a^9b^2c^9)) / (a^9b^12 + 4096a^{15}c^6 \\ & - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) - (((153600A^8a^13c^10 - 5120B^8a^13b^8c^9 \\ & + 6A^8a^6b^14c^3 - 108A^8a^7b^12c^4 + 588A^8a^8b^10c^5 + 792A^8a^9b^8 \\ & *c^6 - 22272A^8a^10b^6c^7 + 100608A^8a^11b^4c^8 - 199680A^8a^12b^2c^9 \\ & - 2B^8a^7b^13c^3 + 36B^8a^8b^11c^4 - 276B^8a^9b^9c^5 + 1216B^8a^10b^7c^6 \\ & - 3456B^8a^11b^5c^7 + 6144B^8a^12b^3c^8)) / (a^9b^12 + 4096a^{15}c^6 \\ & - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) - ((163840a^{16}b^8c^9 - 12a^9b^15c^2 + 328a^{10} \\ & *b^13c^3 - 3840a^{11}b^11c^4 + 24960a^{12}b^9c^5 - 97280a^{13}b^7c^6 + 227328a^{14}b^5c^7 - 294912a^{15}b^3c^8)) * (6A^8b^11 + 2048B^8a^6c^5 - 2B^8 \\ & *a^8b^10 - 120A^8a^8b^9c - 6144A^8a^5b^8c^5 + 40B^8a^2b^8c + 960A^8a^2b^7c^2 \\ & - 3840A^8a^3b^5c^3 + 7680A^8a^4b^3c^4 - 320B^8a^3b^6c^2 + 1280B^8a^4b^4c^3 \\ & - 2560B^8a^5b^2c^4)) / (2*(4a^4b^10 - 4096a^9c^5 - 80a^5b^8c + 640a^6b^6c^2 \\ & - 2560a^7b^4c^3 + 5120a^8b^2c^4) * (a^9b^12 + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 \\ & - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (6A^8b^11 + 2048B^8a^6c^5 \\ & - 2B^8a^8b^10 - 120A^8a^8b^9c - 6144A^8a^5b^8c^5 + 40B^8a^2b^8c + 960A^8a^2b^7c^2 \\ & - 3840A^8a^3b^5c^3 + 7680A^8a^4b^3c^4 - 320B^8a^3b^6c^2 + 1280B^8a^4b^4c^3 \\ & - 2560B^8a^5b^2c^4)) / (2*(4a^4b^10 - 4096a^9c^5 - 80a^5b^8c + 640a^6b^6c^2 \\ & - 2560a^7b^4c^3 + 5120a^8b^2c^4)) * (6A^8b^11 + 2048B^8a^6c^5 - 2B^8a^8b^10 \\ & - 120A^8a^8b^9c - 6144A^8a^5b^8c^5 + 40B^8a^2b^8c + 960A^8a^2b^7c^2 \\ & - 3840A^8a^3b^5c^3 + 7680A^8a^4b^3c^4 - 320B^8a^3b^6c^2 + 1280B^8a^4b^4c^3 \\ & - 2560B^8a^5b^2c^4)) \end{aligned}$$

$$\begin{aligned}
& *c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4)/(2*(4*a^4*b^10 - 4096 \\
& *a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2 \\
& *c^4)) - (((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9 + 6*A*a^6*b^14*c^3 - 1 \\
& 08*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^8*c^6 - 22272*A*a^10*b \\
& ^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^9 - 2*B*a^7*b^13*c^3 + \\
& 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*b^7*c^6 - 3456*B*a^11* \\
& b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + \\
& 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c \\
& ^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11 \\
& *b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - \\
& 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9 \\
& *c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5 \\
& *c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B \\
& *a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c \\
& ^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4))*(a^9*b^12 + 4096*a^15*c^6 - 24*a^ \\
& 10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144 \\
& *a^14*b^2*c^5)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^ \\
& 2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - \\
& ((60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B \\
& a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^ \\
& 10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 \\
& + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2 \\
& *B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b \\
& ^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280 \\
& *B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(4*a^4*b^1 \\
& 0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120 \\
& *a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 \\
& - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(60*A*a^3*c \\
& ^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 9 \\
& 0*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + (((60*A*a^3*c^3 - 3*A*b^6 + \\
& B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2 \\
&)^2*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^ \\
& 11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 29 \\
& 4912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c \\
& - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^ \\
& 3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^ \\
& 5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8 \\
& *c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 409 \\
& 6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a \\
& ^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(9*A*b^8 + 30*A*a^4*c^4 - 3*B*a*b^7 - 99 \\
& *A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^3 + 360*A*a^2*b^4*c^2 - 435*A*a \\
& ^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2*(4*a*c - b^2)^6*(900*A^2*a^6*c^ \\
& 6 - 54*A^2*b^12 - 6*B^2*a^2*b^10 + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640* \\
& A^2*a^2*b^8*c^2 + 34560*A^2*a^3*b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2
\end{aligned}$$

$$\begin{aligned}
& *a^5*b^2*c^5 - 960*B^2*a^4*b^6*c^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^2*c^4 + 1080*A^2*a*b^10*c + 120*B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 2307 \\
& 0*A*B*a^4*b^5*c^3 + 46350*A*B*a^5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a^6*b*c^5)) * (16*a^12*b^12*(4*a*c - b^2)^{(15/2)} + 65536*a^18*c^6*(4*a*c - b^2)^{(15/2)} - 384*a^13*b^10*c*(4*a*c - b^2)^{(15/2)} + 3840*a^14*b^8*c^2*(4*a*c - b^2)^{(15/2)} - 20480*a^15*b^6*c^3*(4*a*c - b^2)^{(15/2)} + 61440*a^16*b^4*c^4*(4*a*c - b^2)^{(15/2)} - 98304*a^17*b^2*c^5*(4*a*c - b^2)^{(15/2)})) / (3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 - 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2*a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4*c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960*A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^11*c^2 + 3600*A*B*a^6*b*c^7) - (((((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6) / (a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6) * (6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4) / (2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) * (4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4))) * (60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2) / (4*a^4*(4*a*c - b^2)^{(5/2)}) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6) * (60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2) * (6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4) / (8*a^4*(4*a*c - b^2)^{(5/2)} * (a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) * (4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4))) * (6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4) / (2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (((900*A^2*a^8*c^8 - 36*A^2*a^3*b^10*c^3 + 549*A^2*a^4*b^8*c^4 - 3078*A^2*a^5*b^6*c^5 + 7533*A^2*a^6*b^4*c^6 - 7020*A^2*a^7*b^2*c^7 - 4*B^2*a^5*b^8*c^3 + 61*B^2*a^6*b^6*c^4 - 302*B^2*a^7*b^4*c^5 + 497*B^2*a^8*b^2*c^6 + 24*A*B*a^4*b^9*c^3 - 366*A*B*a^5*b^7*c^4 + 1932*A*B*a^6*b^5*c^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7) / (a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a
\end{aligned}$$

$$\begin{aligned}
& ^9b^6c^4 - 1312Ba^{10}b^4c^5 + 1472Ba^{11}b^2c^6)/(a^9b^8 + 256a^{13} \\
& *c^4 - 16a^{10}b^6c + 96a^{11}b^4c^2 - 256a^{12}b^2c^3) - ((4a^{10}b^{10} \\
& c^2 - 64a^{11}b^8c^3 + 384a^{12}b^6c^4 - 1024a^{13}b^4c^5 + 1024a^{14}b^2 \\
& 2c^6)*(6A*b^{11} + 2048B*a^6c^5 - 2B*a*b^{10} - 120A*a*b^9c - 6144A*a^5 \\
& *b*c^5 + 40B*a^2b^8c + 960A*a^2b^7c^2 - 3840A*a^3b^5c^3 + 7680A*a \\
& ^4b^3c^4 - 320B*a^3b^6c^2 + 1280B*a^4b^4c^3 - 2560B*a^5b^2c^4))/ \\
& (2*(a^9b^8 + 256a^{13}c^4 - 16a^{10}b^6c + 96a^{11}b^4c^2 - 256a^{12}b^2 \\
& *c^3)*(4a^4b^{10} - 4096a^9c^5 - 80a^5b^8c + 640a^6b^6c^2 - 2560a^7 \\
& b^4c^3 + 5120a^8b^2c^4)))*(6A*b^{11} + 2048B*a^6c^5 - 2B*a*b^{10} - 1 \\
& 20A*a*b^9c - 6144A*a^5b*c^5 + 40B*a^2b^8c + 960A*a^2b^7c^2 - 3840 \\
& *A*a^3b^5c^3 + 7680A*a^4b^3c^4 - 320B*a^3b^6c^2 + 1280B*a^4b^4c^ \\
& 3 - 2560B*a^5b^2c^4))/(2*(4a^4b^{10} - 4096a^9c^5 - 80a^5b^8c + 640 \\
& *a^6b^6c^2 - 2560a^7b^4c^3 + 5120a^8b^2c^4)))*(60A*a^3c^3 - 3A*b \\
& ^6 + B*a*b^5 + 30A*a*b^4c - 10B*a^2b^3c + 30B*a^3b*c^2 - 90A*a^2b^ \\
& 2c^2))/(4a^4*(4a*c - b^2)^(5/2)) + ((4a^{10}b^{10}c^2 - 64a^{11}b^8c^3 + \\
& 384a^{12}b^6c^4 - 1024a^{13}b^4c^5 + 1024a^{14}b^2c^6)*(60A*a^3c^3 - \\
& 3A*b^6 + B*a*b^5 + 30A*a*b^4c - 10B*a^2b^3c + 30B*a^3b*c^2 - 90A*a \\
& ^2b^2c^2)^3)/(64a^{12}*(4a*c - b^2)^(15/2)*(a^9b^8 + 256a^{13}c^4 - 16a \\
& ^{10}b^6c + 96a^{11}b^4c^2 - 256a^{12}b^2c^3)))*(16a^{12}b^{12}*(4a*c - b^ \\
& 2)^(15/2) + 65536a^{18}c^6*(4a*c - b^2)^(15/2) - 384a^{13}b^{10}c*(4a*c - \\
& b^2)^(15/2) + 3840a^{14}b^8c^2*(4a*c - b^2)^(15/2) - 20480a^{15}b^6c^3*(\\
& 4a*c - b^2)^(15/2) + 61440a^{16}b^4c^4*(4a*c - b^2)^(15/2) - 98304a^{17} \\
& b^2c^5*(4a*c - b^2)^(15/2))*(9A*b^9 - 160B*a^5c^4 - 3B*a*b^8 - 117A* \\
& a*b^7c + 570A*a^4b*c^4 + 39B*a^2b^6c + 540A*a^2b^5c^2 - 1005A*a^3 \\
& *b^3c^3 - 180B*a^3b^4c^2 + 325B*a^4b^2c^3))/(8*(4a*c - b^2)^(13/2)* \\
& (900A^2a^9c^8 + 6400B^2a^{10}c^7 - 54A^2a^3b^{12}c^2 - 960a^6b^6c^ \\
& 4*(B^2a - 36A^2c) + 120a^5b^8c^3*(B^2a - 72A^2c) - 6a^4b^{10}c^2* \\
& (B^2a - 180A^2c) - 25a^8b^2c^6*(311B^2a - 2196A^2c) + 25a^7b^4c \\
& ^5*(154B^2a - 2763A^2c) + 36A*B*a^4b^{11}c^2 - 720A*B*a^5b^9c^3 + \\
& 5760A*B*a^6b^7c^4 - 23070A*B*a^7b^5c^5 + 46350A*B*a^8b^3c^6 - 3750 \\
& 0A*B*a^9b*c^7)*(3600A^2a^6c^8 + 9A^2b^{12}c^2 + 1440A^2a^2b^8c^4 \\
& - 5760A^2a^3b^6c^5 + 11700A^2a^4b^4c^6 - 10800A^2a^5b^2c^7 + B^ \\
& 2a^2b^{10}c^2 - 20B^2a^3b^8c^3 + 160B^2a^4b^6c^4 - 600B^2a^5b^4 \\
& *c^5 + 900B^2a^6b^2c^6 - 180A^2a*b^{10}c^3 + 120A*B*a^2b^9c^3 - 960 \\
& *A*B*a^3b^7c^4 + 3720A*B*a^4b^5c^5 - 6600A*B*a^5b^3c^6 - 6A*B*a*b^ \\
& 11c^2 + 3600A*B*a^6b*c^7)) + (((3780A^3a^3b^3c^7 - 1863A^3a^2b^5c \\
& ^6 - 27A^3b^9c^4 + B^3a^3b^6c^4 - 14B^3a^4b^4c^5 + 49B^3a^5b^ \\
& 2c^6 + 900A^2B*a^5c^8 + 378A^3a*b^7c^5 - 2700A^3a^4b*c^8 + 420A* \\
& B^2a^5b*c^7 + 27A^2B*a*b^8c^4 - 9A*B^2a^2b^7c^4 + 126A*B^2a^3b^ \\
& 5c^5 - 501A*B^2a^4b^3c^6 - 378A^2B*a^2b^6c^5 + 1683A^2B*a^3b^4c \\
& ^6 - 2520A^2B*a^4b^2c^7)/(a^9b^8 + 256a^{13}c^4 - 16a^{10}b^6c + 96a \\
& ^{11}b^4c^2 - 256a^{12}b^2c^3) - (((900A^2a^8c^8 - 36A^2a^3b^{10}c^3 \\
& + 549A^2a^4b^8c^4 - 3078A^2a^5b^6c^5 + 7533A^2a^6b^4c^6 - 7020 \\
& *A^2a^7b^2c^7 - 4B^2a^5b^8c^3 + 61B^2a^6b^6c^4 - 302B^2a^7b^4 \\
& *c^5 + 497B^2a^8b^2c^6 + 24A*B*a^4b^9c^3 - 366A*B*a^5b^7c^4 + 193
\end{aligned}$$

$$\begin{aligned}
& 2*A*B*a^6*b^5*c^5 - 4002*A*B*a^7*b^3*c^6 + 2340*A*B*a^8*b*c^7)/(a^9*b^8 + 2 \\
& 56*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - (((1920 \\
& *A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9*c^3 - 1332*A*a^8*b^7*c^4 \\
& + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B*a^7*b^10*c^2 - 68*B*a^8*b^ \\
& 8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + 1472*B*a^11*b^2*c^6)/(a^9 \\
& *b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3) - \\
& ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 \\
& + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9 \\
& *c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5 \\
& *c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B \\
& *a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 \\
& - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^ \\
& 6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - \\
& 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2 \\
& *b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 12 \\
& 80*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80* \\
& a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*A*b \\
& ^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B \\
& *a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - \\
& 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(4*a^4*b^1 \\
& 0 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120 \\
& *a^8*b^2*c^4)) - (((((1920*A*a^11*b*c^7 - 12*A*a^6*b^11*c^2 + 204*A*a^7*b^9 \\
& *c^3 - 1332*A*a^8*b^7*c^4 + 4056*A*a^9*b^5*c^5 - 5376*A*a^10*b^3*c^6 + 4*B* \\
& a^7*b^10*c^2 - 68*B*a^8*b^8*c^3 + 444*B*a^9*b^6*c^4 - 1312*B*a^10*b^4*c^5 + \\
& 1472*B*a^11*b^2*c^6)/(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4 \\
& *c^2 - 256*a^12*b^2*c^3) - ((4*a^10*b^10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b \\
& ^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^6)*(6*A*b^11 + 2048*B*a^6*c^5 \\
& - 2*B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^ \\
& 2*b^7*c^2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1 \\
& 280*B*a^4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(2*(a^9*b^8 + 256*a^13*c^4 - 16*a^ \\
& 10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^9*c^5 - \\
& 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6 \\
& 0*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3* \\
& b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) - ((4*a^10*b^10*c^2 \\
& - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14*b^2*c^ \\
& 6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B \\
& *a^3*b*c^2 - 90*A*a^2*b^2*c^2)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 12 \\
& 0*A*a*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840* \\
& A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 \\
& - 2560*B*a^5*b^2*c^4))/(8*a^4*(4*a*c - b^2)^(5/2)*(a^9*b^8 + 256*a^13*c^4 \\
& - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4096*a^ \\
& 9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^ \\
& 4)))*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30 \\
& *B*a^3*b*c^2 - 90*A*a^2*b^2*c^2))/(4*a^4*(4*a*c - b^2)^(5/2)) + ((4*a^10*b^ \\
& 10*c^2 - 64*a^11*b^8*c^3 + 384*a^12*b^6*c^4 - 1024*a^13*b^4*c^5 + 1024*a^14
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^6)*(60*A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c \\
& + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2)^2*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a* \\
& b^10 - 120*A*a*b^9*c - 6144*A*a^5*b^2*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^ \\
& 2 - 3840*A*a^3*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^ \\
& 4*b^4*c^3 - 2560*B*a^5*b^2*c^4))/(32*a^8*(4*a*c - b^2)^5*(a^9*b^8 + 256*a^1 \\
& 3*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)*(4*a^4*b^10 - 4 \\
& 096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8* \\
& b^2*c^4)))*(16*a^12*b^12*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*(4*a*c - b^2 \\
&)^(15/2) - 384*a^13*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^14*b^8*c^2*(4*a*c \\
& - b^2)^(15/2) - 20480*a^15*b^6*c^3*(4*a*c - b^2)^(15/2) + 61440*a^16*b^4*c^ \\
& 4*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*(4*a*c - b^2)^(15/2))*(9*A*b^8 \\
& + 30*A*a^4*c^4 - 3*B*a*b^7 - 99*A*a*b^6*c + 33*B*a^2*b^5*c + 135*B*a^4*b*c^ \\
& 3 + 360*A*a^2*b^4*c^2 - 435*A*a^3*b^2*c^3 - 120*B*a^3*b^3*c^2))/(8*a^3*c^2* \\
& (4*a*c - b^2)^6*(3600*A^2*a^6*c^8 + 9*A^2*b^12*c^2 + 1440*A^2*a^2*b^8*c^4 - \\
& 5760*A^2*a^3*b^6*c^5 + 11700*A^2*a^4*b^4*c^6 - 10800*A^2*a^5*b^2*c^7 + B^2 \\
& *a^2*b^10*c^2 - 20*B^2*a^3*b^8*c^3 + 160*B^2*a^4*b^6*c^4 - 600*B^2*a^5*b^4* \\
& c^5 + 900*B^2*a^6*b^2*c^6 - 180*A^2*a*b^10*c^3 + 120*A*B*a^2*b^9*c^3 - 960* \\
& A*B*a^3*b^7*c^4 + 3720*A*B*a^4*b^5*c^5 - 6600*A*B*a^5*b^3*c^6 - 6*A*B*a*b^1 \\
& 1*c^2 + 3600*A*B*a^6*b*c^7)*(900*A^2*a^6*c^6 - 54*A^2*b^12 - 6*B^2*a^2*b^10 \\
& + 6400*B^2*a^7*c^5 + 36*A*B*a*b^11 - 8640*A^2*a^2*b^8*c^2 + 34560*A^2*a^3* \\
& b^6*c^3 - 69075*A^2*a^4*b^4*c^4 + 54900*A^2*a^5*b^2*c^5 - 960*B^2*a^4*b^6*c \\
& ^2 + 3850*B^2*a^5*b^4*c^3 - 7775*B^2*a^6*b^2*c^4 + 1080*A^2*a*b^10*c + 120* \\
& B^2*a^3*b^8*c + 5760*A*B*a^3*b^7*c^2 - 23070*A*B*a^4*b^5*c^3 + 46350*A*B*a^ \\
& 5*b^3*c^4 - 720*A*B*a^2*b^9*c - 37500*A*B*a^6*b*c^5)))*(60*A*a^3*c^3 - 3*A* \\
& b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b \\
& ^2*c^2))/(2*a^4*(4*a*c - b^2)^(5/2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.110 \quad \int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=554

$$\frac{\left(-\frac{40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}} + 84a^2Bc^2 - 16aAbc^2 - 27ab^2Bc + Ab^3c + 3b^4B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 11.19, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{8\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{8\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right) + \frac{x(20a^2c^2-24abBc+Ab^3c+3b^4B)}{8c^2(b^2-4ac)^2} + \frac{x^2(-2aB+c^2(-bB-2Ac)+Ab)}{4(b^2-4ac)(a+b^2+c^2)^2} + \frac{x^3(-28aBc+12Ab^2+4Ac-12aB+7Ab^2)}{8(b^2-4ac)^2(a+b^2+c^2)^2} + \frac{x^4(-28aBc+12Ab^2+4Ac-12aB+7Ab^2)}{8(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne

$Q[cd^2 - ae^2, 0] \&\& \text{PosQ}[b^2 - 4ac]$

Rule 1275

$\text{Int}[(f(x))^m((d) + (e)(x)^2)((a) + (b)(x)^2 + (c)(x)^4)^{p_1}, x_Symbol] \rightarrow \text{Simp}[(f(fx))^{m-1}(a + b x^2 + c x^4)^{p+1}(bd - 2ae - (be - 2cd)x^2)/(2(p+1)(b^2 - 4ac)), x] - \text{Dist}[f^2/(2(p+1)(b^2 - 4ac)), \text{Int}[(fx)^{m-2}(a + b x^2 + c x^4)^{p+1} \text{Simp}[(m-1)(bd - 2ae) - (4p+4+m+1)(be - 2cd)x^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4ac, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2p] && (IntegerQ[p] || IntegerQ[m])

Rule 1279

$\text{Int}[(f(x))^m((d) + (e)(x)^2)((a) + (b)(x)^2 + (c)(x)^4)^{p_1}, x_Symbol] \rightarrow \text{Simp}[(ef(fx))^{m-1}(a + b x^2 + c x^4)^{p+1}/(c(m+4p+3)), x] - \text{Dist}[f^2/(c(m+4p+3)), \text{Int}[(fx)^{m-2}(a + b x^2 + c x^4)^p \text{Simp}[ae(m-1) + (be(m+2p+1) - cd(m+4p+3))x^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4ac, 0] && GtQ[m, 1] && NeQ[m+4p+3, 0] && IntegerQ[2p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{x^8 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= -\frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{\int \frac{x^6(7(Ab-2aB)+(-bB+2Ac)x^2)}{(a+bx^2+cx^4)^2} dx}{4(b^2 - 4ac)} \\
&= -\frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{x^5 (7Ab^2 - 12abB - 4aAc + (b^2B + 12Abc - 28aBc))}{8(b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\
&= \frac{(b^2B + 12Abc - 28aBc)x^3}{8c(b^2 - 4ac)^2} - \frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{x^5 (7Ab^2 - 12abB - 4aAc)}{8(b^2 - 4ac)} \\
&= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)x}{8c^2(b^2 - 4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)x^3}{8c(b^2 - 4ac)^2} - \frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} \\
&= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)x}{8c^2(b^2 - 4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)x^3}{8c(b^2 - 4ac)^2} - \frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} \\
&= -\frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)x}{8c^2(b^2 - 4ac)^2} + \frac{(b^2B + 12Abc - 28aBc)x^3}{8c(b^2 - 4ac)^2} - \frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2}
\end{aligned}$$

Mathematica [A] time = 2.40, size = 644, normalized size = 1.16

$$\frac{\frac{1}{8c^2} \frac{(3b^3B + Ab^2c - 24abBc + 20aAc^2)x}{(b^2 - 4ac)^2} + \frac{1}{8c} \frac{(b^2B + 12Abc - 28aBc)x^3}{(b^2 - 4ac)^2} - \frac{x^7 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2}}{1}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c])) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b

- Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) + b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 20.78, size = 9636, normalized size = 17.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 -

$$\begin{aligned}
& 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 129837 \\
& 6*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b \\
& ^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 \\
& - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2* \\
& B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882 \\
& *A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + \\
& 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^ \\
& 3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 + \\
& (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^ \\
& 8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^ \\
& 10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31* \\
& B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c \\
& ^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a \\
& ^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - \\
& 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b \\
& ^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^ \\
& 3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^ \\
& 2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444 \\
& 6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^ \\
& 4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)* \\
& c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280* \\
& a^4*b^2*c^14 - 1024*a^5*c^15))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^ \\
& 3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216* \\
& B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B \\
& *a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b \\
& ^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^ \\
& 10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^ \\
& 2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2* \\
& a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^ \\
& 3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 11 \\
& 6*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^ \\
& 10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^ \\
& 2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640* \\
& a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8 \\
& *a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2 \\
& *(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a \\
& ^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^ \\
& 2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b \\
& ^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c \\
& ^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2 \\
& *A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 \\
& + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50 \\
& *(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95 \\
& 256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - \\
& 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5
\end{aligned}$$

$$\begin{aligned}
&) * c^3 + 27 * (657 * B^4 * a^2 * b^4 - 116 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (33 \\
& * B^4 * a * b^6 - 2 * A * B^3 * b^7) * c) / (b^{10} * c^{10} - 20 * a * b^8 * c^{11} + 160 * a^2 * b^6 * c^{12} \\
& - 640 * a^3 * b^4 * c^{13} + 1280 * a^4 * b^2 * c^{14} - 1024 * a^5 * c^{15})) / (b^{10} * c^5 - 20 * a * \\
& b^8 * c^6 + 160 * a^2 * b^6 * c^7 - 640 * a^3 * b^4 * c^8 + 1280 * a^4 * b^2 * c^9 - 1024 * a^5 * c^{10})) * \log(- (1701 * B^4 * a^2 * b^8 - 945 * A * B^3 * a * b^9 - 10000 * A^4 * a^4 * c^6 + 15000 * \\
& (6 * A^3 * B * a^4 * b - A^4 * a^3 * b^2) * c^5 + 3 * (1037232 * B^4 * a^6 - 1037232 * A * B^3 * a^5 * \\
& b + 287712 * A^2 * B^2 * a^4 * b^2 - 32952 * A^3 * B * a^3 * b^3 + 497 * A^4 * a^2 * b^4) * c^4 - (\\
& 1555848 * B^4 * a^5 * b^2 - 1298376 * A * B^3 * a^4 * b^3 + 238464 * A^2 * B^2 * a^3 * b^4 - 1127 \\
& 7 * A^3 * B * a^2 * b^5 + 35 * A^4 * a * b^6) * c^3 + 9 * (37701 * B^4 * a^4 * b^4 - 26973 * A * B^3 * a^ \\
& 3 * b^5 + 3066 * A^2 * B^2 * a^2 * b^6 - 35 * A^3 * B * a * b^7) * c^2 - 27 * (1341 * B^4 * a^3 * b^6 - \\
& 819 * A * B^3 * a^2 * b^7 + 35 * A^2 * B^2 * a * b^8) * c) * x - 1/2 * \sqrt{1/2} * (27 * B^3 * b^{13} + \\
& 32000 * A^3 * a^5 * c^8 - 640 * (882 * A * B^2 * a^6 - 156 * A^2 * B * a^5 * b + 37 * A^3 * a^4 * b^2) * \\
& c^7 + 64 * (10584 * B^3 * a^6 * b + 5562 * A * B^2 * a^5 * b^2 - 1083 * A^2 * B * a^4 * b^3 + 89 * A^ \\
& 3 * a^3 * b^4) * c^6 - 8 * (93096 * B^3 * a^5 * b^3 + 3816 * A * B^2 * a^4 * b^4 - 1746 * A^2 * B * a^3 \\
& * b^5 + 49 * A^3 * a^2 * b^6) * c^5 + (337392 * B^3 * a^4 * b^5 - 24120 * A * B^2 * a^3 * b^6 - 84 \\
& * A^2 * B * a^2 * b^7 - 17 * A^3 * a * b^8) * c^4 - (81324 * B^3 * a^3 * b^7 - 6993 * A * B^2 * a^2 * b^ \\
& 8 + 195 * A^2 * B * a * b^9 - A^3 * b^{10}) * c^3 + 9 * (1239 * B^3 * a^2 * b^9 - 79 * A * B^2 * a * b^{10} \\
& + A^2 * B * b^{11}) * c^2 - 27 * (31 * B^3 * a * b^{11} - A * B^2 * b^{12}) * c - (3 * B * b^{14} * c^5 - 40 \\
& 96 * (42 * B * a^7 - 13 * A * a^6 * b) * c^{12} + 6144 * (40 * B * a^6 * b^2 - 11 * A * a^5 * b^3) * c^{11} - \\
& 768 * (194 * B * a^5 * b^4 - 45 * A * a^4 * b^5) * c^{10} + 1280 * (39 * B * a^4 * b^6 - 7 * A * a^3 * b^7 \\
&) * c^9 - 240 * (42 * B * a^3 * b^8 - 5 * A * a^2 * b^9) * c^8 + 24 * (52 * B * a^2 * b^{10} - 3 * A * a * b^{11} \\
&) * c^7 - (90 * B * a * b^{12} - A * b^{13}) * c^6) * \sqrt{(81 * B^4 * b^8 + 625 * A^4 * a^2 * c^6 - \\
& 50 * (441 * A^2 * B^2 * a^3 - 108 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (194481 * B^4 * a^4 - \\
& 95256 * A * B^3 * a^3 * b + 17496 * A^2 * B^2 * a^2 * b^2 - 516 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 \\
& - 6 * (14553 * B^4 * a^3 * b^2 - 4446 * A * B^3 * a^2 * b^3 + 324 * A^2 * B^2 * a * b^4 - 2 * A^3 * B * b^ \\
& ^5) * c^3 + 27 * (657 * B^4 * a^2 * b^4 - 116 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (\\
& 33 * B^4 * a * b^6 - 2 * A * B^3 * b^7) * c) / (b^{10} * c^{10} - 20 * a * b^8 * c^{11} + 160 * a^2 * b^6 * c^{12} - 64 \\
& 0 * a^3 * b^4 * c^{13} + 1280 * a^4 * b^2 * c^{14} - 1024 * a^5 * c^{15})) * \sqrt{-(9 * B^2 * b^9 - 1680 * (4 * A * B * a^4 - A^2 * a^3 * b) * c^5 + 280 * \\
& (54 * B^2 * a^4 * b - 12 * A * B * a^3 * b^2 + A^2 * a^2 * b^3) * c^4 - 35 * (216 * B^2 * a^3 * b^3 - 3 \\
& (1701 * B^2 * a^2 * b^5 - 168 * A * B * a * b^6 + A^2 * b^7) * c^2 - 3 * (63 * B^2 * a * b^7 - 2 * A * B \\
& * b^8) * c + (b^{10} * c^5 - 20 * a * b^8 * c^6 + 160 * a^2 * b^6 * c^7 - 640 * a^3 * b^4 * c^8 + 12 \\
& 80 * a^4 * b^2 * c^9 - 1024 * a^5 * c^{10})) * \sqrt{(81 * B^4 * b^8 + 625 * A^4 * a^2 * c^6 - 50 * (44 \\
& 1 * A^2 * B^2 * a^3 - 108 * A^3 * B * a^2 * b + A^4 * a * b^2) * c^5 + (194481 * B^4 * a^4 - 95256 * \\
& A * B^3 * a^3 * b + 17496 * A^2 * B^2 * a^2 * b^2 - 516 * A^3 * B * a * b^3 + A^4 * b^4) * c^4 - 6 * (1 \\
& 4553 * B^4 * a^3 * b^2 - 4446 * A * B^3 * a^2 * b^3 + 324 * A^2 * B^2 * a * b^4 - 2 * A^3 * B * b^5) * c^ \\
& 3 + 27 * (657 * B^4 * a^2 * b^4 - 116 * A * B^3 * a * b^5 + 2 * A^2 * B^2 * b^6) * c^2 - 54 * (33 * B^4 \\
& * a * b^6 - 2 * A * B^3 * b^7) * c) / (b^{10} * c^{10} - 20 * a * b^8 * c^{11} + 160 * a^2 * b^6 * c^{12} - 64 \\
& 0 * a^3 * b^4 * c^{13} + 1280 * a^4 * b^2 * c^{14} - 1024 * a^5 * c^{15})) / (b^{10} * c^5 - 20 * a * b^8 * \\
& c^6 + 160 * a^2 * b^6 * c^7 - 640 * a^3 * b^4 * c^8 + 1280 * a^4 * b^2 * c^9 - 1024 * a^5 * c^{10} \\
&)) - \sqrt{1/2} * ((b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6) * x^8 + a^2 * b^4 * c^2 - 8 * \\
& a^3 * b^2 * c^3 + 16 * a^4 * c^4 + 2 * (b^5 * c^3 - 8 * a * b^3 * c^4 + 16 * a^2 * b * c^5) * x^6 + (\\
& b^6 * c^2 - 6 * a * b^4 * c^3 + 32 * a^3 * c^5) * x^4 + 2 * (a * b^5 * c^2 - 8 * a^2 * b^3 * c^3 + 16 \\
& * a^3 * b * c^4) * x^2) * \sqrt{-(9 * B^2 * b^9 - 1680 * (4 * A * B * a^4 - A^2 * a^3 * b) * c^5 + 280 * \\
& (54 * B^2 * a^4 * b - 12 * A * B * a^3 * b^2 + A^2 * a^2 * b^3) * c^4 - 35 * (216 * B^2 * a^3 * b^3 - 3
\end{aligned}$$

$$\begin{aligned}
& 6*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7) * c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^{10}*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^{10}) * \sqrt{(81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2) * c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4) * c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5) * c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6) * c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7) * c) / (b^{10}*c^{10} - 20*a*b^8*c^{11} + 160*a^2*b^6*c^{12} - 640*a^3*b^4*c^{13} + 1280*a^4*b^2*c^{14} - 1024*a^5*c^{15})) / (b^{10}*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^{10}) * \log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2) * c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4) * c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6) * c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7) * c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8) * c) * x + 1/2 * \sqrt{1/2} * (27*B^3*b^{13} + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2) * c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4) * c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6) * c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8) * c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^{10}) * c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^{10} + A^2*B*b^{11}) * c^2 - 27*(31*B^3*a*b^{11} - A*B^2*b^{12}) * c + (3*B*b^{14}*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b) * c^{12} + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3) * c^{11} - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5) * c^{10} + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7) * c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9) * c^8 + 24*(52*B*a^2*b^{10} - 3*A*a*b^{11}) * c^7 - (90*B*a*b^{12} - A*b^{13}) * c^6) * \sqrt{(81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2) * c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4) * c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5) * c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6) * c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7) * c) / (b^{10}*c^{10} - 20*a*b^8*c^{11} + 160*a^2*b^6*c^{12} - 640*a^3*b^4*c^{13} + 1280*a^4*b^2*c^{14} - 1024*a^5*c^{15})) * \sqrt{-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b) * c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3) * c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5) * c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7) * c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8) * c - (b^{10}*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^{10}) * \sqrt{(81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2) * c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4) * c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5) * c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6) * c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7) * c) / (b^{10}*c^{10} - 20*a*b^8*c^{11} + 160*a^2*b^6*c^{12} - 640*a^3*b^4*c^{13} + 1280*a^4*b^2*c^{14} - 1024*a^5*c^{15})}
\end{aligned}$$

$$\begin{aligned}
& \text{^15)))/(b^{10}c^5 - 20a^2b^8c^6 + 160a^4b^6c^7 - 640a^3b^4c^8 + 1280a^4b^2c^9 - 1024a^5c^{10})) + \text{sqrt}(1/2)*((b^4c^4 - 8a^2b^2c^5 + 16a^2c^6)*x^8 + a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4 + 2*(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5)*x^6 + (b^6c^2 - 6a^2b^4c^3 + 32a^3c^5)*x^4 + 2*(a^2b^5c^2 - 8a^2b^3c^3 + 16a^3b^2c^4)*x^2)*\text{sqrt}(-(9B^2b^9 - 1680(4A^2B^2b^9 - A^2a^3b)*c^5 + 280*(54B^2a^4b - 12A^2B^2a^3b^2 + A^2a^2b^3)*c^4 - 35*(216B^2a^3b^3 - 36A^2B^2a^2b^4 + A^2a^2b^5)*c^3 + (1701B^2a^2b^5 - 168A^2B^2a^2b^6 + A^2b^7)*c^2 - 3*(63B^2a^2b^7 - 2A^2B^2b^8)*c - (b^{10}c^5 - 20a^2b^8c^6 + 160a^4b^6c^7 - 640a^3b^4c^8 + 1280a^4b^2c^9 - 1024a^5c^{10})*\text{sqrt}((81B^4b^8 + 625A^4a^2c^6 - 50*(441A^2B^2a^3 - 108A^3B^2a^2b + A^4a^2b^2)*c^5 + (194481B^4a^4 - 95256A^2B^3a^3b + 17496A^2B^2a^2b^2 - 516A^3B^2a^2b^3 + A^4b^4)*c^4 - 6*(14553B^4a^3b^2 - 4446A^2B^3a^2b^3 + 324A^2B^2a^2b^4 - 2A^3B^2b^5)*c^3 + 27*(657B^4a^2b^4 - 116A^2B^3a^2b^5 + 2A^2B^2b^6)*c^2 - 54*(33B^4a^2b^6 - 2A^2B^3b^7)*c)/(b^{10}c^{10} - 20a^2b^8c^{11} + 160a^4b^6c^{12} - 640a^3b^4c^{13} + 1280a^4b^2c^{14} - 1024a^5c^{15}))/((b^{10}c^5 - 20a^2b^8c^6 + 160a^4b^6c^7 - 640a^3b^4c^8 + 1280a^4b^2c^9 - 1024a^5c^{10}))*\text{log}(-(1701B^4a^2b^8 - 945A^2B^3a^2b^9 - 10000A^4a^4c^6 + 15000*(6A^3B^2a^4b - A^4a^3b^2)*c^5 + 3*(1037232B^4a^6 - 1037232A^2B^3a^5b + 287712A^2B^2a^4b^2 - 32952A^3B^2a^3b^3 + 497A^4a^2b^4)*c^4 - (1555848B^4a^5b^2 - 1298376A^2B^3a^4b^3 + 238464A^2B^2a^3b^4 - 11277A^3B^2a^2b^5 + 35A^4a^2b^6)*c^3 + 9*(37701B^4a^4b^4 - 26973A^2B^3a^3b^5 + 3066A^2B^2a^2b^6 - 35A^3B^2a^2b^7)*c^2 - 27*(1341B^4a^3b^6 - 819A^2B^3a^2b^7 + 35A^2B^2a^2b^8)*c)*x - 1/2*\text{sqrt}(1/2)*(27B^3b^{13} + 32000A^3a^5c^8 - 640*(882A^2B^2a^6 - 156A^2B^2a^5b + 37A^3a^4b^2)*c^7 + 64*(10584B^3a^6b + 5562A^2B^2a^5b^2 - 1083A^2B^2a^4b^3 + 89A^3a^3b^4)*c^6 - 8*(93096B^3a^5b^3 + 3816A^2B^2a^4b^4 - 1746A^2B^2a^3b^5 + 49A^3a^2b^6)*c^5 + (337392B^3a^4b^5 - 24120A^2B^2a^3b^6 - 84A^2B^2a^2b^7 - 17A^3a^2b^8)*c^4 - (81324B^3a^3b^7 - 6993A^2B^2a^2b^8 + 195A^2B^2a^2b^9 - A^3b^{10})*c^3 + 9*(1239B^3a^2b^9 - 79A^2B^2a^2b^{10} + A^2B^2b^{11})*c^2 - 27*(31B^3a^2b^{11} - A^2B^2b^{12})*c + (3B^3b^{14}c^5 - 4096*(42B^2a^7 - 13A^2a^6b)*c^{12} + 6144*(40B^2a^6b^2 - 11A^2a^5b^3)*c^{11} - 768*(194B^2a^5b^4 - 45A^2a^4b^5)*c^{10} + 1280*(39B^2a^4b^6 - 7A^2a^3b^7)*c^9 - 240*(42B^2a^3b^8 - 5A^2a^2b^9)*c^8 + 24*(52B^2a^2b^{10} - 3A^2a^2b^{11})*c^7 - (90B^2a^2b^{12} - A^2b^{13})*c^6)*\text{sqrt}((81B^4b^8 + 625A^4a^2c^6 - 50*(441A^2B^2a^3 - 108A^3B^2a^2b + A^4a^2b^2)*c^5 + (194481B^4a^4 - 95256A^2B^3a^3b + 17496A^2B^2a^2b^2 - 516A^3B^2a^2b^3 + A^4b^4)*c^4 - 6*(14553B^4a^3b^2 - 4446A^2B^3a^2b^3 + 324A^2B^2a^2b^4 - 2A^3B^2b^5)*c^3 + 27*(657B^4a^2b^4 - 116A^2B^3a^2b^5 + 2A^2B^2b^6)*c^2 - 54*(33B^4a^2b^6 - 2A^2B^3b^7)*c)/(b^{10}c^{10} - 20a^2b^8c^{11} + 160a^4b^6c^{12} - 640a^3b^4c^{13} + 1280a^4b^2c^{14} - 1024a^5c^{15}))*\text{sqrt}(-(9B^2b^9 - 1680(4A^2B^2a^4 - A^2a^3b)*c^5 + 280*(54B^2a^4b - 12A^2B^2a^3b^2 + A^2a^2b^3)*c^4 - 35*(216B^2a^3b^3 - 36A^2B^2a^2b^4 + A^2a^2b^5)*c^3 + (1701B^2a^2b^5 - 168A^2B^2a^2b^6 + A^2b^7)*c^2 - 3*(63B^2a^2b^7 - 2A^2B^2b^8)*c - (b^{10}c^5 - 20a^2b^8c^6 + 160a^4b^6c^7 - 640a^3b^4c^8 + 1280a^4b^2c^9 - 10
\end{aligned}$$

$$24*a^5*c^{10}*\sqrt{(81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^{10}*c^{10} - 20*a*b^8*c^{11} + 160*a^2*b^6*c^{12} - 640*a^3*b^4*c^{13} + 1280*a^4*b^2*c^{14} - 1024*a^5*c^{15}))/((b^{10}*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^{10})) + 2*(3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)$$

giac [B] time = 8.28, size = 3987, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $\frac{1}{32} * ((\sqrt{2}) * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^6*c + 12 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^4*c^2 - 2 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^5*c^2 - 2 * b^6*c^2 - 144 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*b^2*c^3 - 32 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^3*c^3 + \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^4*c^3 - 24 * a*b^4*c^3 - 2 * b^5*c^3 + 320 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^3*c^4 + 160 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*b*c^4 + 16 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^2*c^4 + 288 * a^2*b^2*c^4 + 112 * a*b^3*c^4 - 80 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*c^5 - 640 * a^3*c^5 - 416 * a^2*b*c^5 + \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^5*c - 56 * \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^3*c^2 - 2 * \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^4*c^2 + 208 * \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*b*c^3 + 104 * \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^2*c^3 + \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^3*c^3 - 52 * \sqrt{2} * \sqrt{b^2 - 4*a*c}} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b*c^4 + 2 * (b^2 - 4*a*c) * b^4*c^2 + 32 * (b^2 - 4*a*c) * a*b^2*c^3 + 2 * (b^2 - 4*a*c) * b^3*c^3 - 160 * (b^2 - 4*a*c) * a^2*c^4 - 104 * (b^2 - 4*a*c) * a*b*c^4) * A + 3 * (\sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^7 - 16 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^5*c - 2 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^6*c - 2 * b^7*c + 80 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*b^3*c^2 + 24 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^4*c^2 + \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * b^5*c^2 + 32 * a*b^5*c^2 - 2 * b^6*c^2 - 128 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^3*b*c^3 - 64 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a^2*b^2*c^3 - 12 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c) * a*b^3*c^3 - 160 * a^2*b^3*c^3 + 28 * a*b^4*c^3 + 32 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} * c)$

$$\begin{aligned}
&) * a^2 * b * c^4 + 256 * a^3 * b * c^4 - 192 * a^2 * b^2 * c^4 + 448 * a^3 * c^5 + \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c * b^6 - 14 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a * b^4 * c - 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * b^5 * c + 96 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a^2 * b^2 * c^2 + 20 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a * b^3 * c^2 + \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * b^4 * c^2 - 224 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a^3 * c^3 - 112 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a^2 * b * c^3 - 10 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a * b^2 * c^3 + 56 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * a^2 * c^4 + 2 * (b^2 - 4 * a * c) * b^5 * c - 24 * (b^2 - 4 * a * c) * a * b^3 * c^2 + 2 * (b^2 - 4 * a * c) * b^4 * c^2 + 64 * (b^2 - 4 * a * c) * a^2 * b * c^3 - 20 * (b^2 - 4 * a * c) * a * b^2 * c^3 + 112 * (b^2 - 4 * a * c) * a^2 * c^4) * B * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b^5 * c^2 - 8 * a * b^3 * c^3 + 16 * a^2 * b * c^4 + \sqrt{(b^5 * c^2 - 8 * a * b^3 * c^3 + 16 * a^2 * b * c^4)^2 - 4 * (a * b^4 * c^2 - 8 * a^2 * b^2 * c^3 + 16 * a^3 * c^4) * (b^4 * c^3 - 8 * a * b^2 * c^4 + 16 * a^2 * c^5))}) / ((b^4 * c^3 - 8 * a * b^2 * c^4 + 16 * a^2 * c^5)) / ((b^8 * c^2 - 16 * a * b^6 * c^3 - 2 * b^7 * c^3 + 96 * a^2 * b^4 * c^4 + 24 * a * b^5 * c^4 + b^6 * c^4 - 256 * a^3 * b^2 * c^5 - 96 * a^2 * b^3 * c^5 - 12 * a * b^4 * c^5 + 256 * a^4 * c^6 + 128 * a^3 * b * c^6 + 48 * a^2 * b^2 * c^6 - 64 * a^3 * c^7) * \text{abs}(c)) + 1/32 * ((\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * b^6 * c + 12 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^4 * c^2 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^5 * c^2 + 2 * b^6 * c^2 - 144 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b^2 * c^3 - 32 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^3 * c^3 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^4 * c^3 + 24 * a * b^4 * c^3 + 2 * b^5 * c^3 + 320 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^3 * c^4 + 160 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b * c^4 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^2 * c^4 - 288 * a^2 * b^2 * c^4 - 112 * a * b^3 * c^4 - 80 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * c^5 + 640 * a^3 * c^5 + 416 * a^2 * b * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^5 * c + 56 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^3 * c^2 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^4 * c^2 - 208 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b * c^3 - 104 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^2 * c^3 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^3 * c^3 + 52 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b * c^4 - 2 * (b^2 - 4 * a * c) * b^4 * c^2 - 32 * (b^2 - 4 * a * c) * a * b^2 * c^3 - 2 * (b^2 - 4 * a * c) * b^3 * c^3 + 160 * (b^2 - 4 * a * c) * a^2 * c^4 + 104 * (b^2 - 4 * a * c) * a * b * c^4) * A + 3 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * b^7 - 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^5 * c - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^6 * c + 2 * b^7 * c + 80 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b^3 * c^2 + 24 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^4 * c^2 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^5 * c^2 - 32 * a * b^5 * c^2 + 2 * b^6 * c^2 - 128 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^3 * b * c^3 - 64 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b^2 * c^3 - 12 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a * b^3 * c^3 + 160 * a^2 * b^3 * c^3 - 28 * a * b^4 * c^3 + 32 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * a^2 * b * c^4 - 256 * a^3 * b * c^4 + 192 * a^2 * b^2 * c^4 - 448 * a^3 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * b^6 + 1
\end{aligned}$$

$$\begin{aligned}
& b+(-4*a*c+b^2)^{(1/2)}*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^4-21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a^2*B+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*B-3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*B+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^5-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*A*b+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^4+21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a^2*B-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b^2*B+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4*B+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^5
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(5 B b^4 c + 4 (11 B a^2 + 4 A a b) c^2 - (37 B a b^2 + A b^3) c^2 + (3 B b^5 + 36 A a^2 c^3 - (4 B a^2 b - 5 A a b^2) c^2 - (20 B a b^3 - A b^4) c) c^3 + (6 B a b^4 + 28 (B a^2 + A a^2 b) c^2 - (49 B a^2 b^2 - 2 A a b^3) c) c^3 + (3 B a^2 b^3 + 20 A a^2 c^2 - (24 B a^2 b - A a^2 b^2) c) c^3 - \int \frac{3 B a^3 - 20 A a^2 c^2 - (3 B b^5 + 4 (11 B a^2 + 4 A a b) c^2 - (37 B a b^2 + A b^3) c^2 - (24 B a^2 b - A a^2 b^2) c) c^3}{8 (b^2 c^2 - 8 a b^2 c^2 + 16 a^2 c^4)} dx}{8 (b^2 c^2 - 8 a b^2 c^2 + 16 a^2 c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -

$$A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 - 4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)$$

mupad [B] time = 5.05, size = 22911, normalized size = 41.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x)$

[Out] $-\left(\frac{(x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 - 4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 + 44*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^2*b*c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(3*B*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right)/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - \text{atan}\left(\frac{((256*A*a*b^{12}*c^4 - 5242880*A*a^7*c^{10} + 768*B*a*b^{13}*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^{11}*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-9*B^2*b^{19} + A^2*b^{17}*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{1/2} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2} - 288*A*B*a*b^{16}*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{1/2} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{1/2})/(512*(1048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{1/2}*(256*b^{11}*c^5 - 5$

$$\begin{aligned}
& 120*a*b^9*c^6 - 262144*a^5*b*c^{10} + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 \\
& + 327680*a^4*b^3*c^9) / (32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)) * (- (9*B^2*b^{19} + A^2*b^{17}*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{1/2} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2} - 288*A*B*a*b^{16}*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{1/2} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{1/2}) / (512*(1048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{1/2} - (x*(9*B^2*b^{10} + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5)) / (32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)) * (- (9*B^2*b^{19} + A^2*b^{17}*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{1/2} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2} - 288*A*B*a*b^{16}*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{1/2} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{1/2}) / (512*(1048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{1/2} * 1 \\
& i - (((256*A*a*b^{12}*c^4 - 5242880*A*a^7*c^{10} + 768*B*a*b^{13}*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^{11}*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8) / (512*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17 \\
& *c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^{1/2} + 6*A*B*b^18*c + 1140*A^2*a^2*b^ \\
& 13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7 \\
& *c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15 \\
& *c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b \\
& ^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2* \\
& a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^{1/2} + 441*B^2*a^2*c^2*(-(4* \\
& a*c - b^2)^15)^{1/2} + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b \\
& ^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^{1/2} - \\
& 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + \\
& 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 \\
& - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4* \\
& a*c - b^2)^15)^{1/2} - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15) \\
& ^{1/2} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^{1/2}))/((512*(1048576*a^10*c^15 \\
& + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760* \\
& a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6 \\
& *c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^{1/2}*(256*b^11*c^5 \\
& - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c \\
& ^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^ \\
& 2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(\\
& 4*a*c - b^2)^15)^{1/2} + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a \\
& ^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^ \\
& 6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3 \\
& *b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2 \\
& *a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^ \\
& 2*c^2*(-(4*a*c - b^2)^15)^{1/2} + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^{1/2} \\
& + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^ \\
& 2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^{1/2} - 15482880*B^2*a^9*b* \\
& c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10* \\
& c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b \\
& ^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^{1/2} \\
& - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^{1/2} - 108*A*B*a*b \\
& *c^2*(-(4*a*c - b^2)^15)^{1/2}))/((512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b \\
& ^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 25804 \\
& 8*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8* \\
& b^4*c^13 - 2621440*a^9*b^2*c^14)))^{1/2} + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 \\
& + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 20 \\
& 8*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2 \\
& *a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - \\
& 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/((32*(256*a^4* \\
& c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2 \\
& *b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^{1/2} + 6*A*B*b^18*c + \\
& 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 4 \\
& 3776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6 \\
& 921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2
\end{aligned}$$

$$\begin{aligned}
& 851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 \\
& + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 441*B \\
& ^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17} \\
& *c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - \\
& b^2)^{15})^{(1/2)} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B \\
& *a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032 \\
& *A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B \\
& ^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 + 6*A*B*b^3*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1 \\
& 048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b \\
& ^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - \\
& 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)} \\
& *i)/((((256*A*a*b^{12}*c^4 - 5242880*A*a^7*c^{10} + 768*B*a*b^{13}*c^3 + 629145 \\
& 6*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5* \\
& b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^{11}*c^4 + 245760*B*a^3*b^9*c \\
& ^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8) \\
& / (512*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3 \\
& *b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^{19} + A^2*b \\
& ^{17}*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2 \\
& *b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5* \\
& b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b \\
& ^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^ \\
& 5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B \\
& ^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 441*B^2*a^2*c^2*(- \\
& (4*a*c - b^2)^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2* \\
& a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 \\
& + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6* \\
& c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(- \\
& (4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^ \\
& ^{15})^{(1/2)} - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^{10}*c \\
& ^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 537 \\
& 60*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7* \\
& b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)}*(256*b^{11}*c \\
& ^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^{10} + 40960*a^2*b^7*c^7 - 163840*a^3*b^ \\
& 5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96 \\
& *a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^{19} + A^2*b^{17}*c^2 + 9*B^2*b^4* \\
& (- (4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^ \\
& 2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2 \\
& *a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2* \\
& a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416* \\
& B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2 \\
& *b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1 \\
& /2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320 \\
& *A^2*a^8*b*c^{10} - 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 15482880*B^2*a^9
\end{aligned}$$

$$\begin{aligned}
& *b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))^(1/2) + (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7
\end{aligned}$$

$$\begin{aligned}
& c^7 - 1290240* A * B * a^7 * b^4 * c^8 - 5160960 * A * B * a^8 * b^2 * c^9 - 99 * B^2 * a * b^2 * c * (- \\
& (4 * a * c - b^2)^{15})^{(1/2)} - 288 * A * B * a * b^{16} * c^2 + 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^{15})^{(1/2)} - 108 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} / (512 * (1048576 * a^{10} * c^{15} + b^{20} * c^5 - 40 * a * b^{18} * c^6 + 720 * a^2 * b^{16} * c^7 - 7680 * a^3 * b^{14} * c^8 + 53760 * a^4 * b^{12} * c^9 - 258048 * a^5 * b^{10} * c^{10} + 860160 * a^6 * b^8 * c^{11} - 1966080 * a^7 * b^6 * c^{12} + 2949120 * a^8 * b^4 * c^{13} - 2621440 * a^9 * b^2 * c^{14}))^{(1/2)} * (256 * b^{11} * c^5 - 5120 * a * b^9 * c^6 - 262144 * a^5 * b * c^{10} + 40960 * a^2 * b^7 * c^7 - 163840 * a^3 * b^5 * c^8 + 327680 * a^4 * b^3 * c^9) / (32 * (256 * a^4 * c^7 + b^8 * c^3 - 16 * a * b^6 * c^4 + 96 * a^2 * b^4 * c^5 - 256 * a^3 * b^2 * c^6)) * (- (9 * B^2 * b^{19} + A^2 * b^{17} * c^2 + 9 * B^2 * b^4 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 6 * A * B * b^{18} * c + 1140 * A^2 * a^2 * b^{13} * c^4 - 10160 * A^2 * a^3 * b^{11} * c^5 + 34880 * A^2 * a^4 * b^9 * c^6 + 43776 * A^2 * a^5 * b^7 * c^7 - 680960 * A^2 * a^6 * b^5 * c^8 + 1863680 * A^2 * a^7 * b^3 * c^9 + 6921 * B^2 * a^2 * b^{15} * c^2 - 77580 * B^2 * a^3 * b^{13} * c^3 + 570960 * B^2 * a^4 * b^{11} * c^4 - 2851776 * B^2 * a^5 * b^9 * c^5 + 9628416 * B^2 * a^6 * b^7 * c^6 - 21095424 * B^2 * a^7 * b^5 * c^7 + 27095040 * B^2 * a^8 * b^3 * c^8 + A^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 441 * B^2 * a^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 6881280 * A * B * a^9 * c^{10} - 369 * B^2 * a * b^{17} * c - 55 * A^2 * a * b^{15} * c^3 - 1720320 * A^2 * a^8 * b * c^{10} - 25 * A^2 * a * c^3 * (- (4 * a * c - b^2)^{15})^{(1/2)} - 15482880 * B^2 * a^9 * b * c^9 + 5580 * A * B * a^2 * b^{14} * c^3 - 59280 * A * B * a^3 * b^{12} * c^4 + 377280 * A * B * a^4 * b^{10} * c^5 - 1430784 * A * B * a^5 * b^8 * c^6 + 2860032 * A * B * a^6 * b^6 * c^7 - 1290240 * A * B * a^7 * b^4 * c^8 - 5160960 * A * B * a^8 * b^2 * c^9 - 99 * B^2 * a * b^2 * c * (- (4 * a * c - b^2)^{15})^{(1/2)} - 288 * A * B * a * b^{16} * c^2 + 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^{15})^{(1/2)} - 108 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} / (512 * (1048576 * a^{10} * c^{15} + b^{20} * c^5 - 40 * a * b^{18} * c^6 + 720 * a^2 * b^{16} * c^7 - 7680 * a^3 * b^{14} * c^8 + 53760 * a^4 * b^{12} * c^9 - 258048 * a^5 * b^{10} * c^{10} + 860160 * a^6 * b^8 * c^{11} - 1966080 * a^7 * b^6 * c^{12} + 2949120 * a^8 * b^4 * c^{13} - 2621440 * a^9 * b^2 * c^{14}))^{(1/2)} + (x * (9 * B^2 * b^{10} + 800 * A^2 * a^4 * c^6 + A^2 * b^8 * c^2 - 14112 * B^2 * a^5 * c^5 + 6 * A * B * b^9 * c + 314 * A^2 * a^2 * b^4 * c^4 + 208 * A^2 * a^3 * b^2 * c^5 + 1881 * B^2 * a^2 * b^6 * c^2 - 9090 * B^2 * a^3 * b^4 * c^3 + 21312 * B^2 * a^4 * b^2 * c^4 - 198 * B^2 * a * b^8 * c - 36 * A^2 * a * b^6 * c^3 + 1422 * A * B * a^2 * b^5 * c^3 - 4464 * A * B * a^3 * b^3 * c^4 - 174 * A * B * a * b^7 * c^2 + 96 * A * B * a^4 * b * c^5) / (32 * (256 * a^4 * c^7 + b^8 * c^3 - 16 * a * b^6 * c^4 + 96 * a^2 * b^4 * c^5 - 256 * a^3 * b^2 * c^6)) * (- (9 * B^2 * b^{19} + A^2 * b^{17} * c^2 + 9 * B^2 * b^4 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 6 * A * B * b^{18} * c + 1140 * A^2 * a^2 * b^{13} * c^4 - 10160 * A^2 * a^3 * b^{11} * c^5 + 34880 * A^2 * a^4 * b^9 * c^6 + 43776 * A^2 * a^5 * b^7 * c^7 - 680960 * A^2 * a^6 * b^5 * c^8 + 1863680 * A^2 * a^7 * b^3 * c^9 + 6921 * B^2 * a^2 * b^{15} * c^2 - 77580 * B^2 * a^3 * b^{13} * c^3 + 570960 * B^2 * a^4 * b^{11} * c^4 - 2851776 * B^2 * a^5 * b^9 * c^5 + 9628416 * B^2 * a^6 * b^7 * c^6 - 21095424 * B^2 * a^7 * b^5 * c^7 + 27095040 * B^2 * a^8 * b^3 * c^8 + A^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 441 * B^2 * a^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 6881280 * A * B * a^9 * c^{10} - 369 * B^2 * a * b^{17} * c - 55 * A^2 * a * b^{15} * c^3 - 1720320 * A^2 * a^8 * b * c^{10} - 25 * A^2 * a * c^3 * (- (4 * a * c - b^2)^{15})^{(1/2)} - 15482880 * B^2 * a^9 * b * c^9 + 5580 * A * B * a^2 * b^{14} * c^3 - 59280 * A * B * a^3 * b^{12} * c^4 + 377280 * A * B * a^4 * b^{10} * c^5 - 1430784 * A * B * a^5 * b^8 * c^6 + 2860032 * A * B * a^6 * b^6 * c^7 - 1290240 * A * B * a^7 * b^4 * c^8 - 5160960 * A * B * a^8 * b^2 * c^9 - 99 * B^2 * a * b^2 * c * (- (4 * a * c - b^2)^{15})^{(1/2)} - 288 * A * B * a * b^{16} * c^2 + 6 * A * B * b^3 * c * (- (4 * a * c - b^2)^{15})^{(1/2)} - 108 * A * B * a * b * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} / (512 * (1048576 * a^{10} * c^{15} + b^{20} * c^5 - 40 * a * b^{18} * c^6 + 720 * a^2 * b^{16} * c^7 - 7680 * a^3 * b^{14} * c^8 + 53760 * a^4 * b^{12} * c^9 - 258048 * a^5 * b^{10} * c^{10} + 860160 * a^6 * b^8 * c^{11}
\end{aligned}$$

$$\begin{aligned}
& 1 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))^{(1/2)} + (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^3*a^3*b^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a^6*b^2*c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 + 10935*B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*A^2*B*a^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649*A^2*B*a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(2*56*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))) * (- (9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))^{(1/2)} * 2i - atan((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c
\end{aligned}$$

$$\begin{aligned}
&^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 29 \\
&49120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)}*(256*b^{11}*c^5 - 5120*a*b \\
&^9*c^6 - 262144*a^5*b*c^{10} + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32768 \\
&0*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 \\
&- 256*a^3*b^2*c^6)))*(-(9*B^2*b^{19} + A^2*b^{17}*c^2 - 9*B^2*b^4*(-(4*a*c - b^ \\
&2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^ \\
&5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 \\
&+ 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 \\
&+ 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^ \\
&^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4 \\
&*a*c - b^2)^{15})^{(1/2)} - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6881280 \\
&*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^ \\
&10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 15482880*B^2*a^9*b*c^9 + 5580 \\
&*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430 \\
&784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5 \\
&160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B \\
&*a*b^{16}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 108*A*B*a*b*c^2*(-(4* \\
&a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + \\
&720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10} \\
&*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - \\
&2621440*a^9*b^2*c^{14}))^{(1/2)} - (x*(9*B^2*b^{10} + 800*A^2*a^4*c^6 + A^2*b^8 \\
&*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3* \\
&b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^ \\
&^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^ \\
&^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8* \\
&c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^{19} + A^ \\
&2*b^{17}*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2* \\
&a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a \\
&^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^ \\
&2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2 \\
&*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709504 \\
&0*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 441*B^2*a^2*c^2 \\
&*(-(4*a*c - b^2)^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A \\
&^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(\\
&1/2)} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}* \\
&c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b \\
&^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c \\
&*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^ \\
&2)^{15})^{(1/2)} + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^1 \\
&0*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + \\
&53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a \\
&^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)}*1i - (((\\
&256*A*a*b^{12}*c^4 - 5242880*A*a^7*c^{10} + 768*B*a*b^{13}*c^3 + 6291456*B*a^7*b* \\
&c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + \\
&6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^{11}*c^4 + 245760*B*a^3*b^9*c^5 - 14745
\end{aligned}$$

$$\begin{aligned}
& 60*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(409 \\
& 6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + \\
& 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) + (x*(-(9*B^2*b^19 + A^2*b^17*c^2 - \\
& 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 \\
& - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - \\
& 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - \\
& 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 \\
& + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3 \\
& *c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b \\
& ^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 \\
& - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154828 \\
& 80*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280* \\
& A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290 \\
& 240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b \\
& ^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) \\
& + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20 \\
& *c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^1 \\
& 2*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + \\
& 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5120* \\
& a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 32 \\
& 7680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c \\
& ^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - \\
& b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11 \\
& *c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c \\
& ^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c \\
& ^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^ \\
& 7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(\\
& -(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881 \\
& 280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b \\
& *c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5 \\
& 580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1 \\
& 430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 \\
& - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288* \\
& A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(- \\
& (4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 \\
& + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b \\
& ^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^1 \\
& 3 - 2621440*a^9*b^2*c^14)))^(1/2) + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2* \\
& b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a \\
& ^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^ \\
& 2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A* \\
& B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b \\
& ^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + \\
& A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A \\
& ^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^
\end{aligned}$$

$$\begin{aligned}
& 2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2 \\
& *a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776* \\
& B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2709 \\
& 5040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2* \\
& c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 5 \\
& 5*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15 \\
&)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^ \\
& 12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^ \\
& 6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^ \\
& 2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - \\
& b^2)^15)^(1/2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576* \\
& a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 \\
& + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196608 \\
& 0*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*ii)/ \\
& (((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^7 \\
& *b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 \\
& + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 14 \\
& 74560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*(\\
& 4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^ \\
& 6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^2 \\
& - 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c \\
& ^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 \\
& - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 \\
& - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c \\
& ^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8* \\
& b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c \\
& - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15* \\
& c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 154 \\
& 82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772 \\
& 80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1 \\
& 290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c \\
& - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/ \\
& 2) + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b \\
& ^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4* \\
& b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1 \\
& 2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 51 \\
& 20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + \\
& 327680*a^4*b^3*c^9)/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^ \\
& 4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a* \\
& c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b \\
& ^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^ \\
& 5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1 \\
& 3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6 \\
& *b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^ \\
& 2*(-(4*a*c - b^2)^15)^(1/2) - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6
\end{aligned}$$

$$\begin{aligned}
& 881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 15482880*B^2*a^9*b*c^9 \\
& + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 \\
& - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 108*A*B*a*b*c^2 \\
& *(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} \\
& + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)} - (x*(9*B^2*b^{10} + 800*A^2*a^4*c^6 + A^2*b^8*c^2 \\
& - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 \\
& - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 \\
& + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^{19} + A^2*b^{17}*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 \\
& - 10160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 \\
& - 77580*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 \\
& - A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 \\
& - 1720320*A^2*a^8*b*c^{10} + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 \\
& + 377280*A*B*a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c \\
& *(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^{10}*c^{15} \\
& + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} \\
& + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)} + (((256*A*a*b^{12}*c^4 - 5242880*A*a^7*c^{10} + 768*B*a*b^{13}*c^3 + 6291456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 \\
& + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^{11}*c^4 + 245760*B*a^3*b^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 \\
& - 8650752*B*a^6*b^3*c^8)/(512*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) \\
& + (x*(-(9*B^2*b^{19} + A^2*b^{17}*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 10160*A^2*a^3*b^{11}*c^5 \\
& + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 77580*B^2*a^3*b^{13}*c^3 \\
& + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} \\
& - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - 1720320*A^2*a^8*b*c^{10} + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 154
\end{aligned}$$

$$\begin{aligned}
& 82880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 3772 \\
& 80*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1 \\
& 290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c \\
& - b^2)^15)^{(1/2)} - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^{(1/ \\
& 2)} + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^{(1/2))/(512*(1048576*a^10*c^15 + b \\
& ^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4* \\
& b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^1 \\
& 2 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))^{(1/2)}*(256*b^11*c^5 - 51 \\
& 20*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + \\
& 327680*a^4*b^3*c^9)/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^ \\
& 4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a* \\
& c - b^2)^15)^{(1/2)} + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b \\
& ^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^ \\
& 5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^1 \\
& 3*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6 \\
& *b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^ \\
& 2*(-(4*a*c - b^2)^15)^{(1/2)} - 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} + 6 \\
& 881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^ \\
& 8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^{(1/2)} - 15482880*B^2*a^9*b*c^9 \\
& + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 \\
& - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c \\
& ^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 2 \\
& 88*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^{(1/2)} + 108*A*B*a*b*c^2 \\
& *(-(4*a*c - b^2)^15)^{(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18* \\
& c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^ \\
& 5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4* \\
& c^13 - 2621440*a^9*b^2*c^14))^{(1/2)} + (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A \\
& ^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^ \\
& 2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4 \\
& *b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464 \\
& *A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 \\
& + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^1 \\
& 9 + A^2*b^17*c^2 - 9*B^2*b^4*(-(4*a*c - b^2)^15)^{(1/2)} + 6*A*B*b^18*c + 114 \\
& 0*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776 \\
& *A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921* \\
& B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 28517 \\
& 76*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 2 \\
& 7095040*B^2*a^8*b^3*c^8 - A^2*b^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - 441*B^2*a \\
& ^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c \\
& - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 + 25*A^2*a*c^3*(-(4*a*c - b^2) \\
& ^15)^{(1/2)} - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3 \\
& *b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B \\
& *a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a \\
& *b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 288*A*B*a*b^16*c^2 - 6*A*B*b^3*c*(-(4*a* \\
& c - b^2)^15)^{(1/2)} + 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^{(1/2))/(512*(10485
\end{aligned}$$

$$\begin{aligned}
& 76*a^{10}*c^{15} + b^{20}*c^5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}* \\
& c^8 + 53760*a^4*b^{12}*c^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 196 \\
& 6080*a^7*b^6*c^{12} + 2949120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)} + \\
& (35*A^3*a^2*b^7*c^2 - 592704*B^3*a^7*c^4 - 567*B^3*a^3*b^8 - 1176*A^3*a^3*b \\
& ^5*c^3 + 9456*A^3*a^4*b^3*c^4 - 89532*B^3*a^5*b^4*c^2 + 353808*B^3*a^6*b^2* \\
& c^3 + 315*A*B^2*a^2*b^9 - 33600*A^2*B*a^6*c^5 + 6400*A^3*a^5*b*c^5 + 10935* \\
& B^3*a^4*b^6*c - 6552*A*B^2*a^3*b^7*c + 560448*A*B^2*a^6*b*c^4 + 210*A^2*B*a \\
& ^2*b^8*c + 61524*A*B^2*a^4*b^5*c^2 - 280800*A*B^2*a^5*b^3*c^3 - 5649*A^2*B* \\
& a^3*b^6*c^2 + 42516*A^2*B*a^4*b^4*c^3 - 126192*A^2*B*a^5*b^2*c^4)/(256*(409 \\
& 6*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + \\
& 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*(-(9*B^2*b^{19} + A^2*b^{17}*c^2 - 9*B \\
& ^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{18}*c + 1140*A^2*a^2*b^{13}*c^4 - 1 \\
& 0160*A^2*a^3*b^{11}*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680 \\
& 960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^{15}*c^2 - 775 \\
& 80*B^2*a^3*b^{13}*c^3 + 570960*B^2*a^4*b^{11}*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9 \\
& 628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^ \\
& 8 - A^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 441*B^2*a^2*c^2*(-(4*a*c - b^2) \\
& ^{15})^{(1/2)} + 6881280*A*B*a^9*c^{10} - 369*B^2*a*b^{17}*c - 55*A^2*a*b^{15}*c^3 - \\
& 1720320*A^2*a^8*b*c^{10} + 25*A^2*a*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 15482880* \\
& B^2*a^9*b*c^9 + 5580*A*B*a^2*b^{14}*c^3 - 59280*A*B*a^3*b^{12}*c^4 + 377280*A*B \\
& *a^4*b^{10}*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240 \\
& *A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 + 99*B^2*a*b^2*c*(-(4*a*c - b^2) \\
& ^{15})^{(1/2)} - 288*A*B*a*b^{16}*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 1 \\
& 08*A*B*a*b*c^2*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(1048576*a^{10}*c^{15} + b^{20}*c^ \\
& 5 - 40*a*b^{18}*c^6 + 720*a^2*b^{16}*c^7 - 7680*a^3*b^{14}*c^8 + 53760*a^4*b^{12}*c \\
& ^9 - 258048*a^5*b^{10}*c^{10} + 860160*a^6*b^8*c^{11} - 1966080*a^7*b^6*c^{12} + 29 \\
& 49120*a^8*b^4*c^{13} - 2621440*a^9*b^2*c^{14}))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.111 \quad \int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=461

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}} + 12aAc^2 - 16abBc + 3Ab^2c + b^3B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{-40a^2Bc^2+36aAbc^2}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 4.62, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}} + 12aAc^2 - 16abBc + 3Ab^2c + b^3B\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{-40a^2Bc^2+36aAbc^2}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{x^2(-2aB + x^2(-bB - 2Ac)) + Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^3(-x^2(20aBc-12Abc+b^2B) + 4aAc - 12abB + 5Ab^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{x(20aBc-12Abc+b^2B)}{8c(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= -\frac{x^5 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{\int \frac{x^4 (5(Ab - 2aB) + (bB - 2Ac)x^2)}{(a + bx^2 + cx^4)^2} dx}{4 (b^2 - 4ac)} \\
&= -\frac{x^5 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^3 (5Ab^2 - 12abB + 4aAc - (b^2B - 12Abc + 20aBc))}{8 (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\
&= -\frac{(b^2B - 12Abc + 20aBc)x}{8c (b^2 - 4ac)^2} - \frac{x^5 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^3 (5Ab^2 - 12abB + 4aAc)}{8 (b^2 - 4ac)} \\
&= -\frac{(b^2B - 12Abc + 20aBc)x}{8c (b^2 - 4ac)^2} - \frac{x^5 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^3 (5Ab^2 - 12abB + 4aAc)}{8 (b^2 - 4ac)} \\
&= -\frac{(b^2B - 12Abc + 20aBc)x}{8c (b^2 - 4ac)^2} - \frac{x^5 (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x^3 (5Ab^2 - 12abB + 4aAc)}{8 (b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 2.02, size = 543, normalized size = 1.18

$$\frac{4(2a^2b + b^2(4a + 3b)) - 2a^2 + b^2(-8) + 4b^2(a - 3b)}{(b^2 - 4ac)(a + b^2 + cx^2)^3} + \frac{2(b^2(11a + 3bx^2) + 4ac^2(4 - 4b^2) + 12a^2(4x^2 - 3a)) + b^2(24 + b^2) - 2a^2}{(b^2 - 4ac)^2(a + b^2 + cx^2)^2} + \frac{\sqrt{2}\sqrt{4ac^2(4x\sqrt{b^2 - 4ac} + 3a)} + 3b^2(4x\sqrt{b^2 - 4ac} - 6a) - 4ab(4b\sqrt{b^2 - 4ac} + 3a) + 3^2(8\sqrt{b^2 - 4ac} - 3a) + 9(-8)}{16c^2\sqrt{b^2 - 4ac}} + \frac{\sqrt{2}\sqrt{4ac^2(4x\sqrt{b^2 - 4ac} - 3a)} + 3b^2(4x\sqrt{b^2 - 4ac} - 6a) - 4ab(4b\sqrt{b^2 - 4ac} - 3a) + 3^2(8\sqrt{b^2 - 4ac} + 3a) + 9(-8)}{16c^2\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out]
$$\frac{((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3*B*x^2)))/(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2 + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*\text{Sqrt}[b^2 - 4*a*c]) + 4*a*c^2*(10*a*B + 3*A*\text{Sqrt}[b^2 - 4*a*c]) + b^3*(-3*A*c + B*\text{Sqrt}[b^2 - 4*a*c]) - 4*a*b*c*(9*A*c + 4*B*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(5/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*\text{Sqrt}[b^2 - 4*a*c]) + 4*a*c^2*(-10*a*B + 3*A*\text{Sqrt}[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*\text{Sqrt}[b^2 - 4*a*c]) + b^3*(3*A*c + B*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(5/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])/(16*c^2)}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 6.94, size = 7060, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{16}*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2*(B*b^4 + 4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - 2*(2*B*a*b^3 + 4*A*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - \text{sqrt}(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*\text{sqrt}(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*$$

$$\begin{aligned}
& b^3)c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 \\
& - 6*A*B*b^6)*c + (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4* \\
& c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25* \\
& A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2) \\
& *c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2 \\
& *b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11}))/ (b^{10}*c^3 \\
& - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10 \\
& 24*a^5*c^8))*\log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14 \\
& *A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936 \\
& *A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^ \\
& 2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B \\
& ^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*\sqrt{1/2)*(B^3*b^ \\
& 10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4 \\
& *b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284* \\
& A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 \\
& - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^ \\
& 6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - \\
& (B*b^{13}*c^3 - 24576*A*a^6*c^{10} + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536 \\
& *(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - \\
& 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^{10})*c^5 \\
& - 18*(4*B*a*b^{11} - A*b^{12})*c^4)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2 \\
& *a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - \\
& 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^ \\
& 8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11}))*\sqrt{-(B^2*b^7 - \\
& 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3 \\
& *A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^ \\
& 2*a*b^5 - 6*A*B*b^6)*c + (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a \\
& ^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - \\
& 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^ \\
& 2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + \\
& 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11}))/ (b \\
& ^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^ \\
& 7 - 1024*a^5*c^8))) + \sqrt{1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 \\
& + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^ \\
& 2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2* \\
& b^3*c^2 + 16*a^3*b*c^3)*x^2)*\sqrt{-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b) \\
& *c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2 \\
& *b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^{10} \\
& *c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 \\
& - 1024*a^5*c^8)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)* \\
& c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 \\
& - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4* \\
& c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11}))/ (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a \\
& ^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*\log(-(35*B \\
& ^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
\end{aligned}$$

$$\begin{aligned}
& b^2)c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080 \\
& *A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 \\
& + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3* \\
& a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 \\
& - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 \\
& - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2 \\
& *B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + \\
& 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 6 \\
& 3*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*A*a \\
& ^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^ \\
& 4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15* \\
& A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^ \\
& 12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (\\
& 625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B \\
& ^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1 \\
& 280*a^4*b^2*c^10 - 1024*a^5*c^11))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2 \\
& *a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280* \\
& B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c \\
& + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b \\
& ^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^ \\
& 3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4 \\
& *a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a \\
& ^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 \\
& + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - \\
& sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2 \\
& *c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - \\
& 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3) \\
& *x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3* \\
& b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9 \\
& *A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 1 \\
& 60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B \\
& ^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 30 \\
& 0*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10 \\
& *c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 \\
& - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^ \\
& 4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 \\
& - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a \\
& ^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4* \\
& b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - \\
& 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6) \\
& *c)*x + 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64 \\
& *(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - \\
& 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b \\
& ^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A \\
& ^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*
\end{aligned}$$

$$\begin{aligned} & B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^{13}*c^3 - 24576*A*a^6*c^{10} + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^{10})*c^5 - 18*(4*B*a*b^{11} - A*b^{12})*c^4)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11})))*\sqrt{-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11})))/(b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)) + \sqrt{1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*\sqrt{-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11})))/(b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*\log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*\sqrt{1/2)*(B^3*b^{10} - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^{13}*c^3 - 24576*A*a^6*c^{10} + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^{10})*c^5 - 18*(4*B*a*b^{11} - A*b^{12})*c^4)*\sqrt{(B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^{10}*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^{10} - 1024*a^5*c^{11})))*\sqrt{-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 - 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^{10}*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))} \end{aligned}$$

$$\begin{aligned} &^2 + 3A^2ab^3)c^3 + (280B^2a^2b^3 - 60ABab^4 + 9A^2b^5)c^2 - \\ &(35B^2ab^5 - 6A^2Bb^6)c - (b^{10}c^3 - 20ab^8c^4 + 160a^2b^6c^5 - \\ &640a^3b^4c^6 + 1280a^4b^2c^7 - 1024a^5c^8)\sqrt{(B^4b^4 + 81A^4c^4 - 18(25A^2B^2a - 6A^3Bb)c^3 + (625B^4a^2 - 300A^2B^3ab + 54 \\ &A^2B^2b^2)c^2 - 2(25B^4aa^2 - 6A^2B^3b^3)c)/(b^{10}c^6 - 20ab^8c^7 + 160a^2b^6c^8 - 640a^3b^4c^9 + 1280a^4b^2c^{10} - 1024a^5c^{11} \\ &)))/(b^{10}c^3 - 20ab^8c^4 + 160a^2b^6c^5 - 640a^3b^4c^6 + 1280a^4b^2c^7 - 1024a^5c^8)) - 2*(B^2a^2b^2 + 4*(5B^2a^3 - 3A^2ab)c)*x)/(\\ &(b^4c^3 - 8ab^2c^4 + 16a^2c^5)*x^8 + a^2b^4c - 8a^3b^2c^2 + 16a^4c^3 + 2*(b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4)*x^6 + (b^6c - 6ab^4c^2 + 32a^3c^4)*x^4 + 2*(ab^5c - 8a^2b^3c^2 + 16a^3b^2c^3)*x^2) \end{aligned}$$

giac [B] time = 11.66, size = 7578, normalized size = 16.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (3 \cdot (2b^4c^3 - 32a^2c^5 - \sqrt{2})\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + \sqrt{b^2 - 4ac}) \cdot b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + 16\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2c^3 + 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2c^3 - 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2c^3 - 2 \cdot (b^2 - 4ac) \cdot b^2c^3 - 8 \cdot (b^2 - 4ac) \cdot a^2c^4) \cdot (b^4c - 8a^2b^2c^2 + 16a^2c^3)^2 \cdot A + (2b^5c^2 - 40ab^3c^3 + 128a^2b^2c^4 - \sqrt{2})\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + 20\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c - 64\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c - 32\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c + 16\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^3c - 2 \cdot (b^2 - 4ac) \cdot b^3c^2 + 32 \cdot (b^2 - 4ac) \cdot a^2b^3c^3) \cdot (b^4c - 8a^2b^2c^2 + 16a^2c^3)^2 \cdot B - 24 \cdot (\sqrt{2})\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + 12\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 - 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 - 2a^2b^7c^4 + 48\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 + 16\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 + 24a^2b^5c^5 - 64\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 - 32\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 - 96a^3b^3c^6 + 16\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}} + a^2b^5c^4 + 128a^4b^2c^7 + 2 \cdot (b^2 - 4ac) \cdot a^2b^5c^4 - 16 \cdot (b^2 - 4ac) \cdot a^2b^3c^5 + 32 \cdot (b^2 - 4ac) \cdot a^2b^3c^6) \cdot A \cdot \text{abs}(b^4c - 8a^2b^2c^2 + 16a^2c^3) + 2 \cdot (\sqrt{2})\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}$

$$\begin{aligned}
& *c) * a * b^8 * c^2 + 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^6 * c^3 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^7 * c^3 - 2 * a * b^8 * c^3 - 192 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^4 * c^4 - 24 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^5 * c^4 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^6 * c^4 - 16 * a^2 * b^6 * c^4 + 896 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^2 * c^5 + 288 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^3 * c^5 + 12 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^4 * c^5 + 384 * a^3 * b^4 * c^5 - 1280 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * c^6 - 640 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b * c^6 - 144 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^2 * c^6 - 1792 * a^4 * b^2 * c^6 + 320 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * c^7 + 2560 * a^5 * c^7 + 2 * (b^2 - 4 * a * c) * a * b^6 * c^3 + 24 * (b^2 - 4 * a * c) * a^2 * b^4 * c^4 - 288 * (b^2 - 4 * a * c) * a^3 * b^2 * c^5 + 640 * (b^2 - 4 * a * c) * a^4 * c^6) * B * \text{abs}(b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) - 3 * (2 * b^12 * c^5 - 8 * a * b^10 * c^6 - 192 * a^2 * b^8 * c^7 + 1792 * a^3 * b^6 * c^8 - 5632 * a^4 * b^4 * c^9 + 6144 * a^5 * b^2 * c^10 - \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^12 * c^3 + 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^10 * c^4 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^11 * c^4 + 96 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^8 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^10 * c^5 - 896 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^6 * c^6 - 192 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^7 * c^6 + 2816 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^4 * c^7 + 1024 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^5 * c^7 + 96 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^6 * c^7 - 3072 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b^2 * c^8 - 1536 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^3 * c^8 - 512 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^4 * c^8 + 768 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^2 * c^9 - 2 * (b^2 - 4 * a * c) * b^10 * c^5 + 192 * (b^2 - 4 * a * c) * a^2 * b^6 * c^7 - 1024 * (b^2 - 4 * a * c) * a^3 * b^4 * c^8 + 1536 * (b^2 - 4 * a * c) * a^4 * b^2 * c^9) * A - (2 * b^13 * c^4 - 68 * a * b^11 * c^5 + 688 * a^2 * b^9 * c^6 - 2688 * a^3 * b^7 * c^7 + 2048 * a^4 * b^5 * c^8 + 11264 * a^5 * b^3 * c^9 - 20480 * a^6 * b * c^10 - \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^13 * c^2 + 34 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^11 * c^3 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^12 * c^3 - 344 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^9 * c^4 - 60 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^10 * c^4 - \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^11 * c^4 + 1344 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^7 * c^5 + 448 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^8 * c^5 + 30 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^9 * c^5 - 1024 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^5 * c^6 - 896 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^6 * c^6 - 224 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^7 * c^6 - 5632 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b^3 * c^7 - 1536 * \sqrt{2} * \sqrt{b^2 - 4 * a * c}) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^4 * c^7
\end{aligned}$$

$$\begin{aligned}
& + 448\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^5c^7 + 10240\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^8c^8 + 5120\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^2c^8 + 768\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^3c^8 - 2560\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^9c^9 - 2(b^2 - 4ac)b^{11}c^4 + 60(b^2 - 4ac)ab^9c^5 - 448(b^2 - 4ac)a^2b^7c^6 + 896(b^2 - 4ac)a^3b^5c^7 + 1536(b^2 - 4ac)a^4b^3c^8 - 5120(b^2 - 4ac)a^5b^9c^9)B) \arctan(2\sqrt{1/2}x/\sqrt{(b^5c - 8ab^3c^2 + 16a^2b^2c^3 + \sqrt{(b^5c - 8ab^3c^2 + 16a^2b^2c^3)^2 - 4(ab^4c - 8a^2b^2c^2 + 16a^3c^3)(b^4c^2 - 8ab^2c^3 + 16a^2c^4))})/(b^4c^2 - 8ab^2c^3 + 16a^2c^4)))/((ab^{10}c^3 - 20a^2b^8c^4 - 2ab^9c^4 + 160a^3b^6c^5 + 32a^2b^7c^5 + ab^8c^5 - 640a^4b^4c^6 - 192a^3b^5c^6 - 16a^2b^6c^6 + 1280a^5b^2c^7 + 512a^4b^3c^7 + 96a^3b^4c^7 - 1024a^6c^8 - 512a^5b^9c^8 - 256a^4b^2c^8 + 256a^5c^9) \cdot \text{abs}(b^4c - 8ab^2c^2 + 16a^2c^3) \cdot \text{abs}(c)) - 1/64(3(2b^4c^3 - 32a^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^3c^2 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2c^3 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^3c^3 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^2c^3 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2c^4 - 2(b^2 - 4ac) \cdot b^2c^3 - 8(b^2 - 4ac) \cdot a^2c^4) \cdot (b^4c - 8ab^2c^2 + 16a^2c^3)^2A + (2b^5c^2 - 40ab^3c^3 + 128a^2b^2c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^5 + 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^3c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^4c - 64\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^2c^2 - 32\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot b^3c^2 + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^3c^3 - 2(b^2 - 4ac) \cdot b^3c^2 + 32(b^2 - 4ac) \cdot ab^3c^3) \cdot (b^4c - 8ab^2c^2 + 16a^2c^3)^2B + 24(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^7c^3 - 12\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^5c^4 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^6c^4 + 2ab^7c^4 + 48\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^3b^3c^5 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^4c^5 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^5c^5 - 24a^2b^5c^5 - 64\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^4b^2c^6 - 32\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^3b^2c^6 - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^3c^6 + 96a^3b^3c^6 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^3b^3c^7 - 128a^4b^3c^7 - 2(b^2 - 4ac) \cdot ab^5c^4 + 16(b^2 - 4ac) \cdot a^2b^3c^5 - 32(b^2 - 4ac) \cdot a^3b^3c^6)A \cdot \text{abs}(b^4c - 8ab^2c^2 + 16a^2c^3) - 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^8c^2 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^6c^3 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot \sqrt{b^2 - 4ac} \cdot ab^7c^3 + 2ab^8c^3 - 192\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^3b^4c^4 - 24\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot a^2b^5c^4 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c) \cdot ab^6c^4 + 16a^2b^6
\end{aligned}$$

$$\begin{aligned}
& *c^4 + 896*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^5 + 288*\sqrt{2} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^5 + 12*\sqrt{2}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a^2*b^4*c^5 - 384*a^3*b^4*c^5 - 1280*\sqrt{2}*\sqrt{b*c - \sqrt{ \\
& t(b^2 - 4*a*c)*c)}*a^5*c^6 - 640*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4 \\
& *b*c^6 - 144*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^6 + 1792*a^4 \\
& *b^2*c^6 + 320*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*c^7 - 2560*a^5*c \\
& ^7 - 2*(b^2 - 4*a*c)*a*b^6*c^3 - 24*(b^2 - 4*a*c)*a^2*b^4*c^4 + 288*(b^2 - \\
& 4*a*c)*a^3*b^2*c^5 - 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*c - 8*a*b^2*c^2 + \\
& 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*c^7 + 1792*a^3*b^ \\
& 6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*sq \\
& rt(b*c - \sqrt{b^2 - 4*a*c}}*c)*b^12*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b \\
& *c - \sqrt{b^2 - 4*a*c}}*c)*a*b^10*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c \\
& - \sqrt{b^2 - 4*a*c}}*c)*b^11*c^4 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^2*b^8*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{ \\
& t(b^2 - 4*a*c)*c)}*b^10*c^5 - 896*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{ \\
& b^2 - 4*a*c}}*c)*a^3*b^6*c^6 - 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{ \\
& (b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{ \\
& rt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^7 + 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \\
& \sqrt{b^2 - 4*a*c}}*c)*a^2*b^6*c^7 - 3072*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b*c \\
& - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^8 - 1536*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b* \\
& c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^8 - 512*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(b \\
& *c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^8 + 768*\sqrt{2}*\sqrt{b^2 - 4*a*c}}*sqrt(\\
& b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b^10*c^5 + 192*(b^ \\
& 2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 + 1536*(b^2 - 4*a*c \\
&)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3 \\
& *b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - \sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*b^13*c^2 + 34*\sqrt{2})*s \\
& qrt(b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^11*c^3 + 2*\sqrt{2})*sq \\
& rt(b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*b^12*c^3 - 344*\sqrt{2})*sqrt(\\
& b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^9*c^4 - 60*\sqrt{2})*sqrt(\\
& b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^10*c^4 - \sqrt{2})*sqrt(b^2 \\
& - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*b^11*c^4 + 1344*\sqrt{2})*sqrt(b^2 - \\
& 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^7*c^5 + 448*\sqrt{2})*sqrt(b^2 \\
& - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^8*c^5 + 30*\sqrt{2})*sqrt(b^2 \\
& - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^9*c^5 - 1024*\sqrt{2})*sqrt(b^2 \\
& - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^5*c^6 - 896*\sqrt{2})*sqrt(b^2 \\
& - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c^6 - 224*\sqrt{2})*sqrt(b^ \\
& 2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^7*c^6 - 5632*\sqrt{2})*sqrt(\\
& b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^3*c^7 - 1536*\sqrt{2})*sq \\
& rt(b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^4*c^7 + 448*\sqrt{2})*sq \\
& rt(b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^7 + 10240*\sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*b*c^8 + 5120*\sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^8 + 768*\sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}}*sqrt(b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^8 - 2560*\sqrt{2}
\end{aligned}$$

$$(2) \sqrt{b^2 - 4ac} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^5 b^9 c^9 - 2(b^2 - 4ac) b^{11} c^4 + 60(b^2 - 4ac) a^2 b^9 c^5 - 448(b^2 - 4ac) a^2 b^7 c^6 + 896(b^2 - 4ac) a^3 b^5 c^7 + 1536(b^2 - 4ac) a^4 b^3 c^8 - 5120(b^2 - 4ac) a^5 b^9 c^9) B \arctan\left(\frac{2\sqrt{1/2} x / \sqrt{(b^5 c - 8a^2 b^3 c^2 + 16a^2 b^2 c^3 - \sqrt{(b^5 c - 8a^2 b^3 c^2 + 16a^2 b^2 c^3)^2 - 4(a^2 b^4 c - 8a^2 b^2 c^2 + 16a^3 c^3)(b^4 c^2 - 8a^2 b^2 c^3 + 16a^2 c^4))}}}{(a^2 b^{10} c^3 - 20a^2 b^8 c^4 - 2a^2 b^9 c^4 + 160a^3 b^6 c^5 + 32a^2 b^7 c^5 + a^2 b^8 c^5 - 640a^4 b^4 c^6 - 192a^3 b^5 c^6 - 16a^2 b^6 c^6 + 1280a^5 b^2 c^7 + 512a^4 b^3 c^7 + 96a^3 b^4 c^7 - 1024a^6 c^8 - 512a^5 b^2 c^8 - 256a^4 b^2 c^8 + 256a^5 c^9) \operatorname{abs}(b^4 c - 8a^2 b^2 c^2 + 16a^2 c^3) \operatorname{abs}(c)} + \frac{1}{8}(B^2 b^3 c^2 x^7 - 16B^2 a^2 b^2 c^2 x^7 + 3A^2 b^2 c^2 x^7 + 12A^2 a^2 c^3 x^7 - B^2 b^4 x^5 - 5B^2 a^2 b^2 c^2 x^5 + 5A^2 b^3 c^2 x^5 - 36B^2 a^2 c^2 x^5 + 16A^2 a^2 b^2 c^2 x^5 - 2B^2 a^2 b^3 c^2 x^3 - 28B^2 a^2 b^2 c^2 x^3 + 19A^2 a^2 b^2 c^2 x^3 - 4A^2 a^2 c^2 x^3 - B^2 a^2 b^2 c^2 x - 20B^2 a^3 c^2 x + 12A^2 a^2 b^2 c^2 x) / ((b^4 c - 8a^2 b^2 c^2 + 16a^2 c^3)(c^2 x^4 + b^2 x^2 + a)^2)\right)$$

maple [B] time = 0.05, size = 1631, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^6(Bx^2+A)/(cx^4+bx^2+a)^3, x)$

[Out] $\frac{1}{8}(12A^2 a^2 c^2 + 3A^2 b^2 c - 16B^2 a^2 b^2 c + B^2 b^3) / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^7 + \frac{1}{8}(16A^2 a^2 b^2 c^2 + 5A^2 b^3 c - 36B^2 a^2 c^2 - 5B^2 a^2 b^2 c - B^2 b^4) / c / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^5 - \frac{1}{8} a / c (4A^2 a^2 c^2 - 19A^2 b^2 c + 28B^2 a^2 b^2 c + 2B^2 b^3) / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^3 + \frac{1}{8} a^2 (12A^2 b^2 c - 20B^2 a^2 c - B^2 b^2) / c / (16a^2 c^2 - 8a^2 b^2 c + b^4) x / (cx^4 + bx^2 + a)^2 - \frac{3}{4} / (16a^2 c^2 - 8a^2 b^2 c + b^4) c^2 (1/2) / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2}) * c)^{1/2} * c^2 x * a^2 A - 3/16 / (16a^2 c^2 - 8a^2 b^2 c + b^4) * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^2 + 9/4 / (16a^2 c^2 - 8a^2 b^2 c + b^4) * c / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 + 3/16 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 + 1 / (16a^2 c^2 - 8a^2 b^2 c + b^4) * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 + 1 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / c / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 - 1/16 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / c^2 (1/2) / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 - 5/2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) * c / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 - 9/8 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 + 1/16 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / c / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} * c^2 x * a^2 A * b^3 + 3/4$

$$\begin{aligned} & / (16*a^2*c^2-8*a*b^2*c+b^4)*c^{2^{1/2}} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * a * A + 3/16 / (16*a^2*c^2-8*a*b^2*c+b^4) * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * A * b^2 + 9/4 / (16*a^2*c^2-8*a*b^2*c+b^4) * c / (-4*a*c+b^2)^{1/2} * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * a * A * b + 3/16 / (16*a^2*c^2-8*a*b^2*c+b^4) / (-4*a*c+b^2)^{1/2} * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * A * b^3 - 1 / (16*a^2*c^2-8*a*b^2*c+b^4) * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * a * b * B + 1/16 / (16*a^2*c^2-8*a*b^2*c+b^4) / c * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * b^3 * B - 5/2 / (16*a^2*c^2-8*a*b^2*c+b^4) * c / (-4*a*c+b^2)^{1/2} * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * a^2 * B - 9/8 / (16*a^2*c^2-8*a*b^2*c+b^4) / (-4*a*c+b^2)^{1/2} * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * a * b^2 * B + 1/16 / (16*a^2*c^2-8*a*b^2*c+b^4) / c / (-4*a*c+b^2)^{1/2} * 2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * \arctan(2^{1/2} / ((b+(-4*a*c+b^2)^{1/2})*c)^{1/2} * c*x) * b^4 * B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out]
$$\frac{1}{8} * ((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)*x) / ((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) + 1/8 * integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^2)*c)*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c) / (c*x^4 + b*x^2 + a), x) / (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)$$

mupad [B] time = 3.95, size = 19041, normalized size = 41.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out]
$$\operatorname{atan}((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5$$

$$\begin{aligned}
& + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{1/2}) + B^2*b^2*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{1/2} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{1/2} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{1/2}*(256*b^{11}*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{1/2}) + B^2*b^2*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{1/2} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{1/2} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{1/2} - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{1/2}) + B^2*b^2*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{1/2} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{1/2} - 180*A*B*a*b^{14}*c^2)
\end{aligned}$$

$$\begin{aligned}
& 4*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 \\
& - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 \\
& - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*i - (((5242880*B*a^7*c^8 + 3072*A*a*b^{11}*c^3 - 3145728*A*a^6 \\
& *b*c^8 - 256*B*a*b^{12}*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 19 \\
& 66080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360* \\
& B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^{12}*c \\
& + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840 \\
& *a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2 \\
& *c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B* \\
& b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7 \\
& *c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 \\
& - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 \\
& - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 5 \\
& 5*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 \\
& - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 240 \\
& 00*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178 \\
& 1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{1 \\
& 5})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 \\
& + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048 \\
& *a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4 \\
& *c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*(256*b^{11}*c^3 - 5120*a*b^9*c^4 - 2621 \\
& 44*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7) \\
&)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4 \\
& 4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B \\
& ^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 3 \\
& 7440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 5529 \\
& 60*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880 \\
& *B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680 \\
& *B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a \\
& *c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B \\
& ^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a \\
& ^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B* \\
& a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(51 \\
& 2*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a \\
& ^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 \\
& - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1 \\
& /2)} + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A \\
& *B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 \\
& - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c \\
& ^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 \\
& - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a \\
& *c - b^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 50 \\
& 40*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216 \\
& *A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B
\end{aligned}$$

$$\begin{aligned}
&^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*i)/(((5242880*B*a^7*c^8 + 3072*A*a*b^{11}*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^{12}*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*(256*b^{11}*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)} - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2
\end{aligned}$$

$$\begin{aligned}
& *a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 6 \\
& 6*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c \\
& ^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A \\
& ^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A* \\
& B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b \\
& ^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13* \\
& c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^ \\
& 5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - \\
& 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^ \\
& 3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 2 \\
& 4000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1 \\
& 781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2) \\
& ^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a* \\
& b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 2580 \\
& 48*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b \\
& ^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) + (((5242880*B*a^7*c^8 + 3072*A*a*b \\
& ^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 49 \\
& 1520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440* \\
& B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^ \\
& 6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - \\
& 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + \\
& 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - \\
& b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c \\
& ^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 \\
& + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + \\
& 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + \\
& 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) \\
&) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240 \\
& *A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 99225 \\
& 6*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A* \\
& B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^ \\
& 13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 5376 \\
& 0*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6 \\
& *c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 \\
& - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^ \\
& 6 + 327680*a^4*b^3*c^7)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b \\
& ^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a \\
& *c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 50 \\
& 40*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216 \\
& *A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B \\
& ^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^ \\
& 2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^1 \\
& 5*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^ \\
& 2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3* \\
& b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^
\end{aligned}$$

$$\begin{aligned}
& 6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 72 \\
& 0*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c \\
& ^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 262 \\
& 1440*a^9*b^2*c^{12}))^{(1/2)} + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 \\
& + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 \\
& + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b \\
& ^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - \\
& 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15} \\
& *c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1 \\
& /2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680 \\
& *A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2 \\
& *a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2* \\
& a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B \\
& *a^8*c^9 - 55*B^2*a*b^{15}*c - 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2 \\
& *a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^ \\
& 12*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b \\
& ^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4* \\
& a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c \\
& ^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12} \\
& c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 294 \\
& 9120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)} + (1728*A^3*a^4*c^5 - 35* \\
& B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9456*B^3*a^4*b^ \\
& 3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c^2 + 1176*B^3* \\
& a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552*A^2*B*a^4*b*c^ \\
& 4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A^2*B*a^2*b^5*c \\
& ^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^{12}*c + 4096*a^6*c^ \\
& 7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - \\
& 6144*a^5*b^2*c^6)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 + 9*A^2*c^2*(-(4*a*c - b \\
& ^2)^{15})^{(1/2)} + B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2 \\
& *a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a \\
& ^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3 \\
& *b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6* \\
& b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c - \\
& 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7* \\
& b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c \\
& ^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4* \\
& c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A* \\
& B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2* \\
& b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 8 \\
& 60160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a \\
& ^9*b^2*c^{12}))^{(1/2)}*2i - ((x^5*(B*b^4 + 36*B*a^2*c^2 - 5*A*b^3*c - 16*A*a* \\
& b*c^2 + 5*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^7*(B*b^3 + \\
& 12*A*a*c^2 + 3*A*b^2*c - 16*B*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + \\
& (x^3*(4*A*a^2*c^2 + 2*B*a*b^3 - 19*A*a*b^2*c + 28*B*a^2*b*c))/(8*c*(b^4 + 1
\end{aligned}$$

$$\begin{aligned}
& (6a^2c^2 - 8ab^2c) + (a^2x(Bb^2 - 12A^*b^*c + 20B^*a^*c))/(8c(b^4 + \\
& 16a^2c^2 - 8ab^2c)))/(x^4(2a^*c + b^2) + a^2 + c^2x^8 + 2a^*bx^2 + \\
& 2b^*cx^6) + \operatorname{atan}\left(\frac{(5242880B^*a^7c^8 + 3072A^*a^*b^{11}c^3 - 3145728A^*a^6b^*c^8 - 256B^*a^*b^{12}c^2 - 61440A^*a^2b^9c^4 + 491520A^*a^3b^7c^5 - 1966080A^*a^4b^5c^6 + 3932160A^*a^5b^3c^7 + 61440B^*a^3b^8c^4 - 655360B^*a^4b^6c^5 + 2949120B^*a^5b^4c^6 - 6291456B^*a^6b^2c^7)/(512(b^{12}c + 4096a^6c^7 - 24a^*b^{10}c^2 + 240a^2b^8c^3 - 1280a^3b^6c^4 + 3840a^4b^4c^5 - 6144a^5b^2c^6)) - (x(-(B^2b^{17} + 9A^2b^{15}c^2 - 9A^2c^2(-(4a^*c - b^2)^{15})^{1/2}) - B^2b^2(-(4a^*c - b^2)^{15})^{1/2}) + 6A^*B^*b^{16}c - 5040A^2a^2b^{11}c^4 + 37440A^2a^3b^9c^5 - 103680A^2a^4b^7c^6 - 9216A^2a^5b^5c^7 + 552960A^2a^6b^3c^8 + 1140B^2a^2b^{13}c^2 - 10160B^2a^3b^{11}c^3 + 34880B^2a^4b^9c^4 + 43776B^2a^5b^7c^5 - 680960B^2a^6b^5c^6 + 1863680B^2a^7b^3c^7 + 983040A^*B^*a^8c^9 - 55B^2a^*b^{15}c + 25B^2a^*c(-(4a^*c - b^2)^{15})^{1/2}) + 180A^2a^*b^{13}c^3 - 737280A^2a^7b^*c^9 - 1720320B^2a^8b^*c^8 + 240A^*B^*a^2b^{12}c^3 + 24000A^*B^*a^3b^{10}c^4 - 241920A^*B^*a^4b^8c^5 + 992256A^*B^*a^5b^6c^6 - 1781760A^*B^*a^6b^4c^7 + 737280A^*B^*a^7b^2c^8 - 6A^*B^*b^*c(-(4a^*c - b^2)^{15})^{1/2}) - 180A^*B^*a^*b^{14}c^2)/(512(1048576a^{10}c^{13} + b^{20}c^3 - 40a^*b^{18}c^4 + 720a^2b^{16}c^5 - 7680a^3b^{14}c^6 + 53760a^4b^{12}c^7 - 258048a^5b^{10}c^8 + 860160a^6b^8c^9 - 1966080a^7b^6c^{10} + 2949120a^8b^4c^{11} - 2621440a^9b^2c^{12}))^{1/2} * (256b^{11}c^3 - 5120a^*b^9c^4 - 262144a^5b^*c^8 + 40960a^2b^7c^5 - 163840a^3b^5c^6 + 327680a^4b^3c^7) / (32(b^8c + 256a^4c^5 - 16a^*b^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)) * (-(B^2b^{17} + 9A^2b^{15}c^2 - 9A^2c^2(-(4a^*c - b^2)^{15})^{1/2}) - B^2b^2(-(4a^*c - b^2)^{15})^{1/2}) + 6A^*B^*b^{16}c - 5040A^2a^2b^{11}c^4 + 37440A^2a^3b^9c^5 - 103680A^2a^4b^7c^6 - 9216A^2a^5b^5c^7 + 552960A^2a^6b^3c^8 + 1140B^2a^2b^{13}c^2 - 10160B^2a^3b^{11}c^3 + 34880B^2a^4b^9c^4 + 43776B^2a^5b^7c^5 - 680960B^2a^6b^5c^6 + 1863680B^2a^7b^3c^7 + 983040A^*B^*a^8c^9 - 55B^2a^*b^{15}c + 25B^2a^*c(-(4a^*c - b^2)^{15})^{1/2}) + 180A^2a^*b^{13}c^3 - 737280A^2a^7b^*c^9 - 1720320B^2a^8b^*c^8 + 240A^*B^*a^2b^{12}c^3 + 24000A^*B^*a^3b^{10}c^4 - 241920A^*B^*a^4b^8c^5 + 992256A^*B^*a^5b^6c^6 - 1781760A^*B^*a^6b^4c^7 + 737280A^*B^*a^7b^2c^8 - 6A^*B^*b^*c(-(4a^*c - b^2)^{15})^{1/2}) - 180A^*B^*a^*b^{14}c^2)/(512(1048576a^{10}c^{13} + b^{20}c^3 - 40a^*b^{18}c^4 + 720a^2b^{16}c^5 - 7680a^3b^{14}c^6 + 53760a^4b^{12}c^7 - 258048a^5b^{10}c^8 + 860160a^6b^8c^9 - 1966080a^7b^6c^{10} + 2949120a^8b^4c^{11} - 2621440a^9b^2c^{12}))^{1/2} - (x(B^2b^8 - 288A^2a^3c^5 + 9A^2b^6c^2 + 800B^2a^4c^4 + 6A^*B^*b^7c + 576A^2a^2b^2c^4 + 314B^2a^2b^4c^2 + 208B^2a^3b^2c^3 - 36B^2a^*b^6c + 126A^2a^*b^4c^3 - 816A^*B^*a^2b^3c^3 - 66A^*B^*a^*b^5c^2 - 672A^*B^*a^3b^*c^4)) / (32(b^8c + 256a^4c^5 - 16a^*b^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)) * (-(B^2b^{17} + 9A^2b^{15}c^2 - 9A^2c^2(-(4a^*c - b^2)^{15})^{1/2}) - B^2b^2(-(4a^*c - b^2)^{15})^{1/2}) + 6A^*B^*b^{16}c - 5040A^2a^2b^{11}c^4 + 37440A^2a^3b^9c^5 - 103680A^2a^4b^7c^6 - 9216A^2a^5b^5c^7 + 552960A^2a^6b^3c^8 + 1140B^2a^2b^{13}c^2 - 10160B^2a^3b^{11}c^3 + 34880B^2a^4b^9c^4 + 43776B^2a^5b^7c^5 - 680960B
\end{aligned}$$

$$\begin{aligned}
&^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^{(1/2)} + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^{(1/2)} - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12))^{(1/2)}*i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^{(1/2)} + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^{(1/2)} - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12))^{(1/2)}*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^{(1/2)} + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^{(1/2)} - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12))^{(1/2)} + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 -
\end{aligned}$$

$$\begin{aligned}
& (66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*i)/(((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))*(-(B^2*b^17 + 9*A^2*b^15*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c + 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2)
\end{aligned}$$

$$\begin{aligned}
&) - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 \\
& + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - \\
& 2621440*a^9*b^2*c^{12}))^{(1/2)} - (x*(B^2*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c + 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)} + (((5242880*B*a^7*c^8 + 3072*A*a*b^{11}*c^3 - 3145728*A*a^6*b*c^8 - 256*B*a*b^{12}*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c + 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*(256*b^{11}*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}
\end{aligned}$$

$$\begin{aligned}
& *c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 \\
& - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 \\
& - 55*B^2*a*b^{15}*c + 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 \\
& - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + \\
& 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - \\
& 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} \\
& - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a \\
& *b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258 \\
& 048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8* \\
& b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)} + (x*(B^2*b^8 - 288*A^2*a^3*c^5 + \\
& 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 576*A^2*a^2*b^2*c^4 + 314*B \\
& ^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^6*c + 126*A^2*a*b^4*c^3 - \\
& 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*a^3*b*c^4))/(32*(b^8*c + \\
& 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b^1 \\
& 7 + 9*A^2*b^{15}*c^2 - 9*A^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*b^2*(-(4*a*c \\
& - b^2)^{15})^{(1/2)} + 6*A*B*b^{16}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^ \\
& 9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3* \\
& c^8 + 1140*B^2*a^2*b^{13}*c^2 - 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^ \\
& 4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^ \\
& 7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^{15}*c + 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(\\
& 1/2)} + 180*A^2*a*b^{13}*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + \\
& 240*A*B*a^2*b^{12}*c^3 + 24000*A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 99 \\
& 2256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6 \\
& *A*B*b*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10} \\
& *c^{13} + b^{20}*c^3 - 40*a*b^{18}*c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 5 \\
& 3760*a^4*b^{12}*c^7 - 258048*a^5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7* \\
& b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)} + (1728*A^3 \\
& *a^4*c^5 - 35*B^3*a^2*b^7 + 1620*A^3*a^2*b^4*c^3 + 4752*A^3*a^3*b^2*c^4 - 9 \\
& 456*B^3*a^4*b^3*c^2 + 15*A*B^2*a*b^8 + 4800*A*B^2*a^5*c^4 + 135*A^3*a*b^6*c \\
& ^2 + 1176*B^3*a^3*b^5*c - 6400*B^3*a^5*b*c^3 - 705*A*B^2*a^2*b^6*c - 15552* \\
& A^2*B*a^4*b*c^4 + 6084*A*B^2*a^3*b^4*c^2 + 26256*A*B^2*a^4*b^2*c^3 - 1260*A \\
& ^2*B*a^2*b^5*c^2 - 13248*A^2*B*a^3*b^3*c^3 + 90*A^2*B*a*b^7*c)/(256*(b^{12}*c \\
& + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840 \\
& *a^4*b^4*c^5 - 6144*a^5*b^2*c^6))))*(-(B^2*b^{17} + 9*A^2*b^{15}*c^2 - 9*A^2*c^ \\
& 2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*b^{1 \\
& 6}*c - 5040*A^2*a^2*b^{11}*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^ \\
& 6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^{13}*c^2 - \\
& 10160*B^2*a^3*b^{11}*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 6 \\
& 80960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B \\
& ^2*a*b^{15}*c + 25*B^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 180*A^2*a*b^{13}*c^3 - 7 \\
& 37280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^{12}*c^3 + 24000* \\
& A*B*a^3*b^{10}*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178176 \\
& 0*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 - 6*A*B*b*c*(-(4*a*c - b^2)^{15})^{(\\
& 1/2)} - 180*A*B*a*b^{14}*c^2)/(512*(1048576*a^{10}*c^{13} + b^{20}*c^3 - 40*a*b^{18}* \\
& c^4 + 720*a^2*b^{16}*c^5 - 7680*a^3*b^{14}*c^6 + 53760*a^4*b^{12}*c^7 - 258048*a^
\end{aligned}$$

$5*b^{10}*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^{10} + 2949120*a^8*b^4*c^{11} - 2621440*a^9*b^2*c^{12}))^{(1/2)}*2i$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.112 \quad \int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=380

$$\frac{3x \left(x^2 (4aBc - 4Abc + b^2B) - A(4ac + b^2) + 4abB \right) - x^3 \left(-2aB - (x^2(bB - 2Ac)) + Ab \right)}{8(b^2 - 4ac)^2 (a + bx^2 + cx^4)} + \frac{3 \left(-\frac{-8aAc^2 + 12abBc - 6Ab^2c}{\sqrt{b^2 - 4ac}} \right)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3 \left(-\frac{-8aAc^2 + 12abBc - 6Ab^2c}{\sqrt{b^2 - 4ac}} \right)}{8\sqrt{2} \sqrt{c} (b^2 - 4ac)^2 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.41, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1275, 1166, 205}

$$\frac{3 \left(-\frac{-8aAc^2 + 12abBc - 6Ab^2c + b^3B}{\sqrt{b^2 - 4ac}} + 4aBc - 4Abc + b^2B \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + 3 \left(-\frac{-8aAc^2 + 12abBc - 6Ab^2c + b^3B}{\sqrt{b^2 - 4ac}} + 4aBc - 4Abc + b^2B \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right) - \frac{x^3 (-2aB + x^2(-bB - 2Ac) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3x \left(x^2 (4aBc - 4Abc + b^2B) - A(4ac + b^2) + 4abB \right)}{8(b^2 - 4ac)^2 (a + bx^2 + cx^4)}}{8\sqrt{2} \sqrt{c} (b^2 - 4ac)^2 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

[Out] $-(x^3(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c - (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(8*\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2 - 4*a*c)^2*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(8*\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2 - 4*a*c)^2*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= -\frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{\int \frac{x^2(3(Ab - 2aB) + 3(bB - 2Ac)x^2)}{(a + bx^2 + cx^4)^2} dx}{4(b^2 - 4ac)} \\ &= -\frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3x(4abB - A(b^2 + 4ac) + (b^2B - 4Abc + 4aBc)x)}{8(b^2 - 4ac)^2(a + bx^2 + cx^4)} \\ &= -\frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3x(4abB - A(b^2 + 4ac) + (b^2B - 4Abc + 4aBc)x)}{8(b^2 - 4ac)^2(a + bx^2 + cx^4)} \\ &= -\frac{x^3 (Ab - 2aB - (bB - 2Ac)x^2)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{3x(4abB - A(b^2 + 4ac) + (b^2B - 4Abc + 4aBc)x)}{8(b^2 - 4ac)^2(a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 1.70, size = 447, normalized size = 1.18

$$\frac{8ac(A+Bx^2)-4abBx+4b^2(Ac-bB)}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{2(4c(ab-3Acx^2)+4ac^2(A+3Bx^2)+b^2(3Bx^2-7Ac)+2b^3B)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+6Ac\right)-4c\left(A\sqrt{b^2-4ac}+3ab\right)+4ac\left(b\sqrt{b^2-4ac}+2Ac\right)+b^3(-B)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b^2-4ac}}\right) + \frac{3\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-6Ac\right)+4c\left(3aB-A\sqrt{b^2-4ac}\right)+4ac\left(b\sqrt{b^2-4ac}-2Ac\right)+b^3B)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*sqrt[2]*sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*sqrt[b^2 - 4*a*c]) + 4*a*c*(2*A*c + B*sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^5

/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(16*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 5.87, size = 5650, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^2 + (16*B*a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a

$$\begin{aligned}
& ^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)) * \text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 \\
& + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \text{sqrt} \\
& t((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^{10}*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^{10}* \\
& c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) + 3*\text{sqrt}(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + \\
& 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3 \\
& *b*c^2)*x^2) * \text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + \\
& A^2*b^5)*c + (a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2) \\
&)/(a^2*b^{10}*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - \\
& 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (\\
& 16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/2*\text{sqrt}(1/2)*(4*B^3*a^2*b^7 - \\
& A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(2 \\
& 4*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - \\
& 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + \\
& 8*(14*B*a^3*b^{10} - A*a^2*b^{11})*c^2 - (6*B*a^2*b^{12} - A*a*b^{13})*c) * \text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^{10}*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - \\
& 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)) * \text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + \\
& A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \text{sqrt} \\
& ((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^{10}*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - \\
& 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) - 3*\text{sqrt}(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) * \text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^{10}*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^{10}*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)) * \log(-
\end{aligned}$$

$$\begin{aligned}
& 27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4* \\
& a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 \\
& + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*\text{sqrt}(1/2)*(4 \\
& *B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a \\
& ^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b \\
& ^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^ \\
& 3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*(2*B*a^8 - 3*A \\
& *a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5* \\
& A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A* \\
& a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^1 \\
& 3)*c)*\text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c \\
& ^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7))) \\
& *\text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4* \\
& A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c \\
& - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5 \\
& *b^2*c^5 - 1024*a^6*c^6))*\text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10 \\
& *c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^ \\
& 6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4* \\
& b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*\text{sqrt}(1/2)*((b^4*c^2 - 8*a* \\
& b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^ \\
& 2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(\\
& a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - \\
& 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40* \\
& B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160* \\
& a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*\text{sqrt}((B^4* \\
& a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6 \\
& *c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20* \\
& a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a \\
& ^6*c^6))*\log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3* \\
& B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - \\
& 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/ \\
& 2*\text{sqrt}(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a \\
& ^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + \\
& 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^ \\
& 2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096* \\
& (2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2* \\
& B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10* \\
& B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2* \\
& b^12 - A*a*b^13)*c)*\text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 \\
& - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1 \\
& 024*a^7*c^7)))*\text{sqrt}(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2* \\
& B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 \\
& + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4* \\
& c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*\text{sqrt}((B^4*a^2 - 2*A^2*B^2*a*c + A^4* \\
& c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1
\end{aligned}$$

$$\frac{280a^6b^2c^6 - 1024a^7c^7)}{(a^2b^{10}c - 20a^2b^8c^2 + 160a^3b^6c^3 - 640a^4b^4c^4 + 1280a^5b^2c^5 - 1024a^6c^6))} + 6(4B^2a^2b - A^2a^2b^2 - 4A^2a^2c)x) / ((b^4c^2 - 8a^2b^2c^3 + 16a^2c^4)x^8 + 2(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x^6 + a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^6 - 6a^2b^4c + 32a^3c^3)x^4 + 2(a^2b^5 - 8a^2b^3c + 16a^3b^2c^2)x^2)$$

giac [B] time = 8.26, size = 3162, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] $\frac{3}{32} \left(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) b^6 - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^4 c - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) b^5 c - 2 b^6 c - 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c^2 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) b^4 c^2 + 8 a^2 b^4 c^2 + 2 b^5 c^2 + 64 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^3 c^3 + 32 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^2 c^3 + 32 a^2 b^2 c^3 + 16 a^2 b^3 c^3 - 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 c^4 - 128 a^3 c^4 - 96 a^2 b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) b^5 - 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^3 c + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) b^4 c + 48 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c^2 + 24 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^2 c^2 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(b^3 c^2 - 12 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^2 c^3 + 2(b^2 - 4ac) b^4 c - 2(b^2 - 4ac) b^3 c^2 - 32(b^2 - 4ac) a^2 c^3 - 24(b^2 - 4ac) a^2 b^2 c^3 \right) A - 2(2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^5 - 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^3 c - 4 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^4 c - 4 a^2 b^5 c + 32 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^3 b^2 c^2 + 16 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c^2 + 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^3 c^2 + 32 a^2 b^3 c^2 + 6 a^2 b^4 c^2 - 8 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c^3 - 64 a^3 b^2 c^3 - 16 a^2 b^2 c^3 - 32 a^3 c^4 - 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^4 + 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c + 6 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^3 c + 16 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^3 c^2 + 8 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 b^2 c^2 - 3 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \left(a^2 b^2 c^2 - 4 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c \right) a^2 c^3 + 4(b^2 - 4ac) a^2 b^3 c - 16(b^2 - 4ac) a^2 b^2 c^2 - 8(b^2 - 4ac) a^2 c^3) B) \arctan(2 \sqrt{1/2} x / \sqrt{(b^5 - 8 a^2 b^3 c + 16 a^2 b^2 c^2 + \sqrt{2} \sqrt{b^5 - 8 a^2 b^3 c + 16 a^2 b^2 c^2})^2 - 4(a^2 b^4 - 8 a^2 b^2 c + 16 a^3 c^2)})$

$$\begin{aligned} &)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) \\ & /((a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + \\ & 128*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 3/32*((sqrt(2)*sqrt(\\ & b*c - sqrt(b^2 - 4*a*c))*c)*b^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)* \\ & a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*b^5*c + 2*b^6*c - 16*sq \\ & rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt \\ & (b^2 - 4*a*c))*c)*b^4*c^2 - 8*a*b^4*c^2 - 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c - \\ & sqrt(b^2 - 4*a*c))*a^3*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a \\ & ^2*b*c^3 - 32*a^2*b^2*c^3 - 16*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - \\ & 4*a*c))*a^2*c^4 + 128*a^3*c^4 + 96*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c) \\ & *sqrt(b*c - sqrt(b^2 - 4*a*c))*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c \\ & - sqrt(b^2 - 4*a*c))*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq \\ & rt(b^2 - 4*a*c))*b^4*c - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 \\ & - 4*a*c))*a^2*b*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - \\ & 4*a*c))*a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a* \\ & c))*b^3*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))* \\ &)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a \\ & *c)*a^2*c^3 + 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^ \\ & 2 - 4*a*c))*a*b^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^3*c \\ & - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4*c + 4*a*b^5*c + 32*sqrt(2) \\ &)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^ \\ & 2 - 4*a*c))*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3 \\ & *c^2 - 32*a^2*b^3*c^2 - 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c) \\ &)*a^2*b*c^3 + 64*a^3*b*c^3 + 16*a^2*b^2*c^3 + 32*a^3*c^4 + 3*sqrt(2)*sq \\ & rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4 - 8*sqrt(2)*sqrt(b^2 - \\ & 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c - 6*sqrt(2)*sqrt(b^2 - 4* \\ & a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c) \\ & *sqrt(b*c - sqrt(b^2 - 4*a*c))*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt \\ & (b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b* \\ & c - sqrt(b^2 - 4*a*c))*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - \\ & sqrt(b^2 - 4*a*c))*a^2*c^3 - 4*(b^2 - 4*a*c)*a*b^3*c + 16*(b^2 - 4*a*c)* \\ & a^2*b*c^2 + 6*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(\\ & 2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*b^3*c \\ & + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c \\ & ^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^ \\ & 6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2 \\ & *c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a \\ & ^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 1/8*(3*B*b^2*c*x^7 + 12*B*a*c^2*x^7 - 12 \\ & *A*b*c^2*x^7 + 5*B*b^3*x^5 + 16*B*a*b*c*x^5 - 19*A*b^2*c*x^5 + 4*A*a*c^2*x^ \\ & 5 + 19*B*a*b^2*x^3 - 5*A*b^3*x^3 - 4*B*a^2*c*x^3 - 16*A*a*b*c*x^3 + 12*B*a^ \\ & 2*b*x - 3*A*a*b^2*x - 12*A*a^2*c*x)/(c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c \\ & + 16*a^2*c^2)) \end{aligned}$$

maple [B] time = 0.05, size = 1283, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3, x)$

[Out]
$$\begin{aligned} & (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c \\ & ^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A* \\ & a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4 \\ & *A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x/(c*x^4+b*x^2+a)^2+3/4/(\\ & 16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan \\ & h(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b \\ & ^2*c+b^4)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}* \\ & \arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*aA-9/8/(16*a^2*c^2- \\ & 8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/ \\ & 2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2-3/4/(16*a^2 \\ & *c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(\\ & 1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*aB-3/16/(16*a^2*c^2-8*a*b^2*c+ \\ & b^4)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*B+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b \\ & ^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(- \\ & 4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a \\ & *c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(- \\ & 4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*B-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c* \\ & 2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1 \\ & /2)})*c)^{(1/2)}*c*x)*A*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^{(1/2 \\ &)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^ \\ & (1/2)})*c)^{(1/2)}*c*x)*aA-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2 \\ &)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^ \\ & (1/2)})*c)^{(1/2)}*c*x)*A*b^2+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((b+(-4 \\ & *a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*aB+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^ \\ & (1/2)*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*B+9/4/(16*a^ \\ & 2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b*B+3/16/(16 \\ & *a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})* \\ & c)^{(1/2)}*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3*B \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3, x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*b \\ & - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 + \end{aligned}$$

$$3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) - 3/8*\text{integrate}((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 + 4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2)$$

mupad [B] time = 3.49, size = 16688, normalized size = 43.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x)$

[Out] $\text{atan}(\frac{((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^{1/2}) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^{1/2}) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c)))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^{1/2}*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^{1/2}) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^{1/2}) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^{1/2} - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2$

$$\begin{aligned}
& *a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720 \\
& *A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4)) / (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) * (- (9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c)) / (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) * i - (((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6)) / (512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c)) / (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) * (256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6)) / (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) * (- (9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c)) / (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 14
\end{aligned}$$

$$\begin{aligned}
& 4*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4 \\
&)/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) \\
&)*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(\\
& -(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11 \\
& 520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^ \\
& 2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^ \\
& 6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - \\
& 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^ \\
& 2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^ \\
& 6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/ \\
& (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^1 \\
& 4*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 196 \\
& 6080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)) \\
&)^(1/2)*i)/((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + \\
& 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a \\
& ^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7 \\
& *c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6 \\
& *c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2 \\
& *c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) \\
&) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4 \\
& 160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440* \\
& A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a \\
& ^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c \\
& ^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^ \\
& 2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^ \\
& 8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2* \\
& c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3 \\
& *b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + \\
& 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^ \\
& 10*b^2*c^10 + a*b^20*c))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5 \\
& *b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32* \\
& (b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9* \\
& (B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c \\
& - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2 \\
& *a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b \\
& ^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c \\
& ^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920* \\
& A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12* \\
& c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - \\
& 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1 \\
& 048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + \\
& 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^ \\
& 8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) \\
& - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + \\
& 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^
\end{aligned}$$

$$\begin{aligned}
& 2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4)/(32*(b^8 + 256*a^4*c^4 + 96*a^2* \\
& *b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c \\
& c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2 \\
& *a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5 \\
& *b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - \\
& 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 \\
& + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2* \\
& b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - \\
& 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 8 \\
& 1920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b \\
& ^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 25804 \\
& 8*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4 \\
& *c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) + (((3*(1048576*A*a^6*c^8 \\
& - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b* \\
& c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - \\
& 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720 \\
& *B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6 \\
& *c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a \\
& *b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2 \\
&)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b \\
& ^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^ \\
& 2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 6 \\
& 1440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^ \\
& 7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - \\
& 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 14336 \\
& 0*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576 \\
& *a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760 \\
& *a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6* \\
& c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2)*(256* \\
& b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a \\
& ^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - \\
& 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^1 \\
& 5)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11* \\
& c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + \\
& 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152 \\
& 0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A* \\
& B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8 \\
& 1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B \\
& *a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a \\
& ^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + \\
& 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10 \\
& *c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262 \\
& 1440*a^10*b^2*c^10 + a*b^20*c))^(1/2) + (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 \\
& + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 \\
& + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 - 288*A*B*a^2*b
\end{aligned}$$

$$\begin{aligned}
& *c^4)) / (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6 \\
& *c)) * (- (9*(B^2*a*b^{15} + B^2*a*(-(4*a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c - A^2 \\
& *c*(-(4*a*c - b^2)^{15})^{(1/2)} - 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 \\
& - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 56 \\
& 0*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^ \\
& 2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c \\
& ^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A* \\
& B*a^2*b^{12}*c^2 - 64*A*B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^ \\
& 5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}* \\
& c)) / (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 + 720*a^3*b^{16}*c^3 - 7680*a^4 \\
& *b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 + 860160*a^7*b^8*c^7 - \\
& 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} + a*b^{20} \\
& *c))^{(1/2)} + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5*c^3 + 540*B^3*a^2*b^4*c^2 + \\
& 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a*b^6*c + 576*A^2*B*a^3*c^5 \\
& + 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^3*a^2*b*c^5 - 576*A*B^2*a*b \\
& ^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4*c^3 - 3600*A*B^2*a^2*b^3*c \\
& ^3 + 4464*A^2*B*a^2*b^2*c^4)) / (256*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - \\
& 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) * (- \\
& (9*(B^2*a*b^{15} + B^2*a*(-(4*a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c - A^2*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)} - 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520 \\
& *A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a \\
& ^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b \\
& ^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 - 81 \\
& 920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b \\
& ^{12}*c^2 - 64*A*B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c \\
& ^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}*c)) / (51 \\
& 2*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 + 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c \\
& ^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 + 860160*a^7*b^8*c^7 - 196608 \\
& 0*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} + a*b^{20}*c))^{(\\
& 1/2)} * 2i - ((x^3*(5*A*b^3 - 19*B*a*b^2 + 4*B*a^2*c + 16*A*a*b*c)) / (8*(b^4 + \\
& 16*a^2*c^2 - 8*a*b^2*c)) - (x^5*(5*B*b^3 + 4*A*a*c^2 - 19*A*b^2*c + 16*B*a* \\
& b*c)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*x*(A*b^2 + 4*A*a*c - 4*B*a* \\
& b)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*c*x^7*(B*b^2 - 4*A*b*c + 4*B*a* \\
& c)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) / (x^4*(2*a*c + b^2) + a^2 + c^2*x^8 \\
& + 2*a*b*x^2 + 2*b*c*x^6) + \operatorname{atan}((((3*(1048576*A*a^6*c^8 - 256*A*b^{12}*c^2 + \\
& 4096*A*a*b^{10}*c^3 + 1024*B*a*b^{11}*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2* \\
& b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^ \\
& 3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6)) / (\\
& 512*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^ \\
& 4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) - (x*(-(9*(B^2*a*b^{15} - B^2*a*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c + A^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 560* \\
& A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2* \\
& a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b \\
& ^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c \\
& ^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a
\end{aligned}$$

$$\begin{aligned}
&^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 - 64*A*B*a^3*b^{10}*c^3 \\
&- 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 \\
&+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}*c))/ (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 \\
&+ 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 \\
&+ 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} \\
&+ a*b^{20}*c))^{(1/2)}*(256*b^{11}*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 \\
&- 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/ (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) \\
&)*(- (9*(B^2*a*b^{15} - B^2*a*(-(4*a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c + A^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&- 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 \\
&+ 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 \\
&- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 \\
&- 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 \\
&- 64*A*B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 \\
&+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}*c))/ (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 \\
&+ 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 \\
&+ 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} \\
&+ a*b^{20}*c))^{(1/2)} - (x*(9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 \\
&+ 576*B^2*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a*b^3*c^3 \\
&- 288*A*B*a^2*b*c^4))/ (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) \\
&)*(- (9*(B^2*a*b^{15} - B^2*a*(-(4*a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c + A^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&- 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 \\
&+ 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 \\
&- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 \\
&- 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 \\
&- 64*A*B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 \\
&+ 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}*c))/ (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 \\
&+ 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 \\
&+ 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} \\
&+ a*b^{20}*c))^{(1/2)}*i - ((3*(1048576*A*a^6*c^8 - 256*A*b^{12}*c^2 + 4096*A*a*b^{10}*c^3 \\
&+ 1024*B*a*b^{11}*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 \\
&- 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/ (512*(b^{12} \\
&+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) \\
&+ (x*(- (9*(B^2*a*b^{15} - B^2*a*(-(4*a*c - b^2)^{15})^{(1/2)} + A^2*b^{15}*c + A^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&- 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 \\
&+ 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 \\
&- 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 \\
&- 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 +
\end{aligned}$$

$$\begin{aligned}
& 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560* \\
& A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a \\
& *b^14*c)) / (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 76 \\
& 80*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8 \\
& *c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + \\
& a*b^20*c))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960 \\
& *a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6)) / (32*(b^8 + 256*a^4 \\
& *c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - \\
& B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1 \\
& /2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - \\
& 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160 \\
& *B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2 \\
& *a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 \\
& + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a \\
& ^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^ \\
& 6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c)) / (512*(1048576*a^11*c^ \\
& 11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^1 \\
& 2*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 29 \\
& 49120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c))^(1/2) + (x*(9*B^2*b \\
& ^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^2*b^ \\
& 2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A*B*a* \\
& b^3*c^3 - 288*A*B*a^2*b*c^4)) / (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256 \\
& *a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^(\\
& 1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 \\
& + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614 \\
& 40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^ \\
& 2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^ \\
& 8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920 \\
& *B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4 \\
& *b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b \\
& ^2*c^7 - 20*A*B*a*b^14*c)) / (512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720* \\
& a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 \\
& + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440 \\
& *a^10*b^2*c^10 + a*b^20*c))^(1/2)*i) / (((3*(1048576*A*a^6*c^8 - 256*A*b^1 \\
& 2*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480 \\
& *A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2 \\
& *b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3* \\
& c^6)) / (512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840 \\
& *a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 - B^2 \\
& *a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^(1/2) \\
& - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 10 \\
& 24*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^ \\
& 2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^ \\
& 7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 2 \\
& 0*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*
\end{aligned}$$

$$\begin{aligned}
& b^{10}c^3 - 11520*ABa^4b^8c^4 + 66560*ABa^5b^6c^5 - 143360*ABa^6b^4c^6 + 81920*ABa^7b^2c^7 - 20*ABa^8b^{14}c) / (512*(1048576a^{11}c^{11} \\
& - 40a^2b^{18}c^2 + 720a^3b^{16}c^3 - 7680a^4b^{14}c^4 + 53760a^5b^{12}c^5 - 258048a^6b^{10}c^6 + 860160a^7b^8c^7 - 1966080a^8b^6c^8 + 29491 \\
& 20a^9b^4c^9 - 2621440a^{10}b^2c^{10} + a^{20}c))^{(1/2)} * (256b^{11}c^2 - 5120ab^9c^3 - 262144a^5b^7c^4 + 40960a^2b^7c^4 - 163840a^3b^5c^5 \\
& + 327680a^4b^3c^6) / (32*(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c)) * (- (9*(B^2ab^{15} - B^2a*(-(4ac - b^2)^{15})^{(1/2)} + \\
& A^2b^{15}c + A^2c*(-(4ac - b^2)^{15})^{(1/2)} - 560A^2a^2b^{11}c^3 + 4160A^2a^3b^9c^4 - 11520A^2a^4b^7c^5 - 1024A^2a^5b^5c^6 + 61440A^2a^6b^3c^7 - 560B^2a^3b^{11}c^2 + 4160B^2a^4b^9c^3 - 11520B^2a^5b^7c^4 - 1024B^2a^6b^5c^5 + 61440B^2a^7b^3c^6 + 65536*ABa^8c^8 + 20A^2ab^{13}c^2 - 81920A^2a^7b^8c^8 + 20B^2a^2b^{13}c - 81920B^2a^8b^7c^7 + 240*ABa^2b^{12}c^2 - 64*ABa^3b^{10}c^3 - 11520*ABa^4b^8c^4 + 66560*ABa^5b^6c^5 - 143360*ABa^6b^4c^6 + 81920*ABa^7b^2c^7 - 20*ABa^8b^{14}c) / (512*(1048576a^{11}c^{11} - 40a^2b^{18}c^2 + 720a^3b^{16}c^3 - 7680a^4b^{14}c^4 + 53760a^5b^{12}c^5 - 258048a^6b^{10}c^6 + 860160a^7b^8c^7 - 1966080a^8b^6c^8 + 2949120a^9b^4c^9 - 2621440a^{10}b^2c^{10} + a^{20}c))^{(1/2)} - (x*(9B^2b^6c + 288A^2a^2c^5 + 234A^2b^4c^3 - 288B^2a^3c^4 + 576B^2a^2b^2c^3 - 90*ABb^5c^2 + 144A^2ab^2c^4 + 126B^2ab^4c^2 - 720*ABa^3c^3 - 288*ABa^2b^4c^4) / (32*(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c)) * (- (9*(B^2ab^{15} - B^2a*(-(4ac - b^2)^{15})^{(1/2)} + A^2b^{15}c + A^2c*(-(4ac - b^2)^{15})^{(1/2)} - 560A^2a^2b^{11}c^3 + 4160A^2a^3b^9c^4 - 11520A^2a^4b^7c^5 - 1024A^2a^5b^5c^6 + 61440A^2a^6b^3c^7 - 560B^2a^3b^{11}c^2 + 4160B^2a^4b^9c^3 - 11520B^2a^5b^7c^4 - 1024B^2a^6b^5c^5 + 61440B^2a^7b^3c^6 + 65536*ABa^8c^8 + 20A^2ab^{13}c^2 - 81920A^2a^7b^8c^8 + 20B^2a^2b^{13}c - 81920B^2a^8b^7c^7 + 240*ABa^2b^{12}c^2 - 64*ABa^3b^{10}c^3 - 11520*ABa^4b^8c^4 + 66560*ABa^5b^6c^5 - 143360*ABa^6b^4c^6 + 81920*ABa^7b^2c^7 - 20*ABa^8b^{14}c) / (512*(1048576a^{11}c^{11} - 40a^2b^{18}c^2 + 720a^3b^{16}c^3 - 7680a^4b^{14}c^4 + 53760a^5b^{12}c^5 - 258048a^6b^{10}c^6 + 860160a^7b^8c^7 - 1966080a^8b^6c^8 + 2949120a^9b^4c^9 - 2621440a^{10}b^2c^{10} + a^{20}c))^{(1/2)} + (((3*(1048576Aa^6c^8 - 256Ab^{12}c^2 + 4096Aab^{10}c^3 + 1024Baa^{11}c^2 - 1048576Baa^6b^7c^7 - 20480Aa^2b^8c^4 + 327680Aa^4b^4c^6 - 1048576Aa^5b^2c^7 - 20480Baa^2b^9c^3 + 163840Baa^3b^7c^4 - 655360Baa^4b^5c^5 + 1310720Baa^5b^3c^6) / (512*(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + (x*(- (9*(B^2ab^{15} - B^2a*(-(4ac - b^2)^{15})^{(1/2)} + A^2b^{15}c + A^2c*(-(4ac - b^2)^{15})^{(1/2)} - 560A^2a^2b^{11}c^3 + 4160A^2a^3b^9c^4 - 11520A^2a^4b^7c^5 - 1024A^2a^5b^5c^6 + 61440A^2a^6b^3c^7 - 560B^2a^3b^{11}c^2 + 4160B^2a^4b^9c^3 - 11520B^2a^5b^7c^4 - 1024B^2a^6b^5c^5 + 61440B^2a^7b^3c^6 + 65536*ABa^8c^8 + 20A^2ab^{13}c^2 - 81920A^2a^7b^8c^8 + 20B^2a^2b^{13}c - 81920B^2a^8b^7c^7 + 240*ABa^2b^{12}c^2 - 64*ABa^3b^{10}c^3 - 11520*ABa^4b^8c^4 + 66
\end{aligned}$$

$$\begin{aligned}
& 560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A \\
& *B*a*b^{14}*c)/ (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 + 720*a^3*b^{16}*c^3 \\
& - 7680*a^4*b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10}*c^6 + 860160*a^7 \\
& *b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} \\
& + a*b^{20}*c))^{(1/2)}*(256*b^{11}*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 4 \\
& 0960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6)/ (32*(b^8 + 256 \\
& *a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^1 \\
& 5 - B^2*a*(-(4*a*c - b^2)^15)^{(1/2)} + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15 \\
&)^{(1/2)} - 560*A^2*a^2*b^{11}*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 \\
& - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + \\
& 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440 \\
& *B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 - 81920*A^2*a^7*b* \\
& c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 - 64*A \\
& *B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A* \\
& B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^{14}*c)/ (512*(1048576*a^1 \\
& 1*c^{11} - 40*a^2*b^{18}*c^2 + 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c^4 + 53760*a^5 \\
& *b^{12}*c^5 - 258048*a^6*b^{10}*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 \\
& + 2949120*a^9*b^4*c^9 - 2621440*a^{10}*b^2*c^{10} + a*b^{20}*c))^{(1/2)} + (x*(9*B \\
& ^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2*a^ \\
& 2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720*A \\
& B*a*b^3*c^3 - 288*A*B*a^2*b*c^4)/ (32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - \\
& 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^1 \\
& 5)^{(1/2)} + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 560*A^2*a^2*b^{11}* \\
& c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + \\
& 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 1152 \\
& 0*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A* \\
& B*a^8*c^8 + 20*A^2*a*b^{13}*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 8 \\
& 1920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 - 64*A*B*a^3*b^{10}*c^3 - 11520*A*B \\
& *a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a \\
& ^7*b^2*c^7 - 20*A*B*a*b^{14}*c)/ (512*(1048576*a^{11}*c^{11} - 40*a^2*b^{18}*c^2 + \\
& 720*a^3*b^{16}*c^3 - 7680*a^4*b^{14}*c^4 + 53760*a^5*b^{12}*c^5 - 258048*a^6*b^{10} \\
& *c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262 \\
& 1440*a^{10}*b^2*c^{10} + a*b^{20}*c))^{(1/2)} + (3*(576*B^3*a^4*c^4 - 180*A^3*b^5* \\
& c^3 + 540*B^3*a^2*b^4*c^2 + 1584*B^3*a^3*b^2*c^3 - 9*A*B^2*b^7*c + 45*B^3*a \\
& *b^6*c + 576*A^2*B*a^3*c^5 + 81*A^2*B*b^6*c^2 - 1440*A^3*a*b^3*c^4 - 576*A^ \\
& 3*a^2*b*c^5 - 576*A*B^2*a*b^5*c^2 - 3456*A*B^2*a^3*b*c^4 + 1980*A^2*B*a*b^4 \\
& *c^3 - 3600*A*B^2*a^2*b^3*c^3 + 4464*A^2*B*a^2*b^2*c^4)/ (256*(b^{12} + 4096* \\
& a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5* \\
& b^2*c^5 - 24*a*b^{10}*c)))*(-(9*(B^2*a*b^15 - B^2*a*(-(4*a*c - b^2)^15)^{(1/2)} \\
&) + A^2*b^15*c + A^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 560*A^2*a^2*b^{11}*c^3 + 4 \\
& 160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440* \\
& A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^{11}*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a \\
& ^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c \\
& ^8 + 20*A^2*a*b^{13}*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^{13}*c - 81920*B^ \\
& 2*a^8*b*c^7 + 240*A*B*a^2*b^{12}*c^2 - 64*A*B*a^3*b^{10}*c^3 - 11520*A*B*a^4*b^
\end{aligned}$$

$$\frac{8c^4 + 66560ABa^5b^6c^5 - 143360A^2B^2a^6b^4c^6 + 81920A^3B^3a^7b^2c^7 - 20A^4B^4a^8b^0c^8}{(512(1048576a^{11}c^{11} - 40a^2b^{18}c^2 + 720a^3b^{16}c^3 - 7680a^4b^{14}c^4 + 53760a^5b^{12}c^5 - 258048a^6b^{10}c^6 + 860160a^7b^8c^7 - 1966080a^8b^6c^8 + 2949120a^9b^4c^9 - 2621440a^{10}b^2c^{10} + ab^{20}c))^{1/2}} \cdot 2i$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.113 \quad \int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=438

$$\frac{x(-2aB - (x^2(bB - 2Ac)) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{x(-A(8abc + b^3) + cx^2(12abB - A(20ac + b^2)) + aB(7b^2 - 4ac))}{8a(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{\sqrt{c}}{\sqrt{b^2 - 4ac}} \left(A \right)$$

Rubi [A] time = 1.09, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1275, 1178, 1166, 205}

$$\frac{x(-2aB + x^2(-bB - 2Ac) + Ab)}{4(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{x(cx^2(12abB - A(20ac + b^2)) - A(8abc + b^3) + aB(7b^2 - 4ac))}{8a(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(A \left(b^2 \sqrt{b^2 - 4ac} + 20ac \sqrt{b^2 - 4ac} - 52abc + b^3 \right) + 6aB \left(-2b \sqrt{b^2 - 4ac} + 4ac + 3b^2 \right) \tan^{-1} \left(\frac{-\sqrt{c} \sqrt{x}}{\sqrt{b^2 - 4ac}} \right) \right)}{8\sqrt{2}a(b^2 - 4ac)^{5/2} \sqrt{b^2 - 4ac}} - \frac{\sqrt{c} \left(A \left(-b^2 \sqrt{b^2 - 4ac} - 20ac \sqrt{b^2 - 4ac} - 52abc + b^3 \right) + 6aB \left(2b \sqrt{b^2 - 4ac} + 4ac + 3b^2 \right) \tan^{-1} \left(\frac{-\sqrt{c} \sqrt{x}}{\sqrt{b^2 - 4ac}} \right) \right)}{8\sqrt{2}a(b^2 - 4ac)^{5/2} \sqrt{b^2 - 4ac} + b}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2 + 20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1275

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{x^2 (A + Bx^2)}{(a + bx^2 + cx^4)^3} dx &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} + \frac{\int \frac{Ab - 2aB + 5(bB - 2Ac)x^2}{(a + bx^2 + cx^4)^2} dx}{4 (b^2 - 4ac)} \\ &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x (aB (7b^2 - 4ac) - A (b^3 + 8abc) + c (12abB - A (b^3 + 8abc)))}{8a (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\ &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x (aB (7b^2 - 4ac) - A (b^3 + 8abc) + c (12abB - A (b^3 + 8abc)))}{8a (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \\ &= -\frac{x (Ab - 2aB - (bB - 2Ac)x^2)}{4 (b^2 - 4ac) (a + bx^2 + cx^4)^2} - \frac{x (aB (7b^2 - 4ac) - A (b^3 + 8abc) + c (12abB - A (b^3 + 8abc)))}{8a (b^2 - 4ac)^2 (a + bx^2 + cx^4)} \end{aligned}$$

Mathematica [A] time = 1.65, size = 436, normalized size = 1.00

$$\frac{1}{16} \left(\frac{4x (b(2a + bx^2) - A(b + 2cx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)^2}, \frac{2x (A(8abc + 20ac^2x^2 + b^3 + b^2cx^2) + aB(4ac - 7b^2 - 12bcx^2))}{a(b^2 - 4ac)^2 (a + bx^2 + cx^4)}, \frac{\sqrt{2} \sqrt{c} (A (b^2 \sqrt{b^2 - 4ac} + 20ac \sqrt{b^2 - 4ac} - 52abc + b^3) + 6aB (-2b \sqrt{b^2 - 4ac} + 4ac + 3b^2)) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac}} \right)}{a(b^2 - 4ac)^{5/2} \sqrt{b^2 - 4ac}}, \frac{\sqrt{2} \sqrt{c} (A (b^2 \sqrt{b^2 - 4ac} + 20ac \sqrt{b^2 - 4ac} + 52abc - b^3) - 6aB (2b \sqrt{b^2 - 4ac} + 4ac + 3b^2)) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac}} \right)}{a(b^2 - 4ac)^{5/2} \sqrt{b^2 - 4ac} + b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 9.42, size = 7270, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 + 7*A*a*b)*c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a

$$\begin{aligned}
&^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)) * \log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3* \\
&(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 \\
&+ 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 \\
&+ 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*\sqrt{1/2}*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*\sqrt{-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))} - \sqrt{1/2})*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*\sqrt{-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*\log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*\sqrt{1/2}*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*
\end{aligned}$$

$$\begin{aligned}
& a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324 \\
& *A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a \\
& ^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A \\
& *B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3* \\
& b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B* \\
& a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a \\
& ^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a \\
& ^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2 \\
& *a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + \\
& 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - \\
& 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^ \\
& 5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2* \\
& a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^ \\
& 2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a \\
& ^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*sqrt((81*B^4*a^4 + 108*A*B^3* \\
& a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 5 \\
& 0*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + \\
& 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(\\
& a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2* \\
& c^4 - 1024*a^8*c^5)) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4) \\
& *x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16 \\
& *a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a \\
& ^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 \\
& - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7 \\
& *A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (\\
& a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2* \\
& c^4 - 1024*a^8*c^5))*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 \\
& + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B \\
& *a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9 \\
& *b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + \\
& 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((\\
& 10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^ \\
& 5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4* \\
& a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - \\
& 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + \\
& 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B \\
& ^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c \\
& ^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3) \\
& *c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^ \\
& 3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7 \\
&)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a \\
& *b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b \\
& + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^ \\
& 5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 \\
& - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^
\end{aligned}$$

$$\begin{aligned}
& 4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*\sqrt{-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)})*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))} - \sqrt{1/2}*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*\sqrt{-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)})*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)}*\log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*\sqrt{1/2}*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*\sqrt{-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)})*\sqrt{((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
\end{aligned}$$

$$\begin{aligned}
&) * c) * a^3 * b^5 * c^2 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^6 * c^2 + 16 \\
& * a^3 * b^6 * c^2 + 128 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b^2 * c^3 + 32 \\
& * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^3 * c^3 - 4 * \sqrt{2} * \sqrt{b * c + \\
& \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b^4 * c^3 - 256 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} \\
&) * c) * a^6 * c^4 - 128 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * b * c^4 - 16 * s \\
& \text{qrt}(2) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^4 * b^2 * c^4 - 256 * a^5 * b^2 * c^4 + 64 * s \\
& \text{qrt}(2) * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^5 * c^5 + 512 * a^6 * c^5 + 2 * (b^2 - 4 * a \\
& * c) * a^2 * b^6 * c - 8 * (b^2 - 4 * a * c) * a^3 * b^4 * c^2 - 32 * (b^2 - 4 * a * c) * a^4 * b^2 * c^3 \\
& + 128 * (b^2 - 4 * a * c) * a^5 * c^4) * B * \text{abs}(a * b^4 - 8 * a^2 * b^2 * c + 16 * a^3 * c^2) + (2 * a \\
& ^2 * b^12 * c^2 - 136 * a^3 * b^10 * c^3 + 1856 * a^4 * b^8 * c^4 - 10496 * a^5 * b^6 * c^5 + 271 \\
& 36 * a^6 * b^4 * c^6 - 26624 * a^7 * b^2 * c^7 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + s \\
& \text{qrt}(b^2 - 4 * a * c)} * c) * a^2 * b^12 + 68 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^3 * b^10 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^2 * b^11 * c - 928 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^4 * b^8 * c^2 - 128 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^3 * b^9 * c^2 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^2 * b^10 * c^2 + 5248 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}} \\
& (b^2 - 4 * a * c) * c) * a^5 * b^6 * c^3 + 1344 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqr}} \\
& \text{t}(b^2 - 4 * a * c) * c) * a^4 * b^7 * c^3 + 64 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqr}} \\
& \text{t}(b^2 - 4 * a * c) * c) * a^3 * b^8 * c^3 - 13568 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \\
& \text{sqrt}(b^2 - 4 * a * c)} * c) * a^6 * b^4 * c^4 - 5120 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^5 * b^5 * c^4 - 672 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^4 * b^6 * c^4 + 13312 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^7 * b^2 * c^5 + 6656 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^6 * b^3 * c^5 + 2560 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c \\
& + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^5 * b^4 * c^5 - 3328 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} \\
& * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^6 * b^2 * c^6 - 2 * (b^2 - 4 * a * c) * a^2 * b^10 * c^2 \\
& + 128 * (b^2 - 4 * a * c) * a^3 * b^8 * c^3 - 1344 * (b^2 - 4 * a * c) * a^4 * b^6 * c^4 + 5120 * (b \\
& ^2 - 4 * a * c) * a^5 * b^4 * c^5 - 6656 * (b^2 - 4 * a * c) * a^6 * b^2 * c^6) * A + 6 * (6 * a^3 * b^11 \\
& * c^2 - 88 * a^4 * b^9 * c^3 + 448 * a^5 * b^7 * c^4 - 768 * a^6 * b^5 * c^5 - 512 * a^7 * b^3 * c^6 \\
& + 2048 * a^8 * b * c^7 - 3 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} \\
&) * c) * a^3 * b^11 + 44 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) \\
&) * a^4 * b^9 * c + 6 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a \\
& ^3 * b^10 * c - 224 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a \\
& ^5 * b^7 * c^2 - 64 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a \\
& ^4 * b^8 * c^2 - 3 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a^ \\
& 3 * b^9 * c^2 + 384 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * a \\
& ^6 * b^5 * c^3 + 192 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * \\
& a^5 * b^6 * c^3 + 32 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) * \\
& a^4 * b^7 * c^3 + 256 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) \\
& * a^7 * b^3 * c^4 - 96 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) \\
& * a^5 * b^5 * c^4 - 1024 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * \\
& c) * a^8 * b * c^5 - 512 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * c) \\
&) * a^7 * b^2 * c^5 + 256 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \text{sqrt}(b^2 - 4 * a * c)} * \\
& c) * a^7 * b * c^6 - 6 * (b^2 - 4 * a * c) * a^3 * b^9 * c^2 + 64 * (b^2 - 4 * a * c) * a^4 * b^7 * c^3 -
\end{aligned}$$

$$\begin{aligned}
& 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + \sqrt{(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3))})/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*\text{abs}(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*\text{abs}(c) - 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c + 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)^2*B - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^9 - 28*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^7*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^8*c + 2*a*b^9*c + 240*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 + 48*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^6*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^7*c^2 - 56*a^2*b^7*c^2 - 832*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - 288*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 - 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5*c^3 + 480*a^3*b^5*c^3 + 1024*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 + 512*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 + 144*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 - 1664*a^4*b^3*c^4 - 256*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 + 2048*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a*b^7*c + 48*(b^2 - 4*a*c)*a^2*b^5*c^2 - 288*(b^2 - 4*a*c)*a^3*b^3*c^3 + 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*\text{abs}(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) - 6*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^8 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^6*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^7*c + 2*a^2*b^8*c + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^5*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^6*c^2 - 16*a^3*b^6*c^2 + 128*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b^2*c^3 + 32*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^3*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^4*c^3 - 256*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^6*c^4 - 128*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*b*c^4 - 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 + 256*a^5*b^2*c^4 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^5*c^5 - 512*a^6*c^5 - 2*(b^2 - 4*a*c)*a^2*b^6*c + 8*(b^2 - 4*a*c)*a^3*b^4*c^2 + 32*(b^2 - 4*a*c)*a^4*b^2*c
\end{aligned}$$

$$\begin{aligned}
& c^3 - 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + \\
& (2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + \\
& 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c} \\
& - \sqrt{b^2 - 4*a*c})*a^2*b^12 + 68*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c})*a^3*b^10*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^2*b^11*c - 928*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^8*c^2 - 128*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^3*b^9*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^2*b^10*c^2 + 5248*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^6*c^3 + 1344*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^7*c^3 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^3*b^8*c^3 - 13568*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^6*b^4*c^4 - 5120*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^5*c^4 - 672*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^6*c^4 + 13312*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^7*b^2*c^5 + 6656*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^6*b^3*c^5 + 2560*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^4*c^5 - 3328*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^11*c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3*c^6 + 2048*a^8*b*c^7 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^3*b^11 + 44*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^9*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^3*b^10*c - 224*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^7*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^8*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^3*b^9*c^2 + 384*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^6*b^5*c^3 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^6*c^3 + 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^4*b^7*c^3 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^7*b^3*c^4 - 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^5*b^5*c^4 - 1024*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^8*b*c^5 - 512*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^7*b^2*c^5 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*a^7*b*c^6 - 6*(b^2 - 4*a*c)*a^3*b^9*c^2 + 64*(b^2 - 4*a*c)*a^4*b^7*c^3 - 192*(b^2 - 4*a*c)*a^5*b^5*c^4 + 512*(b^2 - 4*a*c)*a^7*b*c^6)*B)*arctan(2*\sqrt{1/2}*x/\sqrt{(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 - \sqrt{(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3))})/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*ab
\end{aligned}$$

$s(c)) - 1/8*(12*B*a*b*c^2*x^7 - A*b^2*c^2*x^7 - 20*A*a*c^3*x^7 + 19*B*a*b^2*c*x^5 - 2*A*b^3*c*x^5 - 4*B*a^2*c^2*x^5 - 28*A*a*b*c^2*x^5 + 5*B*a*b^3*x^3 - A*b^4*x^3 + 16*B*a^2*b*c*x^3 - 5*A*a*b^2*c*x^3 - 36*A*a^2*c^2*x^3 + 3*B*a^2*b^2*x + A*a*b^3*x + 12*B*a^3*c*x - 16*A*a^2*b*c*x)/((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(c*x^4 + b*x^2 + a)^2)$

maple [B] time = 0.05, size = 1335, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)`

[Out] $(1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*c*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^2+13/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^2+5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A+1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*A*b^3-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*B*b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} \left((20Aac^3 - (12Bab - Ab^2)c^2)x^7 + (4(Ba^2 + 7Aab)c^2 - (19Bab^2 - 2Ab^3)c)x^5 - (5Bab^3 - Ab^4 - 36Aa^2c^2 + (16Ba^2b - 5Aab^2)c)x^3 - (3Ba^2b^2 + Aab^3 + 4(3Ba^3 - 4Aa^2b)c)x \right) / ((ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4)x^8 + a^3b^4 - 8a^4b^2c + 16a^5c^2 + 2(ab^5c - 8a^2b^3c^2 + 16a^3b^2c^3)x^6 + (ab^6 - 6a^2b^4c + 32a^4c^3)x^4 + 2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2)x^2 + 1/8 \int ((3Bab^2 + Ab^3 + (20Aac^2 - (12Bab - Ab^2)c)x^2 + 4(3Ba^2 - 4Aab)c) / (c^2x^4 + b^2x^2 + a), x) / (ab^4 - 8a^2b^2c + 16a^3c^2)$

mupad [B] time = 3.92, size = 18992, normalized size = 43.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)

[Out] $\frac{(x^3(Ab^4 + 36Aa^2c^2 - 5Bab^3 + 5Aab^2c - 16Ba^2bc)) / (8a(b^4 + 16a^2c^2 - 8ab^2c)) - (x(Ab^3 + 3Bab^2 + 12Ba^2c - 16Aab^2c)) / (8(b^4 + 16a^2c^2 - 8ab^2c)) + (x^5(4Ba^2c^2 + 2Ab^3c + 28Aab^2c - 19Bab^2c)) / (8a(b^4 + 16a^2c^2 - 8ab^2c)) + (cx^7(20Aac^2 + Ab^2c - 12Bab^2c)) / (8a(b^4 + 16a^2c^2 - 8ab^2c))}{(x^4(2ac + b^2) + a^2 + c^2x^8 + 2abx^2 + 2bcx^6) + \operatorname{atan}\left(\frac{(256Aab^{13}c^2 - 3145728Ba^8c^8 + 4194304Aa^7b^2c^8 - 9216Aa^2b^{11}c^3 + 122880Aa^3b^9c^4 - 819200Aa^4b^7c^5 + 2949120Aa^5b^5c^6 - 5505024Aa^6b^3c^7 + 768Ba^2b^{12}c^2 - 12288Ba^3b^{10}c^3 + 61440Ba^4b^8c^4 - 983040Ba^6b^4c^6 + 3145728Ba^7b^2c^7)}{(512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) - (x(-A^2b^{17} + 9B^2a^2b^{15} + A^2b^2(-4ac - b^2)^{15})^{1/2} + 9B^2a^2(-4ac - b^2)^{15})^{1/2} + 6ABab^{16} + 1140A^2a^2b^{13}c^2 - 10160A^2a^3b^{11}c^3 + 34880A^2a^4b^9c^4 + 43776A^2a^5b^7c^5 - 680960A^2a^6b^5c^6 + 1863680A^2a^7b^3c^7 - 5040B^2a^4b^{11}c^2 + 37440B^2a^5b^9c^3 - 103680B^2a^6b^7c^4 - 9216B^2a^7b^5c^5 + 552960B^2a^8b^3c^6 + 983040ABa^9c^8 - 55A^2ab^{15}c - 25A^2aac(-4ac - b^2)^{15})^{1/2} - 1720320A^2a^8b^2c^8 + 180B^2a^3b^{13}c - 737280B^2a^9b^2c^7 + 240ABa^3b^{12}c^2 + 24000ABa^4b^{10}c^3 - 241920ABa^5b^8c^4 + 992256ABa^6b^6c^5 - 1781760ABa^7b^4c^6 + 737280ABa^8b^2c^7 + 6ABab^2(-4ac - b^2)^{15})^{1/2} - 180ABa^2b^{14}c)}{(512(a^3b^{20} + 1048576a^{13}c^{10} - 40*$

$$\begin{aligned}
& a^4b^{18}c + 720a^5b^{16}c^2 - 7680a^6b^{14}c^3 + 53760a^7b^{12}c^4 - 258048a^8b^{10}c^5 + 860160a^9b^8c^6 - 1966080a^{10}b^6c^7 + 2949120a^{11}b^4c^8 - 2621440a^{12}b^2c^9) \Big)^{(1/2)} * (262144a^7b^7c^7 - 256a^2b^{11}c^2 + 5120a^3b^9c^3 - 40960a^4b^7c^4 + 163840a^5b^5c^5 - 327680a^6b^3c^6) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)) * (-A^2b^{17} + 9B^2a^2b^{15} + A^2b^2 * (-4ac - b^2)^{15})^{(1/2)} + 9B^2a^2 * (-4ac - b^2)^{15})^{(1/2)} + 6A*B*a*b^{16} + 1140A^2a^2b^{13}c^2 - 10160A^2a^3b^{11}c^3 + 34880A^2a^4b^9c^4 + 43776A^2a^5b^7c^5 - 680960A^2a^6b^5c^6 + 1863680A^2a^7b^3c^7 - 5040B^2a^4b^{11}c^2 + 37440B^2a^5b^9c^3 - 103680B^2a^6b^7c^4 - 9216B^2a^7b^5c^5 + 552960B^2a^8b^3c^6 + 983040A*B*a^9c^8 - 55A^2a*b^{15}c - 25A^2a*c * (-4ac - b^2)^{15})^{(1/2)} - 1720320A^2a^8b^8c^8 + 180B^2a^3b^{13}c - 737280B^2a^9b^7c^7 + 240A*B*a^3b^{12}c^2 + 24000A*B*a^4b^{10}c^3 - 241920A*B*a^5b^8c^4 + 992256A*B*a^6b^6c^5 - 1781760A*B*a^7b^4c^6 + 737280A*B*a^8b^2c^7 + 6A*B*a*b * (-4ac - b^2)^{15})^{(1/2)} - 180A*B*a^2b^{14}c) / (512(a^3b^{20} + 1048576a^{13}c^{10} - 40a^4b^{18}c + 720a^5b^{16}c^2 - 7680a^6b^{14}c^3 + 53760a^7b^{12}c^4 - 258048a^8b^{10}c^5 + 860160a^9b^8c^6 - 1966080a^{10}b^6c^7 + 2949120a^{11}b^4c^8 - 2621440a^{12}b^2c^9)) \Big)^{(1/2)} + (x * (A^2b^6c^3 - 800A^2a^3c^6 + 288B^2a^4c^5 + 1472A^2a^2b^2c^5 + 234B^2a^2b^4c^3 + 144B^2a^3b^2c^4 - 34A^2a*b^4c^4 - 1104A*B*a^2b^3c^4 + 6A*B*a*b^5c^3 - 288A*B*a^3b^5c^5)) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)) * (-A^2b^{17} + 9B^2a^2b^{15} + A^2b^2 * (-4ac - b^2)^{15})^{(1/2)} + 9B^2a^2 * (-4ac - b^2)^{15})^{(1/2)} + 6A*B*a*b^{16} + 1140A^2a^2b^{13}c^2 - 10160A^2a^3b^{11}c^3 + 34880A^2a^4b^9c^4 + 43776A^2a^5b^7c^5 - 680960A^2a^6b^5c^6 + 1863680A^2a^7b^3c^7 - 5040B^2a^4b^{11}c^2 + 37440B^2a^5b^9c^3 - 103680B^2a^6b^7c^4 - 9216B^2a^7b^5c^5 + 552960B^2a^8b^3c^6 + 983040A*B*a^9c^8 - 55A^2a*b^{15}c - 25A^2a*c * (-4ac - b^2)^{15})^{(1/2)} - 1720320A^2a^8b^8c^8 + 180B^2a^3b^{13}c - 737280B^2a^9b^7c^7 + 240A*B*a^3b^{12}c^2 + 24000A*B*a^4b^{10}c^3 - 241920A*B*a^5b^8c^4 + 992256A*B*a^6b^6c^5 - 1781760A*B*a^7b^4c^6 + 737280A*B*a^8b^2c^7 + 6A*B*a*b * (-4ac - b^2)^{15})^{(1/2)} - 180A*B*a^2b^{14}c) / (512(a^3b^{20} + 1048576a^{13}c^{10} - 40a^4b^{18}c + 720a^5b^{16}c^2 - 7680a^6b^{14}c^3 + 53760a^7b^{12}c^4 - 258048a^8b^{10}c^5 + 860160a^9b^8c^6 - 1966080a^{10}b^6c^7 + 2949120a^{11}b^4c^8 - 2621440a^{12}b^2c^9)) \Big)^{(1/2)} * i - (((256A*a*b^{13}c^2 - 3145728B*a^8c^8 + 4194304A*a^7b^7c^8 - 9216A*a^2b^{11}c^3 + 122880A*a^3b^9c^4 - 819200A*a^4b^7c^5 + 2949120A*a^5b^5c^6 - 5505024A*a^6b^3c^7 + 768B*a^2b^{12}c^2 - 12288B*a^3b^{10}c^3 + 61440B*a^4b^8c^4 - 983040B*a^6b^4c^6 + 3145728B*a^7b^2c^7) / (512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x * (-A^2b^{17} + 9B^2a^2b^{15} + A^2b^2 * (-4ac - b^2)^{15})^{(1/2)} + 9B^2a^2 * (-4ac - b^2)^{15})^{(1/2)} + 6A*B*a*b^{16} + 1140A^2a^2b^{13}c^2 - 10160A^2a^3b^{11}c^3 + 34880A^2a^4b^9c^4 + 43776A^2a^5b^7c^5 - 680960A^2a^6b^5c^6 + 1863680A^2a^7b^3c^7 - 5040B^2a^4b^{11}c^2 + 37440B^2a^5b^9c^3 - 103680B^2a^6b^7c^4 - 9216B^2a^7b^5c^5 + 552960B^2a^8b^3c^6 + 983040A*B*a^9c^8 - 55A^2a*b^{15}c - 25A^2a*c * (-4ac - b^2)^{15})^{(1/2)} - 1720320A^2a^8b^8c^8 + 180B^2a^3b^{13}c - 737280B^2a^9b^7c^7 + 240A*B*a^3b^{12}c^2 + 24000A*B*a^4b^{10}c^3 - 241920A*B*a^5b^8c^4 + 992256A*B*a^6b^6c^5 - 1781760A*B*a^7b^4c^6 + 737280A*B*a^8b^2c^7 + 6A*B*a*b * (-4ac - b^2)^{15})^{(1/2)} - 180A*B*a^2b^{14}c) / (512(a^3b^{20} + 1048576a^{13}c^{10} - 40a^4b^{18}c + 720a^5b^{16}c^2 - 7680a^6b^{14}c^3 + 53760a^7b^{12}c^4 - 258048a^8b^{10}c^5 + 860160a^9b^8c^6 - 1966080a^{10}b^6c^7 + 2949120a^{11}b^4c^8 - 2621440a^{12}b^2c^9)) \Big)^{(1/2)} * i
\end{aligned}$$

$$\begin{aligned}
& a^6 b^7 c^4 - 9216 B^2 a^7 b^5 c^5 + 552960 B^2 a^8 b^3 c^6 + 983040 A B a^9 c^8 - 55 A^2 a b^{15} c - 25 A^2 a c (-4 a c - b^2)^{15} (1/2) - 1720320 A^2 a^8 b^3 c^8 + 180 B^2 a^3 b^{13} c - 737280 B^2 a^9 b^3 c^7 + 240 A B a^3 b^{12} c^2 + 24000 A B a^4 b^{10} c^3 - 241920 A B a^5 b^8 c^4 + 992256 A B a^6 b^6 c^5 - 1781760 A B a^7 b^4 c^6 + 737280 A B a^8 b^2 c^7 + 6 A B a b (-4 a c - b^2)^{15} (1/2) - 180 A B a^2 b^{14} c / (512 (a^3 b^{20} + 1048576 a^{13} c^{10} - 40 a^4 b^{18} c + 720 a^5 b^{16} c^2 - 7680 a^6 b^{14} c^3 + 53760 a^7 b^{12} c^4 - 258048 a^8 b^{10} c^5 + 860160 a^9 b^8 c^6 - 1966080 a^{10} b^6 c^7 + 2949120 a^{11} b^4 c^8 - 2621440 a^{12} b^2 c^9)) (1/2) * (262144 a^7 b^3 c^7 - 256 a^2 b^{11} c^2 + 5120 a^3 b^9 c^3 - 40960 a^4 b^7 c^4 + 163840 a^5 b^5 c^5 - 327680 a^6 b^3 c^6) / (32 (a^2 b^8 + 256 a^6 c^4 - 16 a^3 b^6 c + 96 a^4 b^4 c^2 - 256 a^5 b^2 c^3)) * (- (A^2 b^{17} + 9 B^2 a^2 b^{15} + A^2 b^2 (-4 a c - b^2)^{15} (1/2) + 9 B^2 a^2 (-4 a c - b^2)^{15} (1/2) + 6 A B a b^{16} + 1140 A^2 a^2 b^{13} c^2 - 10160 A^2 a^3 b^{11} c^3 + 34880 A^2 a^4 b^9 c^4 + 43776 A^2 a^5 b^7 c^5 - 680960 A^2 a^6 b^5 c^6 + 1863680 A^2 a^7 b^3 c^7 - 5040 B^2 a^4 b^{11} c^2 + 37440 B^2 a^5 b^9 c^3 - 103680 B^2 a^6 b^7 c^4 - 9216 B^2 a^7 b^5 c^5 + 552960 B^2 a^8 b^3 c^6 + 983040 A B a^9 c^8 - 55 A^2 a b^{15} c - 25 A^2 a c (-4 a c - b^2)^{15} (1/2) - 1720320 A^2 a^8 b^3 c^8 + 180 B^2 a^3 b^{13} c - 737280 B^2 a^9 b^3 c^7 + 240 A B a^3 b^{12} c^2 + 24000 A B a^4 b^{10} c^3 - 241920 A B a^5 b^8 c^4 + 992256 A B a^6 b^6 c^5 - 1781760 A B a^7 b^4 c^6 + 737280 A B a^8 b^2 c^7 + 6 A B a b (-4 a c - b^2)^{15} (1/2) - 180 A B a^2 b^{14} c) / (512 (a^3 b^{20} + 1048576 a^{13} c^{10} - 40 a^4 b^{18} c + 720 a^5 b^{16} c^2 - 7680 a^6 b^{14} c^3 + 53760 a^7 b^{12} c^4 - 258048 a^8 b^{10} c^5 + 860160 a^9 b^8 c^6 - 1966080 a^{10} b^6 c^7 + 2949120 a^{11} b^4 c^8 - 2621440 a^{12} b^2 c^9)) (1/2) - (x (A^2 b^6 c^3 - 800 A^2 a^3 c^6 + 288 B^2 a^4 c^5 + 1472 A^2 a^2 b^2 c^5 + 234 B^2 a^2 b^4 c^3 + 144 B^2 a^3 b^2 c^4 - 34 A^2 a b^4 c^4 - 1104 A B a^2 b^3 c^4 + 6 A B a b^5 c^3 - 288 A B a^3 b c^5)) / (32 (a^2 b^8 + 256 a^6 c^4 - 16 a^3 b^6 c + 96 a^4 b^4 c^2 - 256 a^5 b^2 c^3)) * (- (A^2 b^{17} + 9 B^2 a^2 b^{15} + A^2 b^2 (-4 a c - b^2)^{15} (1/2) + 9 B^2 a^2 (-4 a c - b^2)^{15} (1/2) + 6 A B a b^{16} + 1140 A^2 a^2 b^{13} c^2 - 10160 A^2 a^3 b^{11} c^3 + 34880 A^2 a^4 b^9 c^4 + 43776 A^2 a^5 b^7 c^5 - 680960 A^2 a^6 b^5 c^6 + 1863680 A^2 a^7 b^3 c^7 - 5040 B^2 a^4 b^{11} c^2 + 37440 B^2 a^5 b^9 c^3 - 103680 B^2 a^6 b^7 c^4 - 9216 B^2 a^7 b^5 c^5 + 552960 B^2 a^8 b^3 c^6 + 983040 A B a^9 c^8 - 55 A^2 a b^{15} c - 25 A^2 a c (-4 a c - b^2)^{15} (1/2) - 1720320 A^2 a^8 b^3 c^8 + 180 B^2 a^3 b^{13} c - 737280 B^2 a^9 b^3 c^7 + 240 A B a^3 b^{12} c^2 + 24000 A B a^4 b^{10} c^3 - 241920 A B a^5 b^8 c^4 + 992256 A B a^6 b^6 c^5 - 1781760 A B a^7 b^4 c^6 + 737280 A B a^8 b^2 c^7 + 6 A B a b (-4 a c - b^2)^{15} (1/2) - 180 A B a^2 b^{14} c) / (512 (a^3 b^{20} + 1048576 a^{13} c^{10} - 40 a^4 b^{18} c + 720 a^5 b^{16} c^2 - 7680 a^6 b^{14} c^3 + 53760 a^7 b^{12} c^4 - 258048 a^8 b^{10} c^5 + 860160 a^9 b^8 c^6 - 1966080 a^{10} b^6 c^7 + 2949120 a^{11} b^4 c^8 - 2621440 a^{12} b^2 c^9)) (1/2) * i) / (((256 A a b^{13} c^2 - 3145728 B a^8 c^8 + 4194304 A a^7 b^3 c^8 - 9216 A a^2 b^{11} c^3 + 122880 A a^3 b^9 c^4 - 819200 A a^4 b^7 c^5 + 2949120 A a^5 b^5 c^6 - 5505024 A a^6 b^3 c^7 + 768 B a^2 b^{12} c^2 - 12288 B a^3 b^{10} c^3 + 61440 B a^4 b^8 c^4 - 983040 B a^6 b^4 c^6 + 3145728 B a^7 b^2 c^7) /
\end{aligned}$$

$$\begin{aligned}
& (512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5* \\
& b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^{17} + 9*B^2*a^2* \\
& b^{15} + A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 348 \\
& 80*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18636 \\
& 80*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680 \\
& *B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A \\
& *B*a^9*c^8 - 55*A^2*a*b^{15}*c - 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 17203 \\
& 20*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3* \\
& b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6 \\
& *b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(\\
& 4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}* \\
& c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{1 \\
& 2}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2 \\
& 949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9)))^{(1/2)}*(262144*a^7*b*c^7 - 256 \\
& *a^2*b^{11}*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - \\
& 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^ \\
& 4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} + A^2*b^2*(-(4*a*c \\
& - b^2)^{15})^{(1/2)} + 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 114 \\
& 0*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776 \\
& *A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040* \\
& B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^ \\
& 2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15} \\
& *c - 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2 \\
& *a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b \\
& ^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7 \\
& *b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 1 \\
& 80*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720 \\
& *a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^ \\
& 5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621 \\
& 440*a^{12}*b^2*c^9)))^{(1/2)} + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4 \\
& *c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 3 \\
& 4*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^ \\
& 5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^ \\
& 2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} + A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + \\
& 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 \\
& - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - \\
& 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + \\
& 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55 \\
& 2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c - 25*A^2*a*c*(- \\
& (4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 7372 \\
& 80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A \\
& *B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280* \\
& A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c) \\
& / (512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 76
\end{aligned}$$

$$\begin{aligned}
& 80*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8 \\
& *c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9) \\
&)^{(1/2)} + (((256*A*a*b^{13}*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9 \\
& 216*A*a^2*b^{11}*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120* \\
& A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^{12}*c^2 - 12288*B*a^3*b^ \\
& 10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7 \\
&)/(512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^ \\
& 5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^{17} + 9*B^2* \\
& a^2*b^{15} + A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 9*B^2*a^2*(-(4*a*c - b^2)^{15} \\
&)^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 3 \\
& 4880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 186 \\
& 3680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 1036 \\
& 80*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040 \\
& *A*B*a^9*c^8 - 55*A^2*a*b^{15}*c - 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 172 \\
& 0320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^ \\
& 3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a \\
& ^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(\\
& -(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^1 \\
& 3*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b \\
& ^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + \\
& 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9)))^{(1/2)}*(262144*a^7*b*c^7 - 2 \\
& 56*a^2*b^{11}*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 \\
& - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4* \\
& b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} + A^2*b^2*(-(4*a* \\
& c - b^2)^{15})^{(1/2)} + 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1 \\
& 140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 437 \\
& 76*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 504 \\
& 0*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216* \\
& B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^ \\
& 15*c - 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B \\
& ^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4 \\
& *b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a \\
& ^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 7 \\
& 20*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}* \\
& c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 26 \\
& 21440*a^{12}*b^2*c^9)))^{(1/2)} - (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a \\
& ^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - \\
& 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b* \\
& c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5* \\
& b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} + A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} \\
& + 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c \\
& ^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 \\
& - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 \\
& + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 +
\end{aligned}$$

$$\begin{aligned}
& 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c* \\
& (-4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 73 \\
& 7280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920 \\
& *A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 73728 \\
& 0*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^14*c \\
& c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - \\
& 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b \\
& ^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9 \\
&))^{(1/2)} + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 54 \\
& 0*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^7 \\
& *c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A \\
& ^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B \\
& ^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^12 + 4096*a^8*c^6 - 2 \\
& 4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144 \\
& *a^7*b^2*c^5))) * (- (A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^{15}) \\
& ^{(1/2)} + 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^16 + 1140*A^2*a^2*b \\
& ^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b \\
& ^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^ \\
& 11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c \\
& ^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^ \\
& 2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c \\
& c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - \\
& 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + \\
& 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2 \\
& *b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c \\
& ^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160 \\
& *a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b \\
& ^2*c^9)))^{(1/2)} * 2i + \operatorname{atan}((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 419430 \\
& 4*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b \\
& ^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 \\
& - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145 \\
& 728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b \\
& ^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(\\
& A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(\\
& -(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2 \\
& *a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2* \\
& a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a \\
& ^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8 \\
& *b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2 \\
&)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b \\
& *c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c \\
& ^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2* \\
& c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^14*c)/(512*(a^3*b \\
& ^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c \\
& ^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 19660
\end{aligned}$$

$$\begin{aligned}
& 80*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)}*(262 \\
& 144*a^7*b*c^7 - 256*a^2*b^{11}*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 1 \\
& 63840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^ \\
& 3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} \\
& - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + \\
& 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2* \\
& a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2* \\
& a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^ \\
& 6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9* \\
& c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2* \\
& a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^ \\
& 2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^ \\
& 5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - \\
& b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - \\
& 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - \\
& 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120* \\
& a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)} + (x*(A^2*b^6*c^3 - 800*A^2*a^ \\
& 3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144* \\
& B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 \\
& - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b \\
& ^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^2*(-(4*a*c \\
& - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 11 \\
& 40*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 4377 \\
& 6*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040 \\
& *B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B \\
& ^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15} \\
& *c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^ \\
& 2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4* \\
& b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^ \\
& 7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 72 \\
& 0*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c \\
& ^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 262 \\
& 1440*a^{12}*b^2*c^9))^{(1/2)}*i - (((256*A*a*b^{13}*c^2 - 3145728*B*a^8*c^8 + 4 \\
& 194304*A*a^7*b*c^8 - 9216*A*a^2*b^{11}*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A* \\
& a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^{1 \\
& 2}*c^2 - 12288*B*a^3*b^{10}*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + \\
& 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240 \\
& *a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (\\
& x*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2* \\
& a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 1016 \\
& 0*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960 \\
& *A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440* \\
& B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^ \\
& 2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2* \\
& a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5* \\
& b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8 \\
& *b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^14*c)/(512*(\\
& a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6* \\
& b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - \\
& 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))^{(1/2)} \\
& *(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^ \\
& 4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - \\
& 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2* \\
& b^15 - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1 \\
& /2)} + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880 \\
& *A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680 \\
& *A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B \\
& ^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B \\
& *a^9*c^8 - 55*A^2*a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320 \\
& *A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^ \\
& 12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b \\
& ^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4* \\
& a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^ \\
& 10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12* \\
& c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 294 \\
& 9120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9))^{(1/2)} - (x*(A^2*b^6*c^3 - 800*A \\
& ^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + \\
& 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^ \\
& 5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96* \\
& a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 - A^2*b^2*(-(\\
& 4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^16 \\
& + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + \\
& 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - \\
& 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9 \\
& 216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2* \\
& a*b^15*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 1 \\
& 80*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B \\
& *a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A \\
& *B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/ \\
& 2)} - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c \\
& + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b \\
& ^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 \\
& - 2621440*a^12*b^2*c^9))^{(1/2)}*i)/((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^ \\
& 8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 8192 \\
& 00*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^ \\
& 2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4* \\
& c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c \\
& + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
\end{aligned}$$

$$\begin{aligned}
&) - (x*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9 \\
& *B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - \\
& 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 6 \\
& 80960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 3 \\
& 7440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 5529 \\
& 60*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280 \\
& *B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B \\
& *a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A \\
& B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(\\
& 512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680 \\
& *a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c \\
& ^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9)))^{(\\
& 1/2)}*(262144*a^7*b*c^7 - 256*a^2*b^{11}*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b \\
& ^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c \\
& ^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2 \\
& *a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^1 \\
& 5)^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + \\
& 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 18 \\
& 63680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103 \\
& 680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 98304 \\
& 0*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 17 \\
& 20320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a \\
& ^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B \\
& a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b* \\
& (- (4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^ \\
& 13*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7* \\
& b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 \\
& + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9)))^{(1/2)} + (x*(A^2*b^6*c^3 - \\
& 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4* \\
& c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B \\
& *a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c \\
& + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^ \\
& 2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a \\
& *b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9* \\
& c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3* \\
& c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^ \\
& 4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55 \\
& *A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^ \\
& 8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 2400 \\
& 0*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781 \\
& 760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15} \\
&)^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b \\
& ^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048* \\
& a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4
\end{aligned}$$

$$\begin{aligned}
& *c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)} + (((256*A*a*b^{13}*c^2 - 3145728*B*a^8* \\
& c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^{11}*c^3 + 122880*A*a^3*b^9*c^4 - 81 \\
& 9200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B* \\
& a^2*b^{12}*c^2 - 12288*B*a^3*b^{10}*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^ \\
& 4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}* \\
& c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^ \\
& 5)) + (x*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 \\
& - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - \\
& 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + \\
& 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 55 \\
& 2960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(- \\
& (4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 7372 \\
& 80*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A \\
& *B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280* \\
& A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c) \\
& / (512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 76 \\
& 80*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8 \\
& *c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9)) \\
&)^{(1/2)} * (262144*a^7*b*c^7 - 256*a^2*b^{11}*c^2 + 5120*a^3*b^9*c^3 - 40960*a^4 \\
& *b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6) / (32*(a^2*b^8 + 256*a^6 \\
& *c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)) * (- (A^2*b^{17} + 9*B \\
& ^2*a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2) \\
& ^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 \\
& + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + \\
& 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 1 \\
& 03680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983 \\
& 040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B \\
& *a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A* \\
& B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a* \\
& b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c) / (512*(a^3*b^{20} + 1048576* \\
& a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^ \\
& 7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^ \\
& 7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)} - (x*(A^2*b^6*c^3 \\
& - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^ \\
& 4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A \\
& *B*a*b^5*c^3 - 288*A*B*a^3*b*c^5) / (32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6* \\
& c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)) * (- (A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2* \\
& b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B \\
& *a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^ \\
& 9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^ \\
& 3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7* \\
& c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - \\
& 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*
\end{aligned}$$

$$\begin{aligned}
& c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)} + (35*A^3*b^6*c^4 - 8000*A^3*a^3*c^7 - 12720*A^3*a^2*b^2*c^6 + 540*B^3*a^2*b^5*c^3 + 4320*B^3*a^3*b^3*c^4 - 2880*A*B^2*a^4*c^6 - 15*A^2*B*b^7*c^3 + 84*A^3*a*b^4*c^5 + 1728*B^3*a^4*b*c^5 + 135*A*B^2*a*b^6*c^3 - 360*A^2*B*a*b^5*c^4 + 26880*A^2*B*a^3*b*c^6 - 5580*A*B^2*a^2*b^4*c^4 - 20592*A*B^2*a^3*b^2*c^5 + 15696*A^2*B*a^2*b^3*c^5)/(256*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(A^2*b^{17} + 9*B^2*a^2*b^{15} - A^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 9*B^2*a^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{16} + 1140*A^2*a^2*b^{13}*c^2 - 10160*A^2*a^3*b^{11}*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^{11}*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^{15}*c + 25*A^2*a*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^{13}*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^{12}*c^2 + 24000*A*B*a^4*b^{10}*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 - 6*A*B*a*b*(-(4*a*c - b^2)^{15})^{(1/2)} - 180*A*B*a^2*b^{14}*c)/(512*(a^3*b^{20} + 1048576*a^{13}*c^{10} - 40*a^4*b^{18}*c + 720*a^5*b^{16}*c^2 - 7680*a^6*b^{14}*c^3 + 53760*a^7*b^{12}*c^4 - 258048*a^8*b^{10}*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^{10}*b^6*c^7 + 2949120*a^{11}*b^4*c^8 - 2621440*a^{12}*b^2*c^9))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.114 \quad \int \frac{A+Bx^2}{(a+bx^2+cx^4)^3} dx$$

Optimal. Leaf size=460

$$\frac{x \left(A \left(28a^2c^2 - 25ab^2c + 3b^4 \right) + cx^2 \left(3A \left(b^3 - 8abc \right) + aB \left(20ac + b^2 \right) \right) + abB \left(8ac + b^2 \right) \right)}{8a^2 \left(b^2 - 4ac \right)^2 \left(a + bx^2 + cx^4 \right)} + \frac{\sqrt{c} \left(\frac{3A \left(56a^2c^2 - 10ab^2c + b^4 \right)}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{b^2 - 4ac}}$$

Rubi [A] time = 1.35, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1178, 1166, 205}

$$\frac{x \left(A \left(28a^2c^2 - 25ab^2c + 3b^4 \right) + cx^2 \left(3A \left(b^3 - 8abc \right) + aB \left(20ac + b^2 \right) \right) + abB \left(8ac + b^2 \right) \right)}{8a^2 \left(b^2 - 4ac \right)^2 \left(a + bx^2 + cx^4 \right)} + \frac{\sqrt{c} \left(\frac{3A \left(56a^2c^2 - 10ab^2c + b^4 \right) + 3A \left(b^3 - 8abc \right) + aB \left(20ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{8\sqrt{2}a^2 \left(b^2 - 4ac \right)^2 \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(\frac{3A \left(56a^2c^2 - 10ab^2c + b^4 \right) + 3A \left(b^3 - 8abc \right) + aB \left(20ac + b^2 \right) \right) \tan^{-1} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{8\sqrt{2}a^2 \left(b^2 - 4ac \right)^2 \sqrt{b^2 - 4ac} + b} + \frac{x \left(cx^2 \left(Ab - 2aB \right) - 2aAc - abB + Ab^2 \right)}{4a \left(b^2 - 4ac \right) \left(a + bx^2 + cx^4 \right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]

[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{(a + bx^2 + cx^4)^3} dx &= \frac{x \left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2 \right)}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} - \frac{\int \frac{-3Ab^2 - abB + 14aAc - 5(Ab - 2aB)cx^2}{(a + bx^2 + cx^4)^2} dx}{4a(b^2 - 4ac)} \\ &= \frac{x \left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2 \right)}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{x \left(abB(b^2 + 8ac) + A(3b^4 - 25ab^2c + 28a^2c^2) \right)}{8a^2(b^2 - 4ac)} \\ &= \frac{x \left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2 \right)}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{x \left(abB(b^2 + 8ac) + A(3b^4 - 25ab^2c + 28a^2c^2) \right)}{8a^2(b^2 - 4ac)} \\ &= \frac{x \left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2 \right)}{4a(b^2 - 4ac)(a + bx^2 + cx^4)^2} + \frac{x \left(abB(b^2 + 8ac) + A(3b^4 - 25ab^2c + 28a^2c^2) \right)}{8a^2(b^2 - 4ac)} \end{aligned}$$

Mathematica [A] time = 2.19, size = 516, normalized size = 1.12

$$\frac{2x(A(2b^2c^2 - 25ab^2c + 28a^2c^2) + 3B(b^2 + 8ac)) + d(8bc + 20a^2c^2 + b^3 + b^2cx^2)}{(b^2 - 4ac)^2(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(5a^2c^2 - 10ab^2c - 8abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} + a^3\right) + d\left(b^2\sqrt{b^2 - 4ac} + 20abc\sqrt{b^2 - 4ac} + 52abc + b^3\right)\operatorname{atan}\left(\frac{\sqrt{2}cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac}} + \frac{\sqrt{2}\sqrt{c}\left(-5a^2c^2 + 10ab^2c - 8abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} + a^3\right) + d\left(b^2\sqrt{b^2 - 4ac} + 20abc\sqrt{b^2 - 4ac} + 52abc + b^3\right)\operatorname{atan}\left(\frac{\sqrt{2}cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac}} - \frac{4cx(d(b^2 + 2cx^2) - A(-2cx^2 + bx^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3, x]

[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2) + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c

$$\begin{aligned}
& A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2) * x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)))
\end{aligned}$$

$$\begin{aligned}
& \wedge^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4* \\
& b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a \\
& ^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + \\
& 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b \\
& ^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1 \\
& 280*a^14*b^2*c^4 - 1024*a^15*c^5))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6* \\
& c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*\log((3111696*A^4 \\
& *a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 900 \\
& 00*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^ \\
& 4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2* \\
& a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + \\
& 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)* \\
& c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)* \\
& c^2)*x - 1/2*\sqrt{1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + \\
& 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + \\
& 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2 \\
& *B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6 \\
& *b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + \\
& 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^ \\
& 3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)* \\
& c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3* \\
& a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 \\
& - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^1 \\
& 0*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 12 \\
& 3*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 \\
& + 93*A*a^6*b^13)*c)*\sqrt{(B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b \\
& ^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^ \\
& 5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b \\
& + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2 \\
& *(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2 \\
& *b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 64 \\
& 0*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5))*\sqrt{-(B^2*a^2*b^7 + \\
& 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a \\
& ^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3 \\
& *b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a* \\
& b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128 \\
& 0*a^9*b^2*c^4 - 1024*a^10*c^5))*\sqrt{(B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^ \\
& 2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25 \\
& *A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400* \\
& A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b \\
& ^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 156 \\
& 6*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b \\
& ^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5))/(a^5*b^10 \\
& - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 102 \\
& 4*a^10*c^5))) - \sqrt{1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 +
\end{aligned}$$

$$\begin{aligned}
& a^4b^4 - 8a^5b^2c + 16a^6c^2 + 2*(a^2b^5c - 8a^3b^3c^2 + 16a^4 \\
& *b^3c^3)*x^6 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*x^4 + 2*(a^3b^5 - 8a^4 \\
& *b^3c + 16a^5b^2c^2)*x^2)*\sqrt{-(B^2a^2b^7 + 6A*B*a^b^8 + 9A^2b^9 - \\
& 1680*(4A*B*a^5 - 9A^2a^4b)*c^4 + 840*(2B^2a^5b - 4A*B*a^4b^2 - 9A \\
& ^2a^3b^3)*c^3 + 7*(40B^2a^4b^3 + 180A*B*a^3b^4 + 243A^2a^2b^5)*c^2 \\
& - 7*(5B^2a^3b^5 + 24A*B*a^2b^6 + 27A^2a*b^7)*c - (a^5b^10 - 20a^6 \\
& *b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^10 \\
& *c^5)*\sqrt{(B^4a^4b^4 + 12A*B^3a^3b^5 + 54A^2B^2a^2b^6 + 108A^3B \\
& *a^b^7 + 81A^4b^8 + 194481A^4a^4c^4 - 882*(25A^2B^2a^5 + 108A^3B*a \\
& ^4b + 99A^4a^3b^2)*c^3 + (625B^4a^6 + 5400A*B^3a^5b + 17496A^2B^ \\
& 2a^4b^2 + 26676A^3B*a^3b^3 + 17739A^4a^2b^4)*c^2 - 2*(25B^4a^5b^ \\
& 2 + 258A*B^3a^4b^3 + 972A^2B^2a^3b^4 + 1566A^3B*a^2b^5 + 891A^4 \\
& a*b^6)*c)/(a^10b^10 - 20a^11b^8c + 160a^12b^6c^2 - 640a^13b^4c^3 \\
& + 1280a^14b^2c^4 - 1024a^15c^5)))/(a^5b^10 - 20a^6b^8c + 160a^7b^ \\
& ^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^10c^5))*\log((3111696 \\
& A^4a^4c^7 - 1555848*(2A^3B*a^4b + A^4a^3b^2)*c^6 - (10000B^4a^6 - \\
& 90000A*B^3a^5b - 863136A^2B^2a^4b^2 - 1298376A^3B*a^3b^3 - 339309 \\
& *A^4a^2b^4)*c^5 - 3*(5000B^4a^5b^2 + 32952A*B^3a^4b^3 + 79488A^2B \\
& ^2a^3b^4 + 80919A^3B*a^2b^5 + 12069A^4a*b^6)*c^4 + 21*(71B^4a^4b^ \\
& 4 + 537A*B^3a^3b^5 + 1314A^2B^2a^2b^6 + 1053A^3B*a^b^7 + 81A^4b^ \\
& 8)*c^3 - 35*(B^4a^3b^6 + 9A*B^3a^2b^7 + 27A^2B^2a*b^8 + 27A^3B*b^ \\
& 9)*c^2)*x + 1/2*\sqrt{1/2)*(B^3a^3b^11 + 9A*B^2a^2b^12 + 27A^2B*a^b^1 \\
& 3 + 27A^3b^14 - 2370816A^3a^7c^7 + 2688*(50A*B^2a^8 + 384A^2B*a^7 \\
& b + 1143A^3a^6b^2)*c^6 - 64*(400B^3a^8b + 4062A*B^2a^7b^2 + 17541 \\
& A^2B*a^6b^3 + 26865A^3a^5b^4)*c^5 + 8*(2728B^3a^7b^3 + 20520A*B^2 \\
& a^6b^4 + 62694A^2B*a^5b^5 + 67797A^3a^4b^6)*c^4 - 7*(976B^3a^6b^5 \\
& + 6744A*B^2a^5b^6 + 16884A^2B*a^4b^7 + 14985A^3a^3b^8)*c^3 + (940 \\
& *B^3a^5b^7 + 6591A*B^2a^4b^8 + 15489A^2B*a^3b^9 + 12528A^3a^2b^1 \\
& 0)*c^2 - (53B^3a^4b^9 + 414A*B^2a^3b^10 + 1053A^2B*a^2b^11 + 864A \\
& ^3a^b^12)*c + (B*a^6b^14 + 3A*a^5b^15 + 4096*(10B*a^13 - 33A*a^12b)* \\
& c^7 - 2048*(16B*a^12b^2 - 99A*a^11b^3)*c^6 + 768*(2B*a^11b^4 - 169A* \\
& a^10b^5)*c^5 + 1280*(5B*a^10b^6 + 36A*a^9b^7)*c^4 - 80*(34B*a^9b^8 + \\
& 123A*a^8b^9)*c^3 + 24*(20B*a^8b^10 + 53A*a^7b^11)*c^2 - (38B*a^7b^ \\
& 12 + 93A*a^6b^13)*c)*\sqrt{(B^4a^4b^4 + 12A*B^3a^3b^5 + 54A^2B^2a^ \\
& 2b^6 + 108A^3B*a^b^7 + 81A^4b^8 + 194481A^4a^4c^4 - 882*(25A^2B^2 \\
& *a^5 + 108A^3B*a^4b + 99A^4a^3b^2)*c^3 + (625B^4a^6 + 5400A*B^3a^ \\
& 5b + 17496A^2B^2a^4b^2 + 26676A^3B*a^3b^3 + 17739A^4a^2b^4)*c^2 \\
& - 2*(25B^4a^5b^2 + 258A*B^3a^4b^3 + 972A^2B^2a^3b^4 + 1566A^3B* \\
& a^2b^5 + 891A^4a*b^6)*c)/(a^10b^10 - 20a^11b^8c + 160a^12b^6c^2 - \\
& 640a^13b^4c^3 + 1280a^14b^2c^4 - 1024a^15c^5)))*\sqrt{-(B^2a^2b^7 \\
& + 6A*B*a^b^8 + 9A^2b^9 - 1680*(4A*B*a^5 - 9A^2a^4b)*c^4 + 840*(2B^ \\
& 2a^5b - 4A*B*a^4b^2 - 9A^2a^3b^3)*c^3 + 7*(40B^2a^4b^3 + 180A*B* \\
& a^3b^4 + 243A^2a^2b^5)*c^2 - 7*(5B^2a^3b^5 + 24A*B*a^2b^6 + 27A^2 \\
& *a^b^7)*c - (a^5b^10 - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + \\
& 1280a^9b^2c^4 - 1024a^10c^5)*\sqrt{(B^4a^4b^4 + 12A*B^3a^3b^5 + 54
\end{aligned}$$

$$\begin{aligned}
& *A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882* \\
& (25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 54 \\
& 00*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^ \\
& 2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + \\
& 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^{10}*b^{10} - 20*a^{11}*b^8*c + 160*a^{1 \\
& 2}*b^6*c^2 - 640*a^{13}*b^4*c^3 + 1280*a^{14}*b^2*c^4 - 1024*a^{15}*c^5))/(a^5*b^ \\
& 10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - \\
& 1024*a^{10}*c^5))) + \text{sqrt}(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^ \\
& 8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16* \\
& a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8* \\
& a^4*b^3*c + 16*a^5*b*c^2)*x^2)*\text{sqrt}(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 \\
& - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - \\
& 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5) \\
& *c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^{10} - 20 \\
& *a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^ \\
& 10*c^5))*\text{sqrt}((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3 \\
& *B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3* \\
& B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2 \\
& *B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5 \\
& *b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A \\
& ^4*a*b^6)*c)/(a^{10}*b^{10} - 20*a^{11}*b^8*c + 160*a^{12}*b^6*c^2 - 640*a^{13}*b^4*c \\
& ^3 + 1280*a^{14}*b^2*c^4 - 1024*a^{15}*c^5))/(a^5*b^{10} - 20*a^6*b^8*c + 160*a^ \\
& 7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^{10}*c^5))*\log((31116 \\
& 96*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 \\
& - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339 \\
& 309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^ \\
& 2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4 \\
& *b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4 \\
& *b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B \\
& *b^9)*c^2)*x - 1/2*\text{sqrt}(1/2)*(B^3*a^3*b^{11} + 9*A*B^2*a^2*b^{12} + 27*A^2*B*a* \\
& b^{13} + 27*A^3*b^{14} - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a \\
& ^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 175 \\
& 41*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B \\
& ^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6* \\
& b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (\\
& 940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2* \\
& b^{10})*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^{10} + 1053*A^2*B*a^2*b^{11} + 86 \\
& 4*A^3*a*b^{12})*c + (B*a^6*b^{14} + 3*A*a^5*b^{15} + 4096*(10*B*a^{13} - 33*A*a^{12} \\
& b)*c^7 - 2048*(16*B*a^{12}*b^2 - 99*A*a^{11}*b^3)*c^6 + 768*(2*B*a^{11}*b^4 - 169 \\
& *A*a^{10}*b^5)*c^5 + 1280*(5*B*a^{10}*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^ \\
& 8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^{10} + 53*A*a^7*b^{11})*c^2 - (38*B*a^7 \\
& *b^{12} + 93*A*a^6*b^{13})*c)*\text{sqrt}((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2 \\
& *a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2* \\
& B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3 \\
& *a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c
\end{aligned}$$

$$\begin{aligned}
& t(2) \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^3 c^3 + 88 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^3 + 11 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^3 c^3 - 44 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^3 + 2(b^2 - 4ac) b^6 c - 26(b^2 - 4ac) a b^4 c^2 - 2(b^2 - 4ac) b^5 c^2 + 128(b^2 - 4ac) a^2 b^2 c^3 + 22(b^2 - 4ac) a b^3 c^3 - 224(b^2 - 4ac) a^3 c^4 - 88(b^2 - 4ac) a^2 b^4 c^4) A + (\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^5 c - 2 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^6 c - 2 a b^7 c + 144 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^3 c^2 + 40 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c^2 + \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^5 c^2 + 48 a^2 b^5 c^2 + 2 a b^6 c^2 - 256 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^4 b^3 c^3 - 128 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^3 - 20 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^3 - 288 a^3 b^3 c^3 - 44 a^2 b^4 c^3 + 64 \sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^4 c^3 + 512 a^4 b^4 c^3 + 64 a^3 b^2 c^4 + 320 a^4 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^6 + 22 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^4 c + 2 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^5 c - 32 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^2 c^2 - 36 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^3 c^2 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a b^4 c^2 - 160 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^4 c^3 - 80 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 b^3 c^3 + 18 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^2 b^2 c^3 + 40 \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}c} a^3 c^4 + 2(b^2 - 4ac) a b^5 c - 40(b^2 - 4ac) a^2 b^3 c^2 - 2(b^2 - 4ac) a b^4 c^2 + 128(b^2 - 4ac) a^3 b^3 c^3 + 36(b^2 - 4ac) a^2 b^2 c^3 + 80(b^2 - 4ac) a^3 c^4) B) \arctan(2\sqrt{1/2} x / \sqrt{(a^2 b^5 - 8a^3 b^3 c + 16a^4 b^2 c^2 + \sqrt{(a^2 b^5 - 8a^3 b^3 c + 16a^4 b^2 c^2)^2 - 4(a^3 b^4 - 8a^4 b^2 c + 16a^5 c^2)(a^2 b^4 c - 8a^3 b^2 c^2 + 16a^4 c^3)}) / (a^2 b^4 c - 8a^3 b^2 c^2 + 16a^4 c^3)}) / ((a^3 b^8 - 16a^4 b^6 c - 2a^3 b^7 c + 96a^5 b^4 c^2 + 24a^4 b^5 c^2 + a^3 b^6 c^2 - 256a^6 b^2 c^3 - 96a^5 b^3 c^3 - 12a^4 b^4 c^3 + 256a^7 c^4 + 128a^6 b^2 c^4 + 48a^5 b^2 c^4 - 64a^6 c^5) \operatorname{abs}(c)) + 1/32(3(\sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^4 c^2 + 26 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a b^5 c^2 + \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} b^6 c^2 - 34 a b^6 c^2 - 2 b^7 c^2 - 368 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^3 b^2 c^3 - 128 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^3 c^3 - 13 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a b^4 c^3 + 232 a^2 b^4 c^3 + 30 a b^5 c^3 + 448 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^4 c^4 + 224 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^3 b^4 c^4 + 64 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^2 b^2 c^4 - 736 a^3 b^2 c^4 - 176 a^2 b^3 c^4 - 112 \sqrt{2} \sqrt{bc - \sqrt{b^2 - 4ac}c} a^3 c^5 + 896 a^4 c^5 + 352 a^3 b^3 c^5 + \sqrt{2} \sqrt{b^2 - 4ac}
\end{aligned}$$

$$\begin{aligned}
& a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^7 - 15*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^5*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^6*c + 88*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^3*c^2 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^4*c^2 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*b^5*c^2 - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b*c^3 - 88*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^2*c^3 - 11*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^3*c^3 + 44*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2 - 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c^3 + 224*(b^2 - 4*a*c)*a^3*c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^7 - 24*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^5*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^6*c + 2*a*b^7*c + 144*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^3*c^2 + 40*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^4*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^5*c^2 - 48*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*b*c^3 - 128*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^2*c^3 - 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^3*c^3 + 288*a^3*b^3*c^3 + 44*a^2*b^4*c^3 + 64*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b*c^4 - 512*a^4*b*c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^6 - 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^4*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^5*c + 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b^2*c^2 + 36*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^3*c^2 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^4*c^2 + 160*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^4*c^3 + 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*b*c^3 - 18*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^2*b^2*c^3 - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^5*c + 40*(b^2 - 4*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 128*(b^2 - 4*a*c)*a^3*b*c^3 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3 - 80*(b^2 - 4*a*c)*a^3*c^4)*B)*\arctan(2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*a*b^5*c - 8*a^3*b^3*c + 16*a^4*b*c^2 - \sqrt{(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)})/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(B*a*b^2*c^2*x^7 + 3*A*b^3*c^2*x^7 + 20*B*a^2*c^3*x^7 - 24*A*a*b*c^3*x^7 + 2*B*a*b^3*c*x^5 + 6*A*b^4*c*x^5 + 28*B*a^2*b*c^2*x^5 - 49*A*a*b^2*c^2*x^5 + 28*A*a^2*c^3*x^5 + B*a*b^4*x^3 + 3*A*b^5*x^3 + 5*B*a^2*b^2*c*x^3 - 20*A*a*b^3*c*x^3 + 36*B*a^3*c^2*x^3 - 4*A*a^2*b*c^2*x^3 - B*a^2*b^3*x + 5*A*a*b^4*x + 16*B*a^3*b*c*x - 37*A*a^2*b^2*c*x + 44*A*a^3*c^2*x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
\end{aligned}$$

$$\begin{aligned}
& c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096* \\
& a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^ \\
& 4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + 9*A^2*b^4*(- \\
& (4*a*c - b^2)^{15})^{1/2} + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2* \\
& a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416* \\
& A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441 \\
& *A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{1/2} \\
& + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 \\
& + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 \\
& + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B \\
& ^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{1/2} \\
& + 5580*A*B*a^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10} \\
& *c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8* \\
& b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2} \\
&) + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{1/2} - 288*A*B*a^2*b^{16}*c - 108*A*B*a^ \\
& 2*b*c*(-(4*a*c - b^2)^{15})^{1/2}))/((512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^ \\
& 6*b^{18}*c + 720*a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 2580 \\
& 48*a^{10}*b^{10}*c^5 + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13} \\
& *b^4*c^8 - 2621440*a^{14}*b^2*c^9)))^{1/2}*(262144*a^9*b*c^7 - 256*a^4*b^{11}* \\
& c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^ \\
& 8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25 \\
& 6*a^7*b^2*c^3)))*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{1/2} \\
& + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + \\
& 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 \\
& - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(- \\
& (4*a*c - b^2)^{15})^{1/2} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{1/2} + 1140*B^2* \\
& a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^ \\
& 7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B \\
& *a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - \\
& 1720320*B^2*a^{10}*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{1/2} + 5580*A*B \\
& *a^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784* \\
& A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 51609 \\
& 60*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2} + 6*A*B*a*b^3 \\
& *(-(4*a*c - b^2)^{15})^{1/2} - 288*A*B*a^2*b^{16}*c - 108*A*B*a^2*b*c*(-(4*a*c \\
& - b^2)^{15})^{1/2}))/((512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720* \\
& a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^ \\
& 5 + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 262 \\
& 1440*a^{14}*b^2*c^9)))^{1/2} + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^ \\
& 2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - \\
& 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^ \\
& 2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/ \\
& (32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 \\
&)))*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{1/2} + 6* \\
& A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^ \\
& 4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A
\end{aligned}$$

$$\begin{aligned}
& ^2a^7b^5c^7 + 27095040A^2a^8b^3c^8 + 441A^2a^2c^2*(-(4ac - b^2)^{15})^{(1/2)} + B^2a^2b^2*(-(4ac - b^2)^{15})^{(1/2)} + 1140B^2a^4b^{13}c^2 \\
& - 10160B^2a^5b^{11}c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^7b^7c^5 - 680960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280ABa^{10}c^9 - 3 \\
& 69A^2ab^{17}c - 15482880A^2a^9b^9c^9 - 55B^2a^3b^{15}c - 1720320B^2a^{10}b^8c^8 - 25B^2a^3c*(-(4ac - b^2)^{15})^{(1/2)} + 5580ABa^3b^{14}c^2 \\
& - 59280ABa^4b^{12}c^3 + 377280ABa^5b^{10}c^4 - 1430784ABa^6b^8c^5 + 2860032ABa^7b^6c^6 - 1290240ABa^8b^4c^7 - 5160960ABa^9b^2 \\
& 2c^8 - 99A^2ab^2c*(-(4ac - b^2)^{15})^{(1/2)} + 6ABa^3b^3*(-(4ac - b^2)^{15})^{(1/2)} - 288ABa^2b^{16}c - 108ABa^2b^3c*(-(4ac - b^2)^{15})^{(1/2)} \\
& (1/2))/(512*(a^5b^{20} + 1048576a^{15}c^{10} - 40a^6b^{18}c + 720a^7b^{16}c^2 - 7680a^8b^{14}c^3 + 53760a^9b^{12}c^4 - 258048a^{10}b^{10}c^5 + 860160a^{11}b^8c^6 \\
& - 1966080a^{12}b^6c^7 + 2949120a^{13}b^4c^8 - 2621440a^{14}b^2c^9)))^{(1/2)}*i - (((4194304B^2a^9b^9c^8 - 22020096A^2a^9c^9 + 768A^2a^2b^{14}c^2 \\
& - 22272A^2a^3b^{12}c^3 + 282624A^2a^4b^{10}c^4 - 2027520A^2a^5b^8c^5 + 8847360A^2a^6b^6c^6 - 23396352A^2a^7b^4c^7 + 34603008A^2a^8b^2c^8 \\
& + 256B^2a^3b^{13}c^2 - 9216B^2a^4b^{11}c^3 + 122880B^2a^5b^9c^4 - 819200B^2a^6b^7c^5 + 2949120B^2a^7b^5c^6 - 5505024B^2a^8b^3c^7)/(512*(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c \\
& + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (x*(-(9A^2b^{19} + B^2a^2b^{17} + 9A^2b^4*(-(4ac - b^2)^{15})^{(1/2)} + 6ABa^3b^{18} \\
& + 6921A^2a^2b^{15}c^2 - 77580A^2a^3b^{13}c^3 + 570960A^2a^4b^{11}c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 - 21095424A^2a^7b^5c^7 + 27095040A^2a^8 \\
& b^3c^8 + 441A^2a^2c^2*(-(4ac - b^2)^{15})^{(1/2)} + B^2a^2b^2*(-(4ac - b^2)^{15})^{(1/2)} + 1140B^2a^4b^{13}c^2 - 10160B^2a^5b^{11}c^3 + 34880B^2a^6b^9c^4 \\
& + 43776B^2a^7b^7c^5 - 680960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280ABa^{10}c^9 - 369A^2ab^{17}c - 15482880A^2a^9b^9c^9 - 55B^2a^3b^{15}c - 1720320B^2a^{10}b^8c^8 \\
& - 25B^2a^3c*(-(4ac - b^2)^{15})^{(1/2)} + 5580ABa^3b^{14}c^2 - 59280ABa^4b^{12}c^3 + 377280ABa^5b^{10}c^4 - 1430784ABa^6b^8c^5 + 2860032ABa^7b^6c^6 - 12 \\
& 90240ABa^8b^4c^7 - 5160960ABa^9b^2c^8 - 99A^2ab^2c*(-(4ac - b^2)^{15})^{(1/2)} + 6ABa^3b^3*(-(4ac - b^2)^{15})^{(1/2)} - 288ABa^2b^{16}c \\
& - 108ABa^2b^3c*(-(4ac - b^2)^{15})^{(1/2)}))/(512*(a^5b^{20} + 1048576a^{15}c^{10} - 40a^6b^{18}c + 720a^7b^{16}c^2 - 7680a^8b^{14}c^3 + 53760a^9b^{12}c^4 - 258048a^{10}b^{10}c^5 \\
& + 860160a^{11}b^8c^6 - 1966080a^{12}b^6c^7 + 2949120a^{13}b^4c^8 - 2621440a^{14}b^2c^9)))^{(1/2)}*(262144a^9b^9c^7 - 256a^4b^{11}c^2 + 5120a^5b^9c^3 - 40960a^6b^7c^4 + 163840a^7b^5c^5 \\
& - 327680a^8b^3c^6))/(32*(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3)))*(-(9A^2b^{19} + B^2a^2b^{17} + 9A^2b^4*(-(4ac - b^2)^{15})^{(1/2)} + 6ABa^3b^{18} \\
& + 6921A^2a^2b^{15}c^2 - 77580A^2a^3b^{13}c^3 + 570960A^2a^4b^{11}c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 - 21095424A^2a^7b^5c^7 + 27095040A^2a^8b^3c^8 + 441A^2a^2c^2 \\
& *(-(4ac - b^2)^{15})^{(1/2)} + B^2a^2b^2*(-(4ac - b^2)^{15})^{(1/2)} + 1140B^2a^4b^{13}c^2 - 10160B^2a^5b^{11}c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^7b^7c^5 - 680960B^2a^8b^5c^6 \\
& + 1863680B^2a^9b^3c^7
\end{aligned}$$

$$\begin{aligned}
& + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^{16}*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)}/(512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720*a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^5 + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 2621440*a^{14}*b^2*c^9))^{(1/2)} - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^{16}*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)}/(512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720*a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^5 + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 2621440*a^{14}*b^2*c^9))^{(1/2)}*i)/((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^{14}*c^2 - 22272*A*a^3*b^{12}*c^3 + 282624*A*a^4*b^{10}*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3*b^{13}*c^2 - 9216*B*a^4*b^{11}*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2
\end{aligned}$$

$$\begin{aligned}
&6*b^{18}*c + 720*a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 2580 \\
&48*a^{10}*b^{10}*c^5 + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 2621440*a^{14}*b^2*c^9))^{(1/2)} - (567*A^3*b^7*c^5 + 8000*B^3*a^5 \\
&*c^7 + 67824*A^3*a^2*b^3*c^7 - 35*B^3*a^2*b^6*c^4 - 84*B^3*a^3*b^4*c^5 + 12 \\
&720*B^3*a^4*b^2*c^6 + 141120*A^2*B*a^4*c^8 - 315*A^2*B*b^8*c^4 - 10368*A^3* \\
&a*b^5*c^6 - 169344*A^3*a^3*b*c^8 - 210*A*B^2*a*b^7*c^4 - 116160*A*B^2*a^4*b \\
&*c^7 + 6237*A^2*B*a*b^6*c^5 + 1764*A*B^2*a^2*b^5*c^5 + 4608*A*B^2*a^3*b^3*c \\
&^6 - 42372*A^2*B*a^2*b^4*c^6 + 96048*A^2*B*a^3*b^2*c^7)/(256*(a^4*b^12 + 40 \\
&96*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8 \\
&*b^4*c^4 - 6144*a^9*b^2*c^5)) + (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 \\
&+ 768*A*a^2*b^{14}*c^2 - 22272*A*a^3*b^{12}*c^3 + 282624*A*a^4*b^{10}*c^4 - 2027 \\
&520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 346030 \\
&08*A*a^8*b^2*c^8 + 256*B*a^3*b^{13}*c^2 - 9216*B*a^4*b^{11}*c^3 + 122880*B*a^5* \\
&b^9*c^4 - 819200*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3* \\
&c^7)/(512*(a^4*b^12 + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 128 \\
&0*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^{19} + \\
&B^2*a^2*b^{17} + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{18} + 6921*A^2 \\
&a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776 \\
&*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 270 \\
&95040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2 \\
&*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11} \\
&*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c \\
&^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15 \\
&482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2* \\
&a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^{14}*c^2 - 59280*A*B*a^4*b^1 \\
&2*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7 \\
&*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2 \\
&*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288* \\
&A*B*a^2*b^{16}*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(a^5*b^{20} \\
&+ 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720*a^7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 \\
&+ 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^5 + 860160*a^{11}*b^8*c^6 - 1966080 \\
&*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 2621440*a^{14}*b^2*c^9))^{(1/2)}*(26214 \\
&4*a^9*b*c^7 - 256*a^4*b^{11}*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163 \\
&840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5* \\
&b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^{19} + B^2*a^2*b^{17} + \\
&9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 \\
&- 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^ \\
&5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b \\
&^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - \\
&b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^ \\
&2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^ \\
&2*a^9*b^3*c^7 + 6881280*A*B*a^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9* \\
&b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^{10}*b*c^8 - 25*B^2*a^3*c*(-(4*a*c \\
&- b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280* \\
&A*B*a^5*b^{10}*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290
\end{aligned}$$

$$\begin{aligned}
& 240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^16*c \\
& - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)}/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 \\
& - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)} - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180 \\
& *A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/((32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)}/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)})))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2)}/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)})*2i + atan((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A*a^6*b^6*c^6 -
\end{aligned}$$

$$\begin{aligned}
& 23396352A^7b^4c^7 + 34603008A^8b^2c^8 + 256B^3a^3b^13c^2 - 921 \\
& 6B^4a^4b^11c^3 + 122880B^5a^5b^9c^4 - 819200B^6a^6b^7c^5 + 2949120B^7 \\
& a^7b^5c^6 - 5505024B^8a^8b^3c^7)/(512*(a^4b^12 + 4096a^10c^6 - 24a^5 \\
& b^10c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9 \\
& b^2c^5)) - (x*(-(9A^2b^19 + B^2a^2b^17 - 9A^2b^4*(-(4ac - b^2)^15 \\
&)^(1/2) + 6A^3B^2a^18 + 6921A^2a^2b^15c^2 - 77580A^2a^3b^13c^3 + 5 \\
& 70960A^2a^4b^11c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 \\
& - 21095424A^2a^7b^5c^7 + 27095040A^2a^8b^3c^8 - 441A^2a^2c^2*(-(\\
& 4ac - b^2)^15)^(1/2) - B^2a^2b^2*(-(4ac - b^2)^15)^(1/2) + 1140B^2a^ \\
& ^4b^13c^2 - 10160B^2a^5b^11c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^ \\
& 7b^7c^5 - 680960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280A^3B^2 \\
& a^10c^9 - 369A^2a^3b^17c - 15482880A^2a^9b^9c^9 - 55B^2a^3b^15c - \\
& 1720320B^2a^10b^8c^8 + 25B^2a^3c*(-(4ac - b^2)^15)^(1/2) + 5580A^3B^2 \\
& a^3b^14c^2 - 59280A^3B^2a^4b^12c^3 + 377280A^3B^2a^5b^10c^4 - 1430784A^3 \\
& B^2a^6b^8c^5 + 2860032A^3B^2a^7b^6c^6 - 1290240A^3B^2a^8b^4c^7 - 516096 \\
& 0A^3B^2a^9b^2c^8 + 99A^2a^3b^2c*(-(4ac - b^2)^15)^(1/2) - 6A^3B^2a^3b^3 \\
& (- (4ac - b^2)^15)^(1/2) - 288A^3B^2a^2b^16c + 108A^3B^2a^2b^3c*(-(4ac - \\
& b^2)^15)^(1/2))/(512*(a^5b^20 + 1048576a^15c^10 - 40a^6b^18c + 720a^ \\
& ^7b^16c^2 - 7680a^8b^14c^3 + 53760a^9b^12c^4 - 258048a^10b^10c^5 \\
& + 860160a^11b^8c^6 - 1966080a^12b^6c^7 + 2949120a^13b^4c^8 - 2621 \\
& 440a^14b^2c^9)))^(1/2)*(262144a^9b^9c^7 - 256a^4b^11c^2 + 5120a^5b^ \\
& ^9c^3 - 40960a^6b^7c^4 + 163840a^7b^5c^5 - 327680a^8b^3c^6))/(32*(\\
& a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) \\
& *(-(9A^2b^19 + B^2a^2b^17 - 9A^2b^4*(-(4ac - b^2)^15)^(1/2) + 6A^3B^2 \\
& a^3b^18 + 6921A^2a^2b^15c^2 - 77580A^2a^3b^13c^3 + 570960A^2a^4b^ \\
& ^11c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 - 21095424A^2a^ \\
& ^7b^5c^7 + 27095040A^2a^8b^3c^8 - 441A^2a^2c^2*(-(4ac - b^2)^15 \\
&)^(1/2) - B^2a^2b^2*(-(4ac - b^2)^15)^(1/2) + 1140B^2a^4b^13c^2 - 1 \\
& 0160B^2a^5b^11c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^7b^7c^5 - 680 \\
& 960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280A^3B^2a^10c^9 - 369A^ \\
& ^2a^3b^17c - 15482880A^2a^9b^9c^9 - 55B^2a^3b^15c - 1720320B^2a^1 \\
& 0b^8c^8 + 25B^2a^3c*(-(4ac - b^2)^15)^(1/2) + 5580A^3B^2a^3b^14c^2 - \\
& 59280A^3B^2a^4b^12c^3 + 377280A^3B^2a^5b^10c^4 - 1430784A^3B^2a^6b^8c^5 \\
& + 2860032A^3B^2a^7b^6c^6 - 1290240A^3B^2a^8b^4c^7 - 5160960A^3B^2a^9b^2c^ \\
& ^8 + 99A^2a^3b^2c*(-(4ac - b^2)^15)^(1/2) - 6A^3B^2a^3b^3*(-(4ac - b^2) \\
& ^15)^(1/2) - 288A^3B^2a^2b^16c + 108A^3B^2a^2b^3c*(-(4ac - b^2)^15)^(1/2) \\
&)/(512*(a^5b^20 + 1048576a^15c^10 - 40a^6b^18c + 720a^7b^16c^2 - 7 \\
& 680a^8b^14c^3 + 53760a^9b^12c^4 - 258048a^10b^10c^5 + 860160a^11b^ \\
& ^8c^6 - 1966080a^12b^6c^7 + 2949120a^13b^4c^8 - 2621440a^14b^2c^ \\
& 9)))^(1/2) + (x*(14112A^2a^4c^7 + 9A^2b^8c^3 - 800B^2a^5c^6 + 1530 \\
& A^2a^2b^4c^5 - 6192A^2a^3b^2c^6 + B^2a^2b^6c^3 - 34B^2a^3b^4c^ \\
& ^4 + 1472B^2a^4b^2c^5 - 180A^2a^3b^6c^4 - 162A^3B^2a^2b^5c^4 + 1104 \\
& A^3B^2a^3b^3c^5 + 6A^3B^2a^3b^7c^3 - 6816A^3B^2a^4b^6c^6))/(32*(a^4b^8 + 25 \\
& 6a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3)))*(-(9A^2b^1 \\
& 9 + B^2a^2b^17 - 9A^2b^4*(-(4ac - b^2)^15)^(1/2) + 6A^3B^2a^3b^18 + 692
\end{aligned}$$

$$\begin{aligned}
& 1*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285 \\
& 1776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + \\
& 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - B^2 \\
& *a^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5* \\
& b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^ \\
& ^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c \\
& - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25* \\
& B^2*a^3*c*(-(4*a*c - b^2)^15)^{(1/2)} + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4 \\
& *b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B \\
& *a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a \\
& *b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^{(1/2)} - \\
& 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^{(1/2)}/(512*(a^5*b \\
& ^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14* \\
& c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 196 \\
& 6080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)}*1i \\
& - (((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 22272 \\
& *A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360*A \\
& *a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^3* \\
& b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*c^ \\
& 5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*a^ \\
& 10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4* \\
& c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4 \\
& *a*c - b^2)^15)^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^ \\
& 3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^ \\
& 2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A \\
& ^2*a^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} \\
&) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 \\
& + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 \\
& + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2 \\
& *a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^{(1/ \\
& 2)} + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c \\
& ^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^ \\
& 4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} \\
& - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^{(1/2)} - 288*A*B*a^2*b^16*c + 108*A*B*a^2* \\
& b*c*(-(4*a*c - b^2)^15)^{(1/2)}/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6* \\
& b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048 \\
& *a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13* \\
& b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)}*(262144*a^9*b*c^7 - 256*a^4*b^11*c^ \\
& 2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8* \\
& b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256* \\
& a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15) \\
& ^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 57 \\
& 0960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - \\
& 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4 \\
& *a*c - b^2)^15)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} + 1140*B^2*a^
\end{aligned}$$

$$\begin{aligned}
& 4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7 \\
& *b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a \\
& ^{10}*c^9 - 369*A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1 \\
& 720320*B^2*a^{10}*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a \\
& ^3*b^{14}*c^2 - 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784*A* \\
& B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960 \\
& *A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 6*A*B*a*b^3*(- \\
& -(4*a*c - b^2)^{15})^{(1/2)} - 288*A*B*a^2*b^{16}*c + 108*A*B*a^2*b*c*(-(4*a*c - \\
& b^2)^{15})^{(1/2)}/(512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720*a^ \\
& 7*b^{16}*c^2 - 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^5 \\
& + 860160*a^{11}*b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 26214 \\
& 40*a^{14}*b^2*c^9))^{(1/2)} - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a \\
& ^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 3 \\
& 4*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2* \\
& b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/((32 \\
& *(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) \\
&)*(-(9*A^2*b^{19} + B^2*a^2*b^{17} - 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A* \\
& B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4* \\
& b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2 \\
& *a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{1 \\
& 5})^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - \\
& 10160*B^2*a^5*b^{11}*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 68 \\
& 0960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^{10}*c^9 - 369 \\
& *A^2*a*b^{17}*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^{15}*c - 1720320*B^2*a^ \\
& 10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^{15})^{(1/2)} + 5580*A*B*a^3*b^{14}*c^2 - \\
& 59280*A*B*a^4*b^{12}*c^3 + 377280*A*B*a^5*b^{10}*c^4 - 1430784*A*B*a^6*b^8*c^5 \\
& + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2* \\
& c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2 \\
&)^{15})^{(1/2)} - 288*A*B*a^2*b^{16}*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^{15})^{(1/2 \\
&))/(512*(a^5*b^{20} + 1048576*a^{15}*c^{10} - 40*a^6*b^{18}*c + 720*a^7*b^{16}*c^2 - \\
& 7680*a^8*b^{14}*c^3 + 53760*a^9*b^{12}*c^4 - 258048*a^{10}*b^{10}*c^5 + 860160*a^{11} \\
& *b^8*c^6 - 1966080*a^{12}*b^6*c^7 + 2949120*a^{13}*b^4*c^8 - 2621440*a^{14}*b^2*c \\
& ^9))^{(1/2)}*ii)/(((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^ \\
& 14*c^2 - 22272*A*a^3*b^{12}*c^3 + 282624*A*a^4*b^{10}*c^4 - 2027520*A*a^5*b^8*c \\
& ^5 + 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^ \\
& 8 + 256*B*a^3*b^{13}*c^2 - 9216*B*a^4*b^{11}*c^3 + 122880*B*a^5*b^9*c^4 - 81920 \\
& 0*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4* \\
& b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + \\
& 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^{19} + B^2*a^2*b^{17} - \\
& 9*A^2*b^4*(-(4*a*c - b^2)^{15})^{(1/2)} + 6*A*B*a*b^{18} + 6921*A^2*a^2*b^{15}*c^2 \\
& - 77580*A^2*a^3*b^{13}*c^3 + 570960*A^2*a^4*b^{11}*c^4 - 2851776*A^2*a^5*b^9*c^ \\
& 5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b \\
& ^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - \\
& b^2)^{15})^{(1/2)} + 1140*B^2*a^4*b^{13}*c^2 - 10160*B^2*a^5*b^{11}*c^3 + 34880*B^ \\
& 2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^
\end{aligned}$$

$$\begin{aligned}
& 2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9* \\
& b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c \\
& - b^2)^15)^{(1/2)} + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280* \\
& A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290 \\
& 240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b \\
& ^2)^15)^{(1/2)} - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^{(1/2)} - 288*A*B*a^2*b^16*c \\
& + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^{(1/2)})/(512*(a^5*b^20 + 1048576*a^15* \\
& c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^1 \\
& 2*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + \\
& 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)}*(262144*a^9*b*c^7 - 2 \\
& 56*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 \\
& - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6* \\
& b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4* \\
& a*c - b^2)^15)^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3 \\
& *b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2 \\
& *a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^ \\
& 2*a^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} \\
& + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + \\
& 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + \\
& 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2* \\
& a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^{(1/2)} \\
&) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^ \\
& 4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4 \\
& *c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - \\
& 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^{(1/2)} - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b \\
& *c*(-(4*a*c - b^2)^15)^{(1/2)})/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b \\
& ^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048* \\
& a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b \\
& ^4*c^8 - 2621440*a^14*b^2*c^9)))^{(1/2)} + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8* \\
& c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a \\
& ^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 \\
& - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a \\
& ^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256 \\
& *a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15 \\
&)^{(1/2)} + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 5 \\
& 70960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 \\
& - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(\\
& 4*a*c - b^2)^15)^{(1/2)} - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^{(1/2)} + 1140*B^2*a \\
& ^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^ \\
& 7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B* \\
& a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - \\
& 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^{(1/2)} + 5580*A*B* \\
& a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A \\
& *B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 516096 \\
& 0*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2)} - 6*A*B*a*b^3*
\end{aligned}$$

$$\begin{aligned}
& \left(-(4ac - b^2)^{15} \right)^{1/2} - 288ABa^2b^{16}c + 108ABa^2b^2c \left(-(4ac - b^2)^{15} \right)^{1/2} \Big/ (512(a^5b^{20} + 1048576a^{15}c^{10} - 40a^6b^{18}c + 720a^7b^{16}c^2 - 7680a^8b^{14}c^3 + 53760a^9b^{12}c^4 - 258048a^{10}b^{10}c^5 + 860160a^{11}b^8c^6 - 1966080a^{12}b^6c^7 + 2949120a^{13}b^4c^8 - 2621440a^{14}b^2c^9))^{1/2} - (567A^3b^7c^5 + 8000B^3a^5c^7 + 67824A^3a^2b^3c^7 - 35B^3a^2b^6c^4 - 84B^3a^3b^4c^5 + 12720B^3a^4b^2c^6 + 141120A^2B^2a^4c^8 - 315A^2B^2b^8c^4 - 10368A^3a^2b^5c^6 - 169344A^3a^3b^2c^8 - 210AB^2a^2b^7c^4 - 116160AB^2a^4b^2c^7 + 6237A^2B^2a^2b^6c^5 + 1764AB^2a^2b^5c^5 + 4608AB^2a^3b^3c^6 - 42372A^2B^2a^2b^4c^6 + 96048A^2B^2a^3b^2c^7) / (256(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (((4194304B^2a^9b^2c^8 - 22020096A^2a^9c^9 + 768A^2a^2b^14c^2 - 22272A^2a^3b^12c^3 + 282624A^2a^4b^10c^4 - 2027520A^2a^5b^8c^5 + 8847360A^2a^6b^6c^6 - 23396352A^2a^7b^4c^7 + 34603008A^2a^8b^2c^8 + 256B^2a^3b^13c^2 - 9216B^2a^4b^11c^3 + 122880B^2a^5b^9c^4 - 819200B^2a^6b^7c^5 + 2949120B^2a^7b^5c^6 - 5505024B^2a^8b^3c^7) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (x \left(-(9A^2b^{19} + B^2a^2b^{17} - 9A^2b^4 \left(-(4ac - b^2)^{15} \right)^{1/2} + 6AB^2a^2b^{18} + 6921A^2a^2b^{15}c^2 - 77580A^2a^3b^{13}c^3 + 570960A^2a^4b^{11}c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 - 21095424A^2a^7b^5c^7 + 27095040A^2a^8b^3c^8 - 441A^2a^2c^2 \left(-(4ac - b^2)^{15} \right)^{1/2} - B^2a^2b^2 \left(-(4ac - b^2)^{15} \right)^{1/2} + 1140B^2a^4b^{13}c^2 - 10160B^2a^5b^{11}c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^7b^7c^5 - 680960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280AB^2a^{10}c^9 - 369A^2a^2b^{17}c - 15482880A^2a^9b^2c^9 - 55B^2a^3b^{15}c - 1720320B^2a^{10}b^2c^8 + 25B^2a^3c \left(-(4ac - b^2)^{15} \right)^{1/2} + 5580AB^2a^3b^{14}c^2 - 59280AB^2a^4b^{12}c^3 + 377280AB^2a^5b^{10}c^4 - 1430784AB^2a^6b^8c^5 + 2860032AB^2a^7b^6c^6 - 1290240AB^2a^8b^4c^7 - 5160960AB^2a^9b^2c^8 + 99A^2a^2b^2c \left(-(4ac - b^2)^{15} \right)^{1/2} - 6AB^2a^2b^3 \left(-(4ac - b^2)^{15} \right)^{1/2} - 288AB^2a^2b^{16}c + 108AB^2a^2b^2c \left(-(4ac - b^2)^{15} \right)^{1/2} \Big/ (512(a^5b^{20} + 1048576a^{15}c^{10} - 40a^6b^{18}c + 720a^7b^{16}c^2 - 7680a^8b^{14}c^3 + 53760a^9b^{12}c^4 - 258048a^{10}b^{10}c^5 + 860160a^{11}b^8c^6 - 1966080a^{12}b^6c^7 + 2949120a^{13}b^4c^8 - 2621440a^{14}b^2c^9))^{1/2} * (262144a^9b^2c^7 - 256a^4b^{11}c^2 + 5120a^5b^9c^3 - 40960a^6b^7c^4 + 163840a^7b^5c^5 - 327680a^8b^3c^6) \Big/ (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3)) \left(-(9A^2b^{19} + B^2a^2b^{17} - 9A^2b^4 \left(-(4ac - b^2)^{15} \right)^{1/2} + 6AB^2a^2b^{18} + 6921A^2a^2b^{15}c^2 - 77580A^2a^3b^{13}c^3 + 570960A^2a^4b^{11}c^4 - 2851776A^2a^5b^9c^5 + 9628416A^2a^6b^7c^6 - 21095424A^2a^7b^5c^7 + 27095040A^2a^8b^3c^8 - 441A^2a^2c^2 \left(-(4ac - b^2)^{15} \right)^{1/2} - B^2a^2b^2 \left(-(4ac - b^2)^{15} \right)^{1/2} + 1140B^2a^4b^{13}c^2 - 10160B^2a^5b^{11}c^3 + 34880B^2a^6b^9c^4 + 43776B^2a^7b^7c^5 - 680960B^2a^8b^5c^6 + 1863680B^2a^9b^3c^7 + 6881280AB^2a^{10}c^9 - 369A^2a^2b^{17}c - 15482880A^2a^9b^2c^9 - 55B^2a^3b^{15}c - 1720320B^2a^{10}b^2c^8 + 25B^2a^3c \left(-(4ac - b^2)^{15} \right)^{1/2} \right)
\end{aligned}$$

$$\begin{aligned}
& + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 \\
& - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - \\
& 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 - 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2))*(-(9*A^2*b^19 + B^2*a^2*b^17 - 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 - 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 + 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 + 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c + 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out

$$3.115 \quad \int \frac{x(-7+4x^2)}{4-5x^2+x^4} dx$$

Optimal. Leaf size=25

$$\frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1247, 632, 31}

$$\frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)$$

Antiderivative was successfully verified.

[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x(-7+4x^2)}{4-5x^2+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-7+4x}{4-5x+x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x} dx, x, x^2 \right) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{-4+x} dx, x, x^2 \right) \\
&= \frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4), x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4), x]

fricas [A] time = 1.14, size = 17, normalized size = 0.68

$$\frac{1}{2} \log(x^2-1) + \frac{3}{2} \log(x^2-4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4), x, algorithm="fricas")

[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)

giac [A] time = 0.31, size = 19, normalized size = 0.76

$$\frac{1}{2} \log(|x^2-1|) + \frac{3}{2} \log(|x^2-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")

[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$\frac{3 \ln(x^2 - 4)}{2} + \frac{\ln(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x)

[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)

maxima [A] time = 0.72, size = 17, normalized size = 0.68

$$\frac{1}{2} \log(x^2 - 1) + \frac{3}{2} \log(x^2 - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")

[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)

mupad [B] time = 0.06, size = 17, normalized size = 0.68

$$\frac{\ln(x^2 - 1)}{2} + \frac{3 \ln(x^2 - 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)

[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2

sympy [A] time = 0.12, size = 17, normalized size = 0.68

$$\frac{3 \log(x^2 - 4)}{2} + \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)

[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2

$$3.116 \quad \int \frac{-7x+4x^3}{4-5x^2+x^4} dx$$

Optimal. Leaf size=25

$$\frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 1247, 632, 31}

$$\frac{1}{2} \log(1-x^2) + \frac{3}{2} \log(4-x^2)$$

Antiderivative was successfully verified.

[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4), x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned}
\int \frac{-7x + 4x^3}{4 - 5x^2 + x^4} dx &= \int \frac{x(-7 + 4x^2)}{4 - 5x^2 + x^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{-7 + 4x}{4 - 5x + x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1 + x} dx, x, x^2 \right) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{-4 + x} dx, x, x^2 \right) \\
&= \frac{1}{2} \log(1 - x^2) + \frac{3}{2} \log(4 - x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2} \log(1 - x^2) + \frac{3}{2} \log(4 - x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4), x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-7x + 4x^3}{4 - 5x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4), x]

fricas [A] time = 0.66, size = 17, normalized size = 0.68

$$\frac{1}{2} \log(x^2 - 1) + \frac{3}{2} \log(x^2 - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4), x, algorithm="fricas")

[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{2} \log(|x^2 - 1|) + \frac{3}{2} \log(|x^2 - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")

[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))

maple [A] time = 0.00, size = 18, normalized size = 0.72

$$\frac{3 \ln(x^2 - 4)}{2} + \frac{\ln(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x)

[Out] 3/2*ln(x^2-4)+1/2*ln(x^2-1)

maxima [A] time = 0.74, size = 25, normalized size = 1.00

$$\frac{3}{2} \log(x + 2) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \log(x - 1) + \frac{3}{2} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")

[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)

mupad [B] time = 0.03, size = 17, normalized size = 0.68

$$\frac{\ln(x^2 - 1)}{2} + \frac{3 \ln(x^2 - 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)

[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2

sympy [A] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3 \log(x^2 - 4)}{2} + \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)

[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2

$$3.117 \quad \int \frac{x(2+x^2)}{1+x^2+x^4} dx$$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + \frac{1}{4} \log(x^4+x^2+1)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1247, 634, 618, 204, 628}

$$\frac{1}{4} \log(x^4+x^2+1) + \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x(2+x^2)}{1+x^2+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+x}{1+x+x^2} dx, x, x^2 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^2 \right) + \frac{3}{4} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^2 \right) \\
 &= \frac{1}{4} \log(1+x^2+x^4) - \frac{3}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^2 \right) \\
 &= \frac{1}{2} \sqrt{3} \tan^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right) + \frac{1}{4} \log(1+x^2+x^4)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2} \sqrt{3} \tan^{-1} \left(\frac{2x^2+1}{\sqrt{3}} \right) + \frac{1}{4} \log(x^4+x^2+1)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4), x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2+x^2)}{1+x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(2 + x^2))/(1 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(x*(2 + x^2))/(1 + x^2 + x^4), x]

fricas [A] time = 0.94, size = 30, normalized size = 0.81

$$\frac{1}{2} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^2+1) \right) + \frac{1}{4} \log(x^4+x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)

giac [A] time = 0.28, size = 30, normalized size = 0.81

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{4} \log(x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{2} + \frac{\ln(x^4 + x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^2+2)/(x^4+x^2+1),x)

[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)

maxima [A] time = 1.40, size = 30, normalized size = 0.81

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{4} \log(x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)

mupad [B] time = 0.21, size = 32, normalized size = 0.86

$$\frac{\ln(x^4 + x^2 + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)`

[Out] `log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2`

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log(x^4 + x^2 + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+2)/(x**4+x**2+1),x)`

[Out] `log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2`

$$3.118 \quad \int \frac{2x+x^3}{1+x^2+x^4} dx$$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + \frac{1}{4} \log(x^4+x^2+1)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1593, 1247, 634, 618, 204, 628}

$$\frac{1}{4} \log(x^4+x^2+1) + \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)/(1 + x^2 + x^4), x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{2x + x^3}{1 + x^2 + x^4} dx &= \int \frac{x(2 + x^2)}{1 + x^2 + x^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + x}{1 + x + x^2} dx, x, x^2 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \frac{1 + 2x}{1 + x + x^2} dx, x, x^2 \right) + \frac{3}{4} \text{Subst} \left(\int \frac{1}{1 + x + x^2} dx, x, x^2 \right) \\
 &= \frac{1}{4} \log(1 + x^2 + x^4) - \frac{3}{2} \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x^2 \right) \\
 &= \frac{1}{2} \sqrt{3} \tan^{-1} \left(\frac{1 + 2x^2}{\sqrt{3}} \right) + \frac{1}{4} \log(1 + x^2 + x^4)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2} \sqrt{3} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + \frac{1}{4} \log(x^4 + x^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4), x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + x^3}{1 + x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*x + x^3)/(1 + x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2*x + x^3)/(1 + x^2 + x^4), x]

fricas [A] time = 1.19, size = 30, normalized size = 0.81

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{4} \log(x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)

giac [A] time = 0.37, size = 30, normalized size = 0.81

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{4} \log(x^4 + x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{2} + \frac{\ln(x^4 + x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+2*x)/(x^4+x^2+1), x)

[Out] 1/2*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/4*ln(x^4+x^2+1)

maxima [A] time = 1.60, size = 53, normalized size = 1.43

$$-\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{4} \log(x^2 + x + 1) + \frac{1}{4} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")

[Out] $-1/2*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/2*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 1/4*\log(x^2 + x + 1) + 1/4*\log(x^2 - x + 1)$

mupad [B] time = 0.03, size = 32, normalized size = 0.86

$$\frac{\ln(x^4 + x^2 + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + x^3)/(x^2 + x^4 + 1), x)`

[Out] $\log(x^2 + x^4 + 1)/4 + (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 + (2*3^{(1/2)}*x^2)/3))/2$

sympy [A] time = 0.12, size = 37, normalized size = 1.00

$$\frac{\log(x^4 + x^2 + 1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+2*x)/(x**4+x**2+1), x)`

[Out] $\log(x**4 + x**2 + 1)/4 + \operatorname{sqrt}(3)*\operatorname{atan}(2*\operatorname{sqrt}(3)*x**2/3 + \operatorname{sqrt}(3)/3)/2$

$$3.119 \quad \int \frac{11x+2x^3}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=45

$$\frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{9x^2+5}{8(x^4+2x^2+3)}$$

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1593, 1247, 638, 618, 204}

$$\frac{9x^2+5}{8(x^4+2x^2+3)} + \frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]

[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1247


```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{11x + 2x^3}{(3 + 2x^2 + x^4)^2} dx &= \int \frac{x(11 + 2x^2)}{(3 + 2x^2 + x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{11 + 2x}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{5 + 9x^2}{8(3 + 2x^2 + x^4)} + \frac{9}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{5 + 9x^2}{8(3 + 2x^2 + x^4)} - \frac{9}{4} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2(1 + x^2) \right) \\
&= \frac{5 + 9x^2}{8(3 + 2x^2 + x^4)} + \frac{9 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{9 \tan^{-1} \left(\frac{x^2+1}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{9x^2 + 5}{8(x^4 + 2x^2 + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2, x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{11x + 2x^3}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 0.83, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}(x^4 + 2x^2 + 3) \arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + 18x^2 + 10}{16(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 + 10)/(x^4 + 2*x^2 + 3)

giac [A] time = 0.94, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) + \frac{9x^2 + 5}{8(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3)

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2} \arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} + \frac{18x^2 + 10}{16x^4 + 32x^2 + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x)

[Out] $1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^{(1/2)}*\arctan(1/4*(2*x^2+2)*2^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^2 + 5}{8(x^4 + 2x^2 + 3)} + \frac{9}{4} \int \frac{x}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")`

[Out] $1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*\integrate(x/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 0.05, size = 41, normalized size = 0.91

$$\frac{\frac{9x^2}{8} + \frac{5}{8}}{x^4 + 2x^2 + 3} + \frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)`

[Out] $((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/16$

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{9x^2 + 5}{8x^4 + 16x^2 + 24} + \frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)`

[Out] $(9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*\operatorname{sqrt}(2)*\operatorname{atan}(\operatorname{sqrt}(2)*x**2/2 + \operatorname{sqrt}(2)/2)/16$

$$3.120 \quad \int x^5 (2 + 3x^2) \sqrt{3 + 5x^2 + x^4} dx$$

Optimal. Leaf size=102

$$\frac{3}{10} (x^4 + 5x^2 + 3)^{3/2} x^4 + \frac{1}{480} (1837 - 510x^2) (x^4 + 5x^2 + 3)^{3/2} - \frac{1633}{256} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} + \frac{21229}{512} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{10} (x^4 + 5x^2 + 3)^{3/2} x^4 + \frac{1}{480} (1837 - 510x^2) (x^4 + 5x^2 + 3)^{3/2} - \frac{1633}{256} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} + \frac{21229}{512} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x

] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 832

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)* (a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int x^5 (2 + 3x^2) \sqrt{3 + 5x^2 + x^4} dx &= \frac{1}{2} \text{Subst} \left(\int x^2 (2 + 3x) \sqrt{3 + 5x + x^2} dx, x, x^2 \right) \\
 &= \frac{3}{10} x^4 (3 + 5x^2 + x^4)^{3/2} + \frac{1}{10} \text{Subst} \left(\int \left(-18 - \frac{85x}{2} \right) x \sqrt{3 + 5x + x^2} dx, x, x \right) \\
 &= \frac{3}{10} x^4 (3 + 5x^2 + x^4)^{3/2} + \frac{1}{480} (1837 - 510x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{1633}{64} \text{Subst} \left(\int \sqrt{3 + 5x + x^2} dx, x, x \right) \\
 &= -\frac{1633}{256} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} + \frac{3}{10} x^4 (3 + 5x^2 + x^4)^{3/2} + \frac{1}{480} (1837 - 510x^2) (3 + 5x^2 + x^4)^{3/2} \\
 &= -\frac{1633}{256} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} + \frac{3}{10} x^4 (3 + 5x^2 + x^4)^{3/2} + \frac{1}{480} (1837 - 510x^2) (3 + 5x^2 + x^4)^{3/2} \\
 &= -\frac{1633}{256} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} + \frac{3}{10} x^4 (3 + 5x^2 + x^4)^{3/2} + \frac{1}{480} (1837 - 510x^2) (3 + 5x^2 + x^4)^{3/2}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.70

$$\frac{318435 \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) + 2\sqrt{x^4+5x^2+3} (1152x^8 + 1680x^6 - 2248x^4 + 12250x^2 - 78387)}{7680}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^8) + 318435*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/7680

IntegrateAlgebraic [A] time = 0.18, size = 69, normalized size = 0.68

$$\frac{\sqrt{x^4+5x^2+3} (1152x^8 + 1680x^6 - 2248x^4 + 12250x^2 - 78387)}{3840} - \frac{21229}{512} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^8))/3840 - (21229*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512

fricas [A] time = 1.36, size = 61, normalized size = 0.60

$$\frac{1}{3840} (1152x^8 + 1680x^6 - 2248x^4 + 12250x^2 - 78387)\sqrt{x^4+5x^2+3} - \frac{21229}{512} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2), x, algorithm="fricas")

[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.46, size = 102, normalized size = 1.00

$$\frac{1}{1280} \sqrt{x^4+5x^2+3} (2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429) + \frac{1}{192} \sqrt{x^4+5x^2+3} (2(4(6x^2+5)x^2-89)x^2+1095) - \frac{21229}{512} \log(2x^2 - 2\sqrt{x^4+5x^2+3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2), x, algorithm="giac")

[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.03, size = 91, normalized size = 0.89

$$\frac{3(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^4}{10} - \frac{17(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^2}{16} + \frac{21229 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{512} + \frac{1837(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{480} - \frac{1633(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2), x)`

[Out] $3/10*x^4*(x^4+5*x^2+3)^{(3/2)} - 17/16*x^2*(x^4+5*x^2+3)^{(3/2)} + 1837/480*(x^4+5*x^2+3)^{(3/2)} - 1633/256*(2*x^2+5)*(x^4+5*x^2+3)^{(1/2)} + 21229/512*\ln(5/2+x^2+(x^4+5*x^2+3)^{(1/2)})$

maxima [A] time = 0.67, size = 104, normalized size = 1.02

$$\frac{3}{10}(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^4 - \frac{17}{16}(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^2 - \frac{1633}{128}\sqrt{x^4 + 5x^2 + 3}x^2 + \frac{1837}{480}(x^4 + 5x^2 + 3)^{\frac{3}{2}} - \frac{8165}{256}\sqrt{x^4 + 5x^2 + 3} + \frac{21229}{512}\log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2), x, algorithm="maxima")`

[Out] $3/10*(x^4 + 5*x^2 + 3)^{(3/2)}*x^4 - 17/16*(x^4 + 5*x^2 + 3)^{(3/2)}*x^2 - 1633/128*\sqrt{x^4 + 5*x^2 + 3}*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^{(3/2)} - 8165/256*\sqrt{x^4 + 5*x^2 + 3} + 21229/512*\log(2*x^2 + 2*\sqrt{x^4 + 5*x^2 + 3} + 5)$

mupad [B] time = 0.49, size = 102, normalized size = 1.00

$$\frac{21229 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{512} - \frac{17x^2(x^4 + 5x^2 + 3)^{3/2}}{16} + \frac{3x^4(x^4 + 5x^2 + 3)^{3/2}}{10} + \frac{51\left(\frac{x^2}{2} + \frac{5}{4}\right)\sqrt{x^4 + 5x^2 + 3}}{16} + \frac{1837\sqrt{x^4 + 5x^2 + 3}(8x^4 + 10x^2 - 51)}{3840}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)`

[Out] $(21229*\log((5*x^2 + x^4 + 3)^{(1/2)} + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^4 + 3)^{(3/2)})/16 + (3*x^4*(5*x^2 + x^4 + 3)^{(3/2)})/10 + (51*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^{(1/2)})/16 + (1837*(5*x^2 + x^4 + 3)^{(1/2)}*(10*x^2 + 8*x^4 - 51))/3840$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (3x^2 + 2) \sqrt{x^4 + 5x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2), x)`

[Out] `Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)`

$$3.121 \quad \int x^3 (2 + 3x^2) \sqrt{3 + 5x^2 + x^4} dx$$

Optimal. Leaf size=81

$$-\frac{1}{48} (59 - 18x^2) (x^4 + 5x^2 + 3)^{3/2} + \frac{259}{128} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} - \frac{3367}{256} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{48} (59 - 18x^2) (x^4 + 5x^2 + 3)^{3/2} + \frac{259}{128} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} - \frac{3367}{256} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]

[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x

] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int x^3 (2 + 3x^2) \sqrt{3 + 5x^2 + x^4} dx &= \frac{1}{2} \text{Subst} \left(\int x(2 + 3x) \sqrt{3 + 5x + x^2} dx, x, x^2 \right) \\
 &= -\frac{1}{48} (59 - 18x^2) (3 + 5x^2 + x^4)^{3/2} + \frac{259}{32} \text{Subst} \left(\int \sqrt{3 + 5x + x^2} dx, x, x^2 \right) \\
 &= \frac{259}{128} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} - \frac{1}{48} (59 - 18x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{3367}{256} \text{Subst} \left(\int \sqrt{3 + 5x + x^2} dx, x, x^2 \right) \\
 &= \frac{259}{128} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} - \frac{1}{48} (59 - 18x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{3367}{128} \text{Subst} \left(\int \sqrt{3 + 5x + x^2} dx, x, x^2 \right) \\
 &= \frac{259}{128} (5 + 2x^2) \sqrt{3 + 5x^2 + x^4} - \frac{1}{48} (59 - 18x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{3367}{256} \text{Subst} \left(\int \sqrt{3 + 5x + x^2} dx, x, x^2 \right)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.81

$$\frac{1}{768} \left(2\sqrt{x^4 + 5x^2 + 3} (144x^6 + 248x^4 - 374x^2 + 2469) - 10101 \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6) - 10101*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768

IntegrateAlgebraic [A] time = 0.15, size = 64, normalized size = 0.79

$$\frac{3367}{256} \log \left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5 \right) + \frac{1}{384} \sqrt{x^4 + 5x^2 + 3} (144x^6 + 248x^4 - 374x^2 + 2469)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6))/384 + (3367*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256

fricas [A] time = 1.28, size = 56, normalized size = 0.69

$$\frac{1}{384} (144x^6 + 248x^4 - 374x^2 + 2469)\sqrt{x^4 + 5x^2 + 3} + \frac{3367}{256} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.36, size = 88, normalized size = 1.09

$$\frac{1}{128} \sqrt{x^4 + 5x^2 + 3} (2(4(6x^2 + 5)x^2 - 89)x^2 + 1095) + \frac{1}{24} \sqrt{x^4 + 5x^2 + 3} (2(4x^2 + 5)x^2 - 51) + \frac{3367}{256} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] 1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 74, normalized size = 0.91

$$\frac{3(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^2}{8} - \frac{3367 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{256} - \frac{59(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{48} + \frac{259(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)

[Out] 3/8*(x^4+5*x^2+3)^(3/2)*x^2-59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-3367/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 0.59, size = 87, normalized size = 1.07

$$\frac{3}{8}(x^4 + 5x^2 + 3)^{\frac{3}{2}}x^2 + \frac{259}{64}\sqrt{x^4 + 5x^2 + 3}x^2 - \frac{59}{48}(x^4 + 5x^2 + 3)^{\frac{3}{2}} + \frac{1295}{128}\sqrt{x^4 + 5x^2 + 3} - \frac{3367}{256} \log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] $\frac{3}{8}(x^4 + 5x^2 + 3)^{3/2}x^2 + \frac{259}{64}\sqrt{x^4 + 5x^2 + 3}x^2 - \frac{59}{48}(x^4 + 5x^2 + 3)^{3/2} + \frac{1295}{128}\sqrt{x^4 + 5x^2 + 3} - \frac{3367}{256}\log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$

mupad [B] time = 0.43, size = 85, normalized size = 1.05

$$\frac{3x^2(x^4 + 5x^2 + 3)^{3/2}}{8} - \frac{3367 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{256} - \frac{9\left(\frac{x^2}{2} + \frac{5}{4}\right)\sqrt{x^4 + 5x^2 + 3}}{8} - \frac{59\sqrt{x^4 + 5x^2 + 3}(8x^4 + 10x^2 - 51)}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)`

[Out] $\frac{3x^2(5x^2 + x^4 + 3)^{3/2}}{8} - \frac{3367 \log((5x^2 + x^4 + 3)^{1/2} + x^2 + 5/2)}{256} - \frac{9(x^2/2 + 5/4)(5x^2 + x^4 + 3)^{1/2}}{8} - \frac{59(5x^2 + x^4 + 3)^{1/2}(10x^2 + 8x^4 - 51)}{384}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (3x^2 + 2) \sqrt{x^4 + 5x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2), x)`

[Out] `Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)`

$$3.122 \quad \int x(2 + 3x^2) \sqrt{3 + 5x^2 + x^4} dx$$

Optimal. Leaf size=74

$$\frac{1}{2} (x^4 + 5x^2 + 3)^{3/2} - \frac{11}{16} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} + \frac{143}{32} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1247, 640, 612, 621, 206}

$$\frac{1}{2} (x^4 + 5x^2 + 3)^{3/2} - \frac{11}{16} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} + \frac{143}{32} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]

[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (143*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int x(2+3x^2)\sqrt{3+5x^2+x^4} dx &= \frac{1}{2} \text{Subst}\left(\int(2+3x)\sqrt{3+5x+x^2} dx, x, x^2\right) \\
 &= \frac{1}{2}(3+5x^2+x^4)^{3/2} - \frac{11}{4} \text{Subst}\left(\int\sqrt{3+5x+x^2} dx, x, x^2\right) \\
 &= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32} \text{Subst}\left(\int\frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2\right) \\
 &= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{16} \text{Subst}\left(\int\frac{1}{4-x^2} dx, x, x^2\right) \\
 &= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32} \tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.82

$$\frac{1}{32} \left(2\sqrt{x^4+5x^2+3} (8x^4+18x^2-31) + 143 \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4) + 143*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32

IntegrateAlgebraic [A] time = 0.15, size = 59, normalized size = 0.80

$$\frac{1}{16} \sqrt{x^4+5x^2+3} (8x^4+18x^2-31) - \frac{143}{32} \log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4], x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4))/16 - (143*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32

fricas [A] time = 1.28, size = 51, normalized size = 0.69

$$\frac{1}{16} (8x^4 + 18x^2 - 31)\sqrt{x^4 + 5x^2 + 3} - \frac{143}{32} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.34, size = 74, normalized size = 1.00

$$\frac{1}{16} \sqrt{x^4 + 5x^2 + 3} (2(4x^2 + 5)x^2 - 51) + \frac{1}{4} \sqrt{x^4 + 5x^2 + 3} (2x^2 + 5) - \frac{143}{32} \log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.01, size = 57, normalized size = 0.77

$$\frac{143 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{32} + \frac{(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{2} - \frac{11(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)

[Out] 1/2*(x^4+5*x^2+3)^(3/2)-11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 0.65, size = 70, normalized size = 0.95

$$-\frac{11}{8} \sqrt{x^4 + 5x^2 + 3} x^2 + \frac{1}{2} (x^4 + 5x^2 + 3)^{\frac{3}{2}} - \frac{55}{16} \sqrt{x^4 + 5x^2 + 3} + \frac{143}{32} \log\left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] $-11/8*\sqrt{x^4 + 5x^2 + 3} * x^2 + 1/2*(x^4 + 5x^2 + 3)^{(3/2)} - 55/16*\sqrt{x^4 + 5x^2 + 3} + 143/32*\log(2x^2 + 2*\sqrt{x^4 + 5x^2 + 3} + 5)$

mupad [B] time = 0.29, size = 67, normalized size = 0.91

$$\frac{143 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{32} + \left(\frac{x^2}{2} + \frac{5}{4}\right) \sqrt{x^4 + 5x^2 + 3} + \frac{\sqrt{x^4 + 5x^2 + 3} (8x^4 + 10x^2 - 51)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)`

[Out] $(143*\log((5*x^2 + x^4 + 3)^{(1/2)} + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^{(1/2)} + ((5*x^2 + x^4 + 3)^{(1/2)}*(10*x^2 + 8*x^4 - 51))/16$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(3x^2 + 2)\sqrt{x^4 + 5x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2), x)`

[Out] `Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)`

$$3.123 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} dx$$

Optimal. Leaf size=94

$$\frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23) + \frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23) + \frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]

[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2


```

) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x^2) \sqrt{3 + 5x^2 + x^4}}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2 + 3x) \sqrt{3 + 5x + x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{8} (23 + 6x^2) \sqrt{3 + 5x^2 + x^4} - \frac{1}{8} \text{Subst} \left(\int \frac{-24 - \frac{x}{2}}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= \frac{1}{8} (23 + 6x^2) \sqrt{3 + 5x^2 + x^4} + \frac{1}{16} \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) + 3 \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= \frac{1}{8} (23 + 6x^2) \sqrt{3 + 5x^2 + x^4} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{4 - x^2} dx, x, \frac{5 + 2x^2}{\sqrt{3 + 5x^2 + x^4}} \right) - 6 \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= \frac{1}{8} (23 + 6x^2) \sqrt{3 + 5x^2 + x^4} + \frac{1}{16} \tanh^{-1} \left(\frac{5 + 2x^2}{2\sqrt{3 + 5x^2 + x^4}} \right) - \sqrt{3} \tanh^{-1} \left(\frac{5 + 2x^2}{2\sqrt{3 + 5x^2 + x^4}} \right) - 6 \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.98

$$\frac{1}{16} \left(2\sqrt{x^4 + 5x^2 + 3} (6x^2 + 23) + \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - 16\sqrt{3} \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]

[Out] (2*(23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]]) - 16*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/16

IntegrateAlgebraic [A] time = 0.20, size = 93, normalized size = 0.99

$$\frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (6x^2 + 23) - \frac{1}{16} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5) + 2\sqrt{3} \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]

[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + 2*Sqrt[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]] - Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16

fricas [A] time = 1.26, size = 95, normalized size = 1.01

$$\frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (6x^2 + 23) + \sqrt{3} \log \left(\frac{25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30}{x^2} \right) - \frac{1}{16} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")

[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.48, size = 98, normalized size = 1.04

$$\frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (6x^2 + 23) + \sqrt{3} \log \left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}} \right) - \frac{1}{16} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")

[Out] $\frac{1}{8}\sqrt{x^4 + 5x^2 + 3}(6x^2 + 23) + \sqrt{3}\log((x^2 + \sqrt{3}) - \sqrt{x^4 + 5x^2 + 3})/(x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}) - \frac{1}{16}\log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$

maple [A] time = 0.02, size = 85, normalized size = 0.90

$$-\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2 + 6)\sqrt{3}}{6\sqrt{x^4 + 5x^2 + 3}}\right) + \frac{\ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{16} + \frac{3(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{8} + \sqrt{x^4 + 5x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((3x^2+2)*(x^4+5x^2+3)^{(1/2)}/x,x)$

[Out] $\frac{3}{8}(2x^2+5)*(x^4+5x^2+3)^{(1/2)} + \frac{1}{16}\ln(x^2+5/2+(x^4+5x^2+3)^{(1/2)}) + (x^4+5x^2+3)^{(1/2)} - \operatorname{arctanh}(1/6*(5x^2+6)*3^{(1/2)}/(x^4+5x^2+3)^{(1/2)}) * 3^{(1/2)}$

maxima [A] time = 1.50, size = 89, normalized size = 0.95

$$\frac{3}{4}\sqrt{x^4 + 5x^2 + 3}x^2 - \sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{23}{8}\sqrt{x^4 + 5x^2 + 3} + \frac{1}{16}\log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((3x^2+2)*(x^4+5x^2+3)^{(1/2)}/x,x, \operatorname{algorithm}="maxima")$

[Out] $\frac{3}{4}\sqrt{x^4 + 5x^2 + 3}x^2 - \sqrt{3}\log(2\sqrt{3}\sqrt{x^4 + 5x^2 + 3})\sqrt{x^4 + 5x^2 + 3} / x^2 + 6/x^2 + 5) + \frac{23}{8}\sqrt{x^4 + 5x^2 + 3} + \frac{1}{16}\log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$

mupad [B] time = 0.43, size = 86, normalized size = 0.91

$$\frac{\ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{16} - \sqrt{3}\ln\left(\frac{3}{x^2} + \frac{\sqrt{3}\sqrt{x^4 + 5x^2 + 3}}{x^2} + \frac{5}{2}\right) + \frac{3\left(\frac{x^2}{2} + \frac{5}{4}\right)\sqrt{x^4 + 5x^2 + 3}}{2} + \sqrt{x^4 + 5x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(((3x^2 + 2)*(5x^2 + x^4 + 3)^{(1/2)}))/x,x)$

[Out] $\log((5x^2 + x^4 + 3)^{(1/2)} + x^2 + 5/2)/16 - 3^{(1/2)}\log(3/x^2 + (3^{(1/2)}*(5x^2 + x^4 + 3)^{(1/2)})/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5x^2 + x^4 + 3)^{(1/2)})/2 + (5x^2 + x^4 + 3)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)\sqrt{x^4 + 5x^2 + 3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

$$3.124 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3} dx$$

Optimal. Leaf size=97

$$-\frac{\sqrt{x^4+5x^2+3}(2-3x^2)}{2x^2} + \frac{19}{4} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{7 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}(2-3x^2)}{2x^2} + \frac{19}{4} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{7 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3, x]

[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 812

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x^2) \sqrt{3 + 5x^2 + x^4}}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2 + 3x) \sqrt{3 + 5x + x^2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{(2 - 3x^2) \sqrt{3 + 5x^2 + x^4}}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{-28 - 19x}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= -\frac{(2 - 3x^2) \sqrt{3 + 5x^2 + x^4}}{2x^2} + \frac{19}{4} \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) + 7 \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= -\frac{(2 - 3x^2) \sqrt{3 + 5x^2 + x^4}}{2x^2} + \frac{19}{2} \text{Subst} \left(\int \frac{1}{4 - x^2} dx, x, \frac{5 + 2x^2}{\sqrt{3 + 5x^2 + x^4}} \right) - 14 \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= -\frac{(2 - 3x^2) \sqrt{3 + 5x^2 + x^4}}{2x^2} + \frac{19}{4} \tanh^{-1} \left(\frac{5 + 2x^2}{2\sqrt{3 + 5x^2 + x^4}} \right) - \frac{7 \tanh^{-1} \left(\frac{6 + 2x^2}{2\sqrt{3 + 5x^2 + x^4}} \right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{\sqrt{x^4 + 5x^2 + 3} (3x^2 - 2)}{2x^2} + \frac{19}{4} \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right) - \frac{7 \tanh^{-1}\left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]

[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3]

IntegrateAlgebraic [A] time = 0.22, size = 96, normalized size = 0.99

$$\frac{\sqrt{x^4 + 5x^2 + 3} (3x^2 - 2)}{2x^2} - \frac{19}{4} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right) + \frac{14 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]

[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (14*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/Sqrt[3] - (19*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4

fricas [A] time = 0.71, size = 112, normalized size = 1.15

$$\frac{56\sqrt{3}x^2 \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30}{x^2}\right) - 114x^2 \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5) + 21x^2 + 12\sqrt{x^4 + 5x^2 + 3}(3x^2 - 2)}{24x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")

[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2

giac [A] time = 0.51, size = 138, normalized size = 1.42

$$\frac{7}{3}\sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{3}{2}\sqrt{x^4 + 5x^2 + 3} + \frac{5x^2 - 5\sqrt{x^4 + 5x^2 + 3} + 6}{(x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3} - \frac{19}{4} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")

[Out] $\frac{7\sqrt{3}\sqrt{3}\log((x^2 + \sqrt{3}) - \sqrt{x^4 + 5x^2 + 3})/(x^2 - \sqrt{3}) - \sqrt{x^4 + 5x^2 + 3}}{3} + \frac{3}{2}\sqrt{x^4 + 5x^2 + 3} + \frac{(5x^2 - 5\sqrt{x^4 + 5x^2 + 3} + 6)/((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3) - 19/4\log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)}{4} - \frac{19/4\log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)}{3} + \frac{(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{6}$

maple [A] time = 0.02, size = 104, normalized size = 1.07

$$-\frac{7\sqrt{3}\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3} + \frac{19\ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{4} - \frac{(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{3x^2} + \frac{7\sqrt{x^4 + 5x^2 + 3}}{3} + \frac{(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x)

[Out] $\frac{7}{3}(x^4+5x^2+3)^{1/2} + \frac{19}{4}\ln(x^2+5/2+(x^4+5x^2+3)^{1/2}) - \frac{7}{3}\operatorname{arctanh}\left(\frac{1}{6}\frac{(5x^2+6)*3^{1/2}}{(x^4+5x^2+3)^{1/2}}\right) - \frac{1}{3}x^{-2}(x^4+5x^2+3)^{3/2} + \frac{1}{6}(2x^2+5)(x^4+5x^2+3)^{1/2}$

maxima [A] time = 1.35, size = 89, normalized size = 0.92

$$-\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{\sqrt{x^4+5x^2+3}}{x^2} + \frac{19}{4}\log\left(2x^2 + 2\sqrt{x^4+5x^2+3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")

[Out] $-\frac{7}{3}\sqrt{3}\log(2\sqrt{3}\sqrt{x^4+5x^2+3}/x^2 + 6/x^2 + 5) + \frac{3}{2}\sqrt{x^4+5x^2+3} - \sqrt{x^4+5x^2+3}/x^2 + \frac{19}{4}\log(2x^2 + 2\sqrt{x^4+5x^2+3} + 5)$

mupad [B] time = 0.88, size = 84, normalized size = 0.87

$$\frac{19\ln\left(\sqrt{x^4+5x^2+3} + x^2 + \frac{5}{2}\right)}{4} - \frac{\sqrt{x^4+5x^2+3}}{x^2} - \frac{7\sqrt{3}\ln\left(\frac{3}{x^2} + \frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{5}{2}\right)}{3} + \frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)

[Out] $\frac{19\log((5x^2 + x^4 + 3)^{1/2} + x^2 + 5/2)}{4} - \frac{(5x^2 + x^4 + 3)^{1/2}}{x^2} - \frac{7\sqrt{3}\log(3/x^2 + (3^{1/2})(5x^2 + x^4 + 3)^{1/2}/x^2 + 5/2)}{3} + \frac{3(5x^2 + x^4 + 3)^{1/2}}{2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)\sqrt{x^4 + 5x^2 + 3}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)

[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)

$$3.125 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} dx$$

Optimal. Leaf size=99

$$-\frac{\sqrt{x^4+5x^2+3}(23x^2+6)}{12x^4} + \frac{3}{2} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{77 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}(23x^2+6)}{12x^4} + \frac{3}{2} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{77 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5, x]

[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/(24*Sqrt[3]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 810

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x))/((e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)))] - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3} dx, x, x^2 \right) \\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24} \text{Subst} \left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) + \frac{77}{24} \text{Subst} \left(\int \frac{1}{x\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3 \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}} \right) - \frac{77}{12} \text{Subst} \left(\int \frac{1}{x\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right) - \frac{77 \tanh^{-1} \left(\frac{6}{2\sqrt{3}\sqrt{3+5x^2+x^4}} \right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.98

$$\frac{1}{72} \left(-\frac{6\sqrt{x^4+5x^2+3}(23x^2+6)}{x^4} + 108 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) - 77\sqrt{3} \tanh^{-1} \left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5, x]

[Out] ((-6*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 108*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]]) - 77*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/72

IntegrateAlgebraic [A] time = 0.27, size = 98, normalized size = 0.99

$$\frac{\sqrt{x^4+5x^2+3}(-23x^2-6)}{12x^4} - \frac{3}{2} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + \frac{77 \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}} \right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5, x]

[Out] ((-6 - 23*x^2)*Sqrt[3 + 5*x^2 + x^4]/(12*x^4) + (77*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(12*Sqrt[3]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]))/2

fricas [A] time = 0.92, size = 112, normalized size = 1.13

$$\frac{77\sqrt{3}x^4 \log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - 108x^4 \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) - 138x^4 - 6\sqrt{x^4+5x^2+3}(23x^2+6)}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")

[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3))*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4

giac [B] time = 0.57, size = 169, normalized size = 1.71

$$\frac{77}{72}\sqrt{3} \log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{127(x^2-\sqrt{x^4+5x^2+3})^3 + 228(x^2-\sqrt{x^4+5x^2+3})^2 - 159x^2 + 159\sqrt{x^4+5x^2+3} - 324}{12((x^2-\sqrt{x^4+5x^2+3})^2)} - \frac{3}{2} \log(2x^2 - 2\sqrt{x^4+5x^2+3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")

[Out] 77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 324)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 121, normalized size = 1.22

$$-\frac{77\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72} + \frac{3 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4+5x^2+3}\right)}{2} - \frac{13(x^4+5x^2+3)^{\frac{3}{2}}}{36x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4} + \frac{77\sqrt{x^4+5x^2+3}}{72} + \frac{13(2x^2+5)\sqrt{x^4+5x^2+3}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x)

[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36*(x^4+5*x^2+3)^(3/2)/x^2+77/72*(x^4+5*x^2+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 1.43, size = 106, normalized size = 1.07

$$-\frac{77}{72}\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{1}{6}\sqrt{x^4+5x^2+3} - \frac{13\sqrt{x^4+5x^2+3}}{12x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4} + \frac{3}{2} \log(2x^2 + 2\sqrt{x^4+5x^2+3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")

[Out] $-77/72*\sqrt{3}*\log(2*\sqrt{3}*\sqrt{x^4 + 5*x^2 + 3})/x^2 + 6/x^2 + 5) + 1/6*\sqrt{3}*\sqrt{x^4 + 5*x^2 + 3} - 13/12*\sqrt{x^4 + 5*x^2 + 3})/x^2 - 1/6*(x^4 + 5*x^2 + 3)^{(3/2)}/x^4 + 3/2*\log(2*x^2 + 2*\sqrt{x^4 + 5*x^2 + 3}) + 5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)

[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)

$$3.126 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7} dx$$

Optimal. Leaf size=90

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4} + \frac{13 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}} - \frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 806, 720, 724, 206}

$$-\frac{(x^4+5x^2+3)^{3/2}}{9x^6} - \frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4} + \frac{13 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7, x]

[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(9*x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x^2) \sqrt{3 + 5x^2 + x^4}}{x^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2 + 3x) \sqrt{3 + 5x + x^2}}{x^4} dx, x, x^2 \right) \\
 &= -\frac{(3 + 5x^2 + x^4)^{3/2}}{9x^6} + \frac{2}{3} \text{Subst} \left(\int \frac{\sqrt{3 + 5x + x^2}}{x^3} dx, x, x^2 \right) \\
 &= -\frac{(6 + 5x^2) \sqrt{3 + 5x^2 + x^4}}{18x^4} - \frac{(3 + 5x^2 + x^4)^{3/2}}{9x^6} - \frac{13}{36} \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{(6 + 5x^2) \sqrt{3 + 5x^2 + x^4}}{18x^4} - \frac{(3 + 5x^2 + x^4)^{3/2}}{9x^6} + \frac{13}{18} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, x^2 \right) \\
 &= -\frac{(6 + 5x^2) \sqrt{3 + 5x^2 + x^4}}{18x^4} - \frac{(3 + 5x^2 + x^4)^{3/2}}{9x^6} + \frac{13 \tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3} \sqrt{3 + 5x^2 + x^4}} \right)}{36\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.82

$$\frac{1}{108} \left(13\sqrt{3} \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3} \sqrt{x^4 + 5x^2 + 3}} \right) - \frac{6\sqrt{x^4 + 5x^2 + 3} (7x^4 + 16x^2 + 6)}{x^6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]

[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 + 13*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/108

IntegrateAlgebraic [A] time = 0.33, size = 75, normalized size = 0.83

$$\frac{(-7x^4 - 16x^2 - 6)\sqrt{x^4 + 5x^2 + 3}}{18x^6} - \frac{13 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]

[Out] ((-6 - 16*x^2 - 7*x^4)*Sqrt[3 + 5*x^2 + x^4])/(18*x^6) - (13*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(18*Sqrt[3])

fricas [A] time = 0.75, size = 90, normalized size = 1.00

$$\frac{13\sqrt{3}x^6 \log\left(\frac{25x^2 + 2\sqrt{3}(5x^2 + 6) + 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} + 6) + 30}{x^2}\right) - 42x^6 - 6(7x^4 + 16x^2 + 6)\sqrt{x^4 + 5x^2 + 3}}{108x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")

[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3))/x^6

giac [B] time = 0.53, size = 189, normalized size = 2.10

$$-\frac{13}{108}\sqrt{3}\log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{67(x^2 - \sqrt{x^4 + 5x^2 + 3})^5 + 306(x^2 - \sqrt{x^4 + 5x^2 + 3})^4 + 430(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 + 90(x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 63x^2 + 63\sqrt{x^4 + 5x^2 + 3} + 108}{18((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")

[Out] -13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 306*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 90*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3

maple [A] time = 0.02, size = 118, normalized size = 1.31

$$\frac{13\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{108} + \frac{5(x^4+5x^2+3)^{\frac{3}{2}}}{54x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6} - \frac{13\sqrt{x^4+5x^2+3}}{108} - \frac{5(2x^2+5)\sqrt{x^4+5x^2+3}}{108}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x)`

[Out] $-1/9*(x^4+5*x^2+3)^{(3/2)}/x^6-1/9*(x^4+5*x^2+3)^{(3/2)}/x^4+5/54*(x^4+5*x^2+3)^{(3/2)}/x^2-13/108*(x^4+5*x^2+3)^{(1/2)}+13/108*\operatorname{arctanh}(1/6*(5*x^2+6)*3^{(1/2)})/(x^4+5*x^2+3)^{(1/2)}*3^{(1/2)}-5/108*(2*x^2+5)*(x^4+5*x^2+3)^{(1/2)}$

maxima [A] time = 1.45, size = 99, normalized size = 1.10

$$\frac{13}{108} \sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{1}{9} \sqrt{x^4+5x^2+3} + \frac{5\sqrt{x^4+5x^2+3}}{18x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")`

[Out] $13/108*\operatorname{sqrt}(3)*\log(2*\operatorname{sqrt}(3)*\operatorname{sqrt}(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*\operatorname{sqrt}(x^4 + 5*x^2 + 3) + 5/18*\operatorname{sqrt}(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 + 3)^{(3/2)}/x^4 - 1/9*(x^4 + 5*x^2 + 3)^{(3/2)}/x^6$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)`

[Out] `int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)`

[Out] `Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)`

$$3.127 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9} dx$$

Optimal. Leaf size=111

$$\frac{67(5x^2+6)\sqrt{x^4+5x^2+3}}{1728x^4} - \frac{871 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}} - \frac{(x^4+5x^2+3)^{3/2}}{12x^8} - \frac{11(x^4+5x^2+3)^{3/2}}{216x^6}$$

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 834, 806, 720, 724, 206}

$$-\frac{11(x^4+5x^2+3)^{3/2}}{216x^6} - \frac{(x^4+5x^2+3)^{3/2}}{12x^8} + \frac{67(5x^2+6)\sqrt{x^4+5x^2+3}}{1728x^4} - \frac{871 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9, x]

[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 834

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(m + 1)*(c*d^2 - b*d*e + a*e^2), x] + Dist[1/(m + 1)*(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5} dx, x, x^2 \right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{1}{24} \text{Subst} \left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4} dx, x, x^2 \right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{67}{144} \text{Subst} \left(\int \frac{\sqrt{3+5x+x^2}}{x^3} dx, x, x^2 \right) \\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} + \frac{871}{1728x^8} \\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871}{1728x^8} \\
&= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871}{1728x^8}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}(247x^6-182x^4-984x^2-432)-871\sqrt{3}x^8 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]

[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6) - 871*Sqrt[3]*x^8*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*x^8)

IntegrateAlgebraic [A] time = 0.37, size = 80, normalized size = 0.72

$$\frac{871 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{1728\sqrt{3}} + \frac{\sqrt{x^4+5x^2+3}(247x^6-182x^4-984x^2-432)}{1728x^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6))/(1728*x^8) + (871*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(1728*Sqrt[3])

fricas [A] time = 1.22, size = 95, normalized size = 0.86

$$\frac{871 \sqrt{3} x^8 \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2+6) - 2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) + 1482x^8 + 6(247x^6 - 182x^4 - 984x^2 - 432)\sqrt{x^4+5x^2+3}}{10368x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")

[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 - 984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8

giac [B] time = 0.46, size = 233, normalized size = 2.10

$$\frac{871 \sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4+5x^2+3}}{x^2 - \sqrt{3} - \sqrt{x^4+5x^2+3}}\right) - 871(x^2 - \sqrt{x^4+5x^2+3})^7 - 5184(x^2 - \sqrt{x^4+5x^2+3})^6 - 57389(x^2 - \sqrt{x^4+5x^2+3})^5 - 165888(x^2 - \sqrt{x^4+5x^2+3})^4 - 204807(x^2 - \sqrt{x^4+5x^2+3})^3 - 93312(x^2 - \sqrt{x^4+5x^2+3})^2 - 2403x^2 + 2403\sqrt{x^4+5x^2+3} - 5184}{1728((x^2 - \sqrt{x^4+5x^2+3})^2 - 3)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")

[Out] 871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 - 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sqrt(x^4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4

maple [A] time = 0.02, size = 135, normalized size = 1.22

$$-\frac{871\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{10368} - \frac{335(x^4+5x^2+3)^{\frac{3}{2}}}{5184x^2} + \frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4} - \frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8} + \frac{871\sqrt{x^4+5x^2+3}}{10368} + \frac{335(2x^2+5)\sqrt{x^4+5x^2+3}}{10368}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x)

[Out] -11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/864*(x^4+5*x^2+3)^(3/2)/x^4-335/5184*(x^4+5*x^2+3)^(3/2)/x^2+871/10368*(x^4+5*x^2+3)^(1/2)-871/10368*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+335/10368*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-1/12*(x^4+5*x^2+3)^(3/2)/x^8

maxima [A] time = 1.74, size = 116, normalized size = 1.05

$$-\frac{871}{10368} \sqrt{3} \log\left(2\frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{67}{864} \sqrt{x^4+5x^2+3} - \frac{335\sqrt{x^4+5x^2+3}}{1728x^2} + \frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4} - \frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")

[Out] $-871/10368*\sqrt{3}*\log(2*\sqrt{3}*\sqrt{x^4 + 5*x^2 + 3})/x^2 + 6/x^2 + 5) - 67/864*\sqrt{x^4 + 5*x^2 + 3} - 335/1728*\sqrt{x^4 + 5*x^2 + 3}/x^2 + 67/864*(x^4 + 5*x^2 + 3)^{(3/2)}/x^4 - 11/216*(x^4 + 5*x^2 + 3)^{(3/2)}/x^6 - 1/12*(x^4 + 5*x^2 + 3)^{(3/2)}/x^8$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)

[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)

$$3.128 \quad \int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}} dx$$

Optimal. Leaf size=132

$$-\frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4} + \frac{2093 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}} - \frac{(x^4+5x^2+3)^{3/2}}{15x^{10}} - \frac{(x^4+5x^2+3)^{3/2}}{36x^8} + \frac{173(x^4+5x^2+3)^{3/2}}{3240x^6}$$

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 834, 806, 720, 724, 206}

$$\frac{173(x^4+5x^2+3)^{3/2}}{3240x^6} - \frac{(x^4+5x^2+3)^{3/2}}{36x^8} - \frac{(x^4+5x^2+3)^{3/2}}{15x^{10}} - \frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4} + \frac{2093 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11, x]

[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4]/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 834

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6} dx, x, x^2 \right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{1}{30} \text{Subst} \left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5} dx, x, x^2 \right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{1}{360} \text{Subst} \left(\int \frac{(-173-10x)\sqrt{3+5x+x^2}}{x^4} dx, x, x^2 \right) \\
&= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{161}{432} \text{Subst} \left(\int \frac{(-173-10x)\sqrt{3+5x+x^2}}{x^3} dx, x, x^2 \right) \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} \\
&= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.64

$$\frac{10465\sqrt{3} \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}(2641x^8-1370x^6+1176x^4+10800x^2+5184)}{x^{10}}}{155520}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11, x]

[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x^8))/x^10 + 10465*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/155520

IntegrateAlgebraic [A] time = 0.45, size = 85, normalized size = 0.64

$$\frac{\sqrt{x^4+5x^2+3}(-2641x^8+1370x^6-1176x^4-10800x^2-5184)}{25920x^{10}} - \frac{2093 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-5184 - 10800*x^2 - 1176*x^4 + 1370*x^6 - 2641*x^8))/(25920*x^10) - (2093*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(5184*Sqrt[3])

fricas [A] time = 0.93, size = 100, normalized size = 0.76

$$\frac{10465 \sqrt{3} x^{10} \log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}(5\sqrt{3}+6)+30}{x^2}\right) - 15846 x^{10} - 6(2641x^8 - 1370x^6 + 1176x^4 + 10800x^2 + 5184)\sqrt{x^4 + 5x^2 + 3}}{155520 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")

[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 1370*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10

giac [B] time = 0.56, size = 255, normalized size = 1.93

$$\frac{2093 \sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{10465(x^2 - \sqrt{x^4 + 5x^2 + 3})^9 - 42830(x^2 - \sqrt{x^4 + 5x^2 + 3})^7 + 1270080(x^2 - \sqrt{x^4 + 5x^2 + 3})^5 + 7060800(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 + 15310080(x^2 - \sqrt{x^4 + 5x^2 + 3}) + 16095870(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 + 7568640(x^2 - \sqrt{x^4 + 5x^2 + 3})^5 + 1096335x^2 - 1096335\sqrt{x^4 + 5x^2 + 3} + 202176}{2920((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)}}{31104}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")

[Out] -2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 + 3))^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568640*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x^2 + 3) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5

maple [A] time = 0.02, size = 152, normalized size = 1.15

$$\frac{2093\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{31104} + \frac{805(x^4+5x^2+3)^{\frac{3}{2}}}{15552x^2} - \frac{161(x^4+5x^2+3)^{\frac{3}{2}}}{2592x^4} + \frac{173(x^4+5x^2+3)^{\frac{3}{2}}}{3240x^6} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{36x^8} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{15x^{10}} - \frac{2093\sqrt{x^4+5x^2+3}}{31104} - \frac{805(2x^2+5)\sqrt{x^4+5x^2+3}}{31104}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x)

[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5*x^2+3)^(3/2)/x^6-161/2592*(x^4+5*x^2+3)^(3/2)/x^4+805/15552*(x^4+5*x^2+3)^(3/2)/x^2-2093/31104*(x^4+5*x^2+3)^(1/2)+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)

maxima [A] time = 1.73, size = 133, normalized size = 1.01

$$\frac{2093}{31104} \sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{161}{2592} \sqrt{x^4+5x^2+3} + \frac{805\sqrt{x^4+5x^2+3}}{5184x^2} - \frac{161(x^4+5x^2+3)^{3/2}}{2592x^4} + \frac{173(x^4+5x^2+3)^{3/2}}{3240x^6} - \frac{(x^4+5x^2+3)^{3/2}}{36x^8} - \frac{(x^4+5x^2+3)^{3/2}}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")

[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 161/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/2592*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/36*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)

[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)

$$3.129 \quad \int x^5 (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx$$

Optimal. Leaf size=127

$$\frac{3}{14} (x^4 + 5x^2 + 3)^{5/2} x^4 + \frac{(3313 - 1070x^2)(x^4 + 5x^2 + 3)^{5/2}}{1680} - \frac{2183}{768} (2x^2 + 5)(x^4 + 5x^2 + 3)^{3/2} + \frac{28379(2x^2 + 5)}{2048} \sqrt{x^4 + 5x^2 + 3} - \frac{368927 \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)}{4096}$$

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{14} (x^4 + 5x^2 + 3)^{5/2} x^4 + \frac{(3313 - 1070x^2)(x^4 + 5x^2 + 3)^{5/2}}{1680} - \frac{2183}{768} (2x^2 + 5)(x^4 + 5x^2 + 3)^{3/2} + \frac{28379(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{2048} - \frac{368927 \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)}{4096}$$

Antiderivative was successfully verified.

[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/4096

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int x^5 (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left(\int x^2 (2 + 3x) (3 + 5x + x^2)^{3/2} dx, x, x^2 \right) \\
&= \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2} + \frac{1}{14} \text{Subst} \left(\int \left(-18 - \frac{107x}{2} \right) x (3 + 5x + x^2)^{3/2} dx, x, x^2 \right) \\
&= \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2} + \frac{(3313 - 1070x^2) (3 + 5x^2 + x^4)^{5/2}}{1680} - \frac{2183}{96} \text{Subst} \left(\int \frac{1}{x} (3 + 5x + x^2)^{3/2} dx, x, x^2 \right) \\
&= -\frac{2183}{768} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} + \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2} + \frac{(3313 - 1070x^2) (3 + 5x^2 + x^4)^{5/2}}{1680} \\
&= \frac{28379 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{2048} - \frac{2183}{768} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} + \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2} \\
&= \frac{28379 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{2048} - \frac{2183}{768} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} + \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2} \\
&= \frac{28379 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{2048} - \frac{2183}{768} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} + \frac{3}{14} x^4 (3 + 5x^2 + x^4)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.64

$$\frac{2\sqrt{x^4 + 5x^2 + 3} (46080x^{12} + 323840x^{10} + 482944x^8 + 154800x^6 + 283304x^4 - 1499570x^2 + 9546951) - 38737335 \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)}{430080}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 + 482944*x^8 + 323840*x^10 + 46080*x^12) - 38737335*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]]))/430080

IntegrateAlgebraic [A] time = 0.30, size = 79, normalized size = 0.62

$$\frac{368927 \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)}{4096} + \frac{\sqrt{x^4 + 5x^2 + 3} (46080x^{12} + 323840x^{10} + 482944x^8 + 154800x^6 + 283304x^4 - 1499570x^2 + 9546951)}{215040}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 + 482944*x^8 + 323840*x^10 + 46080*x^12))/215040 + (368927*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4096

fricas [A] time = 1.00, size = 71, normalized size = 0.56

$$\frac{1}{215040} (46080 x^{12} + 323840 x^{10} + 482944 x^8 + 154800 x^6 + 283304 x^4 - 1499570 x^2 + 9546951) \sqrt{x^4 + 5x^2 + 3} + \frac{368927}{4096} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")

[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 - 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.54, size = 207, normalized size = 1.63

$$\frac{1}{71680} \sqrt{x^4 + 5x^2 + 3} (2(4(2(8(10(12x^2 + 5)^2 - 203)^2 + 7635)^2 - 1627215)^2 - 20756241) + \frac{17}{3072} \sqrt{x^4 + 5x^2 + 3} (2(4(2(8(2x^2 + 1)^2 - 33)^2 + 321) - 6837)^2 + 87147) + \frac{19}{3840} \sqrt{x^4 + 5x^2 + 3} (2(4(6(8x^2 + 5)^2 - 127)^2 + 2635)^2 - 33429) + \frac{1}{64} \sqrt{x^4 + 5x^2 + 3} (2(4(6x^2 + 5)^2 - 89)^2 + 1095) + \frac{368927}{4096} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] 1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2 + 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 87147) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.03, size = 138, normalized size = 1.09

$$\frac{3\sqrt{x^4 + 5x^2 + 3} x^{12}}{14} + \frac{253\sqrt{x^4 + 5x^2 + 3} x^{10}}{168} + \frac{539\sqrt{x^4 + 5x^2 + 3} x^8}{240} + \frac{645\sqrt{x^4 + 5x^2 + 3} x^6}{896} + \frac{5059\sqrt{x^4 + 5x^2 + 3} x^4}{3840} - \frac{149957\sqrt{x^4 + 5x^2 + 3} x^2}{21504} - \frac{368927 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{4096} + \frac{3182317\sqrt{x^4 + 5x^2 + 3}}{71680}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)

[Out] 645/896*x^6*(x^4+5*x^2+3)^(1/2)-149957/21504*x^2*(x^4+5*x^2+3)^(1/2)-368927/4096*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/14*x^12*(x^4+5*x^2+3)^(1/2)+253/168*x^10*(x^4+5*x^2+3)^(1/2)+539/240*x^8*(x^4+5*x^2+3)^(1/2)+3182317/71680*(x^4+5*x^2+3)^(1/2)+5059/3840*x^4*(x^4+5*x^2+3)^(1/2)

maxima [A] time = 0.59, size = 135, normalized size = 1.06

$$\frac{3}{14} (x^4 + 5x^2 + 3)^{\frac{5}{2}} x^4 - \frac{107}{168} (x^4 + 5x^2 + 3)^{\frac{5}{2}} x^2 - \frac{2183}{384} (x^4 + 5x^2 + 3)^{\frac{3}{2}} x^2 + \frac{3313}{1680} (x^4 + 5x^2 + 3)^{\frac{5}{2}} + \frac{28379}{1024} \sqrt{x^4 + 5x^2 + 3} x^2 - \frac{10915}{768} (x^4 + 5x^2 + 3)^{\frac{3}{2}} + \frac{141895}{2048} \sqrt{x^4 + 5x^2 + 3} - \frac{368927}{4096} \log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")


```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 (3x^2 + 2) (x^4 + 5x^2 + 3)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

```
[Out] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (3x^2 + 2) (x^4 + 5x^2 + 3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2), x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

$$3.130 \quad \int x^3 (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx$$

Optimal. Leaf size=106

$$-\frac{1}{40} (27 - 10x^2) (x^4 + 5x^2 + 3)^{5/2} + \frac{123}{128} (2x^2 + 5) (x^4 + 5x^2 + 3)^{3/2} - \frac{4797 (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3}}{1024} + \frac{62361 \operatorname{tanh}^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)}{2048}$$

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{40} (27 - 10x^2) (x^4 + 5x^2 + 3)^{5/2} + \frac{123}{128} (2x^2 + 5) (x^4 + 5x^2 + 3)^{3/2} - \frac{4797 (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3}}{1024} + \frac{62361 \operatorname{tanh}^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)}{2048}$$

Antiderivative was successfully verified.

[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) -

$2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x$
 $] + \text{Dist}[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1251

$\text{Int}[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := \text{Dist}[1/2, \text{Subst}[\text{Int}[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /;$ FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int x^3 (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx &= \frac{1}{2} \text{Subst} \left(\int x(2 + 3x) (3 + 5x + x^2)^{3/2} dx, x, x^2 \right) \\ &= -\frac{1}{40} (27 - 10x^2) (3 + 5x^2 + x^4)^{5/2} + \frac{123}{16} \text{Subst} \left(\int (3 + 5x + x^2)^{3/2} dx, x, x^2 \right) \\ &= \frac{123}{128} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{1}{40} (27 - 10x^2) (3 + 5x^2 + x^4)^{5/2} - \frac{4797}{25} \\ &= -\frac{4797 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{1024} + \frac{123}{128} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{1}{40} \\ &= -\frac{4797 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{1024} + \frac{123}{128} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{1}{40} \\ &= -\frac{4797 (5 + 2x^2) \sqrt{3 + 5x^2 + x^4}}{1024} + \frac{123}{128} (5 + 2x^2) (3 + 5x^2 + x^4)^{3/2} - \frac{1}{40} \end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.72

$$\frac{311805 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) + 2\sqrt{x^4+5x^2+3} (1280x^{10} + 9344x^8 + 14960x^6 + 5064x^4 + 12390x^2 - 77229)}{10240}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x^8 + 1280*x^10) + 311805*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/10240

IntegrateAlgebraic [A] time = 0.24, size = 74, normalized size = 0.70

$$\frac{\sqrt{x^4 + 5x^2 + 3} (1280x^{10} + 9344x^8 + 14960x^6 + 5064x^4 + 12390x^2 - 77229)}{5120} - \frac{62361 \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)}{2048}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x^8 + 1280*x^10))/5120 - (62361*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2048

fricas [A] time = 1.02, size = 66, normalized size = 0.62

$$\frac{1}{5120} (1280x^{10} + 9344x^8 + 14960x^6 + 5064x^4 + 12390x^2 - 77229)\sqrt{x^4 + 5x^2 + 3} - \frac{62361}{2048} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2), x, algorithm="fricas")

[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sqrt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [B] time = 0.59, size = 179, normalized size = 1.69

$$\frac{1}{1024} \sqrt{x^4 + 5x^2 + 3} (2(4(2(8(2x^2 + 1)x^2 - 33)x^2 + 321)x^2 - 6837)x^2 + 87147) + \frac{17}{3840} \sqrt{x^4 + 5x^2 + 3} (2(4(6(8x^2 + 5)x^2 - 127)x^2 + 2635)x^2 - 33429) + \frac{19}{384} \sqrt{x^4 + 5x^2 + 3} (2(4(6x^2 + 5)x^2 - 89)x^2 + 1095) + \frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (2(4x^2 + 5)x^2 - 51) - \frac{62361}{2048} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2), x, algorithm="giac")

[Out] 1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) - 5)

maple [A] time = 0.02, size = 121, normalized size = 1.14

$$\frac{\sqrt{x^4 + 5x^2 + 3} x^{10}}{4} + \frac{73\sqrt{x^4 + 5x^2 + 3} x^8}{40} + \frac{187\sqrt{x^4 + 5x^2 + 3} x^6}{64} + \frac{633\sqrt{x^4 + 5x^2 + 3} x^4}{640} + \frac{1239\sqrt{x^4 + 5x^2 + 3} x^2}{512} + \frac{62361 \ln(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3})}{2048} - \frac{77229\sqrt{x^4 + 5x^2 + 3}}{5120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2), x)

[Out] 187/64*(x^4+5*x^2+3)^(1/2)*x^6+1239/512*(x^4+5*x^2+3)^(1/2)*x^2+62361/2048*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+1/4*(x^4+5*x^2+3)^(1/2)*x^10+73/40*(x^4+5*x

$(x^2+3)^{(1/2)} * x^8 - 77229/5120 * (x^4+5*x^2+3)^{(1/2)} + 633/640 * (x^4+5*x^2+3)^{(1/2)} * x^4$

maxima [A] time = 0.88, size = 118, normalized size = 1.11

$\frac{1}{4}(x^4+5x^2+3)^{\frac{5}{2}}x^2 + \frac{123}{64}(x^4+5x^2+3)^{\frac{3}{2}}x^2 - \frac{27}{40}(x^4+5x^2+3)^{\frac{5}{2}} - \frac{4797}{512}\sqrt{x^4+5x^2+3}x^2 + \frac{615}{128}(x^4+5x^2+3)^{\frac{3}{2}} - \frac{23985}{1024}\sqrt{x^4+5x^2+3} + \frac{62361}{2048}\log(2x^2+2\sqrt{x^4+5x^2+3}+5)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{4}(x^4+5x^2+3)^{(5/2)}x^2 + \frac{123}{64}(x^4+5x^2+3)^{(3/2)}x^2 - \frac{27}{40}(x^4+5x^2+3)^{(5/2)} - \frac{4797}{512}\sqrt{x^4+5x^2+3}x^2 + \frac{615}{128}(x^4+5x^2+3)^{(3/2)} - \frac{23985}{1024}\sqrt{x^4+5x^2+3} + \frac{62361}{2048}\log(2x^2+2\sqrt{x^4+5x^2+3}+5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (3x^2 + 2) (x^4 + 5x^2 + 3)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)*(5*x^2+x^4+3)^(3/2),x)

[Out] int(x^3*(3*x^2+2)*(5*x^2+x^4+3)^(3/2),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (3x^2 + 2) (x^4 + 5x^2 + 3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)

[Out] Integral(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)

$$3.131 \quad \int x (2 + 3x^2) (3 + 5x^2 + x^4)^{3/2} dx$$

Optimal. Leaf size=99

$$\frac{3}{10} (x^4 + 5x^2 + 3)^{5/2} - \frac{11}{32} (2x^2 + 5) (x^4 + 5x^2 + 3)^{3/2} + \frac{429}{256} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} - \frac{5577}{512} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1247, 640, 612, 621, 206}

$$\frac{3}{10} (x^4 + 5x^2 + 3)^{5/2} - \frac{11}{32} (2x^2 + 5) (x^4 + 5x^2 + 3)^{3/2} + \frac{429}{256} (2x^2 + 5) \sqrt{x^4 + 5x^2 + 3} - \frac{5577}{512} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 + x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b

$*e)/(2*c)$, $\text{Int}[(a + b*x + c*x^2)^p, x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e, p\}, x]$
 $\&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

Rule 1247

$\text{Int}[(x_*)*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^{(p_*)}, x_Symbol]$:> $\text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x]$ /; $\text{FreeQ}\{a, b, c, d, e, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int x(2 + 3x^2)(3 + 5x^2 + x^4)^{3/2} dx &= \frac{1}{2} \text{Subst}\left(\int (2 + 3x)(3 + 5x + x^2)^{3/2} dx, x, x^2\right) \\ &= \frac{3}{10}(3 + 5x^2 + x^4)^{5/2} - \frac{11}{4} \text{Subst}\left(\int (3 + 5x + x^2)^{3/2} dx, x, x^2\right) \\ &= -\frac{11}{32}(5 + 2x^2)(3 + 5x^2 + x^4)^{3/2} + \frac{3}{10}(3 + 5x^2 + x^4)^{5/2} + \frac{429}{64} \text{Subst}\left(\int \sqrt{3 + 5x + x^2} dx, x, x^2\right) \\ &= \frac{429}{256}(5 + 2x^2)\sqrt{3 + 5x^2 + x^4} - \frac{11}{32}(5 + 2x^2)(3 + 5x^2 + x^4)^{3/2} + \frac{3}{10}(3 + 5x^2 + x^4)^{5/2} \\ &= \frac{429}{256}(5 + 2x^2)\sqrt{3 + 5x^2 + x^4} - \frac{11}{32}(5 + 2x^2)(3 + 5x^2 + x^4)^{3/2} + \frac{3}{10}(3 + 5x^2 + x^4)^{5/2} \\ &= \frac{429}{256}(5 + 2x^2)\sqrt{3 + 5x^2 + x^4} - \frac{11}{32}(5 + 2x^2)(3 + 5x^2 + x^4)^{3/2} + \frac{3}{10}(3 + 5x^2 + x^4)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.72

$$\frac{2\sqrt{x^4 + 5x^2 + 3}(384x^8 + 2960x^6 + 5304x^4 + 2170x^2 + 7581) - 27885 \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)}{2560}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8) - 27885*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/2560

IntegrateAlgebraic [A] time = 0.24, size = 69, normalized size = 0.70

$$\frac{5577}{512} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right) + \frac{\sqrt{x^4 + 5x^2 + 3}(384x^8 + 2960x^6 + 5304x^4 + 2170x^2 + 7581)}{1280}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8))/1280 + (5577*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512

fricas [A] time = 0.76, size = 61, normalized size = 0.62

$$\frac{1}{1280} (384x^8 + 2960x^6 + 5304x^4 + 2170x^2 + 7581)\sqrt{x^4 + 5x^2 + 3} + \frac{5577}{512} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")

[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 + 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.45, size = 151, normalized size = 1.53

$$\frac{1}{1280}\sqrt{x^4+5x^2+3}(2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429)+\frac{17}{384}\sqrt{x^4+5x^2+3}(2(4(6x^2+5)x^2-89)x^2+1095)+\frac{19}{48}\sqrt{x^4+5x^2+3}(2(4x^2+5)x^2-51)+\frac{3}{4}\sqrt{x^4+5x^2+3}(2x^2+5)+\frac{5577}{512}\log(2x^2-2\sqrt{x^4+5x^2+3}+5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 104, normalized size = 1.05

$$\frac{3\sqrt{x^4+5x^2+3}x^8}{10} + \frac{37\sqrt{x^4+5x^2+3}x^6}{16} + \frac{663\sqrt{x^4+5x^2+3}x^4}{160} + \frac{217\sqrt{x^4+5x^2+3}x^2}{128} - \frac{5577\ln\left(x^2 + \frac{5}{2} + \sqrt{x^4+5x^2+3}\right)}{512} + \frac{7581\sqrt{x^4+5x^2+3}}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)

[Out] 3/10*(x^4+5*x^2+3)^(1/2)*x^8+37/16*(x^4+5*x^2+3)^(1/2)*x^6+663/160*(x^4+5*x^2+3)^(1/2)*x^4+217/128*(x^4+5*x^2+3)^(1/2)*x^2+7581/1280*(x^4+5*x^2+3)^(1/2)-5577/512*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 0.67, size = 101, normalized size = 1.02

$$-\frac{11}{16}(x^4+5x^2+3)^{\frac{3}{2}}x^2 + \frac{3}{10}(x^4+5x^2+3)^{\frac{5}{2}} + \frac{429}{128}\sqrt{x^4+5x^2+3}x^2 - \frac{55}{32}(x^4+5x^2+3)^{\frac{3}{2}} + \frac{2145}{256}\sqrt{x^4+5x^2+3} - \frac{5577}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")

[Out] $-11/16*(x^4 + 5*x^2 + 3)^{(3/2)}*x^2 + 3/10*(x^4 + 5*x^2 + 3)^{(5/2)} + 429/128*\sqrt{x^4 + 5*x^2 + 3}*x^2 - 55/32*(x^4 + 5*x^2 + 3)^{(3/2)} + 2145/256*\sqrt{x^4 + 5*x^2 + 3} - 5577/512*\log(2*x^2 + 2*\sqrt{x^4 + 5*x^2 + 3} + 5)$

mupad [B] time = 0.53, size = 127, normalized size = 1.28

$$\frac{(x^2 + \frac{5}{2})(x^4 + 5x^2 + 3)^{3/2}}{4} - \frac{15x^2(x^4 + 5x^2 + 3)^{3/2}}{16} - \frac{5577 \ln(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2})}{512} + \frac{585(2x^2 + 5)\sqrt{x^4 + 5x^2 + 3}}{256} - \frac{39(\frac{x^2}{2} + \frac{5}{4})\sqrt{x^4 + 5x^2 + 3}}{16} - \frac{75(x^4 + 5x^2 + 3)^{3/2}}{32} + \frac{3(x^4 + 5x^2 + 3)^{5/2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)

[Out] $((x^2 + 5/2)*(5*x^2 + x^4 + 3)^{(3/2)})/4 - (15*x^2*(5*x^2 + x^4 + 3)^{(3/2)})/16 - (5577*\log((5*x^2 + x^4 + 3)^{(1/2)} + x^2 + 5/2))/512 + (585*(2*x^2 + 5)*(5*x^2 + x^4 + 3)^{(1/2)})/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^{(1/2)})/16 - (75*(5*x^2 + x^4 + 3)^{(3/2)})/32 + (3*(5*x^2 + x^4 + 3)^{(5/2)})/10$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(3x^2 + 2)(x^4 + 5x^2 + 3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)

[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)

$$3.132 \quad \int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx$$

Optimal. Leaf size=119

$$\frac{1}{48} (18x^2 + 61) (x^4 + 5x^2 + 3)^{3/2} + \frac{1}{128} (199 - 74x^2) \sqrt{x^4 + 5x^2 + 3} + \frac{2401}{256} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - 3\sqrt{3} \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{48} (18x^2 + 61) (x^4 + 5x^2 + 3)^{3/2} + \frac{1}{128} (199 - 74x^2) \sqrt{x^4 + 5x^2 + 3} + \frac{2401}{256} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - 3\sqrt{3} \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]

[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2

```

) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{48} (61+18x^2)(3+5x^2+x^4)^{3/2} - \frac{1}{16} \text{Subst} \left(\int \frac{(-48+\frac{37x}{2})\sqrt{3+5x+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{128} (199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48} (61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{1}{64} \text{Subst} \left(\int \frac{(-48+\frac{37x}{2})\sqrt{3+5x+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{128} (199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48} (61+18x^2)(3+5x^2+x^4)^{3/2} + 9 \text{Subst} \left(\int \frac{(-48+\frac{37x}{2})\sqrt{3+5x+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{128} (199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48} (61+18x^2)(3+5x^2+x^4)^{3/2} - 18 \text{Subst} \left(\int \frac{(-48+\frac{37x}{2})\sqrt{3+5x+x^2}}{x} dx, x, x^2 \right) \\
&= \frac{1}{128} (199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48} (61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{2401}{256} \text{Subst} \left(\int \frac{(-48+\frac{37x}{2})\sqrt{3+5x+x^2}}{x} dx, x, x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.87

$$\frac{2401}{256} \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) - 3\sqrt{3} \tanh^{-1} \left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}} \right) + \frac{1}{384} \sqrt{x^4+5x^2+3} (144x^6+1208x^4+2650x^2+2061)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x, x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/256 - 3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])

IntegrateAlgebraic [A] time = 0.32, size = 103, normalized size = 0.87

$$-\frac{2401}{256} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + 6\sqrt{3} \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}} \right) + \frac{1}{384} \sqrt{x^4+5x^2+3} (144x^6+1208x^4+2650x^2+2061)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x, x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + 6*Sqrt[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]] - (2401*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256

fricas [A] time = 1.23, size = 106, normalized size = 0.89

$$\frac{1}{384} (144x^6 + 1208x^4 + 2650x^2 + 2061) \sqrt{x^4 + 5x^2 + 3} + 3\sqrt{3} \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30}{x^2}\right) - \frac{2401}{256} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")

[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.50, size = 113, normalized size = 0.95

$$\frac{1}{384} \sqrt{x^4 + 5x^2 + 3} (2(4(18x^2 + 151)x^2 + 1325)x^2 + 2061) + 3\sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) - \frac{2401}{256} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")

[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) + 3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 117, normalized size = 0.98

$$\frac{3\sqrt{x^4 + 5x^2 + 3} x^6}{8} + \frac{151\sqrt{x^4 + 5x^2 + 3} x^4}{48} + \frac{1325\sqrt{x^4 + 5x^2 + 3} x^2}{192} - 3\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2 + 6)\sqrt{3}}{6\sqrt{x^4 + 5x^2 + 3}}\right) + \frac{2401 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{256} + \frac{687\sqrt{x^4 + 5x^2 + 3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x)

[Out] 3/8*(x^4+5*x^2+3)^(1/2)*x^6+151/48*(x^4+5*x^2+3)^(1/2)*x^4+1325/192*(x^4+5*x^2+3)^(1/2)*x^2+687/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)

maxima [A] time = 1.40, size = 120, normalized size = 1.01

$$\frac{3}{8} (x^4 + 5x^2 + 3)^{\frac{3}{2}} x^2 - \frac{37}{64} \sqrt{x^4 + 5x^2 + 3} x^2 + \frac{61}{48} (x^4 + 5x^2 + 3)^{\frac{3}{2}} - 3\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{199}{128} \sqrt{x^4 + 5x^2 + 3} + \frac{2401}{256} \log(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")

[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

+ 6/x² + 5) + 199/128*sqrt(x⁴ + 5*x² + 3) + 2401/256*log(2*x² + 2*sqrt(x⁴ + 5*x² + 3) + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)

[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)

$$3.133 \quad \int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx$$

Optimal. Leaf size=122

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2} + \frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3} + \frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 12\sqrt{3}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 812, 814, 843, 621, 206, 724}

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2} + \frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3} + \frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]

[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 812

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)

```

- d*g*(2*p + 1) + e*g*(m + 1)*x*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
+ 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p
+ 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 814

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^2} dx, x, x^2 \right) \\
&= -\frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x} dx, x, \right. \\
&= \frac{3}{16} (109+18x^2) \sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{1}{16} \text{Subst} \left(\int \right. \\
&= \frac{3}{16} (109+18x^2) \sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32} \text{Subst} \left(\int \right. \\
&= \frac{3}{16} (109+18x^2) \sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{16} \text{Subst} \left(\int \right. \\
&= \frac{3}{16} (109+18x^2) \sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32} \tanh^{-1}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.88

$$\frac{609}{32} \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) - 12\sqrt{3} \tanh^{-1} \left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}} \right) + \frac{\sqrt{x^4+5x^2+3}(8x^6+78x^4+271x^2-48)}{16x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/32 - 12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])

IntegrateAlgebraic [A] time = 0.35, size = 106, normalized size = 0.87

$$-\frac{609}{32} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + 24\sqrt{3} \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}} \right) + \frac{\sqrt{x^4+5x^2+3}(8x^6+78x^4+271x^2-48)}{16x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + 24*Sqrt[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]] - (609*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32

fricas [A] time = 1.04, size = 122, normalized size = 1.00

$$\frac{1536\sqrt{3}x^2 \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2+6) - 2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - 2436x^2 \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + 1541x^2 + 8(8x^6 + 78x^4 + 271x^2 - 48)\sqrt{x^4+5x^2+3}}{128x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")

[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3))*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5*x^2 + 3))/x^2

giac [A] time = 0.56, size = 153, normalized size = 1.25

$$\frac{1}{16}\sqrt{x^4+5x^2+3}(2(4x^2+39)x^2+271)+12\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3(5x^2-5\sqrt{x^4+5x^2+3}+6)}{(x^2-\sqrt{x^4+5x^2+3})^2-3}-\frac{609}{32}\log(2x^2-2\sqrt{x^4+5x^2+3}+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 117, normalized size = 0.96

$$\frac{\sqrt{x^4+5x^2+3}x^4}{2} + \frac{39\sqrt{x^4+5x^2+3}x^2}{8} - 12\sqrt{3}\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right) + \frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32} - \frac{3\sqrt{x^4+5x^2+3}}{x^2} + \frac{271\sqrt{x^4+5x^2+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x)

[Out] 1/2*(x^4+5*x^2+3)^(1/2)*x^4+39/8*(x^4+5*x^2+3)^(1/2)*x^2+271/16*(x^4+5*x^2+3)^(1/2)+609/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-3*(x^4+5*x^2+3)^(1/2)/x^2

maxima [A] time = 1.75, size = 120, normalized size = 0.98

$$\frac{27}{8}\sqrt{x^4+5x^2+3}x^2 + \frac{1}{2}(x^4+5x^2+3)^{\frac{3}{2}} - 12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{327}{16}\sqrt{x^4+5x^2+3} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{x^2} + \frac{609}{32}\log(2x^2+2\sqrt{x^4+5x^2+3}+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")

[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)

[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)

$$3.134 \quad \int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx$$

Optimal. Leaf size=127

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4} - \frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2} + \frac{453}{16} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{127}{8}\sqrt{3} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4} - \frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2} + \frac{453}{16} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{127}{8}\sqrt{3} \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]

[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/8

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 812

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

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Rule 843

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Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^3} dx, x, x^2 \right) \\
&= -\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} - \frac{3}{16} \text{Subst} \left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2} dx, x \right) \\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{3}{32} \text{Subst} \left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2} dx, x \right) \\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16} \text{Subst} \left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2} dx, x \right) \\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{8} \text{Subst} \left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2} dx, x \right) \\
&= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16} \tanh^{-1} \left(\frac{2\sqrt{x^4+5x^2+3}(6x^6+83x^4-86x^2-12)}{x^4} \right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 107, normalized size = 0.84

$$\frac{1}{16} \left(453 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) - 254\sqrt{3} \tanh^{-1} \left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}} \right) + \frac{2\sqrt{x^4+5x^2+3}(6x^6+83x^4-86x^2-12)}{x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]

[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 254*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/16

IntegrateAlgebraic [A] time = 0.39, size = 108, normalized size = 0.85

$$-\frac{453}{16} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + \frac{127}{4} \sqrt{3} \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}} \right) + \frac{\sqrt{x^4+5x^2+3}(6x^6+83x^4-86x^2-12)}{8x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/(8*x^4) + (127*Sqrt[3]*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/4 - (453*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/16

fricas [A] time = 1.15, size = 122, normalized size = 0.96

$$\frac{1016\sqrt{3}x^4 \log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - 1812x^4 \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + 67x^4 + 8(6x^6 + 83x^4 - 86x^2 - 12)\sqrt{x^4+5x^2+3}}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")

[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2 + 3))/x^4

giac [A] time = 0.65, size = 190, normalized size = 1.50

$$\frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+83) + \frac{127}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{227(x^2-\sqrt{x^4+5x^2+3})^3 + 348(x^2-\sqrt{x^4+5x^2+3})^2 - 459x^2 + 459\sqrt{x^4+5x^2+3} - 684}{4((x^2-\sqrt{x^4+5x^2+3})^2-3)^2} - \frac{453}{16}\log(2x^2-2\sqrt{x^4+5x^2+3}+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")

[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(227*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 459*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$\frac{3\sqrt{x^4+5x^2+3}x^2}{4} - \frac{127\sqrt{3}\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8} + \frac{453\ln\left(x^2 + \frac{5}{2} + \sqrt{x^4+5x^2+3}\right)}{16} - \frac{43\sqrt{x^4+5x^2+3}}{4x^2} - \frac{3\sqrt{x^4+5x^2+3}}{2x^4} + \frac{83\sqrt{x^4+5x^2+3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x)

[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-43/4*(x^4+5*x^2+3)^(1/2)/x^2-3/2*(x^4+5*x^2+3)^(1/2)/x^4+3/4*(x^4+5*x^2+3)^(1/2)*x^2

maxima [A] time = 1.46, size = 137, normalized size = 1.08

$$\frac{7}{2}\sqrt{x^4+5x^2+3}x^2 + \frac{1}{6}(x^4+5x^2+3)^{\frac{3}{2}} - \frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{197}{8}\sqrt{x^4+5x^2+3} - \frac{23(x^4+5x^2+3)^{\frac{3}{2}}}{12x^2} - \frac{(x^4+5x^2+3)^{\frac{5}{2}}}{6x^4} + \frac{453}{16}\log(2x^2+2\sqrt{x^4+5x^2+3}+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")

[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)

[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)

[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)

$$3.135 \quad \int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx$$

Optimal. Leaf size=127

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2} + \frac{49}{4} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{527 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}} - \frac{(7x^2+2)(x^4+5x^2)}{6x^6}$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1251, 810, 812, 843, 621, 206, 724}

$$-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6} - \frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2} + \frac{49}{4} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{527 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7, x]

[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 810

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x))/((e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]

```

Rule 812

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/((e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^4} dx, x, x^2 \right) \\
&= -\frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} - \frac{1}{24} \text{Subst} \left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2} dx \right) \\
&= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{1}{48} \text{Subst} \left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2} dx \right) \\
&= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4} \text{Subst} \left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2} dx \right) \\
&= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{2} \text{Subst} \left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2} dx \right) \\
&= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4} \tanh^{-1} \left(\frac{-134-64x}{2\sqrt{3+5x+x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.84

$$\frac{1}{72} \left(882 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) - 527\sqrt{3} \tanh^{-1} \left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}} \right) + \frac{6\sqrt{x^4+5x^2+3}(18x^6-141x^4-62x^2-12)}{x^6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7, x]

[Out] ((6*sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 882*ArcTanh[(5 + 2*x^2)/(2*sqrt[3 + 5*x^2 + x^4]]) - 527*sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*sqrt[3]*sqrt[3 + 5*x^2 + x^4])])/72

IntegrateAlgebraic [A] time = 0.53, size = 108, normalized size = 0.85

$$-\frac{49}{4} \log(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5) + \frac{527 \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}} \right)}{12\sqrt{3}} + \frac{\sqrt{x^4+5x^2+3}(18x^6-141x^4-62x^2-12)}{12x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7, x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/(12*x^6) + (527*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(12*Sqrt[3]) - (49*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4

fricas [A] time = 1.02, size = 122, normalized size = 0.96

$$\frac{527\sqrt{3}x^6 \log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - 882x^6 \log(-2x^2+2\sqrt{x^4+5x^2+3}-5) - 711x^6 + 6(18x^6-141x^4-62x^2-12)\sqrt{x^4+5x^2+3}}{72x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3))*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^2 + 3))/x^6

giac [B] time = 0.68, size = 227, normalized size = 1.79

$$\frac{527\sqrt{3} \log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} + \frac{829(x^2-\sqrt{x^4+5x^2+3})^5 + 1824(x^2-\sqrt{x^4+5x^2+3})^4 - 2200(x^2-\sqrt{x^4+5x^2+3})^3 - 5292(x^2-\sqrt{x^4+5x^2+3})^2 + 2799x^2 - 2799\sqrt{x^4+5x^2+3} + 5724}{12(x^2-\sqrt{x^4+5x^2+3})^3} - \frac{49}{4}\log(2x^2-2\sqrt{x^4+5x^2+3}+5)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")

[Out] 527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*x^2 - 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 117, normalized size = 0.92

$$-\frac{527\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72} + \frac{49 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4+5x^2+3}\right)}{4} - \frac{47\sqrt{x^4+5x^2+3}}{4x^2} - \frac{31\sqrt{x^4+5x^2+3}}{6x^4} - \frac{\sqrt{x^4+5x^2+3}}{x^6} + \frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x)

[Out] 49/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-47/4*(x^4+5*x^2+3)^(1/2)/x^2-(x^4+5*x^2+3)^(1/2)/x^6-31/6*(x^4+5*x^2+3)^(1/2)/x^4+3/2*(x^4+5*x^2+3)^(1/2)

maxima [A] time = 1.58, size = 154, normalized size = 1.21

$$\frac{67\sqrt{x^4+5x^2+3}x^2 + \frac{11}{54}(x^4+5x^2+3)^{\frac{3}{2}} - \frac{527\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{431\sqrt{x^4+5x^2+3}}{36} - \frac{79(x^4+5x^2+3)^{\frac{3}{2}}}{108x^2} - \frac{11(x^4+5x^2+3)^{\frac{5}{2}}}{54x^4} - \frac{(x^4+5x^2+3)^{\frac{5}{2}}}{9x^6} + \frac{49}{4}\log(2x^2+2\sqrt{x^4+5x^2+3}+5)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")`

[Out] $67/36*\sqrt{x^4 + 5x^2 + 3}*x^2 + 11/54*(x^4 + 5x^2 + 3)^{3/2} - 527/72*\sqrt{3}*\log(2*\sqrt{3}*\sqrt{x^4 + 5x^2 + 3}/x^2 + 6/x^2 + 5) + 431/36*\sqrt{x^4 + 5x^2 + 3} - 79/108*(x^4 + 5x^2 + 3)^{3/2}/x^2 - 11/54*(x^4 + 5x^2 + 3)^{5/2}/x^4 - 1/9*(x^4 + 5x^2 + 3)^{5/2}/x^6 + 49/4*\log(2*x^2 + 2*\sqrt{x^4 + 5x^2 + 3} + 5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)`

[Out] `int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)`

[Out] `Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)`

$$3.136 \quad \int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx^2+cx^4} \left(-16aBc - 2cx^2(5bB - 6Ac) - 18Abc + 15b^2B \right)}{48c^3} - \frac{(8aAc^2 - 12abBc - 6Ab^2c + 5b^3B) \tanh^{-1} \left(\frac{bx^2}{2\sqrt{a+bx^2+cx^4}} \right)}{32c^{7/2}}$$

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4} \left(-16aBc - 2cx^2(5bB - 6Ac) - 18Abc + 15b^2B \right)}{48c^3} - \frac{(8aAc^2 - 12abBc - 6Ab^2c + 5b^3B) \tanh^{-1} \left(\frac{bx^2}{2\sqrt{a+bx^2+cx^4}} \right)}{32c^{7/2}} + \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c - 2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5 (A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (A + Bx)}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{Bx^4 \sqrt{a + bx^2 + cx^4}}{6c} + \frac{\text{Subst} \left(\int \frac{x(-2aB - \frac{1}{2}(5bB - 6Ac)x)}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{6c} \\ &= \frac{Bx^4 \sqrt{a + bx^2 + cx^4}}{6c} + \frac{(15b^2B - 18Abc - 16aBc - 2c(5bB - 6Ac)x^2) \sqrt{a + bx^2 + cx^4}}{48c^3} \\ &= \frac{Bx^4 \sqrt{a + bx^2 + cx^4}}{6c} + \frac{(15b^2B - 18Abc - 16aBc - 2c(5bB - 6Ac)x^2) \sqrt{a + bx^2 + cx^4}}{48c^3} \\ &= \frac{Bx^4 \sqrt{a + bx^2 + cx^4}}{6c} + \frac{(15b^2B - 18Abc - 16aBc - 2c(5bB - 6Ac)x^2) \sqrt{a + bx^2 + cx^4}}{48c^3} \end{aligned}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.91

$$\frac{2\sqrt{c} \sqrt{a + bx^2 + cx^4} (4c(-4aB + 3Acx^2 + 2Bcx^4) - 2bc(9A + 5Bx^2) + 15b^2B) - 3(8aAc^2 - 12abBc - 6Ab^2c + 5b^3B) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 2*b*c*(9*A + 5*B*x^2) + 4*c*(-4*a*B + 3*A*c*x^2 + 2*B*c*x^4)) - 3*(5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(96*c^(7/2))

IntegrateAlgebraic [A] time = 0.54, size = 135, normalized size = 0.88

$$\frac{\sqrt{a + bx^2 + cx^4} (-16aBc - 18Abc + 12Ac^2x^2 + 15b^2B - 10bBcx^2 + 8Bc^2x^4)}{48c^3} + \frac{(8aAc^2 - 12abBc - 6Ab^2c + 5b^3B) \log\left(-2\sqrt{c}\sqrt{a + bx^2 + cx^4} + b + 2cx^2\right)}{32c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 18*A*b*c - 16*a*B*c - 10*b*B*c*x^2 + 12*A*c^2*x^2 + 8*B*c^2*x^4))/(48*c^3) + ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]))/(32*c^(7/2))

fricas [A] time = 0.73, size = 315, normalized size = 2.06

$$\frac{3(5Bb^3 + 8Aac^2 - 6(2Bab + Ab^2))\sqrt{c} \log\left(\frac{-8c^2x^4 - 8b^2c^2x^2 - b^2 + 4\sqrt{cx^4 + bx^2 + a}(2cx^2 + b)\sqrt{c} - 4ac}{192c^4}\right) + 4(8Bb^3x^4 + 15Bb^2c - 2(8Ba + 9Ab)c^2 - 2(5Bbc^2 - 6Ac^2)x^2)\sqrt{cx^4 + bx^2 + a}}{192c^4} + \frac{3(5Bb^3 + 8Aac^2 - 6(2Bab + Ab^2))\sqrt{c} \arctan\left(\frac{\sqrt{cx^4 + bx^2 + a}}{2(c^2x^2 + ac)}\right) + 2(8Bc^2x^4 + 15Bb^2c - 2(8Ba + 9Ab)c^2 - 2(5Bbc^2 - 6Ac^2)x^2)\sqrt{cx^4 + bx^2 + a}}{96c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(cx^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(cx^4 + b*x^2 + a)/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(cx^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(cx^4 + b*x^2 + a)/c^4]

giac [A] time = 0.54, size = 138, normalized size = 0.90

$$\frac{1}{48} \sqrt{cx^4 + bx^2 + a} \left(2 \left(\frac{4Bx^2}{c} - \frac{5Bbc - 6Ac^2}{c^3} \right) x^2 + \frac{15Bb^2 - 16Bac - 18Abc}{c^3} \right) + \frac{(5Bb^3 - 12Babc - 6Ab^2c + 8Aac^2) \log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{32c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2), x, algorithm="giac")

[Out] 1/48*sqrt(cx^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 + (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A*b

$^2*c + 8*A*a*c^2)*\log(\text{abs}(-2*(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))*\text{sqrt}(c) - b))/c^{(7/2)}$

maple [B] time = 0.03, size = 286, normalized size = 1.87

$$\frac{\sqrt{cx^4+bx^2+a}Bx^4}{6c} + \frac{\sqrt{cx^4+bx^2+a}Ax^2}{4c} - \frac{5\sqrt{cx^4+bx^2+a}Bbx^2}{24c^2} - \frac{Aa\ln\left(\frac{cx^2+a}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{4c^3} + \frac{3Ab^2\ln\left(\frac{cx^2+a}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{16c^3} + \frac{3Bab\ln\left(\frac{cx^2+a}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{8c^3} - \frac{5Bb^3\ln\left(\frac{cx^2+a}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{32c^3} - \frac{3\sqrt{cx^4+bx^2+a}Ab}{8c^2} - \frac{\sqrt{cx^4+bx^2+a}Ba}{3c^2} + \frac{5\sqrt{cx^4+bx^2+a}Bb^2}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2), x)`

[Out] $\frac{1}{6}Bx^4(c^2x^4+bx^2+a)^{1/2}/c - \frac{5}{24}Bb/c^2x^2(c^2x^4+bx^2+a)^{1/2} + \frac{5}{16}Bb^2/c^3(c^2x^4+bx^2+a)^{1/2} - \frac{5}{32}Bb^3/c^{7/2} \ln\left(\frac{(1/2)b+cx^2}{c^{1/2}} + \frac{(c^2x^4+bx^2+a)^{1/2}}{c^{1/2}}\right) + \frac{3}{8}Bb/c^{5/2} \ln\left(\frac{(1/2)b+cx^2}{c^{1/2}} + \frac{(c^2x^4+bx^2+a)^{1/2}}{c^{1/2}}\right) - \frac{1}{3}Ba/c^2(c^2x^4+bx^2+a)^{1/2} + \frac{1}{4}Ax^2/c(c^2x^4+bx^2+a)^{1/2} - \frac{3}{8}Ab/c^2(c^2x^4+bx^2+a)^{1/2} + \frac{3}{16}Ab^2/c^{5/2} \ln\left(\frac{(1/2)b+cx^2}{c^{1/2}} + \frac{(c^2x^4+bx^2+a)^{1/2}}{c^{1/2}}\right) - \frac{1}{4}Aa/c^{3/2} \ln\left(\frac{(1/2)b+cx^2}{c^{1/2}} + \frac{(c^2x^4+bx^2+a)^{1/2}}{c^{1/2}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 (Bx^2 + A)}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)`

[Out] `int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

$$3.137 \quad \int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=100

$$\frac{(-4aBc - 4Abc + 3b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}} - \frac{\sqrt{a+bx^2+cx^4}(-4Ac + 3bB - 2Bcx^2)}{8c^2}$$

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1251, 779, 621, 206}

$$\frac{(-4aBc - 4Abc + 3b^2B) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}} - \frac{\sqrt{a+bx^2+cx^4}(-4Ac + 3bB - 2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]

[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((3*b^2*B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3 (A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(A + Bx)}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{(3b^2B - 4Abc - 4aBc) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{16c^2} \\ &= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{(3b^2B - 4Abc - 4aBc) \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, x^2 \right)}{8c^2} \\ &= -\frac{(3bB - 4Ac - 2Bcx^2) \sqrt{a + bx^2 + cx^4}}{8c^2} + \frac{(3b^2B - 4Abc - 4aBc) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{16c^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 1.01

$$\frac{(-4aBc - 4Abc + 3b^2B) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right) + 2\sqrt{c} \sqrt{a + bx^2 + cx^4} (4Ac - 3bB + 2Bcx^2)}{16c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (2*Sqrt[c]*(-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2*B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))

IntegrateAlgebraic [A] time = 0.47, size = 104, normalized size = 1.04

$$\frac{(4aBc + 4Abc - 3b^2B) \log \left(-2c^{5/2} \sqrt{a + bx^2 + cx^4} + bc^2 + 2c^3x^2 \right)}{16c^{5/2}} + \frac{\sqrt{a + bx^2 + cx^4} (4Ac - 3bB + 2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]

[Out] $\frac{((-3*b*B + 4*A*c + 2*B*c*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2*B + 4*A*b*c + 4*a*B*c)*\text{Log}[b*c^2 + 2*c^3*x^2 - 2*c^{(5/2)}*\text{Sqrt}[a + b*x^2 + c*x^4]])/(16*c^{(5/2)})}{}$

fricas [A] time = 1.09, size = 233, normalized size = 2.33

$$\left[\frac{(3Bb^2 - 4(Ba + Ab)c)\sqrt{c} \log\left(-8c^2x^4 - 8bcx^2 - b^2 + 4\sqrt{cx^4 + bx^2 + a}(2cx^2 + b)\sqrt{c} - 4ac\right) - 4(2Bc^2x^2 - 3Bbc + 4Ac^2)\sqrt{cx^4 + bx^2 + a} - (3Bb^2 - 4(Ba + Ab)c)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4 + bx^2 + a}(2cx^2 + b)\sqrt{-c}}{2(c^2 + bcc^2 + ac)}\right) - 2(2Bc^2x^2 - 3Bbc + 4Ac^2)\sqrt{cx^4 + bx^2 + a}}{32c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] $[-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*\text{sqrt}(c)*\log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(c) - 4*a*c) - 4*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*\text{sqrt}(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(B*a + A*b)*c)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*\text{sqrt}(c*x^4 + b*x^2 + a))/c^3]$

giac [A] time = 0.45, size = 98, normalized size = 0.98

$$\frac{1}{8} \sqrt{cx^4 + bx^2 + a} \left(\frac{2Bx^2}{c} - \frac{3Bb - 4Ac}{c^2} \right) - \frac{(3Bb^2 - 4Bac - 4Abc) \log\left(\left| -2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b \right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] $1/8*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b^2 - 4*B*a*c - 4*A*b*c)*\log(\text{abs}(-2*(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))*\text{sqrt}(c) - b))/c^{(5/2)}$

maple [B] time = 0.02, size = 176, normalized size = 1.76

$$\frac{\sqrt{cx^4 + bx^2 + a} Bx^2}{4c} - \frac{Ab \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)}{4c^{\frac{3}{2}}} - \frac{Ba \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)}{4c^{\frac{3}{2}}} + \frac{3Bb^2 \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)}{16c^{\frac{5}{2}}} + \frac{\sqrt{cx^4 + bx^2 + a} A}{2c} - \frac{3\sqrt{cx^4 + bx^2 + a} Bb}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $1/4*B*x^2/c*(c*x^4+b*x^2+a)^{(1/2)} - 3/8*B*b/c^2*(c*x^4+b*x^2+a)^{(1/2)} + 3/16*B*b^2/c^{(5/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}) - 1/4*B*a/c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}) + 1/2*A/c*(c*x^4+b*x^2+a)^{(1/2)} - 1/4*A*b/c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 (Bx^2 + A)}{\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)

[Out] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)

$$3.138 \quad \int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1247, 640, 621, 206}

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]

[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{B\sqrt{a + bx^2 + cx^4}}{2c} + \frac{(-bB + 2Ac) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{4c} \\ &= \frac{B\sqrt{a + bx^2 + cx^4}}{2c} + \frac{(-bB + 2Ac) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^2}{\sqrt{a + bx^2 + cx^4}} \right)}{2c} \\ &= \frac{B\sqrt{a + bx^2 + cx^4}}{2c} - \frac{(bB - 2Ac) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{4c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.03

$$\frac{1}{2} \left(\frac{(2Ac - bB) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{2c^{3/2}} + \frac{B\sqrt{a + bx^2 + cx^4}}{c} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]
```

```
[Out] ((B*Sqrt[a + b*x^2 + c*x^4])/c + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

IntegrateAlgebraic [A] time = 0.38, size = 78, normalized size = 1.03

$$\frac{(bB - 2Ac) \log \left(-2c^{3/2} \sqrt{a + bx^2 + cx^4} + bc + 2c^2 x^2 \right)}{4c^{3/2}} + \frac{B\sqrt{a + bx^2 + cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4], x]
```


[Out] $(B\sqrt{a + bx^2 + cx^4})/(2c) + ((bB - 2Ac)\text{Log}[bc + 2c^2x^2 - 2c^{(3/2)}\sqrt{a + bx^2 + cx^4}])/(4c^{(3/2)})$

fricas [A] time = 1.30, size = 178, normalized size = 2.34

$$\left[\frac{4\sqrt{cx^4 + bx^2 + a}Bc - (Bb - 2Ac)\sqrt{c} \log\left(\frac{-8c^2x^4 - 8bcx^2 - b^2 - 4\sqrt{cx^4 + bx^2 + a}(2cx^2 + b)\sqrt{c} - 4ac}{8c^2}\right), \frac{2\sqrt{cx^4 + bx^2 + a}Bc + (Bb - 2Ac)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4 + bx^2 + a}(2cx^2 + b)\sqrt{-c}}{2(c^2x^4 + bcx^2 + ac)}\right)}{4c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")`

[Out] $[1/8*(4*\text{sqrt}(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*\text{sqrt}(c)*\log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(c) - 4*a*c))/c^2, 1/4*(2*\text{sqrt}(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/c^2]$

giac [A] time = 0.46, size = 69, normalized size = 0.91

$$\frac{\sqrt{cx^4 + bx^2 + a}B}{2c} + \frac{(Bb - 2Ac) \log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")`

[Out] $1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*\log(\text{abs}(-2*(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))*\text{sqrt}(c) - b))/c^{(3/2)}$

maple [A] time = 0.01, size = 93, normalized size = 1.22

$$\frac{A \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)}{2\sqrt{c}} - \frac{Bb \ln\left(\frac{cx^2 + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)}{4c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2 + a}B}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)`

[Out] $1/2*B*(c*x^4+b*x^2+a)^{(1/2)}/c - 1/4*B*b/c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}) + 1/2*A*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})/c^{(1/2)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.05, size = 92, normalized size = 1.21

$$\frac{A \ln\left(\sqrt{cx^4 + bx^2 + a} + \frac{cx^2 + \frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}} + \frac{B\sqrt{cx^4 + bx^2 + a}}{2c} - \frac{Bb \ln\left(\sqrt{cx^4 + bx^2 + a} + \frac{cx^2 + \frac{b}{2}}{\sqrt{c}}\right)}{4c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)

[Out] (A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (B*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)

$$3.139 \quad \int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=90

$$\frac{B \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}} - \frac{A \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 843, 621, 206, 724}

$$\frac{B \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}} - \frac{A \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]) + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)
^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} A \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^2 \right) + \frac{1}{2} B \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= - \left(A \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^2}{\sqrt{a + bx^2 + cx^4}} \right) \right) + B \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^2}{\sqrt{a + bx^2 + cx^4}} \right) \\ &= - \frac{A \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{a}} + \frac{B \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 89, normalized size = 0.99

$$\frac{1}{2} \left(\frac{B \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{\sqrt{c}} - \frac{A \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/Sqrt[a])
+ (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/Sqrt[c])/2
```

IntegrateAlgebraic [A] time = 0.33, size = 89, normalized size = 0.99

$$\frac{A \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{B \log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4} + b + 2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (A*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/Sqrt[a] - (B*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])

fricas [A] time = 1.25, size = 517, normalized size = 5.74

$$\frac{B\sqrt{c}\log\left(\frac{8x^4+8bx^2-4\sqrt{c}\sqrt{a+bx^2+cx^4}(2c^2+4a)}{4ac}\right) + A\sqrt{c}\log\left(\frac{(b^2+4ac)\sqrt{a+bx^2+cx^4} + 2bx^2}{4ac}\right) - A\sqrt{c}\log\left(\frac{(b^2+4ac)\sqrt{a+bx^2+cx^4} - 2bx^2}{4ac}\right) + 2A\sqrt{c}\arctan\left(\frac{(b^2+4ac)\sqrt{a+bx^2+cx^4}}{2(b^2+4ac)x}\right) + B\sqrt{c}\log\left(\frac{8x^4+8bx^2-4\sqrt{c}\sqrt{a+bx^2+cx^4}(2c^2+4a)}{4ac}\right) + A\sqrt{c}\arctan\left(\frac{(b^2+4ac)\sqrt{a+bx^2+cx^4}}{2(b^2+4ac)x}\right) - B\sqrt{c}\arctan\left(\frac{(b^2+4ac)\sqrt{a+bx^2+cx^4}}{2(b^2+4ac)x}\right)}{2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.01, size = 76, normalized size = 0.84

$$-\frac{A \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{2\sqrt{a}} + \frac{B \ln\left(\frac{cx^2+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/2*B*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 0.76, size = 81, normalized size = 0.90

$$\frac{B \ln\left(\sqrt{cx^4+bx^2+a} + \frac{cx^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}} - \frac{A \ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{A \ln\left(2a + 2\sqrt{a}\sqrt{cx^4+bx^2+a} + bx^2\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] (B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (A*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)

$$3.140 \quad \int \frac{A+Bx^2}{x^3 \sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=80

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1251, 806, 724, 206}

$$\frac{(Ab - 2aB) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 1251

$\text{Int}[(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(q_.)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] :> \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p}, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m - 1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^3 \sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{2ax^2} - \frac{(Ab - 2aB) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4a} \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{2ax^2} + \frac{(Ab - 2aB) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}} \right)}{2a} \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{2ax^2} + \frac{(Ab - 2aB) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{4a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.02

$$\frac{1}{2} \left(\frac{(Ab - 2aB) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2a^{3/2}} - \frac{A\sqrt{a + bx^2 + cx^4}}{ax^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2

IntegrateAlgebraic [A] time = 0.39, size = 81, normalized size = 1.01

$$\frac{(2aB - Ab) \tanh^{-1} \left(\frac{\sqrt{c}x^2 - \sqrt{a+bx^2+cx^4}}{\sqrt{a}} \right)}{2a^{3/2}} - \frac{A\sqrt{a + bx^2 + cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] $-1/2*(A*\text{Sqrt}[a + b*x^2 + c*x^4])/(a*x^2) + ((-(A*b) + 2*a*B)*\text{ArcTanh}[(\text{Sqrt}[c]*x^2 - \text{Sqrt}[a + b*x^2 + c*x^4])/\text{Sqrt}[a]])/(2*a^(3/2))$

fricas [A] time = 1.33, size = 197, normalized size = 2.46

$$\left[\frac{(2Ba - Ab)\sqrt{a}x^2 \log\left(-\frac{(b^2+4ac)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a+8a^2}}{x^4}\right) + 4\sqrt{cx^4+bx^2+a}Aa(2Ba - Ab)\sqrt{-a}x^2 \arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right) - 2\sqrt{cx^4+bx^2+a}Aa}{8a^2x^2}, \frac{(2Ba - Ab)\sqrt{-a}x^2 \arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right) - 2\sqrt{cx^4+bx^2+a}Aa}{4a^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] $[-1/8*((2*B*a - A*b)*\text{sqrt}(a)*x^2*\log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*\text{sqrt}(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a))*\text{sqrt}(a) + 8*a^2)/x^4) + 4*\text{sqrt}(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*\text{sqrt}(-a)*x^2*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a))*\text{sqrt}(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*\text{sqrt}(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]$

giac [A] time = 0.54, size = 124, normalized size = 1.55

$$\frac{(2Ba - Ab) \arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)Ab + 2Aa\sqrt{c}}{2\left(\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] $1/2*(2*B*a - A*b)*\arctan(-(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))/\text{sqrt}(-a))/(\text{sqrt}(-a)*a) + 1/2*((\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))*A*b + 2*A*a*\text{sqrt}(c))/(((\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))^2 - a)*a)$

maple [A] time = 0.02, size = 104, normalized size = 1.30

$$\frac{Ab \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}} - \frac{B \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{2\sqrt{a}} - \frac{\sqrt{cx^4+bx^2+a}A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $-1/2*B/a^{(1/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2)-1/2*A*(c*x^4+b*x^2+a)^{(1/2)}/a/x^2+1/4*A*b/a^{(3/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 0.78, size = 103, normalized size = 1.29

$$\frac{A b \operatorname{atanh}\left(\frac{\frac{b x^2}{2}+a}{\sqrt{a} \sqrt{c x^4+b x^2+a}}\right)}{4 a^{3/2}} - \frac{B \ln\left(2 a+2 \sqrt{a} \sqrt{c x^4+b x^2+a}+b x^2\right)}{2 \sqrt{a}} - \frac{A \sqrt{c x^4+b x^2+a}}{2 a x^2} - \frac{B \ln\left(\frac{1}{x^2}\right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)`

[Out] $(A*b*\operatorname{atanh}((a + (b*x^2)/2)/(a^{(1/2)}*(a + b*x^2 + c*x^4)^{(1/2)}))/(4*a^{(3/2)}) - (B*\log(2*a + 2*a^{(1/2)}*(a + b*x^2 + c*x^4)^{(1/2)} + b*x^2))/(2*a^{(1/2)}) - (A*(a + b*x^2 + c*x^4)^{(1/2)})/(2*a*x^2) - (B*\log(1/x^2))/(2*a^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3 \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)`

[Out] `Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)`

$$3.141 \quad \int \frac{A+Bx^2}{x^5 \sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=124

$$\frac{(-4aAc - 4abB + 3Ab^2) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{(3Ab - 4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 834, 806, 724, 206}

$$\frac{(-4aAc - 4abB + 3Ab^2) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}} + \frac{(3Ab - 4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 834

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{A + Bx^2}{x^5 \sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^3 \sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{4ax^4} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(3Ab - 4aB) + Acx}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{4a} \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{4ax^4} + \frac{(3Ab - 4aB)\sqrt{a + bx^2 + cx^4}}{8a^2x^2} + \frac{(3Ab^2 - 4abB - 4aAc) \text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{16a^2} \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{4ax^4} + \frac{(3Ab - 4aB)\sqrt{a + bx^2 + cx^4}}{8a^2x^2} - \frac{(3Ab^2 - 4abB - 4aAc) \text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{8a^2} \\ &= -\frac{A\sqrt{a + bx^2 + cx^4}}{4ax^4} + \frac{(3Ab - 4aB)\sqrt{a + bx^2 + cx^4}}{8a^2x^2} - \frac{(3Ab^2 - 4abB - 4aAc) \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{16a^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.86

$$\frac{(4aAc + 4abB - 3Ab^2) \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{16a^{5/2}} + \frac{\sqrt{a + bx^2 + cx^4} (3Abx^2 - 2a(A + 2Bx^2))}{8a^2x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)))/(8*a^2*x^4) + ((-3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))

IntegrateAlgebraic [A] time = 0.64, size = 148, normalized size = 1.19

$$\frac{3Ab^2 \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{(Ac + bB) \tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\sqrt{a+bx^2+cx^4}(-2aA - 4aBx^2 + 3Abx^2)}{8a^2x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-2*a*A + 3*A*b*x^2 - 4*a*B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((b*B + A*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2)) + (3*A*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(8*a^(5/2))

fricas [A] time = 1.59, size = 255, normalized size = 2.06

$$\frac{(4Bab - 3Ab^2 + 4Aac)\sqrt{a}x^4 \log\left(\frac{(b^2+4a)x^4+8abx^2+4\sqrt{c}x^2+\sqrt{a+bx^2+cx^4}(bx^2+2a)\sqrt{a+8a^2}}{x^4}\right) - 4\sqrt{cx^4+bx^2+a}(2Aa^2+(4Ba^2-3Aab)x^2)}{32a^3x^4} + \frac{(4Bab - 3Ab^2 + 4Aac)\sqrt{-a}x^4 \arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right) + 2\sqrt{cx^4+bx^2+a}(2Aa^2+(4Ba^2-3Aab)x^2)}{16a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4), -1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]

giac [B] time = 0.52, size = 339, normalized size = 2.73

$$\frac{(4Bab - 3Ab^2 + 4Aac)\arctan\left(\frac{\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{a}}\right)}{8\sqrt{-a}a^2} + \frac{4\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Bab - 3\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Ab^2 + 4\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Aac + 8\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 Bb^2\sqrt{c} - 4\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)Bb^2b + 5\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)Ab^2 + 4\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)Aa^2\sqrt{c} + 8Aa^2b\sqrt{c}}{8\left(\left(\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}\right)^2 - a\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] -1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)

$$\begin{aligned} & + a))^3 B a b - 3(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a})^3 A b^2 + 4(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a})^3 A a c + 8(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a})^2 B a^2 \sqrt{c} - 4(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a}) B a^2 b + 5(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a}) A a b^2 + 4(\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a}) A a^2 c - 8 B a^3 \sqrt{c} + 8 A a^2 b \sqrt{c} / \\ & ((\sqrt{c} x^2 - \sqrt{c x^4 + b x^2 + a})^2 - a)^2 a^2 \end{aligned}$$

maple [A] time = 0.02, size = 194, normalized size = 1.56

$$\frac{A c \ln\left(\frac{b x^2 + 2 a + 2 \sqrt{c x^4 + b x^2 + a} \sqrt{a}}{x^2}\right)}{4 a^{\frac{3}{2}}} - \frac{3 A b^2 \ln\left(\frac{b x^2 + 2 a + 2 \sqrt{c x^4 + b x^2 + a} \sqrt{a}}{x^2}\right)}{16 a^{\frac{5}{2}}} + \frac{B b \ln\left(\frac{b x^2 + 2 a + 2 \sqrt{c x^4 + b x^2 + a} \sqrt{a}}{x^2}\right)}{4 a^{\frac{3}{2}}} + \frac{3 \sqrt{c x^4 + b x^2 + a} A b}{8 a^2 x^2} - \frac{\sqrt{c x^4 + b x^2 + a} B}{2 a x^2} - \frac{\sqrt{c x^4 + b x^2 + a} A}{4 a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $-1/4 A (c x^4 + b x^2 + a)^{1/2} / a x^4 + 3/8 A b / a^2 x^2 (c x^4 + b x^2 + a)^{1/2} - 3/16 A b^2 / a^{5/2} \ln((b x^2 + 2 a + 2 (c x^4 + b x^2 + a)^{1/2} a^{1/2}) / x^2) + 1/4 A c / a^{3/2} \ln((b x^2 + 2 a + 2 (c x^4 + b x^2 + a)^{1/2} a^{1/2}) / x^2) - 1/2 B / a x^2 (c x^4 + b x^2 + a)^{1/2} + 1/4 B b / a^{3/2} \ln((b x^2 + 2 a + 2 (c x^4 + b x^2 + a)^{1/2} a^{1/2}) / x^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B x^2 + A}{x^5 \sqrt{c x^4 + b x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5 \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)

$$3.142 \quad \int \frac{A+Bx^2}{x^7 \sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=177

$$-\frac{\sqrt{a+bx^2+cx^4}(-16aAc-18abB+15Ab^2)}{48a^3x^2} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{(8a^2Bc-12aAbc-6ab^2B+5Ab^3)}{32a^{7/2}}$$

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx^2+cx^4}(-16aAc-18abB+15Ab^2)}{48a^3x^2} + \frac{(8a^2Bc-12aAbc-6ab^2B+5Ab^3) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^7*sqrt[a + b*x^2 + c*x^4]), x]

[Out] -(A*sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*sqrt[a + b*x^2 + c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 834

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx^2}{x^7 \sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{A + Bx}{x^4 \sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\
 &= -\frac{A\sqrt{a + bx^2 + cx^4}}{6ax^6} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(5Ab - 6aB) + 2Acx}{x^3 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{6a} \\
 &= -\frac{A\sqrt{a + bx^2 + cx^4}}{6ax^6} + \frac{(5Ab - 6aB)\sqrt{a + bx^2 + cx^4}}{24a^2x^4} + \frac{\text{Subst} \left(\int \frac{\frac{1}{4}(15Ab^2 - 18abB - 16aAc)}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{12a^2} \\
 &= -\frac{A\sqrt{a + bx^2 + cx^4}}{6ax^6} + \frac{(5Ab - 6aB)\sqrt{a + bx^2 + cx^4}}{24a^2x^4} - \frac{(15Ab^2 - 18abB - 16aAc)\sqrt{a + bx^2 + cx^4}}{48a^3x^2} \\
 &= -\frac{A\sqrt{a + bx^2 + cx^4}}{6ax^6} + \frac{(5Ab - 6aB)\sqrt{a + bx^2 + cx^4}}{24a^2x^4} - \frac{(15Ab^2 - 18abB - 16aAc)\sqrt{a + bx^2 + cx^4}}{48a^3x^2} \\
 &= -\frac{A\sqrt{a + bx^2 + cx^4}}{6ax^6} + \frac{(5Ab - 6aB)\sqrt{a + bx^2 + cx^4}}{24a^2x^4} - \frac{(15Ab^2 - 18abB - 16aAc)\sqrt{a + bx^2 + cx^4}}{48a^3x^2}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 148, normalized size = 0.84

$$\frac{(8a^2Bc - 12aAbc - 6ab^2B + 5Ab^3) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \sqrt{a+bx^2+cx^4}(-4a^2(2A+3Bx^2) + 2a(5Abx^2 + 8Acx^4 + 9bBx^4) - 15Ab^2x^4)}{32a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}(-4a^2(2A+3Bx^2) + 2a(5Abx^2 + 8Acx^4 + 9bBx^4) - 15Ab^2x^4)}{48a^3x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^7*sqrt[a + b*x^2 + c*x^4]), x]

[Out] (sqrt[a + b*x^2 + c*x^4]*(-15*A*b^2*x^4 - 4*a^2*(2*A + 3*B*x^2) + 2*a*(5*A*b*x^2 + 9*b*B*x^4 + 8*A*c*x^4)))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))

IntegrateAlgebraic [A] time = 0.91, size = 188, normalized size = 1.06

$$-\frac{3(2Abc + b^2B) \tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{(-8a^2Bc - 5Ab^3) \tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}} + \frac{\sqrt{a+bx^2+cx^4}(-8a^2A - 12a^2Bx^2 + 10aAbx^2 + 16aAcx^4 + 18abBx^4 - 15Ab^2x^4)}{48a^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*sqrt[a + b*x^2 + c*x^4]), x]

[Out] (sqrt[a + b*x^2 + c*x^4]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2*x^4 + 18*a*b*B*x^4 + 16*a*A*c*x^4))/(48*a^3*x^6) + (((-5*A*b^3 - 8*a^2*B*c)*ArcTanh[(sqrt[c]*x^2 - sqrt[a + b*x^2 + c*x^4])/sqrt[a]])/(16*a^(7/2)) - (3*(b^2*B + 2*A*b*c)*ArcTanh[(-sqrt[c]*x^2) + sqrt[a + b*x^2 + c*x^4])/sqrt[a]])/(8*a^(5/2))

fricas [A] time = 1.88, size = 339, normalized size = 1.92

$$\frac{3(6Ba^2b - 5Aa^3 - 4(2Ba^2 - 3Aab))\sqrt{a} \log\left(\frac{(a+bx^2+cx^4)\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{a}\right) + 4((18Ba^2b - 15Aa^3 + 16Aa^2c)x^4 - 8Aa^3 - 2(6Ba^2 - 5Aa^2b)c)\sqrt{a+bx^2+cx^4}}{192a^4} + \frac{3(6Ba^2b - 5Aa^3 - 4(2Ba^2 - 3Aab))\sqrt{-a} \arctan\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{2(a^2+bx^2+cx^4)}\right) + 2((18Ba^2b - 15Aa^3 + 16Aa^2c)x^4 - 8Aa^3 - 2(6Ba^2 - 5Aa^2b)c)\sqrt{a+bx^2+cx^4}}{96a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a)/(a^4*x^6), 1/96*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a)/(a^4*x^6)]

giac [B] time = 0.64, size = 571, normalized size = 3.23

$$\frac{3(6Ba^2b - 5Aa^3 - 4(2Ba^2 - 3Aab))\sqrt{a} \log\left(\frac{(a+bx^2+cx^4)\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{a}\right) + 4((18Ba^2b - 15Aa^3 + 16Aa^2c)x^4 - 8Aa^3 - 2(6Ba^2 - 5Aa^2b)c)\sqrt{a+bx^2+cx^4}}{192a^4} + \frac{3(6Ba^2b - 5Aa^3 - 4(2Ba^2 - 3Aab))\sqrt{-a} \arctan\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{2(a^2+bx^2+cx^4)}\right) + 2((18Ba^2b - 15Aa^3 + 16Aa^2c)x^4 - 8Aa^3 - 2(6Ba^2 - 5Aa^2b)c)\sqrt{a+bx^2+cx^4}}{96a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{16}*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*\arctan(-(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})/\sqrt{-a})/(\sqrt{-a})a^3) - \frac{1}{48}*(18*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^5*B*a*b^2 - 15*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^5*A*b^3 - 24*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^5*B*a^2*c + 36*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^5*A*a*b*c - 48*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^3*B*a^2*b^2 + 40*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^3*A*a*b^3 - 96*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^3*A*a^2*b*c - 48*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^2*B*a^3*b*\sqrt{c} - 96*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^2*A*a^3*c^(3/2) + 30*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})*B*a^3*b^2 - 33*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})*A*a^2*b^3 + 24*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})*B*a^4*c - 36*(\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})*A*a^3*b*c + 48*B*a^4*b*\sqrt{c} - 48*A*a^3*b^2*\sqrt{c} + 32*A*a^4*c^(3/2))/(((\sqrt{c})x^2 - \sqrt{c*x^4 + b*x^2 + a})^2 - a)^3*a^3)$

maple [A] time = 0.02, size = 311, normalized size = 1.76

$$-\frac{3Abc \ln\left(\frac{bx^2+2a\sqrt{c^2+bx^2+a}\sqrt{c}}{a^2}\right)}{8a^2} + \frac{5Ab^3 \ln\left(\frac{bx^2+2a\sqrt{c^2+bx^2+a}\sqrt{c}}{a^2}\right)}{32a^2} + \frac{Bc \ln\left(\frac{bx^2+2a\sqrt{c^2+bx^2+a}\sqrt{c}}{a^2}\right)}{4a^2} - \frac{3Bb^2 \ln\left(\frac{bx^2+2a\sqrt{c^2+bx^2+a}\sqrt{c}}{a^2}\right)}{16a^2} + \frac{\sqrt{cx^4+bx^2+a}Ac}{3a^2c^2} - \frac{5\sqrt{cx^4+bx^2+a}Ab^2}{16a^3c^2} + \frac{3\sqrt{cx^4+bx^2+a}Bb}{8a^2c^2} + \frac{5\sqrt{cx^4+bx^2+a}Ab}{24a^2c^2} - \frac{\sqrt{cx^4+bx^2+a}B}{4a^2c} - \frac{\sqrt{cx^4+bx^2+a}A}{6a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $-\frac{1}{6}A*(c*x^4+b*x^2+a)^(1/2)/a/x^6 + \frac{5}{24}A*b/a^2/x^4*(c*x^4+b*x^2+a)^(1/2) - \frac{5}{16}A*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2) + \frac{5}{32}A*b^3/a^(7/2)*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2) - \frac{3}{8}A*b/a^(5/2)*c*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2) + \frac{1}{3}A/a^2*c/x^2*(c*x^4+b*x^2+a)^(1/2) - \frac{1}{4}B/a/x^4*(c*x^4+b*x^2+a)^(1/2) + \frac{3}{8}B*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2) - \frac{3}{16}B*b^2/a^(5/2)*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2) + \frac{1}{4}B*c/a^(3/2)*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Bx^2 + A}{x^7 \sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)

[Out] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^7 \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2), x)

[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)

$$3.143 \quad \int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=98

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4 - \frac{1}{384}(24243-3802x^2)\sqrt{x^4+5x^2+3} + \frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) + \frac{3}{8}\sqrt{x^4+5x^2+3}$$

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 832, 779, 621, 206}

$$\frac{3}{8}\sqrt{x^4+5x^2+3}x^6 - \frac{89}{48}\sqrt{x^4+5x^2+3}x^4 - \frac{1}{384}(24243-3802x^2)\sqrt{x^4+5x^2+3} + \frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/256

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^p, x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 832

```
Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_
)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3(2+3x)}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= \frac{3}{8} x^6 \sqrt{3+5x^2+x^4} + \frac{1}{8} \text{Subst} \left(\int \frac{\left(-27 - \frac{89x}{2}\right) x^2}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{89}{48} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{8} x^6 \sqrt{3+5x^2+x^4} + \frac{1}{24} \text{Subst} \left(\int \frac{x \left(267 + \frac{1901x}{4}\right)}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{89}{48} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{8} x^6 \sqrt{3+5x^2+x^4} - \frac{1}{384} (24243 - 3802x^2) \sqrt{3+5x^2+x^4} + \frac{3}{2} \sqrt{3+5x^2+x^4} \\
&= -\frac{89}{48} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{8} x^6 \sqrt{3+5x^2+x^4} - \frac{1}{384} (24243 - 3802x^2) \sqrt{3+5x^2+x^4} + \frac{3}{2} \sqrt{3+5x^2+x^4} \\
&= -\frac{89}{48} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{8} x^6 \sqrt{3+5x^2+x^4} - \frac{1}{384} (24243 - 3802x^2) \sqrt{3+5x^2+x^4} + \frac{3}{2} \sqrt{3+5x^2+x^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.67

$$\frac{1}{768} \left(98403 \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) + 2\sqrt{x^4 + 5x^2 + 3} (144x^6 - 712x^4 + 3802x^2 - 24243) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6) + 98403*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768

IntegrateAlgebraic [A] time = 0.20, size = 64, normalized size = 0.65

$$\frac{1}{384} \sqrt{x^4 + 5x^2 + 3} (144x^6 - 712x^4 + 3802x^2 - 24243) - \frac{32801}{256} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6))/384 - (32801*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256

fricas [A] time = 0.62, size = 56, normalized size = 0.57

$$\frac{1}{384} (144x^6 - 712x^4 + 3802x^2 - 24243) \sqrt{x^4 + 5x^2 + 3} - \frac{32801}{256} \log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2), x, algorithm="fricas")

[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.37, size = 60, normalized size = 0.61

$$\frac{1}{384} \sqrt{x^4 + 5x^2 + 3} (2(4(18x^2 - 89)x^2 + 1901)x^2 - 24243) - \frac{32801}{256} \log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2), x, algorithm="giac")

[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) - 32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 87, normalized size = 0.89

$$\frac{3\sqrt{x^4 + 5x^2 + 3} x^6}{8} - \frac{89\sqrt{x^4 + 5x^2 + 3} x^4}{48} + \frac{1901\sqrt{x^4 + 5x^2 + 3} x^2}{192} + \frac{32801 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{256} - \frac{8081\sqrt{x^4 + 5x^2 + 3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2), x)

[Out] $\frac{3}{8}(x^4+5x^2+3)^{(1/2)}x^6-89/48*(x^4+5*x^2+3)^{(1/2)}*x^4+1901/192*(x^4+5*x^2+3)^{(1/2)}*x^2-8081/128*(x^4+5*x^2+3)^{(1/2)}+32801/256*\ln(x^2+5/2+(x^4+5*x^2+3)^{(1/2)})$

maxima [A] time = 0.93, size = 90, normalized size = 0.92

$$\frac{3}{8}\sqrt{x^4+5x^2+3}x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4+\frac{1901}{192}\sqrt{x^4+5x^2+3}x^2-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] $\frac{3}{8}\sqrt{x^4+5x^2+3}x^6-89/48*\sqrt{x^4+5x^2+3}x^4+1901/192*\sqrt{x^4+5x^2+3}x^2-8081/128*\sqrt{x^4+5x^2+3}+32801/256*\log(2*x^2+2*\sqrt{x^4+5x^2+3}+5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7(3x^2+2)}{\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(3*x^2+2))/(5*x^2+x^4+3)^(1/2),x)

[Out] int((x^7*(3*x^2+2))/(5*x^2+x^4+3)^(1/2),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(3x^2+2)}{\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)

[Out] Integral(x**7*(3*x**2+2)/sqrt(x**4+5*x**2+3),x)

$$3.144 \quad \int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=77

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4 + \frac{3}{16}(89-14x^2)\sqrt{x^4+5x^2+3} - \frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 832, 779, 621, 206}

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4 + \frac{3}{16}(89-14x^2)\sqrt{x^4+5x^2+3} - \frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} x^4 \sqrt{3+5x^2+x^4} + \frac{1}{6} \text{Subst} \left(\int \frac{\left(-18 - \frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{16} (89-14x^2) \sqrt{3+5x^2+x^4} - \frac{1083}{32} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{16} (89-14x^2) \sqrt{3+5x^2+x^4} - \frac{1083}{16} \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5}{\sqrt{3+5x+x^2}} \right) \\ &= \frac{1}{2} x^4 \sqrt{3+5x^2+x^4} + \frac{3}{16} (89-14x^2) \sqrt{3+5x^2+x^4} - \frac{1083}{32} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.79

$$\frac{1}{32} \left(2\sqrt{x^4+5x^2+3} (8x^4-42x^2+267) - 1083 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]
```

[Out] $(2\sqrt{3 + 5x^2 + x^4})(267 - 42x^2 + 8x^4) - 1083\text{ArcTanh}[(5 + 2x^2)/(2\sqrt{3 + 5x^2 + x^4})])/32$

IntegrateAlgebraic [A] time = 0.18, size = 59, normalized size = 0.77

$$\frac{1}{16}\sqrt{x^4 + 5x^2 + 3}(8x^4 - 42x^2 + 267) + \frac{1083}{32}\log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]

[Out] $(\sqrt{3 + 5x^2 + x^4})(267 - 42x^2 + 8x^4)/16 + (1083\text{Log}[-5 - 2x^2 + 2\sqrt{3 + 5x^2 + x^4}])/32$

fricas [A] time = 0.97, size = 51, normalized size = 0.66

$$\frac{1}{16}(8x^4 - 42x^2 + 267)\sqrt{x^4 + 5x^2 + 3} + \frac{1083}{32}\log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] $1/16*(8*x^4 - 42*x^2 + 267)*\text{sqrt}(x^4 + 5*x^2 + 3) + 1083/32*\log(-2*x^2 + 2*\text{sqrt}(x^4 + 5*x^2 + 3) - 5)$

giac [A] time = 0.34, size = 53, normalized size = 0.69

$$\frac{1}{16}\sqrt{x^4 + 5x^2 + 3}(2(4x^2 - 21)x^2 + 267) + \frac{1083}{32}\log(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] $1/16*\text{sqrt}(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*\log(2*x^2 - 2*\text{sqrt}(x^4 + 5*x^2 + 3) + 5)$

maple [A] time = 0.01, size = 70, normalized size = 0.91

$$\frac{\sqrt{x^4 + 5x^2 + 3} x^4}{2} - \frac{21\sqrt{x^4 + 5x^2 + 3} x^2}{8} - \frac{1083 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{32} + \frac{267\sqrt{x^4 + 5x^2 + 3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)

[Out] $\frac{1}{2}(x^4+5x^2+3)^{1/2}x^4-21/8(x^4+5x^2+3)^{1/2}x^2+267/16(x^4+5x^2+3)^{1/2}-1083/32\ln(x^2+5/2+(x^4+5x^2+3)^{1/2})$

maxima [A] time = 0.99, size = 73, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4-\frac{21}{8}\sqrt{x^4+5x^2+3}x^2+\frac{267}{16}\sqrt{x^4+5x^2+3}-\frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2}\sqrt{x^4+5x^2+3}x^4-21/8\sqrt{x^4+5x^2+3}x^2+267/16\sqrt{x^4+5x^2+3}-1083/32\log(2x^2+2\sqrt{x^4+5x^2+3}+5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5(3x^2+2)}{\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(3*x^2+2))/(5*x^2+x^4+3)^(1/2),x)`

[Out] `int((x^5*(3*x^2+2))/(5*x^2+x^4+3)^(1/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(3x^2+2)}{\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)`

[Out] `Integral(x**5*(3*x**2+2)/sqrt(x**4+5*x**2+3),x)`

$$3.145 \quad \int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=56

$$\frac{149}{16} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - \frac{1}{8} (37 - 6x^2) \sqrt{x^4 + 5x^2 + 3}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1251, 779, 621, 206}

$$\frac{149}{16} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - \frac{1}{8} (37 - 6x^2) \sqrt{x^4 + 5x^2 + 3}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/16

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= -\frac{1}{8} (37-6x^2) \sqrt{3+5x^2+x^4} + \frac{149}{16} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= -\frac{1}{8} (37-6x^2) \sqrt{3+5x^2+x^4} + \frac{149}{8} \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}} \right) \\ &= -\frac{1}{8} (37-6x^2) \sqrt{3+5x^2+x^4} + \frac{149}{16} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$\frac{1}{16} \left(2\sqrt{x^4+5x^2+3} (6x^2-37) + 149 \tanh^{-1} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(2+3*x^2))/Sqrt[3+5*x^2+x^4],x]

[Out] (2*(-37+6*x^2)*Sqrt[3+5*x^2+x^4]+149*ArcTanh[(5+2*x^2)/(2*Sqrt[3+5*x^2+x^4]])/16

IntegrateAlgebraic [A] time = 0.18, size = 54, normalized size = 0.96

$$\frac{1}{8} (6x^2-37) \sqrt{x^4+5x^2+3} - \frac{149}{16} \log(-2x^2+2\sqrt{x^4+5x^2+3}-5)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(2+3*x^2))/Sqrt[3+5*x^2+x^4],x]

[Out] ((-37+6*x^2)*Sqrt[3+5*x^2+x^4])/8 - (149*Log[-5-2*x^2+2*Sqrt[3+5*x^2+x^4]])/16

fricas [A] time = 0.98, size = 46, normalized size = 0.82

$$\frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (6x^2 - 37) - \frac{149}{16} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.35, size = 46, normalized size = 0.82

$$\frac{1}{8} \sqrt{x^4 + 5x^2 + 3} (6x^2 - 37) - \frac{149}{16} \log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.01, size = 53, normalized size = 0.95

$$\frac{3\sqrt{x^4 + 5x^2 + 3} x^2}{4} + \frac{149 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{16} - \frac{37\sqrt{x^4 + 5x^2 + 3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)

[Out] 3/4*(x^4+5*x^2+3)^(1/2)*x^2-37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 1.04, size = 56, normalized size = 1.00

$$\frac{3}{4} \sqrt{x^4 + 5x^2 + 3} x^2 - \frac{37}{8} \sqrt{x^4 + 5x^2 + 3} + \frac{149}{16} \log\left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 (3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)`

[Out] `int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2), x)`

[Out] `Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)`

$$3.146 \quad \int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=49

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} - \frac{11}{4} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1247, 640, 621, 206}

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} - \frac{11}{4} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])))/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+3x}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= \frac{3}{2} \sqrt{3+5x^2+x^4} - \frac{11}{4} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= \frac{3}{2} \sqrt{3+5x^2+x^4} - \frac{11}{2} \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}} \right) \\ &= \frac{3}{2} \sqrt{3+5x^2+x^4} - \frac{11}{4} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} - \frac{11}{4} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4

IntegrateAlgebraic [A] time = 0.17, size = 47, normalized size = 0.96

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} + \frac{11}{4} \log \left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4], x]

[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 + (11*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4

fricas [A] time = 0.75, size = 39, normalized size = 0.80

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} + \frac{11}{4} \log \left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.47, size = 39, normalized size = 0.80

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} + \frac{11}{4} \log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.01, size = 36, normalized size = 0.73

$$-\frac{11 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{4} + \frac{3\sqrt{x^4 + 5x^2 + 3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)

[Out] 3/2*(x^4+5*x^2+3)^(1/2)-11/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

maxima [A] time = 0.92, size = 39, normalized size = 0.80

$$\frac{3}{2} \sqrt{x^4 + 5x^2 + 3} - \frac{11}{4} \log\left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [B] time = 0.52, size = 35, normalized size = 0.71

$$\frac{3\sqrt{x^4 + 5x^2 + 3}}{2} - \frac{11 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)

[Out] $(3*(5*x^2 + x^4 + 3)^{(1/2)})/2 - (11*\log((5*x^2 + x^4 + 3)^{(1/2)} + x^2 + 5/2))/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)

[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)

$$3.147 \quad \int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=69

$$\frac{3}{2} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 843, 621, 206, 724}

$$\frac{3}{2} \tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]), x]

[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/2 - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +

$c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 1251

$\text{Int}[(x_)^{(m_.)*((d_) + (e_.)*(x_)^2)^{(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] :> \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m - 1)/2}*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x\} \&\& \text{IntegerQ}[(m - 1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{2 + 3x^2}{x\sqrt{3 + 5x^2 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\ &= \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) + \text{Subst} \left(\int \frac{1}{x\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\ &= - \left(2 \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{6 + 5x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \right) + 3 \text{Subst} \left(\int \frac{1}{4 - x^2} dx, x, \frac{5 + 2x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \\ &= \frac{3}{2} \tanh^{-1} \left(\frac{5 + 2x^2}{2\sqrt{3 + 5x^2 + x^4}} \right) - \frac{\tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3}\sqrt{3 + 5x^2 + x^4}} \right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$\frac{3}{2} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) - \frac{\tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]), x]

[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4]])/2 - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/Sqrt[3]

IntegrateAlgebraic [A] time = 0.19, size = 68, normalized size = 0.99

$$\frac{2 \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{3}{2} \log \left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] (2*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]]/Sqrt[3] - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]))/2

fricas [A] time = 0.80, size = 75, normalized size = 1.09

$$\frac{1}{3} \sqrt{3} \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30}{x^2}\right) - \frac{3}{2} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3))*(5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

giac [A] time = 0.43, size = 78, normalized size = 1.13

$$\frac{1}{3} \sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) - \frac{3}{2} \log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.01, size = 52, normalized size = 0.75

$$-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{3} + \frac{3 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x)

[Out] 3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)

maxima [A] time = 1.93, size = 58, normalized size = 0.84

$$-\frac{1}{3} \sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{3}{2} \log\left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

mupad [B] time = 1.01, size = 56, normalized size = 0.81

$$\frac{3 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{2} - \frac{\sqrt{3} \left(\ln\left(\frac{1}{x^2}\right) + \ln\left(2\sqrt{3}\sqrt{x^4 + 5x^2 + 3} + 5x^2 + 6\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)

[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x\sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)

[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)

$$3.148 \quad \int \frac{2+3x^2}{x^3 \sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2} - \frac{2 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1251, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2} - \frac{2 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]))/(3*Sqrt[3])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

2*p + 3], 0]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x^3 \sqrt{3 + 5x^2 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^2 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{3x^2} + \frac{2}{3} \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{3x^2} - \frac{4}{3} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{6 + 5x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{3x^2} - \frac{2 \tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3} \sqrt{3 + 5x^2 + x^4}} \right)}{3\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{\sqrt{x^4 + 5x^2 + 3}}{3x^2} - \frac{2 \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3} \sqrt{x^4 + 5x^2 + 3}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]), x]

[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3*Sqrt[3])

IntegrateAlgebraic [A] time = 0.25, size = 63, normalized size = 1.02

$$\frac{4 \tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{3x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] $-1/3*\text{Sqrt}[3 + 5*x^2 + x^4]/x^2 + (4*\text{ArcTanh}[x^2/\text{Sqrt}[3] - \text{Sqrt}[3 + 5*x^2 + x^4]/\text{Sqrt}[3]])/(3*\text{Sqrt}[3])$

fricas [A] time = 0.78, size = 78, normalized size = 1.26

$$\frac{2\sqrt{3}x^2 \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2+6) - 2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - 3x^2 - 3\sqrt{x^4+5x^2+3}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] $1/9*(2*\text{sqrt}(3)*x^2*\log((25*x^2 - 2*\text{sqrt}(3)*(5*x^2 + 6) - 2*\text{sqrt}(x^4 + 5*x^2 + 3))*(5*\text{sqrt}(3) - 6) + 30)/x^2) - 3*x^2 - 3*\text{sqrt}(x^4 + 5*x^2 + 3))/x^2$

giac [B] time = 0.39, size = 101, normalized size = 1.63

$$\frac{2}{9}\sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{5x^2 - 5\sqrt{x^4 + 5x^2 + 3} + 6}{3\left(\left(x^2 - \sqrt{x^4 + 5x^2 + 3}\right)^2 - 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] $2/9*\text{sqrt}(3)*\log((x^2 + \text{sqrt}(3) - \text{sqrt}(x^4 + 5*x^2 + 3))/(x^2 - \text{sqrt}(3) - \text{sqrt}(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*\text{sqrt}(x^4 + 5*x^2 + 3) + 6)/((x^2 - \text{sqrt}(x^4 + 5*x^2 + 3))^2 - 3)$

maple [A] time = 0.02, size = 49, normalized size = 0.79

$$-\frac{2\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9} - \frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x)

[Out] $-2/9*\operatorname{arctanh}(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*x^2+3)^(1/2)/x^2$

maxima [A] time = 2.02, size = 51, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqrt(x^4 + 5*x^2 + 3)/x^2

mupad [B] time = 0.66, size = 83, normalized size = 1.34

$$\frac{5\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{18} - \frac{\sqrt{x^4+5x^2+3}}{3x^2} - \frac{\sqrt{3} \left(\ln\left(\frac{1}{x^2}\right) + \ln\left(2\sqrt{3}\sqrt{x^4+5x^2+3} + 5x^2 + 6\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)

[Out] (5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 - (5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x^3\sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)

[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)

$$3.149 \quad \int \frac{2+3x^2}{x^5 \sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=83

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2} - \frac{\sqrt{x^4+5x^2+3}}{6x^4} + \frac{1}{8}\sqrt{3} \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2} - \frac{\sqrt{x^4+5x^2+3}}{6x^4} + \frac{1}{8}\sqrt{3} \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]), x]

[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m+1)*(a + b*x + c*x^2)^(p+1))/(2*(p+1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m+1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 834

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x^5 \sqrt{3 + 5x^2 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^3 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{6x^4} - \frac{1}{12} \text{Subst} \left(\int \frac{-3 + 2x}{x^2 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{6x^4} - \frac{\sqrt{3 + 5x^2 + x^4}}{12x^2} - \frac{3}{8} \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{6x^4} - \frac{\sqrt{3 + 5x^2 + x^4}}{12x^2} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{6 + 5x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{6x^4} - \frac{\sqrt{3 + 5x^2 + x^4}}{12x^2} + \frac{1}{8} \sqrt{3} \tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3} \sqrt{3 + 5x^2 + x^4}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.81

$$\frac{1}{8} \sqrt{3} \tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3} \sqrt{x^4 + 5x^2 + 3}} \right) - \frac{(x^2 + 2) \sqrt{x^4 + 5x^2 + 3}}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] $-1/12*((2 + x^2)*\text{Sqrt}[3 + 5*x^2 + x^4])/x^4 + (\text{Sqrt}[3]*\text{ArcTanh}[(6 + 5*x^2)/(2*\text{Sqrt}[3]*\text{Sqrt}[3 + 5*x^2 + x^4])])/8$

IntegrateAlgebraic [A] time = 0.28, size = 70, normalized size = 0.84

$$\frac{(-x^2 - 2)\sqrt{x^4 + 5x^2 + 3}}{12x^4} - \frac{1}{4}\sqrt{3} \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] $((-2 - x^2)*\text{Sqrt}[3 + 5*x^2 + x^4])/(12*x^4) - (\text{Sqrt}[3]*\text{ArcTanh}[x^2/\text{Sqrt}[3] - \text{Sqrt}[3 + 5*x^2 + x^4]/\text{Sqrt}[3]])/4$

fricas [A] time = 0.81, size = 83, normalized size = 1.00

$$\frac{3\sqrt{3}x^4 \log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}(5\sqrt{3}+6)+30}{x^2}\right) - 2x^4 - 2\sqrt{x^4+5x^2+3}(x^2+2)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] $1/24*(3*\text{sqrt}(3)*x^4*\log((25*x^2 + 2*\text{sqrt}(3)*(5*x^2 + 6) + 2*\text{sqrt}(x^4 + 5*x^2 + 3)*(5*\text{sqrt}(3) + 6) + 30)/x^2) - 2*x^4 - 2*\text{sqrt}(x^4 + 5*x^2 + 3)*(x^2 + 2))/x^4$

giac [B] time = 0.52, size = 145, normalized size = 1.75

$$-\frac{1}{8}\sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{9(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 + 36(x^2 - \sqrt{x^4 + 5x^2 + 3})^2 + 47x^2 - 47\sqrt{x^4 + 5x^2 + 3} + 12}{12((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] $-1/8*\text{sqrt}(3)*\log((x^2 + \text{sqrt}(3) - \text{sqrt}(x^4 + 5*x^2 + 3))/(x^2 - \text{sqrt}(3) - \text{sqrt}(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - \text{sqrt}(x^4 + 5*x^2 + 3))^3 + 36*(x^2 - \text{sqrt}(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*\text{sqrt}(x^4 + 5*x^2 + 3) + 12)/((x^2 - \text{sqrt}(x^4 + 5*x^2 + 3))^2 - 3)^2$

maple [A] time = 0.01, size = 66, normalized size = 0.80

$$\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{8} - \frac{\sqrt{x^4+5x^2+3}}{12x^2} - \frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x)`

[Out] $\frac{1}{8} \operatorname{arctanh}\left(\frac{1}{6}(5x^2+6)\sqrt{3}\right) \frac{3^{1/2}}{(x^4+5x^2+3)^{1/2}} - \frac{1}{6} \frac{3^{1/2}}{(x^4+5x^2+3)^{1/2}} - \frac{1}{12} \frac{3^{1/2}}{(x^4+5x^2+3)^{1/2}} \frac{1}{x^2}$

maxima [A] time = 2.00, size = 68, normalized size = 0.82

$$\frac{1}{8} \sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{\sqrt{x^4+5x^2+3}}{12x^2} - \frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{8} \sqrt{3} \log(2\sqrt{3}\sqrt{x^4+5x^2+3}/x^2 + 6/x^2 + 5) - \frac{1}{12} \sqrt{3} \sqrt{x^4+5x^2+3}/x^2 - \frac{1}{6} \sqrt{3} \sqrt{x^4+5x^2+3}/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/(x^5*(5*x^2+x^4+3)^(1/2)),x)`

[Out] `int((3*x^2+2)/(x^5*(5*x^2+x^4+3)^(1/2)),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)`

[Out] `Integral((3*x**2+2)/(x**5*sqrt(x**4+5*x**2+3)),x)`

$$3.150 \quad \int \frac{2+3x^2}{x^7 \sqrt{3+5x^2+x^4}} dx$$

Optimal. Leaf size=104

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2} - \frac{\sqrt{x^4+5x^2+3}}{54x^4} - \frac{61 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 834, 806, 724, 206}

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2} - \frac{\sqrt{x^4+5x^2+3}}{54x^4} - \frac{\sqrt{x^4+5x^2+3}}{9x^6} - \frac{61 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^7*sqrt[3 + 5*x^2 + x^4]),x]

[Out] -sqrt[3 + 5*x^2 + x^4]/(9*x^6) - sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*sqrt[3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*sqrt[3]*sqrt[3 + 5*x^2 + x^4])])/(216*sqrt[3])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 834

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x^7 \sqrt{3 + 5x^2 + x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^4 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{9x^6} - \frac{1}{18} \text{Subst} \left(\int \frac{-2 + 4x}{x^3 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{9x^6} - \frac{\sqrt{3 + 5x^2 + x^4}}{54x^4} + \frac{1}{108} \text{Subst} \left(\int \frac{-39 - 2x}{x^2 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{9x^6} - \frac{\sqrt{3 + 5x^2 + x^4}}{54x^4} + \frac{13\sqrt{3 + 5x^2 + x^4}}{108x^2} + \frac{61}{216} \text{Subst} \left(\int \frac{1}{x \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{9x^6} - \frac{\sqrt{3 + 5x^2 + x^4}}{54x^4} + \frac{13\sqrt{3 + 5x^2 + x^4}}{108x^2} - \frac{61}{108} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, x^2 \right) \\
 &= -\frac{\sqrt{3 + 5x^2 + x^4}}{9x^6} - \frac{\sqrt{3 + 5x^2 + x^4}}{54x^4} + \frac{13\sqrt{3 + 5x^2 + x^4}}{108x^2} - \frac{61 \tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3} \sqrt{3 + 5x^2 + x^4}} \right)}{216\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.74

$$\frac{6\sqrt{x^4 + 5x^2 + 3} (13x^4 - 2x^2 - 12) - 61\sqrt{3}x^6 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{648x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4) - 61*Sqrt[3]*x^6*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(648*x^6)

IntegrateAlgebraic [A] time = 0.34, size = 75, normalized size = 0.72

$$\frac{61 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{108\sqrt{3}} + \frac{\sqrt{x^4 + 5x^2 + 3} (13x^4 - 2x^2 - 12)}{108x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4))/(108*x^6) + (61*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(108*Sqrt[3])

fricas [A] time = 0.86, size = 90, normalized size = 0.87

$$\frac{61\sqrt{3}x^6 \log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) + 78x^6 + 6(13x^4 - 2x^2 - 12)\sqrt{x^4 + 5x^2 + 3}}{648x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt(x^4 + 5*x^2 + 3))/x^6

giac [B] time = 0.51, size = 167, normalized size = 1.61

$$\frac{61}{648}\sqrt{3}\log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) - \frac{61(x^2 - \sqrt{x^4 + 5x^2 + 3})^5 - 920(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 - 2052(x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 1449x^2 + 1449\sqrt{x^4 + 5x^2 + 3} - 108}{108((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] $61/648*\sqrt{3}*\log((x^2 + \sqrt{3}) - \sqrt{x^4 + 5x^2 + 3})/(x^2 - \sqrt{3}) - \sqrt{x^4 + 5x^2 + 3}) - 1/108*(61*(x^2 - \sqrt{x^4 + 5x^2 + 3})^5 - 920*(x^2 - \sqrt{x^4 + 5x^2 + 3})^3 - 2052*(x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 1449*x^2 + 1449*\sqrt{x^4 + 5x^2 + 3} - 108)/((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)^3$

maple [A] time = 0.02, size = 83, normalized size = 0.80

$$-\frac{61\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{648} + \frac{13\sqrt{x^4+5x^2+3}}{108x^2} - \frac{\sqrt{x^4+5x^2+3}}{54x^4} - \frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x)

[Out] $-61/648*\operatorname{arctanh}(1/6*(5*x^2+6)*3^{(1/2)/(x^4+5*x^2+3)^{(1/2)})}*3^{(1/2)}-1/9*(x^4+5*x^2+3)^{(1/2)}/x^6-1/54*(x^4+5*x^2+3)^{(1/2)}/x^4+13/108*(x^4+5*x^2+3)^{(1/2)}/x^2$

maxima [A] time = 1.98, size = 85, normalized size = 0.82

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{13\sqrt{x^4+5x^2+3}}{108x^2} - \frac{\sqrt{x^4+5x^2+3}}{54x^4} - \frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] $-61/648*\sqrt{3}*\log(2*\sqrt{3}*\sqrt{x^4 + 5x^2 + 3}/x^2 + 6/x^2 + 5) + 13/108*\sqrt{x^4 + 5x^2 + 3}/x^2 - 1/54*\sqrt{x^4 + 5x^2 + 3}/x^4 - 1/9*\sqrt{x^4 + 5x^2 + 3}/x^6$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2 + 2}{x^7 \sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)

[Out] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x^7 \sqrt{x^4 + 5x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2), x)

[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)

$$3.151 \quad \int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=77

$$-\frac{(47x^2 + 33)x^2}{13\sqrt{x^4 + 5x^2 + 3}} + \frac{133}{26}\sqrt{x^4 + 5x^2 + 3} - \frac{41}{4} \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 818, 640, 621, 206}

$$-\frac{(47x^2 + 33)x^2}{13\sqrt{x^4 + 5x^2 + 3}} + \frac{133}{26}\sqrt{x^4 + 5x^2 + 3} - \frac{41}{4} \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] -(x^2*(33 + 47*x^2))/(13*sqrt[3 + 5*x^2 + x^4]) + (133*sqrt[3 + 5*x^2 + x^4])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*sqrt[3 + 5*x^2 + x^4])])/4

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 818

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p_))

```
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g)*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
- 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))] + e*(b^2*e*g*(m +
p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3,
0])
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_
)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2(2+3x)}{(3+5x+x^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13} \text{Subst} \left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{2} \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}} \right) \\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.94

$$\frac{78x^4 + 1198x^2 - 533\sqrt{x^4 + 5x^2 + 3} \tanh^{-1} \left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}} \right) + 798}{52\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (798 + 1198*x^2 + 78*x^4 - 533*sqrt[3 + 5*x^2 + x^4]*ArcTanh[(5 + 2*x^2)/(2*sqrt[3 + 5*x^2 + x^4])])/(52*sqrt[3 + 5*x^2 + x^4])

IntegrateAlgebraic [A] time = 0.34, size = 59, normalized size = 0.77

$$\frac{39x^4 + 599x^2 + 399}{26\sqrt{x^4 + 5x^2 + 3}} + \frac{41}{4} \log\left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (399 + 599*x^2 + 39*x^4)/(26*sqrt[3 + 5*x^2 + x^4]) + (41*Log[-5 - 2*x^2 + 2*sqrt[3 + 5*x^2 + x^4]])/4

fricas [A] time = 1.01, size = 86, normalized size = 1.12

$$\frac{1811x^4 + 9055x^2 + 1066(x^4 + 5x^2 + 3)\log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5) + 4(39x^4 + 599x^2 + 399)\sqrt{x^4 + 5x^2 + 3} + 5433}{104(x^4 + 5x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")

[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 5433)/(x^4 + 5*x^2 + 3)

giac [A] time = 0.38, size = 52, normalized size = 0.68

$$\frac{(39x^2 + 599)x^2 + 399}{26\sqrt{x^4 + 5x^2 + 3}} + \frac{41}{4} \log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [A] time = 0.02, size = 91, normalized size = 1.18

$$\frac{3x^4}{2\sqrt{x^4 + 5x^2 + 3}} + \frac{41x^2}{4\sqrt{x^4 + 5x^2 + 3}} - \frac{41 \ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{4} - \frac{133}{8\sqrt{x^4 + 5x^2 + 3}} + \frac{\frac{665x^2}{52} + \frac{3325}{104}}{\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)`

[Out] $3/2*x^4/(x^4+5*x^2+3)^{(1/2)}+41/4*x^2/(x^4+5*x^2+3)^{(1/2)}-133/8/(x^4+5*x^2+3)^{(1/2)}+665/104*(2*x^2+5)/(x^4+5*x^2+3)^{(1/2)}-41/4*\ln(x^2+5/2+(x^4+5*x^2+3)^{(1/2)})$

maxima [A] time = 0.92, size = 73, normalized size = 0.95

$$\frac{3x^4}{2\sqrt{x^4+5x^2+3}} + \frac{599x^2}{26\sqrt{x^4+5x^2+3}} + \frac{399}{26\sqrt{x^4+5x^2+3}} - \frac{41}{4} \log\left(2x^2 + 2\sqrt{x^4+5x^2+3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")`

[Out] $3/2*x^4/\text{sqrt}(x^4 + 5*x^2 + 3) + 599/26*x^2/\text{sqrt}(x^4 + 5*x^2 + 3) + 399/26/\text{sqrt}(x^4 + 5*x^2 + 3) - 41/4*\log(2*x^2 + 2*\text{sqrt}(x^4 + 5*x^2 + 3) + 5)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5 (3x^2 + 2)}{(x^4 + 5x^2 + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)`

[Out] `int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (3x^2 + 2)}{(x^4 + 5x^2 + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)`

[Out] `Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)`

$$3.152 \quad \int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=56

$$\frac{-47x^2 - 33}{13\sqrt{x^4 + 5x^2 + 3}} + \frac{3}{2} \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1251, 777, 621, 206}

$$\frac{3}{2} \tanh^{-1}\left(\frac{2x^2 + 5}{2\sqrt{x^4 + 5x^2 + 3}}\right) - \frac{47x^2 + 33}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] -(33 + 47*x^2)/(13*sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2)/(2*sqrt[3 + 5*x^2 + x^4]])/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 777

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(2+3x)}{(3+5x+x^2)^{3/2}} dx, x, x^2 \right) \\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{3+5x+x^2}} dx, x, x^2 \right) \\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + 3 \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}} \right) \\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2} \tanh^{-1} \left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}} \right) \end{aligned}$$

Mathematica [A] time = 0.12, size = 54, normalized size = 0.96

$$\frac{3}{2} \log \left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5 \right) - \frac{47x^2 + 33}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] -1/13*(33 + 47*x^2)/Sqrt[3 + 5*x^2 + x^4] + (3*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2

IntegrateAlgebraic [A] time = 0.27, size = 54, normalized size = 0.96

$$\frac{-47x^2 - 33}{13\sqrt{x^4 + 5x^2 + 3}} - \frac{3}{2} \log \left(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (-33 - 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2

fricas [A] time = 0.99, size = 81, normalized size = 1.45

$$\frac{94x^4 + 470x^2 + 39(x^4 + 5x^2 + 3)\log(-2x^2 + 2\sqrt{x^4 + 5x^2 + 3} - 5) + 2\sqrt{x^4 + 5x^2 + 3}(47x^2 + 33) + 282}{26(x^4 + 5x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")

[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 + 3)

giac [A] time = 0.51, size = 46, normalized size = 0.82

$$-\frac{47x^2 + 33}{13\sqrt{x^4 + 5x^2 + 3}} - \frac{3}{2}\log\left(2x^2 - 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)

maple [B] time = 0.01, size = 95, normalized size = 1.70

$$-\frac{3x^2}{2\sqrt{x^4 + 5x^2 + 3}} + \frac{3\ln\left(x^2 + \frac{5}{2} + \sqrt{x^4 + 5x^2 + 3}\right)}{2} + \frac{15}{4\sqrt{x^4 + 5x^2 + 3}} - \frac{75(2x^2 + 5)}{52\sqrt{x^4 + 5x^2 + 3}} + \frac{\frac{10x^2}{13} + \frac{12}{13}}{\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)

[Out] -3/2/(x^4+5*x^2+3)^(1/2)*x^2+15/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+2/13/(x^4+5*x^2+3)^(1/2)*(5*x^2+6)

maxima [A] time = 0.93, size = 56, normalized size = 1.00

$$-\frac{47x^2}{13\sqrt{x^4 + 5x^2 + 3}} - \frac{33}{13\sqrt{x^4 + 5x^2 + 3}} + \frac{3}{2}\log\left(2x^2 + 2\sqrt{x^4 + 5x^2 + 3} + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")

[Out] $-47/13*x^2/\sqrt{x^4 + 5*x^2 + 3} - 33/13/\sqrt{x^4 + 5*x^2 + 3} + 3/2*\log(2*x^2 + 2*\sqrt{x^4 + 5*x^2 + 3} + 5)$

mupad [B] time = 0.31, size = 52, normalized size = 0.93

$$\frac{3 \ln\left(\sqrt{x^4 + 5x^2 + 3} + x^2 + \frac{5}{2}\right)}{2} - \frac{47x^2}{13\sqrt{x^4 + 5x^2 + 3}} - \frac{33}{13\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)`

[Out] $(3*\log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4 + 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(3x^2 + 2)}{(x^4 + 5x^2 + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2), x)`

[Out] `Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)`

$$3.153 \quad \int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=25

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1247, 636}

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (8 + 11*x^2)/(13*sqrt[3 + 5*x^2 + x^4])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2+3x}{(3+5x+x^2)^{3/2}} dx, x, x^2 \right) \\ &= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 25, normalized size = 1.00

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])

IntegrateAlgebraic [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2), x]

[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])

fricas [B] time = 0.90, size = 46, normalized size = 1.84

$$\frac{11x^4 + 55x^2 + \sqrt{x^4 + 5x^2 + 3}(11x^2 + 8) + 33}{13(x^4 + 5x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2), x, algorithm="fricas")

[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x^2 + 3)

giac [A] time = 0.36, size = 21, normalized size = 0.84

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2), x, algorithm="giac")

[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)`

[Out] `1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)`

maxima [A] time = 0.94, size = 32, normalized size = 1.28

$$\frac{11x^2}{13\sqrt{x^4 + 5x^2 + 3}} + \frac{8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")`

[Out] `11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)`

mupad [B] time = 0.24, size = 21, normalized size = 0.84

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)`

[Out] `(11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(3x^2 + 2)}{(x^4 + 5x^2 + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)`

[Out] `Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)`

$$3.154 \quad \int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=66

$$\frac{-8x^2 - 7}{39\sqrt{x^4 + 5x^2 + 3}} - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 822, 12, 724, 206}

$$-\frac{8x^2 + 7}{39\sqrt{x^4 + 5x^2 + 3}} - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]

[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 822

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +

```

2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
+ b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{2 + 3x^2}{x(3 + 5x^2 + x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x(3 + 5x + x^2)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{7 + 8x^2}{39\sqrt{3 + 5x^2 + x^4}} - \frac{1}{39} \text{Subst} \left(\int -\frac{13}{x\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= -\frac{7 + 8x^2}{39\sqrt{3 + 5x^2 + x^4}} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
&= -\frac{7 + 8x^2}{39\sqrt{3 + 5x^2 + x^4}} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{6 + 5x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \\
&= -\frac{7 + 8x^2}{39\sqrt{3 + 5x^2 + x^4}} - \frac{\tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3}\sqrt{3 + 5x^2 + x^4}} \right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.00

$$-\frac{8x^2 + 7}{39\sqrt{x^4 + 5x^2 + 3}} - \frac{\tanh^{-1} \left(\frac{5x^2 + 6}{2\sqrt{3}\sqrt{x^4 + 5x^2 + 3}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]

[Out] $-\frac{1}{39} \frac{(7 + 8x^2)}{\sqrt{3 + 5x^2 + x^4}} - \frac{\operatorname{ArcTanh}\left[\frac{(6 + 5x^2)}{(2\sqrt{3}\sqrt{3 + 5x^2 + x^4})}\right]}{(3\sqrt{3})}$

IntegrateAlgebraic [A] time = 0.36, size = 67, normalized size = 1.02

$$\frac{-8x^2 - 7}{39\sqrt{x^4 + 5x^2 + 3}} + \frac{2 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4 + 5x^2 + 3}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]

[Out] $\frac{(-7 - 8x^2)/(39\sqrt{3 + 5x^2 + x^4}) + (2\operatorname{ArcTanh}[x^2/\sqrt{3}] - \sqrt{3 + 5x^2 + x^4}/\sqrt{3})}{(3\sqrt{3})}$

fricas [B] time = 0.93, size = 107, normalized size = 1.62

$$\frac{24x^4 - 13\sqrt{3}(x^4 + 5x^2 + 3) \log\left(\frac{25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30}{x^2}\right) + 120x^2 + 3\sqrt{x^4 + 5x^2 + 3}(8x^2 + 7) + 72}{117(x^4 + 5x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")

[Out] $-\frac{1}{117} \frac{(24x^4 - 13\sqrt{3}(x^4 + 5x^2 + 3) \log((25x^2 - 2\sqrt{3}(5x^2 + 6) - 2\sqrt{x^4 + 5x^2 + 3}(5\sqrt{3} - 6) + 30)/x^2) + 120x^2 + 3\sqrt{x^4 + 5x^2 + 3}(8x^2 + 7) + 72)}{(x^4 + 5x^2 + 3)}$

giac [A] time = 0.56, size = 78, normalized size = 1.18

$$-\frac{1}{9} \sqrt{3} \log\left(-x^2 + \sqrt{3} + \sqrt{x^4 + 5x^2 + 3}\right) + \frac{1}{9} \sqrt{3} \log\left(-x^2 - \sqrt{3} + \sqrt{x^4 + 5x^2 + 3}\right) - \frac{8x^2 + 7}{39\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] $-\frac{1}{9} \sqrt{3} \log(-x^2 + \sqrt{3} + \sqrt{x^4 + 5x^2 + 3}) + \frac{1}{9} \sqrt{3} \log(-x^2 - \sqrt{3} + \sqrt{x^4 + 5x^2 + 3}) - \frac{1}{39} \frac{(8x^2 + 7)}{\sqrt{x^4 + 5x^2 + 3}}$

maple [A] time = 0.02, size = 67, normalized size = 1.02

$$-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2 + 6)\sqrt{3}}{6\sqrt{x^4 + 5x^2 + 3}}\right)}{9} - \frac{4(2x^2 + 5)}{39\sqrt{x^4 + 5x^2 + 3}} + \frac{1}{3\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x)`

[Out] $-4/39*(2*x^2+5)/(x^4+5*x^2+3)^{(1/2)}+1/3/(x^4+5*x^2+3)^{(1/2)}-1/9*\operatorname{arctanh}(1/6*(5*x^2+6)*3^{(1/2)}/(x^4+5*x^2+3)^{(1/2)})*3^{(1/2)}$

maxima [A] time = 2.01, size = 65, normalized size = 0.98

$$-\frac{8x^2}{39\sqrt{x^4+5x^2+3}} - \frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{7}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")`

[Out] $-8/39*x^2/\operatorname{sqrt}(x^4+5*x^2+3) - 1/9*\operatorname{sqrt}(3)*\log(2*\operatorname{sqrt}(3)*\operatorname{sqrt}(x^4+5*x^2+3)/x^2 + 6/x^2 + 5) - 7/39/\operatorname{sqrt}(x^4+5*x^2+3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{3x^2 + 2}{x(x^4 + 5x^2 + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+2)/(x*(5*x^2+x^4+3)^(3/2)),x)`

[Out] `int((3*x^2+2)/(x*(5*x^2+x^4+3)^(3/2)),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x(x^4 + 5x^2 + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)`

[Out] `Integral((3*x**2+2)/(x*(x**4+5*x**2+3)**(3/2)),x)`

$$3.155 \quad \int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}} dx$$

Optimal. Leaf size=90

$$\frac{-8x^2 - 7}{39x^2\sqrt{x^4 + 5x^2 + 3}} - \frac{2\sqrt{x^4 + 5x^2 + 3}}{39x^2} + \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1251, 822, 806, 724, 206}

$$-\frac{8x^2 + 7}{39x^2\sqrt{x^4 + 5x^2 + 3}} - \frac{2\sqrt{x^4 + 5x^2 + 3}}{39x^2} + \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]

[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(39*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

Rule 822

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{2 + 3x^2}{x^3 (3 + 5x^2 + x^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{2 + 3x}{x^2 (3 + 5x + x^2)^{3/2}} dx, x, x^2 \right) \\
 &= -\frac{7 + 8x^2}{39x^2 \sqrt{3 + 5x^2 + x^4}} - \frac{1}{39} \text{Subst} \left(\int \frac{-6 + 8x}{x^2 \sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{7 + 8x^2}{39x^2 \sqrt{3 + 5x^2 + x^4}} - \frac{2\sqrt{3 + 5x^2 + x^4}}{39x^2} - \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt{3 + 5x + x^2}} dx, x, x^2 \right) \\
 &= -\frac{7 + 8x^2}{39x^2 \sqrt{3 + 5x^2 + x^4}} - \frac{2\sqrt{3 + 5x^2 + x^4}}{39x^2} + \frac{2}{3} \text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{6 + 5x^2}{\sqrt{3 + 5x^2 + x^4}} \right) \\
 &= -\frac{7 + 8x^2}{39x^2 \sqrt{3 + 5x^2 + x^4}} - \frac{2\sqrt{3 + 5x^2 + x^4}}{39x^2} + \frac{\tanh^{-1} \left(\frac{6 + 5x^2}{2\sqrt{3} \sqrt{3 + 5x^2 + x^4}} \right)}{3\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 0.98

$$\frac{-6x^4 - 54x^2 + 13\sqrt{3}\sqrt{x^4 + 5x^2 + 3}x^2 \tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - 39}{117x^2\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)), x]

[Out] (-39 - 54*x^2 - 6*x^4 + 13*Sqrt[3]*x^2*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(117*x^2*Sqrt[3 + 5*x^2 + x^4])

IntegrateAlgebraic [A] time = 0.42, size = 75, normalized size = 0.83

$$\frac{-2x^4 - 18x^2 - 13}{39x^2\sqrt{x^4 + 5x^2 + 3}} - \frac{2 \tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4+5x^2+3}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)), x]

[Out] (-13 - 18*x^2 - 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(3*Sqrt[3])

fricas [A] time = 1.09, size = 124, normalized size = 1.38

$$\frac{6x^6 + 30x^4 - 13\sqrt{3}(x^6 + 5x^4 + 3x^2) \log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}(5\sqrt{3}+6)+30}{x^2}\right) + 18x^2 + 3(2x^4 + 18x^2 + 13)\sqrt{x^4 + 5x^2 + 3}}{117(x^6 + 5x^4 + 3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2), x, algorithm="fricas")

[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 18*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2)

giac [A] time = 0.49, size = 122, normalized size = 1.36

$$-\frac{1}{9}\sqrt{3} \log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{7x^2 + 11}{117\sqrt{x^4 + 5x^2 + 3}} + \frac{5x^2 - 5\sqrt{x^4 + 5x^2 + 3} + 6}{9\left(\left(x^2 - \sqrt{x^4 + 5x^2 + 3}\right)^2 - 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")

[Out] $-\frac{1}{9}\sqrt{3}\log\left(\frac{x^2 + \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}{x^2 - \sqrt{3} - \sqrt{x^4 + 5x^2 + 3}}\right) + \frac{1}{117}(7x^2 + 11)/\sqrt{x^4 + 5x^2 + 3} + \frac{1}{9}(5x^2 - 5\sqrt{x^4 + 5x^2 + 3} + 6)/((x^2 - \sqrt{x^4 + 5x^2 + 3})^2 - 3)$

maple [A] time = 0.02, size = 84, normalized size = 0.93

$$\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{9} - \frac{1}{3\sqrt{x^4+5x^2+3}x^2} - \frac{1}{3\sqrt{x^4+5x^2+3}} - \frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x)

[Out] $-\frac{1}{3}(x^4+5x^2+3)^{-1/2} - \frac{1}{39}(2x^2+5)/(x^4+5x^2+3)^{1/2} + \frac{1}{9}\operatorname{arctanh}\left(\frac{1}{6}(5x^2+6)\sqrt{3}\right)/(x^4+5x^2+3)^{1/2} - \frac{1}{3x^2}(x^4+5x^2+3)^{-1/2}$

maxima [A] time = 2.09, size = 82, normalized size = 0.91

$$-\frac{2x^2}{39\sqrt{x^4+5x^2+3}} + \frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{6}{13\sqrt{x^4+5x^2+3}} - \frac{1}{3\sqrt{x^4+5x^2+3}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")

[Out] $-\frac{2}{39}x^2/\sqrt{x^4 + 5x^2 + 3} + \frac{1}{9}\sqrt{3}\log(2*\sqrt{3}*\sqrt{x^4 + 5x^2 + 3}/x^2 + 6/x^2 + 5) - 6/13/\sqrt{x^4 + 5x^2 + 3} - 1/3/(\sqrt{x^4 + 5x^2 + 3})*x^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3x^2 + 2}{x^3(x^4 + 5x^2 + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)

[Out] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x^3(x^4 + 5x^2 + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

$$3.156 \quad \int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^3 dx$$

Optimal. Leaf size=243

$$\frac{a^3 d (fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+7} (3a^2 ce + 3ab^2 e + 6abcd + b^3 d)}{f^7(m+7)} + \frac{a^2 (fx)^{m+3} (ae + 3bd)}{f^3(m+3)} + \frac{3c (fx)^{m+11} (ace + b^2 e + bcd)}{f^{11}(m+11)} + \frac{3a(fx)^{m+15} (abce + 3ac^2 d + 3b^2 cd + b^3 e)}{f^{15}(m+15)}$$

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1261}

$$\frac{(fx)^{m+7} (3a^2 ce + 3ab^2 e + 6abcd + b^3 d)}{f^7(m+7)} + \frac{a^2 (fx)^{m+3} (ae + 3bd)}{f^3(m+3)} + \frac{a^3 d (fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+9} (6abce + 3ac^2 d + 3b^2 cd + b^3 e)}{f^9(m+9)} + \frac{3a(fx)^{m+5} (abe + acd + b^2 d)}{f^5(m+5)} + \frac{3c(fx)^{m+11} (ace + b^2 e + bcd)}{f^{11}(m+11)} + \frac{c^2 (fx)^{m+13} (3be + cd)}{f^{13}(m+13)} + \frac{c^3 (fx)^{m+15}}{f^{15}(m+15)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m))

Rule 1261

Int[((f_.)*(x_.))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^3 dx &= \int \left(a^3 d (fx)^m + \frac{a^2 (3bd + ae) (fx)^{2+m}}{f^2} + \frac{3a (b^2 d + acd + abe) (fx)^{4+m}}{f^4} + \right. \\ &= \frac{a^3 d (fx)^{1+m}}{f(1+m)} + \frac{a^2 (3bd + ae) (fx)^{3+m}}{f^3(3+m)} + \frac{3a (b^2 d + acd + abe) (fx)^{5+m}}{f^5(5+m)} + \left. \frac{c^2 (fx)^{m+13} (3be + cd)}{f^{13}(m+13)} + \frac{c^3 ex^{14}}{m+15} \right) dx \end{aligned}$$

Mathematica [A] time = 0.30, size = 191, normalized size = 0.79

$$x(fx)^m \left(\frac{a^3 d}{m+1} + \frac{x^6 (3a^2 ce + 3ab^2 e + 6abcd + b^3 d)}{m+7} + \frac{a^2 x^2 (ae + 3bd)}{m+3} + \frac{3cx^{10} (ace + b^2 e + bcd)}{m+11} + \frac{3ax^4 (abe + acd + b^2 d)}{m+5} + \frac{x^8 (6abce + 3ac^2 d + b^3 e + 3b^2 cd)}{m+9} + \frac{c^2 x^{12} (3be + cd)}{m+13} + \frac{c^3 ex^{14}}{m+15} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] $x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d + a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m) + (3*c*(b*c*d + b^2*e + a*c*e)*x^{10})/(11 + m) + (c^2*(c*d + 3*b*e)*x^{12})/(13 + m) + (c^3*e*x^{14})/(15 + m)$

IntegrateAlgebraic [F] time = 1.21, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3, x]

fricas [B] time = 0.86, size = 1357, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] $((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e*m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^{15} + ((c^3*d + 3*b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5 + 11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(c^3*d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c^2*e)*m*x^{13} + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c^2*d + (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c + a*c^2)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*c + a*c^2)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m*x^{11} + ((3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^2 + 675675*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m*x^9 + ((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*($

$$\begin{aligned}
& (b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*d + 8 \\
& 68725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)* \\
& m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59*(a^2*b*e + (a*b^2 + a^2*c) \\
& c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^ \\
& 2 + a^2*c)*d)*m^4 + 405405*a^2*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m \\
& ^3 + 437121*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d + \\
& 738567*(a^2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61 \\
& *(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(3*a^2*b*d \\
& + a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*(3*a^2*b*d + a^3*e)* \\
& m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 1140855*(3*a^2*b*d + a^3*e)*m)*x^3 + \\
& (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*d \\
& m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m + 2027025*a^3*d)*x*(f*x)^m/(m^8 \\
& + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 + \\
& 4098240*m + 2027025)
\end{aligned}$$

giac [B] time = 0.75, size = 2816, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] ((f*x)^m*c^3*m^7*x^15*e + 49*(f*x)^m*c^3*m^6*x^15*e + (f*x)^m*c^3*d*m^7*x^13 + 3*(f*x)^m*b*c^2*m^7*x^13*e + 973*(f*x)^m*c^3*m^5*x^15*e + 51*(f*x)^m*c^3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*m^6*x^13*e + 10045*(f*x)^m*c^3*m^4*x^15*e + 3*(f*x)^m*b*c^2*d*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3*(f*x)^m*b^2*c*m^7*x^11*e + 3*(f*x)^m*a*c^2*m^7*x^11*e + 3135*(f*x)^m*b*c^2*m^5*x^13*e + 57379*(f*x)^m*c^3*m^3*x^15*e + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 11055*(f*x)^m*c^3*d*m^4*x^13 + 159*(f*x)^m*b^2*c*m^6*x^11*e + 159*(f*x)^m*a*c^2*m^6*x^11*e + 33165*(f*x)^m*b*c^2*m^4*x^13*e + 177331*(f*x)^m*c^3*m^2*x^15*e + 3*(f*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x^13 + (f*x)^m*b^3*m^7*x^9*e + 6*(f*x)^m*a*b*c*m^7*x^9*e + 3375*(f*x)^m*b^2*c*m^5*x^11*e + 3375*(f*x)^m*a*c^2*m^5*x^11*e + 193017*(f*x)^m*b*c^2*m^3*x^13*e + 264207*(f*x)^m*c^3*m*x^15*e + 165*(f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 36795*(f*x)^m*b*c^2*d*m^4*x^11 + 201609*(f*x)^m*c^3*d*m^2*x^13 + 55*(f*x)^m*b^3*m^6*x^9*e + 330*(f*x)^m*a*b*c*m^6*x^9*e + 36795*(f*x)^m*b^2*c*m^4*x^11*e + 36795*(f*x)^m*a*c^2*m^4*x^11*e + 604827*(f*x)^m*b*c^2*m^2*x^13*e + 135135*(f*x)^m*c^3*x^15*e + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3639*(f*x)^m*b^2*c*d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11 + 303255*(f*x)^m*c^3*d*m*x^13 + 3*(f*x)^m*a*b^2*m^7*x^7*e + 3*(f*x)^m*a^2*c*m^7*x^7*e + 1213*(f*x)^m*b^3*m^5*x^9*e + 7278*(f*x)^m*a*b*c*m^5*x^9*e + 219417*(f*x)^m*b^2*c*m^3*x^11*e + 219417*(f*x)^m*a*c^2*m^3*x^11*e + 909765*(f*x)^m*b*c^2*m*x^13*e + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*x^7 + 41169*(f*x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 70046

$$\begin{aligned}
& 1*(f*x)^m*b*c^2*d*m^2*x^11 + 155925*(f*x)^m*c^3*d*x^13 + 171*(f*x)^m*a*b^2* \\
& m^6*x^7*e + 171*(f*x)^m*a^2*c*m^6*x^7*e + 13723*(f*x)^m*b^3*m^4*x^9*e + 823 \\
& 38*(f*x)^m*a*b*c*m^4*x^9*e + 700461*(f*x)^m*b^2*c*m^2*x^11*e + 700461*(f*x) \\
& ^m*a*c^2*m^2*x^11*e + 467775*(f*x)^m*b*c^2*x^13*e + 3*(f*x)^m*a*b^2*d*m^7*x \\
& ^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 1309*(f*x)^m*b^3*d*m^5*x^7 + 7854*(f*x)^m* \\
& a*b*c*d*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f*x)^m*a*c^2*d*m \\
& ^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 3*(f*x)^m*a^2*b*m^7*x^5*e + 3927* \\
& (f*x)^m*a*b^2*m^5*x^7*e + 3927*(f*x)^m*a^2*c*m^5*x^7*e + 84547*(f*x)^m*b^3* \\
& m^3*x^9*e + 507282*(f*x)^m*a*b*c*m^3*x^9*e + 1067445*(f*x)^m*b^2*c*m*x^11*e \\
& + 1067445*(f*x)^m*a*c^2*m*x^11*e + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x) \\
& ^m*a^2*c*d*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*x^7 + 92862*(f*x)^m*a*b*c*d*m^ \\
& 4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 5 \\
& 52825*(f*x)^m*b*c^2*d*x^11 + 177*(f*x)^m*a^2*b*m^6*x^5*e + 46431*(f*x)^m*a* \\
& b^2*m^4*x^7*e + 46431*(f*x)^m*a^2*c*m^4*x^7*e + 277093*(f*x)^m*b^3*m^2*x^9* \\
& e + 1662558*(f*x)^m*a*b*c*m^2*x^9*e + 552825*(f*x)^m*b^2*c*x^11*e + 552825* \\
& (f*x)^m*a*c^2*x^11*e + 3*(f*x)^m*a^2*b*d*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5 \\
& *x^5 + 4239*(f*x)^m*a^2*c*d*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290* \\
& (f*x)^m*a*b*c*d*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a \\
& *c^2*d*m*x^9 + (f*x)^m*a^3*m^7*x^3*e + 4239*(f*x)^m*a^2*b*m^5*x^5*e + 29914 \\
& 5*(f*x)^m*a*b^2*m^3*x^7*e + 299145*(f*x)^m*a^2*c*m^3*x^7*e + 430335*(f*x)^m \\
& *b^3*m*x^9*e + 2582010*(f*x)^m*a*b*c*m*x^9*e + 183*(f*x)^m*a^2*b*d*m^6*x^3 \\
& + 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 340011*(f \\
& *x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 675675*(f*x)^m*b^2* \\
& c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 61*(f*x)^m*a^3*m^6*x^3*e + 52725*(f* \\
& x)^m*a^2*b*m^4*x^5*e + 1020033*(f*x)^m*a*b^2*m^2*x^7*e + 1020033*(f*x)^m*a^ \\
& 2*c*m^2*x^7*e + 225225*(f*x)^m*b^3*x^9*e + 1351350*(f*x)^m*a*b*c*x^9*e + (f \\
& *x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m \\
& ^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264 \\
& 570*(f*x)^m*a*b*c*d*m*x^7 + 1525*(f*x)^m*a^3*m^5*x^3*e + 360537*(f*x)^m*a^2 \\
& *b*m^3*x^5*e + 1632285*(f*x)^m*a*b^2*m*x^7*e + 1632285*(f*x)^m*a^2*c*m*x^7* \\
& e + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 1311363*(f*x)^ \\
& m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 289575*(f*x)^m*b^3*d* \\
& x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 20065*(f*x)^m*a^3*m^4*x^3*e + 1311363*(\\
& f*x)^m*a^2*b*m^2*x^5*e + 868725*(f*x)^m*a*b^2*x^7*e + 868725*(f*x)^m*a^2*c* \\
& x^7*e + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 2215701 \\
& *(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 147859*(f*x)^m*a^3 \\
& *m^3*x^3*e + 2215701*(f*x)^m*a^2*b*m*x^5*e + 22995*(f*x)^m*a^3*d*m^4*x + 17 \\
& 83317*(f*x)^m*a^2*b*d*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x) \\
& ^m*a^2*c*d*x^5 + 594439*(f*x)^m*a^3*m^2*x^3*e + 1216215*(f*x)^m*a^2*b*x^5*e \\
& + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f* \\
& x)^m*a^3*m*x^3*e + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3 \\
& + 675675*(f*x)^m*a^3*x^3*e + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a \\
& ^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2 \\
& 924172*m^2 + 4098240*m + 2027025)
\end{aligned}$$

maple [B] time = 0.01, size = 1935, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x)$

[Out] $x*(c^3*e*m^7*x^{14}+49*c^3*e*m^6*x^{14}+3*b*c^2*e*m^7*x^{12}+c^3*d*m^7*x^{12}+973*c^3*e*m^5*x^{14}+153*b*c^2*e*m^6*x^{12}+51*c^3*d*m^6*x^{12}+10045*c^3*e*m^4*x^{14}+3*a*c^2*e*m^7*x^{10}+3*b^2*c*e*m^7*x^{10}+3*b*c^2*d*m^7*x^{10}+3135*b*c^2*e*m^5*x^{12}+1045*c^3*d*m^5*x^{12}+57379*c^3*e*m^3*x^{14}+159*a*c^2*e*m^6*x^{10}+159*b^2*c*e*m^6*x^{10}+159*b*c^2*d*m^6*x^{10}+33165*b*c^2*e*m^4*x^{12}+11055*c^3*d*m^4*x^{12}+177331*c^3*e*m^2*x^{14}+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5*x^{10}+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^{10}+3375*b*c^2*d*m^5*x^{10}+193017*b*c^2*e*m^3*x^{12}+64339*c^3*d*m^3*x^{12}+264207*c^3*e*m*x^{14}+330*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^{10}+55*b^3*e*m^6*x^8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^{10}+36795*b*c^2*d*m^4*x^{10}+604827*b*c^2*e*m^2*x^{12}+201609*c^3*d*m^2*x^{12}+135135*c^3*e*x^{14}+3*a^2*c*e*m^7*x^6+3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x^8+219417*a*c^2*e*m^3*x^{10}+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^5*x^8+219417*b^2*c*e*m^3*x^{10}+219417*b*c^2*d*m^3*x^{10}+909765*b*c^2*e*m*x^{12}+303255*c^3*d*m*x^{12}+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^{10}+57*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^2*x^{10}+700461*b*c^2*d*m^2*x^{10}+467775*b*c^2*e*x^{12}+155925*c^3*d*x^{12}+3*a^2*b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m^3*x^8+1067445*a*c^2*e*m*x^{10}+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^{10}+1067445*b*c^2*d*m*x^{10}+177*a^2*b*e*m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2*d*m^2*x^8+552825*a*c^2*e*x^{10}+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^{10}+552825*b*c^2*d*x^{10}+a^3*e*m^7*x^2+3*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+675675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+360537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745$

$0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+1140855*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+2027025*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(m+1)/(m+3)/(m+5)/(m+7)/(m+9)/(m+11)/(m+13)/(m+15)$

maxima [A] time = 1.39, size = 408, normalized size = 1.68

$\frac{c^3 f^{m+15} x^m}{m+15} + \frac{c^3 d f^m x^{13} x^m}{m+13} + \frac{3 b^3 c^2 e f^m x^{13} x^m}{m+13} + \frac{3 b^3 c^2 d f^m x^{11} x^m}{m+11} + \frac{3 b^3 c^2 e f^m x^{11} x^m}{m+11} + \frac{3 a^3 c^2 d f^m x^{11} x^m}{m+11} + \frac{3 a^3 c^2 e f^m x^{11} x^m}{m+11} + \frac{3 b^2 c^2 d f^m x^9 x^m}{m+9} + \frac{3 b^2 c^2 e f^m x^9 x^m}{m+9} + \frac{6 a b c^2 d f^m x^9 x^m}{m+9} + \frac{6 a b c^2 e f^m x^9 x^m}{m+9} + \frac{b^3 d f^m x^9 x^m}{m+9} + \frac{6 a^2 b c^2 d f^m x^9 x^m}{m+9} + \frac{6 a^2 b c^2 e f^m x^9 x^m}{m+9} + \frac{3 a b^2 c^2 d f^m x^7 x^m}{m+7} + \frac{3 a b^2 c^2 e f^m x^7 x^m}{m+7} + \frac{3 a^2 b c^2 d f^m x^7 x^m}{m+7} + \frac{3 a^2 b c^2 e f^m x^7 x^m}{m+7} + \frac{3 a^2 c^2 d f^m x^5 x^m}{m+5} + \frac{3 a^2 c^2 e f^m x^5 x^m}{m+5} + \frac{3 a^2 b^2 e f^m x^5 x^m}{m+5} + \frac{3 a^2 b^2 d f^m x^3 x^m}{m+3} + \frac{a^3 e f^m x^3 x^m}{m+3} + \frac{(f x)^{m+1} a^3 d}{f(m+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] $c^3 e f^m x^{15} x^m / (m + 15) + c^3 d f^m x^{13} x^m / (m + 13) + 3 b^3 c^2 e f^m x^{13} x^m / (m + 13) + 3 b^3 c^2 d f^m x^{11} x^m / (m + 11) + 3 b^2 c^2 e f^m x^{11} x^m / (m + 11) + 3 a^3 c^2 e f^m x^{11} x^m / (m + 11) + 3 b^2 c^2 d f^m x^9 x^m / (m + 9) + 3 a^3 c^2 d f^m x^9 x^m / (m + 9) + b^3 e f^m x^9 x^m / (m + 9) + 6 a^2 b c^2 e f^m x^9 x^m / (m + 9) + b^3 d f^m x^9 x^m / (m + 9) + 6 a^2 b c^2 d f^m x^9 x^m / (m + 9) + 3 a^2 b^2 e f^m x^7 x^m / (m + 7) + 3 a^2 c^2 d f^m x^7 x^m / (m + 7) + 3 a^2 c^2 e f^m x^7 x^m / (m + 7) + 3 a^2 b^2 d f^m x^5 x^m / (m + 5) + 3 a^2 c^2 d f^m x^5 x^m / (m + 5) + 3 a^2 b^2 e f^m x^5 x^m / (m + 5) + 3 a^2 b^2 d f^m x^3 x^m / (m + 3) + a^3 e f^m x^3 x^m / (m + 3) + (f*x)^{(m+1)}*a^3*d/(f*(m+1))$

mupad [B] time = 1.06, size = 769, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)

[Out] $(x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 340011*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e)*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 + 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m + 852957*m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m^2 + 120179*m$

$$\begin{aligned} &^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^2 \\ &+ 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) \\ &+ (3*c*x^{11}*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m + 233487*m^2 + 731 \\ &39*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4098240*m + 292417 \\ &2*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 20 \\ &27025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594439*m^2 + 147859*m^3 \\ &+ 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(4098240*m + 2924172*m^2 \\ &+ 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) \\ &+ (c^2*x^{13}*(f*x)^m*(3*b*e + c*d)*(303255*m + 201609*m^2 + 64339*m^3 + 11 \\ &055*m^4 + 1045*m^5 + 51*m^6 + m^7 + 155925))/(4098240*m + 2924172*m^2 + 103 \\ &8016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) \end{aligned}$$

sympy [A] time = 12.38, size = 11538, normalized size = 47.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)

[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b**2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 - a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6)) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b**2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4)) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c**2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x**4) - 3*a**2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x)) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)

$$\begin{aligned}
& + b^{**3}e^{**x**2/2} + 3b^{**2}c^{**d}x^{**2/2} + 3b^{**2}c^{**e}x^{**4/4} + 3b^{**c**2}d^{**x**4/4} \\
& + b^{**c**2}e^{**x**6/2} + c^{**3}d^{**x**6/6} + c^{**3}e^{**x**8/8}/f^{**7}, \text{Eq}(m, -7)), ((-a^{**3}d/(4x^{**4}) - a^{**3}e/(2x^{**2}) - 3a^{**2}b^{**d}/(2x^{**2}) + 3a^{**2}b^{**e}\log(x) + \\
& 3a^{**2}c^{**d}\log(x) + 3a^{**2}c^{**e}x^{**2/2} + 3a^{**b**2}d^{**\log}(x) + 3a^{**b**2}e^{**x**2/2} + 3a^{**b}c^{**d}x^{**2} + 3a^{**b}c^{**e}x^{**4/2} + 3a^{**c**2}d^{**x**4/4} + a^{**c**2}e^{**x**6/2} + \\
& b^{**3}d^{**x**2/2} + b^{**3}e^{**x**4/4} + 3b^{**2}c^{**d}x^{**4/4} + b^{**2}c^{**e}x^{**6/2} + \\
& b^{**c**2}d^{**x**6/2} + 3b^{**c**2}e^{**x**8/8} + c^{**3}d^{**x**8/8} + c^{**3}e^{**x**10/10})/f^{**5}, \text{Eq}(m, -5)), ((-a^{**3}d/(2x^{**2}) + a^{**3}e\log(x) + 3a^{**2}b^{**d}\log(x) + 3a^{**2}b^{**e}x^{**2/2} + 3a^{**2}c^{**d}x^{**2/2} + 3a^{**2}c^{**e}x^{**4/4} + 3a^{**b**2}d^{**x**2/2} + \\
& 3a^{**b**2}e^{**x**4/4} + 3a^{**b}c^{**d}x^{**4/2} + a^{**b}c^{**e}x^{**6} + a^{**c**2}d^{**x**6/2} + 3a^{**c**2}e^{**x**8/8} + b^{**3}d^{**x**4/4} + b^{**3}e^{**x**6/6} + b^{**2}c^{**d}x^{**6/2} + 3b^{**2}c^{**e}x^{**8/8} + 3b^{**c**2}d^{**x**8/8} + 3b^{**c**2}e^{**x**10/10} + c^{**3}d^{**x**10/10} + c^{**3}e^{**x**12/12})/f^{**3}, \text{Eq}(m, -3)), ((a^{**3}d\log(x) + a^{**3}e^{**x**2/2} + 3a^{**2}b^{**d}x^{**2/2} + 3a^{**2}b^{**e}x^{**4/4} + 3a^{**2}c^{**d}x^{**4/4} + a^{**2}c^{**e}x^{**6/2} + 3a^{**b**2}d^{**x**4/4} + a^{**b**2}e^{**x**6/2} + a^{**b}c^{**d}x^{**6} + 3a^{**b}c^{**e}x^{**8/4} + 3a^{**c**2}d^{**x**8/8} + 3a^{**c**2}e^{**x**10/10} + b^{**3}d^{**x**6/6} + b^{**3}e^{**x**8/8} + 3b^{**2}c^{**d}x^{**8/8} + 3b^{**2}c^{**e}x^{**10/10} + 3b^{**c**2}d^{**x**10/10} + b^{**c**2}e^{**x**12/4} + c^{**3}d^{**x**12/12} + c^{**3}e^{**x**14/14})/f, \text{Eq}(m, -1)), (a^{**3}d^{**f**m**m**7}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 63a^{**3}d^{**f**m**m**6}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1645a^{**3}d^{**f**m**m**5}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 22995a^{**3}d^{**f**m**m**4}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 185059a^{**3}d^{**f**m**m**3}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 852957a^{**3}d^{**f**m**m**2}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 2071215a^{**3}d^{**f**m**m}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 2027025a^{**3}d^{**f**m}x^{**x**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + a^{**3}e^{**f**m**m**7}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 61a^{**3}e^{**f**m**m**6}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1525a^{**3}e^{**f**m**m**5}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 20065a^{**3}e^{**f**m**m**4}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 147859a^{**3}e^{**f**m**m**3}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 594439a^{**3}e^{**f**m**m**2}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1140855a^{**3}e^{**f**m**m}x^{**3}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 103
\end{aligned}$$

$$\begin{aligned}
& 8016m^3 + 2924172m^2 + 4098240m + 2027025) + 675675a^3e^3f^3x^3x^3 \\
& **m/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + \\
& 2924172m^2 + 4098240m + 2027025) + 3a^2b^2d^2f^3m^7x^3x^3x^3/(m^8 \\
& + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 \\
& + 4098240m + 2027025) + 183a^2b^2d^2f^3m^6x^3x^3x^3/(m^8 + 64m^7 \\
& + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + \\
& 4098240m + 2027025) + 4575a^2b^2d^2f^3m^5x^3x^3x^3/(m^8 + 64m^7 + \\
& 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 409824 \\
& 0m + 2027025) + 60195a^2b^2d^2f^3m^4x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 \\
& + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + \\
& 2027025) + 443577a^2b^2d^2f^3m^3x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 \\
& + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 202 \\
& 7025) + 1783317a^2b^2d^2f^3m^2x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + \\
& 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 202702 \\
& 5) + 3422565a^2b^2d^2f^3m^1x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 \\
& + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 2 \\
& 027025a^2b^2d^2f^3m^0x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 2 \\
& 08054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 3a^2b^2 \\
& e^3f^3m^7x^5x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 \\
& + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 177a^2b^2e^3f^3m^6 \\
& x^5x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 103 \\
& 8016m^3 + 2924172m^2 + 4098240m + 2027025) + 4239a^2b^2e^3f^3m^5x^5 \\
& x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 \\
& + 2924172m^2 + 4098240m + 2027025) + 52725a^2b^2e^3f^3m^4x^5x^3x^3 \\
& x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + \\
& 2924172m^2 + 4098240m + 2027025) + 360537a^2b^2e^3f^3m^3x^5x^3x^3 \\
& x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 292 \\
& 4172m^2 + 4098240m + 2027025) + 1311363a^2b^2e^3f^3m^2x^5x^3x^3x^3/(m^ \\
& 8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 292417 \\
& 2m^2 + 4098240m + 2027025) + 2215701a^2b^2e^3f^3m^1x^5x^3x^3x^3/(m^8 + 6 \\
& 4m^7 + 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 \\
& + 4098240m + 2027025) + 1216215a^2b^2e^3f^3m^0x^5x^3x^3x^3/(m^8 + 64m^7 + \\
& 1708m^6 + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 40982 \\
& 40m + 2027025) + 3a^2c^2d^2f^3m^7x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 \\
& + 24640m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 20 \\
& 27025) + 177a^2c^2d^2f^3m^6x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 246 \\
& 40m^5 + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) \\
& + 4239a^2c^2d^2f^3m^5x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 \\
& + 208054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 5272 \\
& 5a^2c^2d^2f^3m^4x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 2 \\
& 08054m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 360537a^2 \\
& c^2d^2f^3m^3x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 20805 \\
& 4m^4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 1311363a^2c^2 \\
& d^2f^3m^2x^5x^3x^3x^3/(m^8 + 64m^7 + 1708m^6 + 24640m^5 + 208054m^ \\
& 4 + 1038016m^3 + 2924172m^2 + 4098240m + 2027025) + 2215701a^2c^2d^2
\end{aligned}$$

$$\begin{aligned}
& 054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 1632285*a*b \\
& **2*e*f**m*m*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**} \\
& *4 + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 868725*a*b**2*e*f \\
& **m*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038 \\
& 016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 6*a*b*c*d*f**m*m**7*x**7*x \\
& **m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + \\
& 2924172*m^{**2} + 4098240*m + 2027025) + 342*a*b*c*d*f**m*m**6*x**7*x**m/(m^{**} \\
& 8 + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172 \\
& *m^{**2} + 4098240*m + 2027025) + 7854*a*b*c*d*f**m*m**5*x**7*x**m/(m^{**8} + 64* \\
& m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + \\
& 4098240*m + 2027025) + 92862*a*b*c*d*f**m*m**4*x**7*x**m/(m^{**8} + 64*m^{**7} + \\
& 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 40982 \\
& 40*m + 2027025) + 598290*a*b*c*d*f**m*m**3*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708 \\
& *m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m \\
& + 2027025) + 2040066*a*b*c*d*f**m*m**2*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**} \\
& 6 + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 20 \\
& 27025) + 3264570*a*b*c*d*f**m*m*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 246 \\
& 40*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) \\
& + 1737450*a*b*c*d*f**m*x**7*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + \\
& 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 6*a*b*c \\
& *e*f**m*m**7*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**} \\
& *4 + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 330*a*b*c*e*f**m* \\
& m**6*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 103 \\
& 8016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 7278*a*b*c*e*f**m*m**5*x* \\
& *9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**} \\
& *3 + 2924172*m^{**2} + 4098240*m + 2027025) + 82338*a*b*c*e*f**m*m**4*x**9*x** \\
& m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2 \\
& 924172*m^{**2} + 4098240*m + 2027025) + 507282*a*b*c*e*f**m*m**3*x**9*x**m/(m^{**} \\
& *8 + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 292417 \\
& 2*m^{**2} + 4098240*m + 2027025) + 1662558*a*b*c*e*f**m*m**2*x**9*x**m/(m^{**8} + \\
& 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**} \\
& *2 + 4098240*m + 2027025) + 2582010*a*b*c*e*f**m*m*x**9*x**m/(m^{**8} + 64*m^{**} \\
& 7 + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 40 \\
& 98240*m + 2027025) + 1351350*a*b*c*e*f**m*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708* \\
& m^{**6} + 24640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + \\
& 2027025) + 3*a*c**2*d*f**m*m**7*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24 \\
& 640*m^{**5} + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) \\
& + 165*a*c**2*d*f**m*m**6*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**} \\
& 5 + 208054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 3639 \\
& *a*c**2*d*f**m*m**5*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 20 \\
& 8054*m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 41169*a*c* \\
& *2*d*f**m*m**4*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054* \\
& m^{**4} + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 253641*a*c**2*d \\
& *f**m*m**3*x**9*x**m/(m^{**8} + 64*m^{**7} + 1708*m^{**6} + 24640*m^{**5} + 208054*m^{**4} \\
& + 1038016*m^{**3} + 2924172*m^{**2} + 4098240*m + 2027025) + 831279*a*c**2*d*f**
\end{aligned}$$

$$\begin{aligned}
& m^{**2}x^{**9}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1 \\
& 038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1291005a^{**2}d^{**m}m \\
& x^{**9}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016 \\
& m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 675675a^{**2}d^{**m}x^{**9}x^{**} \\
& m/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2 \\
& 924172m^{**2} + 4098240m + 2027025) + 3a^{**2}e^{**m}m^{**7}x^{**11}x^{**m}/(m^{**8} \\
& + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m \\
& **2 + 4098240m + 2027025) + 159a^{**2}e^{**m}m^{**6}x^{**11}x^{**m}/(m^{**8} + 64m \\
& **7 + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + \\
& 4098240m + 2027025) + 3375a^{**2}e^{**m}m^{**5}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + \\
& 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 40982 \\
& 40m + 2027025) + 36795a^{**2}e^{**m}m^{**4}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + 170 \\
& 8m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m \\
& + 2027025) + 219417a^{**2}e^{**m}m^{**3}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + 1708m \\
& **6 + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + \\
& 2027025) + 700461a^{**2}e^{**m}m^{**2}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} \\
& + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 202 \\
& 7025) + 1067445a^{**2}e^{**m}m^{**1}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24 \\
& 640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) \\
& + 552825a^{**2}e^{**m}x^{**11}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} \\
& + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + b^{**3} \\
& d^{**m}m^{**7}x^{**7}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**} \\
& 4 + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 57b^{**3}d^{**m}m^{**} \\
& 6x^{**7}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 103801 \\
& 6m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1309b^{**3}d^{**m}m^{**5}x^{**7}x \\
& **m/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + \\
& 2924172m^{**2} + 4098240m + 2027025) + 15477b^{**3}d^{**m}m^{**4}x^{**7}x^{**m}/(m \\
& *8 + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 292417 \\
& 2m^{**2} + 4098240m + 2027025) + 99715b^{**3}d^{**m}m^{**3}x^{**7}x^{**m}/(m^{**8} + 64 \\
& m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} \\
& + 4098240m + 2027025) + 340011b^{**3}d^{**m}m^{**2}x^{**7}x^{**m}/(m^{**8} + 64m^{**7} \\
& + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098 \\
& 240m + 2027025) + 544095b^{**3}d^{**m}m^{**1}x^{**7}x^{**m}/(m^{**8} + 64m^{**7} + 1708m \\
& *6 + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2 \\
& 027025) + 289575b^{**3}d^{**m}x^{**7}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640 \\
& m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + b \\
& **3e^{**m}m^{**7}x^{**9}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054 \\
& m^{**4} + 1038016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 55b^{**3}e^{**m} \\
& m^{**6}x^{**9}x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 10 \\
& 38016m^{**3} + 2924172m^{**2} + 4098240m + 2027025) + 1213b^{**3}e^{**m}m^{**5}x \\
& *9x^{**m}/(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m \\
& *3 + 2924172m^{**2} + 4098240m + 2027025) + 13723b^{**3}e^{**m}m^{**4}x^{**9}x^{**m} \\
& /(m^{**8} + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 29 \\
& 24172m^{**2} + 4098240m + 2027025) + 84547b^{**3}e^{**m}m^{**3}x^{**9}x^{**m}/(m^{**8} \\
& + 64m^{**7} + 1708m^{**6} + 24640m^{**5} + 208054m^{**4} + 1038016m^{**3} + 2924172m
\end{aligned}$$

$$\begin{aligned}
& **2 + 4098240*m + 2027025) + 277093*b**3*e*f**m**2*x**9*x**m/(m**8 + 64*m \\
& **7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + \\
& 4098240*m + 2027025) + 430335*b**3*e*f**m**x**9*x**m/(m**8 + 64*m**7 + 170 \\
& 8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m \\
& + 2027025) + 225225*b**3*e*f**m**x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24 \\
& 640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) \\
& + 3*b**2*c*d*f**m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 \\
& + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165*b* \\
& *2*c*d*f**m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805 \\
& 4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*b**2*c*d \\
& *f**m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 \\
& + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*b**2*c*d*f**m \\
& *m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10 \\
& 38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*b**2*c*d*f**m** \\
& 3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801 \\
& 6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*b**2*c*d*f**m**2*x* \\
& *9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m* \\
& *3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*b**2*c*d*f**m**x**9*x** \\
& m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2 \\
& 924172*m**2 + 4098240*m + 2027025) + 675675*b**2*c*d*f**m**x**9*x**m/(m**8 + \\
& 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m* \\
& *2 + 4098240*m + 2027025) + 3*b**2*c*e*f**m**7*x**11*x**m/(m**8 + 64*m**7 \\
& + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409 \\
& 8240*m + 2027025) + 159*b**2*c*e*f**m**6*x**11*x**m/(m**8 + 64*m**7 + 170 \\
& 8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m \\
& + 2027025) + 3375*b**2*c*e*f**m**5*x**11*x**m/(m**8 + 64*m**7 + 1708*m** \\
& 6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20 \\
& 27025) + 36795*b**2*c*e*f**m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + \\
& 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702 \\
& 5) + 219417*b**2*c*e*f**m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246 \\
& 40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) \\
& + 700461*b**2*c*e*f**m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640* \\
& m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1 \\
& 067445*b**2*c*e*f**m**x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 \\
& + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825 \\
& *b**2*c*e*f**m**x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054 \\
& *m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b*c**2*d*f**m \\
& *m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + \\
& 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 159*b*c**2*d*f**m**6 \\
& *x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801 \\
& 6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b*c**2*d*f**m**5*x**1 \\
& 1*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m** \\
& 3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d*f**m**4*x**11*x* \\
& *m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + \\
& 2924172*m**2 + 4098240*m + 2027025) + 219417*b*c**2*d*f**m**3*x**11*x**m/
\end{aligned}$$


```

*2 + 4098240*m + 2027025) + 57379*c**3*e*f**m*m**3*x**15*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 177331*c**3*e*f**m*m**2*x**15*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 264207*c**3*e*f**m*m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 135135*c**3*e*f**m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025), Tr
ue))

```


$$3.157 \quad \int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^2 dx$$

Optimal. Leaf size=155

$$\frac{a^2 d (fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+7} (2ace + b^2e + 2bcd)}{f^7(m+7)} + \frac{(fx)^{m+5} (2abe + 2acd + b^2d)}{f^5(m+5)} + \frac{a(fx)^{m+3} (ae + 2bd)}{f^3(m+3)} + \frac{c(fx)^{m+9} (2be + cd)}{f^9(m+9)}$$

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1261}

$$\frac{a^2 d (fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+5} (2abe + 2acd + b^2d)}{f^5(m+5)} + \frac{(fx)^{m+7} (2ace + b^2e + 2bcd)}{f^7(m+7)} + \frac{a(fx)^{m+3} (ae + 2bd)}{f^3(m+3)} + \frac{c(fx)^{m+9} (2be + cd)}{f^9(m+9)} + \frac{c^2 e (fx)^{m+11}}{f^{11}(m+11)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]

[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^2 dx &= \int \left(a^2 d (fx)^m + \frac{a(2bd + ae)(fx)^{2+m}}{f^2} + \frac{(b^2d + 2acd + 2abe)(fx)^{4+m}}{f^4} + \right. \\ &= \frac{a^2 d (fx)^{1+m}}{f(1+m)} + \frac{a(2bd + ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(b^2d + 2acd + 2abe)(fx)^{5+m}}{f^5(5+m)} + \left. \frac{c^2 e (fx)^{7+m}}{f^7(7+m)} + \frac{c(c^2 d + 2b^2 e)(fx)^{9+m}}{f^9(9+m)} + \frac{c^2 e (fx)^{11+m}}{f^{11}(11+m)} \right) dx \end{aligned}$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.75

$$x(fx)^m \left(\frac{a^2 d}{m+1} + \frac{x^6 (2ace + b^2e + 2bcd)}{m+7} + \frac{x^4 (2abe + 2acd + b^2d)}{m+5} + \frac{ax^2 (ae + 2bd)}{m+3} + \frac{cx^8 (2be + cd)}{m+9} + \frac{c^2 ex^{10}}{m+11} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

IntegrateAlgebraic [F] time = 0.82, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a + bx^2 + cx^4)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2, x]
```

fricas [B] time = 0.70, size = 573, normalized size = 3.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m + 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 262*(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)*m^2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5 + 31*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*m^3 + 4158*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*c)*d + 3489*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x*(f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.40, size = 1178, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*m^5*x^11*e + 25*(f*x)^m*c^2*m^4*x^11*e + (f*x)^m*c^2*d*m^5*x^9
+ 2*(f*x)^m*b*c*m^5*x^9*e + 230*(f*x)^m*c^2*m^3*x^11*e + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*m^4*x^9*e + 950*(f*x)^m*c^2*m^2*x^11*e + 2*(f*x)^m
*b*c*d*m^5*x^7 + 262*(f*x)^m*c^2*d*m^3*x^9 + (f*x)^m*b^2*m^5*x^7*e + 2*(f*x
)^m*a*c*m^5*x^7*e + 524*(f*x)^m*b*c*m^3*x^9*e + 1689*(f*x)^m*c^2*m*x^11*e +
58*(f*x)^m*b*c*d*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 29*(f*x)^m*b^2*m^4
*x^7*e + 58*(f*x)^m*a*c*m^4*x^7*e + 2244*(f*x)^m*b*c*m^2*x^9*e + 945*(f*x)^
m*c^2*x^11*e + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 604*(f*x)^
m*b*c*d*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 2*(f*x)^m*a*b*m^5*x^5*e + 302*
(f*x)^m*b^2*m^3*x^7*e + 604*(f*x)^m*a*c*m^3*x^7*e + 4082*(f*x)^m*b*c*m*x^9*
e + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 2732*(f*x)^m*b*c*
d*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 62*(f*x)^m*a*b*m^4*x^5*e + 1366*(f*x)^
m*b^2*m^2*x^7*e + 2732*(f*x)^m*a*c*m^2*x^7*e + 2310*(f*x)^m*b*c*x^9*e + 2*(
f*x)^m*a*b*d*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^
5 + 5154*(f*x)^m*b*c*d*m*x^7 + (f*x)^m*a^2*m^5*x^3*e + 700*(f*x)^m*a*b*m^3*
x^5*e + 2577*(f*x)^m*b^2*m*x^7*e + 5154*(f*x)^m*a*c*m*x^7*e + 66*(f*x)^m*a*
b*d*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 297
0*(f*x)^m*b*c*d*x^7 + 33*(f*x)^m*a^2*m^4*x^3*e + 3460*(f*x)^m*a*b*m^2*x^5*e
+ 1485*(f*x)^m*b^2*x^7*e + 2970*(f*x)^m*a*c*x^7*e + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 3489*(f*x)^m*b^2*d*m*x^5 + 6978*(f*x)^m*a*c*d*m
*x^5 + 406*(f*x)^m*a^2*m^3*x^3*e + 6978*(f*x)^m*a*b*m*x^5*e + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2079*(f*x)^m*b^2*d*x^5 + 4158*(f*x
)^m*a*c*d*x^5 + 2262*(f*x)^m*a^2*m^2*x^3*e + 4158*(f*x)^m*a*b*x^5*e + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*m*x^3*e +
3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*x^3*e
+ 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

maple [B] time = 0.01, size = 783, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
```

$$3a^2e^m x^4 + 66ab^2d^2 x^2 + 3460a^2 b^2 d^2 x^2 + 3460a^2 b^2 d^2 x^2 + 3460a^2 b^2 d^2 x^2 + 2970a^2 c^2 e^m x^6 + 1730b^2 d^2 m^2 x^4 + 1485b^2 e^m x^6 + 2970b^2 c^2 d^2 x^6 + a^2 d^2 m^5 + 406a^2 e^m x^3 x^2 + 812a^2 b^2 d^2 m^3 x^2 + 6978a^2 b^2 e^m x^4 + 6978a^2 c^2 d^2 m^3 x^4 + 3489b^2 d^2 m^3 x^4 + 35a^2 d^2 m^4 + 2262a^2 e^m x^2 + 4524a^2 b^2 d^2 m^2 x^2 + 4158a^2 b^2 e^m x^4 + 4158a^2 c^2 d^2 x^4 + 2079b^2 d^2 x^4 + 470a^2 d^2 m^3 + 5353a^2 e^m x^2 + 10706a^2 b^2 d^2 m^3 x^2 + 3010a^2 d^2 m^2 + 3465a^2 e^m x^2 + 6930a^2 b^2 d^2 x^2 + 9129a^2 d^2 m + 10395a^2 d^2) (f x)^m / (m+11) / (m+9) / (m+7) / (m+5) / (m+3) / (m+1)$$

maxima [A] time = 1.22, size = 230, normalized size = 1.48

$$\frac{c^2 e^m x^{11} x^m}{m+11} + \frac{c^2 d^2 f^m x^9 x^m}{m+9} + \frac{2bc e^m x^9 x^m}{m+9} + \frac{2bcd^2 f^m x^7 x^m}{m+7} + \frac{b^2 e^m x^7 x^m}{m+7} + \frac{2ace^m x^7 x^m}{m+7} + \frac{b^2 d^2 f^m x^5 x^m}{m+5} + \frac{2acd^2 f^m x^5 x^m}{m+5} + \frac{2ab e^m x^5 x^m}{m+5} + \frac{2abd^2 f^m x^3 x^m}{m+3} + \frac{a^2 e^m x^3 x^m}{m+3} + \frac{(f x)^{m+1} a^2 d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] $c^2 e^m f^m x^{11} x^m / (m+11) + c^2 d^2 f^m x^9 x^m / (m+9) + 2b^2 c^2 e^m f^m x^9 x^m / (m+9) + 2b^2 c^2 d^2 f^m x^7 x^m / (m+7) + b^2 e^m f^m x^7 x^m / (m+7) + 2a^2 c^2 e^m f^m x^7 x^m / (m+7) + b^2 d^2 e^m f^m x^5 x^m / (m+5) + 2a^2 c^2 d^2 f^m x^5 x^m / (m+5) + 2a^2 b^2 e^m f^m x^5 x^m / (m+5) + 2a^2 b^2 d^2 f^m x^3 x^m / (m+3) + a^2 e^m f^m x^3 x^m / (m+3) + (f x)^{m+1} a^2 d / (f(m+1))$

mupad [B] time = 0.60, size = 429, normalized size = 2.77

$$\frac{c^2 e^m (f^m x^{11} x^m + 2c^2 d^2 f^m x^9 x^m + 2bc e^m x^9 x^m + 2bcd^2 f^m x^7 x^m + b^2 e^m x^7 x^m + 2ace^m x^7 x^m + b^2 d^2 f^m x^5 x^m + 2acd^2 f^m x^5 x^m + 2ab e^m x^5 x^m + 2abd^2 f^m x^3 x^m + a^2 e^m x^3 x^m)}{(m+11)(m+9)(m+7)(m+5)(m+3)} + \frac{(f x)^{m+1} a^2 d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)

[Out] $(x^5 (f x)^m (b^2 d + 2a^2 b e + 2a^2 c d) (3489 m + 1730 m^2 + 350 m^3 + 31 m^4 + m^5 + 2079)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395) + (x^7 (f x)^m (b^2 e + 2a^2 c e + 2b^2 c d) (2577 m + 1366 m^2 + 302 m^3 + 29 m^4 + m^5 + 1485)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395) + (a^2 d x x (f x)^m (9129 m + 3010 m^2 + 470 m^3 + 35 m^4 + m^5 + 10395)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395) + (a x^3 (f x)^m (a e + 2 b d) (5353 m + 2262 m^2 + 406 m^3 + 33 m^4 + m^5 + 3465)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395) + (c x^9 (f x)^m (2 b e + c d) (2041 m + 1122 m^2 + 262 m^3 + 27 m^4 + m^5 + 1155)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395) + (c^2 e x^{11} (f x)^m (1689 m + 950 m^2 + 230 m^3 + 25 m^4 + m^5 + 945)) / (19524 m + 12139 m^2 + 3480 m^3 + 505 m^4 + 36 m^5 + m^6 + 10395)$

sympy [A] time = 5.44, size = 4190, normalized size = 27.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)

[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11, Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2) - b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m, -9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 - a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2 + c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(m, -1)), (a**2*d*f**m*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**2*d*f**m*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*f**m*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**2*d*f**m*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*d*f**m*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*f**m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + a**2*e*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 33*a**2*e*f**m*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2262*a**2*e*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*f**m*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465*a**2*e*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*d*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 66*a*b*d*f**m*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 812*a*b*d*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4524*a*b*d*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*f**m*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6930*a*b*d*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*e*f**m*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*f**m*m**4

$$\begin{aligned}
& 4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*b*e*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*b*e*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b*e*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4158*a*b*e*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*d*f**m*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*c*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*c*d*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*c*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4158*a*c*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*e*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 58*a*c*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*a*c*e*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*a*c*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*a*c*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2970*a*c*e*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**2*d*f**m*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 31*b**2*d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b**2*d*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3489*b**2*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**2*e*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29*b**2*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 302*b**2*e*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1366*b**2*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2577*b**2*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1485*b**2*e*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*b*c*d*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 58*b*c*d*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*b*c*d*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*b*c*d*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*b*c*d*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
\end{aligned}$$

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*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
  2970*b*c*d*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 2*b*c*e*f**m**5*x**9*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b*c*e*f**m**4*x**9*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
  + 524*b*c*e*f**m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 2244*b*c*e*f**m**2*x**9*x**m/(m**6 + 36*m*
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4082*b*c*e*f**m
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 2310*b*c*e*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + c**2*d*f**m**5*x**9*x**m/(m**6 + 36*m*
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 27*c**2*d*f**m
**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 262*c**2*d*f**m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 1122*c**2*d*f**m**2*x**9*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 204
1*c**2*d*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
**2 + 19524*m + 10395) + 1155*c**2*d*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m*
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*e*f**m**5*x**11*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
25*c**2*e*f**m**4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 230*c**2*e*f**m**3*x**11*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e*f**m
**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 1689*c**2*e*f**m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 945*c**2*e*f**m*x**11*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))

```

$$3.158 \quad \int (fx)^m (d + ex^2) (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=83

$$\frac{(fx)^{m+3}(ae + bd)}{f^3(m+3)} + \frac{ad(fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+5}(be + cd)}{f^5(m+5)} + \frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$\frac{(fx)^{m+3}(ae + bd)}{f^3(m+3)} + \frac{ad(fx)^{m+1}}{f(m+1)} + \frac{(fx)^{m+5}(be + cd)}{f^5(m+5)} + \frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]

[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7 + m))

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + bx^2 + cx^4) dx &= \int \left(ad(fx)^m + \frac{(bd + ae)(fx)^{2+m}}{f^2} + \frac{(cd + be)(fx)^{4+m}}{f^4} + \frac{ce(fx)^{6+m}}{f^6} \right) dx \\ &= \frac{ad(fx)^{1+m}}{f(1+m)} + \frac{(bd + ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(cd + be)(fx)^{5+m}}{f^5(5+m)} + \frac{ce(fx)^{7+m}}{f^7(7+m)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.71

$$x(fx)^m \left(\frac{x^2(ae + bd)}{m+3} + \frac{ad}{m+1} + \frac{x^4(be + cd)}{m+5} + \frac{cex^6}{m+7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4), x]

[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5 + m) + (c*e*x^6)/(7 + m))

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4), x]

[Out] Defer[IntegrateAlgebraic] [(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4), x]

fricas [B] time = 0.72, size = 171, normalized size = 2.06

$$\frac{(cem^3 + 9cem^2 + 23cem + 15ce)x^7 + ((cd + be)m^3 + 11(cd + be)m^2 + 21cd + 21be + 31(cd + be)m)x^5 + ((bd + ae)m^3 + 13(bd + ae)m^2 + 35bd + 35ae + 47(bd + ae)m)x^3 + (adm^3 + 15adm^2 + 71adm + 105ad)x}{m^4 + 16m^3 + 86m^2 + 176m + 105}(fx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*d + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3 + 13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3 + 15*a*d*m^2 + 71*a*d*m + 105*a*d)*x*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

giac [B] time = 0.42, size = 350, normalized size = 4.22

$$\frac{(c^2em^3 + 9cem^2 + 23cem + 15ce)x^7 + ((cd + be)m^3 + 11(cd + be)m^2 + 21cd + 21be + 31(cd + be)m)x^5 + ((bd + ae)m^3 + 13(bd + ae)m^2 + 35bd + 35ae + 47(bd + ae)m)x^3 + (adm^3 + 15adm^2 + 71adm + 105ad)x}{m^4 + 16m^3 + 86m^2 + 176m + 105}(fx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] ((f*x)^m*c*m^3*x^7*e + 9*(f*x)^m*c*m^2*x^7*e + (f*x)^m*c*d*m^3*x^5 + (f*x)^m*b*m^3*x^5*e + 23*(f*x)^m*c*m*x^7*e + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*b*m^2*x^5*e + 15*(f*x)^m*c*x^7*e + (f*x)^m*b*d*m^3*x^3 + 31*(f*x)^m*c*d*m*x^5 + (f*x)^m*a*m^3*x^3*e + 31*(f*x)^m*b*m*x^5*e + 13*(f*x)^m*b*d*m^2*x^3 + 21*(f*x)^m*c*d*x^5 + 13*(f*x)^m*a*m^2*x^3*e + 21*(f*x)^m*b*x^5*e + (f*x)^m*a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*m*x^3*e + 15*(f*x)^m*a*d*m^2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*x^3*e + 71*(f*x)^m*a*d*m*x + 105*(f*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

maple [B] time = 0.00, size = 221, normalized size = 2.66

$$\frac{(cm^3x^6 + 9cem^2x^6 + be m^3x^4 + cd m^3x^4 + 23cemx^6 + 11be m^2x^4 + 11cd m^2x^4 + 15cemx^6 + ae m^3x^2 + bd m^3x^2 + 31bemx^4 + 31cdm x^4 + 13ae m^2x^2 + 13bd m^2x^2 + 21be x^4 + 21cd x^4 + ad m^3 + 47adm x^2 + 15ad m^2 + 35ae x^2 + 35bd x^2 + 71adm + 105ad)x}{(m+7)(m+5)(m+3)(m+1)}(fx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x)`

[Out] $x*(c*e*m^3*x^6+9*c*e*m^2*x^6+b*e*m^3*x^4+c*d*m^3*x^4+23*c*e*m*x^6+11*b*e*m^2*x^4+11*c*d*m^2*x^4+15*c*e*x^6+a*e*m^3*x^2+b*d*m^3*x^2+31*b*e*m*x^4+31*c*d*m*x^4+13*a*e*m^2*x^2+13*b*d*m^2*x^2+21*b*e*x^4+21*c*d*x^4+a*d*m^3+47*a*e*m*x^2+47*b*d*m*x^2+15*a*d*m^2+35*a*e*x^2+35*b*d*x^2+71*a*d*m+105*a*d)*(f*x)^m/(m+7)/(m+5)/(m+3)/(m+1)$

maxima [A] time = 1.06, size = 104, normalized size = 1.25

$$\frac{cef^m x^7 x^m}{m+7} + \frac{cdf^m x^5 x^m}{m+5} + \frac{bef^m x^5 x^m}{m+5} + \frac{bdf^m x^3 x^m}{m+3} + \frac{aef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] $c*e*f^m*x^7*x^m/(m+7) + c*d*f^m*x^5*x^m/(m+5) + b*e*f^m*x^5*x^m/(m+5) + b*d*f^m*x^3*x^m/(m+3) + a*e*f^m*x^3*x^m/(m+3) + (f*x)^{(m+1)}*a*d/(f*(m+1))$

mupad [B] time = 0.34, size = 171, normalized size = 2.06

$$(fx)^m \left(\frac{x^3 (ae+bd) (m^3+13m^2+47m+35)}{m^4+16m^3+86m^2+176m+105} + \frac{x^5 (be+cd) (m^3+11m^2+31m+21)}{m^4+16m^3+86m^2+176m+105} + \frac{adx (m^3+15m^2+71m+105)}{m^4+16m^3+86m^2+176m+105} + \frac{ce x^7 (m^3+9m^2+23m+15)}{m^4+16m^3+86m^2+176m+105} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)`

[Out] $(f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))$

sympy [A] time = 1.86, size = 1056, normalized size = 12.72



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)`

```
[Out] Piecewise((( -a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) - b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 + c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*f**m*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a*d*f**m*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a*d*f**m*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a*d*f**m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + a*e*f**m*m**3*x*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*f**m*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*f**m*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*f**m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*d*f**m*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*f**m*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*b*d*f**m*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*b*d*f**m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*e*f**m*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b*e*f**m*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*b*e*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*f**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*f**m*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*c*d*f**m*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*c*d*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*c*d*f**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*e*f**m*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*c*e*f**m*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*c*e*f**m*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*c*e*f**m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

$$3.159 \quad \int \frac{x^9}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=134

$$\frac{a^{3/2}d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2 + cd^2)} - \frac{a^2e \log(a + cx^4)}{4c^2(ae^2 + cd^2)} + \frac{d^4 \log(d + ex^2)}{2e^3(ae^2 + cd^2)} - \frac{dx^2}{2ce^2} + \frac{x^4}{4ce}$$

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^2e \log(a + cx^4)}{4c^2(ae^2 + cd^2)} + \frac{a^{3/2}d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2 + cd^2)} + \frac{d^4 \log(d + ex^2)}{2e^3(ae^2 + cd^2)} - \frac{dx^2}{2ce^2} + \frac{x^4}{4ce}$$

Antiderivative was successfully verified.

[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]

[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m + 1)/2]$

Rule 1629

$\text{Int}[(\text{Pq}_.) * ((d_.) + (e_.) * (x_.))^{(m_.)} * ((a_.) + (c_.) * (x_.)^2)^{(p_.)}, x_Symbol]$
 $:\> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m * \text{Pq} * (a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c,$
 $d, e, m\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{IGtQ}[p, -2]$

Rubi steps

$$\begin{aligned} \int \frac{x^9}{(d + ex^2)(a + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(d + ex)(a + cx^2)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{d}{ce^2} + \frac{x}{ce} + \frac{d^4}{e^2(cd^2 + ae^2)(d + ex)} + \frac{a^2(d - ex)}{c(cd^2 + ae^2)(a + cx^2)} \right) dx, x, x^2 \right) \\ &= -\frac{dx^2}{2ce^2} + \frac{x^4}{4ce} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 + ae^2)} + \frac{a^2 \text{Subst} \left(\int \frac{d - ex}{a + cx^2} dx, x, x^2 \right)}{2c(cd^2 + ae^2)} \\ &= -\frac{dx^2}{2ce^2} + \frac{x^4}{4ce} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 + ae^2)} + \frac{(a^2 d) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right)}{2c(cd^2 + ae^2)} - \frac{(a^2 e) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right)}{2c(cd^2 + ae^2)} \\ &= -\frac{dx^2}{2ce^2} + \frac{x^4}{4ce} + \frac{a^{3/2} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2c^{3/2}(cd^2 + ae^2)} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 + ae^2)} - \frac{a^2 e \log(a + cx^4)}{4c^2(cd^2 + ae^2)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 134, normalized size = 1.00

$$\frac{a^{3/2} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2c^{3/2}(ae^2 + cd^2)} - \frac{a^2 e \log(a + cx^4)}{4c^2(ae^2 + cd^2)} + \frac{d^4 \log(d + ex^2)}{2e^3(ae^2 + cd^2)} - \frac{dx^2}{2ce^2} + \frac{x^4}{4ce}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)), x]

[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(d + ex^2)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 11.00, size = 277, normalized size = 2.07

$$\left[\frac{acde^3 \sqrt{\frac{a}{c}} \log\left(\frac{cx^4 + 2cx^2\sqrt{\frac{a}{c}} - a}{cx^4 + a}\right) - a^2e^4 \log(cx^4 + a) + 2c^2d^4 \log(ex^2 + d) + (c^2d^2e^2 + ace^4)x^4 - 2(c^2d^2e + acde^3)x^2}{4(c^3d^2e^3 + ae^5)}, \frac{2acde^3 \sqrt{\frac{a}{c}} \arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) - a^2e^4 \log(cx^4 + a) + 2c^2d^4 \log(ex^2 + d) + (c^2d^2e^2 + ace^4)x^4 - 2(c^2d^2e + acde^3)x^2}{4(c^3d^2e^3 + ae^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4*(2*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]

giac [A] time = 0.31, size = 121, normalized size = 0.90

$$\frac{d^4 \log(|x^2e + d|)}{2(c^2d^2e^3 + ae^5)} - \frac{a^2e \log(cx^4 + a)}{4(c^3d^2 + ac^2e^2)} + \frac{a^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2 + ace^2)\sqrt{ac}} + \frac{(cx^4e - 2cdx^2)e^{(-2)}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-2)/c^2

maple [A] time = 0.01, size = 122, normalized size = 0.91

$$\frac{a^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}c} - \frac{a^2e \ln(cx^4 + a)}{4(ae^2 + cd^2)c^2} + \frac{x^4}{4ce} + \frac{d^4 \ln(ex^2 + d)}{2(ae^2 + cd^2)e^3} - \frac{dx^2}{2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(e*x^2+d)/(c*x^4+a),x)`

[Out] $\frac{1}{4}x^4/c/e - \frac{1}{2}d*x^2/c/e^2 - \frac{1}{4}a^2*e*\ln(cx^4+a)/c^2/(a*e^2+c*d^2) + \frac{1}{2}a^2/(a*e^2+c*d^2)/c*d/(a*c)^{(1/2)}*\arctan(cx^2/(a*c)^{(1/2)}) + \frac{1}{2}d^4*\ln(e*x^2+d)/e^3/(a*e^2+c*d^2)$

maxima [A] time = 2.00, size = 120, normalized size = 0.90

$$\frac{d^4 \log(ex^2 + d)}{2(cd^2e^3 + ae^5)} - \frac{a^2e \log(cx^4 + a)}{4(c^3d^2 + ac^2e^2)} + \frac{a^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2 + ace^2)\sqrt{ac}} + \frac{ex^4 - 2dx^2}{4ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")`

[Out] $\frac{1}{2}d^4*\log(e*x^2 + d)/(c*d^2*e^3 + a*e^5) - \frac{1}{4}a^2*e*\log(cx^4 + a)/(c^3*d^2 + a*c^2*e^2) + \frac{1}{2}a^2*d*\arctan(cx^2/\sqrt{a*c})/((c^2*d^2 + a*c*e^2)*\sqrt{a*c}) + \frac{1}{4}*(e*x^4 - 2*d*x^2)/(c*e^2)$

mupad [B] time = 0.87, size = 181, normalized size = 1.35

$$\frac{\ln(\sqrt{-a^3c^5 + ac^3x^2})(d\sqrt{-a^3c^5} - a^2c^2e)}{4c^5d^2 + 4ac^4e^2} - \frac{\ln(\sqrt{-a^3c^5} - ac^3x^2)(d\sqrt{-a^3c^5} + a^2c^2e)}{4(c^5d^2 + ac^4e^2)} + \frac{d^4 \ln(ex^2 + d)}{2cd^2e^3 + 2ae^5} + \frac{x^4}{4ce} - \frac{dx^2}{2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/((a + c*x^4)*(d + e*x^2)),x)`

[Out] $(\log((-a^3*c^5)^{(1/2)} + a*c^3*x^2)*(d*(-a^3*c^5)^{(1/2)} - a^2*c^2*e))/(4*c^5*d^2 + 4*a*c^4*e^2) - (\log((-a^3*c^5)^{(1/2)} - a*c^3*x^2)*(d*(-a^3*c^5)^{(1/2)} + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*\log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(e*x**2+d)/(c*x**4+a),x)`

[Out] Timed out

$$3.160 \quad \int \frac{x^7}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=118

$$-\frac{a^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2+cd^2)} - \frac{ad \log(a+cx^4)}{4c(ae^2+cd^2)} - \frac{d^3 \log(d+ex^2)}{2e^2(ae^2+cd^2)} + \frac{x^2}{2ce}$$

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2+cd^2)} - \frac{d^3 \log(d+ex^2)}{2e^2(ae^2+cd^2)} - \frac{ad \log(a+cx^4)}{4c(ae^2+cd^2)} + \frac{x^2}{2ce}$$

Antiderivative was successfully verified.

[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]

[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^4])/(4*c*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /;$ FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(d + ex^2)(a + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(d + ex)(a + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2 + ae^2)(d + ex)} - \frac{a(ae + cdx)}{c(cd^2 + ae^2)(a + cx^2)} \right) dx, x, x^2 \right) \\
 &= \frac{x^2}{2ce} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 + ae^2)} - \frac{a \text{Subst} \left(\int \frac{ae + cdx}{a + cx^2} dx, x, x^2 \right)}{2c(cd^2 + ae^2)} \\
 &= \frac{x^2}{2ce} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 + ae^2)} - \frac{(ad) \text{Subst} \left(\int \frac{x}{a + cx^2} dx, x, x^2 \right)}{2(cd^2 + ae^2)} - \frac{(a^2e) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right)}{2c(cd^2 + ae^2)} \\
 &= \frac{x^2}{2ce} - \frac{a^{3/2}e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2c^{3/2}(cd^2 + ae^2)} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 + ae^2)} - \frac{ad \log(a + cx^4)}{4c(cd^2 + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 99, normalized size = 0.84

$$\frac{-\frac{2a^{3/2}e^3 \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{c^{3/2}} + \frac{e(2x^2(ae^2 + cd^2) - ade \log(a + cx^4))}{c} - 2d^3 \log(d + ex^2)}{4e^2(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)), x]

[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 + a*e^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d + ex^2)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 5.18, size = 212, normalized size = 1.80

$$\left[\frac{ae^3 \sqrt{\frac{a}{c}} \log\left(\frac{cx^4 - 2cx^2 \sqrt{\frac{a}{c}} - a}{cx^4 + a}\right) - ade^2 \log(cx^4 + a) - 2cd^3 \log(ex^2 + d) + 2(cd^2e + ae^3)x^2}{4(c^2d^2e^2 + ace^4)}, \frac{2ae^3 \sqrt{\frac{a}{c}} \arctan\left(\frac{cx^2 \sqrt{\frac{a}{c}}}{a}\right) + ade^2 \log(cx^4 + a) + 2cd^3 \log(ex^2 + d) - 2(cd^2e + ae^3)x^2}{4(c^2d^2e^2 + ace^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) - a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2)/(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2)/(c^2*d^2*e^2 + a*c*e^4)]

giac [A] time = 0.31, size = 105, normalized size = 0.89

$$-\frac{d^3 \log(|x^2e + d|)}{2(cd^2e^2 + ae^4)} - \frac{a^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2(c^2d^2 + ace^2)\sqrt{ac}} + \frac{x^2e^{(-1)}}{2c} - \frac{ad \log(cx^4 + a)}{4(c^2d^2 + ace^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqrt(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2)

maple [A] time = 0.01, size = 108, normalized size = 0.92

$$-\frac{a^2e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}c} - \frac{ad \ln(cx^4 + a)}{4(ae^2 + cd^2)c} - \frac{d^3 \ln(ex^2 + d)}{2(ae^2 + cd^2)e^2} + \frac{x^2}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(e*x^2+d)/(c*x^4+a),x)`

[Out] $\frac{1}{2}x^2/c/e - 1/4*a*d*\ln(c*x^4+a)/c/(a*e^2+c*d^2) - 1/2*a^2/(a*e^2+c*d^2)/c*e/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2) - 1/2*d^3*\ln(e*x^2+d)/e^2/(a*e^2+c*d^2)$

maxima [A] time = 2.02, size = 107, normalized size = 0.91

$$-\frac{d^3 \log(ex^2 + d)}{2(cd^2e^2 + ae^4)} - \frac{a^2e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2 + ace^2)\sqrt{ac}} - \frac{ad \log(cx^4 + a)}{4(c^2d^2 + ace^2)} + \frac{x^2}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")`

[Out] $-1/2*d^3*\log(e*x^2 + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*\arctan(c*x^2/\sqrt{a*c})/((c^2*d^2 + a*c*e^2)*\sqrt{a*c}) - 1/4*a*d*\log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*x^2/(c*e)$

mupad [B] time = 0.72, size = 166, normalized size = 1.41

$$\frac{x^2}{2ce} - \frac{d^3 \ln(ex^2 + d)}{2cd^2e^2 + 2ae^4} - \frac{\ln(\sqrt{-a^3c^3} + ac^2x^2)(e\sqrt{-a^3c^3} + ac^2d)}{4(c^4d^2 + ac^3e^2)} + \frac{\ln(\sqrt{-a^3c^3} - ac^2x^2)(e\sqrt{-a^3c^3} - ac^2d)}{4c^4d^2 + 4ac^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/((a + c*x^4)*(d + e*x^2)),x)`

[Out] $x^2/(2*c*e) - (d^3*\log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (\log((-a^3*c^3)^{(1/2)} + a*c^2*x^2)*(e*(-a^3*c^3)^{(1/2)} + a*c^2*d))/(4*(c^4*d^2 + a*c^3*e^2)) + (\log((-a^3*c^3)^{(1/2)} - a*c^2*x^2)*(e*(-a^3*c^3)^{(1/2)} - a*c^2*d))/(4*c^4*d^2 + 4*a*c^3*e^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(e*x**2+d)/(c*x**4+a),x)`

[Out] Timed out

$$3.161 \quad \int \frac{x^5}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=105

$$\frac{ae \log(a+cx^4)}{4c(ae^2+cd^2)} + \frac{d^2 \log(d+ex^2)}{2e(ae^2+cd^2)} - \frac{\sqrt{a} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}(ae^2+cd^2)}$$

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 1629, 635, 205, 260}

$$\frac{d^2 \log(d+ex^2)}{2e(ae^2+cd^2)} + \frac{ae \log(a+cx^4)}{4c(ae^2+cd^2)} - \frac{\sqrt{a} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]

[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[c]*(c*d^2 + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m + 1)/2]$

Rule 1629

$\text{Int}[(Pq_)*((d_)+(e_)*(x_))^{(m_)}*((a_)+(c_)*(x_)^2)^{(p_)}, x_Symbol]$
 $:\> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c,$
 $d, e, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(d+ex^2)(a+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(d+ex)(a+cx^2)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d^2}{(cd^2+ae^2)(d+ex)} - \frac{a(d-ex)}{(cd^2+ae^2)(a+cx^2)} \right) dx, x, x^2 \right) \\ &= \frac{d^2 \log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{a \text{Subst} \left(\int \frac{d-ex}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} \\ &= \frac{d^2 \log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{(ad) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} + \frac{(ae) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} \\ &= -\frac{\sqrt{a} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{c}(cd^2+ae^2)} + \frac{d^2 \log(d+ex^2)}{2e(cd^2+ae^2)} + \frac{ae \log(a+cx^4)}{4c(cd^2+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.73

$$\frac{-2\sqrt{a}\sqrt{c}de \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + ae^2 \log(a+cx^4) + 2cd^2 \log(d+ex^2)}{4ace^3 + 4c^2d^2e}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)), x]

[Out] $(-2*\text{Sqrt}[a]*\text{Sqrt}[c]*d*e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]] + 2*c*d^2*\text{Log}[d + e*x^2] + a*e^2*\text{Log}[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 2.92, size = 170, normalized size = 1.62

$$\left[\frac{cde\sqrt{\frac{a}{c}} \log\left(\frac{cx^4 - 2cx^2\sqrt{\frac{a}{c}} - a}{cx^4 + a}\right) + ae^2 \log(cx^4 + a) + 2cd^2 \log(ex^2 + d)}{4(c^2d^2e + ace^3)}, -\frac{2cde\sqrt{\frac{a}{c}} \arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) - ae^2 \log(cx^4 + a) - 2cd^2 \log(ex^2 + d)}{4(c^2d^2e + ace^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/4*(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]

giac [A] time = 0.36, size = 90, normalized size = 0.86

$$\frac{ae \log(cx^4 + a)}{4(c^2d^2 + ace^2)} + \frac{d^2 \log(|x^2e + d|)}{2(cd^2e + ae^3)} - \frac{ad \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))

maple [A] time = 0.01, size = 92, normalized size = 0.88

$$-\frac{ad \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}} + \frac{ae \ln(cx^4 + a)}{4(ae^2 + cd^2)c} + \frac{d^2 \ln(ex^2 + d)}{2(ae^2 + cd^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e*x^2+d)/(c*x^4+a), x)

[Out] 1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)

maxima [A] time = 2.00, size = 89, normalized size = 0.85

$$\frac{ae \log(cx^4 + a)}{4(c^2d^2 + ace^2)} + \frac{d^2 \log(ex^2 + d)}{2(cd^2e + ae^3)} - \frac{ad \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(e*x^2 + d)/(c*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))

mupad [B] time = 0.99, size = 138, normalized size = 1.31

$$\frac{d^2 \ln(ex^2 + d)}{2cd^2e + 2ae^3} - \frac{\ln(\sqrt{-ac^3} + c^2x^2)(d\sqrt{-ac^3} - ace)}{4(c^3d^2 + ac^2e^2)} + \frac{\ln(\sqrt{-ac^3} - c^2x^2)(d\sqrt{-ac^3} + ace)}{4c^3d^2 + 4ac^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + c*x^4)*(d + e*x^2)),x)

[Out] (d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2)*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1/2) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.162 \quad \int \frac{x^3}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=96

$$\frac{d \log(a+cx^4)}{4(ae^2+cd^2)} - \frac{d \log(d+ex^2)}{2(ae^2+cd^2)} + \frac{\sqrt{a} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}(ae^2+cd^2)}$$

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 801, 635, 205, 260}

$$-\frac{d \log(d+ex^2)}{2(ae^2+cd^2)} + \frac{d \log(a+cx^4)}{4(ae^2+cd^2)} + \frac{\sqrt{a} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]

[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],

`x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]`

Rule 1252

`Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{(d + ex^2)(a + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(d + ex)(a + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{de}{(cd^2 + ae^2)(d + ex)} + \frac{ae + cdx}{(cd^2 + ae^2)(a + cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{d \log(d + ex^2)}{2(cd^2 + ae^2)} + \frac{\text{Subst} \left(\int \frac{ae + cdx}{a + cx^2} dx, x, x^2 \right)}{2(cd^2 + ae^2)} \\
 &= -\frac{d \log(d + ex^2)}{2(cd^2 + ae^2)} + \frac{(cd) \text{Subst} \left(\int \frac{x}{a + cx^2} dx, x, x^2 \right)}{2(cd^2 + ae^2)} + \frac{(ae) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right)}{2(cd^2 + ae^2)} \\
 &= \frac{\sqrt{a} e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{c} (cd^2 + ae^2)} - \frac{d \log(d + ex^2)}{2(cd^2 + ae^2)} + \frac{d \log(a + cx^4)}{4(cd^2 + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.69

$$\frac{d \log(a + cx^4) + \frac{2\sqrt{a} e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{\sqrt{c}} - 2d \log(d + ex^2)}{4ae^2 + 4cd^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/((d + e*x^2)*(a + c*x^4)), x]`

[Out] `((2*sqrt[a]*e*ArcTan[(sqrt[c]*x^2)/sqrt[a]])/sqrt[c] - 2*d*Log[d + e*x^2] + d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)`

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d + ex^2)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 1.33, size = 145, normalized size = 1.51

$$\left[\frac{e\sqrt{\frac{a}{c}} \log\left(\frac{cx^4+2cx^2\sqrt{\frac{a}{c}}-a}{cx^4+a}\right) + d \log(cx^4+a) - 2d \log(ex^2+d)}{4(cd^2+ae^2)}, \frac{2e\sqrt{\frac{a}{c}} \arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) + d \log(cx^4+a) - 2d \log(ex^2+d)}{4(cd^2+ae^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2)]

giac [A] time = 0.39, size = 86, normalized size = 0.90

$$-\frac{de \log(|x^2e + d|)}{2(cd^2e + ae^3)} + \frac{a \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2(cd^2 + ae^2)\sqrt{ac}} + \frac{d \log(cx^4 + a)}{4(cd^2 + ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^2/sqrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{ae \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}} + \frac{d \ln(cx^4 + a)}{4ae^2 + 4cd^2} - \frac{d \ln(ex^2 + d)}{2(ae^2 + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e*x^2+d)/(c*x^4+a), x)

[Out] 1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*a*e/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{ae \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} + \frac{d \log(cx^4 + a)}{4(cd^2 + ae^2)} - \frac{d \log(ex^2 + d)}{2(cd^2 + ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] 1/2*a*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(e*x^2 + d)/(c*d^2 + a*e^2)

mupad [B] time = 1.94, size = 944, normalized size = 9.83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^4)*(d + e*x^2)),x)

[Out] (c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d*log(d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^2*(-a*c)^(1/2)))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

$$3.163 \quad \int \frac{x}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=96

$$-\frac{e \log(a+cx^4)}{4(ae^2+cd^2)} + \frac{e \log(d+ex^2)}{2(ae^2+cd^2)} + \frac{\sqrt{c} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)}$$

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1248, 706, 31, 635, 205, 260}

$$\frac{e \log(d+ex^2)}{2(ae^2+cd^2)} - \frac{e \log(a+cx^4)}{4(ae^2+cd^2)} + \frac{\sqrt{c} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]

[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(d+ex^2)(a+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(d+ex)(a+cx^2)} dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{cd-cex}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} + \frac{e^2 \text{Subst} \left(\int \frac{1}{d+ex} dx, x, x^2 \right)}{2(cd^2+ae^2)} \\ &= \frac{e \log(d+ex^2)}{2(cd^2+ae^2)} + \frac{(cd) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} - \frac{(ce) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} \\ &= \frac{\sqrt{c} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^2+ae^2)} + \frac{e \log(d+ex^2)}{2(cd^2+ae^2)} - \frac{e \log(a+cx^4)}{4(cd^2+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.70

$$\frac{2\sqrt{c}d \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right) - e \log(a+cx^4) + 2e \log(d+ex^2)}{4ae^2 + 4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*sqrt[c]*d*ArcTan[(sqrt[c]*x^2)/sqrt[a]])/sqrt[a] + 2*e*Log[d + e*x^2] -
e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d + ex^2)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 1.50, size = 146, normalized size = 1.52

$$\left[\frac{d\sqrt{-\frac{c}{a}} \log\left(\frac{cx^4 + 2ax^2\sqrt{-\frac{c}{a}} - a}{cx^4 + a}\right) - e \log(cx^4 + a) + 2e \log(ex^2 + d)}{4(cd^2 + ae^2)}, -\frac{2d\sqrt{\frac{c}{a}} \arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) + e \log(cx^4 + a) - 2e \log(ex^2 + d)}{4(cd^2 + ae^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*log(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2)]

giac [A] time = 0.35, size = 85, normalized size = 0.89

$$\frac{cd \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} - \frac{e \log(cx^4 + a)}{4(cd^2 + ae^2)} + \frac{e^2 \log(|x^2e + d|)}{2(cd^2e + ae^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3)

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{cd \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}} - \frac{e \ln(cx^4 + a)}{4(ae^2 + cd^2)} + \frac{e \ln(ex^2 + d)}{2ae^2 + 2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*x^2+d)/(c*x^4+a),x)

[Out] $-1/4*e*\ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*d/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)+1/2*e*\ln(e*x^2+d)/(a*e^2+c*d^2)$

maxima [A] time = 1.98, size = 82, normalized size = 0.85

$$\frac{cd \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} - \frac{e \log(cx^4 + a)}{4(cd^2 + ae^2)} + \frac{e \log(ex^2 + d)}{2(cd^2 + ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $1/2*c*d*\arctan(c*x^2/\sqrt{a*c})/((c*d^2 + a*e^2)*\sqrt{a*c}) - 1/4*e*\log(c*x^4 + a)/(c*d^2 + a*e^2) + 1/2*e*\log(e*x^2 + d)/(c*d^2 + a*e^2)$

mupad [B] time = 1.02, size = 328, normalized size = 3.42

$$\frac{e \ln(cx^2 + d)}{2cd^2 + 2ae^2} - \frac{\ln(a^2 d^2 x^2 - c^2 d^2 (-ac)^2 - 9a^2 d^2 (-ac)^2 + 9a^2 d^2 e^2 x^2 + 19ad^2 e^2 (-ac)^2 + 11cd^2 d^2 (-ac)^2 + 11e^2 d^2 d^2 x^2 + 19a^2 d^2 d^2 e^2 x^2)(ae - d\sqrt{ac})}{4(e^2 d^2 + ca d^2)} - \frac{\ln(9a^2 d^2 (-ac)^2 + c^2 d^2 (-ac)^2 + a^2 d^2 x^2 + 9a^2 d^2 e^2 x^2 - 19ad^2 d^2 (-ac)^2 - 11cd^2 d^2 (-ac)^2 + 11a^2 d^2 d^2 e^2 x^2 + 19a^2 d^2 d^2 e^2 x^2)(ae + d\sqrt{ac})}{4(e^2 d^2 + ca d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + c*x^4)*(d + e*x^2)),x)

[Out] $(e*\log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (\log(a*c^5*d^6*x^2 - c^3*d^6*(-a*c)^{(3/2)} - 9*a^3*e^6*(-a*c)^{(3/2)} + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c)^{(5/2)} + 11*c*d^4*e^2*(-a*c)^{(5/2)} + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e - d*(-a*c)^{(1/2)})/(4*(a^2*e^2 + a*c*d^2)) - (\log(9*a^3*e^6*(-a*c)^{(3/2)} + c^3*d^6*(-a*c)^{(3/2)} + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^2 - 19*a*d^2*e^4*(-a*c)^{(5/2)} - 11*c*d^4*e^2*(-a*c)^{(5/2)} + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^{(1/2)})/(4*(a^2*e^2 + a*c*d^2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.164 \quad \int \frac{1}{x(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=114

$$\frac{cd \log(a+cx^4)}{4a(ae^2+cd^2)} - \frac{e^2 \log(d+ex^2)}{2d(ae^2+cd^2)} - \frac{\sqrt{c} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)} + \frac{\log(x)}{ad}$$

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{e^2 \log(d+ex^2)}{2d(ae^2+cd^2)} - \frac{cd \log(a+cx^4)}{4a(ae^2+cd^2)} - \frac{\sqrt{c} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2+cd^2)} + \frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]

[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*(c*d^2 + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c

*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(d+ex^2)(a+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(d+ex)(a+cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx} - \frac{e^3}{d(cd^2+ae^2)(d+ex)} - \frac{c(ae+cdx)}{a(cd^2+ae^2)(a+cx^2)} \right) dx, x, x^2 \right) \\
 &= \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{c \text{Subst} \left(\int \frac{ae+cdx}{a+cx^2} dx, x, x^2 \right)}{2a(cd^2+ae^2)} \\
 &= \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{(c^2d) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2a(cd^2+ae^2)} - \frac{(ce) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)} \\
 &= -\frac{\sqrt{c} e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^2+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{cd \log(a+cx^4)}{4a(cd^2+ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 134, normalized size = 1.18

$$\frac{-cd^2 \log(a+cx^4) + 2\sqrt{a} \sqrt{c} de \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} \right) + 2\sqrt{a} \sqrt{c} de \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1 \right) - 2ae^2 \log(d+ex^2) + 4ae^2 \log(x) + 4cd^2 \log(x)}{4a^2 de^2 + 4acd^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)), x]

[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*a^2*d*e^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 16.72, size = 201, normalized size = 1.76

$$\left[\frac{ade\sqrt{\frac{c}{a}} \log\left(\frac{cx^4-2ax^2\sqrt{\frac{c}{a}}-a}{cx^4+a}\right) - cd^2 \log(cx^4+a) - 2ae^2 \log(ex^2+d) + 4(cd^2+ae^2) \log(x)}{4(acd^3+a^2de^2)}, \frac{2ade\sqrt{\frac{c}{a}} \arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) - cd^2 \log(cx^4+a) - 2ae^2 \log(ex^2+d) + 4(cd^2+ae^2) \log(x)}{4(acd^3+a^2de^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(a*c*d^3 + a^2*d*e^2)]

giac [A] time = 0.29, size = 102, normalized size = 0.89

$$-\frac{cd \log(cx^4 + a)}{4(acd^2 + a^2e^2)} - \frac{c \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) e}{2(cd^2 + ae^2)\sqrt{ac}} - \frac{e^3 \log(|x^2e + d|)}{2(cd^3e + ade^3)} + \frac{\log(x^2)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e + a*d*e^3) + 1/2*log(x^2)/(a*d)

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$-\frac{ce \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}} - \frac{cd \ln(cx^4 + a)}{4(ae^2 + cd^2)a} - \frac{e^2 \ln(ex^2 + d)}{2(ae^2 + cd^2)d} + \frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x^2+d)/(c*x^4+a),x)`

[Out] $\ln(x)/a/d - 1/4*c*d*\ln(c*x^4+a)/a/(a*e^2+c*d^2) - 1/2*c/(a*e^2+c*d^2)*e/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2) - 1/2*e^2*\ln(e*x^2+d)/d/(a*e^2+c*d^2)$

maxima [A] time = 1.97, size = 101, normalized size = 0.89

$$\frac{cd \log(cx^4 + a)}{4(acd^2 + a^2e^2)} - \frac{e^2 \log(ex^2 + d)}{2(cd^3 + ade^2)} - \frac{ce \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2 + ae^2)\sqrt{ac}} + \frac{\log(x^2)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")`

[Out] $-1/4*c*d*\log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e^2*\log(e*x^2 + d)/(c*d^3 + a*d*e^2) - 1/2*c*e*\arctan(c*x^2/\sqrt{a*c})/((c*d^2 + a*e^2)*\sqrt{a*c}) + 1/2*\log(x^2)/(a*d)$

mupad [B] time = 0.96, size = 527, normalized size = 4.62

⚠️ The following output is a truncated representation of the antiderivative returned by the CAS. It contains a large number of terms and is not fully legible due to truncation.

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + c*x^4)*(d + e*x^2)),x)`

[Out] $(\log(64*a^7*c*e^{10}*x^2 - 64*a^6*e^{10}*(-a^3*c)^{(1/2)} - 25*a*c^5*d^{10}*(-a^3*c)^{(1/2)} + 25*a^2*c^6*d^{10}*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^{(3/2)} - 41*c^2*d^6*e^4*(-a^3*c)^{(3/2)} - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3*c)^{(1/2)} + 109*a*c*d^4*e^6*(-a^3*c)^{(3/2)})*(e*(-a^3*c)^{(1/2)} - a*c*d)/(4*a^3*e^2 + 4*a^2*c*d^2) - (\log(64*a^6*e^{10}*(-a^3*c)^{(1/2)} + 64*a^7*c*e^{10}*x^2 + 25*a*c^5*d^{10}*(-a^3*c)^{(1/2)} + 25*a^2*c^6*d^{10}*x^2 - 180*a^2*d^2*e^8*(-a^3*c)^{(3/2)} + 41*c^2*d^6*e^4*(-a^3*c)^{(3/2)} - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 - 9*a^2*c^4*d^8*e^2*(-a^3*c)^{(1/2)} - 109*a*c*d^4*e^6*(-a^3*c)^{(3/2)})*(e*(-a^3*c)^{(1/2)} + a*c*d)/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*\log(d + e*x^2))/(2*c*d^3 + 2*a*d*e^2) + \log(x)/(a*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

$$3.165 \quad \int \frac{1}{x^3(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=129

$$-\frac{c^{3/2}d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(ae^2+cd^2)} + \frac{ce \log(a+cx^4)}{4a(ae^2+cd^2)} + \frac{e^3 \log(d+ex^2)}{2d^2(ae^2+cd^2)} - \frac{e \log(x)}{ad^2} - \frac{1}{2adx^2}$$

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{c^{3/2}d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(ae^2+cd^2)} + \frac{e^3 \log(d+ex^2)}{2d^2(ae^2+cd^2)} + \frac{ce \log(a+cx^4)}{4a(ae^2+cd^2)} - \frac{e \log(x)}{ad^2} - \frac{1}{2adx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]

[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c

*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (d + ex^2) (a + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (d + ex) (a + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx^2} - \frac{e}{ad^2x} + \frac{e^4}{d^2 (cd^2 + ae^2) (d + ex)} - \frac{c^2 (d - ex)}{a (cd^2 + ae^2) (a + cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{1}{2adx^2} - \frac{e \log(x)}{ad^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 + ae^2)} - \frac{c^2 \text{Subst} \left(\int \frac{d - ex}{a + cx^2} dx, x, x^2 \right)}{2a (cd^2 + ae^2)} \\
 &= -\frac{1}{2adx^2} - \frac{e \log(x)}{ad^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 + ae^2)} - \frac{(c^2 d) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right)}{2a (cd^2 + ae^2)} + \frac{(c^2 e) \text{Subst} \left(\int \frac{x}{a + cx^2} dx, x, x^2 \right)}{2a (cd^2 + ae^2)} \\
 &= -\frac{1}{2adx^2} - \frac{c^{3/2} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2a^{3/2} (cd^2 + ae^2)} - \frac{e \log(x)}{ad^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 + ae^2)} + \frac{ce \log(a + cx^4)}{4a (cd^2 + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 169, normalized size = 1.31

$$\frac{2c^{3/2}d^3x^2 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} \right) + 2c^{3/2}d^3x^2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} + 1 \right) + \sqrt{a} (-4ex^2 \log(x)(ae^2 + cd^2) + cd^2ex^2 \log(a + cx^4) + 2ae^3x^2 \log(d + ex^2) - 2ade^2 - 2cd^3)}{4a^{3/2}d^2x^2 (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]

[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)), x]

fricas [A] time = 101.42, size = 265, normalized size = 2.05

$$\left[\frac{cd^3x^2\sqrt{\frac{c}{a}}\log\left(\frac{cx^4-2ax^2\sqrt{\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log(cx^4+a)+2ae^3x^2\log(ex^2+d)-2cd^3-2ade^2-4(cd^2e+ae^3)x^2\log(x)}{4(acd^4+a^2d^2e^2)x^2}, \frac{2cd^3x^2\sqrt{\frac{e}{a}}\arctan\left(\frac{\sqrt{e}}{cx^2}\right)+cd^2ex^2\log(cx^4+a)+2ae^3x^2\log(ex^2+d)-2cd^3-2ade^2-4(cd^2e+ae^3)x^2\log(x)}{4(acd^4+a^2d^2e^2)x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a), x, algorithm="fricas")

[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2), 1/4*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]

giac [A] time = 0.30, size = 132, normalized size = 1.02

$$-\frac{c^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2 + a^2e^2)\sqrt{ac}} + \frac{ce \log(cx^4 + a)}{4(acd^2 + a^2e^2)} + \frac{e^4 \log(|x^2e + d|)}{2(cd^4e + ad^2e^3)} - \frac{e \log(x^2)}{2ad^2} + \frac{x^2e - d}{2ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a), x, algorithm="giac")

[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e + a*d^2*e^3) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(x^2*e - d)/(a*d^2*x^2)

maple [A] time = 0.01, size = 119, normalized size = 0.92

$$-\frac{c^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}a} + \frac{ce \ln(cx^4 + a)}{4(ae^2 + cd^2)a} + \frac{e^3 \ln(ex^2 + d)}{2(ae^2 + cd^2)d^2} - \frac{e \ln(x)}{ad^2} - \frac{1}{2ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(e*x^2+d)/(c*x^4+a),x)`

[Out]
$$-1/2/a/d/x^2 - e*\ln(x)/a/d^2 + 1/4*c*e*\ln(c*x^4+a)/a/(a*e^2+c*d^2) - 1/2*c^2/(a*e^2+c*d^2)/a*d/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2) + 1/2*e^3*\ln(e*x^2+d)/d^2/(a*e^2+c*d^2)$$

maxima [A] time = 2.00, size = 120, normalized size = 0.93

$$\frac{e^3 \log(ex^2 + d)}{2(cd^4 + ad^2e^2)} - \frac{c^2d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2 + a^2e^2)\sqrt{ac}} + \frac{ce \log(cx^4 + a)}{4(acd^2 + a^2e^2)} - \frac{e \log(x^2)}{2ad^2} - \frac{1}{2adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")`

[Out]
$$1/2*e^3*\log(e*x^2 + d)/(c*d^4 + a*d^2*e^2) - 1/2*c^2*d*\arctan(c*x^2/\sqrt{a*c})/((a*c*d^2 + a^2*e^2)*\sqrt{a*c}) + 1/4*c*e*\log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e*\log(x^2)/(a*d^2) - 1/2/(a*d*x^2)$$

mupad [B] time = 1.38, size = 820, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)`

[Out]
$$\begin{aligned} & (\log(a^6*c^{12}*d^{16}*x^2 + 64*a^{14}*c^4*e^{16}*x^2 + a^2*c^7*d^{16}*(-a^3*c^3)^{(3/2)} - 64*a^{13}*c^2*e^{16}*(-a^3*c^3)^{(1/2)} + 63*a^3*d^8*e^8*(-a^3*c^3)^{(5/2)} + \\ & 224*a^9*d^2*e^{14}*(-a^3*c^3)^{(3/2)} - 28*c^3*d^{14}*e^2*(-a^3*c^3)^{(5/2)} + 28*a^7*c^{11}*d^{14}*e^2*x^2 + 114*a^8*c^{10}*d^{12}*e^4*x^2 + 108*a^9*c^9*d^{10}*e^6*x^2 \\ & - 63*a^{10}*c^8*d^8*e^8*x^2 - 32*a^{11}*c^7*d^6*e^{10}*x^2 + 212*a^{12}*c^6*d^4*e^{12}*x^2 + 224*a^{13}*c^5*d^2*e^{14}*x^2 - 114*a*c^2*d^{12}*e^4*(-a^3*c^3)^{(5/2)} - \\ & 108*a^2*c*d^{10}*e^6*(-a^3*c^3)^{(5/2)} + 212*a^8*c*d^4*e^{12}*(-a^3*c^3)^{(3/2)} - 32*a^7*c^2*d^6*e^{10}*(-a^3*c^3)^{(3/2}))* (d*(-a^3*c^3)^{(1/2)} + a^2*c*e))/ (4*a^4*e^2 + 4*a^3*c*d^2) - (\log(a^6*c^{12}*d^{16}*x^2 + 64*a^{14}*c^4*e^{16}*x^2 - a^2*c^7*d^{16}*(-a^3*c^3)^{(3/2)} + 64*a^{13}*c^2*e^{16}*(-a^3*c^3)^{(1/2)} - 63*a^3*d^8*e^8*(-a^3*c^3)^{(5/2)} - 224*a^9*d^2*e^{14}*(-a^3*c^3)^{(3/2)} + 28*c^3*d^{14}*e^2*(-a^3*c^3)^{(5/2)} + 28*a^7*c^{11}*d^{14}*e^2*x^2 + 114*a^8*c^{10}*d^{12}*e^4*x^2 + 108*a^9*c^9*d^{10}*e^6*x^2 - 63*a^{10}*c^8*d^8*e^8*x^2 - 32*a^{11}*c^7*d^6*e^{10}*x^2 + 212*a^{12}*c^6*d^4*e^{12}*x^2 + 224*a^{13}*c^5*d^2*e^{14}*x^2 + 114*a*c^2*d^{12}*e^4*(-a^3*c^3)^{(5/2)} + 108*a^2*c*d^{10}*e^6*(-a^3*c^3)^{(5/2)} - 212*a^8*c*d^4*e^{12}*(-a^3*c^3)^{(3/2)} + 32*a^7*c^2*d^6*e^{10}*(-a^3*c^3)^{(3/2}))* (d*(-a^3*c^3)^{(1/2)} + a^2*c*e) \end{aligned}$$

$$\frac{\sqrt{a^2 c e}}{4(a^4 e^2 + a^3 c d^2)} + \frac{e^3 \log(d + e x^2)}{2 c d^4 + 2 a d^2 e^2} - \frac{1}{2 a d x^2} - \frac{e \log(x)}{a d^2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.166 \quad \int \frac{1}{x^5(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=156

$$\frac{c^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(ae^2 + cd^2)} + \frac{c^2d \log(a + cx^4)}{4a^2(ae^2 + cd^2)} - \frac{\log(x)(cd^2 - ae^2)}{a^2d^3} - \frac{e^4 \log(d + ex^2)}{2d^3(ae^2 + cd^2)} + \frac{e}{2ad^2x^2} - \frac{1}{4adx^4}$$

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1252, 894, 635, 205, 260}

$$\frac{c^2d \log(a + cx^4)}{4a^2(ae^2 + cd^2)} + \frac{c^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(ae^2 + cd^2)} - \frac{\log(x)(cd^2 - ae^2)}{a^2d^3} - \frac{e^4 \log(d + ex^2)}{2d^3(ae^2 + cd^2)} + \frac{e}{2ad^2x^2} - \frac{1}{4adx^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]

[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x

```

^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))

```

Rule 1252

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (d + ex^2)(a + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 (d + ex)(a + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx^3} - \frac{e}{ad^2x^2} + \frac{-cd^2 + ae^2}{a^2d^3x} - \frac{e^5}{d^3 (cd^2 + ae^2)(d + ex)} + \frac{c^2(ae - e^2)}{a^2 (cd^2 + ae^2)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2 - ae^2) \log(x)}{a^2d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 + ae^2)} + \frac{c^2 \text{Subst} \left(\int \frac{ae + cdx}{a + cx^2} dx, x, x^2 \right)}{2a^2 (cd^2 + ae^2)} \\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2 - ae^2) \log(x)}{a^2d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 + ae^2)} + \frac{(c^3d) \text{Subst} \left(\int \frac{x}{a + cx^2} dx, x, x^2 \right)}{2a^2 (cd^2 + ae^2)} \\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{3/2}e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2} (cd^2 + ae^2)} - \frac{(cd^2 - ae^2) \log(x)}{a^2d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 + ae^2)} +
\end{aligned}$$

Mathematica [A] time = 0.09, size = 209, normalized size = 1.34

$$\frac{a^2d^2c^2 + 2a^2e^4x^4 \log(d + ex^2) - 2a^2de^3x^2 - 4a^2e^4x^4 \log(x) + 2\sqrt{a}c^{3/2}d^3ex^4 \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}} \right) + 2\sqrt{a}c^{3/2}d^3ex^4 \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}} + 1 \right) - c^2d^4x^4 \log(a + cx^4) + acd^4 - 2acd^3ex^2 + 4c^2d^4x^4 \log(x)}{4a^2d^3x^4 (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]
*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3
/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x
] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a
+ c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)

```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 168, normalized size = 1.08

$$\frac{c^2 d \log(cx^4 + a)}{4(a^2 cd^2 + a^3 e^2)} + \frac{c^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) e}{2(acd^2 + a^2 e^2)\sqrt{ac}} - \frac{e^5 \log(|x^2 e + d|)}{2(cd^5 e + ad^3 e^3)} - \frac{(cd^2 - ae^2) \log(x^2)}{2a^2 d^3} + \frac{3cd^2 x^4 - 3ax^4 e^2 + 2adx^2 e - ad^2}{4a^2 d^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4}c^2 d \log(cx^4 + a)/(a^2 c d^2 + a^3 e^2) + \frac{1}{2}c^2 \arctan(cx^2/\sqrt{ac})e/((a^2 c d^2 + a^2 e^2)\sqrt{ac}) - \frac{1}{2}e^5 \log(\text{abs}(x^2 e + d))/(c d^5 e + a d^3 e^3) - \frac{1}{2}(c d^2 - a e^2) \log(x^2)/(a^2 d^3) + \frac{1}{4}(3c d^2 x^4 - 3a x^4 e^2 + 2a d x^2 e - a d^2)/(a^2 d^3 x^4)$

maple [A] time = 0.02, size = 145, normalized size = 0.93

$$\frac{c^2 e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(ae^2 + cd^2)\sqrt{ac}a} + \frac{c^2 d \ln(cx^4 + a)}{4(ae^2 + cd^2)a^2} - \frac{e^4 \ln(e x^2 + d)}{2(ae^2 + cd^2)d^3} + \frac{e^2 \ln(x)}{a d^3} - \frac{c \ln(x)}{a^2 d} + \frac{e}{2a d^2 x^2} - \frac{1}{4a d x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x)

[Out] $-1/4/a/d/x^4 + 1/d^3/a \ln(x) e^2 - 1/d/a^2 \ln(x) c + 1/2 * e/a/d^2/x^2 + 1/4 * c^2 * d \ln(cx^4 + a)/a^2 / (ae^2 + cd^2) + 1/2 * c^2 / (ae^2 + cd^2) / a * e / (ac)^{(1/2)} * \arctan(1/(ac)^{(1/2)} * cx^2) - 1/2 * e^4 * \ln(ex^2 + d) / d^3 / (ae^2 + cd^2)$

maxima [A] time = 2.05, size = 145, normalized size = 0.93

$$\frac{e^4 \log(ex^2 + d)}{2(cd^5 + ad^3e^2)} + \frac{c^2d \log(cx^4 + a)}{4(a^2cd^2 + a^3e^2)} + \frac{c^2e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2 + a^2e^2)\sqrt{ac}} - \frac{(cd^2 - ae^2) \log(x^2)}{2a^2d^3} + \frac{2ex^2 - d}{4ad^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $-\frac{1}{2}e^4 \log(ex^2 + d)/(c^2d^5 + a^3e^2) + \frac{1}{4}c^2d \log(cx^4 + a)/(a^2cd^2 + a^3e^2) + \frac{1}{2}c^2e \arctan(cx^2/\sqrt{ac})/((ac^2d^2 + a^2e^2)\sqrt{ac}) - \frac{1}{2}(cd^2 - ae^2) \log(x^2)/(a^2d^3) + \frac{1}{4}(2ex^2 - d)/(ad^2x^4)$

mupad [B] time = 1.87, size = 1017, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)

[Out] $(\log(25a^2c^9d^{20}(-a^5c^3)^{3/2} - 64a^{19}c^4e^{20}x^2 - 25a^9c^{14}d^{20}x^2 - 64a^{17}c^2e^{20}(-a^5c^3)^{1/2} + 100a^3d^8e^{12}(-a^5c^3)^{5/2} + 128a^{11}d^2e^{18}(-a^5c^3)^{3/2} - 112c^3d^{14}e^6(-a^5c^3)^{5/2} - 76a^{10}c^{13}d^{18}e^2x^2 - 138a^{11}c^{12}d^{16}e^4x^2 - 112a^{12}c^{11}d^{14}e^6x^2 + 55a^{13}c^{10}d^{12}e^8x^2 + 104a^{14}c^9d^{10}e^{10}x^2 + 100a^{15}c^8d^8e^{12}x^2 + 172a^{16}c^7d^6e^{14}x^2 + 32a^{17}c^6d^4e^{16}x^2 - 128a^{18}c^5d^2e^{18}x^2 + 55a^2c^2d^{12}e^8(-a^5c^3)^{5/2} + 104a^2c^2d^{10}e^{10}(-a^5c^3)^{5/2} - 32a^{10}c^4d^4e^{16}(-a^5c^3)^{3/2} + 76a^3c^8d^{18}e^2(-a^5c^3)^{3/2} + 138a^4c^7d^{16}e^4(-a^5c^3)^{3/2} - 172a^9c^2d^6e^{14}(-a^5c^3)^{3/2}) * (e * (-a^5c^3)^{1/2} + a^2c^2d) / (4a^5e^2 + 4a^4cd^2) - (e^4 \log(d + ex^2)) / (2(c^2d^5 + a^3e^2)) - (\log(25a^9c^{14}d^{20}x^2 + 64a^{19}c^4e^{20}x^2 + 25a^2c^9d^{20}(-a^5c^3)^{3/2} - 64a^{17}c^2e^{20}(-a^5c^3)^{1/2} + 100a^3d^8e^{12}(-a^5c^3)^{5/2} + 128a^{11}d^2e^{18}(-a^5c^3)^{3/2} - 112c^3d^{14}e^6(-a^5c^3)^{5/2} + 76a^{10}c^{13}d^{18}e^2x^2 + 138a^{11}c^{12}d^{16}e^4x^2 + 112a^{12}c^{11}d^{14}e^6x^2 - 55a^{13}c^{10}d^{12}e^8x^2 - 104a^{14}c^9d^{10}e^{10}x^2 - 100a^{15}c^8d^8e^{12}x^2 - 172a^{16}c^7d^6e^{14}x^2 - 32a^{17}c^6d^4e^{16}x^2 + 128a^{18}c^5d^2e^{18}x^2 + 55a^2c^2d^{12}e^8(-a^5c^3)^{5/2} + 104a^2c^2d^{10}e^{10}(-a^5c^3)^{5/2} - 32a^{10}c^4d^4e^{16}(-a^5c^3)^{3/2} + 76a^3c^8d^{18}e^2(-a^5c^3)^{3/2} + 138a^4c^7d^{16}e^4(-a^5c^3)^{3/2} - 172a^9c^2d^6e^{14}(-a^5c^3)^{3/2}) * (e * (-a^5c^3)^{1/2} - a^2c^2d) / (4(a^5e^2 + a^4cd^2)) - (1/(4ad) - (ex^2)/(2ad^2))/x^4 + (\log(x) * (ae^2 - cd^2))/(a^2d^3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.167 \quad \int \frac{x^8}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=359

$$\frac{a^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} + \frac{a^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} - \frac{a^{5/4}}{c^{5/2}}$$

Rubi [A] time = 0.35, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{a^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} + \frac{a^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} - \frac{a^{5/4}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(ae^2 + cd^2)} + \frac{a^{5/4}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}c^{7/4}(ae^2 + cd^2)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{5/2}(ae^2 + cd^2)} - \frac{dx}{ce^2} + \frac{x^3}{3ce}$$

Antiderivative was successfully verified.

[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]

[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c^2d^2 + a^2e^2, 0] \ \&\& \ \text{NeQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1288

$\text{Int}[\frac{((f_.)x)^{m_.} \cdot ((d_.) + (e_.)x^2)^{q_ .}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(fx)^m (d + ex^2)^q}{(a + cx^4)}, x], x] \ /; \text{FreeQ}[\{a, c, d, e, f, m\}, x] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(d+ex^2)(a+cx^4)} dx &= \int \left(-\frac{d}{ce^2} + \frac{x^2}{ce} + \frac{d^4}{e^2(cd^2+ae^2)(d+ex^2)} + \frac{a^2(d-ex^2)}{c(cd^2+ae^2)(a+cx^4)} \right) dx \\
&= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{a^2 \int \frac{d-ex^2}{a+cx^4} dx}{c(cd^2+ae^2)} + \frac{d^4 \int \frac{1}{d+ex^2} dx}{e^2(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2c^2(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2c^2(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c^2(cd^2+ae^2)} + \frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c^2(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} - \frac{a^{5/4}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2} + \frac{x^3}{3ce} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} - \frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 344, normalized size = 0.96

$$\frac{-3\sqrt{2}a^{5/2}(a^{3/4}e + \sqrt{a}\sqrt{c}d) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + 3\sqrt{2}a^{5/2}(a^{3/4}e + \sqrt{a}\sqrt{c}d) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + 6\sqrt{2}a^{5/2}e^{3/2}(\sqrt{a}e - \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 6\sqrt{2}a^{5/2}e^{3/2}(\sqrt{a}e - \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) - 24c^{3/4}d\sqrt{c}x(a^2 + ce^2) + 8c^{3/4}e^{3/2}x^3(a^2 + ce^2) + 24c^{7/4}d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{24c^{7/4}e^{5/2}(a^2 + ce^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)), x]

[Out] $(-24*c^{(3/4)}*d*\text{Sqrt}[e]*(c*d^2 + a*e^2)*x + 8*c^{(3/4)}*e^{(3/2)}*(c*d^2 + a*e^2)*x^3 + 24*c^{(7/4)}*d^{(7/2)}*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]] + 6*\text{Sqrt}[2]*a^{(5/4)}*e^{(5/2)}*(-(\text{Sqrt}[c]*d) + \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] - 6*\text{Sqrt}[2]*a^{(5/4)}*e^{(5/2)}*(-(\text{Sqrt}[c]*d) + \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] - 3*\text{Sqrt}[2]*a*e^{(5/2)}*(a^{(1/4)}*\text{Sqrt}[c]*d + a^{(3/4)}*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + 3*\text{Sqrt}[2]*a*e^{(5/2)}*(a^{(1/4)}*\text{Sqrt}[c]*d + a^{(3/4)}*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(24*c^{(7/4)}*e^{(5/2)}*(c*d^2 + a*e^2))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 32.47, size = 4414, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/12*(6*c*d^3*\sqrt{-d/e}*\log((e*x^2 + 2*e*x*\sqrt{-d/e} - d)/(e*x^2 + d)) + \\ & 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*\sqrt{(2*a^3*d*e + (c^5 \\ & *d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 \\ & + a^7*e^4)/(c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2 \\ & *e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)}*\log(-(a^3*c \\ & *d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2 \\ & *e^3 + a^2*c^5*e^5)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11} \\ & *d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^ \\ & 8)))*\sqrt{(2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5 \\ & *c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2 \\ & *c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 \\ & + a^2*c^3*e^4)) + 3*(c^2*d^2*e^2 + a*c*e^4)*\sqrt{(2*a^3*d*e + (c^5*d^4 + \\ & 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e \\ & ^4)/(c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + \\ & a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)}*\log(-(a^3*c*d^2 - \\ & a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 + \\ & a^2*c^5*e^5)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 + 4 \\ & *a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*\sqrt{ \\ & ((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 \\ & - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4 \\ & *e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c \\ & ^3*e^4)) - 3*(c^2*d^2*e^2 + a*c*e^4)*\sqrt{(2*a^3*d*e - (c^5*d^4 + 2*a*c^4 \\ & *d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11} \\ & *d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7 \\ & *e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)}*\log(-(a^3*c*d^2 - a^4*e^ \\ & 2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^ \\ & 5*e^5)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 + 4*a*c^{10} \\ & *d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*\sqrt{(2*a \\ & ^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 - 2*a \\ & ^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + \\ & 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4 \\ &)) + 3*(c^2*d^2*e^2 + a*c*e^4)*\sqrt{(2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^ \\ & 2 + a^2*c^3*e^4)*\sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^{11}*d^8 \end{aligned}$$

$$\begin{aligned}
& + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)) \\
& / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)) * \log(- (a^3*c*d^2 - a^4*e^2) * x - \\
& (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5) * \\
& \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 \\
& + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) * \sqrt{((2*a^3*d*e \\
& - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2 \\
& *e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8 \\
& *d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 1 \\
& 2 * (c*d^3 + a*d*e^2) * x / (c^2*d^2*e^2 + a*c*e^4), 1/12 * (12*c*d^3 * \sqrt{d/e} * \ar \\
& \tan(e*x * \sqrt{d/e} / d) + 4 * (c*d^2*e + a*e^3) * x^3 - 3 * (c^2*d^2*e^2 + a*c*e^4) \\
& * \sqrt{((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{- (a^5*c^2 \\
& *d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9* \\
& d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2 \\
& *c^3*e^4)) * \log(- (a^3*c*d^2 - a^4*e^2) * x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + \\
& (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2 \\
& *e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8 \\
& *d^2*e^6 + a^4*c^7*e^8))) * \sqrt{((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3 \\
& *e^4) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a \\
& *c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5 \\
& *d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3 * (c^2*d^2*e^2 + a*c*e^4) * \sqrt{((\\
& 2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{- (a^5*c^2*d^4 - \\
& 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 \\
& + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3* \\
& e^4)) * \log(- (a^3*c*d^2 - a^4*e^2) * x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^ \\
& 4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + \\
& a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2* \\
& e^6 + a^4*c^7*e^8))) * \sqrt{((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3 \\
& *e^4) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10} \\
& *d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + \\
& 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3 * (c^2*d^2*e^2 + a*c*e^4) * \sqrt{((2*a^3*d \\
& *e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c \\
& *d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^ \\
& 3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)) * \log(- (a^ \\
& 3*c*d^2 - a^4*e^2) * x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2 \\
& *a*c^6*d^2*e^3 + a^2*c^5*e^5) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^ \\
& 4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a \\
& ^4*c^7*e^8))) * \sqrt{((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{ \\
& - (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 \\
& + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^ \\
& 4*d^2*e^2 + a^2*c^3*e^4))) + 3 * (c^2*d^2*e^2 + a*c*e^4) * \sqrt{((2*a^3*d*e - (c \\
& ^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^ \\
& 2 + a^7*e^4) / (c^{11}*d^8 + 4*a*c^{10}*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^ \\
& 2*e^6 + a^4*c^7*e^8))) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)) * \log(- (a^ \\
& 3*c*d^2 - a^4*e^2) * x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6* \\
& d^2*e^3 + a^2*c^5*e^5) * \sqrt{- (a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4) / (c^1
\end{aligned}$$

$$1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)) * \sqrt{(2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4) * \sqrt{-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)}) / (c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)}) / (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)) - 12*(c*d^3 + a*d*e^2)*x / (c^2*d^2*e^2 + a*c*e^4)]$$

giac [A] time = 0.54, size = 363, normalized size = 1.01

$$\frac{d^2 \arctan\left(\frac{\sqrt{x}}{\sqrt{d}}\right) \left(\frac{c}{d}\right)^{\frac{1}{4}} - \left((ac)^{\frac{1}{2}} ac^2 d - (ac^3)^{\frac{1}{2}} ae\right) \arctan\left(\frac{\sqrt{2x+\sqrt{2}(c)^{\frac{1}{2}}}}{2(c)^{\frac{1}{2}}}\right) + \left((ac^3)^{\frac{1}{2}} ac^2 d - (ac^3)^{\frac{1}{2}} ae\right) \arctan\left(\frac{\sqrt{2x-\sqrt{2}(c)^{\frac{1}{2}}}}{2(c)^{\frac{1}{2}}}\right)}{cd^2 + ae^4} + \frac{\left((ac^3)^{\frac{1}{2}} ac^2 d - (ac^3)^{\frac{1}{2}} ae\right) \arctan\left(\frac{\sqrt{2x+\sqrt{2}(c)^{\frac{1}{2}}}}{2(c)^{\frac{1}{2}}}\right)}{2(\sqrt{2}c^3d^2 + \sqrt{2}ac^4e^2)} + \frac{\left((ac^3)^{\frac{1}{2}} ac^2 d - (ac^3)^{\frac{1}{2}} ae\right) \arctan\left(\frac{\sqrt{2x-\sqrt{2}(c)^{\frac{1}{2}}}}{2(c)^{\frac{1}{2}}}\right)}{2(\sqrt{2}c^3d^2 + \sqrt{2}ac^4e^2)} + \frac{\left((ac^3)^{\frac{1}{2}} ac^2 d + (ac^3)^{\frac{1}{2}} ae\right) \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{2}} + \sqrt{\frac{c}{d}}\right) - \left((ac^3)^{\frac{1}{2}} ac^2 d + (ac^3)^{\frac{1}{2}} ae\right) \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{2}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}c^3d^2 + \sqrt{2}ac^4e^2)} - \frac{\left((ac^3)^{\frac{1}{2}} ac^2 d + (ac^3)^{\frac{1}{2}} ae\right) \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{2}} + \sqrt{\frac{c}{d}}\right)}{4(\sqrt{2}c^3d^2 + \sqrt{2}ac^4e^2)} + \frac{(c^2x^3e^2 - 3c^2dx)e^{d-3x}}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

$$[Out] d^{7/2} * \arctan(xe^{1/2}/\sqrt{d}) * e^{-1/2} / (c*d^2*e^2 + a*e^4) + 1/2 * ((ac^3)^{1/4} * a*c^2*d - (ac^3)^{3/4} * a*e) * \arctan(1/2 * \sqrt{2} * (2*x + \sqrt{2}) * (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * c^5*d^2 + \sqrt{2} * a*c^4*e^2) + 1/2 * ((ac^3)^{1/4} * a*c^2*d - (ac^3)^{3/4} * a*e) * \arctan(1/2 * \sqrt{2} * (2*x - \sqrt{2}) * (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * c^5*d^2 + \sqrt{2} * a*c^4*e^2) + 1/4 * ((ac^3)^{1/4} * a*c^2*d + (ac^3)^{3/4} * a*e) * \log(x^2 + \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} * c^5*d^2 + \sqrt{2} * a*c^4*e^2) - 1/4 * ((ac^3)^{1/4} * a*c^2*d + (ac^3)^{3/4} * a*e) * \log(x^2 - \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} * c^5*d^2 + \sqrt{2} * a*c^4*e^2) + 1/3 * (c^2*x^3*e^2 - 3*c^2*d*x*e) * e^{-3} / c^3$$

maple [A] time = 0.02, size = 405, normalized size = 1.13

$$\frac{d^4 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(a^2 + cd^2)\sqrt{de}e^2} - \frac{\sqrt{2} a^2 e \arctan\left(\frac{\sqrt{2}x}{(c)^{\frac{1}{2}}}-1\right)}{4(a^2 + cd^2)(c)^{\frac{1}{2}}c^2} - \frac{\sqrt{2} a^2 e \arctan\left(\frac{\sqrt{2}x}{(c)^{\frac{1}{2}}}+1\right)}{4(a^2 + cd^2)(c)^{\frac{1}{2}}c^2} - \frac{\sqrt{2} a^2 e \ln\left(\frac{x^2-(c)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2+(c)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(a^2 + cd^2)(c)^{\frac{1}{2}}c^2} + \frac{(c)^{\frac{1}{2}}\sqrt{2} ad \arctan\left(\frac{\sqrt{2}x}{(c)^{\frac{1}{2}}}-1\right)}{4(a^2 + cd^2)c} + \frac{(c)^{\frac{1}{2}}\sqrt{2} ad \arctan\left(\frac{\sqrt{2}x}{(c)^{\frac{1}{2}}}+1\right)}{4(a^2 + cd^2)c} + \frac{(c)^{\frac{1}{2}}\sqrt{2} ad \ln\left(\frac{x^2+(c)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-(c)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8(a^2 + cd^2)c} + \frac{x^3}{3ce} - \frac{dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(e*x^2+d)/(c*x^4+a),x)

$$[Out] 1/3 * x^3 / c / e - d * x / c / e^2 + 1/8 * a / (a * e^2 + c * d^2) / c * d * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2})) + 1/4 * a / (a * e^2 + c * d^2) / c * d * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) + 1/4 * a / (a * e^2 + c * d^2) / c * d * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) - 1/8 * a^2 / (a * e^2 + c * d^2) / c^2 * e / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * x * 2^{1/2} + (a/c)^{1/2})) - 1/4 * a^2 / (a * e^2 + c * d^2) / c^2 * e / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) - 1/4 * a^2 / (a * e^2 + c * d^2) / c^2 * e / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) + 1/e^2 * d^4 / (a * e^2 + c * d^2) / (d * e)^{1/2} * \arctan(e * x / (d * e)^{1/2})$$

maxima [A] time = 2.05, size = 294, normalized size = 0.82

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e^2 + ae^4)\sqrt{de}} + \frac{a^2 \left[\frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx} + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{e}} + \frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx} - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{e}} + \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} \right]}{8(c^2d^2 + ace^2)} + \frac{ex^3 - 3dx}{3ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $d^4 \arctan(e*x/\sqrt{d*e})/((c*d^2*e^2 + a*e^4)*\sqrt{d*e}) + 1/8*a^2*(2*\sqrt{2}*(\sqrt{c}*d - \sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{c})})/(\sqrt{a}*\sqrt{(\sqrt{a}*\sqrt{c})}*\sqrt{c}) + 2*\sqrt{2}*(\sqrt{c}*d - \sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{c})})/(\sqrt{a}*\sqrt{(\sqrt{a}*\sqrt{c})}*\sqrt{c}) + \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e)*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)}) - \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e)*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(3/4)})/(c^2*d^2 + a*c*e^2) + 1/3*(e*x^3 - 3*d*x)/(c*e^2)$

mupad [B] time = 2.06, size = 6097, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((a + c*x^4)*(d + e*x^2)),x)

[Out] $(\log(a^7*d^4*e^{26} + 16*c^7*d^{18}*e^{12} - 16*c^7*x*(-d^7*e^5)^{(5/2)} + 2*a^6*c*d^6*e^{24} + 16*a^3*c^4*d^{12}*e^{18} + a^5*c^2*d^8*e^{22} - a^7*e^{24}*x*(-d^7*e^5)^{(1/2)} - a^5*c^2*d^4*e^{20}*x*(-d^7*e^5)^{(1/2)} + 16*a^3*c^4*d*e^{11}*x*(-d^7*e^5)^{(3/2)} - 2*a^6*c*d^2*e^{22}*x*(-d^7*e^5)^{(1/2)}*(-d^7*e^5)^{(1/2)})/(2*a*e^7 + 2*c*d^2*e^5) - \operatorname{atan}\left(\frac{(192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^{(1/2)} - a*e^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)}*(256*a^5*c^7*e^{12} - 256*a^2*c^{10}*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^{10})/(c^3*e^3)*((c*d^2*(-a^5*c^7)^{(1/2)} - a*e^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)} + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/(c^3*e^3)*((c*d^2*(-a^5*c^7)^{(1/2)} - a*e^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)} - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3)*((c*d^2*(-a^5*c^7)^{(1/2)} - a*e^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)} - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3)*((c*d^2*(-a^5*c^7)^{(1/2)} - a*e^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2))$

$$\begin{aligned}
& *d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2))^{(1/2)} + (((((192*a^3*c^8*d^6*e^5 + \\
& 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((a*e^2*(-a^5*c \\
& ^7)^{(1/2)} - c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7* \\
& e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)}*(256*a^5*c^7*e^{12} - 256*a^2*c^{10}*d^6*e^6 - 2 \\
& 56*a^3*c^9*d^4*e^8 + 256*a^4*c^8*d^2*e^{10}))/c^3*e^3))*((a*e^2*(-a^5*c^7)^{(\\
& 1/2)} - c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + \\
& 2*a*c^8*d^2*e^2)))^{(1/2)} - (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a \\
& ^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*e^7))/c^3*e^3))*((a*e^2*(-a^5*c^7)^{(1/2)} - \\
& c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c \\
& ^8*d^2*e^2)))^{(1/2)} - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^ \\
& 2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6))/c^3*e^3))*((a*e^2*(-a^5*c^7)^{(\\
& 1/2)} - c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + \\
& 2*a*c^8*d^2*e^2)))^{(1/2)} + (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/c^3*e^3))*((a* \\
& e^2*(-a^5*c^7)^{(1/2)} - c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4*d*e)/(16*(c^9*d^4 \\
& + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)} + (2*(a^7*d^4*e^3 - a^6*c*d^6*e)) \\
& /c^3*e^3))*((a*e^2*(-a^5*c^7)^{(1/2)} - c*d^2*(-a^5*c^7)^{(1/2)} + 2*a^3*c^4* \\
& d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^{(1/2)}*2i + x^3/(3*c*e) \\
& - (d*x)/c*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(e*x**2+d)/(c*x**4+a), x)

[Out] Timed out

$$3.168 \quad \int \frac{x^6}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=345

$$\frac{a^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{5/4}(ae^2 + cd^2)} + \frac{a^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{5/4}(ae^2 + cd^2)} + \frac{a^{3/4}}{c}$$

Rubi [A] time = 0.30, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{a^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{5/4}(ae^2 + cd^2)} + \frac{a^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{5/4}(ae^2 + cd^2)} + \frac{a^{3/4}(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}(ae^2 + cd^2)} - \frac{a^{3/4}(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}c^{5/4}(ae^2 + cd^2)} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2 + cd^2)} + \frac{x}{ce}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]

[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(e^(3/2)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c^2d^2 + a^2e^2, 0] \ \&\& \ \text{NeQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1288

$\text{Int}[\frac{((f_.)x)^{m_} \cdot ((d_.) + (e_.)x^2)^{q_}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(fx)^m (d + ex^2)^q}{a + cx^4}, x], x] \ /; \text{FreeQ}[\{a, c, d, e, f, m\}, x] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(d+ex^2)(a+cx^4)} dx &= \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2+ae^2)(d+ex^2)} - \frac{a(ae+cdx^2)}{c(cd^2+ae^2)(a+cx^4)} \right) dx \\
&= \frac{x}{ce} - \frac{a \int \frac{ae+cdx^2}{a+cx^4} dx}{c(cd^2+ae^2)} - \frac{d^3 \int \frac{1}{d+ex^2} dx}{e(cd^2+ae^2)} \\
&= \frac{x}{ce} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} + \frac{\left(a\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right)\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2c(cd^2+ae^2)} - \frac{\left(a\left(d + \frac{\sqrt{a}e}{\sqrt{c}}\right)\right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2c(cd^2+ae^2)} \\
&= \frac{x}{ce} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{\left(a^{3/4}\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right)\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2} dx}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} - \frac{\left(a^{3/4}\left(d + \frac{\sqrt{a}e}{\sqrt{c}}\right)\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}-2x}{\sqrt{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} - x^2} dx}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} \\
&= \frac{x}{ce} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{a^{3/4}\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} + \frac{a^{3/4}\left(d + \frac{\sqrt{a}e}{\sqrt{c}}\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} \\
&= \frac{x}{ce} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} + \frac{a^{3/4}\left(\sqrt{c}d + \sqrt{a}e\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)} - \frac{a^{3/4}\left(\sqrt{c}d + \sqrt{a}e\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 373, normalized size = 1.08

$$\frac{(a^{3/4}cd - a^{5/4}\sqrt{c}e) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} + \frac{(a^{3/4}cd - a^{5/4}\sqrt{c}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}c^{7/4}(ae^2 + cd^2)} - \frac{(a^{3/4}cd + a^{5/4}\sqrt{c}e) \tan^{-1}\left(\frac{2\sqrt[4]{c}x - \sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(ae^2 + cd^2)} - \frac{(a^{3/4}cd + a^{5/4}\sqrt{c}e) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a} + 2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(ae^2 + cd^2)} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2 + cd^2)} + \frac{x}{ce}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)), x]

[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))]/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))]/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 4.86, size = 4354, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*c*d^2*\sqrt{-d/e}*\log((e*x^2 - 2*e*x*\sqrt{-d/e} - d)/(e*x^2 + d)) + \\ & (c^2*d^2*e + a*c*e^3)*\sqrt{-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*\log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*\sqrt{-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)) - (c^2*d^2*e + a*c*e^3)*\sqrt{-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*\log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*\sqrt{-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)) + (c^2*d^2*e + a*c*e^3)*\sqrt{-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*\log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*\sqrt{-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)) - (c^2*d^2*e + a*c*e^3)*\sqrt{-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*\sqrt{-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))})/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)) \end{aligned}$$

$e^4 * \text{sqrt}(- (a^3 * c^2 * d^4 - 2 * a^4 * c * d^2 * e^2 + a^5 * e^4) / (c^9 * d^8 + 4 * a * c^8 * d^6 * e^2 + 6 * a^2 * c^7 * d^4 * e^4 + 4 * a^3 * c^6 * d^2 * e^6 + a^4 * c^5 * e^8)) / (c^4 * d^4 + 2 * a * c^3 * d^2 * e^2 + a^2 * c^2 * e^4)) - 4 * (c * d^2 + a * e^2) * x / (c^2 * d^2 * e + a * c * e^3)$
]

giac [A] time = 0.44, size = 333, normalized size = 0.97

$$\frac{d^{\frac{5}{2}} \arctan\left(\frac{ax}{\sqrt{d}}\right) e^{\frac{1}{2}} \left((ac^3)^{\frac{1}{2}} ace + (ac^3)^{\frac{3}{2}} d \right) \arctan\left(\frac{\sqrt{2} \sqrt{2x + \sqrt{d}} \left(\frac{d}{c}\right)^{\frac{1}{4}}}{2 \left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{cd^2e + ae^3} - \frac{\left((ac^3)^{\frac{1}{2}} ace + (ac^3)^{\frac{3}{2}} d \right) \arctan\left(\frac{\sqrt{2} \sqrt{2x - \sqrt{d}} \left(\frac{d}{c}\right)^{\frac{1}{4}}}{2 \left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{2(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2)} + \frac{xe^{(-1)}}{c} - \frac{\left((ac^3)^{\frac{1}{2}} ace - (ac^3)^{\frac{3}{2}} d \right) \log\left(x^2 + \sqrt{2}x \left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2)} + \frac{\left((ac^3)^{\frac{1}{2}} ace - (ac^3)^{\frac{3}{2}} d \right) \log\left(x^2 - \sqrt{2}x \left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $-d^{5/2} * \arctan(x * e^{1/2} / \text{sqrt}(d)) * e^{-1/2} / (c * d^2 * e + a * e^3) - 1/2 * ((a * c^3)^{1/4} * a * c * e + (a * c^3)^{3/4} * d) * \arctan(1/2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2)) * (a/c)^{1/4}) / (a/c)^{1/4} / (\text{sqrt}(2) * c^4 * d^2 + \text{sqrt}(2) * a * c^3 * e^2) - 1/2 * ((a * c^3)^{1/4} * a * c * e + (a * c^3)^{3/4} * d) * \arctan(1/2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2)) * (a/c)^{1/4}) / (a/c)^{1/4} / (\text{sqrt}(2) * c^4 * d^2 + \text{sqrt}(2) * a * c^3 * e^2) + x * e^{-1} / c - 1/4 * ((a * c^3)^{1/4} * a * c * e - (a * c^3)^{3/4} * d) * \log(x^2 + \text{sqrt}(2) * x * (a/c)^{1/4} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * c^4 * d^2 + \text{sqrt}(2) * a * c^3 * e^2) + 1/4 * ((a * c^3)^{1/4} * a * c * e - (a * c^3)^{3/4} * d) * \log(x^2 - \text{sqrt}(2) * x * (a/c)^{1/4} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * c^4 * d^2 + \text{sqrt}(2) * a * c^3 * e^2)$

maple [A] time = 0.01, size = 387, normalized size = 1.12

$$\frac{d^{\frac{5}{2}} \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(ae^2 + cd^2)\sqrt{de}} - \frac{\sqrt{2} ad \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{4}}c} - \frac{\sqrt{2} ad \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{4}}c} - \frac{\sqrt{2} ad \ln\left(\frac{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{4}}c} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} ae \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^2 + cd^2)c} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} ae \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^2 + cd^2)c} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} ae \ln\left(\frac{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(ae^2 + cd^2)c} + \frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(e*x^2+d)/(c*x^4+a),x)

[Out] $x/c/e - 1/4 * a / (a * e^2 + c * d^2) / c * e * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4}) * x - 1/8 * a / (a * e^2 + c * d^2) / c * e * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) - 1/4 * a / (a * e^2 + c * d^2) / c * e * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) - 1/8 * a / (a * e^2 + c * d^2) / c * d / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) - 1/4 * a / (a * e^2 + c * d^2) / c * d / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) - 1/4 * a / (a * e^2 + c * d^2) / c * d / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) - 1/e * d^3 / (a * e^2 + c * d^2) / (d * e)^{1/2} * \arctan(1 / (d * e)^{1/2} * e * x)$

maxima [A] time = 2.09, size = 289, normalized size = 0.84

$$\frac{d^3 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(cd^2e + ae^3)\sqrt{de}} - \frac{a \left(\frac{2\sqrt{2}(\sqrt{a}cd+a\sqrt{c}) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx+\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}})}}}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(\sqrt{a}cd+a\sqrt{c}) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx-\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}})}}}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}(\sqrt{a}cd-a\sqrt{c}) \log\left(\sqrt{c}x^2+\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}(\sqrt{a}cd-a\sqrt{c}) \log\left(\sqrt{c}x^2-\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} \right)}{8(c^2d^2 + ace^2)} + \frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $-d^3 \arctan(e*x/\sqrt{d*e})/((c*d^2*e + a*e^3)*\sqrt{d*e}) - 1/8*a*(2*\sqrt{2}*(\sqrt{a}*c*d + a*\sqrt{c}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*(c*x + \sqrt{2})*a^{1/4}*c^{1/4})/\sqrt{a*\sqrt{a}*c})) / (\sqrt{a}*\sqrt{a*\sqrt{a}*c})*\sqrt{c} + 2*\sqrt{2}*(\sqrt{a}*c*d + a*\sqrt{c}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*(c*x - \sqrt{2})*a^{1/4}*c^{1/4})/\sqrt{a*\sqrt{a}*c})) / (\sqrt{a}*\sqrt{a*\sqrt{a}*c})*\sqrt{c} - \sqrt{2}*(\sqrt{a}*c*d - a*\sqrt{c}*e)*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a}) / (a^{3/4}*c^{3/4}) + \sqrt{2}*(\sqrt{a}*c*d - a*\sqrt{c}*e)*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a}) / (a^{3/4}*c^{3/4})) / (c^2*d^2 + a*c*e^2) + x/(c*e)$

mupad [B] time = 1.83, size = 5908, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((a + c*x^4)*(d + e*x^2)),x)

[Out] $\operatorname{atan}\left(\frac{(64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2}*(256*a^5*c^5*e^{10} - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} * i - \left(\frac{(64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2}*(256*a^5*c^5*e^{10} - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e)*(- (a*e^2*(-a^3*c^5)^{1/2} - c*d^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^{1/2} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)) \right)$

$$\begin{aligned}
& c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}*i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} - (2*(a^4*c*d^5 - a^5*d^3*e^2))/(c*e))*(-a*e^2*(-a^3*c^5)^{(1/2)} - c*d^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}*2 \\
& i + \operatorname{atan}((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-c*d^2*(-a^3*c^5)^{(1/2)} - a*e^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-c*d^2*(-a^3*c^5)^{(1/2)} - a*e^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)} + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-c*d^2*(-a^3*c^5)^{(1/2)} - a*e^2*(-a^3*c^5)^{(1/2)} + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (48a^3c^4d^6e - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5)/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (2x^2(a^6e^6 - 2a^3c^3d^6))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} * i - (((((64a^5c^4d^6e^8 + 64a^3c^6d^5e^4 + 128a^4c^5d^3e^6)/(c^7e) + (2x^2(-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2})*(256a^5c^5e^10 - 256a^2c^8d^6e^4 - 256a^3c^7d^4e^6 + 256a^4c^6d^2e^8))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (2x^2(64a^2c^6d^7e - 56a^5c^3d^6e^7 + 8a^3c^5d^5e^3 + 16a^4c^4d^3e^5))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (48a^3c^4d^6e - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5)/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} + (2x^2(a^6e^6 - 2a^3c^3d^6))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} * i) / (((((64a^5c^4d^6e^8 + 64a^3c^6d^5e^4 + 128a^4c^5d^3e^6)/(c^7e) - (2x^2(-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2})*(256a^5c^5e^10 - 256a^2c^8d^6e^4 - 256a^3c^7d^4e^6 + 256a^4c^6d^2e^8))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} + (2x^2(64a^2c^6d^7e - 56a^5c^3d^6e^7 + 8a^3c^5d^5e^3 + 16a^4c^4d^3e^5))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (48a^3c^4d^6e - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5)/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (2x^2(a^6e^6 - 2a^3c^3d^6))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} + (((((64a^5c^4d^6e^8 + 64a^3c^6d^5e^4 + 128a^4c^5d^3e^6)/(c^7e) + (2x^2(-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2})*(256a^5c^5e^10 - 256a^2c^8d^6e^4 - 256a^3c^7d^4e^6 + 256a^4c^6d^2e^8))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (2x^2(64a^2c^6d^7e - 56a^5c^3d^6e^7 + 8a^3c^5d^5e^3 + 16a^4c^4d^3e^5))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (48a^3c^4d^6e - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5)/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} + (2x^2(a^6e^6 - 2a^3c^3d^6))/(c^7e) * (-c^2d^2(-a^3c^5)^{1/2} - a^2e^2(-a^3c^5)^{1/2} + 2a^2c^3d^2e)/(16(c^7d^4 + a^2c^5e^4 + 2a^2c^6d^2e^2))^{1/2} - (2(a^4
\end{aligned}$$

$$\frac{(c*d^5 - a^5*d^3*e^2)/(c*e)) * (- (c*d^2*(-a^3*c^5)^{1/2} - a*e^2*(-a^3*c^5)^{1/2} + 2*a^2*c^3*d*e) / (16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2))^{1/2} + x/(c*e) - (\log(16*c^5*x*(-d^5*e^3)^{5/2} - 16*c^5*d^{13}*e^7 - a^5*d^3*e^{17} - 2*a^4*c*d^5*e^{15} + 16*a^2*c^3*d^9*e^{11} - a^3*c^2*d^7*e^{13} + a^5*e^{16}*x*(-d^5*e^3)^{1/2} + a^3*c^2*d^4*e^{12}*x*(-d^5*e^3)^{1/2} + 16*a^2*c^3*d*e^7*x*(-d^5*e^3)^{3/2} + 2*a^4*c*d^2*e^{14}*x*(-d^5*e^3)^{1/2})) * (-d^5*e^3)^{1/2}) / (2*(a*e^5 + c*d^2*e^3)) + (\log(a^5*d^3*e^{17} + 16*c^5*d^{13}*e^7 + 16*c^5*x*(-d^5*e^3)^{5/2} + 2*a^4*c*d^5*e^{15} - 16*a^2*c^3*d^9*e^{11} + a^3*c^2*d^7*e^{13} + a^5*e^{16}*x*(-d^5*e^3)^{1/2} + a^3*c^2*d^4*e^{12}*x*(-d^5*e^3)^{1/2} + 16*a^2*c^3*d*e^7*x*(-d^5*e^3)^{3/2} + 2*a^4*c*d^2*e^{14}*x*(-d^5*e^3)^{1/2})) * (-d^5*e^3)^{1/2}) / (2*a*e^5 + 2*c*d^2*e^3)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.169 \quad \int \frac{x^4}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{a} (\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{a} (\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{a} (\sqrt{a}e + \sqrt{c}d)}{\sqrt{c} (ae^2 + cd^2)}$$

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a} (\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{a} (\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{a} (\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} c^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{a} (\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} c^{3/4} (ae^2 + cd^2)} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c} (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]

[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c^2d^2 + a^2e^2, 0] \ \&\& \ \text{NeQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1288

$\text{Int}[\frac{((f_.)x)^{m_} \cdot ((d_.) + (e_.)x^2)^{q_}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(fx)^m (d + ex^2)^q}{a + cx^4}, x], x] \ /; \text{FreeQ}[\{a, c, d, e, f, m\}, x] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex^2)(a+cx^4)} dx &= \int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)} - \frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx \\
&= -\frac{a \int \frac{d-ex^2}{a+cx^4} dx}{cd^2+ae^2} + \frac{d^2 \int \frac{1}{d+ex^2} dx}{cd^2+ae^2} \\
&= \frac{d^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{e}(cd^2+ae^2)} - \frac{\left(a \left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2c(cd^2+ae^2)} - \frac{\left(a \left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c(cd^2+ae^2)} \\
&= \frac{d^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{e}(cd^2+ae^2)} - \frac{\left(a \left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c(cd^2+ae^2)} - \frac{\left(a \left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c(cd^2+ae^2)} \\
&= \frac{d^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{e}(cd^2+ae^2)} + \frac{\sqrt[4]{a}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} - \frac{\sqrt[4]{a}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)} \\
&= \frac{d^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{e}(cd^2+ae^2)} + \frac{a^{3/4} \left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)} - \frac{a^{3/4} \left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 233, normalized size = 0.69

$$\frac{\sqrt{2}\sqrt[4]{a}\sqrt{e} \left((\sqrt{ae} + \sqrt{c}d) \left(\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) + 2(\sqrt{c}d - \sqrt{a}e) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right) + (2\sqrt{a}e - 2\sqrt{c}d) \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1 \right) \right) + 8c^{3/4}d^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{8c^{3/4}\sqrt{e}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]

[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*c^(3/4)*Sqrt[e]*(c*d^2 + a*e^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)} dx$$

giac [A] time = 0.43, size = 327, normalized size = 0.97

$$\frac{d^{\frac{3}{2}} \arctan\left(\frac{x^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{1}{2}} - \frac{\left((ac^3)^{\frac{1}{2}} c^2 d - (ac^3)^{\frac{3}{2}} e\right) \arctan\left(\frac{\sqrt{2}\sqrt{2x+\sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{2}}} - \frac{\left((ac^3)^{\frac{1}{2}} c^2 d - (ac^3)^{\frac{3}{2}} e\right) \arctan\left(\frac{\sqrt{2}\sqrt{2x-\sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{2}}} - \frac{\left((ac^3)^{\frac{1}{2}} c^2 d + (ac^3)^{\frac{3}{2}} e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}c^4 d^2 + \sqrt{2}ac^3 e^2\right)} + \frac{\left((ac^3)^{\frac{1}{2}} c^2 d + (ac^3)^{\frac{3}{2}} e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}c^4 d^2 + \sqrt{2}ac^3 e^2\right)}}{cd^2 + ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $d^{\frac{3}{2}} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{-\frac{1}{2}} / (c d^2 + a e^2) - \frac{1}{2} \left((a c^3)^{\frac{1}{4}} c^2 d - (a c^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{1}{2} \sqrt{2} \left(\frac{2x + \sqrt{2} \left(\frac{d}{c}\right)^{\frac{1}{4}} \right)}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}\right) / \left(\sqrt{2} c^4 d^2 + \sqrt{2} a c^3 e^2 \right) - \frac{1}{2} \left((a c^3)^{\frac{1}{4}} c^2 d - (a c^3)^{\frac{3}{4}} e \right) \arctan\left(\frac{1}{2} \sqrt{2} \left(\frac{2x - \sqrt{2} \left(\frac{d}{c}\right)^{\frac{1}{4}} \right)}{\left(\frac{d}{c}\right)^{\frac{1}{4}}}\right) / \left(\sqrt{2} c^4 d^2 + \sqrt{2} a c^3 e^2 \right) - \frac{1}{4} \left((a c^3)^{\frac{1}{4}} c^2 d + (a c^3)^{\frac{3}{4}} e \right) \log\left(x^2 + \sqrt{2} x \left(\frac{d}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{c}}\right) / \left(\sqrt{2} c^4 d^2 + \sqrt{2} a c^3 e^2 \right) + \frac{1}{4} \left((a c^3)^{\frac{1}{4}} c^2 d + (a c^3)^{\frac{3}{4}} e \right) \log\left(x^2 - \sqrt{2} x \left(\frac{d}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{c}}\right) / \left(\sqrt{2} c^4 d^2 + \sqrt{2} a c^3 e^2 \right)$

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{d^2 \arctan\left(\frac{cx}{\sqrt{de}}\right) + \frac{\sqrt{2} ae \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}} - 1}\right)}{4(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}} c} + \frac{\sqrt{2} ae \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}} + 1}\right)}{4(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}} c} + \frac{\sqrt{2} ae \ln\left(\frac{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(ae^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}} c} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}} - 1}\right)}{4(ae^2 + cd^2)} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}} + 1}\right)}{4(ae^2 + cd^2)} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2} d \ln\left(\frac{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}} \sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(ae^2 + cd^2)}}{(cd^2 + ae^2)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e*x^2+d)/(c*x^4+a),x)

[Out] $-\frac{1}{8} / (a e^2 + c d^2) d (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \ln\left(\frac{x^2 + (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} x + (a/c)^{\frac{1}{2}}}{x^2 - (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} x + (a/c)^{\frac{1}{2}}}\right) - \frac{1}{4} / (a e^2 + c d^2) d (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} x + 1}\right) - \frac{1}{4} / (a e^2 + c d^2) d (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} x - 1}\right) + \frac{1}{8} a / (a e^2 + c d^2) e / (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \ln\left(\frac{x^2 - (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} x + (a/c)^{\frac{1}{2}}}{x^2 + (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} x + (a/c)^{\frac{1}{2}}}\right) + \frac{1}{4} a / (a e^2 + c d^2) e / (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} x + 1}\right) + \frac{1}{4} a / (a e^2 + c d^2) e / (a/c)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan\left(\frac{2^{\frac{1}{2}}}{(a/c)^{\frac{1}{4}} x - 1}\right) + d^2 / (a e^2 + c d^2) / (d e)^{\frac{1}{2}} \arctan\left(\frac{1}{(d e)^{\frac{1}{2}}}\right) e x$

maxima [A] time = 2.63, size = 268, normalized size = 0.80

$$\frac{d^2 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(cd^2 + ae^2)\sqrt{de}} - \frac{\left(\frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}\sqrt{2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}}}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}\sqrt{2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}}}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} \right)}{8(cd^2 + ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $d^2 \arctan(e*x/\sqrt{d*e}) / ((c*d^2 + a*e^2) \sqrt{d*e}) - 1/8 * a * (2*\sqrt{2}) * (\sqrt{c}*d - \sqrt{a}*e) * \arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{1/4}*c^{1/4}) / \sqrt{\sqrt{a}*\sqrt{c}} / (\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + 2*\sqrt{2}*(\sqrt{c}*d - \sqrt{a}*e) * \arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{1/4}*c^{1/4}) / \sqrt{\sqrt{a}*\sqrt{c}} / (\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e) * \log(\sqrt{c}*x^2 + \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a}) / (a^{3/4}*c^{3/4}) - \sqrt{2}*(\sqrt{c}*d + \sqrt{a}*e) * \log(\sqrt{c}*x^2 - \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a}) / (a^{3/4}*c^{3/4}) / (c*d^2 + a*e^2)$

mapad [B] time = 2.20, size = 5111, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + c*x^4)*(d + e*x^2)),x)

[Out] $\operatorname{atan}\left(\frac{((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))^{1/2} * ((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * (64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * (512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6)) * ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e)) * ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * 1i + (((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * ((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * (x*((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * (512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6)) * ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e)) * ((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * 1i) / (((a*e^2*(-a*c^3)^{1/2} - c*d^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} * 1i)$

$$\begin{aligned}
& *d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))^{(1/2)}*((x*(112*a^4*c^3*d*e^6 + 112* \\
& a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a* \\
& c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)} \\
& * (64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} \\
& + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(512*a \\
& ^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) \\
& + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^{(1/2)} - c* \\
& d^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2* \\
& e^2)))^{(1/2)} + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x \\
& *(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} \\
& + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)} - \\
& (((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 \\
& + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*((x*(112*a^4*c^3*d*e^6 + 112*a^2* \\
& c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3) \\
& ^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)} \\
& *(x*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d \\
& ^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7* \\
& d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + \\
& 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2* \\
& (-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2) \\
&))^{(1/2)} - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2* \\
& a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} \\
& + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)} + 2*a \\
& ^3*c*d^2*e^2))*((a*e^2*(-a*c^3)^{(1/2)} - c*d^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e) \\
& / (16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*2i + \operatorname{atan}(-(((c*d^2 \\
& *(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^ \\
& 3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e \\
& ^2 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + \\
& 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(64*a^2* \\
& c^6*d^6*e^2 - x*((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e) \\
&)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - \\
& 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3* \\
& c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} \\
& + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)} \\
& + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 \\
& + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2*a*c^ \\
& 2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*1i + (((c*d^2* \\
& (-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3 \\
& *e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^ \\
& ^2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2 \\
& *a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(x*((c*d^ \\
& 2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c \\
& ^3*e^4 + 2*a*c^4*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - \\
& 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c \\
& ^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^{(1/2)} - a*e^2*(-a*c^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} - \\
& 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 \\
& + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3* \\
& *d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2}*1i)/(((c*d^2*(\\
& -a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3* \\
& e^4 + 2*a*c^4*d^2*e^2)))^{1/2})*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 \\
& - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2* \\
& a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2}*(64*a^2*c^ \\
& 6*d^6*e^2 - x*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/ \\
& (16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2}*(512*a^5*c^4*e^9 - 51 \\
& 2*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*c^ \\
& 5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} \\
& /2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} + \\
& 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + \\
& 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d* \\
& e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} - (((c*d^2*(-a*c \\
& ^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 \\
& + 2*a*c^4*d^2*e^2)))^{1/2})*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 3 \\
& 2*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^ \\
& 2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2}*(x*((c*d^2*(-a \\
& *c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^ \\
& 4 + 2*a*c^4*d^2*e^2)))^{1/2}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a \\
& ^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^ \\
& 4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} \\
& + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} - 16*a \\
& ^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5 + 4*a \\
& ^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e) \\
& /16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2} + 2*a^3*c*d^2*e^2))* \\
& ((c*d^2*(-a*c^3)^{1/2} - a*e^2*(-a*c^3)^{1/2} + 2*a*c^2*d*e)/(16*(c^5*d^4 + \\
& a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^{1/2}*2i - (\log(16*c^3*x*(-d^3*e)^{5/2} + \\
& a^3*d^2*e^8 + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 + a^3*e^ \\
& 8*x*(-d^3*e)^{1/2} - 17*a*c^2*d*e^3*x*(-d^3*e)^{3/2} + 2*a^2*c*d^2*e^6*x*(- \\
& d^3*e)^{1/2}))*(-d^3*e)^{1/2}))/2*(a*e^3 + c*d^2*e)) + (\log(a^3*d^2*e^8 - 16 \\
& *c^3*x*(-d^3*e)^{5/2} + 16*c^3*d^8*e^2 + 17*a*c^2*d^6*e^4 + 2*a^2*c*d^4*e^6 \\
& - a^3*e^8*x*(-d^3*e)^{1/2} + 17*a*c^2*d*e^3*x*(-d^3*e)^{3/2} - 2*a^2*c*d^2 \\
& *e^6*x*(-d^3*e)^{1/2}))*(-d^3*e)^{1/2}))/2*(a*e^3 + 2*c*d^2*e)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.170 \quad \int \frac{x^2}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=337

$$\frac{(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)} - \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)} - \frac{\sqrt{d} \sqrt{e} \tan^{-1}}{ae^2 + cd^2}$$

Rubi [A] time = 0.27, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)} - \frac{(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)} - \frac{\sqrt{d} \sqrt{e} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{ae^2 + cd^2} - \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)} + \frac{(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{2\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]

[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free

$Q\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1288

$\text{Int}[\frac{((f_.)x)^{m_} \cdot ((d_.) + (e_.)x^2)^{q_}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(fx)^m \cdot (d + ex^2)^q / (a + cx^4), x], x] \ /; \text{FreeQ}\{a, c, d, e, f, m\}, x\} \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx &= \int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2+ae^2)(a+cx^4)} \right) dx \\
&= \frac{\int \frac{ae+cdx^2}{a+cx^4} dx}{cd^2+ae^2} - \frac{(de) \int \frac{1}{d+ex^2} dx}{cd^2+ae^2} \\
&= -\frac{\sqrt{d}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^2+ae^2)} + \frac{\left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2(cd^2+ae^2)} \\
&= -\frac{\sqrt{d}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\left(\sqrt[4]{c}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right)\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2} dx}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)} + \frac{\left(\sqrt[4]{c}\left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right)\right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2} dx}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)} \\
&= -\frac{\sqrt{d}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\sqrt[4]{c}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)} \\
&= -\frac{\sqrt{d}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\sqrt[4]{c}\left(d + \frac{\sqrt{ae}}{\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\left(d - \frac{\sqrt{ae}}{\sqrt{c}}\right) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 232, normalized size = 0.69

$$\frac{\sqrt{2} \left((\sqrt{c}d - \sqrt{ae}) \left(\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) - 2(\sqrt{ae} + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2(\sqrt{ae} + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) - 8\sqrt[4]{a}\sqrt[4]{c}\sqrt{d}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) \right)}{8\sqrt[4]{a}\sqrt[4]{c}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)), x]

[Out] $(-8*a^{(1/4)}*c^{(1/4)}*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]] + \text{Sqrt}[2]*(-2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + (\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*(Log[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] - Log[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])))/(8*a^{(1/4)}*c^{(1/4)}*(c*d^2 + a*e^2))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 1.78, size = 3892, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*((c*d^2 + a*e^2)*\sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*} \\ & \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6 \\ & *a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 \\ & + a^2*e^4))*\log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + \\ & 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4))*\sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/} \\ & (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5 \\ & *c*e^8))*\sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 \\ & - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4 \\ & *e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) \\ &) - (c*d^2 + a*e^2)*\sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 \\ & - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4 \\ & *e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) \\ &)*\log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2 \\ & *c^2*d^3*e^2 + a^3*c*d*e^4))*\sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a \\ & *c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c \\ & *e^8))*\sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 \\ & - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4 \\ & *e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) \\ &) + (c*d^2 + a*e^2)*\sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 \\ & - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3 \\ & *c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + \\ & a^2*e^4))*\log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2 \\ & *c^2*d^3*e^2 + a^3*c*d*e^4))*\sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5 \\ & *d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8} \\ &))\sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 - \\ & 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4 \\ & *e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) \\ &) - (c*d^2 + a*e^2)*\sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 \\ & - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3 \\ & *d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2 \\ & *e^4))*\log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2 \\ & *c^2*d^3*e^2 + a^3*c*d*e^4))*\sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5 \\ & *d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8} \\ &))\sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*\sqrt{-(c^2*d^4 - \\ & 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4 \\ & *e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) \\ &) \end{aligned}$$

$$\begin{aligned}
& 8)) * \sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) - 2 \\
& * \sqrt{-d*e} * \log((e*x^2 - 2*\sqrt{-d*e}*x - d) / (e*x^2 + d)) / (c*d^2 + a*e^2), \\
& -1/4 * ((c*d^2 + a*e^2) * \sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) * \sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) - (c*d^2 + a*e^2) * \sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) * \sqrt{-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) + (c*d^2 + a*e^2) * \sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) * \sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) - (c*d^2 + a*e^2) * \sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) * \sqrt{-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) * \sqrt{-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)} / (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)) + 4 * \sqrt{d*e} * \arctan(\sqrt{d*e}*x/d) / (c*d^2 + a*e^2)]
\end{aligned}$$

giac [A] time = 0.38, size = 336, normalized size = 1.00

$$\frac{\sqrt{d} \arctan\left(\frac{x}{\sqrt{d}}\right) e^{\frac{x}{\sqrt{d}}} + \frac{\left((ac^3)^{\frac{1}{4}} ace + (ac^3)^{\frac{3}{4}} d\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{x}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}} ace + (ac^3)^{\frac{3}{4}} d\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{x}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{x}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}} ace - (ac^3)^{\frac{3}{4}} d\right) \log\left(x^2 + \sqrt{2}x\left(\frac{x}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}} ace - (ac^3)^{\frac{3}{4}} d\right) \log\left(x^2 - \sqrt{2}x\left(\frac{x}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2\right)}{cd^2 + ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $-\sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}} / \sqrt{d}}{\sqrt{d}}\right) e^{\frac{1}{2}} / (c d^2 + a e^2) + 1/2 \left((a c^3)^{\frac{1}{4}} \left(\frac{1}{\sqrt{2} a c^3 d^2 + \sqrt{2} a^2 c^2 e^2} \arctan\left(\frac{1/2 \sqrt{2} (2x + \sqrt{2} (x/c)^{1/4})}{(x/c)^{1/4}}\right) + \frac{1/2 \sqrt{2} (2x - \sqrt{2} (x/c)^{1/4})}{(x/c)^{1/4}} \right) \right) / (\sqrt{2} a c^3 d^2 + \sqrt{2} a^2 c^2 e^2) + 1/4 \left((a c^3)^{\frac{1}{4}} a c^3 e - (a c^3)^{\frac{3}{4}} d \right) \log\left(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}\right) / (\sqrt{2} a c^3 d^2 + \sqrt{2} a^2 c^2 e^2) - 1/4 \left((a c^3)^{\frac{1}{4}} a c^3 e - (a c^3)^{\frac{3}{4}} d \right) \log\left(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}\right) / (\sqrt{2} a c^3 d^2 + \sqrt{2} a^2 c^2 e^2)$

maple [A] time = 0.01, size = 351, normalized size = 1.04

$$\frac{d e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(ae^2 + cd^2)\sqrt{de}} + \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^2 + cd^2)\left(\frac{x}{c}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^2 + cd^2)\left(\frac{x}{c}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} d \ln\left(\frac{x^2 - \left(\frac{x}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{x}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{d}{c}}}\right)}{8(ae^2 + cd^2)\left(\frac{x}{c}\right)^{\frac{1}{4}}} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}-1\right)}{4ae^2 + 4cd^2} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{x}{c}\right)^{\frac{1}{4}}}+1\right)}{4ae^2 + 4cd^2} + \frac{\left(\frac{d}{c}\right)^{\frac{1}{4}} \sqrt{2} e \ln\left(\frac{x^2 + \left(\frac{x}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{d}{c}}}{x^2 - \left(\frac{x}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{d}{c}}}\right)}{8ae^2 + 8cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e*x^2+d)/(c*x^4+a),x)

[Out] $1/4 / (ae^2 + cd^2) e (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) + 1/8 / (ae^2 + cd^2) e (a/c)^{1/4} 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) + 1/4 / (ae^2 + cd^2) e (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) + 1/8 / (ae^2 + cd^2) d / (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) + 1/4 / (ae^2 + cd^2) d / (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) + 1/4 / (ae^2 + cd^2) d / (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) - d e / (ae^2 + cd^2) / (d e)^{1/2} \arctan(1 / (d e)^{1/2} e x)$

maxima [A] time = 1.43, size = 275, normalized size = 0.82

$$\frac{d e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2 + ae^2)\sqrt{de}} + \frac{2\sqrt{2}(\sqrt{a}cd + a\sqrt{c}e) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{2\sqrt{2}(\sqrt{a}cd + a\sqrt{c}e) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} - \frac{\sqrt{2}(\sqrt{a}cd - a\sqrt{c}e) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8(cd^2 + ae^2)} + \frac{\sqrt{2}(\sqrt{a}cd - a\sqrt{c}e) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8(cd^2 + ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& (3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)} + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*1i)/(((-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*((-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*(x*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)} + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)} + ((-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*((-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*(192*a^4*c^4*d*e^7 - x*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)} + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)} + 2*a*c^3*d*e^3))*(-a*e^2*(-a*c)^{(1/2)} - c*d^2*(-a*c)^{(1/2)} + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(e*x**2+d)/(c*x**4+a), x)

[Out] Timed out

$$3.171 \quad \int \frac{1}{(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d)}{\sqrt{d} (ae^2 + cd^2)}$$

Rubi [A] time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)} - \frac{\sqrt[4]{c} (\sqrt{c} d - \sqrt{a} e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{\sqrt[4]{c} (\sqrt{c} d - \sqrt{a} e) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} (ae^2 + cd^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{c} x}{\sqrt{d}}\right)}{\sqrt{d} (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]

[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1171

$\text{Int}[\frac{(d_.) + (e_.)x^2)^{q_}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + ex^2)^q/(a + cx^4), x], x] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{IntegerQ}[q]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(a+cx^4)} dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx \\
&= \frac{c \int \frac{d-ex^2}{a+cx^4} dx}{cd^2+ae^2} + \frac{e^2 \int \frac{1}{d+ex^2} dx}{cd^2+ae^2} \\
&= \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)} \\
&= \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4(cd^2+ae^2)} - \dots \\
&= \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{c}d + \dots)}{\dots} \\
&= \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d - \sqrt{a}e) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}(\sqrt{c}d - \sqrt{a}e) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right) + \sqrt{2}\sqrt[4]{c}\sqrt{d} \left(-(\sqrt{a}e + \sqrt{c}d) \left(\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) + (2\sqrt{a}e - 2\sqrt{c}d) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right) + 2(\sqrt{c}d - \sqrt{a}e) \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1 \right) \right)}{8a^{3/4}\sqrt{d}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]

[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[d]*(c*d^2 + a*e^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)(a+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 3.20, size = 4084, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*((c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) + (c*d^2 + a*e^2)*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*\log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})*\sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)}})/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) \end{aligned}$$

$$\begin{aligned}
& \frac{2*a*c^2*d^2*e^2 + a^2*c*e^4}{(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)} * \sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / \\
& \frac{(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)}{(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)} - 2*e*\sqrt{-e/d}*\log((e*x^2 + 2*d*x*\sqrt{-e/d} - d)/(e*x^2 + d)) / (c*d^2 + a*e^2), \frac{1}{4}*(4*e*\sqrt{e/d}*\arctan \\
& (x*\sqrt{e/d}) - (c*d^2 + a*e^2)*\sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \sqrt{(2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) - (c*d^2 + a*e^2) * \sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) + (c*d^2 + a*e^2) * \sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) * \sqrt{(2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) * \sqrt{-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)})} / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)) / (c*d^2 + a*e^2)]
\end{aligned}$$

giac [A] time = 0.42, size = 339, normalized size = 1.01

$$\frac{\left((ac^3)^{\frac{1}{4}}c^2d - (ac^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{e}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{e}{c}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d - (ac^3)^{\frac{3}{4}}e\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{e}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{e}{c}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{e}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d + (ac^3)^{\frac{3}{4}}e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{e}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4(\sqrt{2}ac^3d^2 + \sqrt{2}a^2c^2e^2)} + \frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{a}}\right)e^{\frac{3}{2}}}{(cd^2 + ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 + a*e^2)*sqrt(d))

maple [A] time = 0.01, size = 363, normalized size = 1.08

$$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(ae^2 + cd^2)\sqrt{de}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \arctan\left(\frac{\sqrt{2}x}{\left(\frac{e}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^2 + cd^2)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \arctan\left(\frac{\sqrt{2}x}{\left(\frac{e}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^2 + cd^2)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd \ln\left(\frac{x^2 + \left(\frac{e}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{e}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8(ae^2 + cd^2)a} - \frac{\sqrt{2}e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{e}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^2 + cd^2)\left(\frac{e}{c}\right)^{\frac{1}{4}}} - \frac{\sqrt{2}e \arctan\left(\frac{\sqrt{2}x}{\left(\frac{e}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^2 + cd^2)\left(\frac{e}{c}\right)^{\frac{1}{4}}} - \frac{\sqrt{2}e \ln\left(\frac{x^2 - \left(\frac{e}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 + \left(\frac{e}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8(ae^2 + cd^2)\left(\frac{e}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+a),x)

[Out] 1/8*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a^2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a^2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d*(a/c)^(1/4)/a^2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)

maxima [A] time = 1.09, size = 268, normalized size = 0.80

$$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2 + ae^2)\sqrt{de}} + \frac{\left(2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\left(2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out]
$$e^2 \arctan(e x / \sqrt{d e}) / ((c d^2 + a e^2) \sqrt{d e}) + 1/8 c (2 \sqrt{2}) (\sqrt{c} d - \sqrt{a} e) \arctan(1/2 \sqrt{2} (2 \sqrt{c} x + \sqrt{2} a^{1/4} c^{1/4}) / \sqrt{\sqrt{a} \sqrt{c}}) / (\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}) + 2 \sqrt{2} (\sqrt{c} d - \sqrt{a} e) \arctan(1/2 \sqrt{2} (2 \sqrt{c} x - \sqrt{2} a^{1/4} c^{1/4}) / \sqrt{\sqrt{a} \sqrt{c}}) / (\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}) + \sqrt{2} (\sqrt{c} d + \sqrt{a} e) \log(\sqrt{c} x^2 + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{a}) / (a^{3/4} c^{3/4}) - \sqrt{2} (\sqrt{c} d + \sqrt{a} e) \log(\sqrt{c} x^2 - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{a}) / (a^{3/4} c^{3/4}) / (c d^2 + a e^2)$$

mpad [B] time = 1.67, size = 4802, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)*(d + e*x^2)),x)

[Out]
$$\operatorname{atan}\left(\frac{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (4 c^6 d^3 e^3 - ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (256 a^4 c^4 e^8 + x ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (512 a^5 c^4 e^9 - 512 a^2 c^7 d^6 e^3 - 512 a^3 c^6 d^4 e^5 + 512 a^4 c^5 d^2 e^7) - 64 a^6 c^7 d^6 e^2 + 128 a^2 c^6 d^4 e^4 + 448 a^3 c^5 d^2 e^6) + x (16 c^7 d^5 e^2 + 32 a^6 c^6 d^3 e^4 - 240 a^2 c^5 d e^6))}{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} + 20 a^5 c^5 d e^5 - 6 c^5 e^5 x} \left(\frac{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} * i - ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} * (-i)}{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (4 c^6 d^3 e^3 - ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (256 a^4 c^4 e^8 - x ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (512 a^5 c^4 e^9 - 512 a^2 c^7 d^6 e^3 - 512 a^3 c^6 d^4 e^5 + 512 a^4 c^5 d^2 e^7) - 64 a^6 c^7 d^6 e^2 + 128 a^2 c^6 d^4 e^4 + 448 a^3 c^5 d^2 e^6) - x (16 c^7 d^5 e^2 + 32 a^6 c^6 d^3 e^4 - 240 a^2 c^5 d e^6))}{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} + 20 a^5 c^5 d e^5 + 6 c^5 e^5 x} \right) / \left(\frac{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (4 c^6 d^3 e^3 - ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (256 a^4 c^4 e^8 - x ((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} (512 a^5 c^4 e^9 - 512 a^2 c^7 d^6 e^3 - 512 a^3 c^6 d^4 e^5 + 512 a^4 c^5 d^2 e^7) - 64 a^6 c^7 d^6 e^2 + 128 a^2 c^6 d^4 e^4 + 448 a^3 c^5 d^2 e^6) - x (16 c^7 d^5 e^2 + 32 a^6 c^6 d^3 e^4 - 240 a^2 c^5 d e^6))}{((a e^2 (-a^3 c)^{1/2} - c d^2 (-a^3 c)^{1/2} + 2 a^2 c d e) / (16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2)))^{1/2} + 20 a^5 c^5 d e^5 + 6 c^5 e^5 x} \right)$$

$$\begin{aligned}
 & \sqrt[3]{c}^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^{(1/2)} - \\
 & c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 \\
 & + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((\\
 & (a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a \\
 & *e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + (((a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^ \\
 & 3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(\\
 & 1/2)}*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^ \\
 & 2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(256*a^4*c^4 \\
 & *e^8 - x*((a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(\\
 & a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2 \\
 & *c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^ \\
 & 2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c \\
 & ^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1 \\
 & /2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + \\
 & 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2 \\
 &) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)))*((\\
 & a*e^2*(-a^3*c)^{(1/2)} - c*d^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a \\
 & ^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*2i + atan((((c*d^2*(-a^3*c)^{(1/2)} - \\
 & a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^ \\
 & 2*e^2)))^{(1/2)}*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1 \\
 & /2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(2 \\
 & 56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c* \\
 & d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 \\
 & - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a* \\
 & c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^ \\
 & 2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(\\
 & -a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) \\
 &)^{(1/2)} + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a \\
 & ^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{ \\
 & (1/2)}*1i - (((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(1 \\
 & 6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(4*c^6*d^3*e^3 - (((c*d \\
 & ^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3* \\
 & c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)}*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^{(1 \\
 & /2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^ \\
 & 4*c*d^2*e^2)))^{(1/2)}*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d \\
 & ^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44 \\
 & 8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e \\
 & ^6))*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5* \\
 & e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^{(1/2)} + 20*a*c^5*d*e^5) + 6*c^5*e^5* \\
 & x)*((c*d^2*(-a^3*c)^{(1/2)} - a*e^2*(-a^3*c)^{(1/2)} + 2*a^2*c*d*e)/(16*(a^5*e^
 \end{aligned}$$

$$\begin{aligned}
& \left(4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2 \right)^{1/2} i / \left(\left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \right. \\
& \left. \left(4 c^6 d^3 e^3 - \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \right) \right. \\
& \left. \left(256 a^4 c^4 e^8 + x \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \right) \right. \\
& \left. \left(512 a^5 c^4 e^9 - 512 a^2 c^7 d^6 e^3 - 512 a^3 c^6 d^4 e^5 + 512 a^4 c^5 d^2 e^7 \right) - 64 a^3 c^7 d^6 e^2 + 128 a^2 c^6 d^4 e^4 + 448 a^3 c^5 d^2 e^6 \right) \\
& + x \left(16 c^7 d^5 e^2 + 32 a^3 c^6 d^3 e^4 - 240 a^2 c^5 d e^6 \right) \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \\
& + 20 a^3 c^5 d e^5 - 6 c^5 e^5 x \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \\
& + \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \left(4 c^6 d^3 e^3 - \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \right) \\
& \left(256 a^4 c^4 e^8 - x \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \right) \\
& \left(512 a^5 c^4 e^9 - 512 a^2 c^7 d^6 e^3 - 512 a^3 c^6 d^4 e^5 + 512 a^4 c^5 d^2 e^7 \right) - 64 a^3 c^7 d^6 e^2 + 128 a^2 c^6 d^4 e^4 + 448 a^3 c^5 d^2 e^6 \\
& - x \left(16 c^7 d^5 e^2 + 32 a^3 c^6 d^3 e^4 - 240 a^2 c^5 d e^6 \right) \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \\
& + 20 a^3 c^5 d e^5 + 6 c^5 e^5 x \left(\left(c d^2 (-a^3 c)^{1/2} - a e^2 (-a^3 c)^{1/2} + 2 a^2 c d e \right) / \left(16 (a^5 e^4 + a^3 c^2 d^4 + 2 a^4 c d^2 e^2) \right) \right)^{1/2} \Big) \\
& \left(\log \left(16 a^2 e^2 (-d e^3)^{3/2} + c^2 d^5 e^3 x - c^2 d^5 e^3 (-d e^3)^{1/2} + 16 a^2 d e^7 x + a c d^2 (-d e^3)^{3/2} + a c d^3 e^5 x \right) (-d e^3)^{1/2} \right) \\
& \left(2 (c d^3 + a d e^2) \right) + \left(\log \left(c^2 d^5 e^3 x - 16 a^2 e^2 (-d e^3)^{3/2} + c^2 d^5 e^3 (-d e^3)^{1/2} + 16 a^2 d e^7 x + 4 a c d^2 (-d e^3)^{3/2} \right) \right) \\
& \left(2 (c d^3 + a d e^2) \right) + 5 a c d^3 e^3 (-d e^3)^{1/2} \Big) (-d e^3)^{1/2} / \left(2 c d^3 + 2 a d e^2 \right)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.172 \quad \int \frac{1}{x^2(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=348

$$\frac{c^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{5/4}(ae^2 + cd^2)} + \frac{c^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{5/4}(ae^2 + cd^2)} + \dots$$

Rubi [A] time = 0.30, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{5/4}(ae^2 + cd^2)} + \frac{c^{3/4}(\sqrt{c}d - \sqrt{a}e) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{5/4}(ae^2 + cd^2)} + \frac{c^{3/4}(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(ae^2 + cd^2)} - \frac{c^{3/4}(\sqrt{a}e + \sqrt{c}d) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{5/4}(ae^2 + cd^2)} - \frac{e^{5/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{d^{3/2}(ae^2 + cd^2)} - \frac{1}{adx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)), x]

[Out] $-(1/(a*d*x)) - (e^{5/2} \text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(d^{3/2}*(c*d^2 + a*e^2)) + (c^{3/4}*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/(2*\text{Sqrt}[2]*a^{5/4}*(c*d^2 + a*e^2)) - (c^{3/4}*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/(2*\text{Sqrt}[2]*a^{5/4}*(c*d^2 + a*e^2)) - (c^{3/4}*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{5/4}*(c*d^2 + a*e^2)) + (c^{3/4}*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{5/4}*(c*d^2 + a*e^2))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x]] \ /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1288

$\text{Int}[\frac{((f_.)x)^{m_.} \cdot ((d_.) + (e_.)x^2)^{q_}}{(a_.) + (c_.)x^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(fx)^m \cdot (d + ex^2)^q / (a + cx^4), x], x] \ /; \text{FreeQ}\{a, c, d, e, f, m\}, x\} \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (d + ex^2) (a + cx^4)} dx &= \int \left(\frac{1}{adx^2} - \frac{e^3}{d (cd^2 + ae^2) (d + ex^2)} - \frac{c (ae + cd^2)}{a (cd^2 + ae^2) (a + cx^4)} \right) dx \\
&= -\frac{1}{adx} - \frac{c \int \frac{ae+cdx^2}{a+cx^4} dx}{a (cd^2 + ae^2)} - \frac{e^3 \int \frac{1}{d+ex^2} dx}{d (cd^2 + ae^2)} \\
&= -\frac{1}{adx} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)} + \frac{\left(c \left(d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \right) \int \frac{\sqrt{a} \sqrt{c} - cx^2}{a+cx^4} dx}{2a (cd^2 + ae^2)} - \frac{\left(c \left(d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \right) \int \frac{\sqrt{a} \sqrt{c}}{a+cx^4} dx}{2a (cd^2 + ae^2)} \\
&= -\frac{1}{adx} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)} - \frac{\left(c^{5/4} \left(d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \right) \int \frac{\frac{\sqrt{2} \sqrt[4]{a} + 2x}{\sqrt{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a} x}{\sqrt{c}} - x^2} dx}{4\sqrt{2} a^{5/4} (cd^2 + ae^2)} - \frac{\left(c^{5/4} \left(d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \right) \int \frac{\sqrt{a} \sqrt{c}}{a+cx^4} dx}{4\sqrt{2} a^{5/4} (cd^2 + ae^2)} \\
&= -\frac{1}{adx} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)} - \frac{c^{5/4} \left(d - \frac{\sqrt{ae}}{\sqrt{c}} \right) \log \left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{c} x^2 \right)}{4\sqrt{2} a^{5/4} (cd^2 + ae^2)} + \frac{c^{5/4} \left(d + \frac{\sqrt{ae}}{\sqrt{c}} \right) \log \left(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{c} x^2 \right)}{4\sqrt{2} a^{5/4} (cd^2 + ae^2)} \\
&= -\frac{1}{adx} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)} + \frac{c^{3/4} \left(\sqrt{c} d + \sqrt{ae} \right) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} \right)}{2\sqrt{2} a^{5/4} (cd^2 + ae^2)} - \frac{c^{3/4} \left(\sqrt{c} d + \sqrt{ae} \right) \tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} \right)}{2\sqrt{2} a^{5/4} (cd^2 + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 389, normalized size = 1.12

$$\frac{-\sqrt{d} \left(8a^{5/4} + \sqrt{2} c^{3/4} \log \left(-\sqrt{2} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) - \sqrt{2} c^{3/4} \log \left(\sqrt{2} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) - \sqrt{2} \sqrt{a} c^{3/4} \log \left(-\sqrt{2} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) + \sqrt{2} \sqrt{a} c^{3/4} \log \left(\sqrt{2} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) - 2\sqrt{2} c^{3/4} \left(\sqrt{a} + \sqrt{c} d \right) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} \right) + 2\sqrt{2} c^{3/4} \left(\sqrt{a} + \sqrt{c} d \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt{a}} + 1 \right) + 8\sqrt{2} a^{5/4} \right) \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{8a^{5/4} \sqrt{d} (a^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)), x]

[Out] $(-8a^{5/4}e^{5/2}x \operatorname{ArcTan}[\frac{\sqrt{e}x}{\sqrt{d}}] - \sqrt{d}(8a^{1/4}cd^2 + 8a^{5/4}e^2 - 2\sqrt{2}c^{3/4}d(\sqrt{c}d + \sqrt{a}e))x \operatorname{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}] + 2\sqrt{2}c^{3/4}d(\sqrt{c}d + \sqrt{a}e)x \operatorname{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}] + \sqrt{2}c^{5/4}d^2x \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] - \sqrt{2}c^{5/4}d^2x \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] - \sqrt{2}c^{5/4}d^2x \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + \sqrt{2}c^{5/4}d^2x \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + \sqrt{2}c^{3/4}d(\sqrt{c}d + \sqrt{a}e)x \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] - \sqrt{2}c^{3/4}d(\sqrt{c}d + \sqrt{a}e)x \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / (8a^{5/4}d^{3/2}(cd^2 + ae^2)x)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 9.08, size = 4362, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)))]

$$\begin{aligned}
&^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8))} / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) - (a c d^3 + a^2 d e^2) x \sqrt{-(2 c^2 d e - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8))} / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) \log(-(c^3 d^2 - a c^2 e^2) x - (a^2 c^2 d^2 e - a^3 c e^3 + (a^4 c^2 d^5 + 2 a^5 c d^3 e^2 + a^6 d e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) \sqrt{-(2 c^2 d e - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8))} / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) - 4 c d^2 - 4 a e^2) / ((a c d^3 + a^2 d e^2) x), -1/4 * (4 a e^2 x \sqrt{e/d} * \arctan(x \sqrt{e/d}) - (a c d^3 + a^2 d e^2) x \sqrt{-(2 c^2 d e + (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) \log(-(c^3 d^2 - a c^2 e^2) x + (a^2 c^2 d^2 e - a^3 c e^3 - (a^4 c^2 d^5 + 2 a^5 c d^3 e^2 + a^6 d e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) \sqrt{-(2 c^2 d e + (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8))} / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) + (a c d^3 + a^2 d e^2) x \sqrt{-(2 c^2 d e + (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) \log(-(c^3 d^2 - a c^2 e^2) x - (a^2 c^2 d^2 e - a^3 c e^3 - (a^4 c^2 d^5 + 2 a^5 c d^3 e^2 + a^6 d e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) \sqrt{-(2 c^2 d e + (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8))} / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) - (a c d^3 + a^2 d e^2) x \sqrt{-(2 c^2 d e - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) \log(-(c^3 d^2 - a c^2 e^2) x + (a^2 c^2 d^2 e - a^3 c e^3 + (a^4 c^2 d^5 + 2 a^5 c d^3 e^2 + a^6 d e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) \sqrt{-(2 c^2 d e - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)) + (a c d^3 + a^2 d e^2) x \sqrt{-(2 c^2 d e - (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{-(c^5 d^4 - 2 a^2 c^4 d^2 e^2 + a^2 c^3 e^4)/(a^5 c^4 d^8 + 4 a^6 c^3 d^6 e^2 + 6 a^7 c^2 d^4 e^4 + 4 a^8 c d^2 e^6 + a^9 e^8)}) / (a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4))
\end{aligned}$$

$$\begin{aligned} & \left(6a^6e^2 + 6a^7c^2d^4e^4 + 4a^8c^4d^2e^6 + a^9e^8 \right) / \left(a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 \right) * \log \left(- \left(c^3d^2 - ac^2e^2 \right) * x - \left(a^2c^2d^2e - a^3c^2e^3 + \left(a^4c^2d^5 + 2a^5c^2d^3e^2 + a^6d^2e^4 \right) * \sqrt{- \left(c^5d^4 - 2ac^4d^2e^2 + a^2c^3e^4 \right)} / \left(a^5c^4d^8 + 4a^6c^3d^6e^2 + 6a^7c^2d^4e^4 + 4a^8c^2d^2e^6 + a^9e^8 \right) \right) * \sqrt{- \left(2c^2d^2e - \left(a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 \right) * \sqrt{- \left(c^5d^4 - 2ac^4d^2e^2 + a^2c^3e^4 \right)} / \left(a^5c^4d^8 + 4a^6c^3d^6e^2 + 6a^7c^2d^4e^4 + 4a^8c^2d^2e^6 + a^9e^8 \right)} \right) / \left(a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 \right) \right) + 4c^2d^2 + 4a^2e^2 / \left(\left(ac^3d^3 + a^2d^2e^2 \right) * x \right) \end{aligned}$$

giac [A] time = 0.40, size = 348, normalized size = 1.00

$$\frac{\left((ac^3)^{\frac{1}{4}} ace + (ac^3)^{\frac{3}{4}} d \right) \arctan \left(\frac{\sqrt{2x - \sqrt{2} \left(\frac{d}{c} \right)^{\frac{1}{2}}}}{2 \left(\frac{d}{c} \right)^{\frac{1}{4}}} \right)}{2 \left(\sqrt{2} a^2 c^2 d^2 + \sqrt{2} a^3 c e^2 \right)} - \frac{\left((ac^3)^{\frac{1}{4}} ace + (ac^3)^{\frac{3}{4}} d \right) \arctan \left(\frac{\sqrt{2 \left(2x - \sqrt{2} \left(\frac{d}{c} \right)^{\frac{1}{2}} \right)}}{2 \left(\frac{d}{c} \right)^{\frac{1}{4}}} \right)}{2 \left(\sqrt{2} a^2 c^2 d^2 + \sqrt{2} a^3 c e^2 \right)} - \frac{\left((ac^3)^{\frac{1}{4}} ace - (ac^3)^{\frac{3}{4}} d \right) \log \left(x^2 + \sqrt{2} x \left(\frac{d}{c} \right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}} \right)}{4 \left(\sqrt{2} a^2 c^2 d^2 + \sqrt{2} a^3 c e^2 \right)} + \frac{\left((ac^3)^{\frac{1}{4}} ace - (ac^3)^{\frac{3}{4}} d \right) \log \left(x^2 - \sqrt{2} x \left(\frac{d}{c} \right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}} \right)}{4 \left(\sqrt{2} a^2 c^2 d^2 + \sqrt{2} a^3 c e^2 \right)} - \frac{\arctan \left(\frac{\frac{1}{2} \sqrt{2}}{\sqrt{d}} \right) e^{\frac{5}{2}}}{(cd^3 + ad^2) \sqrt{d}} - \frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

$$\begin{aligned} & \text{[Out]} \quad -1/2 * \left((ac^3)^{\frac{1}{4}} * ac * e + (ac^3)^{\frac{3}{4}} * d \right) * \arctan \left(1/2 * \sqrt{2} * (2x + \sqrt{2}) * (a/c)^{\frac{1}{4}} \right) / \left(a/c \right)^{\frac{1}{4}} / \left(\sqrt{2} * a^2 * c^2 * d^2 + \sqrt{2} * a^3 * c * e^2 \right) - \\ & 1/2 * \left((ac^3)^{\frac{1}{4}} * ac * e + (ac^3)^{\frac{3}{4}} * d \right) * \arctan \left(1/2 * \sqrt{2} * (2x - \sqrt{2}) * (a/c)^{\frac{1}{4}} \right) / \left(a/c \right)^{\frac{1}{4}} / \left(\sqrt{2} * a^2 * c^2 * d^2 + \sqrt{2} * a^3 * c * e^2 \right) - \\ & 1/4 * \left((ac^3)^{\frac{1}{4}} * ac * e - (ac^3)^{\frac{3}{4}} * d \right) * \log \left(x^2 + \sqrt{2} * x * (a/c)^{\frac{1}{4}} + \sqrt{2} * (a/c)^{\frac{1}{4}} \right) / \left(\sqrt{2} * a^2 * c^2 * d^2 + \sqrt{2} * a^3 * c * e^2 \right) + \\ & 1/4 * \left((ac^3)^{\frac{1}{4}} * ac * e - (ac^3)^{\frac{3}{4}} * d \right) * \log \left(x^2 - \sqrt{2} * x * (a/c)^{\frac{1}{4}} + \sqrt{2} * (a/c)^{\frac{1}{4}} \right) / \left(\sqrt{2} * a^2 * c^2 * d^2 + \sqrt{2} * a^3 * c * e^2 \right) - \\ & \arctan \left(x * e^{\frac{1}{2}} / \sqrt{d} \right) * e^{\frac{5}{2}} / \left(\left(c * d^3 + a * d * e^2 \right) * \sqrt{d} \right) - 1 / (a * d * x) \end{aligned}$$

maple [A] time = 0.01, size = 390, normalized size = 1.12

$$\frac{e^3 \arctan \left(\frac{ex}{\sqrt{de}} \right)}{(ae^2 + cd^2) \sqrt{de} d} - \frac{\sqrt{2} cd \arctan \left(\frac{\sqrt{2} x}{\left(\frac{d}{c} \right)^{\frac{1}{4}}} - 1 \right)}{4 \left(ae^2 + cd^2 \right) \left(\frac{d}{c} \right)^{\frac{1}{4}} a} - \frac{\sqrt{2} cd \arctan \left(\frac{\sqrt{2} x}{\left(\frac{d}{c} \right)^{\frac{1}{4}}} + 1 \right)}{4 \left(ae^2 + cd^2 \right) \left(\frac{d}{c} \right)^{\frac{1}{4}} a} - \frac{\sqrt{2} cd \ln \left(\frac{x^2 + \left(\frac{d}{c} \right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{d}{c} \right)^{\frac{1}{2}} \sqrt{2} x - \sqrt{\frac{d}{c}}} \right)}{8 \left(ae^2 + cd^2 \right) \left(\frac{d}{c} \right)^{\frac{1}{4}} a} - \frac{\left(\frac{d}{c} \right)^{\frac{1}{4}} \sqrt{2} ce \arctan \left(\frac{\sqrt{2} x}{\left(\frac{d}{c} \right)^{\frac{1}{4}}} - 1 \right)}{4 \left(ae^2 + cd^2 \right) a} - \frac{\left(\frac{d}{c} \right)^{\frac{1}{4}} \sqrt{2} ce \arctan \left(\frac{\sqrt{2} x}{\left(\frac{d}{c} \right)^{\frac{1}{4}}} + 1 \right)}{4 \left(ae^2 + cd^2 \right) a} - \frac{\left(\frac{d}{c} \right)^{\frac{1}{4}} \sqrt{2} ce \ln \left(\frac{x^2 + \left(\frac{d}{c} \right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{d}{c} \right)^{\frac{1}{2}} \sqrt{2} x - \sqrt{\frac{d}{c}}} \right)}{8 \left(ae^2 + cd^2 \right) a} - \frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x)

$$\begin{aligned} & \text{[Out]} \quad -1/a/d/x - 1/4 * c / (a * e^2 + c * d^2) / a * e * (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left(2^{\frac{1}{2}} / (a/c)^{\frac{1}{4}} * x - 1 \right) - \\ & 1/8 * c / (a * e^2 + c * d^2) / a * e * (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \ln \left((x^2 + (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) / (x^2 - (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) \right) - \\ & 1/4 * c / (a * e^2 + c * d^2) / a * e * (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left(2^{\frac{1}{2}} / (a/c)^{\frac{1}{4}} * x + 1 \right) - \\ & 1/8 * c / (a * e^2 + c * d^2) / a * d / (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \ln \left((x^2 - (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) / (x^2 + (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) \right) - \\ & 1/4 * c / (a * e^2 + c * d^2) / a * d / (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left(2^{\frac{1}{2}} / (a/c)^{\frac{1}{4}} * x + 1 \right) - \\ & 1/4 * c / (a * e^2 + c * d^2) / a * d / (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left(2^{\frac{1}{2}} / (a/c)^{\frac{1}{4}} * x - 1 \right) - \\ & 1/4 * c / (a * e^2 + c * d^2) / a * d / (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \ln \left((x^2 + (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) / (x^2 - (a/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * x + (a/c)^{\frac{1}{2}}) \right) \end{aligned}$$

/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)

maxima [A] time = 1.27, size = 292, normalized size = 0.84

$$\frac{e^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^3 + adc^2)\sqrt{de}} - \frac{\left(\frac{2\sqrt{2}(\sqrt{a}cd+a\sqrt{c}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{e}} + \frac{2\sqrt{2}(\sqrt{a}cd+a\sqrt{c}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}\sqrt{e}} - \frac{\sqrt{2}(\sqrt{a}cd-a\sqrt{c}e)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}(\sqrt{a}cd-a\sqrt{c}e)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} \right)}{8(acd^2 + a^2e^2)} - \frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] -e^3*arctan(e*x/sqrt(d*e))/((c*d^3 + a*d*e^2)*sqrt(d*e)) - 1/8*c*(2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c) + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))/(a*c*d^2 + a^2*e^2) - 1/(a*d*x)

mupad [B] time = 2.00, size = 5761, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)

[Out] atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - ((-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*((-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*((-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*((-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - ((-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(-a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)

$$\begin{aligned}
& \sqrt[3]{c^2 d e} / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2))^{1/2} * (((- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (x * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (512 a^{11} c^7 d^{15} e^3 + 512 a^{12} c^6 d^{13} e^5 - 512 a^{13} c^5 d^{11} e^7 - 512 a^{14} c^4 d^9 e^9) + 192 a^{10} c^7 d^{14} e^3 + 128 a^{11} c^6 d^{12} e^5 - 320 a^{12} c^5 d^{10} e^7 - 256 a^{13} c^4 d^8 e^9) + x * (16 a^8 c^8 d^{14} e^2 + 32 a^9 c^7 d^{12} e^4 - 112 a^{10} c^6 d^{10} e^6 + 128 a^{11} c^5 d^8 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} + 4 a^7 c^8 d^{13} e^2 + 4 a^8 c^7 d^{11} e^4 - 16 a^{10} c^5 d^7 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * i) / ((x * (2 a^7 c^7 d^9 e^5 - 4 a^8 c^6 d^7 e^7) - (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * ((- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (x * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (512 a^{11} c^7 d^{15} e^3 + 512 a^{12} c^6 d^{13} e^5 - 512 a^{13} c^5 d^{11} e^7 - 512 a^{14} c^4 d^9 e^9) - 192 a^{10} c^7 d^{14} e^3 - 128 a^{11} c^6 d^{12} e^5 + 320 a^{12} c^5 d^{10} e^7 + 256 a^{13} c^4 d^8 e^9) + x * (16 a^8 c^8 d^{14} e^2 + 32 a^9 c^7 d^{12} e^4 - 112 a^{10} c^6 d^{10} e^6 + 128 a^{11} c^5 d^8 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} - 4 a^7 c^8 d^{13} e^2 - 4 a^8 c^7 d^{11} e^4 + 16 a^{10} c^5 d^7 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} - (x * (2 a^7 c^7 d^9 e^5 - 4 a^8 c^6 d^7 e^7) - (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * ((- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (x * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (512 a^{11} c^7 d^{15} e^3 + 512 a^{12} c^6 d^{13} e^5 - 512 a^{13} c^5 d^{11} e^7 - 512 a^{14} c^4 d^9 e^9) + 192 a^{10} c^7 d^{14} e^3 + 128 a^{11} c^6 d^{12} e^5 - 320 a^{12} c^5 d^{10} e^7 - 256 a^{13} c^4 d^8 e^9) + x * (16 a^8 c^8 d^{14} e^2 + 32 a^9 c^7 d^{12} e^4 - 112 a^{10} c^6 d^{10} e^6 + 128 a^{11} c^5 d^8 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} + 4 a^7 c^8 d^{13} e^2 + 4 a^8 c^7 d^{11} e^4 - 16 a^{10} c^5 d^7 e^8)) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2})) * (- (a e^2 * (-a^5 c^3)^{1/2} - c d^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * 2i + a \tan(((x * (2 a^7 c^7 d^9 e^5 - 4 a^8 c^6 d^7 e^7) - (- (c d^2 * (-a^5 c^3)^{1/2} - a e^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * ((- (c d^2 * (-a^5 c^3)^{1/2} - a e^2 * (-a^5 c^3)^{1/2} + 2a^3 c^2 d e) / (16(a^7 e^4 + a^5 c^2 d^4 + 2a^6 c d^2 e^2)))^{1/2} * (x * (
\end{aligned}$$

$$\begin{aligned}
& -(c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7 \\
& *e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*(512*a^{11}*c^7*d^{15}*e^3 + 512* \\
& a^{12}*c^6*d^{13}*e^5 - 512*a^{13}*c^5*d^{11}*e^7 - 512*a^{14}*c^4*d^9*e^9) - 192*a^{1 \\
& 0}*c^7*d^{14}*e^3 - 128*a^{11}*c^6*d^{12}*e^5 + 320*a^{12}*c^5*d^{10}*e^7 + 256*a^{13}*c \\
& ^4*d^8*e^9) + x*(16*a^8*c^8*d^{14}*e^2 + 32*a^9*c^7*d^{12}*e^4 - 112*a^{10}*c^6*d \\
& ^{10}*e^6 + 128*a^{11}*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^ \\
& 3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(\\
& 1/2)} - 4*a^7*c^8*d^{13}*e^2 - 4*a^8*c^7*d^{11}*e^4 + 16*a^{10}*c^5*d^7*e^8))*(-(c \\
& *d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^ \\
& 4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*a \\
& ^8*c^6*d^7*e^7) - ((-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^ \\
& 3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*((-c*d^2 \\
& *(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + \\
& a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*(x*(-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2 \\
& *(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2 \\
& *e^2))^{(1/2)}*(512*a^{11}*c^7*d^{15}*e^3 + 512*a^{12}*c^6*d^{13}*e^5 - 512*a^{13}*c^5 \\
& *d^{11}*e^7 - 512*a^{14}*c^4*d^9*e^9) + 192*a^{10}*c^7*d^{14}*e^3 + 128*a^{11}*c^6*d^ \\
& ^{12}*e^5 - 320*a^{12}*c^5*d^{10}*e^7 - 256*a^{13}*c^4*d^8*e^9) + x*(16*a^8*c^8*d^{14} \\
& *e^2 + 32*a^9*c^7*d^{12}*e^4 - 112*a^{10}*c^6*d^{10}*e^6 + 128*a^{11}*c^5*d^8*e^8)) \\
& *(-(c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a \\
& ^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)} + 4*a^7*c^8*d^{13}*e^2 + 4*a^ \\
& 8*c^7*d^{11}*e^4 - 16*a^{10}*c^5*d^7*e^8))*(-(c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(- \\
& a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^ \\
& 2))^{(1/2)}*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - ((-c*d^2*(-a^5 \\
& *c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^ \\
& 2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*((-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5* \\
& c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) \\
& ^{(1/2)}*(x*(-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d* \\
& e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*(512*a^{11}*c^7*d^{15} \\
& *e^3 + 512*a^{12}*c^6*d^{13}*e^5 - 512*a^{13}*c^5*d^{11}*e^7 - 512*a^{14}*c^4*d^9*e^9 \\
&) - 192*a^{10}*c^7*d^{14}*e^3 - 128*a^{11}*c^6*d^{12}*e^5 + 320*a^{12}*c^5*d^{10}*e^7 + \\
& 256*a^{13}*c^4*d^8*e^9) + x*(16*a^8*c^8*d^{14}*e^2 + 32*a^9*c^7*d^{12}*e^4 - 112 \\
& *a^{10}*c^6*d^{10}*e^6 + 128*a^{11}*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^{(1/2)} - a*e \\
& ^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d \\
& ^2*e^2))^{(1/2)} - 4*a^7*c^8*d^{13}*e^2 - 4*a^8*c^7*d^{11}*e^4 + 16*a^{10}*c^5*d^7 \\
& *e^8))*(-(c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/ \\
& (16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)} - (x*(2*a^7*c^7*d^9*e \\
& ^5 - 4*a^8*c^6*d^7*e^7) - ((-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2) \\
&) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*((\\
& (-c*d^2*(-a^5*c^3)^{(1/2)} - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^ \\
& 7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))^{(1/2)}*(x*(-c*d^2*(-a^5*c^3)^{(1/2)} \\
& - a*e^2*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a \\
& ^6*c*d^2*e^2))^{(1/2)}*(512*a^{11}*c^7*d^{15}*e^3 + 512*a^{12}*c^6*d^{13}*e^5 - 512* \\
& a^{13}*c^5*d^{11}*e^7 - 512*a^{14}*c^4*d^9*e^9) + 192*a^{10}*c^7*d^{14}*e^3 + 128*a^{1 \\
& 1}*c^6*d^{12}*e^5 - 320*a^{12}*c^5*d^{10}*e^7 - 256*a^{13}*c^4*d^8*e^9) + x*(16*a^8*
\end{aligned}$$

$$\begin{aligned}
& c^8 d^{14} e^2 + 32 a^9 c^7 d^{12} e^4 - 112 a^{10} c^6 d^{10} e^6 + 128 a^{11} c^5 d^8 e^8) * (- (c d^2 (-a^5 c^3)^{1/2} - a e^2 (-a^5 c^3)^{1/2} + 2 a^3 c^2 d e) / (16 (a^7 e^4 + a^5 c^2 d^4 + 2 a^6 c d^2 e^2))^{1/2} + 4 a^7 c^8 d^{13} e^2 + 4 a^8 c^7 d^{11} e^4 - 16 a^{10} c^5 d^7 e^8) * (- (c d^2 (-a^5 c^3)^{1/2} - a e^2 (-a^5 c^3)^{1/2} + 2 a^3 c^2 d e) / (16 (a^7 e^4 + a^5 c^2 d^4 + 2 a^6 c d^2 e^2))^{1/2})) * (- (c d^2 (-a^5 c^3)^{1/2} - a e^2 (-a^5 c^3)^{1/2} + 2 a^3 c^2 d e) / (16 (a^7 e^4 + a^5 c^2 d^4 + 2 a^6 c d^2 e^2))^{1/2}) * 2i - 1 / (a d x) - (\log(c^4 d^{11} (-d^3 e^5)^{1/2} - 16 a^4 e^3 (-d^3 e^5)^{3/2} + 16 a^4 d^4 e^{11} x + c^4 d^{12} e^3 x + a c^3 d^9 e^2 (-d^3 e^5)^{1/2} + a c^3 d^{10} e^5 x - 16 a^3 c d^6 e^9 x + 16 a^3 c d^2 e (-d^3 e^5)^{3/2}) * (-d^3 e^5)^{1/2}) / (2 (c d^5 + a d^3 e^2)) + (\log(16 a^4 e^3 (-d^3 e^5)^{3/2} - c^4 d^{11} (-d^3 e^5)^{1/2} + 16 a^4 d^4 e^{11} x + c^4 d^{12} e^3 x - a c^3 d^9 e^2 (-d^3 e^5)^{1/2} + a c^3 d^{10} e^5 x - 16 a^3 c d^6 e^9 x - 16 a^3 c d^2 e (-d^3 e^5)^{3/2}) * (-d^3 e^5)^{1/2}) / (2 c d^5 + 2 a d^3 e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.173 \quad \int \frac{1}{x^4(d+ex^2)(a+cx^4)} dx$$

Optimal. Leaf size=360

$$\frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{7/4} (ae^2 + cd^2)} - \frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{7/4} (ae^2 + cd^2)} + \frac{c^{5/4}}{4\sqrt{2} a^{7/4} (ae^2 + cd^2)}$$

Rubi [A] time = 0.30, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{7/4} (ae^2 + cd^2)} - \frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{7/4} (ae^2 + cd^2)} + \frac{c^{5/4}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{7/4} (ae^2 + cd^2)} - \frac{c^{5/4}(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{7/4} (ae^2 + cd^2)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{d^{5/2} (ae^2 + cd^2)} + \frac{e}{ad^2x} - \frac{1}{3adx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)), x]

[Out] $-1/(3*a*d*x^3) + e/(a*d^2*x) + (e^{7/2} * \text{ArcTan}[\text{Sqrt}[e]*x/\text{Sqrt}[d]])/(d^{5/2} * (c*d^2 + a*e^2)) + (c^{5/4} * (\text{Sqrt}[c]*d - \text{Sqrt}[a]*e) * \text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/(2*\text{Sqrt}[2]*a^{7/4} * (c*d^2 + a*e^2)) - (c^{5/4} * (\text{Sqrt}[c]*d - \text{Sqrt}[a]*e) * \text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/(2*\text{Sqrt}[2]*a^{7/4} * (c*d^2 + a*e^2)) + (c^{5/4} * (\text{Sqrt}[c]*d + \text{Sqrt}[a]*e) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{7/4} * (c*d^2 + a*e^2)) - (c^{5/4} * (\text{Sqrt}[c]*d + \text{Sqrt}[a]*e) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{7/4} * (c*d^2 + a*e^2))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1288

```
Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} dx &= \int \left(\frac{1}{adx^4} - \frac{e}{ad^2x^2} + \frac{e^4}{d^2 (cd^2 + ae^2) (d + ex^2)} - \frac{c^2 (d - ex^2)}{a (cd^2 + ae^2) (a + cx^4)} \right) dx \\
&= -\frac{1}{3adx^3} + \frac{e}{ad^2x} - \frac{c^2 \int \frac{d-ex^2}{a+cx^4} dx}{a (cd^2 + ae^2)} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{d^2 (cd^2 + ae^2)} \\
&= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)} - \frac{\left(c \left(\frac{\sqrt{c}d}{\sqrt{a}} - e \right) \right) \int \frac{\sqrt{a} \sqrt{c+cx^2}}{a+cx^4} dx}{2a (cd^2 + ae^2)} - \frac{\left(c \left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4a (cd^2 + ae^2)} - \frac{\left(c \left(\frac{\sqrt{c}d}{\sqrt{a}} + e \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4a (cd^2 + ae^2)} \\
&= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)} + \frac{c^{5/4} (\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x)}{4\sqrt{2} a^{7/4} (cd^2 + ae^2)} \\
&= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)} + \frac{c^{5/4} (\sqrt{c}d - \sqrt{a}e) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2} a^{7/4} (cd^2 + ae^2)} - \frac{c^{5/4} (\sqrt{c}d + \sqrt{a}e) \tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2} a^{7/4} (cd^2 + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 367, normalized size = 1.02

$$\frac{3\sqrt{2}c^{5/4}d^{5/2}x^3(a^{3/4}e + \sqrt{a}\sqrt{c}d)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - 3\sqrt{2}c^{5/4}d^{5/2}x^3(a^{3/4}e + \sqrt{a}\sqrt{c}d)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + 24a^{7/4}d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + 6\sqrt{2}\sqrt[4]{a}c^{5/4}d^{5/2}\sqrt{c}d - \sqrt{a}e)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 6\sqrt{2}\sqrt[4]{a}c^{5/4}d^{5/2}\sqrt{c}d - \sqrt{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) + 24a\sqrt{d}cx^2(a^2 + cd) - 8a^{7/2}(a^2 + cd)}{24a^{7/2}d^{5/2}(a^2 + cd)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)), x]

[Out] $(-8*a*d^{(3/2)}*(c*d^2 + a*e^2) + 24*a*\text{Sqrt}[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2*e^{(7/2)}*x^3*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]] + 6*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*d^{(5/2)}*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*x^3*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 6*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*d^{(5/2)}*(-(\text{Sqrt}[c]*d) + \text{Sqrt}[a]*e)*x^3*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 3*\text{Sqrt}[2]*c^{(5/4)}*d^{(5/2)}*(a^{(1/4)}*\text{Sqrt}[c]*d + a^{(3/4)}*e)*x^3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] - 3*\text{Sqrt}[2]*c^{(5/4)}*d^{(5/2)}*(a^{(1/4)}*\text{Sqrt}[c]*d + a^{(3/4)}*e)*x^3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(24*a^2*d^{(5/2)}*(c*d^2 + a*e^2)*x^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)), x]

fricas [B] time = 26.63, size = 4442, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)))] + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (

$$2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*\sqrt{-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8))}/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*\log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*\sqrt{-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*\sqrt{((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*\sqrt{-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))} - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3]$$

giac [A] time = 0.40, size = 364, normalized size = 1.01

$$\frac{\left((ac^3)^{\frac{1}{2}}c^2d - (ac^3)^{\frac{3}{2}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{2}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{2\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{2}}c^2d - (ac^3)^{\frac{3}{2}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{2}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{2\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} - \frac{\left((ac^3)^{\frac{1}{2}}c^2d + (ac^3)^{\frac{3}{2}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} + \frac{\left((ac^3)^{\frac{1}{2}}c^2d + (ac^3)^{\frac{3}{2}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{d}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} + \frac{\arctan\left(\frac{\frac{d}{c}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(ad^4 + ad^2e^2\right)\sqrt{d}} + \frac{3x^2e - d}{3ad^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")

[Out] $-1/2*((a*c^3)^{(1/4)}*c^2*d - (a*c^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^2*c^2*d^2 + \sqrt{2}*a^3*c*e^2) - 1/2*((a*c^3)^{(1/4)}*c^2*d - (a*c^3)^{(3/4)}*e)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^2*c^2*d^2 + \sqrt{2}*a^3*c*e^2) - 1/4*((a*c^3)^{(1/4)}*c^2*d + (a*c^3)^{(3/4)}*e)*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^2*c^2*d^2 + \sqrt{2}*a^3*c*e^2) + 1/4*((a*c^3)^{(1/4)}*c^2*d + (a*c^3)^{(3/4)}*e)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^2*c^2*d^2 + \sqrt{2}*a^3*c*e^2) + \arctan(x*e^{(1/2)}/\sqrt{d})*e^{(7/2)}/((c*d^4 + a*d^2*e^2)*\sqrt{d}) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)$

maple [A] time = 0.01, size = 406, normalized size = 1.13

$$\frac{e^4 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{(a^2 + cd^2)\sqrt{de}d^2} + \frac{\sqrt{2}ce \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{4(a^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}}a} + \frac{\sqrt{2}ce \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{4(a^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}}a} + \frac{\sqrt{2}ce \ln\left(\frac{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(a^2 + cd^2)\left(\frac{d}{c}\right)^{\frac{1}{2}}a} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}c^2d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{4(a^2 + cd^2)a^2} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}c^2d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{d}{c}\right)^{\frac{1}{2}}}\right)}{4(a^2 + cd^2)a^2} - \frac{\left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}c^2d \ln\left(\frac{x^2 + \left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}x + \sqrt{\frac{d}{c}}}{x^2 - \left(\frac{d}{c}\right)^{\frac{1}{2}}\sqrt{2}x + \sqrt{\frac{d}{c}}}\right)}{8(a^2 + cd^2)a^2} + \frac{e}{ad^2x} - \frac{1}{3ad^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x)

[Out] $-1/3/a/d/x^3 + e/a/d^2/x - 1/8*c^2/(a*e^2 + c*d^2)/a^2*d*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2 + (a/c)^{(1/4)}*2^{(1/2)}*x + (a/c)^{(1/2)})/(x^2 - (a/c)^{(1/4)}*2^{(1/2)}*x + (a/c)^{(1/2)})) - 1/4*c^2/(a*e^2 + c*d^2)/a^2*d*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x + 1) - 1/4*c^2/(a*e^2 + c*d^2)/a^2*d*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x - 1) + 1/8*c/(a*e^2 + c*d^2)/a*e/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2 - (a/c)^{(1/4)}*2^{(1/2)}*x + (a/c)^{(1/2)})/(x^2 + (a/c)^{(1/4)}*2^{(1/2)}*x + (a/c)^{(1/2)})) + 1/4*c/($

$$a * e^{2+cd^2} / a * e / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) + 1/4 * c / (a * e^{2+cd^2}) / a * e / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) + 1/d^2 * e^4 / (a * e^{2+cd^2}) / (d * e)^{1/2} * \arctan(1 / (d * e)^{1/2} * e * x)$$

maxima [A] time = 2.12, size = 297, normalized size = 0.82

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^4 + ad^2e^2)\sqrt{de}} - \left[\frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx + \sqrt{2}a^{1/4}c^{1/4}})}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}\sqrt{c}}} + \frac{2\sqrt{2}(\sqrt{c}d - \sqrt{a}e) \arctan\left(\frac{\sqrt{2}(2\sqrt{cx - \sqrt{2}a^{1/4}c^{1/4}})}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}\sqrt{c}}} + \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})}{a^{3/4}c^{3/4}} - \frac{\sqrt{2}(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})}{a^{3/4}c^{3/4}} \right] + \frac{3ex^2 - d}{3ad^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a), x, algorithm="maxima")

[Out] $e^4 * \arctan(e * x / \sqrt{d * e}) / ((c * d^4 + a * d^2 * e^2) * \sqrt{d * e}) - 1/8 * c^2 * (2 * \sqrt{2} * (\sqrt{c} * d - \sqrt{a} * e) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x + \sqrt{2} * a^{1/4} * c^{1/4}) * c^{1/4}) / \sqrt{\sqrt{a} * \sqrt{c}}) / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c})) + 2 * \sqrt{2} * (\sqrt{c} * d - \sqrt{a} * e) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{1/4} * c^{1/4}) * c^{1/4}) / \sqrt{\sqrt{a} * \sqrt{c}}) / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c})) * \sqrt{c} + \sqrt{2} * (\sqrt{c} * d + \sqrt{a} * e) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4}) - \sqrt{2} * (\sqrt{c} * d + \sqrt{a} * e) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4}) / (a * c * d^2 + a^2 * e^2) + 1/3 * (3 * e * x^2 - d) / (a * d^2 * x^3)$

mupad [B] time = 2.26, size = 5972, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + c*x^4)*(d + e*x^2)), x)

[Out] $\operatorname{atan}\left(\frac{(x * (2 * a^5 * c^9 * d^{18} * e^5 + 4 * a^7 * c^7 * d^{14} * e^9) - ((a * e^2 * (-a^7 * c^5)^{1/2})^{1/2} - c * d^2 * (-a^7 * c^5)^{1/2} + 2 * a^4 * c^3 * d * e) / (16 * (a^9 * e^4 + a^7 * c^2 * d^4 + 2 * a^8 * c * d^2 * e^2)))^{1/2} * (((a * e^2 * (-a^7 * c^5)^{1/2})^{1/2} - c * d^2 * (-a^7 * c^5)^{1/2} + 2 * a^4 * c^3 * d * e) / (16 * (a^9 * e^4 + a^7 * c^2 * d^4 + 2 * a^8 * c * d^2 * e^2)))^{1/2} * (x * ((a * e^2 * (-a^7 * c^5)^{1/2})^{1/2} - c * d^2 * (-a^7 * c^5)^{1/2} + 2 * a^4 * c^3 * d * e) / (16 * (a^9 * e^4 + a^7 * c^2 * d^4 + 2 * a^8 * c * d^2 * e^2)))^{1/2} * (512 * a^{11} * c^7 * d^{24} * e^3 + 512 * a^{12} * c^6 * d^{22} * e^5 - 512 * a^{13} * c^5 * d^{20} * e^7 - 512 * a^{14} * c^4 * d^{18} * e^9) - 64 * a^9 * c^8 * d^{24} * e^2 + 128 * a^{10} * c^7 * d^{22} * e^4 + 192 * a^{11} * c^6 * d^{20} * e^6 - 256 * a^{12} * c^5 * d^{18} * e^8 - 256 * a^{13} * c^4 * d^{16} * e^{10} - x * (16 * a^7 * c^9 * d^{23} * e^2 + 32 * a^8 * c^8 * d^{21} * e^4 - 112 * a^9 * c^7 * d^{19} * e^6 - 128 * a^{11} * c^5 * d^{15} * e^{10}) * ((a * e^2 * (-a^7 * c^5)^{1/2})^{1/2} - c * d^2 * (-a^7 * c^5)^{1/2} + 2 * a^4 * c^3 * d * e) / (16 * (a^9 * e^4 + a^7 * c^2 * d^4 + 2 * a^8 * c * d^2 * e^2)))^{1/2} - 4 * a^6 * c^9 * d^{21} * e^3 - 4 * a^7 * c^8 * d^{19} * e^5 + 4 * 8 * a^9 * c^6 * d^{15} * e^9) * ((a * e^2 * (-a^7 * c^5)^{1/2})^{1/2} - c * d^2 * (-a^7 * c^5)^{1/2} + 2 * a^4 * c^3 * d * e) / (16 * (a^9 * e^4 + a^7 * c^2 * d^4 + 2 * a^8 * c * d^2 * e^2)))^{1/2} * i + (x * \dots$

$$\begin{aligned}
& (2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (x*((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2))))^{(1/2)} * (512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * i) / ((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (x*((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2))))^{(1/2)} * (512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} - (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * (x*((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2))))^{(1/2)} * (512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10)) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9) * ((a*e^2*(-a^7*c^5)^{(1/2)} - c*d^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} * 2i + \operatorname{atan}(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3
\end{aligned}$$

$$\begin{aligned}
& *d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(((c*d^2*(-a^7 \\
& *c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^ \\
& 2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(x*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7* \\
& c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2))) \\
& ^{(1/2)}*(512*a^{11}*c^7*d^{24}*e^3 + 512*a^{12}*c^6*d^{22}*e^5 - 512*a^{13}*c^5*d^{20}*e \\
& ^7 - 512*a^{14}*c^4*d^{18}*e^9) - 64*a^9*c^8*d^{24}*e^2 + 128*a^{10}*c^7*d^{22}*e^4 + \\
& 192*a^{11}*c^6*d^{20}*e^6 - 256*a^{12}*c^5*d^{18}*e^8 - 256*a^{13}*c^4*d^{16}*e^{10}) - \\
& x*(16*a^7*c^9*d^{23}*e^2 + 32*a^8*c^8*d^{21}*e^4 - 112*a^9*c^7*d^{19}*e^6 - 128*a \\
& ^{11}*c^5*d^{15}*e^{10}))*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a \\
& ^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} - 4*a^6*c \\
& ^9*d^{21}*e^3 - 4*a^7*c^8*d^{19}*e^5 + 48*a^9*c^6*d^{15}*e^9))*((c*d^2*(-a^7*c^5) \\
& ^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 \\
& + 2*a^8*c*d^2*e^2)))^{(1/2)}*1i + (x*(2*a^5*c^9*d^{18}*e^5 + 4*a^7*c^7*d^{14}*e^ \\
& 9) - ((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16 \\
& *(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(((c*d^2*(-a^7*c^5)^{(1/ \\
& 2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2 \\
& *a^8*c*d^2*e^2)))^{(1/2)}*(x*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2) \\
&) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(5 \\
& 12*a^{11}*c^7*d^{24}*e^3 + 512*a^{12}*c^6*d^{22}*e^5 - 512*a^{13}*c^5*d^{20}*e^7 - 512* \\
& a^{14}*c^4*d^{18}*e^9) + 64*a^9*c^8*d^{24}*e^2 - 128*a^{10}*c^7*d^{22}*e^4 - 192*a^{11} \\
& *c^6*d^{20}*e^6 + 256*a^{12}*c^5*d^{18}*e^8 + 256*a^{13}*c^4*d^{16}*e^{10}) - x*(16*a^7 \\
& *c^9*d^{23}*e^2 + 32*a^8*c^8*d^{21}*e^4 - 112*a^9*c^7*d^{19}*e^6 - 128*a^{11}*c^5*d \\
& ^{15}*e^{10}))*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d* \\
& e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} + 4*a^6*c^9*d^{21}*e \\
& ^3 + 4*a^7*c^8*d^{19}*e^5 - 48*a^9*c^6*d^{15}*e^9))*((c*d^2*(-a^7*c^5)^{(1/2)} - \\
& a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8* \\
& c*d^2*e^2)))^{(1/2)}*1i)/((x*(2*a^5*c^9*d^{18}*e^5 + 4*a^7*c^7*d^{14}*e^9) - ((c \\
& d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 \\
& + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^ \\
& 2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^ \\
& 2*e^2)))^{(1/2)}*(x*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4 \\
& *c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(512*a^{11}* \\
& ^7*d^{24}*e^3 + 512*a^{12}*c^6*d^{22}*e^5 - 512*a^{13}*c^5*d^{20}*e^7 - 512*a^{14}*c^4 \\
& ^{18}*e^9) - 64*a^9*c^8*d^{24}*e^2 + 128*a^{10}*c^7*d^{22}*e^4 + 192*a^{11}*c^6*d^{20} \\
& *e^6 - 256*a^{12}*c^5*d^{18}*e^8 - 256*a^{13}*c^4*d^{16}*e^{10}) - x*(16*a^7*c^9*d^{23} \\
& *e^2 + 32*a^8*c^8*d^{21}*e^4 - 112*a^9*c^7*d^{19}*e^6 - 128*a^{11}*c^5*d^{15}*e^{10} \\
&)*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a \\
& ^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/2)} - 4*a^6*c^9*d^{21}*e^3 - 4*a^ \\
& 7*c^8*d^{19}*e^5 + 48*a^9*c^6*d^{15}*e^9))*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a \\
& ^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 \\
&)))^{(1/2)} - (x*(2*a^5*c^9*d^{18}*e^5 + 4*a^7*c^7*d^{14}*e^9) - ((c*d^2*(-a^7*c^ \\
& 5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d \\
& ^4 + 2*a^8*c*d^2*e^2)))^{(1/2)}*(((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5) \\
& ^{(1/2)} + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^{(1/ \\
& 2)}*(x*((c*d^2*(-a^7*c^5)^{(1/2)} - a*e^2*(-a^7*c^5)^{(1/2)} + 2*a^4*c^3*d*e)/(1
\end{aligned}$$

$$\begin{aligned}
& 6*(a^9e^4 + a^7c^2d^4 + 2a^8cd^2e^2))^{(1/2)}*(512a^{11}c^7d^{24}e^3 \\
& + 512a^{12}c^6d^{22}e^5 - 512a^{13}c^5d^{20}e^7 - 512a^{14}c^4d^{18}e^9) + \\
& 64a^9c^8d^{24}e^2 - 128a^{10}c^7d^{22}e^4 - 192a^{11}c^6d^{20}e^6 + 256a^{12}c^5d^{18}e^8 + 256a^{13}c^4d^{16}e^{10}) - x*(16a^7c^9d^{23}e^2 + 32a^8c^8d^{21}e^4 - 112a^9c^7d^{19}e^6 - 128a^{11}c^5d^{15}e^{10}))*((cd^2*(-a^7c^5)^{(1/2)} - a*e^2*(-a^7c^5)^{(1/2)} + 2a^4c^3d*e)/(16*(a^9e^4 + a^7c^2d^4 + 2a^8cd^2e^2)))^{(1/2)} + 4a^6c^9d^{21}e^3 + 4a^7c^8d^{19}e^5 - 48a^9c^6d^{15}e^9))*((cd^2*(-a^7c^5)^{(1/2)} - a*e^2*(-a^7c^5)^{(1/2)} + 2a^4c^3d*e)/(16*(a^9e^4 + a^7c^2d^4 + 2a^8cd^2e^2)))^{(1/2)} + 2a^5c^8d^{14}e^8))*((cd^2*(-a^7c^5)^{(1/2)} - a*e^2*(-a^7c^5)^{(1/2)} + 2a^4c^3d*e)/(16*(a^9e^4 + a^7c^2d^4 + 2a^8cd^2e^2)))^{(1/2)}*2i - (1/(3*a*d) - (e*x^2)/(a*d^2))/x^3 - (log(16*a^7*d^13*e^20 + c^7*d^27*e^6 + 2*a*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*e^14 + 16*a^7*e^3*x*(-d^5*e^7)^{(5/2)} - a^2*c^5*d^15*x*(-d^5*e^7)^{(3/2)} + c^7*d^24*e^3*x*(-d^5*e^7)^{(1/2)} - 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^{(3/2)} + 2*a*c^6*d^22*e^5*x*(-d^5*e^7)^{(1/2)}*(-d^5*e^7)^{(1/2)))/(2*(c*d^7 + a*d^5*e^2)) + (log(16*a^7*d^13*e^20 + c^7*d^27*e^6 + 2*a*c^6*d^25*e^8 + a^2*c^5*d^23*e^10 + 16*a^4*c^3*d^19*e^14 - 16*a^7*e^3*x*(-d^5*e^7)^{(5/2)} + a^2*c^5*d^15*x*(-d^5*e^7)^{(3/2)} - c^7*d^24*e^3*x*(-d^5*e^7)^{(1/2)} + 16*a^4*c^3*d^11*e^4*x*(-d^5*e^7)^{(3/2)} - 2*a*c^6*d^22*e^5*x*(-d^5*e^7)^{(1/2)}*(-d^5*e^7)^{(1/2)))/(2*c*d^7 + 2*a*d^5*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)

[Out] Timed out

$$3.174 \quad \int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=169

$$-\frac{\sqrt{a} d (ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2} (ae^2 + cd^2)^2} + \frac{ae (ae^2 + 2cd^2) \log(a + cx^4)}{4c^2 (ae^2 + cd^2)^2} + \frac{a (ae + cd^2)}{4c^2 (a + cx^4) (ae^2 + cd^2)} + \frac{d^4 \log(d + ex^2)}{2e (ae^2 + cd^2)^2}$$

Rubi [A] time = 0.37, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 1647, 1629, 635, 205, 260}

$$\frac{a (ae + cd^2)}{4c^2 (a + cx^4) (ae^2 + cd^2)} + \frac{ae (ae^2 + 2cd^2) \log(a + cx^4)}{4c^2 (ae^2 + cd^2)^2} - \frac{\sqrt{a} d (ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2} (ae^2 + cd^2)^2} + \frac{d^4 \log(d + ex^2)}{2e (ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) + (d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1252

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],

$x, x^2], x] /; \text{FreeQ}[\{a, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m + 1)/2]$

Rule 1629

$\text{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)}*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol]$
 $:\> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c,$
 $d, e, m\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 1647

$\text{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)}*((a_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] :$
 $> \text{With}[\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + c*x^2, x], f = \text{Coeff}[\text{Pol}$
 $\text{ynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{Polynomial}$
 $\text{Remainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]\}, \text{Simp}[(a*g - c*f*x)*(a + c$
 $*x^2)^{(p + 1)}/(2*a*c*(p + 1)), x] + \text{Dist}[1/(2*a*c*(p + 1)), \text{Int}[(d + e*x)^$
 $m*(a + c*x^2)^{(p + 1)*\text{ExpandToSum}[(2*a*c*(p + 1)*Q]/(d + e*x)^m + (c*f*(2*p$
 $+ 3))/d + e*x)^m, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&$
 $\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
&= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \frac{\frac{a^2d^2}{cd^2+ae^2} - \frac{a^2dex}{cd^2+ae^2} - 2ax^2}{(d+ex)(a+cx^2)} dx, x, x^2 \right)}{4ac} \\
&= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2(d(3cd^2+ae^2)-2e(2cd^2+ae^2))x}{(cd^2+ae^2)^2(a+cx^2)} \right) dx, x, x^2 \right)}{4ac} \\
&= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4 \log(d+ex^2)}{2e(cd^2+ae^2)^2} - \frac{a \text{Subst} \left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{a+cx^2} dx, x, x^2 \right)}{4c(cd^2+ae^2)^2} \\
&= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4 \log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{(ae(2cd^2+ae^2)) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2c(cd^2+ae^2)^2} \\
&= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}d(3cd^2+ae^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4 \log(d+ex^2)}{2e(cd^2+ae^2)^2} + \dots
\end{aligned}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.80

$$\frac{-\frac{\sqrt{a}d(ae^2+3cd^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{c^{3/2}} + \frac{ae(ae^2+2cd^2) \log(a+cx^4)}{c^2} + \frac{a(ae^2+cd^2)(ae+cdx^2)}{c^2(a+cx^4)} + \frac{2d^4 \log(d+ex^2)}{e}}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*d^4*Log[d + e*x^2])/e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^2)/(4*(c*d^2 + a*e^2)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [A] time = 35.28, size = 555, normalized size = 3.28

$$\frac{2a^2cd^2e^2 + 2a^2e^4 + 2(a^2d^2e + a^2de^2 + (3a^2d^2e + a^2de^2 + (3a^2d^2e + a^2de^2)^2)^{\frac{1}{2}} \log\left(\frac{a^2 - 2a^2d^2e + d^2}{a^2 + 2a^2d^2e + d^2}\right) + 2(2a^2d^2e^2 + a^2e^4 + (2a^2d^2e + a^2de^2) \log(cx^4 + a) + 4(a^2d^2e + a^2de^2) \log(e^2x^2 + d)) \sqrt{a/c} \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) + (2a^2d^2e + a^2e^4 + (2a^2d^2e + a^2de^2) \log(cx^4 + a) + 2(a^2d^2e + a^2de^2) \log(e^2x^2 + d))}{8(a^2d^2e + 2a^2d^2e^2 + a^2e^4 + (3a^2d^2e + a^2de^2)^2)^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 2*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]

giac [A] time = 0.36, size = 251, normalized size = 1.49

$$\frac{d^4 \log(|x^2e + d|)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} + \frac{(2acd^2e + a^2e^3) \log(cx^4 + a)}{4(c^4d^4 + 2ac^3d^2e^2 + a^2c^2e^4)} - \frac{(3acd^3 + a^2de^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} - \frac{2acd^2x^4e - acd^3x^2 + a^2x^4e^3 - a^2dx^2e^2 + a^2d^2e}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)(cx^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/2*d^4*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^2*x^4*e^3 - a^2*d*x^2*e^2 + a^2*d^2*e)/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a)

maple [A] time = 0.02, size = 305, normalized size = 1.80

$$\frac{a^2d^2e^2}{4(a^2 + cd^2)^2(cx^4 + a)c} + \frac{ad^3x^2}{4(a^2 + cd^2)^2(cx^4 + a)} - \frac{a^2d^2e^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2 + cd^2)^2\sqrt{ac}c} - \frac{3ad^3 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2 + cd^2)^2\sqrt{ac}} + \frac{a^2e^3}{4(a^2 + cd^2)^2(cx^4 + a)c^2} + \frac{a^2d^2e}{4(a^2 + cd^2)^2(cx^4 + a)c} + \frac{a^2e^3 \ln(cx^4 + a)}{4(a^2 + cd^2)^2c^2} + \frac{ad^2e \ln(cx^4 + a)}{2(a^2 + cd^2)^2c} + \frac{d^4 \ln(e^2x^2 + d)}{2(a^2 + cd^2)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(e*x^2+d)/(c*x^4+a)^2,x)`

[Out] $\frac{1}{4}a^2/(a^2e^2+c^2d^2)^2/(c^2x^4+a)^2d/c^2x^2e^2+1/4a/(a^2e^2+c^2d^2)^2/(c^2x^4+a)^2x^2d^3+1/4a^3/(a^2e^2+c^2d^2)^2/(c^2x^4+a)^2e^3/c^2+1/4a^2/(a^2e^2+c^2d^2)^2/(c^2x^4+a)^2e/c^2d^2+1/4a^2/(a^2e^2+c^2d^2)^2/c^2\ln(c^2x^4+a)^2e^3+1/2a/(a^2e^2+c^2d^2)^2/c^2\ln(c^2x^4+a)^2d^2e-1/4a^2/(a^2e^2+c^2d^2)^2/c^2(a^2c)^{(1/2)}\arctan(1/(a^2c)^{(1/2)}c^2x^2)d^2e-3/4a/(a^2e^2+c^2d^2)^2/(a^2c)^{(1/2)}\arctan(1/(a^2c)^{(1/2)}c^2x^2)d^3+1/2d^4\ln(e^2x^2+d)/e/(a^2e^2+c^2d^2)^2$

maxima [A] time = 2.08, size = 220, normalized size = 1.30

$$\frac{d^4 \log(ex^2 + d)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} + \frac{(2acd^2e + a^2e^3) \log(cx^4 + a)}{4(c^4d^4 + 2ac^3d^2e^2 + a^2c^2e^4)} - \frac{(3acd^3 + a^2de^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} + \frac{acdx^2 + a^2e}{4(ac^3d^2 + a^2c^2e^2 + (c^4d^2 + ac^3e^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{2}d^4\log(e^2x^2 + d)/(c^2d^4e + 2a^2c^2d^2e^3 + a^2e^5) + \frac{1}{4}*(2a^2c^2d^2e + a^2e^3)\log(c^2x^4 + a)/(c^4d^4 + 2a^2c^3d^2e^2 + a^2c^2e^4) - \frac{1}{4}*(3a^2c^2d^3 + a^2d^2e^2)\arctan(cx^2/\sqrt{ac})/((c^3d^4 + 2a^2c^2d^2e^2 + a^2c^2e^4)\sqrt{ac}) + \frac{1}{4}*(a^2c^2d^2x^2 + a^2e)/(a^2c^3d^2 + a^2c^2e^2 + (c^4d^2 + a^2c^3e^2)x^4)$

mupad [B] time = 1.30, size = 305, normalized size = 1.80

$$\frac{\frac{d^4 \ln(ex^2 + d)}{4c^2(d^4 + ae^2)} + \frac{ad^2x^2}{4c(d^4 + ae^2)} - \frac{\ln(\sqrt{-ac^3} + c^3x^2)(3cd^3\sqrt{-ac^3} - 2a^2c^2e^3 - 4ac^3d^2e + ad^2e^2\sqrt{-ac^3})}{8(a^2c^4e^4 + 2ac^5d^2e^2 + c^6d^4)}}{c^4 + a} + \frac{\ln(\sqrt{-ac^3} - c^3x^2)(3cd^3\sqrt{-ac^3} + 2a^2c^2e^3 + 4ac^3d^2e + ad^2e^2\sqrt{-ac^3})}{8(a^2c^4e^4 + 2ac^5d^2e^2 + c^6d^4)}}{c^4 + a} + \frac{d^4 \ln(ex^2 + d)}{2a^2e^5 + 4ac^2d^2e^3 + 2c^2d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)`

[Out] $\frac{(a^2e)/(4c^2(a^2e^2 + c^2d^2)) + (ad^2x^2)/(4c(a^2e^2 + c^2d^2))}{(a + c^2x^4)} - \frac{(\log((-a^2c^5)^{(1/2)} + c^3x^2)*(3c^2d^3*(-a^2c^5)^{(1/2)} - 2a^2c^2e^3 - 4a^2c^3d^2e + ad^2e^2*(-a^2c^5)^{(1/2)}))}{(8(c^6d^4 + a^2c^4e^4 + 2a^2c^5d^2e^2))} + \frac{(\log((-a^2c^5)^{(1/2)} - c^3x^2)*(3c^2d^3*(-a^2c^5)^{(1/2)} + 2a^2c^2e^3 + 4a^2c^3d^2e + ad^2e^2*(-a^2c^5)^{(1/2)}))}{(8(c^6d^4 + a^2c^4e^4 + 2a^2c^5d^2e^2))} + \frac{d^4\log(d + e^2x^2)}{(2a^2e^5 + 2c^2d^4e + 4a^2c^2d^2e^3)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)`

[Out] Timed out

$$3.175 \quad \int \frac{x^7}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{a} e (ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2} (ae^2 + cd^2)^2} + \frac{a(d - ex^2)}{4c(a + cx^4)(ae^2 + cd^2)} + \frac{d^3 \log(a + cx^4)}{4(ae^2 + cd^2)^2} - \frac{d^3 \log(d + ex^2)}{2(ae^2 + cd^2)^2}$$

Rubi [A] time = 0.25, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 1647, 801, 635, 205, 260}

$$\frac{\sqrt{a} e (ae^2 + 3cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2} (ae^2 + cd^2)^2} + \frac{a(d - ex^2)}{4c(a + cx^4)(ae^2 + cd^2)} - \frac{d^3 \log(d + ex^2)}{2(ae^2 + cd^2)^2} + \frac{d^3 \log(a + cx^4)}{4(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801


```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
&= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \frac{\frac{a^2de}{cd^2+ae^2} - \frac{a(2cd^2+ae^2)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)} dx, x, x^2 \right)}{4ac} \\
&= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{2acd^3e}{(cd^2+ae^2)^2(d+ex)} - \frac{a(3acd^2e+a^2e^3+2c^2d^3x)}{(cd^2+ae^2)^2(a+cx^2)} \right) dx, x, x^2 \right)}{4ac} \\
&= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{\text{Subst} \left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2} dx, x, x^2 \right)}{4c(cd^2+ae^2)^2} \\
&= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cd^3) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)^2} + \frac{(ae^3)}{4c(cd^2+ae^2)^2} \\
&= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}e(3cd^2+ae^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3}{4c(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.95

$$\frac{\sqrt{a}e(a+cx^4)(ae^2+3cd^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right) + \sqrt{c}(-2cd^3(a+cx^4) \log(d+ex^2) + cd^3(a+cx^4) \log(a+cx^4) + a(d-ex^2)(ae^2+cd^2))}{4c^{3/2}(a+cx^4)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((d+e*x^2)*(a+c*x^4)^2),x]

[Out] (Sqrt[a]*e*(3*c*d^2+a*e^2)*(a+c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]+Sqrt[c]*(a*(c*d^2+a*e^2)*(d-e*x^2)-2*c*d^3*(a+c*x^4)*Log[d+e*x^2]+c*d^3*(a+c*x^4)*Log[a+c*x^4]))/(4*c^(3/2)*(c*d^2+a*e^2)^2*(a+c*x^4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [A] time = 17.04, size = 457, normalized size = 3.05

$$\frac{2acd^3 + 2a^2d^2 - 2(acd^2e + a^2e^3)^2 + (3acd^2e + a^2e^3 + (3c^2d^2e + acd^3)^2)\sqrt{-c}\log\left(\frac{c^2+2cx^2+\sqrt{-c}}{c^2+2cx^2-\sqrt{-c}}\right) + 2(c^2d^3x^4 + acd^3)\log(cx^4 + a) - 4(c^2d^3x^4 + acd^3)\log(cx^2 + d)}{8(ac^3d^4 + 2a^2c^2d^2e^2 + a^2ce^4 + (c^4d^4 + 2ac^2d^2e^2 + a^2c^2e^4)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 4*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)]

giac [A] time = 0.35, size = 223, normalized size = 1.49

$$\frac{d^3e\log(|x^2e + d|)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} + \frac{d^3\log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{(3acd^2e + a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} - \frac{c^2d^3x^4 + acd^2x^2e + a^2x^2e^3 - a^2de^2}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)(cx^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] -1/2*d^3*e*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))

maple [A] time = 0.02, size = 260, normalized size = 1.73

$$\frac{a^2e^3x^2}{4(ae^2 + cd^2)^2(cx^4 + a)c} - \frac{ad^2ex^2}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{a^2e^3\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}c} + \frac{3ad^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}} + \frac{a^2de^2}{4(ae^2 + cd^2)^2(cx^4 + a)c} + \frac{ad^3}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{d^3\ln(cx^4 + a)}{4(ae^2 + cd^2)^2} - \frac{d^3\ln(ex^2 + d)}{2(ae^2 + cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*e*x^2*d^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*d/c*e^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*d^3+1/4*d^3*\ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/c/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)*a^2*e^3+3/4/(a*e^2+c*d^2)^2/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)*a*d^2*e-1/2*d^3*\ln(e*x^2+d)/(a*e^2+c*d^2)^2$

maxima [A] time = 2.05, size = 197, normalized size = 1.31





















$$\frac{d^3 \log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} - \frac{d^3 \log(ex^2 + d)}{2(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{(3acd^2e + a^2e^3) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} - \frac{aex^2 - ad}{4(ac^2d^2 + a^2ce^2 + (c^3d^2 + ac^2e^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $1/4*d^3*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*\log(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*\arctan(c*x^2/\sqrt{a*c})/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*\sqrt{a*c}) - 1/4*(a*e*x^2 - a*d)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)$

mupad [B] time = 1.49, size = 647, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] $((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (d^3*\log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (\log(36*c^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^{(1/2)} + a^5*c*e^10*(-a*c^3)^{(1/2)} + a^5*c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^{(3/2)} - 81*c^2*d^8*e^2*(-a*c^3)^{(3/2)} + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^{(1/2)} - 60*a*c*d^6*e^4*(-a*c^3)^{(3/2)} + 81*a*c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^{(1/2)} + 3*c*d^2*e*(-a*c^3)^{(1/2)}))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (\log(36*c^8*d^10*x^2 - 36*c^6*d^10*(-a*c^3)^{(1/2)} - a^5*c*e^10*(-a*c^3)^{(1/2)} + a^5*c^3*e^10*x^2 + 22*a^2*d^4*e^6*(-a*c^3)^{(3/2)} + 81*c^2*d^8*e^2*(-a*c^3)^{(3/2)} + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 - 8*a^4*c^2*d^2*e^8*(-a*c^3)^{(1/2)} + 60*a*c*d^6*e^4*(-a*c^3)^{(3/2)} + 81*a*c^7*d^8*e^2*x^2)*(a*e^3*(-a*c^3)^{(1/2)} - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^{(1/2)}))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

$$3.176 \quad \int \frac{x^5}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=155

$$-\frac{d^2 e \log(a+cx^4)}{4(ae^2+cd^2)^2} + \frac{d^2 e \log(d+ex^2)}{2(ae^2+cd^2)^2} + \frac{d(cd^2-ae^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2} + \frac{-ae-cdx^2}{4c(a+cx^4)(ae^2+cd^2)}$$

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 1647, 801, 635, 205, 260}

$$-\frac{ae+cdx^2}{4c(a+cx^4)(ae^2+cd^2)} + \frac{d^2 e \log(d+ex^2)}{2(ae^2+cd^2)^2} - \frac{d^2 e \log(a+cx^4)}{4(ae^2+cd^2)^2} + \frac{d(cd^2-ae^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
&= \frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \frac{-\frac{acd^2}{cd^2+ae^2} + \frac{acdex}{cd^2+ae^2}}{(d+ex)(a+cx^2)} dx, x, x^2 \right)}{4ac} \\
&= \frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2acd^2e^2}{(cd^2+ae^2)^2(d+ex)} + \frac{acd(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)^2(a+cx^2)} \right) dx, x, x^2 \right)}{4ac} \\
&= \frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e \log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d \text{Subst} \left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2} dx, x, x^2 \right)}{4(cd^2+ae^2)^2} \\
&= \frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e \log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(cd^2e) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)^2} + \dots \\
&= \frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d(cd^2-ae^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} + \frac{d^2e \log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d^2e \log(d+ex^2)}{4(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 120, normalized size = 0.77

$$\frac{\frac{d(cd^2-ae^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} - \frac{(ae^2+cd^2)(ae+cdx^2)}{c(a+cx^4)} - d^2e \log(a+cx^4) + 2d^2e \log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^2*e*Log[d + e*x^2] - d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [A] time = 6.20, size = 487, normalized size = 3.14

$$\frac{2a^2d^2e + 2a^2b^2 + 2(a^2d^2 + a^2cd^2)^2 - (acd^3 - a^2d^2 + (c^2d^3 - acd^2)^2)\sqrt{-ac} \log\left(\frac{cx^2 + \sqrt{-ac}}{cx^2}\right) + 2(a^2d^2cx^4 + a^2cd^2)\log(cx^4 + a) - 4(a^2d^2cx^4 + a^2cd^2)\log(cx^2 + d)}{8(a^2c^3d^4 + 2a^2c^2d^2e + a^2cd^4 + (ac^4d^4 + 2a^2c^2d^2e + a^2c^4d^4)^2)} - \frac{a^2cd^2e + a^2b^2 + (a^2d^2 + a^2cd^2)^2 + (acd^3 - a^2d^2 + (c^2d^3 - acd^2)^2)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}}{cx^2}\right) + (a^2d^2cx^4 + a^2cd^2)\log(cx^4 + a) - 2(a^2d^2cx^4 + a^2cd^2)\log(cx^2 + d)}{4(a^2c^3d^4 + 2a^2c^2d^2e + a^2cd^4 + (ac^4d^4 + 2a^2c^2d^2e + a^2c^4d^4)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*\text{sqrt}(-a*c)*\log((c*x^4 + 2*\text{sqrt}(-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*\log(c*x^4 + a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*\log(e*x^2 + d)]/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4), \\ & -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a*c*d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*\text{sqrt}(a*c)*\arctan(\text{sqrt}(a*c)/(c*x^2)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*\log(c*x^4 + a) - 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*\log(e*x^2 + d)]/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4) \end{aligned}$$

giac [A] time = 0.33, size = 220, normalized size = 1.42

$$-\frac{d^2e \log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{d^2e^2 \log(|x^2e + d|)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} + \frac{(cd^3 - ade^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{ac}} + \frac{c^2d^2x^4e - c^2d^3x^2 - acdx^2e^2 - a^2e^3}{4(c^3d^4 + 2ac^2d^2e^2 + a^2ce^4)(cx^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4*d^2*e*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e^2*\log(\text{abs}(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c*d^3 - a*d*e^2)*\arctan(c*x^2/\text{sqrt}(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\text{sqrt}(a*c)) \\ & + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a)) \end{aligned}$$

maple [A] time = 0.02, size = 252, normalized size = 1.63

$$-\frac{ad^2e^2x^2}{4(a^2 + cd^2)^2(cx^4 + a)} - \frac{cd^3x^2}{4(a^2 + cd^2)^2(cx^4 + a)} - \frac{ad^2e^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2 + cd^2)^2\sqrt{ac}} + \frac{cd^3 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2 + cd^2)^2\sqrt{ac}} - \frac{a^2e^3}{4(a^2 + cd^2)^2(cx^4 + a)c} - \frac{ad^2e}{4(a^2 + cd^2)^2(cx^4 + a)} - \frac{d^2e \ln(cx^4 + a)}{4(a^2 + cd^2)^2} + \frac{d^2e \ln(ex^2 + d)}{2(a^2 + cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*e*d^2-1/4*d^2*e*\ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4/(a*e^2+c*d^2)^2*d/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)*a*e^2+1/4/(a*e^2+c*d^2)^2/(a*c)^{(1/2)}*a*\operatorname{rctan}(1/(a*c)^{(1/2)}*c*x^2)*c*d^3+1/2*d^2*e*\ln(e*x^2+d)/(a*e^2+c*d^2)^2$

maxima [A] time = 2.08, size = 192, normalized size = 1.24

$$\frac{d^2 e \log(cx^4 + a)}{4(c^2 d^4 + 2acd^2 e^2 + a^2 e^4)} + \frac{d^2 e \log(ex^2 + d)}{2(c^2 d^4 + 2acd^2 e^2 + a^2 e^4)} + \frac{(cd^3 - ade^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2 d^4 + 2acd^2 e^2 + a^2 e^4) \sqrt{ac}} - \frac{cdx^2 + ae}{4(ac^2 d^2 + a^2 ce^2 + (c^3 d^2 + ac^2 e^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] $-1/4*d^2*e*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*\log(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)*a*\operatorname{rctan}(c*x^2/\sqrt{a*c})/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\sqrt{a*c}) - 1/4*(c*d*x^2 + a*e)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)$

mupad [B] time = 1.52, size = 528, normalized size = 3.41

$$\frac{d^2 e \log(cx^4 + a)}{4(c^2 d^4 + 2acd^2 e^2 + a^2 e^4)} + \frac{d^2 e \log(ex^2 + d)}{2(c^2 d^4 + 2acd^2 e^2 + a^2 e^4)} + \frac{(cd^3 - ade^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2 d^4 + 2acd^2 e^2 + a^2 e^4) \sqrt{ac}} - \frac{cdx^2 + ae}{4(ac^2 d^2 + a^2 ce^2 + (c^3 d^2 + ac^2 e^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)`

[Out] $(\log(a^4*e^8*(-a*c)^{(1/2)} + c^4*d^8*(-a*c)^{(1/2)} + 70*d^4*e^4*(-a*c)^{(5/2)} + c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^{(3/2)} - 36*c^2*d^6*e^2*(-a*c)^{(3/2)} + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3*(-a*c)^{(1/2)})/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)^{(1/2)})/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e^2 + c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (\log(c^5*d^8*x^2 - c^4*d^8*(-a*c)^{(1/2)} - 70*d^4*e^4*(-a*c)^{(5/2)} - a^4*e^8*(-a*c)^{(1/2)} + a^4*c*e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^{(3/2)} + 36*c^2*d^6*e^2*(-a*c)^{(3/2)} + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3*(-a*c)^{(1/2)})/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^{(1/2)})/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*\log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

$$3.177 \quad \int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=149

$$\frac{de^2 \log(a+cx^4)}{4(ae^2+cd^2)^2} - \frac{de^2 \log(d+ex^2)}{2(ae^2+cd^2)^2} - \frac{e(cd^2-ae^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2} + \frac{ex^2-d}{4(a+cx^4)(ae^2+cd^2)}$$

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 823, 801, 635, 205, 260}

$$-\frac{d-ex^2}{4(a+cx^4)(ae^2+cd^2)} - \frac{de^2 \log(d+ex^2)}{2(ae^2+cd^2)^2} + \frac{de^2 \log(a+cx^4)}{4(ae^2+cd^2)^2} - \frac{e(cd^2-ae^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
  x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
&= \frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \frac{acde-ace^2x}{(d+ex)(a+cx^2)} dx, x, x^2 \right)}{4ac(cd^2+ae^2)} \\
&= \frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{2acde^3}{(cd^2+ae^2)(d+ex)} - \frac{ace(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)(a+cx^2)} \right) dx, x, x^2 \right)}{4ac(cd^2+ae^2)} \\
&= \frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{e \text{Subst} \left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2} dx, x, x^2 \right)}{4(cd^2+ae^2)^2} \\
&= \frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cde^2) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)^2} - \frac{e}{4(cd^2+ae^2)^2} \\
&= \frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} - \frac{de^2 \log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{de^2 \log}{4(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 114, normalized size = 0.77

$$\frac{\frac{e(ae^2-cd^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} + \frac{(ex^2-d)(ae^2+cd^2)}{a+cx^4} + de^2 \log(a+cx^4) - 2de^2 \log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [A] time = 7.09, size = 492, normalized size = 3.30

$$\frac{2a^2d^3 + 2a^2cd^2 - 2(ac^2d^2 + a^2c^2) - (ac^2d^2 + a^2c^2)\sqrt{ac} \log\left(\frac{cx^2 + d}{\sqrt{ac}}\right) - 2(ac^2d^2 + a^2c^2) \log(cx^2 + a) + 4(ac^2d^2 + a^2c^2) \log(cx^2 + d)}{8(a^2d^3 + 2a^2cd^2 + a^2c^2) + (ac^2d^2 + a^2c^2)x^2} - \frac{ac^2d^3 + a^2cd^2 - (ac^2d^2 + a^2c^2)x^2 - (ac^2d^2 + a^2c^2)\sqrt{ac} \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) - (ac^2d^2 + a^2c^2) \log(cx^2 + a) + 2(ac^2d^2 + a^2c^2) \log(cx^2 + d)}{4(a^2d^3 + 2a^2cd^2 + a^2c^2) + (ac^2d^2 + a^2c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] $[-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*\text{sqrt}(-a*c)*\log((c*x^4 - 2*\text{sqrt}(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*\log(c*x^4 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*\log(e*x^2 + d)]/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*\text{sqrt}(a*c)*\arctan(\text{sqrt}(a*c)/(c*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*\log(c*x^4 + a) + 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*\log(e*x^2 + d)]/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]$

giac [A] time = 0.28, size = 188, normalized size = 1.26

$$\frac{de^2 \log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} - \frac{de^3 \log(|x^2e + d|)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} - \frac{(cd^2e - ae^3) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{ac}} - \frac{cd^3 - (cd^2e + ae^3)x^2 + ade^2}{4(cx^4 + a)(cd^2 + ae^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $1/4*d*e^2*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^3*\log(\text{abs}(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*(c*d^2*e - a*e^3)*\arctan(c*x^2/\text{sqrt}(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\text{sqrt}(a*c)) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e^2)^2)$

maple [A] time = 0.02, size = 247, normalized size = 1.66

$$\frac{ae^3x^2}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{cd^2ex^2}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{ae^3 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}} - \frac{cd^2e \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}} - \frac{ade^2}{4(ae^2 + cd^2)^2(cx^4 + a)} - \frac{cd^3}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{de^2 \ln(cx^4 + a)}{4(ae^2 + cd^2)^2} - \frac{de^2 \ln(ex^2 + d)}{2(ae^2 + cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e*c*d^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*d*e^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c*d^3+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*a*e^3-1/4/(a*e^2+c*d^2)^2*e/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)*c*d^2-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2

maxima [A] time = 2.03, size = 186, normalized size = 1.25

$$\frac{d e^2 \log \left(c x^4+a\right)}{4\left(c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)}-\frac{d e^2 \log \left(e x^2+d\right)}{2\left(c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)}-\frac{\left(c d^2 e-a e^3\right) \arctan \left(\frac{c x^2}{\sqrt{a c}}\right)}{4\left(c^2 d^4+2 a c d^2 e^2+a^2 e^4\right) \sqrt{a c}}+\frac{e x^2-d}{4\left(\left(c^2 d^2+a c e^2\right) x^4+a c d^2+a^2 e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*log(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4*(e*x^2 - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)

mupad [B] time = 1.41, size = 527, normalized size = 3.54

$$\frac{\ln \left(\frac{d e^2 \sqrt{c x^4+a}}{c^2 d^4+2 a c d^2 e^2+a^2 e^4}\right)+\ln \left(\frac{d e^2 \sqrt{e x^2+d}}{c^2 d^4+2 a c d^2 e^2+a^2 e^4}\right)-\frac{\left(c d^2 e-a e^3\right) \arctan \left(\frac{c x^2}{\sqrt{a c}}\right)}{\left(c^2 d^4+2 a c d^2 e^2+a^2 e^4\right) \sqrt{a c}}+\frac{e x^2-d}{\left(\left(c^2 d^2+a c e^2\right) x^4+a c d^2+a^2 e^2\right)}}{c^2 d^4+2 a c d^2 e^2+a^2 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2) + c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c*d^2)) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

$$3.178 \quad \int \frac{x}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=151

$$\frac{\sqrt{c} d (3ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2} (ae^2 + cd^2)^2} + \frac{ae + cdx^2}{4a (a + cx^4) (ae^2 + cd^2)} - \frac{e^3 \log(a + cx^4)}{4 (ae^2 + cd^2)^2} + \frac{e^3 \log(d + ex^2)}{2 (ae^2 + cd^2)^2}$$

Rubi [A] time = 0.18, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1248, 741, 801, 635, 205, 260}

$$\frac{\sqrt{c} d (3ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2} (ae^2 + cd^2)^2} + \frac{ae + cdx^2}{4a (a + cx^4) (ae^2 + cd^2)} + \frac{e^3 \log(d + ex^2)}{2 (ae^2 + cd^2)^2} - \frac{e^3 \log(a + cx^4)}{4 (ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]

[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2

+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
 c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
 2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 801

Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2),
 x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
 := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
 [{a, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(d + ex^2)(a + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(d + ex)(a + cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{ae + cd x^2}{4a(cd^2 + ae^2)(a + cx^4)} - \frac{\text{Subst} \left(\int \frac{-cd^2 - 2ae^2 - cdex}{(d+ex)(a+cx^2)} dx, x, x^2 \right)}{4a(cd^2 + ae^2)} \\
 &= \frac{ae + cd x^2}{4a(cd^2 + ae^2)(a + cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2ae^4}{(cd^2 + ae^2)(d+ex)} - \frac{c(cd^3 + 3ade^2 - 2ae^3x)}{(cd^2 + ae^2)(a+cx^2)} \right) dx, x, x^2 \right)}{4a(cd^2 + ae^2)} \\
 &= \frac{ae + cd x^2}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^3 \log(d + ex^2)}{2(cd^2 + ae^2)^2} + \frac{c \text{Subst} \left(\int \frac{cd^3 + 3ade^2 - 2ae^3x}{a+cx^2} dx, x, x^2 \right)}{4a(cd^2 + ae^2)^2} \\
 &= \frac{ae + cd x^2}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{e^3 \log(d + ex^2)}{2(cd^2 + ae^2)^2} - \frac{(ce^3) \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^2 + ae^2)^2} + \frac{(cd^3 + 3ade^2 - 2ae^3x)}{4a(cd^2 + ae^2)^2} \\
 &= \frac{ae + cd x^2}{4a(cd^2 + ae^2)(a + cx^4)} + \frac{\sqrt{c} d (cd^2 + 3ae^2) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{3/2} (cd^2 + ae^2)^2} + \frac{e^3 \log(d + ex^2)}{2(cd^2 + ae^2)^2} - \frac{e^3}{4}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 117, normalized size = 0.77

$$\frac{\frac{\sqrt{c}d(3ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(ae^2+cd^2)(ae+cdx^2)}{a(a+cx^4)} - e^3 \log(a+cx^4) + 2e^3 \log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]

[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*Log[d + e*x^2] - e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)^2),x]

[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [A] time = 16.31, size = 458, normalized size = 3.03

$$\frac{2ac^2e + 2a^2e^3 + 2(c^2d^2 + acd^2)^2 + (acd^2 + 3a^2d^2 + (c^2d^2 + 3acd^2)^2)\sqrt{\frac{c}{a}} \log\left(\frac{c^2+2a^2\sqrt{\frac{c}{a}}}{a^2+cx^4}\right) - 2(ac^2e^3 + a^2e^3) \log(cx^4 + a) + 4(ac^2e^3 + a^2e^3) \log(ex^2 + d) - acd^2e + a^2e^3 + (c^2d^2 + acd^2)^2 - (acd^2 + 3a^2d^2 + (c^2d^2 + 3acd^2)^2)\sqrt{\frac{c}{a}} \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) - (ac^2e^3 + a^2e^3) \log(cx^4 + a) + 2(ac^2e^3 + a^2e^3) \log(ex^2 + d)}{8(a^2c^2d^2 + 2a^2cd^2 + a^4 + (ac^2d^2 + 2a^2cd^2 + a^2c^2)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 4*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]

giac [A] time = 0.38, size = 199, normalized size = 1.32

$$\frac{e^3 \log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{e^4 \log(|x^2e + d|)}{2(c^2d^4e + 2acd^2e^3 + a^2e^5)} + \frac{(c^2d^3 + 3acde^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4 + 2a^2cd^2e^2 + a^3e^4)\sqrt{ac}} + \frac{acd^2e + (c^2d^3 + acde^2)x^2 + a^2e^3}{4(cx^4 + a)(cd^2 + ae^2)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $-1/4*e^3*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^4*\log(\text{abs}(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)*\arctan(c*x^2/\text{sqrt}(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\text{sqrt}(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 + a)*(c*d^2 + a*e^2)^2*a)$

maple [A] time = 0.02, size = 255, normalized size = 1.69

$$\frac{c^2d^3x^2}{4(ae^2 + cd^2)^2(cx^4 + a)a} + \frac{cd^2e^2x^2}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{c^2d^3 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}a} + \frac{3cd^2e^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ae^2 + cd^2)^2\sqrt{ac}} + \frac{ae^3}{4(ae^2 + cd^2)^2(cx^4 + a)} + \frac{cd^2e}{4(ae^2 + cd^2)^2(cx^4 + a)} - \frac{e^3 \ln(cx^4 + a)}{4(ae^2 + cd^2)^2} + \frac{e^3 \ln(e^2x^2 + d)}{2(ae^2 + cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x^2*e^2+1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d^3/a*x^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*a+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e*d^2-1/4*e^3*\ln(c*x^4+a)/(a*e^2+c*d^2)^2+3/4*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2+1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^3*\ln(e*x^2+d)/(a*e^2+c*d^2)^2$

maxima [A] time = 2.03, size = 196, normalized size = 1.30

$$\frac{e^3 \log(cx^4 + a)}{4(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{e^3 \log(ex^2 + d)}{2(c^2d^4 + 2acd^2e^2 + a^2e^4)} + \frac{(c^2d^3 + 3acde^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4 + 2a^2cd^2e^2 + a^3e^4)\sqrt{ac}} + \frac{cdx^2 + ae}{4(a^2cd^2 + a^3e^2 + (ac^2d^2 + a^2ce^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $-1/4*e^3*\log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*\log(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)*\arctan(c*x^2/\text{sqrt}(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\text{sqrt}(a*c)) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4)$

mupad [B] time = 1.49, size = 649, normalized size = 4.30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((a + c*x^4)^2*(d + e*x^2)),x)`

[Out]
$$\frac{e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2))}{(a + c*x^4)} + (e^3*\log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (\log(36*a^6*e^{10}*(-a^3*c)^{(1/2)} + 36*a^7*c*e^{10}*x^2 + a*c^5*d^{10}*(-a^3*c)^{(1/2)} + a^2*c^6*d^{10}*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^{(3/2)} - 22*c^2*d^6*e^4*(-a^3*c)^{(3/2)} + 8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^{(1/2)} - 60*a*c*d^4*e^6*(-a^3*c)^{(3/2)})*(c*d^3*(-a^3*c)^{(1/2)} - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^{(1/2)}))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (\log(36*a^7*c*e^{10}*x^2 - 36*a^6*e^{10}*(-a^3*c)^{(1/2)} - a*c^5*d^{10}*(-a^3*c)^{(1/2)} + a^2*c^6*d^{10}*x^2 + 81*a^2*d^2*e^8*(-a^3*c)^{(3/2)} + 22*c^2*d^6*e^4*(-a^3*c)^{(3/2)} + 8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^{(1/2)} + 60*a*c*d^4*e^6*(-a^3*c)^{(3/2)})*(2*a^3*e^3 + c*d^3*(-a^3*c)^{(1/2)} + 3*a*d*e^2*(-a^3*c)^{(1/2)}))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)`

[Out] Timed out

$$3.179 \quad \int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=209

$$\frac{\sqrt{c} e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4a^{3/2}(ae^2 + cd^2)} - \frac{cd(2ae^2 + cd^2) \log(a + cx^4)}{4a^2(ae^2 + cd^2)^2} + \frac{\log(x)}{a^2 d} + \frac{c(d - ex^2)}{4a(a + cx^4)(ae^2 + cd^2)} - \frac{e^4 \log(d + ex^2)}{2d(ae^2 + cd^2)^2} - \frac{\sqrt{c} e^3 \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2 + cd^2)^2}$$

Rubi [A] time = 0.24, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 894, 639, 205, 635, 260}

$$\frac{cd(2ae^2 + cd^2) \log(a + cx^4)}{4a^2(ae^2 + cd^2)^2} - \frac{\sqrt{c} e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4a^{3/2}(ae^2 + cd^2)} + \frac{\log(x)}{a^2 d} + \frac{c(d - ex^2)}{4a(a + cx^4)(ae^2 + cd^2)} - \frac{e^4 \log(d + ex^2)}{2d(ae^2 + cd^2)^2} - \frac{\sqrt{c} e^3 \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a^2 dx} - \frac{e^5}{d(cd^2+ae^2)^2(d+ex)} - \frac{c(ae+cdx)}{a(cd^2+ae^2)(a+cx^2)^2} + \frac{c(-a^2e^3)}{a^2(cd^2+ae^2)} \right) dx, x, x^2 \right) \\
 &= \frac{\log(x)}{a^2 d} - \frac{e^4 \log(d+ex^2)}{2d(cd^2+ae^2)^2} + \frac{c \text{Subst} \left(\int \frac{-a^2e^3 - cd(cd^2+2ae^2)x}{a+cx^2} dx, x, x^2 \right)}{2a^2(cd^2+ae^2)^2} - \frac{c \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2a(cd^2+ae^2)} \\
 &= \frac{c(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\log(x)}{a^2 d} - \frac{e^4 \log(d+ex^2)}{2d(cd^2+ae^2)^2} - \frac{(ce^3) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^2+ae^2)^2} \\
 &= \frac{c(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{c}e^3 \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^2+ae^2)^2} - \frac{\sqrt{c}e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}(cd^2+ae^2)} + \frac{\log(x)}{a^2 d} - \frac{e^4}{2d}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 241, normalized size = 1.15

$$\frac{-2a^2e^4(a+cx^4)\log(d+ex^2)+4\log(x)(a+cx^4)(a^2+cd^2)^2-cd^2(a+cx^4)(2a^2+cd^2)\log(a+cx^4)+\sqrt{a}\sqrt{c}de(a+cx^4)(3a^2+cd^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{c}de(a+cx^4)(3a^2+cd^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}}+1\right)+acd(d-cx^2)(a^2+cd^2)}{4a^2d(a+cx^4)(a^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] - c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^2)^2*(a + c*x^4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.37, size = 279, normalized size = 1.33

$$\frac{(c^2d^3+2acde^2)\log(cx^4+a)}{4(a^2c^2d^4+2a^3cd^2e^2+a^4e^4)} - \frac{e^5\log(|x^2e+d|)}{2(c^2d^3e+2acd^3e^3+a^2de^5)} - \frac{(c^2d^2e+3ace^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{c^3d^3x^4+2ac^2dx^4e^2-ac^2d^2x^2e+2ac^2d^3-a^2cx^2e^3+3a^2cde^2}{4(a^2c^2d^4+2a^3cd^2e^2+a^4e^4)(cx^4+a)} + \frac{\log(x^2)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - 1/2*e^5*log(abs(x^2*e + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d

$*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*\arctan(c*x^2/\sqrt{a*c})/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{a*c}) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2 - a*c^2*d^2*x^2*e + 2*a*c^2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*\log(x^2)/(a^2*d)$

maple [A] time = 0.02, size = 309, normalized size = 1.48

$$\frac{c^2 d^2 e x^2}{4(a^2 + c d^2)^2 (c x^4 + a) a} - \frac{c e^3 x^2}{4(a^2 + c d^2)^2 (c x^4 + a)} - \frac{c^2 d^2 e \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c} a} - \frac{3 c e^3 \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c}} + \frac{c^2 d^3}{4(a^2 + c d^2)^2 (c x^4 + a) a} - \frac{c d e^2 \ln(c x^4 + a)}{2(a^2 + c d^2)^2 a} - \frac{c^2 d^3 \ln(c x^4 + a)}{4(a^2 + c d^2)^2 a^2} + \frac{c d e^2}{4(a^2 + c d^2)^2 (c x^4 + a)} - \frac{e^4 \ln(e x^2 + d)}{2(a^2 + c d^2)^2 d} + \frac{\ln(x)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x^2+d)/(c*x^4+a)^2,x)`

[Out] $\ln(x)/a^2/d - 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^2 - 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^2*e*d^2 + 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*e^2 + 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*d^3 - 1/2*c/(a*e^2+c*d^2)^2/a*\ln(c*x^4+a)*d*e^2 - 1/4*c^2/(a*e^2+c*d^2)^2/a^2*\ln(c*x^4+a)*d^3 - 3/4*c/(a*e^2+c*d^2)^2/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)*e^3 - 1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x^2)*e*d^2 - 1/2*e^4*\ln(e*x^2+d)/d/(a*e^2+c*d^2)^2$

maxima [A] time = 2.05, size = 228, normalized size = 1.09

$$\frac{e^4 \log(e x^2 + d)}{2(c^2 d^5 + 2 a c d^3 e^2 + a^2 d e^4)} - \frac{(c^2 d^3 + 2 a c d e^2) \log(c x^4 + a)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)} - \frac{(c^2 d^2 e + 3 a c e^3) \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a c^2 d^4 + 2 a^2 c d^2 e^2 + a^3 e^4) \sqrt{a c}} - \frac{c e x^2 - c d}{4(a^2 c d^2 + a^3 e^2 + (a c^2 d^2 + a^2 c e^2) x^4)} + \frac{\log(x^2)}{2 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] $-1/2*e^4*\log(e*x^2 + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) - 1/4*(c^2*d^3 + 2*a*c*d*e^2)*\log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*\arctan(c*x^2/\sqrt{a*c})/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{a*c}) - 1/4*(c*e*x^2 - c*d)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4) + 1/2*\log(x^2)/(a^2*d)$

mupad [B] time = 2.58, size = 1082, normalized size = 5.18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)`

[Out] $((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) - (\log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^{(7/2)} - 1024*a^12*e^20*(-a^5*c)^{(3/2)} + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)^{(3/2)} + 5840*a^6*d^2*e^18*(-a^5*c)^{(5/2)} + 33710*c^6*d^14*e^6*(-a^5*c)^{(5/2)})$

$$\begin{aligned}
& + 4104*a^{10}*c^{11}*d^{18}*e^2*x^2 + 16689*a^{11}*c^{10}*d^{16}*e^4*x^2 + 33710*a^{12}*c^9*d^{14}*e^6*x^2 + 33391*a^{13}*c^8*d^{12}*e^8*x^2 + 10748*a^{14}*c^7*d^{10}*e^{10}*x^2 \\
& - 3585*a^{15}*c^6*d^8*e^{12}*x^2 + 3998*a^{16}*c^5*d^6*e^{14}*x^2 + 10481*a^{17}*c^4*d^4*e^{16}*x^2 + 5840*a^{18}*c^3*d^2*e^{18}*x^2 + 10748*a^2*c^4*d^{10}*e^{10}*(-a^5*c)^{(5/2)} \\
& - 3585*a^3*c^3*d^8*e^{12}*(-a^5*c)^{(5/2)} + 3998*a^4*c^2*d^6*e^{14}*(-a^5*c)^{(5/2)} - 4104*a^3*c^9*d^{18}*e^2*(-a^5*c)^{(3/2)} - 16689*a^4*c^8*d^{16}*e^4*(-a^5*c)^{(3/2)} \\
& + 33391*a*c^5*d^{12}*e^8*(-a^5*c)^{(5/2)}*(3*a*e^3*(-a^5*c)^{(1/2)} + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^{(1/2)})) / (8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) \\
& + (\log(1024*a^{12}*e^{20}*(-a^5*c)^{(3/2)} + 10481*d^4*e^{16}*(-a^5*c)^{(7/2)} + 400*a^9*c^{12}*d^{20}*x^2 + 1024*a^{19}*c^2*e^{20}*x^2 \\
& + 400*a^2*c^{10}*d^{20}*(-a^5*c)^{(3/2)} - 5840*a^6*d^2*e^{18}*(-a^5*c)^{(5/2)} - 33710*c^6*d^{14}*e^6*(-a^5*c)^{(5/2)} + 4104*a^{10}*c^{11}*d^{18}*e^2*x^2 + 16689*a^{11}*c^{10}*d^{16}*e^4*x^2 \\
& + 33710*a^{12}*c^9*d^{14}*e^6*x^2 + 33391*a^{13}*c^8*d^{12}*e^8*x^2 + 10748*a^{14}*c^7*d^{10}*e^{10}*x^2 - 3585*a^{15}*c^6*d^8*e^{12}*x^2 + 3998*a^{16}*c^5*d^6*e^{14}*x^2 \\
& + 10481*a^{17}*c^4*d^4*e^{16}*x^2 + 5840*a^{18}*c^3*d^2*e^{18}*x^2 - 10748*a^2*c^4*d^{10}*e^{10}*(-a^5*c)^{(5/2)} + 3585*a^3*c^3*d^8*e^{12}*(-a^5*c)^{(5/2)} \\
& - 3998*a^4*c^2*d^6*e^{14}*(-a^5*c)^{(5/2)} + 4104*a^3*c^9*d^{18}*e^2*(-a^5*c)^{(3/2)} + 16689*a^4*c^8*d^{16}*e^4*(-a^5*c)^{(3/2)} - 33391*a*c^5*d^{12}*e^8*(-a^5*c)^{(5/2)} \\
& *(3*a*e^3*(-a^5*c)^{(1/2)} - 2*a^2*c^2*d^3 - 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^{(1/2)})) / (8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) - (e^4 * \log(d + e*x^2)) / (2*c^2*d^5 + 2*a^2*d*e^4 + 4*a*c*d^3*e^2) + \log(x) / (a^2*d)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.180 \quad \int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=236

$$-\frac{c^{3/2}d(2ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2+cd^2)^2} - \frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)} + \frac{ce(2ae^2+cd^2)\log(a+cx^4)}{4a^2(ae^2+cd^2)^2} - \frac{c(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)} - \frac{e^5\log(d+ex^2)}{2d^2(ae^2+cd^2)^2}$$

Rubi [A] time = 0.26, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^{3/2}d(2ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2+cd^2)^2} - \frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)} - \frac{c(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)} + \frac{ce(2ae^2+cd^2)\log(a+cx^4)}{4a^2(ae^2+cd^2)^2} - \frac{e\log(x)}{a^2d^2} - \frac{1}{2a^2dx^2} + \frac{e^5\log(d+ex^2)}{2d^2(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $-1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (c^{(3/2)}*d*\text{ArcTan}[\text{Sqrt}[c]*x^2]/\text{Sqrt}[a])/(4*a^{(5/2)}*(c*d^2 + a*e^2)) - (c^{(3/2)}*d*(c*d^2 + 2*a*e^2)*\text{ArcTan}[\text{Sqrt}[c]*x^2]/\text{Sqrt}[a])/(2*a^{(5/2)}*(c*d^2 + a*e^2)^2) - (e*\text{Log}[x])/(a^2*d^2) + (e^5*\text{Log}[d + e*x^2])/(2*d^2*(c*d^2 + a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*\text{Log}[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^2)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 894

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1252

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(d+ex)(a+cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a^2 dx^2} - \frac{e}{a^2 d^2 x} + \frac{e^6}{d^2 (cd^2 + ae^2)^2 (d+ex)} - \frac{c^2(d-ex)}{a(cd^2 + ae^2)(a+cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{1}{2a^2 dx^2} - \frac{e \log(x)}{a^2 d^2} + \frac{e^5 \log(d+ex^2)}{2d^2 (cd^2 + ae^2)^2} - \frac{c^2 \text{Subst} \left(\int \frac{d-ex}{(a+cx^2)^2} dx, x, x^2 \right)}{2a(cd^2 + ae^2)} - \frac{(c^2 d) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{4a^2} \\
 &= -\frac{1}{2a^2 dx^2} - \frac{c(ae + cd^2)}{4a^2 (cd^2 + ae^2)(a+cx^4)} - \frac{e \log(x)}{a^2 d^2} + \frac{e^5 \log(d+ex^2)}{2d^2 (cd^2 + ae^2)^2} - \frac{(c^2 d) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{4a^2} \\
 &= -\frac{1}{2a^2 dx^2} - \frac{c(ae + cd^2)}{4a^2 (cd^2 + ae^2)(a+cx^4)} - \frac{c^{3/2} d \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{5/2} (cd^2 + ae^2)} - \frac{c^{3/2} d (cd^2 + 2ae^2) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2a^{5/2} (cd^2 + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.43, size = 248, normalized size = 1.05

$$\frac{1}{4} \left(\frac{c^{3/2} d (5ae^2 + 3cd^2) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} \right)}{a^{5/2} (ae^2 + cd^2)^2} + \frac{c^{3/2} d (5ae^2 + 3cd^2) \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1 \right)}{a^{5/2} (ae^2 + cd^2)^2} - \frac{c (ae + cd^2)}{a^2 (a + cx^4) (ae^2 + cd^2)} + \frac{c (2ae^3 + cd^2 e) \log(a + cx^4)}{a^2 (ae^2 + cd^2)^2} - \frac{4e \log(x)}{a^2 d^2} - \frac{2}{a^2 dx^2} + \frac{2e^5 \log(d + ex^2)}{(ade^2 + cd^3)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $(-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (c^{3/2}*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(a^{5/2}*(c*d^2 + a*e^2)^2) + (c^{3/2}*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(a^{5/2}*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2) + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 344, normalized size = 1.46

$$\frac{(c^2 d^2 e + 2 a c e^3) \log(cx^4 + a)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)} + \frac{e^6 \log(|x^2 e + d|)}{2(c^2 d^2 e + 2 a c d^4 e^3 + a^2 d^2 e^5)} - \frac{(3 c^3 d^3 + 5 a c^2 d e^2) \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{ac}} - \frac{9 c^3 d^3 x^4 + 15 a c^2 d^3 x^4 e^2 - 2 a^2 c x^6 e^5 + 3 a c^2 d^4 x^2 e + 6 a^2 c d x^4 e^4 + 6 a c^2 d^3 + 3 a^2 c d^2 x^2 e^3 + 12 a^2 c d^3 e^2 - 2 a^3 x^2 e^5 + 6 a^3 d e^4}{12(a^2 c^2 d^6 + 2 a^3 c d^4 e^2 + a^4 d^2 e^4)(cx^6 + ax^2)} - \frac{e \log(x^2)}{2 a^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $1/4*(c^2*d^2*e + 2*a*c*e^3)*\log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) + 1/2*e^6*\log(\text{abs}(x^2*e + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2$

$$2e^5) - 1/4*(3c^3d^3 + 5a*c^2*d*e^2)*\arctan(c*x^2/\sqrt{a*c})/((a^2*c^2*d^4 + 2a^3*c*d^2*e^2 + a^4*e^4)*\sqrt{a*c}) - 1/12*(9c^3*d^5*x^4 + 15a*c^2*d^3*x^4*e^2 - 2a^2*c*x^6*e^5 + 3a*c^2*d^4*x^2*e + 6a^2*c*d*x^4*e^4 + 6a*c^2*d^5 + 3a^2*c*d^2*x^2*e^3 + 12a^2*c*d^3*e^2 - 2a^3*x^2*e^5 + 6a^3*d*e^4)/((a^2*c^2*d^6 + 2a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1/2*e*\log(x^2)/(a^2*d^2)$$

maple [A] time = 0.02, size = 332, normalized size = 1.41

$$\frac{c^2 d e^2 x^2}{4(a^2 + c d^2)(c x^4 + a) a} - \frac{c^2 d^2 x^2}{4(a^2 + c d^2)^2 (c x^4 + a) a^2} - \frac{5 c^2 d^2 e^2 \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c} a} - \frac{3 c^3 d^3 \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c} a^2} - \frac{c^2 d^2 e}{4(a^2 + c d^2)^2 (c x^4 + a) a} + \frac{c^2 \ln(c x^4 + a)}{2(a^2 + c d^2)^2 a} + \frac{c^2 d^2 e \ln(c x^4 + a)}{4(a^2 + c d^2)^2 a^2} - \frac{c^2 e^3}{4(a^2 + c d^2)^2 (c x^4 + a)} + \frac{e^5 \ln(e x^2 + d)}{2(a^2 + c d^2)^2 d^2} - \frac{e \ln(x)}{a^2 d^2} - \frac{1}{2 a^2 d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x)

$$[Out] -1/2/a^2/d/x^2-e*\ln(x)/a^2/d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^2*d*e^2-2-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^2*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e*d^2+1/2*c/(a*e^2+c*d^2)^2/a*\ln(c*x^4+a)*e^3+1/4*c^2/(a*e^2+c*d^2)^2/a^2*\ln(c*x^4+a)*e*d^2-5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*d*e^2-3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*d^3+1/2*e^5*\ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2$$

maxima [A] time = 2.01, size = 278, normalized size = 1.18

$$\frac{e^5 \log(e x^2 + d)}{2(c^2 d^6 + 2 a c d^4 e^2 + a^2 d^2 e^4)} + \frac{(c^2 d^2 e + 2 a c e^3) \log(c x^4 + a)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4)} - \frac{(3 c^3 d^3 + 5 a c^2 d e^2) \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{a c}} - \frac{a c d e x^2 + (3 c^2 d^2 + 2 a c e^2) x^4 + 2 a c d^2 + 2 a^2 e^2}{4((a^2 c^2 d^3 + a^3 c d e^2) x^6 + (a^3 c d^3 + a^4 d e^2) x^2)} - \frac{e \log(x^2)}{2 a^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

$$[Out] 1/2*e^5*\log(e*x^2 + d)/((c^2*d^6 + 2a*c*d^4*e^2 + a^2*d^2*e^4) + 1/4*(c^2*d^2*e + 2a*c*e^3)*\log(c*x^4 + a)/((a^2*c^2*d^4 + 2a^3*c*d^2*e^2 + a^4*e^4) - 1/4*(3c^3*d^3 + 5a*c^2*d*e^2)*\arctan(c*x^2/\sqrt{a*c})/((a^2*c^2*d^4 + 2a^3*c*d^2*e^2 + a^4*e^4)*\sqrt{a*c}) - 1/4*(a*c*d*e*x^2 + (3c^2*d^2 + 2a*c*e^2)*x^4 + 2a*c*d^2 + 2a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 + (a^3*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*\log(x^2)/(a^2*d^2)$$

mupad [B] time = 2.94, size = 1337, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)

$$[Out] (\log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(-a$$

$$\begin{aligned}
& ^5c^3)^{(5/2)} - 5120*a^{14}*d^2*e^{22}*(-a^5*c^3)^{(3/2)} + 11647*c^6*d^{20}*e^4*(- \\
& a^5*c^3)^{(5/2)} + 1638*a^{11}*c^{15}*d^{22}*e^2*x^2 + 11647*a^{12}*c^{14}*d^{20}*e^4*x^2 \\
& + 43524*a^{13}*c^{13}*d^{18}*e^6*x^2 + 97311*a^{14}*c^{12}*d^{16}*e^8*x^2 + 133334*a^{15} \\
& *c^{11}*d^{14}*e^{10}*x^2 + 103633*a^{16}*c^{10}*d^{12}*e^{12}*x^2 + 29456*a^{17}*c^9*d^{10} \\
& *e^{14}*x^2 - 14496*a^{18}*c^8*d^8*e^{16}*x^2 - 7984*a^{19}*c^7*d^6*e^{18}*x^2 + 5888 \\
& *a^{20}*c^6*d^4*e^{20}*x^2 + 5120*a^{21}*c^5*d^2*e^{22}*x^2 + 43524*a*c^5*d^{18}*e^6* \\
& (-a^5*c^3)^{(5/2)} + 29456*a^5*c*d^{10}*e^{14}*(-a^5*c^3)^{(5/2)} - 5888*a^{13}*c*d^4 \\
& *e^{20}*(-a^5*c^3)^{(3/2)} + 97311*a^2*c^4*d^{16}*e^8*(-a^5*c^3)^{(5/2)} + 133334*a \\
& ^3*c^3*d^{14}*e^{10}*(-a^5*c^3)^{(5/2)} + 103633*a^4*c^2*d^{12}*e^{12}*(-a^5*c^3)^{(5/ \\
& 2)} - 1638*a^4*c^{10}*d^{22}*e^2*(-a^5*c^3)^{(3/2)} + 7984*a^{12}*c^2*d^6*e^{18}*(-a^5 \\
& *c^3)^{(3/2)}*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d^2*e - 5* \\
& a*d*e^2*(-a^5*c^3)^{(1/2)}))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) - \\
& (1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e^2 + 3*c*d^2))/ \\
& (4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (\log(81*a^{10}*c^{16}*d^{24}*x^2 + 1 \\
& 024*a^{22}*c^4*e^{24}*x^2 + 81*a^3*c^{11}*d^{24}*(-a^5*c^3)^{(3/2)} - 1024*a^{20}*c^2*e \\
& ^{24}*(-a^5*c^3)^{(1/2)} + 14496*a^6*d^8*e^{16}*(-a^5*c^3)^{(5/2)} + 5120*a^{14}*d^2* \\
& e^{22}*(-a^5*c^3)^{(3/2)} - 11647*c^6*d^{20}*e^4*(-a^5*c^3)^{(5/2)} + 1638*a^{11}*c^1 \\
& 5*d^{22}*e^2*x^2 + 11647*a^{12}*c^{14}*d^{20}*e^4*x^2 + 43524*a^{13}*c^{13}*d^{18}*e^6*x^ \\
& 2 + 97311*a^{14}*c^{12}*d^{16}*e^8*x^2 + 133334*a^{15}*c^{11}*d^{14}*e^{10}*x^2 + 103633* \\
& a^{16}*c^{10}*d^{12}*e^{12}*x^2 + 29456*a^{17}*c^9*d^{10}*e^{14}*x^2 - 14496*a^{18}*c^8*d^8 \\
& *e^{16}*x^2 - 7984*a^{19}*c^7*d^6*e^{18}*x^2 + 5888*a^{20}*c^6*d^4*e^{20}*x^2 + 5120* \\
& a^{21}*c^5*d^2*e^{22}*x^2 - 43524*a*c^5*d^{18}*e^6*(-a^5*c^3)^{(5/2)} - 29456*a^5*c \\
& *d^{10}*e^{14}*(-a^5*c^3)^{(5/2)} + 5888*a^{13}*c*d^4*e^{20}*(-a^5*c^3)^{(3/2)} - 97311 \\
& *a^2*c^4*d^{16}*e^8*(-a^5*c^3)^{(5/2)} - 133334*a^3*c^3*d^{14}*e^{10}*(-a^5*c^3)^{(5 \\
& /2)} - 103633*a^4*c^2*d^{12}*e^{12}*(-a^5*c^3)^{(5/2)} + 1638*a^4*c^{10}*d^{22}*e^2*(- \\
& a^5*c^3)^{(3/2)} - 7984*a^{12}*c^2*d^6*e^{18}*(-a^5*c^3)^{(3/2)}*(4*a^4*c*e^3 + 3* \\
& c*d^3*(-a^5*c^3)^{(1/2)} + 2*a^3*c^2*d^2*e + 5*a*d*e^2*(-a^5*c^3)^{(1/2)}))/(8* \\
& (a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) + (e^5*\log(d + e*x^2))/(2*c^2*d^ \\
& 6 + 2*a^2*d^2*e^4 + 4*a*c*d^4*e^2) - (e*\log(x))/(a^2*d^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.181 \quad \int \frac{1}{x^5(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=265

$$\frac{c^{3/2}e(2ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2 + cd^2)^2} + \frac{c^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2 + cd^2)} + \frac{c^2d(3ae^2 + 2cd^2) \log(a + cx^4)}{4a^3(ae^2 + cd^2)^2} - \frac{\log(x)(2cd^2 - ae^2)}{a^3d^3} - \frac{1}{4a^2}$$

Rubi [A] time = 0.33, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^2(d - ex^2)}{4a^2(a + cx^4)(ae^2 + cd^2)} + \frac{c^2d(3ae^2 + 2cd^2) \log(a + cx^4)}{4a^3(ae^2 + cd^2)^2} + \frac{c^{3/2}e(2ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2 + cd^2)^2} + \frac{c^{3/2}e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2 + cd^2)} - \frac{\log(x)(2cd^2 - ae^2)}{a^3d^3} + \frac{e}{2a^2d^2x^2} - \frac{1}{4a^2dx^4} - \frac{e^6 \log(d + ex^2)}{2d^3(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $-\frac{1}{4a^2d^2x^4} + \frac{e}{2a^2d^2x^2} - \frac{c^2(d - ex^2)}{4a^2(c^2d^2 + ae^2)(a + cx^4)} + \frac{c^{3/2}e \operatorname{ArcTan}\left[\frac{\sqrt{c}x^2}{\sqrt{a}}\right]}{4a^{5/2}(c^2d^2 + ae^2)} + \frac{c^{3/2}e(c^2d^2 + 2ae^2) \operatorname{ArcTan}\left[\frac{\sqrt{c}x^2}{\sqrt{a}}\right]}{(2a^{5/2}(c^2d^2 + ae^2)^2) - ((2c^2d^2 - ae^2) \operatorname{Log}[x])/(a^3d^3) - (e^6 \operatorname{Log}[d + ex^2])/(2d^3(c^2d^2 + ae^2)^2)} + \frac{c^2d(2c^2d^2 + 3ae^2) \operatorname{Log}[a + cx^4]}{4a^3(c^2d^2 + ae^2)^2}$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

```
Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e
- c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^
2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_S
ymbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^5 (d + ex^2) (a + cx^4)^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 (d + ex) (a + cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{a^2 dx^3} - \frac{e}{a^2 d^2 x^2} + \frac{-2cd^2 + ae^2}{a^3 d^3 x} - \frac{e^7}{d^3 (cd^2 + ae^2)^2 (d + ex)} + \frac{c^2}{a^2 (cd^2 + ae^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{1}{4a^2 dx^4} + \frac{e}{2a^2 d^2 x^2} - \frac{(2cd^2 - ae^2) \log(x)}{a^3 d^3} - \frac{e^6 \log(d + ex^2)}{2d^3 (cd^2 + ae^2)^2} + \frac{c^2 \text{Subst} \left(\int \frac{ae^{ca}}{a^2 (cd^2 + ae^2)} dx, x, x^2 \right)}{2a^2 (cd^2 + ae^2)} \\
 &= -\frac{1}{4a^2 dx^4} + \frac{e}{2a^2 d^2 x^2} - \frac{c^2 (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{(2cd^2 - ae^2) \log(x)}{a^3 d^3} - \frac{e^6 \log(d + ex^2)}{2d^3 (cd^2 + ae^2)^2} \\
 &= -\frac{1}{4a^2 dx^4} + \frac{e}{2a^2 d^2 x^2} - \frac{c^2 (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{c^{3/2} e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{5/2} (cd^2 + ae^2)} + \frac{c^{3/2} e (cd^2 + ae^2)}{2a^2 (cd^2 + ae^2)^2}
 \end{aligned}$$

Mathematica [A] time = 0.39, size = 278, normalized size = 1.05

$$\frac{1}{4} \left(-\frac{c^{3/2} e (5ae^2 + 3cd^2) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{ex}}{\sqrt{a}} \right)}{a^{5/2} (ae^2 + cd^2)^2} - \frac{c^{3/2} e (5ae^2 + 3cd^2) \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{ex}}{\sqrt{a}} + 1 \right)}{a^{5/2} (ae^2 + cd^2)^2} + \frac{c^2 (3ade^2 + 2cd^3) \log(a + cx^4)}{a^3 (ae^2 + cd^2)^2} + \frac{4 \log(x) (ae^2 - 2cd^2)}{a^3 d^3} + \frac{c^2 (ex^2 - d)}{a^2 (a + cx^4) (ae^2 + cd^2)} + \frac{2e}{a^2 d^2 x^2} - \frac{1}{a^2 dx^4} - \frac{2e^6 \log(d + ex^2)}{d^3 (ae^2 + cd^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $(-1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (c^{(3/2)}*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(a^{(5/2)}*(c*d^2 + a*e^2)^2) - (c^{(3/2)}*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(a^{(5/2)}*(c*d^2 + a*e^2)^2) + (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a*e^2)^2)/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.37, size = 350, normalized size = 1.32

$$\frac{(2c^3d^3 + 3ac^2d^2) \log(cx^4 + a)}{4(a^3c^2d^4 + 2a^4cd^2e + a^5e^4)} - \frac{e^2 \log(|x^2e + d|)}{2(c^2de + 2acd^2e^3 + a^2d^3e^5)} + \frac{(3c^3d^2e + 5ac^2e^2) \arctan\left(\frac{cx^2}{\sqrt{ae}}\right)}{4(a^2c^2d^4 + 2a^3cd^2e + a^4e^4)\sqrt{ac}} - \frac{2c^4d^3x^4 + 3ac^3dx^4e^2 - ac^3d^2x^2e + 3ac^3d^3 - a^2c^2x^2e^2 + 4a^2c^2de^2}{4(a^3c^2d^4 + 2a^4cd^2e + a^5e^4)(cx^4 + a)} - \frac{(2cd^2 - ae^2) \log(x^2)}{2a^3d^3} + \frac{6cd^2x^4 - 3ax^4e^2 + 2adx^2e - ad^2}{4a^3d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*\log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) - 1/2*e^7*\log(\text{abs}(x^2*e + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^5*e^7)$

$$2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*\arctan(c*x^2/\sqrt{a*c})/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*\sqrt{a*c}) - 1/4*(2*c^4*d^3*x^4 + 3*a*c^3*d*x^4*e^2 - a*c^3*d^2*x^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*d^2 - a*e^2)*\log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)/(a^3*d^3*x^4)$$

maple [A] time = 0.02, size = 363, normalized size = 1.37

$$\frac{c^2 e^2 x^2}{4(a^2 + c d^2)^2 (c x^4 + a) a} + \frac{c^2 d^2 x^2}{4(a^2 + c d^2)^2 (c x^4 + a) a^2} + \frac{5 c^2 a^3 \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c} a} + \frac{3 c^3 d^2 e \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 + c d^2)^2 \sqrt{a c} a^2} - \frac{c^2 d e^2}{4(a^2 + c d^2)^2 (c x^4 + a) a} - \frac{c^2 d^3}{4(a^2 + c d^2)^2 (c x^4 + a) a^2} + \frac{3 c^2 d e^2 \ln(c x^4 + a)}{4(a^2 + c d^2)^2 a^2} + \frac{c^3 d^3 \ln(c x^4 + a)}{2(a^2 + c d^2)^2 a^3} - \frac{e^6 \ln(c x^2 + d)}{2(a^2 + c d^2)^2 d^3} + \frac{e^2 \ln(x)}{a^2 d^3} - \frac{2 c \ln(x)}{a^3 d} + \frac{e}{2 a^2 d^2 e^2} - \frac{1}{4 a^2 d^4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4/a^2/d/x^4 + 1/a^2/d^3*\ln(x)*e^2 - 2/a^3/d*\ln(x)*c + 1/2*e/a^2/d^2/x^2 + 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e^3*x^2 + 1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^2*e*d^2 - 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*d*e^2 - 1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*d^3 + 3/4*c^2/(a*e^2+c*d^2)^2/a^2*\ln(c*x^4+a)*d*e^2 + 1/2*c^3/(a*e^2+c*d^2)^2/a^3*\ln(c*x^4+a)*d^3 + 5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*e^3 + 3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x^2)*e*d^2 - 1/2*e^6*\ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2$

maxima [A] time = 2.08, size = 332, normalized size = 1.25

$$\frac{e^6 \log(c x^2 + d)}{2(c^2 d^7 + 2 a c d^5 e^2 + a^2 d^3 e^4)} + \frac{(2 c^3 d^3 + 3 a c^2 d e^2) \log(c x^4 + a)}{4(a^3 c^2 d^4 + 2 a^4 c d^2 e^2 + a^5 e^4)} + \frac{(3 c^3 d^2 e + 5 a^2 c^2 e^3) \arctan\left(\frac{c x^2}{\sqrt{a c}}\right)}{4(a^2 c^2 d^4 + 2 a^3 c d^2 e^2 + a^4 e^4) \sqrt{a c}} + \frac{(3 c^2 d^2 e + 2 a c e^3) x^6 - a c d^3 - a^2 d e^2 - (2 c^2 d^3 + a c d e^2) x^4 + 2(a c d^2 e + a^2 e^3) x^2}{4((a^2 c^2 d^4 + a^3 c d^2 e^2) x^8 + (a^3 c d^4 + a^4 d^2 e^2) x^4)} - \frac{(2 c d^2 - a e^2) \log(x^2)}{2 a^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $-1/2*e^6*\log(e*x^2 + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*\log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*\arctan(c*x^2/\sqrt{a*c})/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*\sqrt{a*c}) + 1/4*((3*c^2*d^2*e + 2*a*c*e^3)*x^6 - a*c*d^3 - a^2*d*e^2 - (2*c^2*d^3 + a*c*d*e^2)*x^4 + 2*(a*c*d^2*e + a^2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2*e^2)*x^4) - 1/2*(2*c*d^2 - a*e^2)*\log(x^2)/(a^3*d^3)$

mupad [B] time = 3.48, size = 1545, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)

```
[Out] (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28*
(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^20
*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^20*e
^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*
e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 4
65092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20
*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e
^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*
a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*d^18*e^10
*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584*a^15*c*d^
4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) + 52822
*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5
/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^11*d^24*e^4*(
-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) + 10288*a^14*c^
2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*c^3)^(1/2) + 6*
a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*
a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^2 + a*c*e^2))/(4*a
^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4*a^2*d^2*(a*e^2 + c
*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28
*x^2 + 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) - 1024*a^24*c^2*e^28*(-a^7*c^3)^(
1/2) + 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) + 2048*a^16*d^2*e^26*(-a^7*c^3)
^(3/2) - 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^
2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*
a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*
d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x
^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^2
4*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^
2 - 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) + 27472*a^5*c*d^10*e^18*(-a^7*c
^3)^(5/2) - 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) - 256991*a^2*c^4*d^16*e^1
2*(-a^7*c^3)^(5/2) - 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) + 37423*a^4*c
^2*d^12*e^16*(-a^7*c^3)^(5/2) + 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) +
200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) + 413414*a^6*c^10*d^22*e^6*(-a^7*
c^3)^(3/2) - 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 - 5*a
*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 - 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(
a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + (log(x)*(a*e^2 - 2*c*d^2))/(a^3
*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.182 \quad \int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=712

$$\frac{\sqrt[4]{a} d^2 (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)^2} - \frac{\sqrt[4]{a} d^2 (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)^2} +$$

Rubi [A] time = 0.67, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1314, 1276, 1280, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$\frac{\sqrt[4]{a} d^2 (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)^2} - \frac{\sqrt[4]{a} d^2 (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} c^{3/4} (ae^2 + cd^2)^2} +$$

Antiderivative was successfully verified.

[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1276

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m-1)*(a + c*x^4)^(p+1)*(a*e - c*d*x^2))/(4*a*c*(p+1)), x] - Dist[f^2/(4*a*c*(p+1)), Int[(f*x)^(m-2)*(a + c*x^4)^(p+1)*(a*e*(m-1) - c*d*(4*p+4+m+1)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I

ntegerQ[m])

Rule 1280

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m - 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1288

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]

Rule 1314

Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ[p, -1] && GtQ[m, 2]

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx &= -\frac{a \int \frac{x^4(d-ex^2)}{(a+cx^4)^2} dx}{cd^2+ae^2} + \frac{d^2 \int \frac{x^4}{(d+ex^2)(a+cx^4)} dx}{cd^2+ae^2} \\
&= -\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{x^2(-3ae-cdx^2)}{a+cx^4} dx}{4c(cd^2+ae^2)} + \frac{d^2 \int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)} - \frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx}{cd^2+ae^2} \\
&= \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{(ad^2) \int \frac{d-ex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{d^4 \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} + \frac{\int -a}{4c^2} \\
&= \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2} - \frac{(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)) \int \sqrt{e}}{2c(cd^2+ae^2)} \\
&= \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2} - \frac{(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)) \int \frac{1}{\sqrt{e}}}{4c(cd^2+ae^2)} \\
&= \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2} + \frac{4\sqrt{a}d^2(\sqrt{c}d+\sqrt{a}e)}{4\sqrt{2}} \\
&= \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2} + \frac{a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 431, normalized size = 0.61

$$\frac{\sqrt{2} \sqrt{e} \left(3a^{3/4} d^2 + 7\sqrt{e} a d^2 + a\sqrt{e} d^2 + 5c^{3/4} d^2 \right) \log\left(-\sqrt{2} \sqrt{e} \sqrt{c} x + \sqrt{e} + \sqrt{c} x^2\right) - \sqrt{2} \sqrt{e} \left(3a^{3/4} d^2 + 7\sqrt{e} a d^2 + a\sqrt{e} d^2 + 5c^{3/4} d^2 \right) \log\left(\sqrt{2} \sqrt{e} \sqrt{c} x + \sqrt{e} + \sqrt{c} x^2\right) - 2\sqrt{2} \sqrt{e} \left(3a^{3/4} d^2 + 7\sqrt{e} a d^2 - a\sqrt{e} d^2 - 5c^{3/4} d^2 \right) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt{e} x}{\sqrt{d}}\right) + 2\sqrt{2} \sqrt{e} \left(3a^{3/4} d^2 + 7\sqrt{e} a d^2 - a\sqrt{e} d^2 - 5c^{3/4} d^2 \right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} x}{\sqrt{d}} + 1\right) + \frac{8a \tan(d - ex^2) (ae^2 + cd^2)}{(e+cx^4)} + \frac{32d^{7/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{32(a^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)

$$\frac{1}{c^{7/4}} + \frac{(\sqrt{2} a^{1/4} (5c^{3/2} d^3 + 7\sqrt{a} c d^2 e + a \sqrt{c} d e^2 + 3a^{3/2} e^3) \operatorname{Log}[\sqrt{a} - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2])}{c^{7/4}} - \frac{(\sqrt{2} a^{1/4} (5c^{3/2} d^3 + 7\sqrt{a} c d^2 e + a \sqrt{c} d e^2 + 3a^{3/2} e^3) \operatorname{Log}[\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2])}{c^{7/4}} \Big/ (32(c d^2 + a e^2)^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^8/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 43.13, size = 9856, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\frac{-1/16(4*(a*c*d^2*e + a^2*e^3)*x^3 - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\operatorname{sqrt}((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\operatorname{sqrt}(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12))/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16))}{(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\operatorname{log}(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11))*\operatorname{sqrt}(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12))/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16))*\operatorname{sqrt}((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\operatorname{sqrt}(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12))/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28$$

$$\begin{aligned}
& *a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})) / (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^{10}*d^{10}*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^{11})*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})))*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^{10}*d^{10}*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^{11})*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16})))*\sqrt{((
\end{aligned}$$

$$\begin{aligned}
& 70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)) - 8*(c^2*d^3*x^4 + a*c*d^3)*\sqrt{-d/e}*\log((e*x^2 + 2*e*x*\sqrt{-d/e} - d)/(e*x^2 + d)) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), -1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - 16*(c^2*d^3*x^4 + a*c*d^3)*\sqrt{d/e}*\arctan(e*x*\sqrt{d/e}/d) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a
\end{aligned}$$

$$\begin{aligned}
& *c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (\\
& 125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 \\
& - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 \\
& + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*\sqrt{-(625*a*c^6*d^12 \\
& - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 \\
& + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8* \\
& a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11 \\
& *d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 \\
& + a^8*c^7*e^16)))*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (\\
& c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3 \\
& *e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + \\
& 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7* \\
& e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10* \\
& e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 \\
& + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4* \\
& e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)) + (a*c^3*d^4 + 2*a^2*c^2*d^2* \\
& e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70 \\
& *a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 \\
& + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 \\
& - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 23 \\
& 83*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14* \\
& d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8* \\
& e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8 \\
& *c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2* \\
& e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4* \\
& e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 \\
& - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10* \\
& e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4 \\
& *c^6*d^2*e^9 + 3*a^5*c^5*e^11)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 \\
& - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738 \\
& *a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13 \\
& *d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 \\
& + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*\sqrt{((70*a* \\
& c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6 \\
& *a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - \\
& 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383* \\
& a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14* \\
& e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 \\
& + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7* \\
& e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 \\
& + a^4*c^3*e^8)) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 \\
& + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 \\
& + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 \\
& + a^4*c^3*e^8)*\sqrt{-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 5 \\
& 29*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*
\end{aligned}$$

$$\begin{aligned}
& c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12} \\
& *e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + \\
& 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16}))/((c^7*d^8 + 4*a*c^6 \\
& *d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(62 \\
& 5*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - \\
& 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86 \\
& *a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^{10}*d^{10}*e + 31*a*c^9*d^8*e^3 + 54 \\
& *a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^{11} \\
&)*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 274 \\
& 8*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12} \\
&)/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10} \\
& *e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8* \\
& a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16}))*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 \\
& + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4* \\
& d^2*e^6 + a^4*c^3*e^8))*\sqrt{-(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529* \\
& a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2 \\
& *e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 \\
& + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28* \\
& a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16}))/((c^7*d^8 + 4*a*c^6* \\
& d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 \\
& + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) \\
& *x^4)*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + \\
& 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\sqrt{ \\
& -(625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4* \\
& c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15} \\
& *d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + \\
& 70*a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8 \\
& *d^2*e^{14} + a^8*c^7*e^{16}))/((c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 \\
& + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 \\
& - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 \\
& - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d \\
& *e^8 - (7*c^{10}*d^{10}*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7 \\
& *d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^{11})*\sqrt{-(625*a*c^6*d^{12} - 195 \\
& 0*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5* \\
& c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15}*d^{16} + 8*a*c^{14}*d^{14} \\
& *e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70*a^4*c^{11}*d^8*e^8 + 5 \\
& 6*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8*d^2*e^{14} + a^8*c^7*e^{16} \\
&))*\sqrt{((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4* \\
& a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*\sqrt{-(\\
& 625*a*c^6*d^{12} - 1950*a^2*c^5*d^{10}*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^ \\
& 3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^{10} + 81*a^7*e^{12})/(c^{15} \\
& *d^{16} + 8*a*c^{14}*d^{14}*e^2 + 28*a^2*c^{13}*d^{12}*e^4 + 56*a^3*c^{12}*d^{10}*e^6 + 70 \\
& *a^4*c^{11}*d^8*e^8 + 56*a^5*c^{10}*d^6*e^{10} + 28*a^6*c^9*d^4*e^{12} + 8*a^7*c^8* \\
& d^2*e^{14} + a^8*c^7*e^{16}))/((c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + \\
& 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4
\end{aligned}$$

+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)]

giac [A] time = 0.57, size = 581, normalized size = 0.82

$$\frac{d^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{2d^2 + 2d^2e + d^2e^2} \cdot \frac{\left(5(a^2)^{1/4}c^2d^2 + (a^2)^{1/4}ac^2d^2 - 7(a^2)^{1/4}cd^2e - 3(a^2)^{1/4}d^2e^2\right) \arctan\left(\frac{ax - d}{2cd}\right)}{8(\sqrt{2}c^2d^2 + 2\sqrt{2}ac^2d^2 + \sqrt{2}d^2c^2d^2)} \cdot \frac{\left(5(a^2)^{1/4}c^2d^2 + (a^2)^{1/4}ac^2d^2 - 7(a^2)^{1/4}cd^2e - 3(a^2)^{1/4}d^2e^2\right) \arctan\left(\frac{ax - d}{2cd}\right)}{8(\sqrt{2}c^2d^2 + 2\sqrt{2}ac^2d^2 + \sqrt{2}d^2c^2d^2)} \cdot \frac{\left(\frac{ax - d}{2cd}\right)}{2cd^2} \cdot \frac{\left(5(a^2)^{1/4}c^2d^2 + (a^2)^{1/4}ac^2d^2 + 7(a^2)^{1/4}cd^2e + 3(a^2)^{1/4}d^2e^2\right) \log\left(\frac{c + \sqrt{2}c(\frac{d}{c})^2 + \sqrt{d}}{c - \sqrt{2}c(\frac{d}{c})^2 + \sqrt{d}}\right)}{16(\sqrt{2}c^2d^2 + 2\sqrt{2}ac^2d^2 + \sqrt{2}d^2c^2d^2)} \cdot \frac{\left(5(a^2)^{1/4}c^2d^2 + (a^2)^{1/4}ac^2d^2 + 7(a^2)^{1/4}cd^2e + 3(a^2)^{1/4}d^2e^2\right) \log\left(\frac{c - \sqrt{2}c(\frac{d}{c})^2 + \sqrt{d}}{c + \sqrt{2}c(\frac{d}{c})^2 + \sqrt{d}}\right)}{16(\sqrt{2}c^2d^2 + 2\sqrt{2}ac^2d^2 + \sqrt{2}d^2c^2d^2)} \cdot \frac{ax - d}{4(d^2 + 2d^2e + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $d^{7/2} \arctan(x \cdot e^{1/2} / \sqrt{d}) \cdot e^{-1/2} / (c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4) - 1/8 \cdot (5(a c^3)^{1/4} c^3 d^3 + (a c^3)^{1/4} a c^2 d e^2 - 7(a c^3)^{3/4} c d^2 e - 3(a c^3)^{3/4} a e^3) \arctan(1/2 \sqrt{2} (2 x + \sqrt{2}(a/c)^{1/4})) / (a/c)^{1/4} / (\sqrt{2} c^6 d^4 + 2 \sqrt{2} a c^5 d^2 e^2 + \sqrt{2} a^2 c^4 e^4) - 1/8 \cdot (5(a c^3)^{1/4} c^3 d^3 + (a c^3)^{1/4} a c^2 d e^2 - 7(a c^3)^{3/4} c d^2 e - 3(a c^3)^{3/4} a e^3) \arctan(1/2 \sqrt{2} (2 x - \sqrt{2}(a/c)^{1/4})) / (a/c)^{1/4} / (\sqrt{2} c^6 d^4 + 2 \sqrt{2} a c^5 d^2 e^2 + \sqrt{2} a^2 c^4 e^4) - 1/16 \cdot (5(a c^3)^{1/4} c^3 d^3 + (a c^3)^{1/4} a c^2 d e^2 + 7(a c^3)^{3/4} c d^2 e + 3(a c^3)^{3/4} a e^3) \log(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} c^6 d^4 + 2 \sqrt{2} a c^5 d^2 e^2 + \sqrt{2} a^2 c^4 e^4) + 1/16 \cdot (5(a c^3)^{1/4} c^3 d^3 + (a c^3)^{1/4} a c^2 d e^2 + 7(a c^3)^{3/4} c d^2 e + 3(a c^3)^{3/4} a e^3) \log(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} c^6 d^4 + 2 \sqrt{2} a c^5 d^2 e^2 + \sqrt{2} a^2 c^4 e^4) - 1/4 \cdot (a x^3 e - a d x) / ((c x^4 + a)(c^2 d^2 + a c e^2))$

maple [A] time = 0.02, size = 873, normalized size = 1.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4 a^2 / (a e^2 + c d^2)^2 / (c x^4 + a) e^3 / c x^3 - 1/4 a / (a e^2 + c d^2)^2 / (c x^4 + a) e x^3 d^2 + 1/4 a^2 / (a e^2 + c d^2)^2 / (c x^4 + a) d^3 x - 1/16 a / (a e^2 + c d^2)^2 / c (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) d e^2 - 5/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) d^3 - 1/32 a / (a e^2 + c d^2)^2 / c (a/c)^{1/4} 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d e^2 - 5/32 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d^3 - 1/16 a / (a e^2 + c d^2)^2 / c (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) d e^2 - 5/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) d^3 + 3/32 a^2 / (a e^2 + c d^2)^2 / c^2 / (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) e^3 + 7/32 a / (a$

$$\begin{aligned}
& 6*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)}*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)} + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)}*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)} + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)}*i - (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14))/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^{(1/2)} - 9*a^3*e^6*(-a*c^7)^{(1/2)} + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^{(1/2)} - 41*a^2*c*d^2*e^4*(-a*c^7)^{(1/2)})/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^{(1/2)}*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4
\end{aligned}$$

$$\begin{aligned}
& d^2 e^6)) * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} - (x * (1920a^8 c^4 d e^{14} + 13184a^2 c^{10} d^{13} e^2 + 16640a^3 c^9 d^{11} e^4 + 18560a^4 c^8 d^9 e^6 + 56832a^5 c^7 d^7 e^8 + 60544a^6 c^6 d^5 e^{10} + 20736a^7 c^5 d^3 e^{12})) / (128(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6))) * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} - (x * (81a^8 e^{13} + 800a^2 c^6 d^{12} e + 612a^7 c^5 d^2 e^{11} + 832a^3 c^5 d^{10} e^3 + 913a^4 c^4 d^8 e^5 + 1700a^5 c^3 d^6 e^7 + 1606a^6 c^2 d^4 e^9)) / (128(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6))) * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} * i) / ((81a^6 d^4 e^8 + 450a^5 c^3 d^6 e^6 + 300a^3 c^3 d^{10} e^2 + 733a^4 c^2 d^8 e^4) / (128(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6)) + (((5120a^2 c^8 d^{13} e + 432a^8 c^2 d e^{13} - 17232a^3 c^7 d^{11} e^3 - 37776a^4 c^6 d^9 e^5 - 13600a^5 c^5 d^7 e^7 + 4320a^6 c^4 d^5 e^9 + 2928a^7 c^3 d^3 e^{11}) / (256(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6)) - (((81920a^5 c^9 d^8 e^8 - 73728a^3 c^{11} d^{12} e^4 - 61440a^4 c^{10} d^{10} e^6 - 20480a^2 c^{12} d^{14} e^2 + 184320a^6 c^8 d^6 e^{10} + 122880a^7 c^7 d^4 e^{12} + 28672a^8 c^6 d^2 e^{14}) / (256(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6)) - (x * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} * (65536a^9 c^7 e^{17} - 65536a^2 c^{14} d^{14} e^3 - 327680a^3 c^{13} d^{12} e^5 - 589824a^4 c^{12} d^{10} e^7 - 327680a^5 c^{11} d^8 e^9 + 327680a^6 c^{10} d^6 e^{11} + 589824a^7 c^9 d^4 e^{13} + 327680a^8 c^8 d^2 e^{15})) / (128(c^7 d^8 + a^4 c^3 e^8 + 4a^3 c^6 d^6 e^2 + 6a^2 c^5 d^4 e^4 + 4a^3 c^4 d^2 e^6))) * ((25c^3 d^6 (-ac^7)^{(1/2)} - 9a^3 e^6 (-ac^7)^{(1/2)} + 6a^3 c^4 d e^5 + 44a^2 c^5 d^3 e^3 + 70a^3 c^6 d^5 e - 39a^2 c^2 d^4 e^2 (-ac^7)^{(1/2)} - 41a^2 c^2 d^2 e^4 (-ac^7)^{(1/2)}) / (256(c^{11} d^8 + a^4 c^7 e^8 + 4a^3 c^10 d^6 e^2 + 6a^2 c^9 d^4 e^4 + 4a^3 c^8 d^2 e^6)))^{(1/2)} + (x * (1920a^8 c^4 d e^{14} + 13184a^2 c^{10} d^{13} e^2 + 16640a^3 c^9 d^{11} e^4 + 18560a^4 c^8 d^9 e^6 + 56832a^5 c^7 d^7 e^8 + 60544a^6 c^6 d^5 e^{10} + 20736a^7 c^5 d^3 e^{12})) / (128(c^7 d^8
\end{aligned}$$

$$\begin{aligned}
& + a^4c^3e^8 + 4a^2c^6d^6e^2 + 6a^2c^5d^4e^4 + 4a^3c^4d^2e^6)) \\
& *((25c^3d^6(-ac^7)^{(1/2)} - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + \\
& 44a^2c^5d^3e^3 + 70a^2c^6d^5e - 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41 \\
& a^2c^2d^2e^4(-ac^7)^{(1/2)})/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 \\
& e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} * ((25c^3d^6(-ac^7)^{(1/2)} \\
& - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + 44a^2c^5d^3e^3 + \\
& 70a^2c^6d^5e - 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41a^2c^2d^2e^4(-ac^7)^{(1/2)}) \\
&)/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} \\
& + (x*(81a^8e^{13} + 800a^2c^6d^{12}e + 612a^7c^2d^2e^{11} + 832a^3c^5d^{10}e^3 + 913a^4c^4d^8e^5 + 1700a^5c^3 \\
& d^6e^7 + 1606a^6c^2d^4e^9))/(128*(c^7d^8 + a^4c^3e^8 + 4a^2c^6d^6e^2 + 6a^2c^5d^4e^4 + 4a^3c^4d^2e^6))) \\
& * ((25c^3d^6(-ac^7)^{(1/2)} - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + 44a^2c^5d^3e^3 + 70a^2c^6d^5e - \\
& 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41a^2c^2d^2e^4(-ac^7)^{(1/2)})/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} \\
& + (((5120a^2c^8d^{13}e + 432a^8c^2d^6e^{13} - 17232a^3c^7d^{11}e^3 - 37776a^4c^6d^9e^5 - 13600a^5c^5d^7e^7 + 4320 \\
& a^6c^4d^5e^9 + 2928a^7c^3d^3e^{11}))/((81920a^5c^9d^8e^8 - 73728a^3c^{11}d^{12}e^4 - 61440a^4c^{10}d^{10}e^6 - 20480a^2c^{12} \\
& d^{14}e^2 + 184320a^6c^8d^6e^{10} + 122880a^7c^7d^4e^{12} + 28672a^8c^6d^2e^{14}))/((256*(c^7d^8 + a^4c^3e^8 + 4a^2c^6d^6e^2 + 6a^2c^5d^4e^4 + 4a^3c^4d^2e^6))) \\
& + (x*((25c^3d^6(-ac^7)^{(1/2)} - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + 44a^2c^5d^3e^3 + 70a^2c^6d^5e - 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41a^2c^2d^2e^4(-ac^7)^{(1/2)}) \\
&)/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} * (65536a^9c^7e^{17} - 65536a^2c^{14}d^{14}e^3 - 327680a^3c^{13}d^{12}e^5 - 589824a^4c^{12}d^{10}e^7 - 327680a^5c^{11}d^8e^9 + 327680a^6c^{10}d^6e^{11} + 589824a^7c^9d^4e^{13} + 327680a^8c^8d^2e^{15}))/((128*(c^7d^8 + a^4c^3e^8 + 4a^2c^6d^6e^2 + 6a^2c^5d^4e^4 + 4a^3c^4d^2e^6))) \\
& * ((25c^3d^6(-ac^7)^{(1/2)} - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + 44a^2c^5d^3e^3 + 70a^2c^6d^5e - 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41a^2c^2d^2e^4(-ac^7)^{(1/2)})/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} - (x*(1920a^8c^4d^5e^{14} + 13184a^2c^{10}d^{13}e^2 + 16640a^3c^9d^{11}e^4 + 18560a^4c^8d^9e^6 + 56832a^5c^7d^7e^8 + 60544a^6c^6d^5e^{10} + 20736a^7c^5d^3e^{12}))/((128*(c^7d^8 + a^4c^3e^8 + 4a^2c^6d^6e^2 + 6a^2c^5d^4e^4 + 4a^3c^4d^2e^6))) * ((25c^3d^6(-ac^7)^{(1/2)} - 9a^3e^6(-ac^7)^{(1/2)} + 6a^3c^4d^5e^5 + 44a^2c^5d^3e^3 + 70a^2c^6d^5e - 39a^2c^2d^4e^2(-ac^7)^{(1/2)} - 41a^2c^2d^2e^4(-ac^7)^{(1/2)})/(256*(c^{11}d^8 + a^4c^7e^8 + 4a^2c^{10}d^6e^2 + 6a^2c^9d^4e^4 + 4a^3c^8d^2e^6)))^{(1/2)} - (x*(81a^8e^{13} + 800
\end{aligned}$$

$$\begin{aligned}
& *a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d \\
& ^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9)/(128*(c^7*d^8 + a^4* \\
& c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c \\
& ^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2 \\
& *c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c* \\
& d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6 \\
& *a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) \\
& - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c \\
& ^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2 \\
&))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4* \\
& a^3*c^8*d^2*e^6)))^(1/2)*2i + atan((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d* \\
& e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e \\
& ^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3* \\
& e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - ((81920* \\
& a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480 \\
& *a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28 \\
& 672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2 \\
& *c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (x*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3 \\
& *d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e \\
& + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256* \\
& (c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8* \\
& d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3 \\
& *c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 32768 \\
& 0*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(\\
& 128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^ \\
& 4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3 \\
& *c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7 \\
&)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4 \\
& *a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920 \\
& *a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560* \\
& a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^ \\
& 7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5* \\
& d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a* \\
& c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c \\
& ^2*d^4*e^2*(-a*c^7)^(1/2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 \\
& + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)) \\
&)^(1/2))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4 \\
& *d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1 \\
& /2) + 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c \\
& ^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e \\
& ^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913* \\
& a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d \\
& ^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6) \\
&)))*((9*a^3*e^6*(-a*c^7)^(1/2) - 25*c^3*d^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 \\
& + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e + 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) +
\end{aligned}$$

$$\begin{aligned}
& e^{12}) / (256 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + \\
& 4 * a^3 * c^4 * d^2 * e^6))) / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (-d^7 * e)^{1/2} \\
& (1/2) / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (-d^7 * e)^{1/2} * i / (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3) + \\
& (((x * (81 * a^8 * e^{13} + 800 * a^2 * c^6 * d^{12} * e + 612 * a^7 * c * d^2 * e^{11} + 832 * a^3 * c^5 * d^{10} * e^3 + 913 * a^4 * c^4 * d^8 * e^5 + 1700 * a^5 * c^3 * d^6 * e^7 + 1606 * a^6 * c^2 * d^4 * e^9))) / (256 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) - \\
& (((20 * a^2 * c^8 * d^{13} * e + (27 * a^8 * c^2 * d * e^{13}) / 16 - (1077 * a^3 * c^7 * d^{11} * e^3) / 16 - (2361 * a^4 * c^6 * d^9 * e^5) / 16 - (425 * a^5 * c^5 * d^7 * e^7) / 8 + (135 * a^6 * c^4 * d^5 * e^9) / 8 + (183 * a^7 * c^3 * d^3 * e^{11}) / 16) / (2 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) - \\
& ((-d^7 * e)^{1/2}) * (((-d^7 * e)^{1/2}) * ((320 * a^5 * c^9 * d^8 * e^8 - 288 * a^3 * c^{11} * d^{12} * e^4 - 240 * a^4 * c^{10} * d^{10} * e^6 - 80 * a^2 * c^{12} * d^{14} * e^2 + 720 * a^6 * c^8 * d^6 * e^{10} + 480 * a^7 * c^7 * d^4 * e^{12} + 112 * a^8 * c^6 * d^2 * e^{14}) / (2 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) + \\
& (x * (-d^7 * e)^{1/2}) * (65536 * a^9 * c^7 * e^{17} - 65536 * a^2 * c^{14} * d^{14} * e^3 - 327680 * a^3 * c^{13} * d^{12} * e^5 - 589824 * a^4 * c^{12} * d^{10} * e^7 - 327680 * a^5 * c^{11} * d^8 * e^9 + 327680 * a^6 * c^{10} * d^6 * e^{11} + 589824 * a^7 * c^9 * d^4 * e^{13} + 327680 * a^8 * c^8 * d^2 * e^{15})) / (512 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) - \\
& (x * (1920 * a^8 * c^4 * d * e^{14} + 13184 * a^2 * c^{10} * d^{13} * e^2 + 16640 * a^3 * c^9 * d^{11} * e^4 + 18560 * a^4 * c^8 * d^9 * e^6 + 56832 * a^5 * c^7 * d^7 * e^8 + 60544 * a^6 * c^6 * d^5 * e^{10} + 20736 * a^7 * c^5 * d^3 * e^{12})) / (256 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (-d^7 * e)^{1/2} / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (-d^7 * e)^{1/2} * i / (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3) / \\
& (((81 * a^6 * d^4 * e^8) / 128 + (225 * a^5 * c * d^6 * e^6) / 64 + (75 * a^3 * c^3 * d^{10} * e^2) / 32 + (733 * a^4 * c^2 * d^8 * e^4) / 128) / (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6) + \\
& (((x * (81 * a^8 * e^{13} + 800 * a^2 * c^6 * d^{12} * e + 612 * a^7 * c * d^2 * e^{11} + 832 * a^3 * c^5 * d^{10} * e^3 + 913 * a^4 * c^4 * d^8 * e^5 + 1700 * a^5 * c^3 * d^6 * e^7 + 1606 * a^6 * c^2 * d^4 * e^9))) / (256 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) + \\
& (((20 * a^2 * c^8 * d^{13} * e + (27 * a^8 * c^2 * d * e^{13}) / 16 - (1077 * a^3 * c^7 * d^{11} * e^3) / 16 - (2361 * a^4 * c^6 * d^9 * e^5) / 16 - (425 * a^5 * c^5 * d^7 * e^7) / 8 + (135 * a^6 * c^4 * d^5 * e^9) / 8 + (183 * a^7 * c^3 * d^3 * e^{11}) / 16) / (2 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) - \\
& ((-d^7 * e)^{1/2}) * (((-d^7 * e)^{1/2}) * ((320 * a^5 * c^9 * d^8 * e^8 - 288 * a^3 * c^{11} * d^{12} * e^4 - 240 * a^4 * c^{10} * d^{10} * e^6 - 80 * a^2 * c^{12} * d^{14} * e^2 + 720 * a^6 * c^8 * d^6 * e^{10} + 480 * a^7 * c^7 * d^4 * e^{12} + 112 * a^8 * c^6 * d^2 * e^{14}) / (2 * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) - \\
& (x * (-d^7 * e)^{1/2}) * (65536 * a^9 * c^7 * e^{17} - 65536 * a^2 * c^{14} * d^{14} * e^3 - 327680 * a^3 * c^{13} * d^{12} * e^5 - 589824 * a^4 * c^{12} * d^{10} * e^7 - 327680 * a^5 * c^{11} * d^8 * e^9 + 327680 * a^6 * c^{10} * d^6 * e^{11} + 589824 * a^7 * c^9 * d^4 * e^{13} + 327680 * a^8 * c^8 * d^2 * e^{15})) / (512 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) * (c^7 * d^8 + a^4 * c^3 * e^8 + 4 * a * c^6 * d^6 * e^2 + 6 * a^2 * c^5 * d^4 * e^4 + 4 * a^3 * c^4 * d^2 * e^6)) / (2 * (a^2 * e^5 + c^2 * d^4 * e + 2 * a * c * d^2 * e^3)) + \\
& (x * (1920 * a^8 * c^4 * d * e^{14} + 13184 * a^2 * c^{10} * d^{13} * e^2 + 16640 * a^3 * c^9 * d^{11} * e^4 + 18560 * a^4 * c^8 * d^9 * e^6 + 56832 * a^5 *
\end{aligned}$$

$$3.183 \quad \int \frac{x^6}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=687

$$\frac{(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} \sqrt[4]{a} c^{5/4} (ae^2 + cd^2)} + \frac{(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} \sqrt[4]{a} c^{5/4} (ae^2 + cd^2)} + \frac{(\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2} \sqrt[4]{a} c^{5/4}}$$

Rubi [A] time = 0.60, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1314, 1276, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$\frac{(\sqrt{a}e + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} \sqrt[4]{a} c^{5/4} (ae^2 + cd^2)} + \frac{(\sqrt{a}e + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} \sqrt[4]{a} c^{5/4} (ae^2 + cd^2)} + \frac{(\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2} \sqrt[4]{a} c^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] $-(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^{(5/2)}*\text{Sqrt}[e]*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(c*d^2 + a*e^2)^2 - (d^2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*(c*d^2 + a*e^2)^2) + ((\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*(c*d^2 + a*e^2)) + (d^2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*(c*d^2 + a*e^2)^2) - ((\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*(c*d^2 + a*e^2)) + (d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*(c*d^2 + a*e^2)^2) - ((\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*(c*d^2 + a*e^2)) - (d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*(c*d^2 + a*e^2)^2) + ((\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(1/4)}*c^{(5/4)}*(c*d^2 + a*e^2))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1276

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
  x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
  x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_.))/((d_.) + (e_.)*(x_)^2),
  x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
  (a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
  (a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
  Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(d+ex^2)(a+cx^4)^2} dx &= -\frac{a \int \frac{x^2(d-ex^2)}{(a+cx^4)^2} dx}{cd^2+ae^2} + \frac{d^2 \int \frac{x^2}{(d+ex^2)(a+cx^4)} dx}{cd^2+ae^2} \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-ae+cdx^2}{a+cx^4} dx}{4c(cd^2+ae^2)} + \frac{d^2 \int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2+ae^2)(a+cx^4)} \right) dx}{cd^2+ae^2} \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2 \int \frac{ae+cdx^2}{a+cx^4} dx}{(cd^2+ae^2)^2} - \frac{(d^3e) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} - \frac{\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{8c(cd^2+ae^2)} \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d^2\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right)\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^2+ae^2)^2} + \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a})}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+a)} \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d - \frac{\sqrt{a}e}{\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}(cd^2+ae^2)} - \\
&= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}d^2\left(d + \frac{\sqrt{a}e}{\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 428, normalized size = 0.62

$$\frac{\sqrt{2}(a^{3/2}e^2+5\sqrt{a}cd^2+ae\sqrt{c}d^2-3c^{3/2}d^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}(a^{3/2}e^2+5\sqrt{a}cd^2+ae\sqrt{c}d^2-3c^{3/2}d^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+2\sqrt{2}(a^{3/2}e^2+5\sqrt{a}cd^2+ae\sqrt{c}d^2+3c^{3/2}d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-2\sqrt{2}(a^{3/2}e^2+5\sqrt{a}cd^2+ae\sqrt{c}d^2+3c^{3/2}d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+8(a^2+cd^2)(ae+cdx^2)+32d^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{32(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] -1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(

$$\begin{aligned}
& (3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) / (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^{12} - 558*a*c^5*d^{10}*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) / (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) * \log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^{11} + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^{10})*\sqrt{-(81*c^6*d^{12} - 558*a*c^5*d^{10}*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) * \sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^{12} - 558*a*c^5*d^{10}*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) / (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^{12} - 558*a*c^5*d^{10}*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) / (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) * \log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^{11} + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^{10})*\sqrt{-(81*c^6*d^{12} - 558*a*c^5*d^{10}*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^{10} + a^6*e^{12}) / (a*c^{13}*d^{16} + 8*a^2*c^{12}*d^{14}*e^2 + 28*a^3*c^{11}*d^{12}*e^4 + 56*a^4*c^{10}*d^{10}*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^{10} + 28*a^7*c^7*d^4*e^{12} + 8*a^8*c^6*d^2*e^{14} + a^9*c^5*e^{16})) / (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))
\end{aligned}$$

$$\begin{aligned}
& \left(5e^{16}\right) \sqrt{\left(-30c^2d^5e - 4aac^3d^3e^3 - 2a^2d^5e - (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)\right) \sqrt{\left(-81c^6d^{12} - 558aac^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^2d^2e^{10} + a^6e^{12}\right) / (ac^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})} / (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)) - (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4 + (c^4d^4 + 2aac^3d^2e^2 + a^2c^2e^4) * x^4) \sqrt{\left(-30c^2d^5e - 4aac^3d^3e^3 - 2a^2d^5e - (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)\right) \sqrt{\left(-81c^6d^{12} - 558aac^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^2d^2e^{10} + a^6e^{12}\right) / (ac^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})} / (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)) * \log\left(-81c^4d^8 - 270aac^3d^6e^2 - 112a^2c^2d^4e^4 - 18a^3c^2d^2e^6 - a^4e^8\right) * x - (45aac^5d^8e - 146a^2c^4d^6e^3 - 76a^3c^3d^4e^5 - 14a^4c^2d^2e^7 - a^5c^2e^9 + (3aac^9d^{11} + 11a^2c^8d^9e^2 + 14a^3c^7d^7e^4 + 6a^4c^6d^5e^6 - a^5c^5d^3e^8 - a^6c^4d^2e^{10}) \sqrt{\left(-81c^6d^{12} - 558aac^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^2d^2e^{10} + a^6e^{12}\right) / (ac^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})} \sqrt{\left(-30c^2d^5e - 4aac^3d^3e^3 - 2a^2d^5e - (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)\right) \sqrt{\left(-81c^6d^{12} - 558aac^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^2d^2e^{10} + a^6e^{12}\right) / (ac^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})} / (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)) - 8(c^2d^2x^4 + acd^2) \sqrt{-d} * \log\left(\frac{ex^2 - 2\sqrt{-d} * x - d}{ex^2 + d}\right) + 4(acd^2e + a^2e^3) * x / (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4 + (c^4d^4 + 2aac^3d^2e^2 + a^2c^2e^4) * x^4), -1/16(4(c^2d^3 + acd^2e^2) * x^3 + 16(c^2d^2 * x^4 + acd^2) \sqrt{d} * \arctan(\sqrt{d} * x / d) + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4 + (c^4d^4 + 2aac^3d^2e^2 + a^2c^2e^4) * x^4) \sqrt{\left(-30c^2d^5e - 4aac^3d^3e^3 - 2a^2d^5e - (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)\right) \sqrt{\left(-81c^6d^{12} - 558aac^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^2d^2e^{10} + a^6e^{12}\right) / (ac^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})} / (c^6d^8 + 4aac^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)) * \log\left(-81c^4d^8 - 270aac^3d^6e^2 - 112a^2c^2d^4e^4 - 18a^3c^2d^2e^6 - a^4e^8\right) *
\end{aligned}$$

$$\begin{aligned}
& x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/((c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/((c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*\log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/((c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*\sqrt{-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*\sqrt{-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)})/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/((c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))
\end{aligned}$$

$$\begin{aligned}
&^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})) / (c^6d^8 + 4a^5c^5d^6e^2 + \\
&6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8) * \log(- (81c^4d^8 - 2 \\
&70a^3c^3d^6e^2 - 112a^2c^2d^4e^4 - 18a^3c^3d^2e^6 - a^4e^8) * x + (4 \\
&5a^5c^5d^8e - 146a^2c^4d^6e^3 - 76a^3c^3d^4e^5 - 14a^4c^2d^2e \\
&^7 - a^5c^5e^9 + (3a^9c^9d^{11} + 11a^2c^8d^9e^2 + 14a^3c^7d^7e^4 + \\
&6a^4c^6d^5e^6 - a^5c^5d^3e^8 - a^6c^4d^4e^{10}) * \sqrt{-(81c^6d^{12} - \\
&558a^5c^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 \\
&+ 18a^5c^3d^2e^{10} + a^6e^{12}) / (a^3c^{13}d^{16} + 8a^2c^{12}d^{14}e^2 \\
&+ 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} \\
&+ 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})) \\
&)* \sqrt{-(30c^2d^5e - 4a^3c^3d^3e^3 - 2a^2d^2e^5 - (c^6d^8 + 4a^5c^5d^6e^2 \\
&+ 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8) * \sqrt{-(81c^6d^{12} - 558a^5c^5d^{10}e^2 \\
&+ 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^3d^2e^{10} \\
&+ a^6e^{12}) / (a^3c^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 \\
&+ 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})) \\
&)) / (c^6d^8 + 4a^5c^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8))) - (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^3e^4 + (c^4d^4 + \\
&2a^3c^3d^2e^2 + a^2c^2e^4) * x^4) * \sqrt{-(30c^2d^5e - 4a^3c^3d^3e^3 - \\
&2a^2d^2e^5 - (c^6d^8 + 4a^5c^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8) * \sqrt{-(81c^6d^{12} - 558a^5c^5d^{10}e^2 \\
&+ 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^3d^2e^{10} + a^6e^{12}) / (a^3c^{13}d^{16} + 8a^2c^{12}d^{14}e^2 \\
&+ 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})) \\
&)) / (c^6d^8 + 4a^5c^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8)) * \log(- (81c^4d^8 - 270a^3c^3d^6e^2 - 112a^2c^2d^4e^4 - 18a^3c^3d^2e^6 - a^4e^8) * x - (45a^5c^5d^8e - 146a^2c^4d^6e^3 - 76a^3c^3d^4e^5 - 14a^4c^2d^2e^7 - a^5c^5e^9 + (3a^9c^9d^{11} + 11a^2c^8d^9e^2 + 14a^3c^7d^7e^4 + 6a^4c^6d^5e^6 - a^5c^5d^3e^8 - a^6c^4d^4e^{10}) * \sqrt{-(81c^6d^{12} - 558a^5c^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^3d^2e^{10} + a^6e^{12}) / (a^3c^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16})) * \sqrt{-(30c^2d^5e - 4a^3c^3d^3e^3 - 2a^2d^2e^5 - (c^6d^8 + 4a^5c^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8) * \sqrt{-(81c^6d^{12} - 558a^5c^5d^{10}e^2 + 799a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 143a^4c^2d^4e^8 + 18a^5c^3d^2e^{10} + a^6e^{12}) / (a^3c^{13}d^{16} + 8a^2c^{12}d^{14}e^2 + 28a^3c^{11}d^{12}e^4 + 56a^4c^{10}d^{10}e^6 + 70a^5c^9d^8e^8 + 56a^6c^8d^6e^{10} + 28a^7c^7d^4e^{12} + 8a^8c^6d^2e^{14} + a^9c^5e^{16}))} \\
&)) / (c^6d^8 + 4a^5c^5d^6e^2 + 6a^2c^4d^4e^4 + 4a^3c^3d^2e^6 + a^4c^2e^8))) + 4 * (a^3c^3d^2e + a^2e^3) * x) / (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^3e^4 + (c^4d^4 + 2a^3c^3d^2e^2 + a^2c^2e^4) * x^4)]
\end{aligned}$$

giac [A] time = 0.59, size = 595, normalized size = 0.87

$$\frac{d^2 \arctan\left(\frac{x}{\sqrt{d}}\right)}{dx^2} \frac{\left(5(a^2)^2 a^2 d^2 + 3(a^2)^2 d^2 + (a^2)^2 d^2 - (a^2)^2 d^2\right) \arctan\left(\frac{\sqrt{d} - \sqrt{d} \sqrt{d}}{2 \sqrt{d}}\right)}{8(\sqrt{d} a^2 d^2 + 2 \sqrt{d} a^2 d^2 + \sqrt{d} a^2 d^2)} + \frac{\left(5(a^2)^2 a^2 d^2 + 3(a^2)^2 d^2 + (a^2)^2 d^2 - (a^2)^2 d^2\right) \arctan\left(\frac{\sqrt{d} - \sqrt{d} \sqrt{d}}{2 \sqrt{d}}\right)}{8(\sqrt{d} a^2 d^2 + 2 \sqrt{d} a^2 d^2 + \sqrt{d} a^2 d^2)} + \frac{\left(5(a^2)^2 a^2 d^2 - 3(a^2)^2 d^2 + (a^2)^2 d^2 + (a^2)^2 d^2\right) \log\left(x^2 + \sqrt{d} \sqrt{d} + \sqrt{d}\right)}{16(\sqrt{d} a^2 d^2 + 2 \sqrt{d} a^2 d^2 + \sqrt{d} a^2 d^2)} + \frac{\left(5(a^2)^2 a^2 d^2 - 3(a^2)^2 d^2 + (a^2)^2 d^2 + (a^2)^2 d^2\right) \log\left(x^2 - \sqrt{d} \sqrt{d} + \sqrt{d}\right)}{16(\sqrt{d} a^2 d^2 + 2 \sqrt{d} a^2 d^2 + \sqrt{d} a^2 d^2)} + \frac{a d^2 + a d^2}{4(a^2 + d)(c^2 d^2 + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $-d^{5/2} \arctan(xe^{1/2}/\sqrt{d}) e^{1/2} / (c^2 d^4 + 2 a c^2 d^2 e^2 + a^2 e^4) + 1/8 (5 (a c^3)^{1/4} a c^2 d^2 e + 3 (a c^3)^{3/4} c d^3 + (a c^3)^{1/4} a^2 c^2 e^3 - (a c^3)^{3/4} a d e^2) \arctan(1/2 \sqrt{2} (2 x + \sqrt{2}) (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) + 1/8 (5 (a c^3)^{1/4} a c^2 d^2 e + 3 (a c^3)^{3/4} c d^3 + (a c^3)^{1/4} a^2 c^2 e^3 - (a c^3)^{3/4} a d e^2) \arctan(1/2 \sqrt{2} (2 x - \sqrt{2}) (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) + 1/16 (5 (a c^3)^{1/4} a c^2 d^2 e - 3 (a c^3)^{3/4} c d^3 + (a c^3)^{1/4} a^2 c^2 e^3 + (a c^3)^{3/4} a d e^2) \log(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{2} a/c) / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) - 1/16 (5 (a c^3)^{1/4} a c^2 d^2 e - 3 (a c^3)^{3/4} c d^3 + (a c^3)^{1/4} a^2 c^2 e^3 + (a c^3)^{3/4} a d e^2) \log(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{2} a/c) / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) - 1/4 (c d x^3 + a x e) / ((c x^4 + a) (c^2 d^2 + a c e^2))$

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x^3 a d e^2 - 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x^3 c d^3 - 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) a^2 e^3 / c x - 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) e a d^2 x + 1/16 / (a e^2 + c d^2)^2 / c (a/c)^{1/4} a^2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) e^3 + 5/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) d^2 e + 1/32 / (a e^2 + c d^2)^2 / c (a/c)^{1/4} a^2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) e^3 + 5/32 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d^2 e + 1/16 / (a e^2 + c d^2)^2 / c (a/c)^{1/4} a^2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) e^3 + 5/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x + 1) d^2 e - 1/32 / (a e^2 + c d^2)^2 / c (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) a d e^2 + 3/32 / (a e^2 + c d^2)^2 / (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d^3 - 1/16 / (a e^2 + c d^2)^2 / c (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4} x - 1) a d e^2 + 3/16 / (a e^2 + c d^2)^2 / (a/c)^{1/4}$

$$\begin{aligned}
& 9 + 327680*a^6*c^8*d^6*e^{11} + 589824*a^7*c^7*d^4*e^{13} + 327680*a^8*c^6*d^2* \\
& e^{15})/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4* \\
& a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} - 2 \\
& *a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a \\
& c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c^5*e^8 \\
& + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} + (x*(\\
& 1152*a*c^9*d^{13}*e^2 + 1152*a^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472* \\
& a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c \\
& ^4*d^3*e^{12}))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e \\
& ^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1 \\
& /2)} - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e \\
& ^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c \\
& ^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} \\
&)*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} - 2*a^3*c^3*d*e^5 - \\
& 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^ \\
& 2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e \\
& ^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} - (x*(a^6*e^{13} - 288*a* \\
& c^5*d^{10}*e^3 + 20*a^5*c*d^2*e^{11} + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 \\
& + 118*a^4*c^2*d^4*e^9))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^ \\
& 2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6* \\
& (-a*c^5)^{(1/2)} - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31* \\
& a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9* \\
& d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e \\
& ^6))^{(1/2)}*1i - (((432*a*c^7*d^{12}*e^2 + 13040*a^2*c^6*d^{10}*e^4 + 12000*a^3 \\
& *c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^{10} + 48*a^6*c^2*d^2 \\
& *e^{12}))/((256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4* \\
& a^3*c^2*d^2*e^6)) + (((45056*a^2*c^{10}*d^{13}*e^3 - 4096*a^8*c^4*d*e^{15} + 2211 \\
& 84*a^3*c^9*d^{11}*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184 \\
& 320*a^6*c^6*d^5*e^{11} + 24576*a^7*c^5*d^3*e^{13}))/((256*(c^5*d^8 + a^4*c*e^8 + \\
& 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(\\
& -a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3* \\
& e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(- \\
& a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7 \\
& *d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}*(65536*a^9*c^5*e^{17} - 65536*a^2*c^{12}* \\
& d^{14}*e^3 - 327680*a^3*c^{11}*d^{12}*e^5 - 589824*a^4*c^{10}*d^{10}*e^7 - 327680*a^5 \\
& *c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^{11} + 589824*a^7*c^7*d^4*e^{13} + 327680*a \\
& ^8*c^6*d^2*e^{15}))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d \\
& ^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5 \\
&)^{(1/2)} - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d \\
& ^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a \\
& ^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(\\
& 1/2)} - (x*(1152*a*c^9*d^{13}*e^2 + 1152*a^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}* \\
& ^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + \\
& 4864*a^6*c^4*d^3*e^{12}))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^ \\
& 2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*
\end{aligned}$$

$$\begin{aligned}
& (-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2} / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} \\
& * (-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} \\
& + (x(a^6e^{13} - 288a^5c^5d^10e^3 + 20a^5c^5d^2e^{11} + 17a^2c^4d^8e^5 + 148a^3c^3d^6e^7 + 118a^4c^2d^4e^9)) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) * (-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} * i) / (((432a^7c^7d^12e^2 + 13040a^2c^6d^10e^4 + 12000a^3c^5d^8e^6 - 1056a^4c^4d^6e^8 - 400a^5c^3d^4e^10 + 48a^6c^2d^2e^12) / (256(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) + ((45056a^2c^10d^13e^3 - 4096a^8c^4d^8e^15 + 221184a^3c^9d^11e^5 + 430080a^4c^8d^9e^7 + 409600a^5c^7d^7e^9 + 184320a^6c^6d^5e^11 + 24576a^7c^5d^3e^13) / (256(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) - (x(-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} * (65536a^9c^5e^17 - 65536a^2c^12d^14e^3 - 327680a^3c^11d^12e^5 - 589824a^4c^10d^10e^7 - 327680a^5c^9d^8e^9 + 327680a^6c^8d^6e^11 + 589824a^7c^7d^4e^13 + 327680a^8c^6d^2e^15)) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))) * (-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} + (x(1152a^9c^9d^13e^2 + 1152a^7c^3d^8e^14 + 21248a^2c^8d^11e^4 + 25472a^3c^7d^9e^6 - 5632a^4c^6d^7e^8 - 7296a^5c^5d^5e^10 + 4864a^6c^4d^3e^12)) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) * (-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2} - (x(a^6e^{13} - 288a^5c^5d^10e^3 + 20a^5c^5d^2e^{11} + 17a^2c^4d^8e^5 + 148a^3c^3d^6e^7 + 118a^4c^2d^4e^9)) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) * (-a^3e^6(-a^5)^{1/2} - 9c^3d^6(-a^5)^{1/2} - 2a^3c^3d^3e^5 - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-a^5)^{1/2} + 9a^2c^2d^2e^4(-a^5)^{1/2}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& ^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 \\
& + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} + (((432a^7c^7d^12e^2 + 13040a^2c^6d^10e^4 + 12000a^3c^5d^8e^6 - 1056a^4c^4d^6e^8 - 400a^5c^3d^4e^{10} + 48a^6c^2d^2e^{12}) / (256(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) + (((45056a^2c^10d^13e^3 - 4096a^8c^4d^8e^{15} + 221184a^3c^9d^11e^5 + 430080a^4c^8d^9e^7 + 409600a^5c^7d^7e^9 + 184320a^6c^6d^5e^{11} + 24576a^7c^5d^3e^{13}) / (256(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6)) + (x*(-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} * (65536a^9c^5e^{17} - 65536a^2c^{12}d^{14}e^3 - 327680a^3c^{11}d^{12}e^5 - 589824a^4c^{10}d^{10}e^7 - 327680a^5c^9d^8e^9 + 327680a^6c^8d^6e^{11} + 589824a^7c^7d^4e^{13} + 327680a^8c^6d^2e^{15})) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))) * (-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} - (x*(1152a^9c^9d^{13}e^2 + 1152a^7c^3d^3e^{14} + 21248a^2c^8d^{11}e^4 + 25472a^3c^7d^9e^6 - 5632a^4c^6d^7e^8 - 7296a^5c^5d^5e^{10} + 4864a^6c^4d^3e^{12})) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))) * (-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} * (-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} + (x*(a^6e^{13} - 288a^5c^5d^{10}e^3 + 20a^5c^5d^2e^{11} + 17a^2c^4d^8e^5 + 148a^3c^3d^6e^7 + 118a^4c^2d^4e^9)) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))) * (-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} + (a^4d^3e^9 + 108a^3c^3d^9e^3 + 18a^3c^3d^5e^7 + 93a^2c^2d^7e^5) / (128(c^5d^8 + a^4c^8e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))) * (-a^3e^6(-ac^5)^{(1/2)} - 9c^3d^6(-ac^5)^{(1/2)} - 2a^3c^3d^5e - 4a^2c^4d^3e^3 + 30a^5c^5d^5e + 31a^2c^2d^4e^2(-ac^5)^{(1/2)} + 9a^2c^2d^2e^4(-ac^5)^{(1/2)}) / (256(a^9c^9d^8 + a^5c^5e^8 + 4a^2c^8d^6e^2 + 6a^3c^7d^4e^4 + 4a^4c^6d^2e^6))^{(1/2)} * 2i - ((dx^3) / (4*(ae^2 + cd^2))) + (aex) /
\end{aligned}$$

$$\begin{aligned}
& (4*c*(a*e^2 + c*d^2)))/(a + c*x^4) + \operatorname{atan}\left(\frac{((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^{10} + 48*a^6*c^2*d^2*e^{12})/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^{10}*d^{13}*e^3 - 4096*a^8*c^4*d*e^{15} + 221184*a^3*c^9*d^{11}*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^{11} + 24576*a^7*c^5*d^3*e^{13})/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}*(65536*a^9*c^5*e^{17} - 65536*a^2*c^{12}*d^{14}*e^3 - 327680*a^3*c^{11}*d^{12}*e^5 - 589824*a^4*c^{10}*d^{10}*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^{11} + 589824*a^7*c^7*d^4*e^{13} + 327680*a^8*c^6*d^2*e^{15})/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} + (x*(1152*a*c^9*d^{13}*e^2 + 1152*a^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c^4*d^3*e^{12}))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} + (x*(1152*a*c^9*d^{13}*e^2 + 1152*a^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c^4*d^3*e^{12}))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} - (x*(a^6*e^{13} - 288*a*c^5*d^{10}*e^3 + 20*a^5*c*d^2*e^{11} + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/((128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^{10} + 48*a^6*c^2*d^2*e^{12})/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^{10}*d^{13}*e^3 - 4096*a^8*c^4*d*e^{15} + 221184*a^3*c^9*d^{11}*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^{11} + 24576*a^7*c^5*d^3*e^{13})/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)}))/(256*(a*c^9*d^8 + a^5*c^5*e^8
\end{aligned}$$

$$\begin{aligned}
& + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}*(65536 \\
& *a^9*c^5*e^{17} - 65536*a^2*c^{12}*d^{14}*e^3 - 327680*a^3*c^{11}*d^{12}*e^5 - 589824 \\
& *a^4*c^{10}*d^{10}*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^{11} + 589 \\
& 824*a^7*c^7*d^4*e^{13} + 327680*a^8*c^6*d^2*e^{15}))/((128*(c^5*d^8 + a^4*c*e^8 \\
& + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^6*(-a* \\
& c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 \\
& - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c \\
& ^5)^{(1/2)))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^ \\
& 4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} - (x*(1152*a*c^9*d^{13}*e^2 + 1152*a^7*c^3 \\
& *d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7 \\
& *e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c^4*d^3*e^{12}))/((128*(c^5*d^8 + a^4* \\
& c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^ \\
& 6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d \\
& ^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4 \\
& *(-a*c^5)^{(1/2)))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3* \\
& c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)})*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3* \\
& d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + \\
& 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)))/(256*(a* \\
& c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d \\
& ^2*e^6))^{(1/2)} + (x*(a^6*e^{13} - 288*a*c^5*d^{10}*e^3 + 20*a^5*c*d^2*e^{11} + 1 \\
& 7*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/((128*(c^5*d \\
& ^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6))) \\
& *((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4* \\
& a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2* \\
& c*d^2*e^4*(-a*c^5)^{(1/2)))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 \\
& + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}*1i)/((((432*a*c^7*d^{12}*e^ \\
& 2 + 13040*a^2*c^6*d^{10}*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - \\
& 400*a^5*c^3*d^4*e^{10} + 48*a^6*c^2*d^2*e^{12}))/((256*(c^5*d^8 + a^4*c*e^8 + 4* \\
& a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^{10} \\
& *d^{13}*e^3 - 4096*a^8*c^4*d*e^{15} + 221184*a^3*c^9*d^{11}*e^5 + 430080*a^4*c^8* \\
& d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^{11} + 24576*a^7*c^5* \\
& d^3*e^{13}))/((256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + \\
& 4*a^3*c^2*d^2*e^6)) - (x*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/ \\
& 2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^ \\
& 2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)))/(256*(a*c^9*d^8 + a^5*c^ \\
& 5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)}* \\
& (65536*a^9*c^5*e^{17} - 65536*a^2*c^{12}*d^{14}*e^3 - 327680*a^3*c^{11}*d^{12}*e^5 - \\
& 589824*a^4*c^{10}*d^{10}*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^{11} \\
& + 589824*a^7*c^7*d^4*e^{13} + 327680*a^8*c^6*d^2*e^{15}))/((128*(c^5*d^8 + a^4* \\
& c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((a^3*e^ \\
& 6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d \\
& ^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4 \\
& *(-a*c^5)^{(1/2)))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3* \\
& c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6))^{(1/2)} + (x*(1152*a*c^9*d^{13}*e^2 + 1152*a \\
& ^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c
\end{aligned}$$

$$\begin{aligned}
& \left(6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c^4*d^3*e^{12} \right) / \left(128*(c^5*d^8 \right. \\
& + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6) \left. \right) * \left(\left(\right. \right. \\
& a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2 \\
& *c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d \\
& ^2*e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + \right. \\
& 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} * \left(\left(a^3*e^6*(-a*c^5)^{(1/2)} - \right. \right. \\
& 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d \\
& ^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(2 \right. \\
& 56*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4 \\
& *c^6*d^2*e^6) \left. \right)^{(1/2)} - \left(x*(a^6*e^{13} - 288*a*c^5*d^{10}*e^3 + 20*a^5*c*d^2*e^{11} \right. \\
& + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9) \left. \right) / \left(128* \right. \\
& (c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2* \\
& e^6) \left. \right) * \left(\left(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 \right. \right. \\
& + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + \\
& 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 \right. \\
& + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} + \left(\left((432*a*c^7*d^{12} \right. \right. \\
& *e^2 + 13040*a^2*c^6*d^{10}*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 \\
& - 400*a^5*c^3*d^4*e^{10} + 48*a^6*c^2*d^2*e^{12}) \left. \right) / \left(256*(c^5*d^8 + a^4*c*e^8 + \right. \\
& 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6) \left. \right) + \left(\left((45056*a^2*c \right. \right. \\
& ^{10}*d^{13}*e^3 - 4096*a^8*c^4*d*e^{15} + 221184*a^3*c^9*d^{11}*e^5 + 430080*a^4*c^8 \\
& *d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^{11} + 24576*a^7*c^5 \\
& *d^3*e^{13}) \left. \right) / \left(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 \right. \\
& + 4*a^3*c^2*d^2*e^6) \left. \right) + \left(x*\left(\left(a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} \right. \right. \right. \\
& + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4 \\
& *e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(256*(a*c^9*d^8 + a^5 \right. \\
& *c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} * \left(65536*a^9*c^5*e^{17} \right. \\
& - 65536*a^2*c^{12}*d^{14}*e^3 - 327680*a^3*c^{11}*d^{12}*e^5 \\
& - 589824*a^4*c^{10}*d^{10}*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e \\
& ^{11} + 589824*a^7*c^7*d^4*e^{13} + 327680*a^8*c^6*d^2*e^{15} \left. \right) / \left(128*(c^5*d^8 + a \right. \\
& ^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6) \left. \right) * \left(\left(a^3 \right. \right. \\
& *e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4 \\
& *d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2* \\
& e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a \right. \\
& ^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} - \left(x*\left(1152*a*c^9*d^{13}*e^2 + 115 \right. \right. \\
& 2*a^7*c^3*d*e^{14} + 21248*a^2*c^8*d^{11}*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4 \\
& *c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^{10} + 4864*a^6*c^4*d^3*e^{12}) \left. \right) / \left(128*(c^5*d \right. \\
& ^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6) \left. \right) * \left(\left(a^3 \right. \right. \\
& *e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4* \\
& a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c \\
& *d^2*e^4*(-a*c^5)^{(1/2)} \left. \right) / \left(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 \right. \\
& + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} * \left(\left(a^3*e^6*(-a*c^5)^{(1/2)} \right. \right. \\
& - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5 \\
& *d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)} \left. \right) \\
& / \left(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4* \right. \\
& a^4*c^6*d^2*e^6) \left. \right)^{(1/2)} + \left(x*(a^6*e^{13} - 288*a*c^5*d^{10}*e^3 + 20*a^5*c*d^2 \right.
\end{aligned}$$

$$\begin{aligned}
& *e^{11} + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9)/(1 \\
& 28*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d \\
& ^2*e^6)))*((a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d \\
& *e^5 + 4*a^2*c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} \\
& + 9*a^2*c*d^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^ \\
& 8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^{(1/2)} + (a^4*d^3*e^9 + \\
& 108*a*c^3*d^9*e^3 + 18*a^3*c*d^5*e^7 + 93*a^2*c^2*d^7*e^5)/(128*(c^5*d^8 + \\
& a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*((\\
& a^3*e^6*(-a*c^5)^{(1/2)} - 9*c^3*d^6*(-a*c^5)^{(1/2)} + 2*a^3*c^3*d*e^5 + 4*a^2 \\
& *c^4*d^3*e^3 - 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^{(1/2)} + 9*a^2*c*d \\
& ^2*e^4*(-a*c^5)^{(1/2)})/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + \\
& 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^{(1/2)}*2i - (\operatorname{atan}(-(((-d^5*e)^{(1/2)} \\
& *(((d^5*e)^{(1/2)}*((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (3 \\
& 75*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + \\
& (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2 \\
& *c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c \\
& ^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d \\
& ^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^ \\
& 4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((17 \\
& 6*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c \\
& ^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e \\
& ^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3* \\
& c^2*d^2*e^6)) - (x*(-d^5*e)^{(1/2)}*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14 \\
& *e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9 \\
& *d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c \\
& ^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 \\
& + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^{(1/2} \\
&))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^{(1/2)})/(2*(a^2*e^4 + c \\
& ^2*d^4 + 2*a*c*d^2*e^2)))/((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 2547 \\
& 2*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6 \\
& *c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4 \\
& *e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + \\
& 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a \\
& ^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^ \\
& 6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5*e)^{(1/2)}*(65536* \\
& a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824* \\
& a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 5898
\end{aligned}$$

$$\begin{aligned}
& (24*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2 \\
& *a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + \\
& 4*a^3*c^2*d^2*e^6)))*(-d^5*e)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) \\
&))*(-d^5*e)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/((2*(a^2*e^4 + \\
& c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^ \\
& 2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9)))/(\\
& 256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2* \\
& d^2*e^6))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(((a^4*d^3*e^9)/128 + (\\
& 27*a*c^3*d^9*e^3)/32 + (9*a^3*c*d^5*e^7)/64 + (93*a^2*c^2*d^7*e^5)/128)/(c^ \\
& 5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6 \\
&) + ((-d^5*e)^{(1/2)}*(((d^5*e)^{(1/2)}*(((27*a*c^7*d^12*e^2)/16 + (815*a^2*c^ \\
& 6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (33*a^4*c^4*d^6*e^8)/8 - (25*a^5 \\
& *c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a* \\
& c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((x*(1152*a*c^9*d^ \\
& 13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e \\
& ^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12)) \\
& /((256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^ \\
& 2*d^2*e^6)) + (((176*a^2*c^10*d^13*e^3 - 16*a^8*c^4*d*e^15 + 864*a^3*c^9*d^ \\
& 11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7*d^7*e^9 + 720*a^6*c^6*d^5*e^11 \\
& + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c \\
& ^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-d^5*e)^{(1/2)}*(65536*a^9*c^5*e^17 - \\
& 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e \\
& ^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4* \\
& e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(\\
& c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e \\
& ^6)))*(-d^5*e)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^5*e)^{(1/ \\
& 2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/((2*(a^2*e^4 + c^2*d^4 + 2*a*c \\
& *d^2*e^2)) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2 \\
& *c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(256*(c^5*d^8 + \\
& a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))/((a^ \\
& 2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((-d^5*e)^{(1/2)}*(((d^5*e)^{(1/2)}*(((27*a \\
& *c^7*d^12*e^2)/16 + (815*a^2*c^6*d^10*e^4)/16 + (375*a^3*c^5*d^8*e^6)/8 - (\\
& 33*a^4*c^4*d^6*e^8)/8 - (25*a^5*c^3*d^4*e^10)/16 + (3*a^6*c^2*d^2*e^12)/16) \\
& /((2*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2* \\
& d^2*e^6)) - (((x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8 \\
& *d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5 \\
& *e^10 + 4864*a^6*c^4*d^3*e^12))/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 \\
& + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (((176*a^2*c^10*d^13*e^3 - 16* \\
& a^8*c^4*d*e^15 + 864*a^3*c^9*d^11*e^5 + 1680*a^4*c^8*d^9*e^7 + 1600*a^5*c^7 \\
& *d^7*e^9 + 720*a^6*c^6*d^5*e^11 + 96*a^7*c^5*d^3*e^13)/(2*(c^5*d^8 + a^4*c* \\
& e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-d^5* \\
& e)^{(1/2)}*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^ \\
& 12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8 \\
& *d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(512*(a^2*e \\
& ^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^
\end{aligned}$$

$$\frac{(2c^3d^4e^4 + 4a^3c^2d^2e^6)(-d^5e)^{1/2}}{(2(a^2e^4 + c^2d^4 + 2ac^2d^2e^2))(-d^5e)^{1/2}} \cdot \frac{1}{(2(a^2e^4 + c^2d^4 + 2ac^2d^2e^2)) + (x(a^6e^{13} - 288ac^5d^{10}e^3 + 20a^5c^2d^2e^{11} + 17a^2c^4d^8e^5 + 148a^3c^3d^6e^7 + 118a^4c^2d^4e^9)) / (256(c^5d^8 + a^4c^2e^8 + 4a^3c^4d^6e^2 + 6a^2c^3d^4e^4 + 4a^3c^2d^2e^6))} \cdot \frac{1}{(a^2e^4 + c^2d^4 + 2ac^2d^2e^2)} (-d^5e)^{1/2} i$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.184 \quad \int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=685

$$\frac{(\sqrt{a}e + 3\sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} c^{3/4} (ae^2 + cd^2)} - \frac{(\sqrt{a}e + 3\sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} c^{3/4} (ae^2 + cd^2)} + \frac{(3\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2}}$$

Rubi [A] time = 0.61, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1314, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{(\sqrt{e} + \sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} c^{3/4} (ae^2 + cd^2)} - \frac{(\sqrt{e} + \sqrt{c}d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{16\sqrt{2} a^{3/4} c^{3/4} (ae^2 + cd^2)} + \frac{(3\sqrt{c}d - \sqrt{a}e)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] $-(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^{(3/2)}*e^{(3/2)}*\text{ArcTan}[\text{Sqrt}[e]*x/\text{Sqrt}[d]])/(c*d^2 + a*e^2)^2 - (c^{(1/4)}*d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^2 + a*e^2)^2) + ((3*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*c^{(3/4)}*(c*d^2 + a*e^2)) + (c^{(1/4)}*d^2*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^2 + a*e^2)^2) - ((3*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(3/4)}*c^{(3/4)}*(c*d^2 + a*e^2)) - (c^{(1/4)}*d^2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^2 + a*e^2)^2) + ((3*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*c^{(3/4)}*(c*d^2 + a*e^2)) + (c^{(1/4)}*d^2*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^2 + a*e^2)^2) - ((3*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(3/4)}*c^{(3/4)}*(c*d^2 + a*e^2))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
```



```
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1314

```
Int[(((f_.)*(x_)^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} dx &= -\frac{a \int \frac{d-ex^2}{(a+cx^4)^2} dx}{cd^2+ae^2} + \frac{d^2 \int \frac{1}{(d+ex^2)(a+cx^4)} dx}{cd^2+ae^2} \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{\int \frac{-3d+ex^2}{a+cx^4} dx}{4(cd^2+ae^2)} + \frac{d^2 \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx}{cd^2+ae^2} \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{(cd^2) \int \frac{d-ex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{(d^2e^2) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} - \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}} - e \right) \int \frac{\sqrt{a}\sqrt{cx^2}}{a+cx^4} dx}{8c(cd^2+ae^2)} \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right)\right) \int \frac{\sqrt{a}\sqrt{cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)^2} + \dots \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(3\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2} \sqrt[4]{ax})}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(3\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{cx}}{\sqrt{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} \\
&= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}d^2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{cx}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 423, normalized size = 0.62

$$\frac{\sqrt{2}(a^{3/2}e^2-3\sqrt{a}cd^2+3a\sqrt{c}d^2-c^2d^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{c}x^2)}{a^{3/4}c^{3/4}} + \frac{\sqrt{2}(-a^{3/2}e^2+3\sqrt{a}cd^2-3a\sqrt{c}d^2+c^2d^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx}+\sqrt{c}x^2)}{a^{3/4}c^{3/4}} - \frac{2\sqrt{2}(a^{3/2}e^2-3\sqrt{a}cd^2-3a\sqrt{c}d^2+c^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right)}{a^{3/4}c^{3/4}} + \frac{2\sqrt{2}(a^{3/2}e^2-3\sqrt{a}cd^2-3a\sqrt{c}d^2+c^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}+1\right)}{a^{3/4}c^{3/4}} + \frac{8(c^2d-d)(ae^2+cd^2)}{a+cx^4} + \frac{32d^3e^{3/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{32(a^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(

$\frac{3}{4}c^{3/4}) + (2\sqrt{2}*(c^{3/2}*d^3 - 3\sqrt{a}*c*d^2*e - 3*a*\sqrt{c}*d*e^2 + a^{3/2}*e^3)*\text{ArcTan}[1 + (\sqrt{2}*c^{1/4}*x)/a^{1/4}])/(a^{3/4}*c^{3/4}) + (\sqrt{2}*(-(c^{3/2}*d^3) - 3\sqrt{a}*c*d^2*e + 3*a*\sqrt{c}*d*e^2 + a^{3/2}*e^3)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/(a^{3/4}) * c^{3/4}) + (\sqrt{2}*(c^{3/2}*d^3 + 3\sqrt{a}*c*d^2*e - 3*a*\sqrt{c}*d*e^2 - a^{3/2}*e^3)*\text{Log}[\sqrt{a} + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/(a^{3/4}*c^{3/4}))/ (32*(c*d^2 + a*e^2)^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 24.28, size = 9678, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] $[1/16*(4*(c*d^2*e + a*e^3)*x^3 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*\text{sqrt}((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*\text{sqrt}(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12))/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*\text{log}(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*\text{sqrt}(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12))/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*\text{sqrt}((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*\text{sqrt}(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c$

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $d^{3/2} \arctan(x e^{1/2} / \sqrt{d}) e^{3/2} / (c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4) + 1/8 * ((a c^3)^{1/4} c^3 d^3 - 3 (a c^3)^{1/4} a c^2 d e^2 - 3 (a c^3)^{3/4} c d^2 e + (a c^3)^{3/4} a e^3) \arctan(1/2 \sqrt{2} (2 x + \sqrt{2}) (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) + 1/8 * ((a c^3)^{1/4} c^3 d^3 - 3 (a c^3)^{1/4} a c^2 d e^2 - 3 (a c^3)^{3/4} c d^2 e + (a c^3)^{3/4} a e^3) \arctan(1/2 \sqrt{2} (2 x - \sqrt{2}) (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) + 1/16 * ((a c^3)^{1/4} c^3 d^3 - 3 (a c^3)^{1/4} a c^2 d e^2 + 3 (a c^3)^{3/4} c d^2 e - (a c^3)^{3/4} a e^3) \log(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) - 1/16 * ((a c^3)^{1/4} c^3 d^3 - 3 (a c^3)^{1/4} a c^2 d e^2 + 3 (a c^3)^{3/4} c d^2 e - (a c^3)^{3/4} a e^3) \log(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (\sqrt{2} a c^5 d^4 + 2 \sqrt{2} a^2 c^4 d^2 e^2 + \sqrt{2} a^3 c^3 e^4) + 1/4 * (x^3 e - d x) / ((c x^4 + a) (c d^2 + a e^2))$

maple [A] time = 0.02, size = 848, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x^3 e^3 a + 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x^3 e c d^2 - 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x a d e^2 - 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) x c d^3 - 3/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x - 1) d e^2 + 1/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x - 1) c d^3 - 3/32 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d e^2 + 1/32 / (a e^2 + c d^2)^2 (a/c)^{1/4} / a 2^{1/2} \ln((x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) c d^3 - 3/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x + 1) d e^2 + 1/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x + 1) c d^3 + 1/32 / (a e^2 + c d^2)^2 c / (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) a e^3 - 3/32 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \ln((x^2 - (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} 2^{1/2} x + (a/c)^{1/2})) d^2 e + 1/16 / (a e^2 + c d^2)^2 c / (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x - 1) a e^3 - 3/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x - 1) d^2 e + 1/16 / (a e^2 + c d^2)^2 c / (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x + 1) a e^3 - 3/16 / (a e^2 + c d^2)^2 (a/c)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/c)^{1/4}) x + 1) d^2 e + e^2 / (a e^2 + c d^2)^2 d^2 / (d e)^{1/2} \arctan(1 / (d e)^{1/2} e x)$

maxima [A] time = 2.09, size = 490, normalized size = 0.72

$$\frac{d^2 e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2 d^2 + 2acd^2 + a^2 e^2)\sqrt{de}} + \frac{(cd^2 + ad^2)^2 - (cd^2 + ad^2)^2}{4(ad^2 d^2 + 2a^2 cd^2 + a^2 e^2 + (c^2 d^2 + 2acd^2 + a^2 e^2)d^2)} + \frac{2\sqrt{\frac{d^2 d^2 - 3\sqrt{cd^2 - 3a}\sqrt{ad^2 + d^2 d^2}}{2\sqrt{de}}}}{\sqrt{de}\sqrt{de}\sqrt{de}} + \frac{2\sqrt{\frac{d^2 d^2 - 3\sqrt{cd^2 - 3a}\sqrt{ad^2 + d^2 d^2}}{2\sqrt{de}}}}{\sqrt{de}\sqrt{de}\sqrt{de}} + \frac{\sqrt{\frac{d^2 d^2 - 3\sqrt{cd^2 - 3a}\sqrt{ad^2 + d^2 d^2}}{2\sqrt{de}}}}{\frac{d^2 d^2}{2\sqrt{de}}} + \frac{\sqrt{\frac{d^2 d^2 - 3\sqrt{cd^2 - 3a}\sqrt{ad^2 + d^2 d^2}}{2\sqrt{de}}}}{\frac{d^2 d^2}{2\sqrt{de}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $d^2 e^2 \arctan(ex/\sqrt{de}) / ((c^2 d^4 + 2a^2 c d^2 e^2 + a^2 e^4) \sqrt{de}) + 1/4 * ((c d^2 e + a e^3) x^3 - (c d^3 + a d e^2) x) / (a^2 c^2 d^4 + 2a^2 c d^2 e^2 + a^3 e^4 + (c^3 d^4 + 2a^2 c^2 d^2 e^2 + a^2 c e^4) x^4) + 1/32 * (2 * \sqrt{2} * (c^{3/2} d^3 - 3 * \sqrt{a} * c d^2 e - 3 * a * \sqrt{c} * d e^2 + a^{3/2} e^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x + \sqrt{2} * a^{1/4} * c^{1/4}) / \sqrt{\sqrt{a} * \sqrt{c}})) / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c}) + 2 * \sqrt{2} * (c^{3/2} d^3 - 3 * \sqrt{a} * c d^2 e - 3 * a * \sqrt{c} * d e^2 + a^{3/2} e^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{1/4} * c^{1/4}) / \sqrt{\sqrt{a} * \sqrt{c}})) / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c}) + \sqrt{2} * (c^{3/2} d^3 + 3 * \sqrt{a} * c d^2 e - 3 * a * \sqrt{c} * d e^2 - a^{3/2} e^3) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4}) - \sqrt{2} * (c^{3/2} d^3 + 3 * \sqrt{a} * c d^2 e - 3 * a * \sqrt{c} * d e^2 - a^{3/2} e^3) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{3/4})) / (c^2 d^4 + 2a^2 c d^2 e^2 + a^2 e^4)$

mupad [B] time = 4.87, size = 17180, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] $- \operatorname{atan}\left(\frac{(((((28672a^2c^8d^{10}e^4 - 4096a^2c^9d^{12}e^2 + 155648a^3c^7d^8e^6 + 253952a^4c^6d^6e^8 + 176128a^5c^5d^4e^{10} + 45056a^6c^4d^2e^{12}) / (256(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) - (x((a^3e^6(-a^3c^3)^{1/2} - c^3d^6(-a^3c^3)^{1/2} + 6a^2c^4d^5e + 6a^4c^2d^5e - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{1/2} - 15a^2c^2d^2e^4(-a^3c^3)^{1/2}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6)))^{1/2} * (65536a^9c^4e^{17} - 65536a^2c^{11}d^{14}e^3 - 327680a^3c^{10}d^{12}e^5 - 589824a^4c^9d^{10}e^7 - 327680a^5c^8d^8e^9 + 327680a^6c^7d^6e^{11} + 589824a^7c^6d^4e^{13} + 327680a^8c^5d^2e^{15})) / (128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a^2c^2d^4e^4))) * ((a^3e^6(-a^3c^3)^{1/2} - c^3d^6(-a^3c^3)^{1/2} + 6a^2c^4d^5e + 6a^4c^2d^5e - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{1/2} - 15a^2c^2d^2e^4(-a^3c^3)^{1/2}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6)))^{1/2} + (x(256a^2c^8d^{11}e^4 - 128c^9d^{13}e^2 + 2944a^6c^3d^5e^{14} + 21632a^2c^7d^9e^6 + 32256a^3c^6d^7e^8 +$

$$\begin{aligned}
& (4224a^4c^5d^5e^{10} - 3840a^5c^4d^3e^{12}) / (128(a^4e^8 + c^4d^8 + 4 \\
& *a^3c^3d^6e^2 + 4a^3c^3d^2e^6 + 6a^2c^2d^4e^4)) * ((a^3e^6(-a^3c^3) \\
&)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - 20 \\
& *a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a \\
& ^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c \\
& ^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (16c^6d^9e^3 - 960a^5c^5d^7e \\
& ^5 + 16a^4c^2d^11 + 8288a^2c^4d^5e^7 - 3008a^3c^3d^3e^9) / (256 \\
& *(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) * ((a^3e^6(-a^3c \\
& ^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - \\
& 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(- \\
& a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a \\
& ^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} - (x*(a^4c^13 + 33c^5d^8e \\
& ^5 - 188a^4c^4d^6e^7 + 38a^2c^3d^4e^9 + 4a^3c^2d^2e^11)) / (128(a^ \\
& 4e^8 + c^4d^8 + 4a^3c^3d^6e^2 + 4a^3c^3d^2e^6 + 6a^2c^2d^4e^4)) * \\
& ((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6 \\
& *a^4c^2d^5e^5 - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 1 \\
& 5a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c \\
& ^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} * i - (((((28672 \\
& *a^2c^8d^10e^4 - 4096a^2c^9d^12e^2 + 155648a^3c^7d^8e^6 + 253952a \\
& ^4c^6d^6e^8 + 176128a^5c^5d^4e^10 + 45056a^6c^4d^2e^12) / (256(a^ \\
& 3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) + (x*((a^3e^6(-a^3c \\
& ^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - \\
& 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(- \\
& a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a \\
& ^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} * (65536a^9c^4e^17 - 65536a^2 \\
& *c^11d^14e^3 - 327680a^3c^10d^12e^5 - 589824a^4c^9d^10e^7 - 32768 \\
& 0a^5c^8d^8e^9 + 327680a^6c^7d^6e^11 + 589824a^7c^6d^4e^13 + 327 \\
& 680a^8c^5d^2e^15)) / (128(a^4e^8 + c^4d^8 + 4a^3c^3d^6e^2 + 4a^3c^3 \\
& d^2e^6 + 6a^2c^2d^4e^4)) * ((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c \\
& ^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - 20a^3c^3d^3e^3 + 15a^2c \\
& ^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c \\
& ^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d \\
& ^2e^6))^{(1/2)} - (x*(256a^8c^8d^11e^4 - 128c^9d^13e^2 + 2944a^6c^3 \\
& *d^14 + 21632a^2c^7d^9e^6 + 32256a^3c^6d^7e^8 + 4224a^4c^5d^5e \\
& ^10 - 3840a^5c^4d^3e^12)) / (128(a^4e^8 + c^4d^8 + 4a^3c^3d^6e^2 + 4 \\
& *a^3c^3d^2e^6 + 6a^2c^2d^4e^4)) * ((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(- \\
& a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - 20a^3c^3d^3e^3 + \\
& 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (25 \\
& 6(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d \\
& ^2e^6))^{(1/2)} + (16c^6d^9e^3 - 960a^5c^5d^7e^5 + 16a^4c^2d \\
& *e^11 + 8288a^2c^4d^5e^7 - 3008a^3c^3d^3e^9) / (256(a^3e^6 + c^3d^ \\
& 6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) * ((a^3e^6(-a^3c^3)^{(1/2)} - c^3d \\
& ^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 - 20a^3c^3d^3e^ \\
& 3 + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / \\
& (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4
\end{aligned}$$

$$\begin{aligned}
& *a^6*c^4*d^2*e^6)))^{(1/2)} + (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 188*a*c^4*d^6 \\
& *e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((128*(a^4*e^8 + c^4*d^8 + \\
& 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^ \\
& 3)^{(1/2)} - c^3*d^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 2 \\
& 0*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4*(- \\
& a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5 \\
& *c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}*1i)/((5*c^2*d^4*e^6 + a*c*d^2*e^8 \\
&)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((((2867 \\
& 2*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952* \\
& a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^{10} + 45056*a^6*c^4*d^2*e^{12}))/((256*(a \\
& ^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((a^3*e^6*(-a^3 \\
& *c^3)^{(1/2)} - c^3*d^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 \\
& - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4 \\
& *(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6* \\
& a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}*(65536*a^9*c^4*e^{17} - 65536*a^ \\
& 2*c^{11}*d^{14}*e^3 - 327680*a^3*c^{10}*d^{12}*e^5 - 589824*a^4*c^9*d^{10}*e^7 - 3276 \\
& 80*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^{11} + 589824*a^7*c^6*d^4*e^{13} + 32 \\
& 7680*a^8*c^5*d^2*e^{15}))/((128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c \\
& *d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^{(1/2)} - c^3*d^6*(-a^3* \\
& c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a* \\
& c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3 \\
& *c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4* \\
& d^2*e^6)))^{(1/2)} + (x*(256*a*c^8*d^{11}*e^4 - 128*c^9*d^{13}*e^2 + 2944*a^6*c^3 \\
& *d*e^{14} + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5* \\
& e^{10} - 3840*a^5*c^4*d^3*e^{12}))/((128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + \\
& 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^{(1/2)} - c^3*d^6 \\
& *(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 \\
& + 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(2 \\
& 56*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a \\
& ^6*c^4*d^2*e^6)))^{(1/2)} + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2* \\
& d*e^{11} + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d \\
& ^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^{(1/2)} - c^3* \\
& d^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e \\
& ^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)) \\
& /((256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + \\
& 4*a^6*c^4*d^2*e^6)))^{(1/2)} - (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 188*a*c^4*d^ \\
& 6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((128*(a^4*e^8 + c^4*d^8 + \\
& 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c \\
& ^3)^{(1/2)} - c^3*d^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - \\
& 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} - 15*a^2*c*d^2*e^4*(\\
& -a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^ \\
& 5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (((((28672*a^2*c^8*d^{10}*e^4 - \\
& 4096*a*c^9*d^{12}*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176 \\
& 128*a^5*c^5*d^4*e^{10} + 45056*a^6*c^4*d^2*e^{12}))/((256*(a^3*e^6 + c^3*d^6 + 3* \\
& a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*c^3)^{(1/2)} - c^3*d^6
\end{aligned}$$

$$\begin{aligned}
& *(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 \\
& + 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(2 \\
& 56(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a \\
& ^6c^4d^2e^6))^{(1/2)}(65536a^9c^4e^{17} - 65536a^2c^{11}d^{14}e^3 - 327 \\
& 680a^3c^{10}d^{12}e^5 - 589824a^4c^9d^{10}e^7 - 327680a^5c^8d^8e^9 + \\
& 327680a^6c^7d^6e^{11} + 589824a^7c^6d^4e^{13} + 327680a^8c^5d^2e^{15} \\
&))/(128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a^2c^2 \\
& d^4e^4)))*((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6a^2c^4 \\
& d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(-a^3c^3 \\
&)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256(a^3c^7d^8 + a^7 \\
& c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} - (x \\
& *(256a^8c^8d^{11}e^4 - 128c^9d^{13}e^2 + 2944a^6c^3d^3e^{14} + 21632a^2c \\
& ^7d^9e^6 + 32256a^3c^6d^7e^8 + 4224a^4c^5d^5e^{10} - 3840a^5c^4d \\
& ^3e^{12}))/((128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a \\
& ^2c^2d^4e^4)))*((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} + 6 \\
& a^2c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2(- \\
& a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256(a^3c^7d^8 + a^7 \\
& c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (16c^6d^9e^3 \\
& - 960a^5c^5d^7e^5 + 16a^4c^2de^{11} + 8288a^2c^4 \\
& d^5e^7 - 3008a^3c^3d^3e^9)/(256(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 \\
& + 3a^2c^2d^2e^4)))*((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} \\
& + 6a^2c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2 \\
& *(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256(a^3c^7d^8 + \\
& a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (x \\
& *(a^4c^4e^{13} + 33c^5d^8e^5 - 188a^2c^4d^6e^7 + 38a^2c^3d^4 \\
& e^9 + 4a^3c^2d^2e^{11}))/((128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4 \\
& a^3c^2d^2e^6 + 6a^2c^2d^4e^4)))*((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(- \\
& a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 + \\
& 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256 \\
& *(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6 \\
& c^4d^2e^6))^{(1/2)}))*((a^3e^6(-a^3c^3)^{(1/2)} - c^3d^6(-a^3c^3)^{(1/2)} \\
& + 6a^2c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 + 15a^2c^2d^4e^2 \\
& *(-a^3c^3)^{(1/2)} - 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256(a^3c^7d^8 \\
& + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6) \\
&))^{(1/2)}*2i - \operatorname{atan}(((((((28672a^2c^8d^{10}e^4 - 4096a^2c^9d^{12}e^2 + 155 \\
& 648a^3c^7d^8e^6 + 253952a^4c^6d^6e^8 + 176128a^5c^5d^4e^{10} + 45 \\
& 056a^6c^4d^2e^{12}))/((256(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2 \\
& ^2e^4)) - (x*((c^3d^6(-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2 \\
& c^4d^5e + 6a^4c^2de^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} \\
& + 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)})/(256(a^3c^7d^8 + a^7c^3 \\
& e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)}*(\\
& 65536a^9c^4e^{17} - 65536a^2c^{11}d^{14}e^3 - 327680a^3c^{10}d^{12}e^5 - 5 \\
& 89824a^4c^9d^{10}e^7 - 327680a^5c^8d^8e^9 + 327680a^6c^7d^6e^{11} + \\
& 589824a^7c^6d^4e^{13} + 327680a^8c^5d^2e^{15}))/((128(a^4e^8 + c^4d^8 \\
& + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a^2c^2d^4e^4)))*((c^3d^6(-a^3
\end{aligned}$$

$$\begin{aligned}
& 3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^5e^5 \\
& - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} + 15a^2c^2d^2e^4 \\
& 4(-a^3c^3)^{(1/2)} / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6 \\
& a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (x(256a^8d^{11}e^4 - 12 \\
& 8c^9d^{13}e^2 + 2944a^6c^3d^5e^{14} + 21632a^2c^7d^9e^6 + 32256a^3c^6 \\
& d^7e^8 + 4224a^4c^5d^5e^{10} - 3840a^5c^4d^3e^{12})) / (128(a^4e^8 + \\
& c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a^2c^2d^4e^4)) * ((c^3d^6 \\
& (-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2 \\
& d^2e^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} + 15a^2c^2 \\
& d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + \\
& 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (16c^6d^9e^3 - 96 \\
& 0a^2c^5d^7e^5 + 16a^4c^2d^5e^{11} + 8288a^2c^4d^5e^7 - 3008a^3c^3d^3 \\
& e^9) / (256(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) * ((c^3 \\
& d^6(-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2 \\
& d^2e^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} + 15a^2c^2 \\
& d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + \\
& 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} - (x(a^4c^5e^{13} + \\
& 33c^5d^8e^5 - 188a^2c^4d^6e^7 + 38a^2c^3d^4e^9 + 4a^3c^2d^2e^{11} \\
& 1)) / (128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 + 6a^2c^2 \\
& d^4e^4)) * ((c^3d^6(-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4 \\
& d^5e + 6a^4c^2d^2e^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} \\
& + 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6 \\
& d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} * 1i \\
& - (((((28672a^2c^8d^{10}e^4 - 4096a^2c^9d^{12}e^2 + 155648a^3c^7d^8e^6 \\
& + 253952a^4c^6d^6e^8 + 176128a^5c^5d^4e^{10} + 45056a^6c^4d^2e^{12}) / (256(a^3e^6 \\
& + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) + (x(((c^3d^6(-a^3c^3)^{(1/2)} \\
& - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4d^5e + 6a^4c^2d^2e^5 - 20a^3c^3d^3e^3 \\
& - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} + 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 \\
& + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} * (65536a^9c^4e^{17} \\
& - 65536a^2c^{11}d^{14}e^3 - 327680a^3c^{10}d^{12}e^5 - 589824a^4c^9d^{10} \\
& e^7 - 327680a^5c^8d^8e^9 + 327680a^6c^7d^6e^{11} + 589824a^7c^6d^4 \\
& e^{13} + 327680a^8c^5d^2e^{15})) / (128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 \\
& + 4a^3c^2d^2e^6 + 6a^2c^2d^4e^4)) * ((c^3d^6(-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} \\
& + 6a^2c^4d^5e + 6a^4c^2d^2e^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} \\
& + 15a^2c^2d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 \\
& + 6a^5c^5d^4e^4 + 4a^6c^4d^2e^6))^{(1/2)} - (x(256a^8d^{11}e^4 - 128c^9d^{13}e^2 + 2 \\
& 944a^6c^3d^5e^{14} + 21632a^2c^7d^9e^6 + 32256a^3c^6d^7e^8 + 4224a^4c^5d^5e^{10} \\
& - 3840a^5c^4d^3e^{12})) / (128(a^4e^8 + c^4d^8 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 \\
& + 6a^2c^2d^4e^4)) * ((c^3d^6(-a^3c^3)^{(1/2)} - a^3e^6(-a^3c^3)^{(1/2)} + 6a^2c^4 \\
& d^5e + 6a^4c^2d^2e^5 - 20a^3c^3d^3e^3 - 15a^2c^2d^4e^2(-a^3c^3)^{(1/2)} + 15a^2c^2 \\
& d^2e^4(-a^3c^3)^{(1/2)}) / (256(a^3c^7d^8 + a^7c^3e^8 + 4a^4c^6d^6e^2 + 6a^5c^5d^4 \\
& e^4 + 4a^6c^4d^2e^6))^{(1/2)} + (16c^6d^9e^3 - 960a^2c^5d^7e^5 +
\end{aligned}$$

$$\begin{aligned}
& 16*a^4*c^2*d*e^{11} + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3* \\
& e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^ \\
& 3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5 \\
& *d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 1 \\
& 88*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((128*(a^4*e^8 \\
& + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6 \\
& *e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}*i)/((5*c^2*d^4*e^6 + \\
& a*c*d^2*e^8)/(128*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) \\
& + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^{10} + 45056*a^6*c^4*d^2*e^{12}))/256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}*(65536*a^9*c^4*e^{17} - 65536*a^2*c^{11}*d^{14}*e^3 - 327680*a^3*c^{10}*d^{12}*e^5 - 589824*a^4*c^9*d^{10}*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^{11} + 589824*a^7*c^6*d^4*e^{13} + 327680*a^8*c^5*d^2*e^{15}))/((128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (x*(256*a*c^8*d^{11}*e^4 - 128*c^9*d^{13}*e^2 + 2944*a^6*c^3*d*e^{14} + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^{10} - 3840*a^5*c^4*d^3*e^{12}))/((128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^{11} + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} - (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& *c*d^2*e^4*(-a^3*c^3)^{(1/2)})/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)}))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)}*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)}))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)}))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)}))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((c^3*d^6*(-a^3*c^3)^{(1/2)} - a^3*e^6*(-a^3*c^3)^{(1/2)} + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 - 15*a*c^2*d^4*e^2*(-a^3*c^3)^{(1/2)} + 15*a^2*c*d^2*e^4*(-a^3*c^3)^{(1/2)}))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^{(1/2)} + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/
\end{aligned}$$

$$\begin{aligned}
& *i - ((dx)/(4*(a*e^2 + c*d^2)) - (e*x^3)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (\operatorname{atan}((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d^5*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + ((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/
\end{aligned}$$

$$\begin{aligned}
& (256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2)) * (65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)*i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - ((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^(1/2))*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^(1/2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^(1/2)*i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)/(((5*c^2*d^4*e^6)/128 + (a*c*d^2*e^8)/128)/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d*e^11)/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (((x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (((112*a^2*c^8*d^10*e^4 - 16*a*c^9*d^12*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^10 + 176*a^6*c^4*d^2*e^12)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(-d^3*e^3)^(1/2))*(65536*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15)))/
\end{aligned}$$

$$\begin{aligned}
& (512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^{(1/2)})/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) + ((((((c^6*d^9*e^3)/16 - (15*a*c^5*d^7*e^5)/4 + (a^4*c^2*d^5*e^{11})/16 + (259*a^2*c^4*d^5*e^7)/8 - (47*a^3*c^3*d^3*e^9)/4)/(2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (((x*(256*a*c^8*d^{11}*e^4 - 128*c^9*d^{13}*e^2 + 2944*a^6*c^3*d^5*e^{14} + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^{10} - 3840*a^5*c^4*d^3*e^{12}))/((256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (((112*a^2*c^8*d^{10}*e^4 - 16*a*c^9*d^{12}*e^2 + 608*a^3*c^7*d^8*e^6 + 992*a^4*c^6*d^6*e^8 + 688*a^5*c^5*d^4*e^{10} + 176*a^6*c^4*d^2*e^{12}))/((2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*(-d^3*e^3)^{(1/2)}*(65536*a^9*c^4*e^{17} - 65536*a^2*c^{11}*d^{14}*e^3 - 327680*a^3*c^{10}*d^{12}*e^5 - 589824*a^4*c^9*d^{10}*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^{11} + 589824*a^7*c^6*d^4*e^{13} + 327680*a^8*c^5*d^2*e^{15}))/((512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))))*(-d^3*e^3)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d^3*e^3)^{(1/2)})/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(a^4*c*e^{13} + 33*c^5*d^8*e^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^{11}))/((256*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(-d^3*e^3)^{(1/2)})/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))*(-d^3*e^3)^{(1/2)}*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.185 \quad \int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=685

$$\frac{\sqrt[4]{c} de (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} - \frac{\sqrt[4]{c} de (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} + \dots$$

Rubi [A] time = 0.56, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1316, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\sqrt[4]{c} de (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} - \frac{\sqrt[4]{c} de (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
```

```
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1316

```
Int[(((f_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^2 + a*e^2), Int[((f*x)^(m - 2)*(
a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx &= \frac{\int \frac{ae+cdx^2}{(a+cx^4)^2} dx}{cd^2+ae^2} - \frac{(de) \int \frac{1}{(d+ex^2)(a+cx^4)} dx}{cd^2+ae^2} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4} dx}{4a(cd^2+ae^2)} - \frac{(de) \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)} \right) dx}{cd^2+ae^2} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{(cde) \int \frac{d-ex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} - \frac{(de^3) \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} - \frac{\left(d - \frac{3\sqrt{a}e}{\sqrt{c}}\right) \int \frac{\sqrt{a}\sqrt{c}}{a+cx^4} dx}{8a(cd^2+ae^2)} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right)e\right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2(cd^2+ae^2)^2} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}\left(d - \frac{3\sqrt{a}e}{\sqrt{c}}\right) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt{c}\right)}{16\sqrt{2}a^{5/4}(cd^2+ae^2)} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\left(d + \frac{3\sqrt{a}e}{\sqrt{c}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{8\sqrt{2}a^{5/4}(cd^2+ae^2)} \\
&= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}de(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 428, normalized size = 0.62

$$\frac{\sqrt{2}(-3a^{3/2}e^2 + \sqrt{a}cd^2 + 5a\sqrt{c}d^2 + c^{3/2}d^2) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) - \sqrt{2}(-3a^{3/2}e^2 + \sqrt{a}cd^2 + 5a\sqrt{c}d^2 + c^{3/2}d^2) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) - \frac{2\sqrt{2}(3a^{3/2}e^2 - \sqrt{a}cd^2 + 5a\sqrt{c}d^2 + c^{3/2}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{2\sqrt{2}(3a^{3/2}e^2 - \sqrt{a}cd^2 + 5a\sqrt{c}d^2 + c^{3/2}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} + 1\right)}{a^{5/4}\sqrt[4]{c}} + \frac{8(ae^2 + cd^2)(ae^2 + cd^2)}{a(a+cx^4)} - 32\sqrt{d}e^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{32(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])

$$\frac{(a^{5/4}c^{1/4}) + (2\sqrt{2}(c^{3/2}d^3 - \sqrt{a}cd^2e + 5a\sqrt{c}de^2 + 3a^{3/2}e^3)\text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}])}{(a^{5/4}c^{1/4}) + (\sqrt{2}(c^{3/2}d^3 + \sqrt{a}cd^2e + 5a\sqrt{c}de^2 - 3a^{3/2}e^3)\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])} - \frac{(\sqrt{2}(c^{3/2}d^3 + \sqrt{a}cd^2e + 5a\sqrt{c}de^2 - 3a^{3/2}e^3)\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])}{(a^{5/4}c^{1/4})} / (32(c^2d^2 + ae^2)^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 26.34, size = 9774, normalized size = 14.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] [1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))]

$$\begin{aligned}
& 6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16})) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^1e^4) * x^4) * \text{sqrt}((2c^2d^5e + 4aacd^3e^3 - 30a^2d^5e^5 + (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \text{sqrt}(-(c^6d^{12} + 18aac^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) * \log(-(c^4d^8 + 18aac^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8) * x - (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5cd^2e^7 + 27a^6e^9 - (a^4c^6d^{11} + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9cd^1e^{10}) * \text{sqrt}(-(c^6d^{12} + 18aac^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16}))) * \text{sqrt}((2c^2d^5e + 4aacd^3e^3 - 30a^2d^5e^5 + (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \text{sqrt}(-(c^6d^{12} + 18aac^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) + (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^1e^4) * x^4) * \text{sqrt}((2c^2d^5e + 4aacd^3e^3 - 30a^2d^5e^5 - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \text{sqrt}(-(c^6d^{12} + 18aac^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) * \log(-(c^4d^8 + 18aac^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8) * x + (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5cd^2e^7 + 27a^6e^9 + (a^4c^6d^{11} + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9cd^1e^{10}) * \text{sqrt}(-(c^6d^{12} + 18aac^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12}) / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^1e^{16})))
\end{aligned}$$

$$\begin{aligned}
&)\sqrt{((2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\sqrt{-(c^6d^{12} \\
& + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + \\
& 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + \\
& 2a^2c^2d^2e^2 + a^3ce^4)*x^4)\sqrt{((2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + \\
& 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + \\
& 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\log(-(c^4d^8 + 18a^5c^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8)*x - (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5cd^2e^7 + 27a^6e^9 + (a^4c^6d^{11} + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9cd^1e^{10}))\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16}))\sqrt{((2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) + 8*(ac^2e^2x^4 + a^2e^2)\sqrt{-d^2e}\log((e^2x^2 - 2\sqrt{-d^2e})x - d)/(e^2x^2 + d)) + 4*(ac^2d^2e + a^2e^3)x/(a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3ce^4)*x^4), 1/16*(4*(c^2d^3 + ac^2d^2e^2)*x^3 - 16*(ac^2e^2x^4 + a^2e^2)\sqrt{d^2e}\arctan(\sqrt{d^2e}x/d) + (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3ce^4)*x^4)\sqrt{((2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e + (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))\log(-(c^4d^8 + 18a^5c^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8)*x + (a^2c^4d^8
\end{aligned}$$

$$\begin{aligned}
& 8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5c^2d^2e^7 + 27a^6e^9 - (a^4c^6d^{11} + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9c^2d^3e^{10})\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16}))\sqrt{((2c^2d^5e + 4a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8)\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8))} - (a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8)\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8))\log(-(c^4d^8 + 18a^3c^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3c^2d^2e^6 - 81a^4e^8)x - (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5c^2d^2e^7 + 27a^6e^9 - (a^4c^6d^{11} + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9c^2d^3e^{10})\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16}))\sqrt{((2c^2d^5e + 4a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8)\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8))} + (a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8)\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8))\sqrt{((2c^2d^5e + 4a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8)\sqrt{-(c^6d^{12} + 18a^5c^5d^{10}e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5c^2d^2e^{10} + 81a^6e^{12})/(a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^2e^{16})))/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5c^2d^2e^6 + a^6e^8))}
\end{aligned}$$

$$\begin{aligned}
& d^2e^{14} + a^{13}c^3e^{16})) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \log(-(c^4d^8 + 18a^3c^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8) * x + (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5cd^2e^7 + 27a^6e^9 + (a^4c^6d^11 + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9cd^10) * \sqrt{-(c^6d^12 + 18a^3c^5d^10e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})} / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^3e^{16}))) * \sqrt{(2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e^5 - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \sqrt{-(c^6d^12 + 18a^3c^5d^10e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})} / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^3e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^3e^4) * x^4) * \sqrt{(2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e^5 - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \sqrt{-(c^6d^12 + 18a^3c^5d^10e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})} / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^3e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)) * \log(-(c^4d^8 + 18a^3c^3d^6e^2 + 112a^2c^2d^4e^4 + 270a^3cd^2e^6 - 81a^4e^8) * x - (a^2c^4d^8e + 6a^3c^3d^6e^3 + 4a^4c^2d^4e^5 - 102a^5cd^2e^7 + 27a^6e^9 + (a^4c^6d^11 + 9a^5c^5d^9e^2 + 26a^6c^4d^7e^4 + 34a^7c^3d^5e^6 + 21a^8c^2d^3e^8 + 5a^9cd^10) * \sqrt{-(c^6d^12 + 18a^3c^5d^10e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})} / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^3e^{16}))) * \sqrt{(2c^2d^5e + 4a^3cd^3e^3 - 30a^2d^5e^5 - (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8) * \sqrt{-(c^6d^12 + 18a^3c^5d^10e^2 + 143a^2c^4d^8e^4 + 540a^3c^3d^6e^6 + 799a^4c^2d^4e^8 - 558a^5cd^2e^{10} + 81a^6e^{12})} / (a^5c^9d^{16} + 8a^6c^8d^{14}e^2 + 28a^7c^7d^{12}e^4 + 56a^8c^6d^{10}e^6 + 70a^9c^5d^8e^8 + 56a^{10}c^4d^6e^{10} + 28a^{11}c^3d^4e^{12} + 8a^{12}c^2d^2e^{14} + a^{13}c^3e^{16}))) / (a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8))) + 4 * (a^3cd^2e + a^2e^3) * x / (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^3e^4) * x^4)]
\end{aligned}$$

giac [A] time = 0.50, size = 603, normalized size = 0.88

$$\frac{\sqrt{d} \arctan\left(\frac{x}{\sqrt{d}}\right)}{c^2 d^2 + 2 a c d^2 e^2 + a^2 e^4} \left(\frac{(a c^3)^{1/4} a^2 d^2 e^2 - 3 (a c^3)^{3/4} a^2 d^2 e^2 - 5 (a c^3)^{1/4} a^2 d^2 e^2}{8 (\sqrt{2} a^2 c^3 d^2 + 2 \sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^3 d^2)} \arctan\left(\frac{\sqrt{2} x - \sqrt{d}}{\sqrt{2} d}\right) \right) \left(\frac{(a c^3)^{1/4} a^2 d^2 e^2 - 3 (a c^3)^{3/4} a^2 d^2 e^2 - 5 (a c^3)^{1/4} a^2 d^2 e^2}{8 (\sqrt{2} a^2 c^3 d^2 + 2 \sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^3 d^2)} \arctan\left(\frac{\sqrt{2} x + \sqrt{d}}{\sqrt{2} d}\right) \right) \left(\frac{(a c^3)^{1/4} a^2 d^2 e^2 + 3 (a c^3)^{3/4} a^2 d^2 e^2 + 5 (a c^3)^{1/4} a^2 d^2 e^2}{16 (\sqrt{2} a^2 c^3 d^2 + 2 \sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^3 d^2)} \log\left(\frac{x^2 + \sqrt{2} x + \sqrt{d}}{\sqrt{2} d}\right) \right) \left(\frac{(a c^3)^{1/4} a^2 d^2 e^2 + 3 (a c^3)^{3/4} a^2 d^2 e^2 + 5 (a c^3)^{1/4} a^2 d^2 e^2}{16 (\sqrt{2} a^2 c^3 d^2 + 2 \sqrt{2} a^2 c^3 d^2 + \sqrt{2} a^2 c^3 d^2)} \log\left(\frac{x^2 - \sqrt{2} x + \sqrt{d}}{\sqrt{2} d}\right) \right) \frac{c d^2 + a x}{4 (c x^4 + a) (a c d^2 + a^2 e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $-\sqrt{d} \arctan(x e^{1/2} / \sqrt{d}) e^{5/2} / (c^2 d^4 + 2 a c d^2 e^2 + a^2 e^4) - 1/8 * ((a c^3)^{1/4} a^2 c^2 d^2 e - (a c^3)^{3/4} c^2 d^2 e - 3 * (a c^3)^{1/4} a^2 c^2 e^3 - 5 * (a c^3)^{3/4} a^2 d e^2) \arctan(1/2 * \sqrt{2} * (2 x + \sqrt{2}) * (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * a^2 c^4 d^4 + 2 * \sqrt{2} * a^3 c^3 d^2 e^2 + \sqrt{2} * a^4 c^2 e^4) - 1/8 * ((a c^3)^{1/4} a^2 c^2 d^2 e - (a c^3)^{3/4} c^2 d^2 e - 3 * (a c^3)^{1/4} a^2 c^2 e^3 - 5 * (a c^3)^{3/4} a^2 d e^2) \arctan(1/2 * \sqrt{2} * (2 x - \sqrt{2}) * (a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * a^2 c^4 d^4 + 2 * \sqrt{2} * a^3 c^3 d^2 e^2 + \sqrt{2} * a^4 c^2 e^4) - 1/16 * ((a c^3)^{1/4} a^2 c^2 d^2 e + (a c^3)^{3/4} c^2 d^2 e - 3 * (a c^3)^{1/4} a^2 c^2 e^3 + 5 * (a c^3)^{3/4} a^2 d e^2) * \log(x^2 + \sqrt{2} * x * (a/c)^{1/4} + \sqrt{2} * (a/c)) / (\sqrt{2} * a^2 c^4 d^4 + 2 * \sqrt{2} * a^3 c^3 d^2 e^2 + \sqrt{2} * a^4 c^2 e^4) + 1/16 * ((a c^3)^{1/4} a^2 c^2 d^2 e + (a c^3)^{3/4} c^2 d^2 e - 3 * (a c^3)^{1/4} a^2 c^2 e^3 + 5 * (a c^3)^{3/4} a^2 d e^2) * \log(x^2 - \sqrt{2} * x * (a/c)^{1/4} + \sqrt{2} * (a/c)) / (\sqrt{2} * a^2 c^4 d^4 + 2 * \sqrt{2} * a^3 c^3 d^2 e^2 + \sqrt{2} * a^4 c^2 e^4) + 1/4 * (c d x^3 + a x e) / ((c x^4 + a) * (a c d^2 + a^2 e^2))$

maple [A] time = 0.02, size = 852, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) * c d x^3 e^2 + 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) * c^2 d^3 / a x^3 + 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) * x e^3 a + 1/4 / (a e^2 + c d^2)^2 / (c x^4 + a) * x e^2 c d^2 + 3/16 / (a e^2 + c d^2)^2 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * e^3 - 1/16 / (a e^2 + c d^2)^2 / a * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * c d^2 e + 3/32 / (a e^2 + c d^2)^2 * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * e^3 - 1/32 / (a e^2 + c d^2)^2 / a * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * c d^2 e + 3/16 / (a e^2 + c d^2)^2 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * e^3 - 1/16 / (a e^2 + c d^2)^2 / a * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * c d^2 e + 5/32 / (a e^2 + c d^2)^2 / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d e^2 + 1/32 / (a e^2 + c d^2)^2 / a * c / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^3 + 5/16 / (a e^2 + c d^2)^2 / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d e^2 + 1/16 / (a e^2 + c d^2)^2 / a * c / (a/c)^{1/4}$

$$4) * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^3 + 5/16 / (a * e^2 + c * d^2)^{2/3} / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d * e^2 + 1/16 / (a * e^2 + c * d^2)^{2/3} * a * c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^3 - d * e^3 / (a * e^2 + c * d^2)^{2/3} / (d * e)^{(1/2)} * \arctan(1 / (d * e)^{(1/2)} * e * x)$$

maxima [A] time = 2.15, size = 472, normalized size = 0.69

$$\frac{d^3 \arctan\left(\frac{ax}{\sqrt{ax^2+bx+c}}\right)}{(a^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{de}} + \frac{cdx^3 + aex}{4(a^2cd^2 + a^2e^2 + (ac^2d^2 + a^2e^2)^2)} + \frac{2\sqrt{2}\sqrt{a^2d^2 - ac^2}\sqrt{a^2d^2 + ac^2}\sqrt{c}}{\sqrt{a}\sqrt{d}\sqrt{e}} \arctan\left(\frac{\sqrt{2}\sqrt{a^2d^2 - ac^2}}{2\sqrt{cd}}\right) + \frac{2\sqrt{2}\sqrt{a^2d^2 - ac^2}\sqrt{a^2d^2 + ac^2}\sqrt{c}}{\sqrt{a}\sqrt{d}\sqrt{e}} \arctan\left(\frac{\sqrt{2}\sqrt{a^2d^2 + ac^2}}{2\sqrt{cd}}\right) + \frac{\sqrt{2}\sqrt{a^2d^2 + ac^2}\sqrt{a^2d^2 - ac^2}\sqrt{c}}{\sqrt{a}\sqrt{d}\sqrt{e}} \log\left(\sqrt{a^2d^2 + ac^2}\sqrt{a^2d^2 - ac^2}\sqrt{c}\right) + \frac{\sqrt{2}\sqrt{a^2d^2 + ac^2}\sqrt{a^2d^2 - ac^2}\sqrt{c}}{\sqrt{a}\sqrt{d}\sqrt{e}} \log\left(\sqrt{a^2d^2 - ac^2}\sqrt{a^2d^2 + ac^2}\sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $-d * e^3 * \arctan(e * x / \sqrt{d * e}) / ((c^2 * d^4 + 2 * a * c * d^2 * e^2 + a^2 * e^4) * \sqrt{d * e}) + 1/4 * (c * d * x^3 + a * e * x) / (a^2 * c * d^2 + a^3 * e^2 + (a * c^2 * d^2 + a^2 * c * e^2) * x^4) + 1/32 * (2 * \sqrt{2}) * (\sqrt{a} * c^2 * d^3 - a * c^{(3/2)} * d^2 * e + 5 * a^{(3/2)} * c * d * e^2 + 3 * a^2 * \sqrt{c} * e^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x + \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c} / (\sqrt{a} * \sqrt{c} * \sqrt{a} * \sqrt{c} * \sqrt{c}) + 2 * \sqrt{2} * (\sqrt{a} * c^2 * d^3 - a * c^{(3/2)} * d^2 * e + 5 * a^{(3/2)} * c * d * e^2 + 3 * a^2 * \sqrt{c} * e^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a} * \sqrt{c} / (\sqrt{a} * \sqrt{c} * \sqrt{a} * \sqrt{c} * \sqrt{c}) - \sqrt{2} * (\sqrt{a} * c^2 * d^3 + a * c^{(3/2)} * d^2 * e + 5 * a^{(3/2)} * c * d * e^2 - 3 * a^2 * \sqrt{c} * e^3) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) + \sqrt{2} * (\sqrt{a} * c^2 * d^3 + a * c^{(3/2)} * d^2 * e + 5 * a^{(3/2)} * c * d * e^2 - 3 * a^2 * \sqrt{c} * e^3) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) / (a * c^2 * d^4 + 2 * a^2 * c * d^2 * e^2 + a^3 * e^4)$

mupad [B] time = 2.87, size = 17812, normalized size = 26.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] $((e * x) / (4 * (a * e^2 + c * d^2)) + (c * d * x^3) / (4 * a * (a * e^2 + c * d^2))) / (a + c * x^4) + \operatorname{atan}\left(\frac{(((((53248 * a^9 * c^4 * d * e^{15} + 4096 * a^3 * c^{10} * d^{13} * e^3 + 73728 * a^4 * c^9 * d^{11} * e^5 + 307200 * a^5 * c^8 * d^9 * e^7 + 573440 * a^6 * c^7 * d^7 * e^9 + 552960 * a^7 * c^6 * d^5 * e^{11} + 270336 * a^8 * c^5 * d^3 * e^{13})) / (256 * (a^6 * e^8 + a^2 * c^4 * d^8 + 4 * a^5 * c * d^2 * e^6 + 4 * a^3 * c^3 * d^6 * e^2 + 6 * a^4 * c^2 * d^4 * e^4)) - (x * (-c^3 * d^6 * (-a^5 * c)^{(1/2)} - 9 * a^3 * e^6 * (-a^5 * c)^{(1/2)} - 2 * a^3 * c^3 * d^5 * e - 4 * a^4 * c^2 * d^3 * e^3 + 30 * a^5 * c * d * e^5 + 9 * a * c^2 * d^4 * e^2 * (-a^5 * c)^{(1/2)} + 31 * a^2 * c * d^2 * e^4 * (-a^5 * c)^{(1/2}))) / (256 * (a^9 * c * e^8 + a^5 * c^5 * d^8 + 4 * a^6 * c^4 * d^6 * e^2 + 6 * a^7 * c^3 * d^4 * e^4 + 4 * a^8 * c^2 * d^2 * e^6))^{(1/2)} * (65536 * a^{11} * c^4 * e^{17} - 65536 * a^4 * c^{11} * d^{14} * e^3 - 327680 * a^5 * c^{10} * d^{12} * e^5 - 589824 * a^6 * c^9 * d^{10} * e^7 - 327680 * a^7 * c^8 * d^8 * e^9 + 327680 * a^8 * c^7 * d^6 * e^{11} + 589824 * a^9 * c^6 * d^4 * e^{13} + 327680 * a^{10} * c^5 * d^2 * e^{15})) / (128 * (a^6 * e^8 + a^2 * c^4 * d^8 + 4 * a^5 * c * d^2 * e^6 + 4 * a^3 * c^3 * d^6 * e^2$

$$\begin{aligned}
& + 6a^4c^2d^4e^4)) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} \\
& - 2a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2d^4e^2 * \\
& (-a^5c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 + a^5c^5 \\
& *d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} + \\
& (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 39 \\
& 68*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424* \\
& a^6*c^5*d^3*e^12)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2*e^6 + 4a^3*c^ \\
& 3*d^6*e^2 + 6a^4*c^2*d^4*e^4))) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^ \\
& 5c)^{1/2} - 2a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2 \\
& *d^4e^2*(-a^5c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 \\
& + a^5c^5*d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)) \\
&)^{1/2} + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928 \\
& *a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12) / (256*(a \\
& ^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2*e^6 + 4a^3*c^3*d^6*e^2 + 6a^4*c^2*d^4 \\
& *e^4))) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} - 2a^3c^3d^5 \\
& *e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2d^4e^2*(-a^5c)^{1/2} + \\
& 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 + a^5c^5*d^8 + 4a^6c^4d^ \\
& 6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} - (x*(81*a^4c^3e \\
& ^13 + c^7*d^8e^5 - 12*a*c^6*d^6e^7 + 54*a^2c^5*d^4e^9 - 108*a^3c^4*d^2 \\
& *e^11)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2*e^6 + 4a^3*c^3*d^6e^2 + \\
& 6a^4c^2d^4e^4))) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} \\
& - 2a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2d^4e^2*(- \\
& a^5c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 + a^5c^5*d^ \\
& 8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} * 1i \\
& - (((((53248*a^9c^4*d*e^15 + 4096*a^3c^10*d^13e^3 + 73728*a^4c^9*d^11e \\
& ^5 + 307200*a^5c^8*d^9e^7 + 573440*a^6c^7*d^7e^9 + 552960*a^7c^6*d^5e \\
& ^11 + 270336*a^8c^5*d^3e^13) / (256*(a^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2e^ \\
& 6 + 4a^3c^3*d^6e^2 + 6a^4c^2*d^4e^4)) + (x*(-c^3d^6(-a^5c)^{1/2} \\
& - 9a^3e^6(-a^5c)^{1/2} - 2a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c \\
& *d*e^5 + 9a*c^2d^4e^2*(-a^5c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / \\
& (256*(a^9c*e^8 + a^5c^5*d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a \\
& ^8c^2d^2e^6)))^{1/2} * (65536*a^11*c^4e^17 - 65536*a^4c^11*d^14e^3 - 32 \\
& 7680*a^5c^10*d^12e^5 - 589824*a^6c^9*d^10e^7 - 327680*a^7c^8*d^8e^9 + \\
& 327680*a^8c^7*d^6e^11 + 589824*a^9c^6*d^4e^13 + 327680*a^10c^5*d^2e^ \\
& 15)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2e^6 + 4a^3c^3*d^6e^2 + 6 \\
& a^4c^2*d^4e^4))) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} - 2 \\
& *a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2d^4e^2*(-a^5 \\
& *c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 + a^5c^5*d^8 \\
& + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} - (x*(\\
& 128*a*c^10*d^13e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11e^4 + 3968*a^ \\
& 3*c^8*d^9e^6 + 27136*a^4*c^7*d^7e^8 + 30592*a^5*c^6*d^5e^10 - 7424*a^6*c \\
& ^5*d^3e^12)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4a^5*c*d^2e^6 + 4a^3c^3*d^6 \\
& *e^2 + 6a^4c^2*d^4e^4))) * (-c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{ \\
& 1/2} - 2a^3c^3d^5e - 4a^4c^2d^3e^3 + 30a^5c*d*e^5 + 9a*c^2d^4e^2 \\
& *(-a^5c)^{1/2} + 31a^2c*d^2e^4(-a^5c)^{1/2}) / (256*(a^9c*e^8 + a^5
\end{aligned}$$

$$\begin{aligned}
& *c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} \\
& + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) \\
&)*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} + (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11)) \\
&)/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)}*i)/((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)}*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^15))/ \\
& (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/ \\
& (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} - (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))
\end{aligned}$$

$$\begin{aligned}
& d^4 e^4)) * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} + (((((53248 a^9 c^4 d e^{15} + 4096 a^3 c^{10} d^{13} e^3 + 73728 a^4 c^9 d^{11} e^5 + 307200 a^5 c^8 d^9 e^7 + 573440 a^6 c^7 d^7 e^9 + 552960 a^7 c^6 d^5 e^{11} + 270336 a^8 c^5 d^3 e^{13}) / (256 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) + (x * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} * (65536 a^{11} c^4 e^{17} - 65536 a^4 c^{11} d^{14} e^3 - 327680 a^5 c^{10} d^{12} e^5 - 589824 a^6 c^9 d^{10} e^7 - 327680 a^7 c^8 d^8 e^9 + 327680 a^8 c^7 d^6 e^{11} + 589824 a^9 c^6 d^4 e^{13} + 327680 a^{10} c^5 d^2 e^{15}) / (128 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} - (x * (128 a c^{10} d^{13} e^2 - 14208 a^7 c^4 d e^{14} + 768 a^2 c^9 d^{11} e^4 + 3968 a^3 c^8 d^9 e^6 + 27136 a^4 c^7 d^7 e^8 + 30592 a^5 c^6 d^5 e^{10} - 7424 a^6 c^5 d^3 e^{12})) / (128 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} + (16 c^9 d^{12} e^2 + 208 a c^8 d^{10} e^4 + 672 a^2 c^7 d^8 e^6 + 928 a^3 c^6 d^6 e^8 + 12880 a^4 c^5 d^4 e^{10} + 12432 a^5 c^4 d^2 e^{12}) / (256 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} + (x * (81 a^4 c^3 e^{13} + c^7 d^8 e^5 - 12 a c^6 d^6 e^7 + 54 a^2 c^5 d^4 e^9 - 108 a^3 c^4 d^2 e^{11})) / (128 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * (- (c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} - 2 a^3 c^3 d^5 e - 4 a^4 c^2 d^3 e^3 + 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} * 2i + \operatorname{atan}((((((53248 a^9 c^4 d e^{15} + 4096 a^3 c^3
\end{aligned}$$

$$\begin{aligned}
& \cdot 10 \cdot d^{13} \cdot e^3 + 73728 \cdot a^4 \cdot c^9 \cdot d^{11} \cdot e^5 + 307200 \cdot a^5 \cdot c^8 \cdot d^9 \cdot e^7 + 573440 \cdot a^6 \\
& \cdot c^7 \cdot d^7 \cdot e^9 + 552960 \cdot a^7 \cdot c^6 \cdot d^5 \cdot e^{11} + 270336 \cdot a^8 \cdot c^5 \cdot d^3 \cdot e^{13}) / (256 \cdot (a^6 \\
& \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4) \\
& - (x \cdot ((c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e \\
& + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} + \\
& 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) / (256 \cdot (a^9 \cdot c \cdot e^8 + a^5 \cdot c^5 \cdot d^8 + 4 \cdot a^6 \cdot c^4 \cdot d^6 \cdot e^2 \\
& \cdot d^6 \cdot e^2 + 6 \cdot a^7 \cdot c^3 \cdot d^4 \cdot e^4 + 4 \cdot a^8 \cdot c^2 \cdot d^2 \cdot e^6)))^{(1/2)} \cdot (65536 \cdot a^{11} \cdot c^4 \cdot e^{17} \\
& - 65536 \cdot a^4 \cdot c^{11} \cdot d^{14} \cdot e^3 - 327680 \cdot a^5 \cdot c^{10} \cdot d^{12} \cdot e^5 - 589824 \cdot a^6 \cdot c^9 \cdot d^{10} \cdot e^7 \\
& - 327680 \cdot a^7 \cdot c^8 \cdot d^8 \cdot e^9 + 327680 \cdot a^8 \cdot c^7 \cdot d^6 \cdot e^{11} + 589824 \cdot a^9 \cdot c^6 \cdot d^4 \cdot e^{13} \\
& + 327680 \cdot a^{10} \cdot c^5 \cdot d^2 \cdot e^{15})) / (128 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 \\
& + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4))) \cdot ((c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) / (256 \cdot (a^9 \cdot c \cdot e^8 + a^5 \cdot c^5 \cdot d^8 + 4 \cdot a^6 \cdot c^4 \cdot d^6 \cdot e^2 + 6 \cdot a^7 \cdot c^3 \cdot d^4 \cdot e^4 \\
& + 4 \cdot a^8 \cdot c^2 \cdot d^2 \cdot e^6)))^{(1/2)} + (x \cdot (128 \cdot a \cdot c^{10} \cdot d^{13} \cdot e^2 - 14208 \cdot a^7 \cdot c^4 \cdot d \cdot e^{14} \\
& + 768 \cdot a^2 \cdot c^9 \cdot d^{11} \cdot e^4 + 3968 \cdot a^3 \cdot c^8 \cdot d^9 \cdot e^6 + 27136 \cdot a^4 \cdot c^7 \cdot d^7 \cdot e^8 + 30 \\
& 592 \cdot a^5 \cdot c^6 \cdot d^5 \cdot e^{10} - 7424 \cdot a^6 \cdot c^5 \cdot d^3 \cdot e^{12})) / (128 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 \\
& + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4))) \cdot ((c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) / (256 \cdot (a^9 \cdot c \cdot e^8 + a^5 \cdot c^5 \cdot d^8 + 4 \cdot a^6 \cdot c^4 \cdot d^6 \cdot e^2 + 6 \cdot a^7 \cdot c^3 \cdot d^4 \cdot e^4 \\
& + 4 \cdot a^8 \cdot c^2 \cdot d^2 \cdot e^6)))^{(1/2)} + (16 \cdot c^9 \cdot d^{12} \cdot e^2 + 208 \cdot a \cdot c^8 \cdot d^{10} \cdot e^4 \\
& + 672 \cdot a^2 \cdot c^7 \cdot d^8 \cdot e^6 + 928 \cdot a^3 \cdot c^6 \cdot d^6 \cdot e^8 + 12880 \cdot a^4 \cdot c^5 \cdot d^4 \cdot e^{10} + 1243 \\
& 2 \cdot a^5 \cdot c^4 \cdot d^2 \cdot e^{12}) / (256 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 \\
& + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4))) \cdot ((c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e \\
& + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} + 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) \\
&)^{(1/2)} - (x \cdot (81 \cdot a^4 \cdot c^3 \cdot e^{13} + c^7 \cdot d^8 \cdot e^5 - 12 \cdot a \cdot c^6 \cdot d^6 \cdot e^7 + 54 \cdot a^2 \cdot c^5 \cdot d^4 \cdot e^9 \\
& - 108 \cdot a^3 \cdot c^4 \cdot d^2 \cdot e^{11})) / (128 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 \\
& + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4))) \cdot ((c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e \\
& + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} + 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) / (2 \\
& 56 \cdot (a^9 \cdot c \cdot e^8 + a^5 \cdot c^5 \cdot d^8 + 4 \cdot a^6 \cdot c^4 \cdot d^6 \cdot e^2 + 6 \cdot a^7 \cdot c^3 \cdot d^4 \cdot e^4 + 4 \cdot a^8 \cdot c^2 \cdot d^2 \cdot e^6)))^{(1/2)} \cdot 1i - (((((53248 \cdot a^9 \cdot c^4 \cdot d \cdot e^{15} \\
& + 4096 \cdot a^3 \cdot c^{10} \cdot d^{13} \cdot e^3 + 73728 \cdot a^4 \cdot c^9 \cdot d^{11} \cdot e^5 + 307200 \cdot a^5 \cdot c^8 \cdot d^9 \cdot e^7 + 573440 \cdot a^6 \cdot c^7 \cdot d^7 \cdot e^9 \\
& + 552960 \cdot a^7 \cdot c^6 \cdot d^5 \cdot e^{11} + 270336 \cdot a^8 \cdot c^5 \cdot d^3 \cdot e^{13}) / (256 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6 \\
& + 4 \cdot a^3 \cdot c^3 \cdot d^6 \cdot e^2 + 6 \cdot a^4 \cdot c^2 \cdot d^4 \cdot e^4)) + (x \cdot (c^3 \cdot d^6 \cdot (-a^5 \cdot c)^{(1/2)} - 9 \cdot a^3 \cdot e^6 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 2 \cdot a^3 \cdot c^3 \cdot d^5 \cdot e + 4 \cdot a^4 \cdot c^2 \cdot d^3 \cdot e^3 - 30 \cdot a^5 \cdot c \cdot d \cdot e^5 + 9 \cdot a \cdot c^2 \cdot d^4 \cdot e^2 \cdot (-a^5 \cdot c)^{(1/2)} \\
& + 31 \cdot a^2 \cdot c \cdot d^2 \cdot e^4 \cdot (-a^5 \cdot c)^{(1/2}))) / (256 \cdot (a^9 \cdot c \cdot e^8 + a^5 \cdot c^5 \cdot d^8 + 4 \cdot a^6 \cdot c^4 \cdot d^6 \cdot e^2 + 6 \cdot a^7 \cdot c^3 \cdot d^4 \cdot e^4 \\
& + 4 \cdot a^8 \cdot c^2 \cdot d^2 \cdot e^6)))^{(1/2)} \cdot (65536 \cdot a^{11} \cdot c^4 \cdot e^{17} - 6553 \\
& 6 \cdot a^4 \cdot c^{11} \cdot d^{14} \cdot e^3 - 327680 \cdot a^5 \cdot c^{10} \cdot d^{12} \cdot e^5 - 589824 \cdot a^6 \cdot c^9 \cdot d^{10} \cdot e^7 - 327680 \cdot a^7 \cdot c^8 \cdot d^8 \cdot e^9 \\
& + 327680 \cdot a^8 \cdot c^7 \cdot d^6 \cdot e^{11} + 589824 \cdot a^9 \cdot c^6 \cdot d^4 \cdot e^{13} + 327680 \cdot a^{10} \cdot c^5 \cdot d^2 \cdot e^{15})) / (128 \cdot (a^6 \cdot e^8 + a^2 \cdot c^4 \cdot d^8 + 4 \cdot a^5 \cdot c \cdot d^2 \cdot e^6
\end{aligned}$$

$$\begin{aligned}
& + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3 \\
& *e^6*(-a^5*c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 \\
& + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) / (256*(a \\
& ^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2 \\
& *d^2*e^6))^{(1/2)} - (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2 \\
& *c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^ \\
& 6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c \\
& *d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} \\
& - 9*a^3*e^6*(-a^5*c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5 \\
& *c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) \\
& / (256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4* \\
& a^8*c^2*d^2*e^6))^{(1/2)} + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c \\
& ^7*d^8*e^6 + 928*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4 \\
& *d^2*e^12) / (256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 \\
& + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} \\
& + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(\\
& -a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) / (256*(a^9*c*e^8 + a^5*c^5 \\
& *d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} + \\
& (x*(81*a^4*c^3*e^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - \\
& 108*a^3*c^4*d^2*e^11)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a \\
& ^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6 \\
& (-a^5*c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a \\
& *c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) / (256*(a^9*c \\
& *e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2 \\
& *e^6))^{(1/2)} * i) / ((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728 \\
& *a^4*c^9*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 55296 \\
& 0*a^7*c^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13) / (256*(a^6*e^8 + a^2*c^4*d^8 + \\
& 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*((c^3*d^6*(\\
& -a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5*c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3 \\
& *e^3 - 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(\\
& -a^5*c)^{(1/2)}) / (256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7 \\
& *c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))^{(1/2)} * (65536*a^11*c^4*e^17 - 65536*a^4*c^11 \\
& *d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7 \\
& *c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a \\
& ^10*c^5*d^2*e^15)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^ \\
& 3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} - 9*a^3*e^6*(-a^5 \\
& *c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c*d*e^5 + 9*a*c^2 \\
& *d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) / (256*(a^9*c \\
& *e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2 \\
& *e^6))^{(1/2)} + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9 \\
& *d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5 \\
& *e^10 - 7424*a^6*c^5*d^3*e^12)) / (128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2 \\
& *e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) * ((c^3*d^6*(-a^5*c)^{(1/2)} - 9 \\
& *a^3*e^6*(-a^5*c)^{(1/2)} + 2*a^3*c^3*d^5*e + 4*a^4*c^2*d^3*e^3 - 30*a^5*c \\
& *d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^5*c)^{(1/2)}) / (256*(a^9
\end{aligned}$$

$$\begin{aligned} & *c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6))^{1/2} + (16c^9d^{12}e^2 + 208a^8c^8d^{10}e^4 + 672a^2c^7d^8e^6 + 928a^3c^6d^6e^8 + 12880a^4c^5d^4e^{10} + 12432a^5c^4d^2e^{12})/ \\ & (256(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 4a^4c^2d^4e^4)) * ((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} - (x*(81a^4c^3e^{13} + c^7d^8e^5 - 12a^6c^6d^6e^7 + 54a^2c^5d^4e^9 - 108a^3c^4d^2e^{11})) / (128(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4)) * ((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} + (((((53248a^9c^4d^15 + 4096a^3c^10d^13e^3 + 73728a^4c^9d^11e^5 + 307200a^5c^8d^9e^7 + 573440a^6c^7d^7e^9 + 552960a^7c^6d^5e^{11} + 270336a^8c^5d^3e^{13}) / (256(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4)) + (x*((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} * (65536a^{11}c^4e^{17} - 65536a^4c^{11}d^{14}e^3 - 327680a^5c^{10}d^{12}e^5 - 589824a^6c^9d^{10}e^7 - 327680a^7c^8d^8e^9 + 327680a^8c^7d^6e^{11} + 589824a^9c^6d^4e^{13} + 327680a^{10}c^5d^2e^{15}) / (128(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4)) * ((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} - (x*(128a^8c^{10}d^{13}e^2 - 14208a^7c^4d^14e^{14} + 768a^2c^9d^{11}e^4 + 3968a^3c^8d^9e^6 + 27136a^4c^7d^7e^8 + 30592a^5c^6d^5e^{10} - 7424a^6c^5d^3e^{12})) / (128(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4)) * ((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} + (16c^9d^{12}e^2 + 208a^8c^8d^{10}e^4 + 672a^2c^7d^8e^6 + 928a^3c^6d^6e^8 + 12880a^4c^5d^4e^{10} + 12432a^5c^4d^2e^{12}) / (256(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4)) * ((c^3d^6(-a^5c)^{1/2} - 9a^3e^6(-a^5c)^{1/2} + 2a^3c^3d^5e + 4a^4c^2d^3e^3 - 30a^5c^2d^4e^2(-a^5c)^{1/2} + 31a^2cd^2e^4(-a^5c)^{1/2}) / (256(a^9c^8 + a^5c^5d^8 + 4a^6c^4d^6e^2 + 6a^7c^3d^4e^4 + 4a^8c^2d^2e^6)))^{1/2} + (x*(81a^4c^3e^{13} + c^7d^8e^5 - 12a^6c^6d^6e^7 + 54a^2c^5d^4e^9 - 108a^3c^4d^2e^{11})) / (128(a^6e^8 + a^2c^4d^8 + 4a^5c^3d^6e^2 + 6a^4c^2d^4e^4))$$

$$\begin{aligned}
& c^2 d^4 e^4)) * ((c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} + 2 a^3 c^3 d^5 e + 4 a^4 c^2 d^3 e^3 - 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} + (5 c^5 d^5 e^7 + 54 a c^4 d^3 e^9 + 81 a^2 c^3 d e^{11}) / (128 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * ((c^3 d^6 (-a^5 c)^{1/2} - 9 a^3 e^6 (-a^5 c)^{1/2} + 2 a^3 c^3 d^5 e + 4 a^4 c^2 d^3 e^3 - 30 a^5 c d e^5 + 9 a c^2 d^4 e^2 (-a^5 c)^{1/2} + 31 a^2 c d^2 e^4 (-a^5 c)^{1/2}) / (256 (a^9 c e^8 + a^5 c^5 d^8 + 4 a^6 c^4 d^6 e^2 + 6 a^7 c^3 d^4 e^4 + 4 a^8 c^2 d^2 e^6)))^{1/2} * i - (\operatorname{atan}(\frac{((-d e^5)^{1/2} * ((x * (81 a^4 c^3 e^{13} + c^7 d^8 e^5 - 12 a c^6 d^6 e^7 + 54 a^2 c^5 d^4 e^9 - 108 a^3 c^4 d^2 e^{11}))) / (256 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) - (((c^9 d^{12} e^2) / 16 + (13 a c^8 d^{10} e^4) / 16 + (21 a^2 c^7 d^8 e^6) / 8 + (29 a^3 c^6 d^6 e^8) / 8 + (805 a^4 c^5 d^4 e^{10}) / 16 + (777 a^5 c^4 d^2 e^{12}) / 16) / (2 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) + (((-d e^5)^{1/2} * ((208 a^9 c^4 d e^{15} + 16 a^3 c^{10} d^{13} e^3 + 288 a^4 c^9 d^{11} e^5 + 1200 a^5 c^8 d^9 e^7 + 2240 a^6 c^7 d^7 e^9 + 2160 a^7 c^6 d^5 e^{11} + 1056 a^8 c^5 d^3 e^{13})) / (2 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) - (x * (-d e^5)^{1/2} * (65536 a^{11} c^4 e^{17} - 65536 a^4 c^{11} d^{14} e^3 - 327680 a^5 c^{10} d^{12} e^5 - 589824 a^6 c^9 d^{10} e^7 - 327680 a^7 c^8 d^8 e^9 + 327680 a^8 c^7 d^6 e^{11} + 589824 a^9 c^6 d^4 e^{13} + 327680 a^{10} c^5 d^2 e^{15})) / (512 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2) * (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)))) / (2 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2)) + (x * (128 a c^{10} d^{13} e^2 - 14208 a^7 c^4 d e^{14} + 768 a^2 c^9 d^{11} e^4 + 3968 a^3 c^8 d^9 e^6 + 27136 a^4 c^7 d^7 e^8 + 30592 a^5 c^6 d^5 e^{10} - 7424 a^6 c^5 d^3 e^{12})) / (256 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4))) * (-d e^5)^{1/2} / (2 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2))) * (-d e^5)^{1/2} / (2 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2))) * i) / (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2) + ((-d e^5)^{1/2} * ((x * (81 a^4 c^3 e^{13} + c^7 d^8 e^5 - 12 a c^6 d^6 e^7 + 54 a^2 c^5 d^4 e^9 - 108 a^3 c^4 d^2 e^{11})) / (256 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) + (((c^9 d^{12} e^2) / 16 + (13 a c^8 d^{10} e^4) / 16 + (21 a^2 c^7 d^8 e^6) / 8 + (29 a^3 c^6 d^6 e^8) / 8 + (805 a^4 c^5 d^4 e^{10}) / 16 + (777 a^5 c^4 d^2 e^{12}) / 16) / (2 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) + (((-d e^5)^{1/2} * ((208 a^9 c^4 d e^{15} + 16 a^3 c^{10} d^{13} e^3 + 288 a^4 c^9 d^{11} e^5 + 1200 a^5 c^8 d^9 e^7 + 2240 a^6 c^7 d^7 e^9 + 2160 a^7 c^6 d^5 e^{11} + 1056 a^8 c^5 d^3 e^{13})) / (2 (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)) + (x * (-d e^5)^{1/2} * (65536 a^{11} c^4 e^{17} - 65536 a^4 c^{11} d^{14} e^3 - 327680 a^5 c^{10} d^{12} e^5 - 589824 a^6 c^9 d^{10} e^7 - 327680 a^7 c^8 d^8 e^9 + 327680 a^8 c^7 d^6 e^{11} + 589824 a^9 c^6 d^4 e^{13} + 327680 a^{10} c^5 d^2 e^{15})) / (512 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2) * (a^6 e^8 + a^2 c^4 d^8 + 4 a^5 c d^2 e^6 + 4 a^3 c^3 d^6 e^2 + 6 a^4 c^2 d^4 e^4)))) / (2 (a^2 e^4 + c^2 d^4 + 2 a c d^2 e^2)) - (x * (128 a c^{10} d^{13} e^2 -
\end{aligned}$$

$$\begin{aligned}
& 14208*a^7*c^4*d*e^{14} + 768*a^2*c^9*d^{11}*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136* \\
& a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^{10} - 7424*a^6*c^5*d^3*e^{12})/(256*(a^6* \\
& e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) \\
& *(-d*e^5)^{(1/2)}/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^{(1/2)} \\
&)/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))*1i)/(a^2*e^4 + c^2*d^4 + 2*a*c \\
& *d^2*e^2))/(((5*c^5*d^5*e^7)/128 + (27*a*c^4*d^3*e^9)/64 + (81*a^2*c^3*d*e^ \\
& 11)/128)/(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& - ((-d*e^5)^{(1/2))*((x*(81*a^4*c^3*e^{13} + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 \\
& - 108*a^3*c^4*d^2*e^{11}))/((256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& - ((c^9*d^{12}*e^2)/16 + (13*a*c^8*d^{10}*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29 \\
& *a^3*c^6*d^6*e^8)/8 + (805*a^4*c^5*d^4*e^{10})/16 + (777*a^5*c^4*d^2*e^{12})/16) \\
&)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& + (((-d*e^5)^{(1/2))*((208*a^9*c^4*d*e^{15} + 16*a^3*c^{10}*d^{13}*e^3 + 288*a^4*c^9*d^{11}*e^5 \\
& + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^{11} + 1056*a^8*c^5*d^3*e^{13}) \\
&)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& - (x*(-d*e^5)^{(1/2))*((65536*a^{11}*c^4*e^{17} - 65536*a^4*c^{11}*d^{14}*e^3 - 327680*a^5*c^{10}*d^{12}*e^5 \\
& - 589824*a^6*c^9*d^{10}*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^{11} + 589824*a^9*c^6*d^4*e^{13} \\
& + 327680*a^{10}*c^5*d^2*e^{15}))/((512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 \\
& + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) \\
& + (x*(128*a*c^{10}*d^{13}*e^2 - 14208*a^7*c^4*d*e^{14} + 768*a^2*c^9*d^{11}*e^4 + 396 \\
& 8*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^{10} - 7424*a^6*c^5*d^3*e^{12}))/ \\
& (256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) \\
&)*(-d*e^5)^{(1/2)}/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))*(-d*e^5)^{(1/2)}/(2*(a^2*e^4 + c^2*d^4 + 2*a \\
& *c*d^2*e^2)))/(((-d*e^5)^{(1/2))*((x*(81*a^4*c^3*e^{13} + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 \\
& - 108*a^3*c^4*d^2*e^{11}))/((256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& + (((c^9*d^{12}*e^2)/16 + (13*a*c^8*d^{10}*e^4)/16 + (21*a^2*c^7*d^8*e^6)/8 + (29*a^3*c^6*d^6*e^8)/8 \\
& + (805*a^4*c^5*d^4*e^{10})/16 + (777*a^5*c^4*d^2*e^{12})/16) \\
&)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& + (((-d*e^5)^{(1/2))*((208*a^9*c^4*d*e^{15} + 16*a^3*c^{10}*d^{13}*e^3 + 288*a^4*c^9*d^{11}*e^5 \\
& + 1200*a^5*c^8*d^9*e^7 + 2240*a^6*c^7*d^7*e^9 + 2160*a^7*c^6*d^5*e^{11} + 1056*a^8*c^5*d^3*e^{13}) \\
&)/(2*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4) \\
& + (x*(-d*e^5)^{(1/2))*((65536*a^{11}*c^4*e^{17} - 65536*a^4*c^{11}*d^{14}*e^3 - 327680*a^5*c^{10}*d^{12}*e^5 \\
& - 589824*a^6*c^9*d^{10}*e^7 - 327680*a^7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^{11} + 589824*a^9*c^6*d^4*e^{13} \\
& + 327680*a^{10}*c^5*d^2*e^{15}))/((512*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)*(a^6*e^8 + a^2*c^4*d^8 \\
& + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))))/(2*(a^2*e^4 + c^2*d^4 + 2 \\
& *a*c*d^2*e^2)) - (x*(128*a*c^{10}*d^{13}*e^2 - 14208*a^7*c^4*d*e^{14} + 768*a^2*c^9*d^{11}*e^4 \\
& + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^{10} - 7424*a^6*c^5*d^3*e^{12}))/ \\
& (256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))
\end{aligned}$$

$$\frac{(2e^6 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4))(-de^5)^{1/2}}{(2(a^2e^4 + c^2d^4 + 2ac^2d^2e^2))(-de^5)^{1/2}} \frac{(2(a^2e^4 + c^2d^4 + 2ac^2d^2e^2))(-de^5)^{1/2}}{(a^2e^4 + c^2d^4 + 2ac^2d^2e^2))(-de^5)^{1/2}} \frac{1}{(a^2e^4 + c^2d^4 + 2ac^2d^2e^2)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

3.186 $\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=689

$$\frac{\sqrt[4]{c} e^2 (\sqrt{a} e + \sqrt{c} d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2} + \frac{\sqrt[4]{c} e^2 (\sqrt{a} e + \sqrt{c} d) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^2 + cd^2)^2}$$

Rubi [A] time = 0.60, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$\frac{\sqrt{c}(\sqrt{a} + \sqrt{c}d)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}, \frac{\sqrt{c}(\sqrt{a} + \sqrt{c}d)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}, \frac{\sqrt{c}(\sqrt{a} + \sqrt{c}d)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}, \frac{\sqrt{c}(\sqrt{a} + \sqrt{c}d)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}(ae^2 + cd^2)^2}, \frac{e^2(\sqrt{c}d - \sqrt{a})\log^2(1 - \frac{\sqrt{2}\sqrt{c}}{\sqrt{a}}x)}{2\sqrt{2}a^{3/4}(ae^2 + cd^2)}, \frac{e^2(\sqrt{c}d - \sqrt{a})\log^2(\frac{\sqrt{2}\sqrt{c}}{\sqrt{a}}x + 1)}{2\sqrt{2}a^{3/4}(ae^2 + cd^2)}, \frac{e^2(\sqrt{c}d - \sqrt{a})\log^2(1 - \frac{\sqrt{2}\sqrt{c}}{\sqrt{a}}x)}{8\sqrt{2}a^{3/4}(ae^2 + cd^2)}, \frac{e^2(\sqrt{c}d - \sqrt{a})\log^2(\frac{\sqrt{2}\sqrt{c}}{\sqrt{a}}x + 1)}{8\sqrt{2}a^{3/4}(ae^2 + cd^2)}, \frac{cd(a - cd)}{4a(a + cd)\sqrt{ae^2 + cd^2}}, \frac{e^2 \log^2(\frac{\sqrt{c}}{\sqrt{a}})}{\sqrt{c}(ae^2 + cd^2)}$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + c*x^4)^2), x]

[Out] $(c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^{7/2}*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^{1/4}*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{3/4}*(c*d^2 + a*e^2)^2) - (c^{1/4}*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*(c*d^2 + a*e^2)) + (c^{1/4}*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{3/4}*(c*d^2 + a*e^2)^2) + (c^{1/4}*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{7/4}*(c*d^2 + a*e^2)) - (c^{1/4}*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{3/4}*(c*d^2 + a*e^2)^2) - (c^{1/4}*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{7/4}*(c*d^2 + a*e^2)) + (c^{1/4}*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{3/4}*(c*d^2 + a*e^2)^2) + (c^{1/4}*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{7/4}*(c*d^2 + a*e^2))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1179

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx &= \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)^2} - \frac{ce^2(-d+ex^2)}{(cd^2+ae^2)^2(a+cx^4)} \right) dx \\
&= -\frac{(ce^2) \int \frac{-d+ex^2}{a+cx^4} dx}{(cd^2+ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^2} dx}{(cd^2+ae^2)^2} + \frac{c \int \frac{d-ex^2}{(a+cx^4)^2} dx}{cd^2+ae^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right) e^2\right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2(cd^2+ae^2)^2} + \frac{e^2}{\sqrt{d}(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}} - e\right) e^2\right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}} + x^2} dx}{4(cd^2+ae^2)^2} + \frac{e^2}{\sqrt{d}(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d + \sqrt{a}e) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}x)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{e^2}{\sqrt{d}(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{e^2}{\sqrt{d}(cd^2+ae^2)^2} \\
&= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}e^2(\sqrt{c}d - \sqrt{a}e) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{e^2}{\sqrt{d}(cd^2+ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 429, normalized size = 0.62

$$\frac{\sqrt{2}\sqrt[4]{c}(\sqrt{a}\sqrt[4]{c}d + \sqrt{a}e)\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}x)}{a^{3/4}(cd^2+ae^2)^2} + \frac{\sqrt{2}\sqrt[4]{c}(\sqrt{a}\sqrt[4]{c}d - \sqrt{a}e)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{a^{3/4}(cd^2+ae^2)^2} + \frac{2\sqrt{2}\sqrt[4]{c}(\sqrt{a}\sqrt[4]{c}d + \sqrt{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{a^{3/4}(cd^2+ae^2)^2} - \frac{2\sqrt{2}\sqrt[4]{c}(\sqrt{a}\sqrt[4]{c}d - \sqrt{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}}\right)}{a^{3/4}(cd^2+ae^2)^2} + \frac{8c(d-cx^4)(a^2+cd^2)}{a^{3/4}(cd^2+ae^2)^2} + \frac{32e^{7/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]

[Out]
$$\left(\frac{8*c*(c*d^2 + a*e^2)*x*(d - e*x^2)}{a*(a + c*x^4)} + (32*e^{7/2}) * \text{ArcTan}\left[\frac{\text{Sqrt}[e]*x}{\text{Sqrt}[d]}\right] / \text{Sqrt}[d] + (2*\text{Sqrt}[2]*c^{1/4}*(-3*c^{3/2}*d^3 + \text{Sqrt}[a]*c*d^2*e - 7*a*\text{Sqrt}[c]*d*e^2 + 5*a^{3/2}*e^3)*\text{ArcTan}\left[1 - \frac{\text{Sqrt}[2]*c^{1/4}*x}{a^{1/4}}\right]) / a^{7/4} - (2*\text{Sqrt}[2]*c^{1/4}*(-3*c^{3/2}*d^3 + \text{Sqrt}[a]*c*d^2*e - 7*a*\text{Sqrt}[c]*d*e^2 + 5*a^{3/2}*e^3)*\text{ArcTan}\left[1 + \frac{\text{Sqrt}[2]*c^{1/4}*x}{a^{1/4}}\right]) / a^{7/4} - (\text{Sqrt}[2]*c^{1/4}*(3*c^{3/2}*d^3 + \text{Sqrt}[a]*c*d^2*e + 7*a*\text{Sqrt}[c]*d*e^2 + 5*a^{3/2}*e^3)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2]) / a^{7/4} + (\text{Sqrt}[2]*c^{1/4}*(3*c^{3/2}*d^3 + \text{Sqrt}[a]*c*d^2*e + 7*a*\text{Sqrt}[c]*d*e^2 + 5*a^{3/2}*e^3)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2]) / a^{7/4} \right) / (32*(c*d^2 + a*e^2)^2$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)^2),x]

[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 41.28, size = 9892, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\frac{-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*\text{sqrt}((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*\text{sqrt}(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12))/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/ (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*\text{log}(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*\text{sqrt}(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12))/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16))$$

$$\begin{aligned}
& d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} \\
& 0 + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))\sqrt{(6c^3d^5 \\
& *e + 44ac^2d^3e^3 + 70a^2c^2de^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + \\
& 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + \\
& 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + \\
& 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + \\
& 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + \\
& 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + \\
& 4a^6cd^2e^6 + a^7e^8)) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 \\
& + a^3c^2e^4)*x^4)\sqrt{(6c^3d^5e + 44ac^2d^3e^3 + 70a^2c^2de^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 \\
& + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + 2383a^2c^5d^8e^4 \\
& + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + \\
& 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + \\
& 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 \\
& + 4a^6cd^2e^6 + a^7e^8))\log(-(81c^5d^8 + 594ac^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - \\
& 625a^4c^2e^8)*x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - \\
& 175a^6cd^2e^8 + (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + \\
& 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - \\
& 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + \\
& 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} \\
& + 8a^{14}cd^2e^{14} + a^{15}e^{16}))\sqrt{(6c^3d^5e + 44ac^2d^3e^3 + 70a^2c^2de^5 + (a^3c^4d^8 + \\
& 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + \\
& 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + \\
& 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + \\
& 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + \\
& 4a^6cd^2e^6 + a^7e^8)) + (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)*x^4) \\
& \sqrt{(6c^3d^5e + 44ac^2d^3e^3 + 70a^2c^2de^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + \\
& a^7e^8)\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - \\
& 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + \\
& 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + \\
& 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\log(-(81c^5d^8 + 594ac^4d^6e^2 + 1376a^2c^3d^4e^4 + \\
& 750a^3c^2d^2e^6 - 625a^4c^2e^8)*x + (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - \\
& 175a^6cd^2e^8 + (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738ac^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))
\end{aligned}$$

$$\begin{aligned}
& 6 - 175a^6cd^8e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))\sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2cd^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) - (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)x^4)\sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2cd^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\log(-(81c^5d^8 + 594a^4c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^2e^8)x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6cd^2e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}cd^2e^9 + 5a^{11}e^{11})\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16}))\sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2cd^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8)) - 8*(ac^3e^3x^4 + a^2e^3)\sqrt{-e/d}\log((ex^2 + 2dx\sqrt{-e/d} - d)/(ex^2 + d)) - 4*(c^2d^3 + ac^2d^2e^2)x/(a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)x^4), -1/16*(4*(c^2d^2e + ac^2e^3)x^3 - 16*(ac^2e^3x^4 + a^2e^3)\sqrt{e/d}\arctan(x\sqrt{e/d})) + (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4 + (ac^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4)x^4)\sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2cd^2e^5 + (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))\sqrt{-(81c^7d^{12} + 738a^6c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}cd^2e^{14} + a^{15}e^{16})))/(a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6cd^2e^6 + a^7e^8))
\end{aligned}$$

$$\begin{aligned}
& (a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) + (a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4) * x^4) * \sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594a^2c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^2e^8) * x + (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6c^2d^2e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^2d^2e^9 + 5a^{11}e^{11}) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) * \sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) - (a^2c^2d^4 + 2a^3c^2d^2e^2 + a^4e^4 + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^2e^4) * x^4) * \sqrt{((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8)) * \log(-(81c^5d^8 + 594a^2c^4d^6e^2 + 1376a^2c^3d^4e^4 + 750a^3c^2d^2e^6 - 625a^4c^2e^8) * x - (27a^2c^5d^9 + 186a^3c^4d^7e^2 + 404a^4c^3d^5e^4 + 198a^5c^2d^3e^6 - 175a^6c^2d^2e^8 - (a^6c^5d^{10}e + 9a^7c^4d^8e^3 + 26a^8c^3d^6e^5 + 34a^9c^2d^4e^7 + 21a^{10}c^2d^2e^9 + 5a^{11}e^{11}) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16}))/((6c^3d^5e + 44a^2c^2d^3e^3 + 70a^2c^2d^2e^5 - (a^3c^4d^8 + 4a^4c^3d^6e^2 + 6a^5c^2d^4e^4 + 4a^6c^2d^2e^6 + a^7e^8) * \sqrt{-(81c^7d^{12} + 738a^2c^6d^{10}e^2 + 2383a^2c^5d^8e^4 + 2748a^3c^4d^6e^6 - 529a^4c^3d^4e^8 - 1950a^5c^2d^2e^{10} + 625a^6c^2e^{12})/(a^7c^8d^{16} + 8a^8c^7d^{14}e^2 + 28a^9c^6d^{12}e^4 + 56a^{10}c^5d^{10}e^6 + 70a^{11}c^4d^8e^8 + 56a^{12}c^3d^6e^{10} + 28a^{13}c^2d^4e^{12} + 8a^{14}c^2d^2e^{14} + a^{15}e^{16})))
\end{aligned}$$

$$\begin{aligned} & *e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8) * \text{sqrt}(- (81*c^7*d^12 + \\ & 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4* \\ & c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12) / (a^7*c^8*d^16 + 8*a^8 \\ & *c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^ \\ & 8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a \\ & ^15*e^16)) / (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c* \\ & d^2*e^6 + a^7*e^8)) - 4*(c^2*d^3 + a*c*d*e^2)*x / (a^2*c^2*d^4 + 2*a^3*c*d^ \\ & 2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4) \end{aligned}$$

giac [A] time = 0.47, size = 603, normalized size = 0.88

$$\frac{\left(\frac{1}{8} \frac{(a^2 c^2 d^2 + 2 a^2 c^2 d^2 e^2 - (a^2 c^2 d^2 e^2 - 5 (a^2 c^2 d^2 e^2)) \arctan\left(\frac{d^2 c^2 + a^2 d^2}{2 d^2}\right)}{8 (\sqrt{2} a^2 c^2 d^2 + 2 \sqrt{2} a^2 c^2 d^2 e^2 + \sqrt{2} a^2 c^2 d^2)} \right) \left(\frac{1}{8} \frac{(a^2 c^2 d^2 + 7 (a^2 c^2 d^2 e^2 - (a^2 c^2 d^2 e^2 - 5 (a^2 c^2 d^2 e^2)) \arctan\left(\frac{d^2 c^2 + a^2 d^2}{2 d^2}\right))}{8 (\sqrt{2} a^2 c^2 d^2 + 2 \sqrt{2} a^2 c^2 d^2 e^2 + \sqrt{2} a^2 c^2 d^2)} \right) \left(\frac{1}{16} \frac{(a^2 c^2 d^2 + 7 (a^2 c^2 d^2 e^2 + (a^2 c^2 d^2 e^2 + 5 (a^2 c^2 d^2 e^2)) \log(\sqrt{2} + \sqrt{2} x) + \sqrt{2})}{16 (\sqrt{2} a^2 c^2 d^2 + 2 \sqrt{2} a^2 c^2 d^2 e^2 + \sqrt{2} a^2 c^2 d^2)} \right) \left(\frac{1}{16} \frac{(a^2 c^2 d^2 + 7 (a^2 c^2 d^2 e^2 + (a^2 c^2 d^2 e^2 + 5 (a^2 c^2 d^2 e^2)) \log(\sqrt{2} - \sqrt{2} x) + \sqrt{2})}{16 (\sqrt{2} a^2 c^2 d^2 + 2 \sqrt{2} a^2 c^2 d^2 e^2 + \sqrt{2} a^2 c^2 d^2)} \right) \frac{\arctan\left(\frac{d^2 c^2 + a^2 d^2}{2 d^2}\right)^2}{4 (a^2 + 2 a^2 c^2 d^2 + a^2 d^2)} \frac{c^2 d^2 - a^2 d^2}{4 (a^2 + 2 a^2 c^2 d^2 + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $\frac{1}{8} * (3 * (a * c^3)^{(1/4)} * c^3 * d^3 + 7 * (a * c^3)^{(1/4)} * a * c^2 * d * e^2 - (a * c^3)^{(3/4)} * c * d^2 * e - 5 * (a * c^3)^{(3/4)} * a * e^3) * \arctan(1/2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2)) * (a/c)^{(1/4)}) / (a/c)^{(1/4)} / (\text{sqrt}(2) * a^2 * c^4 * d^4 + 2 * \text{sqrt}(2) * a^3 * c^3 * d^2 * e^2 + \text{sqrt}(2) * a^4 * c^2 * e^4) + 1/8 * (3 * (a * c^3)^{(1/4)} * c^3 * d^3 + 7 * (a * c^3)^{(1/4)} * a * c^2 * d * e^2 - (a * c^3)^{(3/4)} * c * d^2 * e - 5 * (a * c^3)^{(3/4)} * a * e^3) * \arctan(1/2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2)) * (a/c)^{(1/4)}) / (a/c)^{(1/4)} / (\text{sqrt}(2) * a^2 * c^4 * d^4 + 2 * \text{sqrt}(2) * a^3 * c^3 * d^2 * e^2 + \text{sqrt}(2) * a^4 * c^2 * e^4) + 1/16 * (3 * (a * c^3)^{(1/4)} * c^3 * d^3 + 7 * (a * c^3)^{(1/4)} * a * c^2 * d * e^2 + (a * c^3)^{(3/4)} * c * d^2 * e + 5 * (a * c^3)^{(3/4)} * a * e^3) * \log(x^2 + \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * a^2 * c^4 * d^4 + 2 * \text{sqrt}(2) * a^3 * c^3 * d^2 * e^2 + \text{sqrt}(2) * a^4 * c^2 * e^4) - 1/16 * (3 * (a * c^3)^{(1/4)} * c^3 * d^3 + 7 * (a * c^3)^{(1/4)} * a * c^2 * d * e^2 + (a * c^3)^{(3/4)} * c * d^2 * e + 5 * (a * c^3)^{(3/4)} * a * e^3) * \log(x^2 - \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * a^2 * c^4 * d^4 + 2 * \text{sqrt}(2) * a^3 * c^3 * d^2 * e^2 + \text{sqrt}(2) * a^4 * c^2 * e^4) + \arctan(x * e^{(1/2)} / \text{sqrt}(d)) * e^{(7/2)} / ((c^2 * d^4 + 2 * a * c * d^2 * e^2 + a^2 * e^4) * \text{sqrt}(d)) - 1/4 * (c * x^3 * e - c * d * x) / ((c * x^4 + a) * (a * c * d^2 + a^2 * e^2))$

maple [A] time = 0.02, size = 873, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)

[Out] $-1/4 * c / (a * e^2 + c * d^2)^2 / (c * x^4 + a) * e^3 * x^3 - 1/4 * c^2 / (a * e^2 + c * d^2)^2 / (c * x^4 + a) * e / a * x^3 * d^2 + 1/4 * c / (a * e^2 + c * d^2)^2 / (c * x^4 + a) * d * x * e^2 + 1/4 * c^2 / (a * e^2 + c * d^2)^2 / (c * x^4 + a) * d^3 / a * x + 7/16 * c / (a * e^2 + c * d^2)^2 / a * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d * e^2 + 3/16 * c^2 / (a * e^2 + c * d^2)^2 / a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^3 + 7/32 * c / (a * e^2 + c * d^2)^2 / a * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}))$

$$\begin{aligned}
& x + (a/c)^{(1/2)}) * d * e^{2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)*2^{(1/2)}} * \ln((x^2+(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})) * d^3+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x+1}) * d * e^{2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x+1}) * d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^{(1/4)*2^{(1/2)}} * \ln((x^2-(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})) * e^3-1/32*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)*2^{(1/2)}} * \ln((x^2-(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)*2^{(1/2)}}*x+(a/c)^{(1/2)})) * d^2 * e^{-5/16}/(a*e^2+c*d^2)^2/(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x-1}) * e^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x-1}) * d^2 * e^{-5/16}/(a*e^2+c*d^2)^2/(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x+1}) * e^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)*2^{(1/2)}} * \arctan(2^{(1/2)}/(a/c)^{(1/4)*x+1}) * d^2 * e + e^4/(a*e^2+c*d^2)^2/(d*e)^{(1/2)} * \arctan(1/(d*e)^{(1/2)} * e * x)
\end{aligned}$$

maxima [A] time = 2.06, size = 506, normalized size = 0.73

$$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{de}} + \frac{2\sqrt{2}\sqrt{3c^{3/2}d^3 - \sqrt{a}cd^2e + 7a\sqrt{c}d^2e^2 - 5a^{3/2}e^3} \arctan\left(\frac{\sqrt{2}\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4}}{\sqrt{a}\sqrt{c}}\right) + 2\sqrt{2}\sqrt{3c^{3/2}d^3 - \sqrt{a}cd^2e + 7a\sqrt{c}d^2e^2 - 5a^{3/2}e^3} \arctan\left(\frac{\sqrt{2}\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4}}{\sqrt{a}\sqrt{c}}\right) + \sqrt{2}\sqrt{3c^{3/2}d^3 + \sqrt{a}cd^2e + 7a\sqrt{c}d^2e^2 + 5a^{3/2}e^3} \log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})}{32(a^2c^2d^4 + 2a^2c^2d^2e^2 + a^4e^4)} - \frac{\sqrt{2}\sqrt{3c^{3/2}d^3 + \sqrt{a}cd^2e + 7a\sqrt{c}d^2e^2 + 5a^{3/2}e^3} \log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})}{4(c^2d^4 + 2acd^2e^2 + a^2e^4) + (a^3c^3d^4 + 2a^2c^2d^2e^2 + a^3c^3e^4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $e^4 * \arctan(ex/\sqrt{de}) / ((c^2d^4 + 2a*c*d^2*e^2 + a^2*e^4) * \sqrt{de}) + 1/32*c*(2*\sqrt{2}*(3*c^{(3/2)}*d^3 - \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d^2*e^2 - 5*a^{(3/2)}*e^3)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*\sqrt{c}*x + \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})) / (\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + 2*\sqrt{2}*(3*c^{(3/2)}*d^3 - \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d^2*e^2 - 5*a^{(3/2)}*e^3)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*\sqrt{c}*x - \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})) / (\sqrt{a}*\sqrt{\sqrt{a}*\sqrt{c}}*\sqrt{c}) + \sqrt{2}*(3*c^{(3/2)}*d^3 + \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d^2*e^2 + 5*a^{(3/2)}*e^3)*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a}) / (a^{(3/4)}*c^{(3/4)}) - \sqrt{2}*(3*c^{(3/2)}*d^3 + \sqrt{a}*c*d^2*e + 7*a*\sqrt{c}*d^2*e^2 + 5*a^{(3/2)}*e^3)*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a}) / (a^{(3/4)}*c^{(3/4)}) / (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d^2*e^2)*x) / (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c^3*e^4)*x^4)$

mupad [B] time = 2.73, size = 17945, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)

[Out] $((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) - \operatorname{atan}(\frac{((65536*a^{11}*c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c$

$$\begin{aligned}
& ^{10}d^{12}e^4 - 36864a^6c^9d^{10}e^6 + 245760a^7c^8d^8e^8 + 634880a^8 \\
& *c^7d^6e^{10} + 663552a^9c^6d^4e^{12} + 331776a^{10}c^5d^2e^{14}) / (256*(a \\
& ^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e \\
& e^4)) - (x*((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c \\
& ^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c)^{(1/2)} \\
& (1/2) + 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4a \\
& ^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} * (65536a^{13} \\
& c^4e^{17} - 65536a^6c^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8 \\
& c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a \\
& ^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15})) / (128*(a^8e^8 + a^4c^4d^8 + \\
& 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a \\
& ^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3 \\
& e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c)^{(1/2)} + 39a^2c^2d^2e^4 * \\
& (-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3 \\
& d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} - (x*(1152a^2c^{11}d^{13}e^2 - 49024a \\
& ^8c^5d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12})) / (128*(a^8e \\
& ^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4) \\
&)) * ((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e \\
& + 44a^5c^2d^3e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c)^{(1/2)} + \\
& 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^ \\
& 2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} - (720a^2c^{10}d^{11}e \\
& ^3 + 20432a^6c^5d^9e^{13} + 4880a^2c^9d^9e^5 + 12320a^3c^8d^7e^7 + \\
& 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}) / (256*(a^8e^8 + a^4c^4d^8 \\
& + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(- \\
& a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^ \\
& 3e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c)^{(1/2)} + 39a^2c^2d^2e^4 \\
& * (-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^ \\
& 3d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} - (x*(1425a^4c^5e^{13} + 81c^9d^8 \\
& e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11})) / (\\
& 128*(a^8e^8 + a^4c^4d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2 \\
& d^4e^4)) * ((9c^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4 \\
& c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c) \\
&)^{(1/2)} + 39a^2c^2d^2e^4(-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4 \\
& a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} * i - (((((\\
& 65536a^{11}c^4e^{16} - 12288a^4c^{11}d^{14}e^2 - 57344a^5c^{10}d^{12}e^4 - 3 \\
& 6864a^6c^9d^{10}e^6 + 245760a^7c^8d^8e^8 + 634880a^8c^7d^6e^{10} + \\
& 663552a^9c^6d^4e^{12} + 331776a^{10}c^5d^2e^{14}) / (256*(a^8e^8 + a^4c^4 \\
& d^8 + 4a^7c^3d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) + (x*((9c \\
& ^3d^6(-a^7c)^{(1/2)} - 25a^3e^6(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^ \\
& 5c^2d^3e^3 + 70a^6c^2d^5e^5 + 41a^7c^2d^4e^2(-a^7c)^{(1/2)} + 39a^2c \\
& ^2d^2e^4(-a^7c)^{(1/2)}) / (256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + \\
& 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4)))^{(1/2)} * (65536a^{13}c^4e^{17} - 65536 \\
& a^6c^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 3 \\
& 27680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13}
\end{aligned}$$

$$\begin{aligned}
& + 327680*a^{12}*c^5*d^2*e^{15})/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 \\
& + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^{(1/2)} - 25 \\
& *a^3*e^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d \\
& *e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(\\
& 256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9* \\
& c^2*d^4*e^4))^{(1/2)} + (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + \\
& 7936*a^3*c^{10}*d^{11}*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666 \\
& 88*a^6*c^7*d^5*e^{10} - 110848*a^7*c^6*d^3*e^{12}))/((128*(a^8*e^8 + a^4*c^4*d^8 \\
& + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(\\
& -a^7*c)^{(1/2)} - 25*a^3*e^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^ \\
& 3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} + 39*a^2*c*d^2*e^4 \\
& *(-a^7*c)^{(1/2)}))/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^ \\
& 3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} - (720*a*c^{10}*d^{11}*e^3 + 20432*a^6*c \\
& ^5*d*e^{13} + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^ \\
& 5*e^9 + 33296*a^5*c^6*d^3*e^{11}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e \\
& ^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^{(1/2)} - \\
& 25*a^3*e^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c \\
& *d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) \\
& /((256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^ \\
& 9*c^2*d^4*e^4))^{(1/2)} + (x*(1425*a^4*c^5*e^{13} + 81*c^9*d^8*e^5 + 612*a*c^8 \\
& *d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^{11}))/((128*(a^8*e^8 + a \\
& ^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9 \\
& *c^3*d^6*(-a^7*c)^{(1/2)} - 25*a^3*e^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44* \\
& a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} + 39*a^2 \\
& *c*d^2*e^4*(-a^7*c)^{(1/2)}))/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 \\
& + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)}*i)/((125*a^2*c^5*e^{12} + 8 \\
& 1*c^7*d^4*e^8 + 270*a*c^6*d^2*e^{10}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d \\
& ^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^{11}*c^4*e^{16} \\
& - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e \\
& ^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9*c^6*d^4* \\
& e^{12} + 331776*a^{10}*c^5*d^2*e^{14}))/((256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2* \\
& e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^{(1/2)} \\
& - 25*a^3*e^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70* \\
& a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^7*c)^{(\\
& 1/2)}))/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + \\
& 6*a^9*c^2*d^4*e^4))^{(1/2)}*(65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}*d^{14}*e^3 \\
& - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9*c^8*d^8*e \\
& ^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680*a^{12}*c^5* \\
& d^2*e^{15}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^ \\
& 2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^{(1/2)} - 25*a^3*e^6*(-a^7*c)^{(\\
& 1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4 \\
& *e^2*(-a^7*c)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}))/(256*(a^{11}*e^8 + a^7 \\
& *c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} \\
&) - (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + 7936*a^3*c^{10}*d^{11} \\
& e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^{10}
\end{aligned}$$

$$\begin{aligned}
& - 110848a^7c^6d^3e^{12}) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 \\
& + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (2 \\
& 56(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} - (720a^2c^9d^9e^5 + 12320a^3c^8d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} - (x * (1425a^4c^5e^{13} + 81c^9d^8e^5 + 612a^2c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11}) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} + (((((65536a^{11}c^4e^{16} - 12288a^4c^{11}d^{14}e^2 - 57344a^5c^{10}d^{12}e^4 - 36864a^6c^9d^{10}e^6 + 245760a^7c^8d^8e^8 + 634880a^8c^7d^6e^{10} + 663552a^9c^6d^4e^{12} + 331776a^{10}c^5d^2e^{14}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) + (x * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} * (65536a^{13}c^4e^{17} - 65536a^6c^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} + (x * (1152a^2c^{11}d^{13}e^2 - 49024a^8c^5d^5e^{14} + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12})) / (128(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2} - (720a^2c^9d^9e^5 + 12320a^3c^8d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * ((9c^3d^6(-a^7c)^{1/2} - 25a^3e^6(-a^7c)^{1/2} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c^2d^4e^5 + 41a^6c^2d^4e^5 * (-a^7c)^{1/2} + 39a^2c^2d^2e^4 * (-a^7c)^{1/2}) / (256(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c^2d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 + 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& + 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10} \\
& *c*d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} + (x*(1425a^4c^5e^{13} \\
& + 81c^9d^8e^5 + 612*a*c^8*d^6e^7 + 1894*a^2*c^7*d^4e^9 + 2532 \\
& *a^3*c^6*d^2e^{11}))/((128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)))*((9c^3d^6*(-a^7c)^{(1/2)} - 25a^3e^6*(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 + 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& + 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}*c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)})))*((9c^3d^6*(-a^7c)^{(1/2)} - 25a^3e^6*(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 + 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& + 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}*c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)})*2i - \operatorname{atan} \\
& (((((((65536a^{11}c^4e^{16} - 12288a^4c^{11}d^{14}e^2 - 57344a^5c^{10}d^{12}e^4 \\
& - 36864a^6c^9d^{10}e^6 + 245760a^7c^8d^8e^8 + 634880a^8c^7d^6e^{10} + 663552a^9c^6d^4e^{12} \\
& + 331776a^{10}c^5d^2e^{14}))/((256*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)) - (x*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}*c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)}*(65536a^{13}c^4e^{17} - 65536a^6c^{11}d^{14}e^3 \\
& - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} \\
& + 589824a^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15}))/((128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 \\
& + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)))*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}*c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (x*(1152a^2c^{11}d^{13}e^2 - 49024a^8c^5d^e^{14} \\
& + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} \\
& - 110848a^7c^6d^3e^{12}))/((128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2d^4e^4)))*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} \\
& + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} \\
& - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}*c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (720a*c^{10}d^{11}e^3 + 20432a^6c^5d^e^{13} \\
& + 4880a^2c^9d^9e^5 + 12320a^3c^8d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11}))/ \\
& (256*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)))*((25a^3e^6 \\
& (-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 \\
& - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 \\
& + a^7c^4d^8 + 4a^{10}*c*d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (x*(1425a^4c^5e^{13} \\
& + 81c^9d^8e^5 + 612*a*c^8*d^6e^7 + 1894*a^2*c^7*d^4e^9 + 2532*a^3*c^6*d^2e^{11}))/ \\
& ((128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)
\end{aligned}$$

$$\begin{aligned}
&)) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e \\
& + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - \\
& 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^ \\
& 2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * i - (((((65536*a^{11} \\
& *c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c \\
& ^9*d^{10}*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9 \\
& *c^6*d^4*e^{12} + 331776*a^{10}*c^5*d^2*e^{14}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a \\
& ^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(- \\
& a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3* \\
& e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(\\
& -a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3* \\
& d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * (65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}* \\
& d^{14}*e^3 - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9* \\
& c^8*d^8*e^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680* \\
& a^{12}*c^5*d^2*e^{15}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c \\
& ^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(\\
& -a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41* \\
& a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{11} \\
& e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^ \\
& 4))^{(1/2)} + (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + 7936*a^3*c \\
& ^{10}*d^{11}*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7 \\
& *d^5*e^{10} - 110848*a^7*c^6*d^3*e^{12}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c \\
& *d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) * ((25*a^3*e^6*(-a^7*c)^{(\\
& 1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70 \\
& *a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(\\
& 1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 \\
& + 6*a^9*c^2*d^4*e^4))^{(1/2)} - (720*a*c^{10}*d^{11}*e^3 + 20432*a^6*c^5*d*e^{13} \\
& + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33 \\
& 296*a^5*c^6*d^3*e^{11}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5 \\
& *c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) * ((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6 \\
& *(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4 \\
& 1*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)}) / (256*(a^{1 \\
& 1}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4* \\
& e^4))^{(1/2)} + (x*(1425*a^4*c^5*e^{13} + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + \\
& 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^{11}) / (128*(a^8*e^8 + a^4*c^4*d^8 \\
& + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) * ((25*a^3*e^6* \\
& (-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^ \\
& 3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4 \\
& *(-a^7*c)^{(1/2)}) / (256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^ \\
& 3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^{(1/2)} * i) / ((125*a^2*c^5*e^{12} + 81*c^7*d^4* \\
& e^8 + 270*a*c^6*d^2*e^{10}) / (128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4 \\
& *a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^{11}*c^4*e^{16} - 12288*a \\
& ^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e^6 + 24576 \\
& 0*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9*c^6*d^4*e^{12} + 331 \\
& 776*a^{10}*c^5*d^2*e^{14}) / (256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^
\end{aligned}$$

$$\begin{aligned}
& 5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)}*(65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}*d^{14}*e^3 - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680*a^{12}*c^5*d^2*e^{15}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)} - (x*(1152*a^2*c^{11}*d^{13}*e^2 - 49024*a^8*c^5*d*e^{14} + 7936*a^3*c^{10}*d^{11}*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^{10} - 110848*a^7*c^6*d^3*e^{12}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)} - (720*a*c^{10}*d^{11}*e^3 + 20432*a^6*c^5*d*e^{13} + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^{11}))/((256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)} - (x*(1425*a^4*c^5*e^{13} + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^{11}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)} + (((((65536*a^{11}*c^4*e^{16} - 12288*a^4*c^{11}*d^{14}*e^2 - 57344*a^5*c^{10}*d^{12}*e^4 - 36864*a^6*c^9*d^{10}*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^{10} + 663552*a^9*c^6*d^4*e^{12} + 331776*a^{10}*c^5*d^2*e^{14}))/((256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^7*c)^{(1/2)})/(256*(a^{11}*e^8 + a^7*c^4*d^8 + 4*a^{10}*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^{(1/2)}*(65536*a^{13}*c^4*e^{17} - 65536*a^6*c^{11}*d^{14}*e^3 - 327680*a^7*c^{10}*d^{12}*e^5 - 589824*a^8*c^9*d^{10}*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^{10}*c^7*d^6*e^{11} + 589824*a^{11}*c^6*d^4*e^{13} + 327680*a^{12}*c^5*d^2*e^{15}))/((128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^{(1/2)} - 9*c^3*d^6*(-a^7*c)^{(1/2)} + 6*a^4*c^3*d^5*e + 44*a
\end{aligned}$$

$$\begin{aligned}
& ^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} - 39a^2* \\
& c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c*d^2e^6 + \\
& 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} + (x*(1152a^2c^{11}d^{13}e^2 \\
& - 49024a^8c^5d^7e^{14} + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + \\
& 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12}))/ \\
& (128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2 \\
& ^2*d^4e^4)))*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} + 6a^4 \\
& c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c) \\
& ^{(1/2)} - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + \\
& 4a^{10}c*d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} - (720*a* \\
& c^{10}d^{11}e^3 + 20432a^6c^5d^7e^{13} + 4880a^2c^9d^9e^5 + 12320a^3c^8 \\
& *d^7e^7 + 21024a^4c^7d^5e^9 + 33296a^5c^6d^3e^{11})/(256*(a^8e^8 + \\
& a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2*d^4e^4)))*((\\
& 25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} + 6a^4c^3d^5e + 44 \\
& a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} - 39a^2* \\
& c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c*d^2e^6 \\
& + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} + (x*(1425a^4c^5e^{13} + \\
& 81c^9d^8e^5 + 612a*c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2 \\
& ^2e^{11}))/((128*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 \\
& + 6a^6c^2*d^4e^4)))*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1 \\
& /2)} + 6a^4c^3d^5e + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4* \\
& e^2*(-a^7c)^{(1/2)} - 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7* \\
& c^4d^8 + 4a^{10}c*d^2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)} \\
&))*((25a^3e^6*(-a^7c)^{(1/2)} - 9c^3d^6*(-a^7c)^{(1/2)} + 6a^4c^3d^5e \\
& + 44a^5c^2d^3e^3 + 70a^6c*d*e^5 - 41a*c^2d^4e^2*(-a^7c)^{(1/2)} - \\
& 39a^2*c*d^2e^4*(-a^7c)^{(1/2)})/(256*(a^{11}e^8 + a^7c^4d^8 + 4a^{10}c*d^ \\
& 2e^6 + 4a^8c^3d^6e^2 + 6a^9c^2d^4e^4))^{(1/2)}*2i + (atan(-((((((4 \\
& 5a*c^{10}d^{11}e^3)/16 + (1277a^6c^5d^7e^{13})/16 + (305a^2c^9d^9e^5)/16 \\
& + (385a^3c^8d^7e^7)/8 + (657a^4c^7d^5e^9)/8 + (2081a^5c^6d^3e^ \\
& 11)/16))/(2*(a^8e^8 + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6 \\
& a^6c^2*d^4e^4)) - (((((256a^{11}c^4e^{16} - 48a^4c^{11}d^{14}e^2 - 224a^ \\
& 5c^{10}d^{12}e^4 - 144a^6c^9d^{10}e^6 + 960a^7c^8d^8e^8 + 2480a^8c^7 \\
& *d^6e^{10} + 2592a^9c^6d^4e^{12} + 1296a^{10}c^5d^2e^{14}))/((2*(a^8e^8 + a \\
& ^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2*d^4e^4)) - (x \\
& *(-d*e^7)^{(1/2)}*(65536a^{13}c^4e^{17} - 65536a^6c^{11}d^{14}e^3 - 327680a^7 \\
& *c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680* \\
& a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15}))/ \\
& (512*(c^2d^5 + a^2d^4e^4 + 2a*c*d^3e^2))*(a^8e^8 + a^4c^4d^8 + 4a^7c* \\
& d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2*d^4e^4))*(-d*e^7)^{(1/2)})/(2*(c^2* \\
& d^5 + a^2d^4e^4 + 2a*c*d^3e^2)) - (x*(1152a^2c^{11}d^{13}e^2 - 49024a^8* \\
& c^5d^7e^{14} + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8* \\
& d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12}))/((256*(a^8e^8 \\
& + a^4c^4d^8 + 4a^7c*d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2*d^4e^4)))* \\
& (-d*e^7)^{(1/2)})/(2*(c^2d^5 + a^2d^4e^4 + 2a*c*d^3e^2))*(-d*e^7)^{(1/2)})/ \\
& (2*(c^2d^5 + a^2d^4e^4 + 2a*c*d^3e^2)) + (x*(1425a^4c^5e^{13} + 81c^9*
\end{aligned}$$

$$\begin{aligned}
& d^8e^5 + 612a^8c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11} \\
&) / (256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} * i / (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2) - \\
& (((((((45a^8c^10d^11e^3)/16 + (1277a^6c^5d^5e^13)/16 + (305a^2c^9d^9e^5)/16 + (385a^3c^8d^7e^7)/8 + (657a^4c^7d^5e^9)/8 + (2081a^5c^6d^3e^11)/16) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) - \\
& ((((((256a^11c^4e^16 - 48a^4c^11d^14e^2 - 224a^5c^10d^12e^4 - 144a^6c^9d^10e^6 + 960a^7c^8d^8e^8 + 2480a^8c^7d^6e^10 + 2592a^9c^6d^4e^12 + 1296a^10c^5d^2e^14) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) + \\
& (x * (-d^7e)^{(1/2)} * (65536a^13c^4e^17 - 65536a^6c^11d^14e^3 - 327680a^7c^10d^12e^5 - 589824a^8c^9d^10e^7 - 327680a^9c^8d^8e^9 + 327680a^10c^7d^6e^11 + 589824a^11c^6d^4e^13 + 327680a^12c^5d^2e^15)) / (512 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2) * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) + \\
& (x * (1152a^2c^11d^13e^2 - 49024a^8c^5d^5e^14 + 7936a^3c^10d^11e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^10 - 110848a^7c^6d^3e^12)) / (256 * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) - \\
& (x * (1425a^4c^5e^13 + 81c^9d^8e^5 + 612a^8c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^11)) / (256 * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} * i / (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2) / (((((((45a^8c^10d^11e^3)/16 + (1277a^6c^5d^5e^13)/16 + (305a^2c^9d^9e^5)/16 + (385a^3c^8d^7e^7)/8 + (657a^4c^7d^5e^9)/8 + (2081a^5c^6d^3e^11)/16) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) - \\
& ((((((256a^11c^4e^16 - 48a^4c^11d^14e^2 - 224a^5c^10d^12e^4 - 144a^6c^9d^10e^6 + 960a^7c^8d^8e^8 + 2480a^8c^7d^6e^10 + 2592a^9c^6d^4e^12 + 1296a^10c^5d^2e^14) / (2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) - \\
& (x * (-d^7e)^{(1/2)} * (65536a^13c^4e^17 - 65536a^6c^11d^14e^3 - 327680a^7c^10d^12e^5 - 589824a^8c^9d^10e^7 - 327680a^9c^8d^8e^9 + 327680a^10c^7d^6e^11 + 589824a^11c^6d^4e^13 + 327680a^12c^5d^2e^15)) / (512 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2) * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) - \\
& (x * (1152a^2c^11d^13e^2 - 49024a^8c^5d^5e^14 + 7936a^3c^10d^11e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^10 - 110848a^7c^6d^3e^12)) / (256 * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) * (-d^7e)^{(1/2)} / (2 * (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)) + \\
& (x * (1425a^4c^5e^13 + 81c^9d^8e^5 + 612a^8c^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^11)) / (256 * (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e)^{(1/2)} / (c^2d^5 + a^2d^4e + 2a^3c^2d^3e^2)
\end{aligned}$$

$$d^3e^2) - \left(\frac{(125a^2c^5e^{12})}{128} + \frac{(81c^7d^4e^8)}{128} + \frac{(135ac^6d^2e^{10})}{64} \right) / (a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4) + \left(\frac{(((((45ac^{10}d^{11}e^3)/16 + (1277a^6c^5d^5e^{13})/16 + (305a^2c^9d^9e^5)/16 + (385a^3c^8d^7e^7)/8 + (657a^4c^7d^5e^9)/8 + (2081a^5c^6d^3e^{11})/16)))/2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)} - \left(\frac{(((((256a^{11}c^4e^{16} - 48a^4c^{11}d^{14}e^2 - 224a^5c^{10}d^{12}e^4 - 144a^6c^9d^{10}e^6 + 960a^7c^8d^8e^8 + 2480a^8c^7d^6e^{10} + 2592a^9c^6d^4e^{12} + 1296a^{10}c^5d^2e^{14}))/2(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)} + (x(-d^7e^7)^{1/2})(65536a^{13}c^4e^{17} - 65536a^6c^{11}d^{14}e^3 - 327680a^7c^{10}d^{12}e^5 - 589824a^8c^9d^{10}e^7 - 327680a^9c^8d^8e^9 + 327680a^{10}c^7d^6e^{11} + 589824a^{11}c^6d^4e^{13} + 327680a^{12}c^5d^2e^{15})))/(512(c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2))(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)} \right) * (-d^7e^7)^{1/2} / (2(c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2)) + (x(1152a^2c^{11}d^{13}e^2 - 49024a^8c^5d^5e^{14} + 7936a^3c^{10}d^{11}e^4 + 20352a^4c^9d^9e^6 + 8704a^5c^8d^7e^8 - 66688a^6c^7d^5e^{10} - 110848a^7c^6d^3e^{12}))/256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e^7)^{1/2} / (2(c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2)) * (-d^7e^7)^{1/2} / (2(c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2)) - (x(1425a^4c^5e^{13} + 81c^9d^8e^5 + 612ac^8d^6e^7 + 1894a^2c^7d^4e^9 + 2532a^3c^6d^2e^{11}))/256(a^8e^8 + a^4c^4d^8 + 4a^7c^2d^2e^6 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4)) * (-d^7e^7)^{1/2} / (c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2)) * (-d^7e^7)^{1/2} * i) / (c^2d^5 + a^2d^4e^4 + 2ac^3d^3e^2)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.187 \quad \int \frac{1}{x^2(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=745

$$\frac{c^{3/4}(\sqrt{c}d - 3\sqrt{a}e) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{9/4}(ae^2 + cd^2)} + \frac{c^{3/4}(\sqrt{c}d - 3\sqrt{a}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{9/4}(ae^2 + cd^2)} + \dots$$

Rubi [A] time = 0.77, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, number of rules / integrand size = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4}(\sqrt{c}d - 3\sqrt{a}e) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{9/4}(ae^2 + cd^2)} + \frac{c^{3/4}(\sqrt{c}d - 3\sqrt{a}e) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{9/4}(ae^2 + cd^2)} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $-(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (e^{9/2}*ArcTan[Sqrt[e]*x]/Sqrt[d])/(d^{3/2}*(c*d^2 + a*e^2)^2) + (c^{3/4}*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)) + (c^{3/4}*(a^{3/2}*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)^2) - (c^{3/4}*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(8*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)) - (c^{3/4}*(a^{3/2}*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)^2) - (c^{3/4}*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)) + (c^{3/4}*(a^{3/2}*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)^2) + (c^{3/4}*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)) - (c^{3/4}*(a^{3/2}*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{9/4}*(c*d^2 + a*e^2)^2)$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2

*p]

Rule 1336

Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] | IntegersQ[m, q])

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} dx &= \int \left(\frac{1}{a^2 dx^2} - \frac{e^5}{d (cd^2 + ae^2)^2 (d + ex^2)} - \frac{c (ae + cdx^2)}{a (cd^2 + ae^2) (a + cx^4)^2} + \frac{c (-a^2 e^3 - cd^2)}{a^2 (cd^2 + ae^2)} \right) dx \\
&= -\frac{1}{a^2 dx} + \frac{c \int \frac{-a^2 e^3 - cd^2 - 2ae^2 x^2}{a + cx^4} dx}{a^2 (cd^2 + ae^2)^2} - \frac{e^5 \int \frac{1}{d + ex^2} dx}{d (cd^2 + ae^2)^2} - \frac{c \int \frac{ae + cdx^2}{(a + cx^4)^2} dx}{a (cd^2 + ae^2)} \\
&= -\frac{1}{a^2 dx} - \frac{cx (ae + cdx^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{e^{9/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)^2} + \frac{c \int \frac{-3ae - cdx^2}{a + cx^4} dx}{4a^2 (cd^2 + ae^2)} + \frac{c}{4a^2 (cd^2 + ae^2)} \\
&= -\frac{1}{a^2 dx} - \frac{cx (ae + cdx^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{e^{9/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)^2} + \frac{\left(c \left(d - \frac{3\sqrt{a} e}{\sqrt{c}} \right) \right) \int \frac{\sqrt{a} \sqrt{c}}{a + cx^4} dx}{8a^2 (cd^2 + ae^2)} \\
&= -\frac{1}{a^2 dx} - \frac{cx (ae + cdx^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{e^{9/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)^2} - \frac{c^{5/4} \left(cd^3 + 2ade^2 - \frac{a^{3/2}}{\sqrt{c}} \right)}{4\sqrt{2} a^2 (cd^2 + ae^2)} \\
&= -\frac{1}{a^2 dx} - \frac{cx (ae + cdx^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{e^{9/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)^2} + \frac{c^{5/4} \left(cd^3 + 2ade^2 + \frac{a^{3/2}}{\sqrt{c}} \right)}{2\sqrt{2} a^{9/4} (cd^2 + ae^2)} \\
&= -\frac{1}{a^2 dx} - \frac{cx (ae + cdx^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} - \frac{e^{9/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 + ae^2)^2} + \frac{c^{3/4} (\sqrt{c} d + 3\sqrt{a} e) \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{8\sqrt{2} a^{9/4} (cd^2 + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 499, normalized size = 0.67

$$\frac{1}{32} \left(\frac{\sqrt{2} e^{2a} (2a^2 d^2 + 3\sqrt{d} a d^2 - 9a\sqrt{d} d^2 - 5a^2 d^2) \log(-\sqrt{2} \sqrt{d} \sqrt{e} x + \sqrt{d} + \sqrt{e} x^2)}{a^{14} (a^2 + c d^2)^2}, \frac{\sqrt{2} e^{2a} (-2a^2 d^2 - 3\sqrt{d} a d^2 + 9a\sqrt{d} d^2 + 5a^2 d^2) \log(\sqrt{2} \sqrt{d} \sqrt{e} x + \sqrt{d} + \sqrt{e} x^2)}{a^{14} (a^2 + c d^2)^2}, \frac{2\sqrt{2} e^{2a} (7a^2 d^2 + 3\sqrt{d} a d^2 + 9a\sqrt{d} d^2 + 5a^2 d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{d} x}{\sqrt{e}}\right)}{a^{14} (a^2 + c d^2)^2}, \frac{2\sqrt{2} e^{2a} (7a^2 d^2 + 3\sqrt{d} a d^2 + 9a\sqrt{d} d^2 + 5a^2 d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{d} x}{\sqrt{e}} + 1\right)}{a^{14} (a^2 + c d^2)^2}, \frac{8c x (a + c d^2)}{a^2 (a + c d^2) (a^2 + c d^2)}, \frac{32}{a^{14} (a^2 + c d^2)^2}, \frac{32 e^{2a} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{d} x}{\sqrt{e}}\right)}{a^{14} (a^2 + c d^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2), x]

[Out]
$$\begin{aligned} & (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) \\ & - (32*e^{(9/2)}*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^{(3/2)}*(c*d^2 + a*e^2)^2) + (2 \\ & *Sqrt[2]*c^{(3/4)}*(5*c^{(3/2)}*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7 \\ & *a^{(3/2)}*e^3)*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}]/(a^{(9/4)}*(c*d^2 + a* \\ & e^2)^2) - (2*Sqrt[2]*c^{(3/4)}*(5*c^{(3/2)}*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[\\ & c]*d*e^2 + 7*a^{(3/2)}*e^3)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}]/(a^{(9/4)} \\ & *(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^{(3/4)}*(-5*c^{(3/2)}*d^3 + 3*Sqrt[a]*c*d^2*e \\ & - 9*a*Sqrt[c]*d*e^2 + 7*a^{(3/2)}*e^3)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}* \\ & x + Sqrt[c]*x^2]/(a^{(9/4)}*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^{(3/4)}*(5*c^{(3/2)} \\ & *d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^{(3/2)}*e^3)*Log[Sqrt[a] + \\ & Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2]/(a^{(9/4)}*(c*d^2 + a*e^2)^2))/32 \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + c*x^4)^2), x]

fricas [B] time = 111.35, size = 10188, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d^4 + 9*a*c \\ & ^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - ((a^2*c \\ & ^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3* \\ & e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d \\ & *e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e \\ & ^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8* \\ & e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + \\ & 2401*a^6*c^3*e^12))/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12* \end{aligned}$$

$$\begin{aligned}
& e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16})) / (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8)) * \log(-(625c^6d^8 + 3250a^5c^5d^6e^2 + 4944a^2c^4d^4e^4 + 686a^3c^3d^2e^6 - 2401a^4c^2e^8) * x + (75a^3c^5d^8e + 418a^4c^4d^6e^3 + 684a^5c^3d^4e^5 + 126a^6c^2d^2e^7 - 343a^7c^2e^9 - (5a^7c^5d^{11} + 29a^8c^4d^9e^2 + 66a^9c^3d^7e^4 + 74a^{10}c^2d^5e^6 + 41a^{11}cd^3e^8 + 9a^{12}de^{10}) * \sqrt{-(625c^9d^{12} + 4050a^8c^8d^{10}e^2 + 8511a^2c^7d^8e^4 + 3868a^3c^6d^6e^6 - 6417a^4c^5d^4e^8 - 3822a^5c^4d^2e^{10} + 2401a^6c^3e^{12})} / (a^9c^8d^{16} + 8a^{10}c^7d^{14}e^2 + 28a^{11}c^6d^{12}e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16}))) * \sqrt{-(30c^4d^5e + 124a^3c^3d^3e^3 + 126a^2c^2de^5 + (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8) * \sqrt{-(625c^9d^{12} + 4050a^8c^8d^{10}e^2 + 8511a^2c^7d^8e^4 + 3868a^3c^6d^6e^6 - 6417a^4c^5d^4e^8 - 3822a^5c^4d^2e^{10} + 2401a^6c^3e^{12})} / (a^9c^8d^{16} + 8a^{10}c^7d^{14}e^2 + 28a^{11}c^6d^{12}e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16}))) / (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8))) + ((a^2c^3d^5 + 2a^3c^2d^3e^2 + a^4cd^2e^4) * x^5 + (a^3c^2d^5 + 2a^4cd^3e^2 + a^5de^4) * x) * \sqrt{-(30c^4d^5e + 124a^3c^3d^3e^3 + 126a^2c^2de^5 + (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8) * \sqrt{-(625c^9d^{12} + 4050a^8c^8d^{10}e^2 + 8511a^2c^7d^8e^4 + 3868a^3c^6d^6e^6 - 6417a^4c^5d^4e^8 - 3822a^5c^4d^2e^{10} + 2401a^6c^3e^{12})} / (a^9c^8d^{16} + 8a^{10}c^7d^{14}e^2 + 28a^{11}c^6d^{12}e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16}))) / (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8)) * \log(-(625c^6d^8 + 3250a^5c^5d^6e^2 + 4944a^2c^4d^4e^4 + 686a^3c^3d^2e^6 - 2401a^4c^2e^8) * x - (75a^3c^5d^8e + 418a^4c^4d^6e^3 + 684a^5c^3d^4e^5 + 126a^6c^2d^2e^7 - 343a^7c^2e^9 - (5a^7c^5d^{11} + 29a^8c^4d^9e^2 + 66a^9c^3d^7e^4 + 74a^{10}c^2d^5e^6 + 41a^{11}cd^3e^8 + 9a^{12}de^{10}) * \sqrt{-(625c^9d^{12} + 4050a^8c^8d^{10}e^2 + 8511a^2c^7d^8e^4 + 3868a^3c^6d^6e^6 - 6417a^4c^5d^4e^8 - 3822a^5c^4d^2e^{10} + 2401a^6c^3e^{12})} / (a^9c^8d^{16} + 8a^{10}c^7d^{14}e^2 + 28a^{11}c^6d^{12}e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16}))) * \sqrt{-(30c^4d^5e + 124a^3c^3d^3e^3 + 126a^2c^2de^5 + (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8) * \sqrt{-(625c^9d^{12} + 4050a^8c^8d^{10}e^2 + 8511a^2c^7d^8e^4 + 3868a^3c^6d^6e^6 - 6417a^4c^5d^4e^8 - 3822a^5c^4d^2e^{10} + 2401a^6c^3e^{12})} / (a^9c^8d^{16} + 8a^{10}c^7d^{14}e^2 + 28a^{11}c^6d^{12}e^4 + 56a^{12}c^5d^{10}e^6 + 70a^{13}c^4d^8e^8 + 56a^{14}c^3d^6e^{10} + 28a^{15}c^2d^4e^{12} + 8a^{16}cd^2e^{14} + a^{17}e^{16}))) / (a^4c^4d^8 + 4a^5c^3d^6e^2 + 6a^6c^2d^4e^4 + 4a^7cd^2e^6 + a^8e^8))) - ((a^2c^3d^5
\end{aligned}$$

$$\begin{aligned}
& + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10))*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10))*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)
\end{aligned}$$

$$\begin{aligned}
&)\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)) - 8*(a^2*c*e^4*x^5 + a^3*e^4*x)*\sqrt{-e/d}*\log((e*x^2 - 2*d*x*\sqrt{-e/d} - d)/(e*x^2 + d))/((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x), -1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + 16*(a^2*c*e^4*x^5 + a^3*e^4*x)*\sqrt{e/d}*\arctan(x*\sqrt{e/d})) \\
& - ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\log(- (625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
\end{aligned}$$

$$\begin{aligned}
& + 4*a^7*c*d^2*e^6 + a^8*e^8)) * \log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 494 \\
& 4*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5 \\
& *d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - \\
& 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + \\
& 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*\sqrt{-(625*c^9*d^ \\
& 12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 64 \\
& 17*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^ \\
& 16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70 \\
& *a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c* \\
& d^2*e^14 + a^17*e^16))) * \sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c \\
& ^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d \\
& ^2*e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7* \\
& d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^ \\
& 10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d \\
& ^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 \\
& + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4 \\
& *a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2 \\
& *c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^ \\
& 3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2 \\
& *d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2 \\
& *e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^ \\
& 8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 \\
& + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^1 \\
& 2*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + \\
& 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a \\
& ^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*\log(-(625* \\
& c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - \\
& 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^ \\
& 3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8* \\
& c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 \\
& + 9*a^12*d*e^10)*\sqrt{-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d \\
& ^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^1 \\
& 0 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^ \\
& 12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 \\
& + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*\sqrt{-(30*c^4*d^5 \\
& *e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e \\
& ^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*\sqrt{-(625*c^9*d^12 + 4 \\
& 050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4 \\
& *c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8 \\
& *a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13* \\
& c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^ \\
& 14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4* \\
& a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^ \\
& 4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*\sqrt{-(30*c^4*d^5*e \\
& + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
\end{aligned}$$

$$\begin{aligned}
& + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 405 \\
& 0*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c \\
& ^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a \\
& ^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^ \\
& 4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 \\
& + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^ \\
& 7*c*d^2*e^6 + a^8*e^8)*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c \\
& ^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e \\
& + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7 \\
& *c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^1 \\
& 0*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 40 \\
& 50*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c \\
& ^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8* \\
& a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^ \\
& ^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^1 \\
& 4 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^ \\
& 5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 \\
& + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 \\
& + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 24 \\
& 01*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 \\
& + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a \\
& ^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^ \\
& 3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)))/((a^2*c^3*d^5 \\
& + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + \\
& a^5*d*e^4)*x)]
\end{aligned}$$

giac [A] time = 0.45, size = 639, normalized size = 0.86

$$\frac{\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}x + \sqrt{2}}{a/c}\right) \sqrt{a^3 c^3 d^4 + 2 \sqrt{2} a^4 c^2 d^2 e^2 + \sqrt{2} a^5 c e^4} + \frac{1}{\sqrt{2}} \arctan\left(\frac{\sqrt{2}x - \sqrt{2}}{a/c}\right) \sqrt{a^3 c^3 d^4 + 2 \sqrt{2} a^4 c^2 d^2 e^2 + \sqrt{2} a^5 c e^4} - \frac{1}{16} \left(3 (a^3 c^3)^{\frac{1}{4}} a^2 c^2 d^2 e - 5 (a^3 c^3)^{\frac{3}{4}} c d^3 + 7 (a^3 c^3)^{\frac{1}{4}} a^2 c e^3 - 9 (a^3 c^3)^{\frac{3}{4}} a d e^2\right) \log(x^2 + \sqrt{2} x \sqrt{a/c} + \sqrt{a/c})}{\sqrt{2} a^3 c^3 d^4 + 2 \sqrt{2} a^4 c^2 d^2 e^2 + \sqrt{2} a^5 c e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/8*(3*(a*c^3)^{(1/4)}*a*c^2*d^2*e + 5*(a*c^3)^{(3/4)}*c*d^3 + 7*(a*c^3)^{(1/4)} \\
& *a^2*c*e^3 + 9*(a*c^3)^{(3/4)}*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/ \\
& c)^{(1/4)))/(a/c)^{(1/4)))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + s \\
& qrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^{(1/4)}*a*c^2*d^2*e + 5*(a*c^3)^{(3/4)}*c*d^ \\
& 3 + 7*(a*c^3)^{(1/4)}*a^2*c*e^3 + 9*(a*c^3)^{(3/4)}*a*d*e^2)*arctan(1/2*sqrt(2) \\
& *(2*x - sqrt(2)*(a/c)^{(1/4)))/(a/c)^{(1/4)))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)* \\
& a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^{(1/4)}*a*c^2*d^2*e - \\
& 5*(a*c^3)^{(3/4)}*c*d^3 + 7*(a*c^3)^{(1/4)}*a^2*c*e^3 - 9*(a*c^3)^{(3/4)}*a*d*e^2 \\
&)*log(x^2 + sqrt(2)*x*(a/c)^{(1/4)} + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqr \\
& t(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^{(1/4)}*a*c^2*d^2 \\
& e - 5*(a*c^3)^{(3/4)}*c*d^3 + 7*(a*c^3)^{(1/4)}*a^2*c*e^3 - 9*(a*c^3)^{(3/4)}*a*
\end{aligned}$$

$$d*e^2)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^3*c^3*d^4 + 2*\sqrt{2}*a^4*c^2*d^2*e^2 + \sqrt{2}*a^5*c*e^4) - \arctan(x*e^{(1/2)}/\sqrt{d})*e^{(9/2)}/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*\sqrt{d}) - 1/4*(5*c^2*d^2*x^4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*d*e^2)*(c*x^5 + a*x))$$

maple [A] time = 0.02, size = 911, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x)

[Out]
$$-1/a^2/d/x - 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*d*e^2 - 1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^3*d^3 - 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3 - 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x*e*d^2 - 7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*e^3 - 3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^2*e - 7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^3 - 3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2*e - 7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^3 - 3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2*e - 9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d*e^2 - 5/32*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^3 - 9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d*e^2 - 5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^3 - 9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^2 - 5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3 - 1/d*e^5/(a*e^2+c*d^2)^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)})*e*x$$

maxima [A] time = 2.11, size = 521, normalized size = 0.70

$$\frac{e^5 \arctan\left(\frac{x}{\sqrt{d}}\right)}{(2d^5 + 2acd^3 + a^2d^2)\sqrt{d}} \left(\frac{2\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}\sqrt{d^5+2acd^3+a^2d^2}}{\sqrt{d}\sqrt{c}\sqrt{e}} \arctan\left(\frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}}{2\sqrt{d}\sqrt{c}}\right) + \frac{2\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}\sqrt{d^5+2acd^3+a^2d^2}}{\sqrt{d}\sqrt{c}\sqrt{e}} \arctan\left(\frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}}{2\sqrt{d}\sqrt{c}}\right)}{\sqrt{d}\sqrt{c}\sqrt{e}} \right) - \frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}\sqrt{d^5+2acd^3+a^2d^2}}{d^{3/2}} \arctan\left(\frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}}{2\sqrt{d}\sqrt{c}}\right) + \frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}\sqrt{d^5+2acd^3+a^2d^2}}{d^{3/2}} \arctan\left(\frac{\sqrt{2}\sqrt{5}\sqrt{d}^{1/4}\sqrt{c}^{3/4}\sqrt{e}^{3/2}}{2\sqrt{d}\sqrt{c}}\right)}{4((a^2c^2d^5 + a^2cd^3)e^2 + (a^2cd^2 + a^2d^2)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out]
$$-e^5*\arctan(e*x/\sqrt{d*e})/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*\sqrt{d*e}) - 1/32*c*(2*\sqrt{2})*(5*\sqrt{2}*\sqrt{a}*c^2*d^3 + 3*a*c^{(3/2)}*d^2*e + 9*a^{(3/2)}*c*d*e^2 + 7*a^2*\sqrt{c}*e^3)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}$$

$$\begin{aligned} &) * c^{(1/4)} / \sqrt{\sqrt{a} * \sqrt{c}} / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c}) + \\ & 2 * \sqrt{2} * (5 * \sqrt{a} * c^2 * d^3 + 3 * a * c^{(3/2)} * d^2 * e + 9 * a^{(3/2)} * c * d * e^2 + 7 * a \\ & ^2 * \sqrt{c} * e^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \\ & \sqrt{\sqrt{a} * \sqrt{c}}) / (\sqrt{a} * \sqrt{\sqrt{a} * \sqrt{c}} * \sqrt{c}) - \sqrt{2} * (5 \\ & * \sqrt{a} * c^2 * d^3 - 3 * a * c^{(3/2)} * d^2 * e + 9 * a^{(3/2)} * c * d * e^2 - 7 * a^2 * \sqrt{c} * e^3) \\ & * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) \\ & + \sqrt{2} * (5 * \sqrt{a} * c^2 * d^3 - 3 * a * c^{(3/2)} * d^2 * e + 9 * a^{(3/2)} * c * d * e^2 - 7 * a \\ & ^2 * \sqrt{c} * e^3) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) \\ & / (a^2 * c^2 * d^4 + 2 * a^3 * c * d^2 * e^2 + a^4 * e^4) - 1/4 * (a * c * d * e * x^2 \\ & + (5 * c^2 * d^2 + 4 * a * c * e^2) * x^4 + 4 * a * c * d^2 + 4 * a^2 * e^2) / ((a^2 * c^2 * d^3 + a^3 \\ & * c * d * e^2) * x^5 + (a^3 * c * d^3 + a^4 * d * e^2) * x) \end{aligned}$$

mupad [B] time = 5.16, size = 24015, normalized size = 32.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)

[Out]
$$\begin{aligned} & - (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))/ \\ & (4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - \operatorname{atan}\left(\frac{(11875*a^5*c^{10}*d^{15}*e - a^9*c^3*(72128*a^3*d*e^{15} + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^{11} - 1785 \right. \\ & 85*a^2*c*d^3*e^{13}) + 68800*a^6*c^9*d^{13}*e^3 + 89403*a^7*c^8*d^{11}*e^5 - 1264 \\ & 88*a^8*c^7*d^9*e^7) * (a^{25}*d^2*e^{19}*x * (- (49*a^3*e^6 * (-a^9*c^3)^{(1/2)} - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3 \\ & *d^3*e^3 - 81*a*c^2*d^4*e^2 * (-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4 * (-a^9*c^3)^{(1/2)}) / (a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6* \\ & a^{11}*c^2*d^4*e^4))^{(5/2)} * 2i - a^{15}*c^2*e^{17}*x * (- (49*a^3*e^6 * (-a^9*c^3)^{(1/2)} \\ &) - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 12 \\ & 4*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2 * (-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4 * (- \\ & a^9*c^3)^{(1/2)}) / (a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6 \\ & *e^2 + 6*a^{11}*c^2*d^4*e^4))^{(1/2)} * 3136i - a^{11}*c^{10}*d^{19}*x * (- (49*a^3*e^6 * (- \\ & a^9*c^3)^{(1/2)} - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c \\ & ^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2 * (-a^9*c^3)^{(1/2)} - 39*a^2 \\ & *c*d^2*e^4 * (-a^9*c^3)^{(1/2)}) / (a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4 \\ & *a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(3/2)} * 25i - a^{16}*c^9*d^{20}*e*x * (- (4 \\ & 9*a^3*e^6 * (-a^9*c^3)^{(1/2)} - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e \\ & + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2 * (-a^9*c^3)^{(1/2)} \\ & - 39*a^2*c*d^2*e^4 * (-a^9*c^3)^{(1/2)}) / (a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c \\ & *d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(5/2)} * 2i + a^{24}*c*d^4* \\ & e^{17}*x * (- (49*a^3*e^6 * (-a^9*c^3)^{(1/2)} - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} + 30*a^5 \\ & *c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2 * (- \\ & a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4 * (-a^9*c^3)^{(1/2)}) / (a^{13}*e^8 + a^9*c^4*d^8 \\ & + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(5/2)} * 14i + \\ & a^8*c^9*d^{14}*e^3*x * (- (49*a^3*e^6 * (-a^9*c^3)^{(1/2)} - 25*c^3*d^6 * (-a^9*c^3)^{(1/2)} \end{aligned}$$

$$\begin{aligned}
& (1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*1250i + a^9*c^8*d^{12}*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*9900i + a^{10}*c^7*d^{10}*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*31902i + a^{11}*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*52008i + a^{12}*c^5*d^6*e^{11}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*42238i + a^{13}*c^4*d^4*e^{13}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*10924i - a^{14}*c^3*d^2*e^{15}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(1/2)}*5694i - a^{12}*c^9*d^{17}*e^{21}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*216i - a^{13}*c^8*d^{15}*e^{14}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*700i - a^{14}*c^7*d^{13}*e^{16}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*808i + a^{15}*c^6*d^{11}*e^{18}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*808i + a^{15}*c^6*d^{11}*e^{18}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)}))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*808i
\end{aligned}$$

$$\begin{aligned}
& 8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4) \\
& ^{(1/2)} * 52008i + a^{12}c^5d^6e^{11} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(1/2)} * 42238i + a^{13}c^4d^4e^{13} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(1/2)} * 10924i - a^{14}c^3d^2e^{15} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(1/2)} * 5694i - a^{12}c^9d^{17}e^2 * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 216i - a^{13}c^8d^{15}e^4 * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 700i - a^{14}c^7d^{13}e^6 * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 808i + a^{15}c^6d^{11}e^8 * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 778i + a^{16}c^5d^9e^{10} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 3224i + a^{17}c^4d^7e^{12} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 3460i + a^{18}c^3d^5e^{14} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 - 81a^2c^2d^4e^2 * (-a^9c^3)^{(1/2)} - 39a^2c^2d^2e^4 * (-a^9c^3)^{(1/2)}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4))^{(3/2)} * 1384i - a^{19}c^2d^3e^{16} * x * (- (49a^3e^6 * (-a^9c^3)^{(1/2)} - 25c^3d^6 * (-a^9c^3)^{(1/2)} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 1
\end{aligned}$$

$$\begin{aligned}
& 24*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(\\
& -a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^ \\
& 6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*57i - a^{17}*c^8*d^{18}*e^3*x*(-(49*a^3*e^6* \\
& (-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7 \\
& *c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a \\
& ^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + \\
& 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)}*14i - a^{18}*c^7*d^{16}*e^5*x* \\
& (- (49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d \\
& ^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3 \\
&)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^ \\
& 12*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)}*40i - a^{19}*c \\
& ^6*d^{14}*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} \\
& + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4 \\
& *e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9* \\
& c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)} \\
&)*56i - a^{20}*c^5*d^{12}*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a \\
& ^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 \\
& - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a \\
& ^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2* \\
& d^4*e^4)^{(5/2)}*28i + a^{21}*c^4*d^{10}*e^{11}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - \\
& 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a \\
& ^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9 \\
& *c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^ \\
& 2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)}*28i + a^{22}*c^3*d^8*e^{13}*x*(-(49*a^3*e^6*(-a^ \\
& 9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2 \\
& *d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} - 39*a^2*c \\
& *d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a \\
& ^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)}*56i + a^{23}*c^2*d^6*e^{15}*x*(-(4 \\
& 9*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e \\
& + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1 \\
& /2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c \\
& *d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(5/2)}*40i - a^{20}*c*d*e \\
& ^{18}*x*(-(49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} + 30*a^5 \\
& *c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a \\
& ^9*c^3)^{(1/2)} - 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)})/(a^{13}*e^8 + a^9*c^4*d^8 \\
& + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4)^{(3/2)}*128i) \\
& / (9765625*a^9*c^{21}*d^{32} + 481890304*a^{25}*c^5*e^{32} + 159765625*a^{10}*c^{20}*d^3 \\
& 0*e^2 + 1159031250*a^{11}*c^{19}*d^{28}*e^4 + 4879001250*a^{12}*c^{18}*d^{26}*e^6 + 130 \\
& 43411775*a^{13}*c^{17}*d^{24}*e^8 + 22507897839*a^{14}*c^{16}*d^{22}*e^{10} + 23209461788 \\
& *a^{15}*c^{15}*d^{20}*e^{12} + 7790140604*a^{16}*c^{14}*d^{18}*e^{14} - 15160518297*a^{17}*c^{ \\
& 13}*d^{16}*e^{16} - 24964288057*a^{18}*c^{12}*d^{14}*e^{18} - 11511478798*a^{19}*c^{11}*d^{12} \\
& *e^{20} + 8613907074*a^{20}*c^{10}*d^{10}*e^{22} + 11397074817*a^{21}*c^9*d^8*e^{24} + 58 \\
& 6708977*a^{22}*c^8*d^6*e^{26} - 3576733440*a^{23}*c^7*d^4*e^{28} - 521228288*a^{24}*c \\
& ^6*d^2*e^{30})*(- (49*a^3*e^6*(-a^9*c^3)^{(1/2)} - 25*c^3*d^6*(-a^9*c^3)^{(1/2)} \\
& + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
\end{aligned}$$

$$\begin{aligned}
& e^2 * (-a^9 * c^3)^{(1/2)} - 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)} / (256 * (a^{13} * e^8 + \\
& a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4)) \\
&)^{(1/2)} * 2i - \operatorname{atan}\left(\frac{(11875 * a^5 * c^{10} * d^{15} * e - a^9 * c^3 * (72128 * a^3 * d * e^{15} + 265 \right.}{655 * c^3 * d^7 * e^9 - 76440 * a * c^2 * d^5 * e^{11} - 178585 * a^2 * c * d^3 * e^{13}) + 68800 * a^6} \\
& * c^9 * d^{13} * e^3 + 89403 * a^7 * c^8 * d^{11} * e^5 - 126488 * a^8 * c^7 * d^9 * e^7) * (a^{25} * d^2 * \\
& e^{19} * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 \\
& * c^4 * d^5 * e + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (- \\
& a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 \\
& + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(5/2)} * 2i - \\
& a^{15} * c^2 * e^{17} * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} \\
&) + 30 * a^5 * c^4 * d^5 * e + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (- \\
& a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 \\
& + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(1/2)} * 3136i - a^{11} * c^{10} * d^{19} * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (- \\
& a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 \\
& + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (\\
& a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(3/2)} * 25i - \\
& a^{16} * c^9 * d^{20} * e * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e \\
& + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (- \\
& a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 \\
& + 6 * a^{11} * c^2 * d^4 * e^4))^{(5/2)} * 2i + a^{24} * c * d^4 * e^{17} * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (- \\
& a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (- \\
& a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 \\
& + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(5/2)} * 14i + a^8 * c^9 * d^{14} * e^3 * x * (-25 * c^3 * \\
& d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e + 126 \\
& * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + \\
& 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 \\
& + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(1/2)} * 1250i + a^9 * c^8 * d^{12} * e^5 * x * (-25 * c^3 * d^6 * (- \\
& a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 \\
& + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (-a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 \\
& + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(1/2)} * 9900i + \\
& a^{10} * c^7 * d^{10} * e^7 * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e \\
& + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (- \\
& a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(1/2)} * 31902i \\
& + a^{11} * c^6 * d^8 * e^9 * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e \\
& + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 * e^4 * (- \\
& a^9 * c^3)^{(1/2)}) / (a^{13} * e^8 + a^9 * c^4 * d^8 + 4 * a^{12} * c * d^2 * e^6 + 4 * a^{10} * c^3 * d^6 * e^2 + 6 * a^{11} * c^2 * d^4 * e^4))^{(1/2)} * 52008i \\
& + a^{12} * c^5 * d^6 * e^{11} * x * (-25 * c^3 * d^6 * (-a^9 * c^3)^{(1/2)} - 49 * a^3 * e^6 * (-a^9 * c^3)^{(1/2)} + 30 * a^5 * c^4 * d^5 * e \\
& + 126 * a^7 * c^2 * d * e^5 + 124 * a^6 * c^3 * d^3 * e^3 + 81 * a * c^2 * d^4 * e^2 * (-a^9 * c^3)^{(1/2)} + 39 * a^2 * c * d^2 *
\end{aligned}$$

$$\begin{aligned}
& \cdot 2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2 \\
& *c*d^2*e^4*(-a^9*c^3)^{(1/2))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4 \\
& *a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(3/2)}*25i - a^{16}*c^9*d^{20}*e*x*(-(2 \\
& 5*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e \\
& + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1 \\
& /2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c \\
& *d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(5/2)}*2i + a^{24}*c*d^4* \\
& e^{17}*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5 \\
& *c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(- \\
& a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d^8 \\
& + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(5/2)}*14i + \\
& a^8*c^9*d^{14}*e^3*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(\\
& 1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c \\
& ^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 \\
& + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4) \\
&)^{(1/2)}*1250i + a^9*c^8*d^{12}*e^5*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3* \\
& e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d \\
& ^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1 \\
& /2)))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^ \\
& 11*c^2*d^4*e^4))^{(1/2)}*9900i + a^{10}*c^7*d^{10}*e^7*x*(-(25*c^3*d^6*(-a^9*c^3) \\
& ^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 \\
& + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e \\
& ^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^ \\
& 3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(1/2)}*31902i + a^{11}*c^6*d^8*e^9*x*(-(25*c^ \\
& 3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 1 \\
& 26*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} \\
& + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2 \\
& *e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(1/2)}*52008i + a^{12}*c^5*d^ \\
& 6*e^{11}*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30* \\
& a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2* \\
& (-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d \\
& ^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(1/2)}*422 \\
& 38i + a^{13}*c^4*d^4*e^{13}*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9 \\
& *c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + \\
& 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^1 \\
& 3*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^ \\
& 4*e^4))^{(1/2)}*10924i - a^{14}*c^3*d^2*e^{15}*x*(-(25*c^3*d^6*(-a^9*c^3)^{(1/2)} - \\
& 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a \\
& ^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2*c*d^2*e^4*(-a^9 \\
& *c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4*a^{10}*c^3*d^6*e^ \\
& 2 + 6*a^{11}*c^2*d^4*e^4))^{(1/2)}*5694i - a^{12}*c^9*d^{17}*e^2*x*(-(25*c^3*d^6*(- \\
& a^9*c^3)^{(1/2)} - 49*a^3*e^6*(-a^9*c^3)^{(1/2)} + 30*a^5*c^4*d^5*e + 126*a^7*c \\
& ^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^{(1/2)} + 39*a^2 \\
& *c*d^2*e^4*(-a^9*c^3)^{(1/2)))/(a^{13}*e^8 + a^9*c^4*d^8 + 4*a^{12}*c*d^2*e^6 + 4 \\
& *a^{10}*c^3*d^6*e^2 + 6*a^{11}*c^2*d^4*e^4))^{(3/2)}*216i - a^{13}*c^8*d^{15}*e^4*x*(
\end{aligned}$$

$$\begin{aligned}
& -(25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 700i - a^{14}c^7d^{13}e^6 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 808i + a^{15}c^6d^{11}e^8 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 778i + a^{16}c^5d^9e^{10} * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 3224i + a^{17}c^4d^7e^{12} * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 3460i + a^{18}c^3d^5e^{14} * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 1384i - a^{19}c^2d^3e^{16} * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{3/2} * 57i - a^{17}c^8d^{18}e^3 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{5/2} * 14i - a^{18}c^7d^{16}e^5 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{5/2} * 40i - a^{19}c^6d^{14}e^7 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}c^2d^2e^6 + 4a^{10}c^3d^6e^2 + 6a^{11}c^2d^4e^4)^{5/2} * 56i - a^{20}c^5d^{12}e^9 * x * (-25c^3d^6(-a^9c^3)^{1/2} - 49a^3e^6(-a^9c^3)^{1/2} + 30a^5c^4d^5e + 126a^7c^2d^2e^5 + 124a^6c^3d^3e^3 + 81a^2c^2d^4e^2(-a^9c^3)^{1/2} + 39a^2c^2d^2e^4(-a^9c^3)^{1/2}) / (a^{13}e^8 + a^9c^4d^8 + 4a^{12}
\end{aligned}$$

```

*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*28i + a^21*c^4
*d^10*e^11*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) +
  30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*
e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2)))/(a^13*e^8 + a^9*c
^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)
*28i + a^22*c^3*d^8*e^13*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^
9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 +
  81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2)))/(a^
13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d
^4*e^4))^(5/2)*56i + a^23*c^2*d^6*e^15*x*(-(25*c^3*d^6*(-a^9*c^3)^(1/2) - 4
9*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6
*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e^4*(-a^9*c
^3)^(1/2)))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2
+ 6*a^11*c^2*d^4*e^4))^(5/2)*40i - a^20*c*d*e^18*x*(-(25*c^3*d^6*(-a^9*c^3)
^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5
+ 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2*c*d^2*e
^4*(-a^9*c^3)^(1/2)))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^
3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*128i))/(9765625*a^9*c^21*d^32 + 4818
90304*a^25*c^5*e^32 + 159765625*a^10*c^20*d^30*e^2 + 1159031250*a^11*c^19*d
^28*e^4 + 4879001250*a^12*c^18*d^26*e^6 + 13043411775*a^13*c^17*d^24*e^8 +
22507897839*a^14*c^16*d^22*e^10 + 23209461788*a^15*c^15*d^20*e^12 + 7790140
604*a^16*c^14*d^18*e^14 - 15160518297*a^17*c^13*d^16*e^16 - 24964288057*a^1
8*c^12*d^14*e^18 - 11511478798*a^19*c^11*d^12*e^20 + 8613907074*a^20*c^10*d
^10*e^22 + 11397074817*a^21*c^9*d^8*e^24 + 586708977*a^22*c^8*d^6*e^26 - 35
76733440*a^23*c^7*d^4*e^28 - 521228288*a^24*c^6*d^2*e^30))*(-(25*c^3*d^6*(-
a^9*c^3)^(1/2) - 49*a^3*e^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 + 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2)))/(256*(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^
6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*2i - (atan((a^9*e^3*x*
(-d^3*e^9)^(5/2)*4096i - a^3*c^6*d^15*x*(-d^3*e^9)^(3/2)*26804i + c^9*d^24*
e^3*x*(-d^3*e^9)^(1/2)*625i - a^4*c^5*d^13*e^2*x*(-d^3*e^9)^(3/2)*24831i -
a^5*c^4*d^11*e^4*x*(-d^3*e^9)^(3/2)*8214i + a^6*c^3*d^9*e^6*x*(-d^3*e^9)^(3
/2)*13471i + a^7*c^2*d^7*e^8*x*(-d^3*e^9)^(3/2)*16128i + a^2*c^7*d^20*e^7*x
*(-d^3*e^9)^(1/2)*15951i + a*c^8*d^22*e^5*x*(-d^3*e^9)^(1/2)*4950i)/(4096*a
^9*d^8*e^25 + 625*c^9*d^26*e^7 + 4950*a*c^8*d^24*e^9 + 15951*a^2*c^7*d^22*e
^11 + 26804*a^3*c^6*d^20*e^13 + 24831*a^4*c^5*d^18*e^15 + 8214*a^5*c^4*d^16
*e^17 - 13471*a^6*c^3*d^14*e^19 - 16128*a^7*c^2*d^12*e^21))*(-d^3*e^9)^(1/2
)*1i)/(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.188 \quad \int \frac{1}{x^4(d+ex^2)(a+cx^4)^2} dx$$

Optimal. Leaf size=751

$$\frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d)(2ae^2 + cd^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{11/4}(ae^2 + cd^2)^2} + \frac{c^{5/4}(\sqrt{a}e + 3\sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{11/4}(ae^2 + cd^2)}$$

Rubi [A] time = 0.69, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}(\sqrt{a}e + \sqrt{c}d)(2ae^2 + cd^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{11/4}(ae^2 + cd^2)^2} + \frac{c^{5/4}(\sqrt{a}e + 3\sqrt{c}d) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{11/4}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2), x]

[Out]
$$-1/(3a^2d*x^3) + e/(a^2d^2*x) - (c^2*x*(d - e*x^2))/(4a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^{(11/2)*\text{ArcTan}[\text{Sqrt}[e]*x]/\text{Sqrt}[d]})/(d^{(5/2)*(c*d^2 + a*e^2)^2}) + (c^{(5/4)*(3*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})])/(8*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)}) + (c^{(5/4)*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})])/(2*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)^2}) - (c^{(5/4)*(3*\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})])/(8*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)}) - (c^{(5/4)*(\text{Sqrt}[c]*d - \text{Sqrt}[a]*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})])/(2*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)^2}) + (c^{(5/4)*(3*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2]})/(16*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)}) + (c^{(5/4)*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2]})/(4*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)^2}) - (c^{(5/4)*(3*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2]})/(16*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)}) - (c^{(5/4)*(\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2]})/(4*\text{Sqrt}[2]*a^{(11/4)*(c*d^2 + a*e^2)^2})$$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1179

Int[((d_) + (e_)*(x_)^2)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2

*p]

Rule 1336

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] | IntegersQ[m, q])

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (d + ex^2) (a + cx^4)^2} dx &= \int \left(\frac{1}{a^2 dx^4} - \frac{e}{a^2 d^2 x^2} + \frac{e^6}{d^2 (cd^2 + ae^2)^2 (d + ex^2)} - \frac{c^2 (d - ex^2)}{a (cd^2 + ae^2) (a + cx^4)^2} - \frac{c^2}{a^2} \right) dx \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} + \frac{e^6 \int \frac{1}{d+ex^2} dx}{d^2 (cd^2 + ae^2)^2} - \frac{c^2 \int \frac{d-ex^2}{(a+cx^4)^2} dx}{a (cd^2 + ae^2)} - \frac{(c^2 (cd^2 + 2ae^2)) \int \frac{d-ex^2}{a+cx^4} dx}{a^2 (cd^2 + ae^2)^2} \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{e^{11/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)^2} + \frac{c^2 \int \frac{-3d+ex^2}{a+cx^4} dx}{4a^2 (cd^2 + ae^2)^2} \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{e^{11/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)^2} - \frac{c \left(\frac{3\sqrt{c} d}{\sqrt{a}} - e \right)}{8a^2 (cd^2 + ae^2)^2} \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{e^{11/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)^2} + \frac{c^{5/4} \left(\frac{\sqrt{c} d}{\sqrt{a}} + e \right)}{8a^2 (cd^2 + ae^2)^2} \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{e^{11/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)^2} + \frac{c^{5/4} \left(\frac{\sqrt{c} d}{\sqrt{a}} - e \right)}{8a^2 (cd^2 + ae^2)^2} \\
&= -\frac{1}{3a^2 dx^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x (d - ex^2)}{4a^2 (cd^2 + ae^2) (a + cx^4)} + \frac{e^{11/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{5/2} (cd^2 + ae^2)^2} + \frac{c^{5/4} \left(\frac{3\sqrt{c} d}{\sqrt{a}} - e \right)}{8\sqrt{2} a^2 (cd^2 + ae^2)^2}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 513, normalized size = 0.68

$$\frac{1}{96} \left(\frac{3\sqrt{2}e^{2x}(5a^2d^2 + 5\sqrt{d}cd^2 + 11a\sqrt{d}d^2 + 7c^2d^2) \log(-\sqrt{2}\sqrt{d}\sqrt{x + \sqrt{d}} + \sqrt{d}e^x)}{a^{11/4}(a^2 + cd^2)} - \frac{3\sqrt{2}e^{2x}(5a^2d^2 + 5\sqrt{d}cd^2 + 11a\sqrt{d}d^2 + 7c^2d^2) \log(\sqrt{2}\sqrt{d}\sqrt{x + \sqrt{d}} + \sqrt{d}e^x)}{a^{11/4}(a^2 + cd^2)} + \frac{6\sqrt{2}e^{2x}(-5a^2d^2 - 5\sqrt{d}cd^2 + 11a\sqrt{d}d^2 + 7c^2d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{d}}{\sqrt{x + \sqrt{d}}}\right)}{a^{11/4}(a^2 + cd^2)} - \frac{6\sqrt{2}e^{2x}(5a^2d^2 + 5\sqrt{d}cd^2 + 11a\sqrt{d}d^2 + 7c^2d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}}{\sqrt{x + \sqrt{d}}}\right)}{a^{11/4}(a^2 + cd^2)} - \frac{24cdx(d - cd^2)}{d^2(d + cd^2)(a^2 + cd^2)} + \frac{96c}{d^2d^2} - \frac{32}{d^2d^2} + \frac{96e^{11/2} \tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{x + \sqrt{d}}}\right)}{d^{11/4}(a^2 + cd^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] $(-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (96*e^{(11/2)}*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^{(5/2)}*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^{(5/4)}*(7*c^{(3/2)}*d^3 - 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 - 9*a^{(3/2)}*e^3)*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(a^{(11/4)}*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^{(5/4)}*(-7*c^{(3/2)}*d^3 + 5*Sqrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^{(3/2)}*e^3)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(a^{(11/4)}*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^{(5/4)}*(7*c^{(3/2)}*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^{(3/2)}*e^3)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(a^{(11/4)}*(c*d^2 + a*e^2)^2) - (3*Sqrt[2]*c^{(5/4)}*(7*c^{(3/2)}*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^{(3/2)}*e^3)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(a^{(11/4)}*(c*d^2 + a*e^2)^2))/96$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.50, size = 628, normalized size = 0.84

$$\frac{\left(\frac{7}{8}(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} - 5(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d} - 9(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d}\right) \arctan\left(\frac{\sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d}}{\sqrt{d} \sqrt{d}}\right)}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} - \frac{\left(\frac{7}{8}(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} - 5(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d} - 9(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d}\right) \arctan\left(\frac{\sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d}}{\sqrt{d} \sqrt{d}}\right)}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} + \frac{\left(\frac{7}{8}(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} - 5(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d} - 9(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d}\right) \log\left(\frac{d^2 + \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d}}{\sqrt{d}}\right)}{16(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} + \frac{\left(\frac{7}{8}(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} - 5(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d} - 9(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d} \sqrt{d}\right) \log\left(\frac{d^2 - \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d}}{\sqrt{d}}\right)}{16(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} + \frac{\arctan\left(\frac{\sqrt{d}}{\sqrt{d}}\right)}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} + \frac{\frac{d^2 + 5d \sqrt{d} \sqrt{d}}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}}}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}} + \frac{\frac{d^2 - 5d \sqrt{d} \sqrt{d}}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}}}{8(\sqrt{d}x^2 + 1) \sqrt{d} \sqrt{d^2 + 5d} \sqrt{d} \sqrt{d} \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$-1/8*(7*(a*c^3)^{(1/4)}*c^3*d^3 + 11*(a*c^3)^{(1/4)}*a*c^2*d*e^2 - 5*(a*c^3)^{(3/4)}*c*d^2*e - 9*(a*c^3)^{(3/4)}*a*e^3)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^3*c^3*d^4 + 2*\sqrt{2}*a^4*c^2*d^2*e^2 + \sqrt{2}*a^5*c*e^4) - 1/8*(7*(a*c^3)^{(1/4)}*c^3*d^3 + 11*(a*c^3)^{(1/4)}*a*c^2*d*e^2 - 5*(a*c^3)^{(3/4)}*c*d^2*e - 9*(a*c^3)^{(3/4)}*a*e^3)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^3*c^3*d^4 + 2*\sqrt{2}*a^4*c^2*d^2*e^2 + \sqrt{2}*a^5*c*e^4) - 1/16*(7*(a*c^3)^{(1/4)}*c^3*d^3 + 11*(a*c^3)^{(1/4)}*a*c^2*d*e^2 + 5*(a*c^3)^{(3/4)}*c*d^2*e + 9*(a*c^3)^{(3/4)}*a*e^3)*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^3*c^3*d^4 + 2*\sqrt{2}*a^4*c^2*d^2*e^2 + \sqrt{2}*a^5*c*e^4) + 1/16*(7*(a*c^3)^{(1/4)}*c^3*d^3 + 11*(a*c^3)^{(1/4)}*a*c^2*d*e^2 + 5*(a*c^3)^{(3/4)}*c*d^2*e + 9*(a*c^3)^{(3/4)}*a*e^3)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(\sqrt{2}*a^3*c^3*d^4 + 2*\sqrt{2}*a^4*c^2*d^2*e^2 + \sqrt{2}*a^5*c*e^4) + \arctan(x*e^{(1/2)}/\sqrt{d})*e^{(1/2)}/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*\sqrt{d}) + 1/4*(c^2*x^3*e - c^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*e - d)/(a^2*d^2*x^3)$$

maple [A] time = 0.02, size = 932, normalized size = 1.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x)

[Out]
$$-1/3/a^2/d/x^3 + e/a^2/d^2/x + 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*e^3 + 1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^3*e*d^2 - 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x*d*e^2 - 1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x*d^3 - 11/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d*e^2 - 7/16*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^3 - 11/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d*e^2 - 7/32*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^3 - 11/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^2 - 7/16*c^3/(a*e^2+c*d^2)^2/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3 + 9/32*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*e^3 + 5/32*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^2*e + 9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^3 + 5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2*e + 9/16*c/(a*e^2+c*d^2)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^3 + 5/16*c^2/(a*e^2+c*d^2)^2/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2*e + 1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)})*e*x$$

)

maxima [A] time = 2.11, size = 543, normalized size = 0.72

$$\frac{e^6 \arctan\left(\frac{ex}{\sqrt{d+e}}\right)}{(c^2e^6 + 2ac^3d^2e^2 + a^2d^4e^4)\sqrt{d+e}} - \frac{2\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}}{\sqrt{d+e}\sqrt{d+e}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}}{2\sqrt{d+e}}\right) + \frac{2\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}\sqrt{d+e}}{\sqrt{d+e}\sqrt{d+e}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}}{2\sqrt{d+e}}\right) + \frac{2\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}\sqrt{d+e}}{\sqrt{d+e}\sqrt{d+e}} \arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e}\sqrt{d+11e}\sqrt{d+e}}{2\sqrt{d+e}}\right)}{32(c^2d^6 + 2a^3cd^2e^2 + a^4e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $e^6 \arctan\left(\frac{ex}{\sqrt{d+e}}\right) / ((c^2d^6 + 2a^3cd^2e^2 + a^2d^4e^4)\sqrt{d+e}) - \frac{1}{32}c^2(2\sqrt{2})(7c^{3/2}d^3 - 5\sqrt{a}cd^2e + 11a\sqrt{c}d^2e^2 - 9a^{3/2}e^3) \arctan\left(\frac{1/2\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4})}{\sqrt{a}\sqrt{c}}\right) / (\sqrt{a}\sqrt{c}) + 2\sqrt{2}(7c^{3/2}d^3 - 5\sqrt{a}cd^2e + 11a\sqrt{c}d^2e^2 - 9a^{3/2}e^3) \arctan\left(\frac{1/2\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4})}{\sqrt{a}\sqrt{c}}\right) / (\sqrt{a}\sqrt{c}) + \sqrt{2}(7c^{3/2}d^3 + 5\sqrt{a}cd^2e + 11a\sqrt{c}d^2e^2 + 9a^{3/2}e^3) \log\left(\frac{\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a}}{a^{3/4}c^{3/4}}\right) - \sqrt{2}(7c^{3/2}d^3 + 5\sqrt{a}cd^2e + 11a\sqrt{c}d^2e^2 + 9a^{3/2}e^3) \log\left(\frac{\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a}}{a^{3/4}c^{3/4}}\right) / (a^2c^2d^4 + 2a^3cd^2e^2 + a^4e^4) + \frac{1}{12}(3(5c^2d^2e + 4ac^3e^3)x^6 - 4a^2cd^3 - 4a^2d^2e^2 - (7c^2d^3 + 4ac^3d^2e^2)x^4 + 12(a^2cd^2e + a^2e^3)x^2) / ((a^2c^2d^4 + a^3cd^2e^2)x^7 + (a^3cd^4 + a^4d^2e^2)x^3)$

mupad [B] time = 5.22, size = 20828, normalized size = 27.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)

[Out] $\operatorname{atan}\left(\frac{(x(4917248a^{10}c^{18}d^{36}e^5 + 50677760a^{11}c^{17}d^{34}e^7 + 230498304a^{12}c^{16}d^{32}e^9 + 607559680a^{13}c^{15}d^{30}e^{11} + 1026486272a^{14}c^{14}d^{28}e^{13} + 1166602240a^{15}c^{13}d^{26}e^{15} + 923508736a^{16}c^{12}d^{24}e^{17} + 539500544a^{17}c^{11}d^{22}e^{19} + 259409920a^{18}c^{10}d^{20}e^{21} + 109709312a^{19}c^9d^{18}e^{23} + 34537472a^{20}c^8d^{16}e^{25} + 5308416a^{21}c^7d^{14}e^{27}) - ((81a^3e^6(-a^{11}c^5)^{1/2} - 49c^3d^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} - 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))\right)^{1/2} \cdot \left(\frac{x(1787297792a^{19}c^{13}d^{31}e^{12} - 147587072a^{15}c^{17}d^{39}e^4 - 698089472a^{16}c^{16}d^{37}e^6 - 1660157952a^{17}c^{15}d^{35}e^8 - 1588068352a^{18}c^{14}d^{33}e^{10} - 12845056a^{14}c^{18}d^{41}e^2 + 7839678464a^{20}c^{12}e^8)}{(a^2c^2d^4 + a^3cd^2e^2)x^7 + (a^3cd^4 + a^4d^2e^2)x^3}\right)$

$$\begin{aligned}
& d^{29}e^{14} + 11879841792a^{21}c^{11}d^{27}e^{16} + 10631249920a^{22}c^{10}d^{25}e^{18} + 6274940928a^{23}c^9d^{23}e^{20} + 2652110848a^{24}c^8d^{21}e^{22} + 891027456a^{25}c^7d^{19}e^{24} + 234881024a^{26}c^6d^{17}e^{26} + 33554432a^{27}c^5d^{15}e^{28}) + ((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)))/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}(x*((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)))/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}(134217728a^{20}c^{16}d^{42}e^3 + 1342177280a^{21}c^{15}d^{40}e^5 + 5905580032a^{22}c^{14}d^{38}e^7 + 14763950080a^{23}c^{13}d^{36}e^9 + 22145925120a^{24}c^{12}d^{34}e^{11} + 17716740096a^{25}c^{11}d^{32}e^{13} - 17716740096a^{27}c^9d^{28}e^{17} - 22145925120a^{28}c^8d^{26}e^{19} - 14763950080a^{29}c^7d^{24}e^{21} - 5905580032a^{30}c^6d^{22}e^{23} - 1342177280a^{31}c^5d^{20}e^{25} - 134217728a^{32}c^4d^{18}e^{27}) + 29360128a^{17}c^{17}d^{42}e^2 + 239075328a^{18}c^{16}d^{40}e^4 + 708837376a^{19}c^{15}d^{38}e^6 + 465567744a^{20}c^{14}d^{36}e^8 - 2726297600a^{21}c^{13}d^{34}e^{10} - 9084862464a^{22}c^{12}d^{32}e^{12} - 13614710784a^{23}c^{11}d^{30}e^{14} - 10745806848a^{24}c^{10}d^{28}e^{16} - 2403336192a^{25}c^9d^{26}e^{18} + 3879731200a^{26}c^8d^{24}e^{20} + 4517265408a^{27}c^7d^{22}e^{22} + 2294284288a^{28}c^6d^{20}e^{24} + 603979776a^{29}c^5d^{18}e^{26} + 67108864a^{30}c^4d^{16}e^{28}))((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)))/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} + 7225344a^{12}c^{18}d^{39}e^3 + 76972032a^{13}c^{17}d^{37}e^5 + 367607808a^{14}c^{16}d^{35}e^7 + 1036910592a^{15}c^{15}d^{33}e^9 + 1876983808a^{16}c^{14}d^{31}e^{11} + 2115436544a^{17}c^{13}d^{29}e^{13} + 1052803072a^{18}c^{12}d^{27}e^{15} - 848429056a^{19}c^{11}d^{25}e^{17} - 2105458688a^{20}c^{10}d^{23}e^{19} - 1909030912a^{21}c^9d^{21}e^{21} - 959037440a^{22}c^8d^{19}e^{23} - 262144000a^{23}c^7d^{17}e^{25} - 30408704a^{24}c^6d^{15}e^{27}))((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)))/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*i + (x*(4917248a^{10}c^{18}d^{36}e^5 + 50677760a^{11}c^{17}d^{34}e^7 + 230498304a^{12}c^{16}d^{32}e^9 + 607559680a^{13}c^{15}d^{30}e^{11} + 1026486272a^{14}c^{14}d^{28}e^{13} + 1166602240a^{15}c^{13}d^{26}e^{15} + 923508736a^{16}c^{12}d^{24}e^{17} + 539500544a^{17}c^{11}d^{22}e^{19} + 259409920a^{18}c^{10}d^{20}e^{21} + 109709312a^{19}c^9d^{18}e^{23} + 34537472a^{20}c^8d^{16}e^{25} + 5308416a^{21}c^7d^{14}e^{27}) - ((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)))/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}((x*(1
\end{aligned}$$

$$\begin{aligned}
& 787297792*a^{19}*c^{13}*d^{31}*e^{12} - 147587072*a^{15}*c^{17}*d^{39}*e^4 - 698089472*a^{16}*c^{16}*d^{37}*e^6 - 1660157952*a^{17}*c^{15}*d^{35}*e^8 - 1588068352*a^{18}*c^{14}*d^{33}*e^{10} - 12845056*a^{14}*c^{18}*d^{41}*e^2 + 7839678464*a^{20}*c^{12}*d^{29}*e^{14} + 11879841792*a^{21}*c^{11}*d^{27}*e^{16} + 10631249920*a^{22}*c^{10}*d^{25}*e^{18} + 6274940928*a^{23}*c^9*d^{23}*e^{20} + 2652110848*a^{24}*c^8*d^{21}*e^{22} + 891027456*a^{25}*c^7*d^{19}*e^{24} + 234881024*a^{26}*c^6*d^{17}*e^{26} + 33554432*a^{27}*c^5*d^{15}*e^{28}) - ((81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} - 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)})/(256*(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4)))^{(1/2)}*(29360128*a^{17}*c^{17}*d^{42}*e^2 - x*((81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} - 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)}))/(256*(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4)))^{(1/2)}*(134217728*a^{20}*c^{16}*d^{42}*e^3 + 1342177280*a^{21}*c^{15}*d^{40}*e^5 + 5905580032*a^{22}*c^{14}*d^{38}*e^7 + 14763950080*a^{23}*c^{13}*d^{36}*e^9 + 22145925120*a^{24}*c^{12}*d^{34}*e^{11} + 17716740096*a^{25}*c^{11}*d^{32}*e^{13} - 17716740096*a^{27}*c^9*d^{28}*e^{17} - 22145925120*a^{28}*c^8*d^{26}*e^{19} - 14763950080*a^{29}*c^7*d^{24}*e^{21} - 5905580032*a^{30}*c^6*d^{22}*e^{23} - 1342177280*a^{31}*c^5*d^{20}*e^{25} - 134217728*a^{32}*c^4*d^{18}*e^{27}) + 239075328*a^{18}*c^{16}*d^{40}*e^4 + 708837376*a^{19}*c^{15}*d^{38}*e^6 + 465567744*a^{20}*c^{14}*d^{36}*e^8 - 2726297600*a^{21}*c^{13}*d^{34}*e^{10} - 9084862464*a^{22}*c^{12}*d^{32}*e^{12} - 13614710784*a^{23}*c^{11}*d^{30}*e^{14} - 10745806848*a^{24}*c^{10}*d^{28}*e^{16} - 2403336192*a^{25}*c^9*d^{26}*e^{18} + 3879731200*a^{26}*c^8*d^{24}*e^{20} + 4517265408*a^{27}*c^7*d^{22}*e^{22} + 2294284288*a^{28}*c^6*d^{20}*e^{24} + 603979776*a^{29}*c^5*d^{18}*e^{26} + 67108864*a^{30}*c^4*d^{16}*e^{28}))*((81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} - 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)})/(256*(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4)))^{(1/2)} - 7225344*a^{12}*c^{18}*d^{39}*e^3 - 76972032*a^{13}*c^{17}*d^{37}*e^5 - 367607808*a^{14}*c^{16}*d^{35}*e^7 - 1036910592*a^{15}*c^{15}*d^{33}*e^9 - 1876983808*a^{16}*c^{14}*d^{31}*e^{11} - 2115436544*a^{17}*c^{13}*d^{29}*e^{13} - 1052803072*a^{18}*c^{12}*d^{27}*e^{15} + 848429056*a^{19}*c^{11}*d^{25}*e^{17} + 2105458688*a^{20}*c^{10}*d^{23}*e^{19} + 1909030912*a^{21}*c^9*d^{21}*e^{21} + 959037440*a^{22}*c^8*d^{19}*e^{23} + 262144000*a^{23}*c^7*d^{17}*e^{25} + 30408704*a^{24}*c^6*d^{15}*e^{27}))*((81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} - 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)})/(256*(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4)))^{(1/2)}*1i)/((x*(4917248*a^{10}*c^{18}*d^{36}*e^5 + 50677760*a^{11}*c^{17}*d^{34}*e^7 + 230498304*a^{12}*c^{16}*d^{32}*e^9 + 607559680*a^{13}*c^{15}*d^{30}*e^{11} + 1026486272*a^{14}*c^{14}*d^{28}*e^{13} + 1166602240*a^{15}*c^{13}*d^{26}*e^{15} + 923508736*a^{16}*c^{12}*d^{24}*e^{17} + 539500544*a^{17}*c^{11}*d^{22}*e^{19} + 259409920*a^{18}*c^{10}*d^{20}*e^{21} + 109709312*a^{19}*c^9*d^{18}*e^{23} + 34537472*a^{20}*c^8*d^{16}*e^{25} + 5308416*a^{21}*c^7*d^{14}*e^{27}) - ((81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*
\end{aligned}$$

$$\begin{aligned}
& d^5 e^5 + 236 a^7 c^4 d^3 e^3 - 129 a^2 c^2 d^4 e^2 (-a^{11} c^5)^{(1/2)} - 31 a^2 c^2 d^2 e^4 (-a^{11} c^5)^{(1/2)} / (256 (a^{15} e^8 + a^{11} c^4 d^8 + 4 a^{14} c^2 d^2 e^6 + 4 a^{12} c^3 d^6 e^2 + 6 a^{13} c^2 d^4 e^4))^{(1/2)} * ((x (1787297792 a^{19} c^{13} d^31 e^{12} - 147587072 a^{15} c^{17} d^39 e^4 - 698089472 a^{16} c^{16} d^37 e^6 - 1660157952 a^{17} c^{15} d^35 e^8 - 1588068352 a^{18} c^{14} d^33 e^{10} - 12845056 a^{14} c^{18} d^41 e^2 + 7839678464 a^{20} c^{12} d^29 e^{14} + 11879841792 a^{21} c^{11} d^27 e^{16} + 10631249920 a^{22} c^{10} d^25 e^{18} + 6274940928 a^{23} c^9 d^23 e^{20} + 2652110848 a^{24} c^8 d^21 e^{22} + 891027456 a^{25} c^7 d^19 e^{24} + 234881024 a^{26} c^6 d^17 e^{26} + 33554432 a^{27} c^5 d^15 e^{28}) - ((81 a^3 e^6 (-a^{11} c^5)^{(1/2)} - 49 c^3 d^6 (-a^{11} c^5)^{(1/2)} + 70 a^6 c^5 d^5 e + 198 a^8 c^3 d^3 e^5 + 236 a^7 c^4 d^3 e^3 - 129 a^2 c^2 d^4 e^2 (-a^{11} c^5)^{(1/2)} - 31 a^2 c^2 d^2 e^4 (-a^{11} c^5)^{(1/2)}) / (256 (a^{15} e^8 + a^{11} c^4 d^8 + 4 a^{14} c^2 d^2 e^6 + 4 a^{12} c^3 d^6 e^2 + 6 a^{13} c^2 d^4 e^4))^{(1/2)} * (29360128 a^{17} c^{17} d^42 e^2 - x ((81 a^3 e^6 (-a^{11} c^5)^{(1/2)} - 49 c^3 d^6 (-a^{11} c^5)^{(1/2)} + 70 a^6 c^5 d^5 e + 198 a^8 c^3 d^3 e^5 + 236 a^7 c^4 d^3 e^3 - 129 a^2 c^2 d^4 e^2 (-a^{11} c^5)^{(1/2)} - 31 a^2 c^2 d^2 e^4 (-a^{11} c^5)^{(1/2)}) / (256 (a^{15} e^8 + a^{11} c^4 d^8 + 4 a^{14} c^2 d^2 e^6 + 4 a^{12} c^3 d^6 e^2 + 6 a^{13} c^2 d^4 e^4))^{(1/2)} * (134217728 a^{20} c^{16} d^42 e^3 + 1342177280 a^{21} c^{15} d^40 e^5 + 5905580032 a^{22} c^{14} d^38 e^7 + 14763950080 a^{23} c^{13} d^36 e^9 + 22145925120 a^{24} c^{12} d^34 e^{11} + 17716740096 a^{25} c^{11} d^32 e^{13} - 17716740096 a^{27} c^9 d^28 e^{17} - 22145925120 a^{28} c^8 d^26 e^{19} - 14763950080 a^{29} c^7 d^24 e^{21} - 5905580032 a^{30} c^6 d^22 e^{23} - 1342177280 a^{31} c^5 d^20 e^{25} - 134217728 a^{32} c^4 d^18 e^{27}) + 239075328 a^{18} c^{16} d^40 e^4 + 708837376 a^{19} c^{15} d^38 e^6 + 465567744 a^{20} c^{14} d^36 e^8 - 2726297600 a^{21} c^{13} d^34 e^{10} - 9084862464 a^{22} c^{12} d^32 e^{12} - 13614710784 a^{23} c^{11} d^30 e^{14} - 10745806848 a^{24} c^{10} d^28 e^{16} - 2403336192 a^{25} c^9 d^26 e^{18} + 3879731200 a^{26} c^8 d^24 e^{20} + 4517265408 a^{27} c^7 d^22 e^{22} + 2294284288 a^{28} c^6 d^20 e^{24} + 603979776 a^{29} c^5 d^18 e^{26} + 67108864 a^{30} c^4 d^16 e^{28})) * ((81 a^3 e^6 (-a^{11} c^5)^{(1/2)} - 49 c^3 d^6 (-a^{11} c^5)^{(1/2)} + 70 a^6 c^5 d^5 e + 198 a^8 c^3 d^3 e^5 + 236 a^7 c^4 d^3 e^3 - 129 a^2 c^2 d^4 e^2 (-a^{11} c^5)^{(1/2)} - 31 a^2 c^2 d^2 e^4 (-a^{11} c^5)^{(1/2)}) / (256 (a^{15} e^8 + a^{11} c^4 d^8 + 4 a^{14} c^2 d^2 e^6 + 4 a^{12} c^3 d^6 e^2 + 6 a^{13} c^2 d^4 e^4))^{(1/2)} - 7225344 a^{12} c^{18} d^39 e^3 - 76972032 a^{13} c^{17} d^37 e^5 - 367607808 a^{14} c^{16} d^35 e^7 - 1036910592 a^{15} c^{15} d^33 e^9 - 1876983808 a^{16} c^{14} d^31 e^{11} - 2115436544 a^{17} c^{13} d^29 e^{13} - 1052803072 a^{18} c^{12} d^27 e^{15} + 848429056 a^{19} c^{11} d^25 e^{17} + 2105458688 a^{20} c^{10} d^23 e^{19} + 1909030912 a^{21} c^9 d^21 e^{21} + 959037440 a^{22} c^8 d^19 e^{23} + 262144000 a^{23} c^7 d^17 e^{25} + 30408704 a^{24} c^6 d^15 e^{27})) * ((81 a^3 e^6 (-a^{11} c^5)^{(1/2)} - 49 c^3 d^6 (-a^{11} c^5)^{(1/2)} + 70 a^6 c^5 d^5 e + 198 a^8 c^3 d^3 e^5 + 236 a^7 c^4 d^3 e^3 - 129 a^2 c^2 d^4 e^2 (-a^{11} c^5)^{(1/2)} - 31 a^2 c^2 d^2 e^4 (-a^{11} c^5)^{(1/2)}) / (256 (a^{15} e^8 + a^{11} c^4 d^8 + 4 a^{14} c^2 d^2 e^6 + 4 a^{12} c^3 d^6 e^2 + 6 a^{13} c^2 d^4 e^4))^{(1/2)} - (x (4917248 a^{10} c^{18} d^36 e^5 + 50677760 a^{11} c^{17} d^34 e^7 + 230498304 a^{12} c^{16} d^32 e^9 + 607559680 a^{13} c^{15} d^30 e^{11} + 1026486272 a^{14} c^{14} d^28 e^{13} + 1166602240 a^{15} c^{13} d^26 e^{15} + 923508736 a^{16} c^{12} d^24 e^{17} + 539500544 a^{17} c^{11} d^22 e^{19} + 259409920 a
\end{aligned}$$

$$\begin{aligned}
& ^{18}c^{10}d^{20}e^{21} + 109709312a^{19}c^9d^{18}e^{23} + 34537472a^{20}c^8d^{16}e^{25} + 5308416a^{21}c^7d^{14}e^{27} - ((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2)(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)})/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} * ((x*(1787297792a^{19}c^{13}d^{31}e^{12} - 147587072a^{15}c^{17}d^{39}e^4 - 698089472a^{16}c^{16}d^{37}e^6 - 1660157952a^{17}c^{15}d^{35}e^8 - 1588068352a^{18}c^{14}d^{33}e^{10} - 12845056a^{14}c^{18}d^{41}e^2 + 7839678464a^{20}c^{12}d^{29}e^{14} + 11879841792a^{21}c^{11}d^{27}e^{16} + 10631249920a^{22}c^{10}d^{25}e^{18} + 6274940928a^{23}c^9d^{23}e^{20} + 2652110848a^{24}c^8d^{21}e^{22} + 891027456a^{25}c^7d^{19}e^{24} + 234881024a^{26}c^6d^{17}e^{26} + 33554432a^{27}c^5d^{15}e^{28}) + ((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2)(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)})/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} * (x*((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2)(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)})/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} * (134217728a^{20}c^{16}d^{42}e^3 + 1342177280a^{21}c^{15}d^{40}e^5 + 5905580032a^{22}c^{14}d^{38}e^7 + 14763950080a^{23}c^{13}d^{36}e^9 + 22145925120a^{24}c^{12}d^{34}e^{11} + 17716740096a^{25}c^{11}d^{32}e^{13} - 17716740096a^{27}c^9d^{28}e^{17} - 22145925120a^{28}c^8d^{26}e^{19} - 14763950080a^{29}c^7d^{24}e^{21} - 5905580032a^{30}c^6d^{22}e^{23} - 1342177280a^{31}c^5d^{20}e^{25} - 134217728a^{32}c^4d^{18}e^{27}) + 29360128a^{17}c^{17}d^{42}e^2 + 239075328a^{18}c^{16}d^{40}e^4 + 708837376a^{19}c^{15}d^{38}e^6 + 465567744a^{20}c^{14}d^{36}e^8 - 2726297600a^{21}c^{13}d^{34}e^{10} - 9084862464a^{22}c^{12}d^{32}e^{12} - 13614710784a^{23}c^{11}d^{30}e^{14} - 10745806848a^{24}c^{10}d^{28}e^{16} - 2403336192a^{25}c^9d^{26}e^{18} + 3879731200a^{26}c^8d^{24}e^{20} + 4517265408a^{27}c^7d^{22}e^{22} + 2294284288a^{28}c^6d^{20}e^{24} + 603979776a^{29}c^5d^{18}e^{26} + 67108864a^{30}c^4d^{16}e^{28}))*((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2)(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)})/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} + 7225344a^{12}c^{18}d^39e^3 + 76972032a^{13}c^{17}d^{37}e^5 + 367607808a^{14}c^{16}d^{35}e^7 + 1036910592a^{15}c^{15}d^{33}e^9 + 1876983808a^{16}c^{14}d^{31}e^{11} + 2115436544a^{17}c^{13}d^{29}e^{13} + 1052803072a^{18}c^{12}d^{27}e^{15} - 848429056a^{19}c^{11}d^{25}e^{17} - 2105458688a^{20}c^{10}d^{23}e^{19} - 1909030912a^{21}c^9d^{21}e^{21} - 959037440a^{22}c^8d^{19}e^{23} - 262144000a^{23}c^7d^{17}e^{25} - 30408704a^{24}c^6d^{15}e^{27}))*((81a^3e^6(-a^{11}c^5)^{(1/2)} - 49c^3d^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 - 129a^2c^2d^4e^2)(-a^{11}c^5)^{(1/2)} - 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)})/(256(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)} + 4917248a^{10}c^{16}d^{30}e^{10} + 40843264a^{11}c^{15}d^{28}e^{12} +
\end{aligned}$$

$$\begin{aligned}
& 147507200*a^{12}*c^{14}*d^{26}*e^{14} + 302962688*a^{13}*c^{13}*d^{24}*e^{16} + 387512320* \\
& a^{14}*c^{12}*d^{22}*e^{18} + 316418048*a^{15}*c^{11}*d^{20}*e^{20} + 161224704*a^{16}*c^{10}*d \\
& ^{18}*e^{22} + 46909440*a^{17}*c^9*d^{16}*e^{24} + 5971968*a^{18}*c^8*d^{14}*e^{26}))*((81* \\
& a^3*e^6*(-a^{11}*c^5)^{(1/2)} - 49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e \\
& + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} - \\
& 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(256*(a^{15}*e^8 + a^{11}*c^4*d^8 + \\
& 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(1/2)}*2i - (\\
& 1/(3*a*d) - (e*x^2)/(a*d^2) + (x^4*(7*c^2*d^2 + 4*a*c*e^2))/(12*a^2*d*(a*e^ \\
& 2 + c*d^2)) - (c*x^6*(4*a*e^3 + 5*c*d^2*e))/(4*a^2*d^2*(a*e^2 + c*d^2)))/(a \\
& *x^3 + c*x^7) + \operatorname{atan}(((a^{11}*c^5*(156627*c^2*d^6*e^{12} - 245952*a^2*d^2*e^{16} \\
& + 324032*a*c*d^4*e^{14}) - 16807*a^5*c^{13}*d^{18} + 46656*a^{14}*c^4*e^{18} + 24696* \\
& a^6*c^{12}*d^{16}*e^2 + 455609*a^7*c^{11}*d^{14}*e^4 + 856936*a^8*c^{10}*d^{12}*e^6 - 2 \\
& 7429*a^9*c^9*d^{10}*e^8 - 805344*a^{10}*c^8*d^8*e^{10})*(a^{13}*c^{11}*d^{21}*x*((49*c^ \\
& 3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + \\
& 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + \\
& 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14} \\
& *c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(3/2)}*49i + a^{17}*c^3 \\
& *e^{19}*x*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70* \\
& a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2 \\
& *(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(a^{15}*e^8 + a^{11}*c \\
& ^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(1/2)} \\
& *5184i + a^{28}*d^4*e^{19}*x*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11} \\
& *c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + \\
& 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(\\
& a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^ \\
& 2*d^4*e^4))^{(5/2)}*2i - a^{19}*c^9*d^{22}*e*x*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 8 \\
& 1*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^ \\
& 7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^ \\
& 11*c^5)^{(1/2)))/(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6 \\
& *e^2 + 6*a^{13}*c^2*d^4*e^4))^{(5/2)}*2i + a^{27}*c*d^6*e^{17}*x*((49*c^3*d^6*(-a^1 \\
& 1*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^ \\
& 3*d^5*e + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^ \\
& 2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 \\
& + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(5/2)}*14i + a^9*c^{11}*d^{16}*e^3*x \\
& *((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5 \\
& *d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^{11} \\
& *c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(a^{15}*e^8 + a^{11}*c^4*d^8 \\
& + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(1/2)}*4802i \\
& + a^{10}*c^{10}*d^{14}*e^5*x*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c \\
& ^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 12 \\
& 9*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)))/(a^ \\
& 15*e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2* \\
& d^4*e^4))^{(1/2)}*35084i + a^{11}*c^9*d^{12}*e^7*x*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} \\
& - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 23 \\
& 6*a^7*c^4*d^3*e^3 + 129*a*c^2*d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*
\end{aligned}$$

$$\begin{aligned}
& (-a^{11}c^5)^{(1/2)} / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3 \\
& * d^6e^2 + 6a^{13}c^2d^4e^4)^{(1/2)} * 105438i + a^{12}c^8d^{10}e^9 * x * ((49c^3 \\
& * d^6(-a^{11}c^5)^{(1/2)} - 81a^3e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + \\
& 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2 \\
& * c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14} \\
& * cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4)^{(1/2)} * 166952i + a^{13} \\
& * c^7d^8e^{11} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3e^6(-a^{11}c^5)^{(1/2)} + 70a^6 \\
& * c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2 \\
& * d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 \\
& + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \\
&)^{(1/2)} * 150174i + a^{14}c^6d^6e^{13} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3 \\
& * e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4 \\
& * d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11} \\
& * c^5)^{(1/2)}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 \\
& + 6a^{13}c^2d^4e^4)^{(1/2)} * 82444i + a^{15}c^5d^4e^{15} * x * ((49c^3d^6(- \\
& -a^{11}c^5)^{(1/2)} - 81a^3e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8 \\
& * c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 3 \\
& 1a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 \\
& + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4)^{(1/2)} * 37058i + a^{16}c^4d^2 \\
& * e^{17} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3e^6(-a^{11}c^5)^{(1/2)} + 70a^6 \\
& * c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2 \\
& * (-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 + a^{11}c^4 \\
& * d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4)^{(1/2)} \\
& * 18176i + a^{14}c^{10}d^{19}e^{20} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3e^6(- \\
& -a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 \\
& + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 \\
& + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \\
&)^{(3/2)} * 416i + a^{15}c^9d^{17}e^{14} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3 \\
& * e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 \\
& + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 \\
& + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \\
&)^{(3/2)} * 1268i + a^{16}c^8d^{15}e^{16} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3 \\
& * e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 \\
& + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 \\
& + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \\
&)^{(3/2)} * 1232i - a^{17}c^7d^{13}e^{18} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81a^3e^6(- \\
& -a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2 \\
& * d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11}c^5)^{(1/2)}) / (a^{15}e^8 \\
& + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \\
&)^{(3/2)} * 1858i - a^{18}c^6d^{11}e^{10} * x * ((49c^3d^6(-a^{11}c^5)^{(1/2)} - 81 \\
& * a^3e^6(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7 \\
& * c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4(-a^{11} \\
& * c^5)^{(1/2)}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 \\
& + 6a^{13}c^2d^4e^4) \\
&)^{(3/2)} * 6208i - a^{19}c^5d^9e^{12} * x * ((49c^3d^6(-
\end{aligned}$$

$$\begin{aligned}
&)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + \\
&6a^{13}c^2d^4e^4)^{(5/2)}*40i - a^{23}c^2d^2e^20*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(3/2)}*128i - (-a^{11}c^5)^{(1/2)}*(69629c^{10}d^{17}e + 286944a^9c^9d^{15}e^3 + 150336a^8c^2d^2e^{17} + 110645a^2c^8d^{13}e^5 - 770024a^3c^7d^{11}e^7 - 606089a^4c^6d^9e^9 + 566984a^5c^5d^7e^{11} + 157207a^6c^4d^5e^{13} - 327104a^7c^3d^3e^{15})*(a^{13}c^{11}d^{21}*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*5184i + a^{28}d^4e^{19}*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(5/2)}*2i - a^{19}c^9d^{22}*e*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(5/2)}*2i + a^{27}c^2d^6e^{17}*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(5/2)}*14i + a^9c^{11}d^{16}e^3*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*4802i + a^{10}c^{10}d^{14}e^5*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*35084i + a^{11}c^9d^{12}e^7*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*105438i + a^{12}c^8d^{10}e^9*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*105438i + a^{12}c^8d^{10}e^9*x*((49c^3d^6*(-a^{11}c^5)^{(1/2)} - 81a^3e^6*(-a^{11}c^5)^{(1/2)} + 70a^6c^5d^5e + 198a^8c^3d^3e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2*(-a^{11}c^5)^{(1/2)} + 31a^2c^2d^2e^4*(-a^{11}c^5)^{(1/2)})/(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4))^{(1/2)}*105438i
\end{aligned}$$

$$\begin{aligned}
& \left((-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2} \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 166952i + a^{13}c^7d^8e^{11}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 150174i + a^{14}c^6d^6e^{13}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 82444i + a^{15}c^5d^4e^{15}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 37058i + a^{16}c^4d^2e^{17}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 18176i + a^{14}c^{10}d^{19}e^2x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 416i + a^{15}c^9d^{17}e^4x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 1268i + a^{16}c^8d^{15}e^6x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 1232i - a^{17}c^7d^{13}e^8x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 1858i - a^{18}c^6d^{11}e^{10}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 6208i - a^{19}c^5d^9e^{12}x \cdot \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2cd^2e^4(-a^{11}c^5)^{1/2}) \right) / \left(a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}cd^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4 \right)^{1/2} \\
& \cdot 6940
\end{aligned}$$

$$\begin{aligned}
& i - a^{20}c^4d^7e^{14}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{3/2} \\
& 4016i - a^{21}c^3d^5e^{16}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{3/2} \\
& 1479i - a^{22}c^2d^3e^{18}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{3/2} \\
& 512i - a^{20}c^8d^{20}e^3x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 14i - a^{21}c^7d^{18}e^5x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 40i - a^{22}c^6d^{16}e^7x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 56i - a^{23}c^5d^{14}e^9x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 28i + a^{24}c^4d^{12}e^{11}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 28i + a^{25}c^3d^{10}e^{13}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 56i + a^{26}c^2d^8e^{15}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2} \\
& 40i - a^{23}c^3d^6e^{20}x \left((49c^3d^6(-a^{11}c^5)^{1/2} - 81a^3e^6(-a^{11}c^5)^{1/2} + 70a^6c^5d^5e + 198a^8c^3d^5e^5 + 236a^7c^4d^3e^3 + 129a^2c^2d^4e^2(-a^{11}c^5)^{1/2} + 31a^2c^2d^2e^4(-a^{11}c^5)^{1/2}) / (a^{15}e^8 + a^{11}c^4d^8 + 4a^{14}c^2d^2e^6 + 4a^{12}c^3d^6e^2 + 6a^{13}c^2d^4e^4) \right)^{5/2}
\end{aligned}$$

$$\begin{aligned}
& 2) + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)}/(a^{15}*e^8 + a^{11}*c^4*d^8 + 4*a^{14}* \\
& c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4*e^4))^{(3/2)*128i}))/((2824752 \\
& 49*a^{10}*c^{26}*d^{36} + 2176782336*a^{28}*c^8*e^{36} + 4018066297*a^{11}*c^{25}*d^{34}*e^ \\
& 2 + 25254299042*a^{12}*c^{24}*d^{32}*e^4 + 91443453570*a^{13}*c^{23}*d^{30}*e^6 + 20709 \\
& 3177767*a^{14}*c^{22}*d^{28}*e^8 + 292503608847*a^{15}*c^{21}*d^{26}*e^{10} + 22503434162 \\
& 8*a^{16}*c^{20}*d^{24}*e^{12} + 22083537020*a^{17}*c^{19}*d^{22}*e^{14} - 108969417553*a^{18} \\
& *c^{18}*d^{20}*e^{16} - 43670306041*a^{19}*c^{17}*d^{18}*e^{18} + 58023955010*a^{20}*c^{16}*d \\
& ^{16}*e^{20} + 18862267874*a^{21}*c^{15}*d^{14}*e^{22} - 60676266279*a^{22}*c^{14}*d^{12}*e^{2} \\
& 4 - 33348619375*a^{23}*c^{13}*d^{10}*e^{26} + 20433166080*a^{24}*c^{12}*d^8*e^{28} + 9487 \\
& 311616*a^{25}*c^{11}*d^6*e^{30} - 7622553600*a^{26}*c^{10}*d^4*e^{32} - 349360128*a^{27}* \\
& c^9*d^2*e^{34}))*((49*c^3*d^6*(-a^{11}*c^5)^{(1/2)} - 81*a^3*e^6*(-a^{11}*c^5)^{(1/2)} \\
&) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 + 129*a*c^2* \\
& d^4*e^2*(-a^{11}*c^5)^{(1/2)} + 31*a^2*c*d^2*e^4*(-a^{11}*c^5)^{(1/2)})/(256*(a^{15}* \\
& e^8 + a^{11}*c^4*d^8 + 4*a^{14}*c*d^2*e^6 + 4*a^{12}*c^3*d^6*e^2 + 6*a^{13}*c^2*d^4 \\
& *e^4)))^{(1/2)*2i} + (atan((a^{11}*e^3*x*(-d^5*e^{11})^{(5/2)}*4096i - a^4*c^7*d^{19} \\
& *x*(-d^5*e^{11})^{(3/2)}*73519i + c^{11}*d^{32}*e^3*x*(-d^5*e^{11})^{(1/2)}*2401i - a^5 \\
& *c^6*d^{17}*e^2*x*(-d^5*e^{11})^{(3/2)}*34182i - a^6*c^5*d^{15}*e^4*x*(-d^5*e^{11})^{(\\
& 3/2)}*15521i - a^7*c^4*d^{13}*e^6*x*(-d^5*e^{11})^{(3/2)}*30208i - a^8*c^3*d^{11}*e^ \\
& 8*x*(-d^5*e^{11})^{(3/2)}*25344i + a^2*c^9*d^{28}*e^7*x*(-d^5*e^{11})^{(1/2)}*52719i \\
& + a^3*c^8*d^{26}*e^9*x*(-d^5*e^{11})^{(1/2)}*83476i + a*c^{10}*d^{30}*e^5*x*(-d^5*e^{1} \\
& 1)^{(1/2)}*17542i)/(4096*a^{11}*d^{13}*e^{30} + 2401*c^{11}*d^{35}*e^8 + 17542*a*c^{10}*d \\
& ^{33}*e^{10} + 52719*a^2*c^9*d^{31}*e^{12} + 83476*a^3*c^8*d^{29}*e^{14} + 73519*a^4*c^ \\
& 7*d^{27}*e^{16} + 34182*a^5*c^6*d^{25}*e^{18} + 15521*a^6*c^5*d^{23}*e^{20} + 30208*a^7 \\
& *c^4*d^{21}*e^{22} + 25344*a^8*c^3*d^{19}*e^{24}))*(-d^5*e^{11})^{(1/2)*1i})/(c^2*d^9 + \\
& a^2*d^5*e^4 + 2*a*c*d^7*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.189 \quad \int x^2 \sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Optimal. Leaf size=243

$$\frac{c^2 \sqrt{a^2 + 2abx^2 + b^2x^4} (bc - 2ad) \tanh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}} \right)}{16d^{5/2} (a + bx^2)} - \frac{cx \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2} (bc - 2ad)}{16d^2 (a + bx^2)} + \frac{bx^3 \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2} (bc - 2ad)}{6d^2 (a + bx^2)}$$

Rubi [A] time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1250, 459, 279, 321, 217, 206}

$$\frac{c^2 \sqrt{a^2 + 2abx^2 + b^2x^4} (bc - 2ad) \tanh^{-1} \left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}} \right)}{16d^{5/2} (a + bx^2)} - \frac{cx \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2} (bc - 2ad)}{16d^2 (a + bx^2)} + \frac{bx^3 \sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{3/2}}{6d (a + bx^2)} - \frac{x^3 \sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2} (bc - 2ad)}{8d (a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*d^2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4} dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^2 (ab+b^2x^2) \sqrt{c+dx^2} dx}{ab+b^2x^2} \\
&= \frac{bx^3 (c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} - \frac{(b(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4})}{2d(ab+b^2x^2)} \\
&= -\frac{(bc-2ad)x^3 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3 (c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} \\
&= -\frac{c(bc-2ad)x \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} \\
&= -\frac{c(bc-2ad)x \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} \\
&= -\frac{c(bc-2ad)x \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3 \sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 142, normalized size = 0.58

$$\frac{\sqrt{(a+bx^2)^2} \sqrt{c+dx^2} \left(3c^{3/2}(bc-2ad) \sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + \sqrt{d}x \sqrt{\frac{dx^2}{c}+1} (6ad(c+2dx^2) + b(-3c^2+2cdx^2+8d^2x^4)) \right)}{48d^{5/2}(a+bx^2) \sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[c+d*x^2]*Sqrt[a^2+2*a*b*x^2+b^2*x^4],x]

[Out] (Sqrt[(a+b*x^2)^2]*Sqrt[c+d*x^2]*(Sqrt[d]*x*Sqrt[1+(d*x^2)/c]*(6*a*d*(c+2*d*x^2)+b*(-3*c^2+2*c*d*x^2+8*d^2*x^4))+3*c^(3/2)*(b*c-2*a*d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/(48*d^(5/2)*(a+b*x^2)*Sqrt[1+(d*x^2)/c])

IntegrateAlgebraic [A] time = 0.20, size = 126, normalized size = 0.52

$$\frac{\sqrt{(a+bx^2)^2} \left(\frac{\sqrt{c+dx^2} (6acd^2x+12ad^2x^3-3bc^2x+2bcdx^3+8bd^2x^5)}{48d^2} + \frac{(2ac^2d-bc^3) \log(\sqrt{c+dx^2}-\sqrt{d}x)}{16d^{5/2}} \right)}{a+bx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*sqrt[c + d*x^2]*sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (sqrt[(a + b*x^2)^2]*((sqrt[c + d*x^2]*(-3*b*c^2*x + 6*a*c*d*x + 2*b*c*d*x^3 + 12*a*d^2*x^3 + 8*b*d^2*x^5))/(48*d^2) + ((-(b*c^3) + 2*a*c^2*d)*Log[-(sqrt[d]*x) + sqrt[c + d*x^2]])/(16*d^(5/2))))/(a + b*x^2)

fricas [A] time = 0.85, size = 206, normalized size = 0.85

$$\frac{3(bc^3 - 2ac^2d)\sqrt{d} \log(-2dx^2 + 2\sqrt{dx^2 + c}\sqrt{dx - c}) - 2(8bd^3x^5 + 2(bcd^2 + 6ad^3)x^3 - 3(bc^2d - 2acd^2)x)\sqrt{dx^2 + c} - 3(bc^3 - 2ac^2d)\sqrt{-d} \arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2 + c}}\right) - (8bd^3x^5 + 2(bcd^2 + 6ad^3)x^3 - 3(bc^2d - 2acd^2)x)\sqrt{dx^2 + c}}{96d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x, algorithm="fricas")

[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]

giac [A] time = 0.38, size = 156, normalized size = 0.64

$$\frac{1}{48} \left(2 \left(4bx^2\operatorname{sgn}(bx^2 + a) + \frac{bcd^3\operatorname{sgn}(bx^2 + a) + 6ad^4\operatorname{sgn}(bx^2 + a)}{d^4} \right) x^2 - \frac{3(bc^2d^2\operatorname{sgn}(bx^2 + a) - 2acd^3\operatorname{sgn}(bx^2 + a))}{d^4} \right) \sqrt{dx^2 + cx} - \frac{(bc^3\operatorname{sgn}(bx^2 + a) - 2acd^2\operatorname{sgn}(bx^2 + a)) \log\left(\frac{-\sqrt{d}x + \sqrt{dx^2 + c}}{16d^{\frac{5}{2}}}\right)}{16d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x, algorithm="giac")

[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2 + a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)

maple [A] time = 0.01, size = 159, normalized size = 0.65

$$\frac{\sqrt{(bx^2 + a)^2} \left(8(dx^2 + c)^{\frac{3}{2}} bd^{\frac{3}{2}} x^3 - 6a^2 d \ln(\sqrt{d}x + \sqrt{dx^2 + c}) + 3bc^3 \ln(\sqrt{d}x + \sqrt{dx^2 + c}) - 6\sqrt{dx^2 + c} acd^{\frac{3}{2}} x + 3\sqrt{d} dx^2 + c b c^2 \sqrt{d} x + 12(dx^2 + c)^{\frac{3}{2}} a d^{\frac{3}{2}} x - 6(dx^2 + c)^{\frac{3}{2}} bc \sqrt{d} x \right)}{48(bx^2 + a)d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x)

[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*d^(3/2)*(d*x^2+c)^(3/2)*x^3*b+12*d^(3/2)*(d*x^2+c)^(3/2)*x*a-6*d^(1/2)*(d*x^2+c)^(3/2)*x*b*c-6*d^(3/2)*(d*x^2+c)^(1/2)*x*a*c+3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c^2-6*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*d+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^3)/(b*x^2+a)/d^(5/2)

maxima [A] time = 0.92, size = 124, normalized size = 0.51

$$\frac{(dx^2 + c)^{\frac{3}{2}}bx^3}{6d} - \frac{(dx^2 + c)^{\frac{3}{2}}bcx}{8d^2} + \frac{\sqrt{dx^2 + c}bc^2x}{16d^2} + \frac{(dx^2 + c)^{\frac{3}{2}}ax}{4d} - \frac{\sqrt{dx^2 + c}acx}{8d} + \frac{bc^3 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16d^{\frac{5}{2}}} - \frac{ac^2 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqrt(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 + c)*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{dx^2 + c} \sqrt{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)

[Out] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)

[Out] Timed out

$$3.190 \quad \int x \sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Optimal. Leaf size=108

$$\frac{b\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{5/2}}{5d^2 (a + bx^2)} - \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{3/2} (bc - ad)}{3d^2 (a + bx^2)}$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1247, 646, 43}

$$\frac{b\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{5/2}}{5d^2 (a + bx^2)} - \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{3/2} (bc - ad)}{3d^2 (a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^2*(a + b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*(a + b*x^2))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4} dx &= \frac{1}{2} \text{Subst}\left(\int \sqrt{c+dx}\sqrt{a^2+2abx+b^2x^2} dx, x, x^2\right) \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4} \text{Subst}\left(\int (ab+b^2x)\sqrt{c+dx} dx, x, x^2\right)}{2(ab+b^2x^2)} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4} \text{Subst}\left(\int \left(-\frac{b(bc-ad)\sqrt{c+dx}}{d} + \frac{b^2(c+dx)^{3/2}}{d}\right) dx, x, x^2\right)}{2(ab+b^2x^2)} \\
&= -\frac{(bc-ad)(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2(a+bx^2)} + \frac{b(c+dx^2)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^2(a+bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.52

$$\frac{\sqrt{(a+bx^2)^2}(c+dx^2)^{3/2}(5ad-2bc+3bdx^2)}{15d^2(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^2*(a + b*x^2))

IntegrateAlgebraic [A] time = 0.09, size = 56, normalized size = 0.52

$$\frac{\sqrt{(a+bx^2)^2}(c+dx^2)^{3/2}(5ad-2bc+3bdx^2)}{15d^2(a+bx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^2*(a + b*x^2))

fricas [A] time = 0.60, size = 50, normalized size = 0.46

$$\frac{(3bd^2x^4 - 2bc^2 + 5acd + (bcd + 5ad^2)x^2)\sqrt{dx^2 + c}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 + c)/d^2

giac [A] time = 0.34, size = 68, normalized size = 0.63

$$\frac{3(dx^2 + c)^{\frac{5}{2}} b \operatorname{sgn}(bx^2 + a) - 5(dx^2 + c)^{\frac{3}{2}} bc \operatorname{sgn}(bx^2 + a) + 5(dx^2 + c)^{\frac{3}{2}} ad \operatorname{sgn}(bx^2 + a)}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b*x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2

maple [A] time = 0.00, size = 51, normalized size = 0.47

$$\frac{(dx^2 + c)^{\frac{3}{2}} (3bdx^2 + 5ad - 2bc) \sqrt{(bx^2 + a)^2}}{15(bx^2 + a)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)

[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)

maxima [A] time = 0.90, size = 50, normalized size = 0.46

$$\frac{(dx^2 + c)^{\frac{3}{2}} bx^2}{5d} - \frac{2(dx^2 + c)^{\frac{3}{2}} bc}{15d^2} + \frac{(dx^2 + c)^{\frac{3}{2}} a}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^2 + c)^(3/2)*a/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{dx^2 + c} \sqrt{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

$$3.191 \quad \int \sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4} dx$$

Optimal. Leaf size=178

$$-\frac{c\sqrt{a^2 + 2abx^2 + b^2x^4} (bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx^2)} + \frac{bx\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{3/2}}{4d(a + bx^2)} - \frac{x\sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2}}{8d(a + bx^2)}$$

Rubi [A] time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1148, 388, 195, 217, 206}

$$-\frac{c\sqrt{a^2 + 2abx^2 + b^2x^4} (bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a + bx^2)} + \frac{bx\sqrt{a^2 + 2abx^2 + b^2x^4} (c + dx^2)^{3/2}}{4d(a + bx^2)} - \frac{x\sqrt{a^2 + 2abx^2 + b^2x^4} \sqrt{c + dx^2} (bc - 4ad)}{8d(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a + b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4} dx &= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int (ab + b^2x^2) \sqrt{c + dx^2} dx}{ab + b^2x^2} \\
&= \frac{bx(c + dx^2)^{3/2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)} - \frac{(b(bc - 4ad)\sqrt{a^2 + 2abx^2 + b^2x^4})}{4d(ab + b^2x^2)} \\
&= -\frac{(bc - 4ad)x\sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{8d(a + bx^2)} + \frac{bx(c + dx^2)^{3/2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)} \\
&= -\frac{(bc - 4ad)x\sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{8d(a + bx^2)} + \frac{bx(c + dx^2)^{3/2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)} \\
&= -\frac{(bc - 4ad)x\sqrt{c + dx^2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{8d(a + bx^2)} + \frac{bx(c + dx^2)^{3/2} \sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.68

$$\frac{\sqrt{(a + bx^2)^2} \sqrt{c + dx^2} \left(\sqrt{d} x \sqrt{\frac{dx^2}{c} + 1} (4ad + b(c + 2dx^2)) - \sqrt{c} (bc - 4ad) \sinh^{-1} \left(\frac{\sqrt{d} x}{\sqrt{c}} \right) \right)}{8d^{3/2} (a + bx^2) \sqrt{\frac{dx^2}{c} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]
```

[Out] $(\text{Sqrt}[(a + b*x^2)^2]*\text{Sqrt}[c + d*x^2]*(\text{Sqrt}[d]*x*\text{Sqrt}[1 + (d*x^2)/c]*(4*a*d + b*(c + 2*d*x^2)) - \text{Sqrt}[c]*(b*c - 4*a*d)*\text{ArcSinh}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]))/ (8*d^{3/2}*(a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c])$

IntegrateAlgebraic [A] time = 0.16, size = 100, normalized size = 0.56

$$\frac{\sqrt{(a + bx^2)^2} \left(\frac{(bc^2 - 4acd) \log(\sqrt{c+dx^2} - \sqrt{d}x)}{8d^{3/2}} + \frac{\sqrt{c+dx^2}(4adx + bcx + 2bdx^3)}{8d} \right)}{a + bx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4], x]

[Out] $(\text{Sqrt}[(a + b*x^2)^2]*((\text{Sqrt}[c + d*x^2]*(b*c*x + 4*a*d*x + 2*b*d*x^3))/(8*d) + ((b*c^2 - 4*a*c*d)*\text{Log}[-(\text{Sqrt}[d]*x) + \text{Sqrt}[c + d*x^2]])/(8*d^{3/2}))) / (a + b*x^2)$

fricas [A] time = 0.94, size = 155, normalized size = 0.87

$$\left[\frac{(bc^2 - 4acd)\sqrt{d} \log(-2dx^2 - 2\sqrt{dx^2 + c}\sqrt{d}x - c) - 2(2bd^2x^3 + (bcd + 4ad^2)x)\sqrt{dx^2 + c}}{16d^2}, \frac{(bc^2 - 4acd)\sqrt{-d} \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2 + c}}\right) + (2bd^2x^3 + (bcd + 4ad^2)x)\sqrt{dx^2 + c}}{8d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x, algorithm="fricas")

[Out] $[-1/16*((b*c^2 - 4*a*c*d)*\text{sqrt}(d)*\log(-2*d*x^2 - 2*\text{sqrt}(d*x^2 + c)*\text{sqrt}(d)*x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*\text{sqrt}(d*x^2 + c))/d^2, 1/8*((b*c^2 - 4*a*c*d)*\text{sqrt}(-d)*\text{arctan}(\text{sqrt}(-d)*x/\text{sqrt}(d*x^2 + c)) + (2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*\text{sqrt}(d*x^2 + c))/d^2]$

giac [A] time = 0.46, size = 109, normalized size = 0.61

$$\frac{1}{8} \left(2bx^2 \text{sgn}(bx^2 + a) + \frac{bcd \text{sgn}(bx^2 + a) + 4ad^2 \text{sgn}(bx^2 + a)}{d^2} \right) \sqrt{dx^2 + cx} + \frac{(bc^2 \text{sgn}(bx^2 + a) - 4acd \text{sgn}(bx^2 + a)) \log(|-\sqrt{d}x + \sqrt{dx^2 + c}|)}{8d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x, algorithm="giac")

[Out] $1/8*(2*b*x^2*\text{sgn}(b*x^2 + a) + (b*c*d*\text{sgn}(b*x^2 + a) + 4*a*d^2*\text{sgn}(b*x^2 + a))/d^2)*\text{sqrt}(d*x^2 + c)*x + 1/8*(b*c^2*\text{sgn}(b*x^2 + a) - 4*a*c*d*\text{sgn}(b*x^2 + a))*\log(\text{abs}(-\text{sqrt}(d)*x + \text{sqrt}(d*x^2 + c)))/d^{3/2}$

maple [A] time = 0.01, size = 119, normalized size = 0.67

$$\frac{\sqrt{(bx^2+a)^2} \left(4acd \ln(\sqrt{d}x + \sqrt{dx^2+c}) - bc^2 \ln(\sqrt{d}x + \sqrt{dx^2+c}) + 4\sqrt{dx^2+c} ad^{\frac{3}{2}}x - \sqrt{dx^2+c} bc\sqrt{d}x + 2(dx^2+c)^{\frac{3}{2}} b\sqrt{d}x \right)}{8(bx^2+a)d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x)

[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*d^(1/2)*(d*x^2+c)^(3/2)*x*b+4*d^(3/2)*(d*x^2+c)^(1/2)*x*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b*c+4*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/(b*x^2+a)/d^(3/2)

maxima [A] time = 1.08, size = 81, normalized size = 0.46

$$\frac{1}{2} \sqrt{dx^2+c} ax + \frac{(dx^2+c)^{\frac{3}{2}} bx}{4d} - \frac{\sqrt{dx^2+c} bcx}{8d} - \frac{bc^2 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8d^{\frac{3}{2}}} + \frac{ac \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2), x, algorithm="maxima")

[Out] 1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c)*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x/sqrt(c*d))/sqrt(d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{dx^2+c} \sqrt{(bx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)

[Out] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \sqrt{(a + bx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2), x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)

$$3.192 \quad \int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x} dx$$

Optimal. Leaf size=152

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{3d(a+bx^2)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1250, 446, 80, 50, 63, 208}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{3d(a+bx^2)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]

[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x^2)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x} dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x} dx}{ab+b^2x^2} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4} \operatorname{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x} dx, x, x^2\right)}{2(ab+b^2x^2)} \\
&= \frac{b(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{(ab\sqrt{a^2+2abx^2+b^2x^4}) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)}{2(ab+b^2x^2)} \\
&= \frac{a\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{ab\sqrt{a^2+2abx^2+b^2x^4}}{2(ab+b^2x^2)} \\
&= \frac{a\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{ab\sqrt{a^2+2abx^2+b^2x^4}}{2(ab+b^2x^2)} \\
&= \frac{a\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{ab\sqrt{a^2+2abx^2+b^2x^4}}{2(ab+b^2x^2)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.55

$$\frac{\sqrt{(a+bx^2)^2} \left(\sqrt{c+dx^2} (3ad+b(c+dx^2)) - 3a\sqrt{c}d \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right)}{3d(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]

[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))

IntegrateAlgebraic [A] time = 0.11, size = 82, normalized size = 0.54

$$\frac{\sqrt{(a+bx^2)^2} \left(\frac{\sqrt{c+dx^2}(3ad+bc+bdx^2)}{3d} - a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right)}{a+bx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]

[Out] (Sqrt[(a + b*x^2)^2]*((Sqrt[c + d*x^2]*(b*c + 3*a*d + b*d*x^2))/(3*d) - a*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x^2)

fricas [A] time = 0.95, size = 123, normalized size = 0.81

$$\left[\frac{3a\sqrt{c}d \log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c+2c}}{x^2}\right) + 2(bdx^2 + bc + 3ad)\sqrt{dx^2+c}}{6d}, \frac{3a\sqrt{-c}d \arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + (bdx^2 + bc + 3ad)\sqrt{dx^2+c}}{3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")

[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]

giac [A] time = 0.45, size = 84, normalized size = 0.55

$$\frac{ac \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right) \operatorname{sgn}(bx^2 + a)}{\sqrt{-c}} + \frac{(dx^2 + c)^{\frac{3}{2}} bd^2 \operatorname{sgn}(bx^2 + a) + 3\sqrt{dx^2 + c} ad^3 \operatorname{sgn}(bx^2 + a)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")

[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2 + c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d^3

maple [A] time = 0.01, size = 80, normalized size = 0.53

$$\frac{\sqrt{(bx^2 + a)^2} \left(3a\sqrt{c} d \ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) - 3\sqrt{dx^2+c} ad - (dx^2 + c)^{\frac{3}{2}} b \right)}{3(bx^2 + a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)

[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*c^(1/2)*a*d - b*(d*x^2+c)^(3/2) - 3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d

maxima [A] time = 1.45, size = 45, normalized size = 0.30

$$-a\sqrt{c} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \sqrt{dx^2 + c}a + \frac{(dx^2 + c)^{\frac{3}{2}}b}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")

[Out] -a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2 + c)^(3/2)*b/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{dx^2 + c} \sqrt{(bx^2 + a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)

[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2} \sqrt{(a + bx^2)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)

$$3.193 \quad \int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx$$

Optimal. Leaf size=177

$$\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{cx(a+bx^2)} + \frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)}{2\sqrt{d}(a+bx^2)}$$

Rubi [A] time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1250, 453, 195, 217, 206}

$$\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{cx(a+bx^2)} + \frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2, x]

[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a + b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2} dx}{ab+b^2x^2} \\ &= -\frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} + \frac{\left((-b^2c-2abd) \sqrt{a^2+2abx^2+b^2x^4}\right)}{c(ab+b^2x^2)} \\ &= \frac{(bc+2ad)x\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} \\ &= \frac{(bc+2ad)x\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} \\ &= \frac{(bc+2ad)x\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{cx(a+bx^2)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.69

$$\frac{\sqrt{(a+bx^2)^2} \sqrt{c+dx^2} \left(\sqrt{c} \sqrt{d} (bx^2-2a) \sqrt{\frac{dx^2}{c}+1} + x(2ad+bc) \sinh^{-1}\left(\frac{\sqrt{dx^2}}{\sqrt{c}}\right) \right)}{2\sqrt{c} \sqrt{d} x (a+bx^2) \sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]

[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[c]*Sqrt[d]*(-2*a + b*x^2)*Sqrt[1 + (d*x^2)/c] + (b*c + 2*a*d)*x*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])

IntegrateAlgebraic [A] time = 0.20, size = 90, normalized size = 0.51

$$\frac{\sqrt{(a + bx^2)^2} \left(\frac{(bx^2 - 2a)\sqrt{c + dx^2}}{2x} + \frac{(-2ad - bc) \log(\sqrt{c + dx^2} - \sqrt{d}x)}{2\sqrt{d}} \right)}{a + bx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]

[Out] (Sqrt[(a + b*x^2)^2]*(((-2*a + b*x^2)*Sqrt[c + d*x^2])/(2*x) + ((-(b*c) - 2*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(a + b*x^2)

fricas [A] time = 0.88, size = 134, normalized size = 0.76

$$\left[\frac{(bc + 2ad)\sqrt{d}x \log(-2dx^2 - 2\sqrt{dx^2 + c}\sqrt{d}x - c) + 2(bdx^2 - 2ad)\sqrt{dx^2 + c}}{4dx}, -\frac{(bc + 2ad)\sqrt{-d}x \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2 + c}}\right) - (bdx^2 - 2ad)\sqrt{dx^2 + c}}{2dx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x)]

giac [A] time = 0.47, size = 116, normalized size = 0.66

$$\frac{1}{2} \sqrt{dx^2 + c} b x \operatorname{sgn}(bx^2 + a) + \frac{2ac\sqrt{d} \operatorname{sgn}(bx^2 + a)}{(\sqrt{d}x - \sqrt{dx^2 + c})^2 - c} - \frac{(bc\sqrt{d} \operatorname{sgn}(bx^2 + a) + 2ad^{\frac{3}{2}} \operatorname{sgn}(bx^2 + a)) \log\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*sgn(b*x^2 + a) + 2*a*d^(3/2)*sgn(b*x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d

maple [A] time = 0.01, size = 128, normalized size = 0.72

$$\frac{\sqrt{(bx^2+a)^2} \left(2acdx \ln(\sqrt{d}x + \sqrt{dx^2+c}) + bc^2x \ln(\sqrt{d}x + \sqrt{dx^2+c}) + 2\sqrt{dx^2+c} ad^{\frac{3}{2}}x^2 + \sqrt{dx^2+c} bc\sqrt{d}x^2 - 2(dx^2+c)^{\frac{3}{2}}a\sqrt{d} \right)}{2(bx^2+a)c\sqrt{d}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)

[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*d^(3/2)*(d*x^2+c)^(1/2)*x^2*a+d^(1/2)*(d*x^2+c)^(1/2)*x^2*b*c-2*d^(1/2)*(d*x^2+c)^(3/2)*a+2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*a*c*d+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*b*c^2)/(b*x^2+a)/c/x/d^(1/2)

maxima [A] time = 1.08, size = 59, normalized size = 0.33

$$\frac{1}{2}\sqrt{dx^2+c}bx + \frac{bc \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}} + a\sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^2+c}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")

[Out] 1/2*sqrt(d*x^2+c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2+c)*a/x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{dx^2+c} \sqrt{(bx^2+a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c+d*x^2)^(1/2)*((a+b*x^2)^2)^(1/2))/x^2,x)

[Out] int(((c+d*x^2)^(1/2)*((a+b*x^2)^2)^(1/2))/x^2,x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2} \sqrt{(a+bx^2)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)

[Out] Integral(sqrt(c+d*x**2)*sqrt((a+b*x**2)**2)/x**2,x)

$$3.194 \quad \int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^3} dx$$

Optimal. Leaf size=177

$$\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{2cx^2(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c(a+bx^2)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)}{2\sqrt{c}(a+bx^2)}$$

Rubi [A] time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1250, 446, 78, 50, 63, 208}

$$\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2)^{3/2}}{2cx^2(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c(a+bx^2)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]

[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a + b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1250

```
Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{x^3} dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^3} dx}{ab+b^2x^2} \\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4} \operatorname{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x^2} dx, x, x^2\right)}{2(ab+b^2x^2)} \\
&= -\frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} + \frac{\left(b^2c + \frac{abd}{2}\right) \sqrt{a^2+2abx^2+b^2x^4}}{2c(ab+b^2x^2)} \\
&= \frac{(2bc+ad)\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} \\
&= \frac{(2bc+ad)\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} \\
&= \frac{(2bc+ad)\sqrt{c+dx^2} \sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2} \sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.51

$$\frac{\sqrt{(a+bx^2)^2} \left(\sqrt{c} (a-2bx^2) \sqrt{c+dx^2} + x^2(ad+2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) \right)}{2\sqrt{c} x^2 (a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]

[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c + a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))

IntegrateAlgebraic [A] time = 0.15, size = 88, normalized size = 0.50

$$\frac{\sqrt{(a+bx^2)^2} \left(\frac{(2bx^2-a)\sqrt{c+dx^2}}{2x^2} + \frac{(-ad-2bc) \tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}} \right)}{a+bx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]

[Out] (Sqrt[(a + b*x^2)^2]*((-a + 2*b*x^2)*Sqrt[c + d*x^2])/(2*x^2) + ((-2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(2*Sqrt[c]))/(a + b*x^2)

fricas [A] time = 0.87, size = 141, normalized size = 0.80

$$\left[\frac{(2bc + ad)\sqrt{c}x^2 \log\left(-\frac{dx^2 - 2\sqrt{dx^2+c}\sqrt{c+2c}}{x^2}\right) + 2(2bcx^2 - ac)\sqrt{dx^2+c} (2bc + ad)\sqrt{-c}x^2 \arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + (2bcx^2 - ac)\sqrt{dx^2+c}}{4cx^2}, \frac{(2bc + ad)\sqrt{-c}x^2 \arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + (2bcx^2 - ac)\sqrt{dx^2+c}}{2cx^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2)]

giac [A] time = 0.44, size = 100, normalized size = 0.56

$$\frac{2\sqrt{dx^2+c}bd\operatorname{sgn}(bx^2+a) + \frac{(2bcd\operatorname{sgn}(bx^2+a)+ad^2\operatorname{sgn}(bx^2+a))\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right) - \frac{\sqrt{dx^2+c}ad\operatorname{sgn}(bx^2+a)}{x^2}}{\sqrt{-c}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")

[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c)*a*d*sgn(b*x^2 + a)/x^2)/d

maple [A] time = 0.01, size = 133, normalized size = 0.75

$$\frac{\sqrt{(bx^2+a)^2} \left(a\sqrt{c} dx^2 \ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) + 2bc^{\frac{3}{2}}x^2 \ln\left(\frac{2c+2\sqrt{dx^2+c}\sqrt{c}}{x}\right) - \sqrt{dx^2+c} adx^2 - 2\sqrt{dx^2+c} bcx^2 + (dx^2+c)^{\frac{3}{2}}a \right)}{2(bx^2+a)cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)

[Out] $-1/2*((b*x^2+a)^2)^{(1/2)}*(c^{(1/2)}*\ln(2*(c+(d*x^2+c)^{(1/2)}*c^{(1/2))}/x)*x^2*a$
 $*d+2*c^{(3/2)}*\ln(2*(c+(d*x^2+c)^{(1/2)}*c^{(1/2))}/x)*x^2*b-(d*x^2+c)^{(1/2)}*x^2*$
 $a*d-2*(d*x^2+c)^{(1/2)}*x^2*b*c+(d*x^2+c)^{(3/2)}*a)/(b*x^2+a)/c/x^2$

maxima [A] time = 1.23, size = 83, normalized size = 0.47

$$-b\sqrt{c} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{ad \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\sqrt{c}} + \sqrt{dx^2+c}b + \frac{\sqrt{dx^2+c}ad}{2c} - \frac{(dx^2+c)^{\frac{3}{2}}a}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")`

[Out] $-b*\sqrt{c}*\operatorname{arcsinh}(c/(\sqrt{c*d}*\operatorname{abs}(x))) - 1/2*a*d*\operatorname{arcsinh}(c/(\sqrt{c*d}*\operatorname{abs}(x)))/\sqrt{c} + \sqrt{d*x^2+c}*b + 1/2*\sqrt{d*x^2+c}*a*d/c - 1/2*(d*x^2+c)^{(3/2)}*a/(c*x^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{dx^2+c} \sqrt{(bx^2+a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((c+d*x^2)^(1/2))*((a+b*x^2)^2)^(1/2)/x^3,x)`

[Out] `int(((c+d*x^2)^(1/2))*((a+b*x^2)^2)^(1/2)/x^3,x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)`

[Out] Timed out

$$3.195 \quad \int x^3 (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=78

$$\frac{1}{8}x^8 (e(ae + 2bd) + cd^2) + \frac{1}{6}dx^6(2ae + bd) + \frac{1}{4}ad^2x^4 + \frac{1}{10}ex^{10}(be + 2cd) + \frac{1}{12}ce^2x^{12}$$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 771}

$$\frac{1}{8}x^8 (e(ae + 2bd) + cd^2) + \frac{1}{6}dx^6(2ae + bd) + \frac{1}{4}ad^2x^4 + \frac{1}{10}ex^{10}(be + 2cd) + \frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12

Rule 771

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2)^2 (a + bx^2 + cx^4) dx &= \frac{1}{2} \text{Subst} \left(\int x(d + ex)^2 (a + bx + cx^2) dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (ad^2x + d(bd + 2ae)x^2 + (cd^2 + e(2bd + ae))x^3 + e(2cd + be)x^4) dx, x, x^2 \right) \\ &= \frac{1}{4}ad^2x^4 + \frac{1}{6}d(bd + 2ae)x^6 + \frac{1}{8}(cd^2 + e(2bd + ae))x^8 + \frac{1}{10}e(2cd + be)x^{10} + \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4(15x^4(e(ae + 2bd) + cd^2) + 20dx^2(2ae + bd) + 30ad^2 + 12ex^6(be + 2cd) + 10ce^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4 + 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8)/120

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.83, size = 79, normalized size = 1.01

$$\frac{1}{12}x^{12}e^2c + \frac{1}{5}x^{10}edc + \frac{1}{10}x^{10}e^2b + \frac{1}{8}x^8d^2c + \frac{1}{4}x^8edb + \frac{1}{8}x^8e^2a + \frac{1}{6}x^6d^2b + \frac{1}{3}x^6eda + \frac{1}{4}x^4d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/12*x^12*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/8*x^8*d^2*c + 1/4*x^8*e*d*b + 1/8*x^8*e^2*a + 1/6*x^6*d^2*b + 1/3*x^6*e*d*a + 1/4*x^4*d^2*a

giac [A] time = 0.27, size = 79, normalized size = 1.01

$$\frac{1}{12}cx^{12}e^2 + \frac{1}{5}cdx^{10}e + \frac{1}{10}bx^{10}e^2 + \frac{1}{8}cd^2x^8 + \frac{1}{4}bdx^8e + \frac{1}{8}ax^8e^2 + \frac{1}{6}bd^2x^6 + \frac{1}{3}adx^6e + \frac{1}{4}ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^8 + 1/4*b*d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{c e^2 x^{12}}{12} + \frac{(e^2 b + 2 d e c) x^{10}}{10} + \frac{(a e^2 + 2 d e b + c d^2) x^8}{8} + \frac{a d^2 x^4}{4} + \frac{(2 d e a + d^2 b) x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)`

[Out] $1/12*c*e^2*x^{12}+1/10*(b*e^2+2*c*d*e)*x^{10}+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4$

maxima [A] time = 1.11, size = 72, normalized size = 0.92

$$\frac{1}{12}ce^2x^{12} + \frac{1}{10}(2cde + be^2)x^{10} + \frac{1}{8}(cd^2 + 2bde + ae^2)x^8 + \frac{1}{4}ad^2x^4 + \frac{1}{6}(bd^2 + 2ade)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] $1/12*c*e^2*x^{12} + 1/10*(2*c*d*e + b*e^2)*x^{10} + 1/8*(c*d^2 + 2*b*d*e + a*e^2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6$

mupad [B] time = 0.04, size = 73, normalized size = 0.94

$$x^8 \left(\frac{cd^2}{8} + \frac{bde}{4} + \frac{ae^2}{8} \right) + x^6 \left(\frac{bd^2}{6} + \frac{aed}{3} \right) + x^{10} \left(\frac{be^2}{10} + \frac{cde}{5} \right) + \frac{ad^2x^4}{4} + \frac{ce^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)`

[Out] $x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^{10}*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^{12})/12$

sympy [A] time = 0.08, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4} + \frac{ce^2x^{12}}{12} + x^{10} \left(\frac{be^2}{10} + \frac{cde}{5} \right) + x^8 \left(\frac{ae^2}{8} + \frac{bde}{4} + \frac{cd^2}{8} \right) + x^6 \left(\frac{ade}{3} + \frac{bd^2}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)`

[Out] $a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)$

$$3.196 \quad \int x^2 (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=78

$$\frac{1}{7}x^7 (e(ae + 2bd) + cd^2) + \frac{1}{5}dx^5(2ae + bd) + \frac{1}{3}ad^2x^3 + \frac{1}{9}ex^9(be + 2cd) + \frac{1}{11}ce^2x^{11}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$\frac{1}{7}x^7 (e(ae + 2bd) + cd^2) + \frac{1}{5}dx^5(2ae + bd) + \frac{1}{3}ad^2x^3 + \frac{1}{9}ex^9(be + 2cd) + \frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2)^2 (a + bx^2 + cx^4) dx &= \int (ad^2x^2 + d(bd + 2ae)x^4 + (cd^2 + e(2bd + ae))x^6 + e(2cd + be)x^8 + ce^2x^{10}) dx \\ &= \frac{1}{3}ad^2x^3 + \frac{1}{5}d(bd + 2ae)x^5 + \frac{1}{7}(cd^2 + e(2bd + ae))x^7 + \frac{1}{9}e(2cd + be)x^9 + \frac{1}{11}ce^2x^{11} \end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.00

$$\frac{1}{7}x^7 (ae^2 + 2bde + cd^2) + \frac{1}{5}dx^5(2ae + bd) + \frac{1}{3}ad^2x^3 + \frac{1}{9}ex^9(be + 2cd) + \frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] $(a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.79, size = 79, normalized size = 1.01

$$\frac{1}{11}x^{11}e^2c + \frac{2}{9}x^9edc + \frac{1}{9}x^9e^2b + \frac{1}{7}x^7d^2c + \frac{2}{7}x^7edb + \frac{1}{7}x^7e^2a + \frac{1}{5}x^5d^2b + \frac{2}{5}x^5eda + \frac{1}{3}x^3d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] $1/11*x^{11}*e^2*c + 2/9*x^9*e*d*c + 1/9*x^9*e^2*b + 1/7*x^7*d^2*c + 2/7*x^7*e*d*b + 1/7*x^7*e^2*a + 1/5*x^5*d^2*b + 2/5*x^5*e*d*a + 1/3*x^3*d^2*a$

giac [A] time = 0.39, size = 79, normalized size = 1.01

$$\frac{1}{11}cx^{11}e^2 + \frac{2}{9}cdx^9e + \frac{1}{9}bx^9e^2 + \frac{1}{7}cd^2x^7 + \frac{2}{7}bdx^7e + \frac{1}{7}ax^7e^2 + \frac{1}{5}bd^2x^5 + \frac{2}{5}adx^5e + \frac{1}{3}ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] $1/11*c*x^{11}*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3$

maple [A] time = 0.00, size = 73, normalized size = 0.94

$$\frac{c e^2 x^{11}}{11} + \frac{(e^2 b + 2 d e c) x^9}{9} + \frac{(a e^2 + 2 d e b + c d^2) x^7}{7} + \frac{a d^2 x^3}{3} + \frac{(2 d e a + d^2 b) x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a), x)

[Out] $1/11*c*e^2*x^{11}+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3$

maxima [A] time = 1.19, size = 72, normalized size = 0.92

$$\frac{1}{11} ce^2 x^{11} + \frac{1}{9} (2cde + be^2) x^9 + \frac{1}{7} (cd^2 + 2bde + ae^2) x^7 + \frac{1}{3} ad^2 x^3 + \frac{1}{5} (bd^2 + 2ade) x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5

mupad [B] time = 0.03, size = 73, normalized size = 0.94

$$x^7 \left(\frac{cd^2}{7} + \frac{2bde}{7} + \frac{ae^2}{7} \right) + x^5 \left(\frac{bd^2}{5} + \frac{2aed}{5} \right) + x^9 \left(\frac{be^2}{9} + \frac{2cde}{9} \right) + \frac{ad^2 x^3}{3} + \frac{ce^2 x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)

[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5) + x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11

sympy [A] time = 0.08, size = 82, normalized size = 1.05

$$\frac{ad^2 x^3}{3} + \frac{ce^2 x^{11}}{11} + x^9 \left(\frac{be^2}{9} + \frac{2cde}{9} \right) + x^7 \left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7} \right) + x^5 \left(\frac{2ade}{5} + \frac{bd^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)

[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)

$$3.197 \quad \int x (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=75

$$\frac{(d + ex^2)^3 (ae^2 - bde + cd^2)}{6e^3} - \frac{(d + ex^2)^4 (2cd - be)}{8e^3} + \frac{c(d + ex^2)^5}{10e^3}$$

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1247, 698}

$$\frac{(d + ex^2)^3 (ae^2 - bde + cd^2)}{6e^3} - \frac{(d + ex^2)^4 (2cd - be)}{8e^3} + \frac{c(d + ex^2)^5}{10e^3}$$

Antiderivative was successfully verified.

[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]

[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x(d+ex^2)^2(a+bx^2+cx^4)dx &= \frac{1}{2} \text{Subst} \left(\int (d+ex)^2(a+bx+cx^2)dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{(cd^2 - bde + ae^2)(d+ex)^2}{e^2} + \frac{(-2cd + be)(d+ex)^3}{e^2} + \frac{c(d+ex)^4}{e^2} \right) dx, x, x^2 \right) \\
&= \frac{(cd^2 - bde + ae^2)(d+ex^2)^3}{6e^3} - \frac{(2cd - be)(d+ex^2)^4}{8e^3} + \frac{c(d+ex^2)^5}{10e^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2(20x^4(e(ae+2bd)+cd^2)+30dx^2(2ae+bd)+60ad^2+15ex^6(be+2cd)+12ce^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e))*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4 + 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8)/120

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(d+ex^2)^2(a+bx^2+cx^4)dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.81, size = 79, normalized size = 1.05

$$\frac{1}{10}x^{10}e^2c + \frac{1}{4}x^8edc + \frac{1}{8}x^8e^2b + \frac{1}{6}x^6d^2c + \frac{1}{3}x^6edb + \frac{1}{6}x^6e^2a + \frac{1}{4}x^4d^2b + \frac{1}{2}x^4eda + \frac{1}{2}x^2d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/10*x^10*e^2*c + 1/4*x^8*e*d*c + 1/8*x^8*e^2*b + 1/6*x^6*d^2*c + 1/3*x^6*e*d*b + 1/6*x^6*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + 1/2*x^2*d^2*a

giac [A] time = 0.27, size = 79, normalized size = 1.05

$$\frac{1}{10}cx^{10}e^2 + \frac{1}{4}cdx^8e + \frac{1}{8}bx^8e^2 + \frac{1}{6}cd^2x^6 + \frac{1}{3}bdx^6e + \frac{1}{6}ax^6e^2 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adx^4e + \frac{1}{2}ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $1/10*c*x^{10}*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 + 1/3*b*d*x^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2$

maple [A] time = 0.00, size = 73, normalized size = 0.97

$$\frac{ce^2x^{10}}{10} + \frac{(e^2b + 2dec)x^8}{8} + \frac{(ae^2 + 2deb + cd^2)x^6}{6} + \frac{ad^2x^2}{2} + \frac{(2dea + d^2b)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)

[Out] $1/10*c*e^2*x^{10} + 1/8*(b*e^2 + 2*c*d*e)*x^8 + 1/6*(a*e^2 + 2*b*d*e + c*d^2)*x^6 + 1/4*(2*a*d*e + b*d^2)*x^4 + 1/2*a*d^2*x^2$

maxima [A] time = 1.21, size = 72, normalized size = 0.96

$$\frac{1}{10}ce^2x^{10} + \frac{1}{8}(2cde + be^2)x^8 + \frac{1}{6}(cd^2 + 2bde + ae^2)x^6 + \frac{1}{2}ad^2x^2 + \frac{1}{4}(bd^2 + 2ade)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $1/10*c*e^2*x^{10} + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4$

mupad [B] time = 0.03, size = 73, normalized size = 0.97

$$x^6 \left(\frac{cd^2}{6} + \frac{bde}{3} + \frac{ae^2}{6} \right) + x^4 \left(\frac{bd^2}{4} + \frac{aed}{2} \right) + x^8 \left(\frac{be^2}{8} + \frac{cde}{4} \right) + \frac{ad^2x^2}{2} + \frac{ce^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)

[Out] $x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^8*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^{10})/10$

sympy [A] time = 0.08, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2} + \frac{ce^2x^{10}}{10} + x^8 \left(\frac{be^2}{8} + \frac{cde}{4} \right) + x^6 \left(\frac{ae^2}{6} + \frac{bde}{3} + \frac{cd^2}{6} \right) + x^4 \left(\frac{ade}{2} + \frac{bd^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

$$3.198 \quad \int (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Optimal. Leaf size=73

$$\frac{1}{5}x^5 (e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1153}

$$\frac{1}{5}x^5 (e(ae + 2bd) + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + bx^2 + cx^4) dx &= \int (ad^2 + d(bd + 2ae)x^2 + (cd^2 + e(2bd + ae))x^4 + e(2cd + be)x^6 + ce^2x^8) dx \\ &= ad^2x + \frac{1}{3}d(bd + 2ae)x^3 + \frac{1}{5}(cd^2 + e(2bd + ae))x^5 + \frac{1}{7}e(2cd + be)x^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{5}x^5 (ae^2 + 2bde + cd^2) + \frac{1}{3}dx^3(2ae + bd) + ad^2x + \frac{1}{7}ex^7(be + 2cd) + \frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] $a*d^2*x + (d*(b*d + 2*a*e))*x^3/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*(2*c*d + b*e))*x^7/7 + (c*e^2*x^9)/9$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex^2)^2 (a + bx^2 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

[Out] IntegrateAlgebraic[(d + e*x^2)^2*(a + b*x^2 + c*x^4), x]

fricas [A] time = 0.52, size = 76, normalized size = 1.04

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{7}x^7e^2b + \frac{1}{5}x^5d^2c + \frac{2}{5}x^5edb + \frac{1}{5}x^5e^2a + \frac{1}{3}x^3d^2b + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] $1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a$

giac [A] time = 0.35, size = 76, normalized size = 1.04

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] $1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x$

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{ce^2x^9}{9} + \frac{(e^2b + 2dec)x^7}{7} + \frac{(ae^2 + 2deb + cd^2)x^5}{5} + ad^2x + \frac{(2dea + d^2b)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a), x)

[Out] $1/9*c*e^2*x^9 + 1/7*(b*e^2 + 2*c*d*e)*x^7 + 1/5*(a*e^2 + 2*b*d*e + c*d^2)*x^5 + 1/3*(2*a*d*e + b*d^2)*x^3 + a*d^2*x$

maxima [A] time = 1.17, size = 69, normalized size = 0.95

$$\frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde + be^2)x^7 + \frac{1}{5}(cd^2 + 2bde + ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2 + 2ade)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3

mupad [B] time = 0.03, size = 70, normalized size = 0.96

$$x^5 \left(\frac{cd^2}{5} + \frac{2bde}{5} + \frac{ae^2}{5} \right) + x^3 \left(\frac{bd^2}{3} + \frac{2aed}{3} \right) + x^7 \left(\frac{be^2}{7} + \frac{2cde}{7} \right) + \frac{ce^2x^9}{9} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)

[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) + x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x

sympy [A] time = 0.08, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7 \left(\frac{be^2}{7} + \frac{2cde}{7} \right) + x^5 \left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5} \right) + x^3 \left(\frac{2ade}{3} + \frac{bd^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)

[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)

$$3.199 \quad \int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx$$

Optimal. Leaf size=74

$$\frac{1}{4}x^4(eae + 2bd) + cd^2 + \frac{1}{2}dx^2(2ae + bd) + ad^2 \log(x) + \frac{1}{6}ex^6(be + 2cd) + \frac{1}{8}ce^2x^8$$

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 893}

$$\frac{1}{4}x^4(eae + 2bd) + cd^2 + \frac{1}{2}dx^2(2ae + bd) + ad^2 \log(x) + \frac{1}{6}ex^6(be + 2cd) + \frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]

[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)^2(a+bx+cx^2)}{x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(d(bd+2ae) + \frac{ad^2}{x} + (cd^2 + e(2bd+ae))x + e(2cd+be)x^2 + ce^2x^3 \right) dx, x, x^2 \right) \\ &= \frac{1}{2}d(bd+2ae)x^2 + \frac{1}{4}(cd^2 + e(2bd+ae))x^4 + \frac{1}{6}e(2cd+be)x^6 + \frac{1}{8}ce^2x^8 + ad^2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{1}{4}x^4(ae^2 + 2bde + cd^2) + \frac{1}{2}dx^2(2ae + bd) + ad^2 \log(x) + \frac{1}{6}ex^6(be + 2cd) + \frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]

[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]

[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x, x]

fricas [A] time = 0.90, size = 70, normalized size = 0.95

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}(2cde + be^2)x^6 + \frac{1}{4}(cd^2 + 2bde + ae^2)x^4 + ad^2 \log(x) + \frac{1}{2}(bd^2 + 2ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")

[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2

giac [A] time = 0.26, size = 79, normalized size = 1.07

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")

[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1/2*b*d*x^4*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*d^2*log(x^2)

maple [A] time = 0.00, size = 77, normalized size = 1.04

$$\frac{ce^2x^8}{8} + \frac{be^2x^6}{6} + \frac{cde x^6}{3} + \frac{ae^2x^4}{4} + \frac{bdex^4}{2} + \frac{cd^2x^4}{4} + adex^2 + \frac{bd^2x^2}{2} + ad^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x)

[Out] 1/8*c*e^2*x^8+1/6*x^6*b*e^2+1/3*x^6*c*d*e+1/4*x^4*a*e^2+1/2*x^4*b*d*e+1/4*x^4*c*d^2+x^2*a*d*e+1/2*x^2*b*d^2+a*d^2*ln(x)

maxima [A] time = 1.12, size = 73, normalized size = 0.99

$$\frac{1}{8}ce^2x^8 + \frac{1}{6}(2cde + be^2)x^6 + \frac{1}{4}(cd^2 + 2bde + ae^2)x^4 + \frac{1}{2}ad^2\log(x^2) + \frac{1}{2}(bd^2 + 2ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")

[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2

mupad [B] time = 0.03, size = 70, normalized size = 0.95

$$x^4 \left(\frac{cd^2}{4} + \frac{bde}{2} + \frac{ae^2}{4} \right) + x^2 \left(\frac{bd^2}{2} + aed \right) + x^6 \left(\frac{be^2}{6} + \frac{cde}{3} \right) + \frac{ce^2x^8}{8} + ad^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)

[Out] x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((b*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)

sympy [A] time = 0.17, size = 73, normalized size = 0.99

$$ad^2 \log(x) + \frac{ce^2x^8}{8} + x^6 \left(\frac{be^2}{6} + \frac{cde}{3} \right) + x^4 \left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4} \right) + x^2 \left(ade + \frac{bd^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)

[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4 + b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)

$$3.200 \quad \int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx$$

Optimal. Leaf size=71

$$\frac{1}{3}x^3(e(ae+2bd)+cd^2)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1261}

$$\frac{1}{3}x^3(e(ae+2bd)+cd^2)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2, x]

[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7

Rule 1261

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx &= \int \left(d(bd+2ae) + \frac{ad^2}{x^2} + (cd^2 + e(2bd+ae))x^2 + e(2cd+be)x^4 + ce^2x^6 \right) dx \\ &= -\frac{ad^2}{x} + d(bd+2ae)x + \frac{1}{3}(cd^2 + e(2bd+ae))x^3 + \frac{1}{5}e(2cd+be)x^5 + \frac{1}{7}ce^2x^7 \end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.00

$$\frac{1}{3}x^3(ae^2+2bde+cd^2)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]

[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2 (a + bx^2 + cx^4)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]

[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2, x]

fricas [A] time = 0.84, size = 74, normalized size = 1.04

$$\frac{15ce^2x^8 + 21(2cde + be^2)x^6 + 35(cd^2 + 2bde + ae^2)x^4 - 105ad^2 + 105(bd^2 + 2ade)x^2}{105x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")

[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e^2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x

giac [A] time = 0.36, size = 74, normalized size = 1.04

$$\frac{1}{7}cx^7e^2 + \frac{2}{5}cdx^5e + \frac{1}{5}bx^5e^2 + \frac{1}{3}cd^2x^3 + \frac{2}{3}bdx^3e + \frac{1}{3}ax^3e^2 + bd^2x + 2adxe - \frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")

[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2/3*b*d*x^3*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x

maple [A] time = 0.00, size = 75, normalized size = 1.06

$$\frac{ce^2x^7}{7} + \frac{b e^2x^5}{5} + \frac{2cde x^5}{5} + \frac{a e^2x^3}{3} + \frac{2bde x^3}{3} + \frac{c d^2x^3}{3} + 2adex + b d^2x - \frac{a d^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x)

[Out] $1/7*c*e^2*x^7+1/5*x^5*b*e^2+2/5*x^5*c*d*e+1/3*x^3*a*e^2+2/3*x^3*b*d*e+1/3*x^3*c*d^2+2*d*e*a*x+d^2*b*x-a*d^2/x$

maxima [A] time = 1.07, size = 69, normalized size = 0.97

$$\frac{1}{7}ce^2x^7 + \frac{1}{5}(2cde + be^2)x^5 + \frac{1}{3}(cd^2 + 2bde + ae^2)x^3 - \frac{ad^2}{x} + (bd^2 + 2ade)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")`

[Out] $1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x$

mupad [B] time = 0.03, size = 70, normalized size = 0.99

$$x^3 \left(\frac{cd^2}{3} + \frac{2bde}{3} + \frac{ae^2}{3} \right) + x (bd^2 + 2aed) + x^5 \left(\frac{be^2}{5} + \frac{2cde}{5} \right) - \frac{ad^2}{x} + \frac{ce^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)`

[Out] $x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*e^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7$

sympy [A] time = 0.16, size = 73, normalized size = 1.03

$$-\frac{ad^2}{x} + \frac{ce^2x^7}{7} + x^5 \left(\frac{be^2}{5} + \frac{2cde}{5} \right) + x^3 \left(\frac{ae^2}{3} + \frac{2bde}{3} + \frac{cd^2}{3} \right) + x(2ade + bd^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)`

[Out] $-a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 + 2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)$

$$3.201 \quad \int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3} dx$$

Optimal. Leaf size=74

$$\frac{1}{2}x^2(e(ae+2bd)+cd^2) + d \log(x)(2ae+bd) - \frac{ad^2}{2x^2} + \frac{1}{4}ex^4(be+2cd) + \frac{1}{6}ce^2x^6$$

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1251, 893}

$$\frac{1}{2}x^2(e(ae+2bd)+cd^2) + d \log(x)(2ae+bd) - \frac{ad^2}{2x^2} + \frac{1}{4}ex^4(be+2cd) + \frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]

[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + bx^2 + cx^4)}{x^3} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d + ex^2)^2 (a + bx + cx^2)}{x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(cd^2 \left(1 + \frac{e(2bd + ae)}{cd^2} \right) + \frac{ad^2}{x^2} + \frac{d(bd + 2ae)}{x} + e(2cd + be)x + c \right) dx, x, x^2 \right) \\ &= -\frac{ad^2}{2x^2} + \frac{1}{2} (cd^2 + e(2bd + ae)) x^2 + \frac{1}{4} e(2cd + be)x^4 + \frac{1}{6} ce^2 x^6 + d(bd + 2ae) \log(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.96

$$\frac{1}{12} \left(6x^2 (e(ae + 2bd) + cd^2) + 12d \log(x)(2ae + bd) - \frac{6ad^2}{x^2} + 3ex^4(be + 2cd) + 2ce^2x^6 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]

[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 + 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2 (a + bx^2 + cx^4)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]

[Out] IntegrateAlgebraic[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3, x]

fricas [A] time = 0.89, size = 76, normalized size = 1.03

$$\frac{2ce^2x^8 + 3(2cde + be^2)x^6 + 6(cd^2 + 2bde + ae^2)x^4 + 12(bd^2 + 2ade)x^2 \log(x) - 6ad^2}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")

[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*x^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2

giac [A] time = 0.38, size = 97, normalized size = 1.31

$$\frac{1}{6}cx^6e^2 + \frac{1}{2}cdx^4e + \frac{1}{4}bx^4e^2 + \frac{1}{2}cd^2x^2 + bdx^2e + \frac{1}{2}ax^2e^2 + \frac{1}{2}(bd^2 + 2ade)\log(x^2) - \frac{bd^2x^2 + 2adx^2e + ad^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")

[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b*d*x^2*e + 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*x^2*e + a*d^2)/x^2

maple [A] time = 0.01, size = 76, normalized size = 1.03

$$\frac{ce^2x^6}{6} + \frac{be^2x^4}{4} + \frac{cde x^4}{2} + \frac{ae^2x^2}{2} + bde x^2 + \frac{cd^2x^2}{2} + 2ade \ln(x) + bd^2 \ln(x) - \frac{ad^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x)

[Out] 1/6*c*e^2*x^6+1/4*x^4*b*e^2+1/2*x^4*c*d*e+1/2*x^2*a*e^2+x^2*b*d*e+1/2*x^2*c*d^2+2*ln(x)*a*d*e+ln(x)*b*d^2-1/2*a*d^2/x^2

maxima [A] time = 1.09, size = 73, normalized size = 0.99

$$\frac{1}{6}ce^2x^6 + \frac{1}{4}(2cde + be^2)x^4 + \frac{1}{2}(cd^2 + 2bde + ae^2)x^2 + \frac{1}{2}(bd^2 + 2ade)\log(x^2) - \frac{ad^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")

[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2

mupad [B] time = 0.04, size = 70, normalized size = 0.95

$$x^2 \left(\frac{cd^2}{2} + bde + \frac{ae^2}{2} \right) + x^4 \left(\frac{be^2}{4} + \frac{cde}{2} \right) + \ln(x) (bd^2 + 2aed) - \frac{ad^2}{2x^2} + \frac{ce^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)

[Out] x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)*(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6

sympy [A] time = 0.26, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2} + \frac{ce^2x^6}{6} + d(2ae + bd)\log(x) + x^4\left(\frac{be^2}{4} + \frac{cde}{2}\right) + x^2\left(\frac{ae^2}{2} + bde + \frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)

[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4 + c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)

$$3.202 \quad \int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=168

$$-\frac{dx(4cd^2 - e(3bd - 2ae))}{e^5} + \frac{x^3(3cd^2 - e(2bd - ae))}{3e^4} - \frac{d^2x(ae^2 - bde + cd^2)}{2e^5(d + ex^2)} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(9cd^2 - e(7bd - 5ae))}{2e^{11/2}}$$

Rubi [A] time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1257, 1810, 205}

$$\frac{x^3(3cd^2 - e(2bd - ae))}{3e^4} - \frac{d^2x(ae^2 - bde + cd^2)}{2e^5(d + ex^2)} - \frac{dx(4cd^2 - e(3bd - 2ae))}{e^5} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(9cd^2 - e(7bd - 5ae))}{2e^{11/2}} - \frac{x^5(2cd - be)}{5e^3} + \frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1257

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^6 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx &= -\frac{d^2 (cd^2 - bde + ae^2) x}{2e^5 (d + ex^2)} - \frac{\int \frac{-d^2(cd^2 - bde + ae^2) + 2de(cd^2 - bde + ae^2)x^2 - 2e^2(cd^2 - bde + ae^2)x^4 + 2e^3(cd - bde + ae^2)x^6}{d + ex^2} dx}{2e^5} \\
&= -\frac{d^2 (cd^2 - bde + ae^2) x}{2e^5 (d + ex^2)} - \frac{\int (2d(4cd^2 - e(3bd - 2ae)) - 2e(3cd^2 - e(2bd - ae)) x^2 + 2e^3(cd - bde + ae^2) x^4 - 2e^5 a x^6) dx}{2e^5} \\
&= -\frac{d(4cd^2 - e(3bd - 2ae)) x}{e^5} + \frac{(3cd^2 - e(2bd - ae)) x^3}{3e^4} - \frac{(2cd - be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2 (cd^2 - bde + ae^2) x}{2e^5} \\
&= -\frac{d(4cd^2 - e(3bd - 2ae)) x}{e^5} + \frac{(3cd^2 - e(2bd - ae)) x^3}{3e^4} - \frac{(2cd - be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2 (cd^2 - bde + ae^2) x}{2e^5}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 0.98

$$-\frac{dx(2ae^2 - 3bde + 4cd^2)}{e^5} + \frac{x^3(ae^2 - 2bde + 3cd^2)}{3e^4} + \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(5ae^2 - 7bde + 9cd^2)}{2e^{11/2}} - \frac{x(ad^2e^2 - bd^3e + cd^4)}{2e^5(d + ex^2)} + \frac{x^5(be - 2cd)}{5e^3} + \frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]

[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + (((3*c*d^2 - 2*b*d*e + a*e^2)*x^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2))) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]

[Out] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]

fricas [A] time = 0.99, size = 426, normalized size = 2.54

$$\frac{d^2 (cd^2 - bde + ae^2) x}{2e^5 (d + ex^2)} - \frac{\int \frac{-d^2(cd^2 - bde + ae^2) + 2de(cd^2 - bde + ae^2)x^2 - 2e^2(cd^2 - bde + ae^2)x^4 + 2e^3(cd - bde + ae^2)x^6}{d + ex^2} dx}{2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 - 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 - 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5)]

giac [A] time = 0.32, size = 160, normalized size = 0.95

$$\frac{(9cd^4 - 7bd^3e + 5ad^2e^2) \arctan\left(\frac{x\sqrt{d}}{\sqrt{d}}\right) e^{\left(-\frac{11}{2}\right)}}{2\sqrt{d}} + \frac{1}{105} (15cx^7e^{12} - 42cdx^5e^{11} + 21bx^5e^{12} + 105cd^2x^3e^{10} - 70bdx^3e^{11} - 420cd^3xe^9 + 35ax^3e^{12} + 315bd^2xe^{10} - 210adx^{11})e^{(-14)} - \frac{(cd^4x - bd^3xe + ad^2xe^2)e^{(-5)}}{2(x^2e + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*b*x^5*e^12 + 105*c*d^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e^9 + 35*a*x^3*e^12 + 315*b*d^2*x*e^10 - 210*a*d*x*e^11)*e^(-14) - 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)*e^(-5)/(x^2*e + d)

maple [A] time = 0.01, size = 214, normalized size = 1.27

$$\frac{cx^7}{7e^2} + \frac{bx^5}{5e^2} - \frac{2cdx^5}{5e^3} + \frac{ax^3}{3e^2} - \frac{2bdx^3}{3e^3} + \frac{cd^2x^3}{e^4} - \frac{ad^2x}{2(e^2x^2 + d)e^3} + \frac{5ad^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^3} + \frac{bd^3x}{2(e^2x^2 + d)e^4} - \frac{7bd^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^4} - \frac{cd^4x}{2(e^2x^2 + d)e^5} + \frac{9cd^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^5} - \frac{2adx}{e^3} + \frac{3bd^2x}{e^4} - \frac{4cd^3x}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] 1/7*c*x^7/e^2+1/5/e^2*x^5*b-2/5/e^3*x^5*c*d+1/3/e^2*x^3*a-2/3/e^3*x^3*b*d+1/e^4*x^3*c*d^2-2/e^3*a*d*x+3/e^4*d^2*b*x-4/e^5*c*d^3*x-1/2*d^2/e^3*x/(e*x^2+d)*a+1/2*d^3/e^4*x/(e*x^2+d)*b-1/2*d^4/e^5*x/(e*x^2+d)*c+5/2*d^2/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+9/2*d^4/e^5/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c

maxima [A] time = 2.49, size = 165, normalized size = 0.98

$$\frac{(cd^4 - bd^3e + ad^2e^2)x}{2(e^6x^2 + de^5)} + \frac{(9cd^4 - 7bd^3e + 5ad^2e^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^5} + \frac{15ce^3x^7 - 21(2cde^2 - be^3)x^5 + 35(3cd^2e - 2bde^2 + ae^3)x^3 - 105(4cd^3 - 3bd^2e + 2ade^2)x}{105e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $-1/2*(c*d^4 - b*d^3*e + a*d^2*e^2)*x/(e^6*x^2 + d*e^5) + 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*e^5) + 1/105*(15*c*e^3*x^7 - 21*(2*c*d*e^2 - b*e^3)*x^5 + 35*(3*c*d^2*e - 2*b*d*e^2 + a*e^3)*x^3 - 105*(4*c*d^3 - 3*b*d^2*e + 2*a*d*e^2)*x)/e^5$

mupad [B] time = 0.33, size = 251, normalized size = 1.49

$$x^5 \left(\frac{b}{5e^2} - \frac{2cd}{5e^3} \right) - x^3 \left(\frac{cd^2}{3e^4} - \frac{a}{3e^2} + \frac{2d \left(\frac{b}{2} - \frac{2cd}{e^2} \right)}{3e} \right) + x \left(\frac{2d \left(\frac{cd^2}{e^4} - \frac{a}{e^2} + \frac{2d \left(\frac{b}{2} - \frac{2cd}{e^2} \right)}{e} \right)}{e} - d^2 \left(\frac{b}{e^2} - \frac{2cd}{e^3} \right) \right) - \frac{x \left(\frac{cd^4}{2} - \frac{bd^3e}{2} + \frac{ad^2e^2}{2} \right)}{e^6 x^2 + de^5} + \frac{cx^7}{7e^2} + \frac{d^{3/2} \operatorname{atan} \left(\frac{d^{3/2} \sqrt{e} x (9cd^2 - 7bde + 5ae^2)}{9cd^4 - 7bd^3e + 5ae^2} \right)}{2e^{11/2}} (9cd^2 - 7bde + 5ae^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)

[Out] $x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*d*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2 - (x*((c*d^4)/2 + (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d^(3/2)*\operatorname{atan}((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5*a*d^2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))$

sympy [B] time = 1.18, size = 320, normalized size = 1.90

$$\frac{cx^7}{7e^2} + x^5 \left(\frac{b}{5e^2} - \frac{2cd}{5e^3} \right) + x^3 \left(\frac{a}{3e^2} - \frac{2bd}{3e^3} + \frac{cd^2}{e^4} \right) + x \left(-\frac{2nd}{e^3} + \frac{3bd^2}{e^4} - \frac{4cd^3}{e^5} \right) + \frac{x(-ad^2e^2 + bd^3e - cd^4)}{2de^5 + 2e^6x^2} - \frac{\sqrt{-\frac{d^3}{21}}(5ae^2 - 7bde + 9cd^2) \log \left(-\frac{e^5 \sqrt{\frac{d^3}{21}}(5ae^2 - 7bde + 9cd^2)}{5ad^2 - 7bd^2e + 9cd^3} + x \right)}{4} + \frac{\sqrt{-\frac{d^3}{21}}(5ae^2 - 7bde + 9cd^2) \log \left(\frac{e^5 \sqrt{\frac{d^3}{21}}(5ae^2 - 7bde + 9cd^2)}{5ad^2 - 7bd^2e + 9cd^3} + x \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)

[Out] $c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) - 2*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/e**5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - \sqrt{-d**3/e**11}*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*\log(-e**5*\sqrt{-d**3/e**11}*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 + \sqrt{-d**3/e**11}*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*\log(e**5*\sqrt{-d**3/e**11}*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4$

$$3.203 \quad \int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=135

$$-\frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(7cd^2 - e(5bd - 3ae))}{2e^{9/2}} + \frac{x(3cd^2 - e(2bd - ae))}{e^4} + \frac{dx(ae^2 - bde + cd^2)}{2e^4(d + ex^2)} - \frac{x^3(2cd - be)}{3e^3} + \frac{cx^5}{5e^2}$$

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1257, 1810, 205}

$$\frac{dx(ae^2 - bde + cd^2)}{2e^4(d + ex^2)} + \frac{x(3cd^2 - e(2bd - ae))}{e^4} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(7cd^2 - e(5bd - 3ae))}{2e^{9/2}} - \frac{x^3(2cd - be)}{3e^3} + \frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx &= \frac{d (cd^2 - bde + ae^2) x}{2e^4 (d + ex^2)} - \frac{\int \frac{d(cd^2 - bde + ae^2) - 2e(cd^2 - bde + ae^2)x^2 + 2e^2(cd - be)x^4 - 2ce^3x^6}{d + ex^2} dx}{2e^4} \\
&= \frac{d (cd^2 - bde + ae^2) x}{2e^4 (d + ex^2)} - \frac{\int \left(-2 (3cd^2 - 2bde + ae^2) + 2e(2cd - be)x^2 - 2ce^2x^4 + \frac{7cd^3 - e(3cd^2 - 2bde + ae^2)}{d + ex^2} \right) dx}{2e^4} \\
&= \frac{(3cd^2 - e(2bd - ae)) x}{e^4} - \frac{(2cd - be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d (cd^2 - bde + ae^2) x}{2e^4 (d + ex^2)} - \frac{d (7cd^2 - e(3cd^2 - 2bde + ae^2))}{2e^4 (d + ex^2)} \\
&= \frac{(3cd^2 - e(2bd - ae)) x}{e^4} - \frac{(2cd - be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d (cd^2 - bde + ae^2) x}{2e^4 (d + ex^2)} - \frac{\sqrt{d} (7cd^2 - e(3cd^2 - 2bde + ae^2))}{2e^4 (d + ex^2)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.99

$$-\frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3ae^2 - 5bde + 7cd^2)}{2e^{9/2}} + \frac{x(ae^2 - 2bde + 3cd^2)}{e^4} + \frac{x(ade^2 - bd^2e + cd^3)}{2e^4(d + ex^2)} + \frac{x^3(be - 2cd)}{3e^3} + \frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]* (7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]

fricas [A] time = 0.99, size = 350, normalized size = 2.59

$$\frac{12cd^3x^7 - 4(7cd^2 - 5be^2)x^6 + 20(7cd^2e - 5bd^2 + 3ae^2)x^5 + 15(7cd^3 - 5bd^2e + 3ad^2 + (7cd^2e - 5bd^2 + 3ae^2)^2)\sqrt{-\frac{e}{d}} \log\left(\frac{e^2 - 2ex + d}{-ex + d}\right) + 30(7cd^3 - 5bd^2e + 3ad^2)ex - 6cx^5d^2 - 2(7cd^2 - 5be^2)x^4 + 10(7cd^2e - 5bd^2 + 3ae^2)x^3 - 15(7cd^3 - 5bd^2e + 3ad^2 + (7cd^2e - 5bd^2 + 3ae^2)^2)\sqrt{-\frac{e}{d}} \arctan\left(\frac{ex}{\sqrt{d}}\right) + 15(7cd^3 - 5bd^2e + 3ad^2)ex}{30(e^2x^2 + d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]

giac [A] time = 0.28, size = 125, normalized size = 0.93

$$-\frac{(7cd^3 - 5bd^2e + 3ade^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{2\sqrt{d}} + \frac{1}{15} (3cx^5e^8 - 10cdx^3e^7 + 5bx^3e^8 + 45cd^2xe^6 - 30bdxe^7 + 15axe^8) e^{(-10)} + \frac{(cd^3x - bd^2xe + adxe^2) e^{(-4)}}{2(x^2e + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e^8 + 45*c*d^2*x*e^6 - 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c*d^3*x - b*d^2*x*e + a*d*x*e^2)*e^(-4)/(x^2*e + d)

maple [A] time = 0.01, size = 176, normalized size = 1.30

$$\frac{cx^5}{5e^2} + \frac{bx^3}{3e^2} - \frac{2cdx^3}{3e^3} + \frac{adx}{2(e^2x^2 + d)e^2} - \frac{3ad \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} - \frac{bd^2x}{2(e^2x^2 + d)e^3} + \frac{5bd^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^3} + \frac{cd^3x}{2(e^2x^2 + d)e^4} - \frac{7cd^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^4} + \frac{ax}{e^2} - \frac{2bdx}{e^3} + \frac{3cd^2x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] 1/5*c*x^5/e^2+1/3/e^2*x^3*b-2/3/e^3*x^3*c*d+1/e^2*a*x-2/e^3*d*b*x+3/e^4*c*d^2*x+1/2*d/e^2*x/(e*x^2+d)*a-1/2*d^2/e^3*x/(e*x^2+d)*b+1/2*d^3/e^4*x/(e*x^2+d)*c-3/2*d/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*d^2/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-7/2*d^3/e^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c

maxima [A] time = 2.45, size = 130, normalized size = 0.96

$$\frac{(cd^3 - bd^2e + ade^2)x}{2(e^5x^2 + de^4)} - \frac{(7cd^3 - 5bd^2e + 3ade^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^4} + \frac{3ce^2x^5 - 5(2cde - be^2)x^3 + 15(3cd^2 - 2bde + ae^2)x}{15e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(c*d^3 - b*d^2*e + a*d*e^2)*x/(e^5*x^2 + d*e^4) - \frac{1}{2}*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*e^4) + \frac{1}{15}*(3*c*e^2*x^5 - 5*(2*c*d*e - b*e^2)*x^3 + 15*(3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4$

mupad [B] time = 0.32, size = 179, normalized size = 1.33

$$x^3 \left(\frac{b}{3e^2} - \frac{2cd}{3e^3} \right) - x \left(\frac{cd^2}{e^4} - \frac{a}{e^2} + \frac{2d \left(\frac{b}{e^2} - \frac{2cd}{e^3} \right)}{e} \right) + \frac{cx^5}{5e^2} + \frac{x \left(\frac{cd^3}{2} - \frac{bd^2e}{2} + \frac{ade^2}{2} \right)}{e^5x^2 + de^4} - \frac{\sqrt{d} \operatorname{atan} \left(\frac{\sqrt{d} \sqrt{e} x (7cd^2 - 5bde + 3ae^2)}{7cd^3 - 5bd^2e + 3ade^2} \right) (7cd^2 - 5bde + 3ae^2)}{2e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)

[Out] $x^3*(b/(3e^2) - (2*c*d)/(3e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 - (2*c*d)/e^3))/e) + (c*x^5)/(5e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2*e)/2))/(d*e^4 + e^5*x^2) - (d^{(1/2)}*\operatorname{atan}((d^{(1/2)}*e^{(1/2)}*x*(3*a*e^2 + 7*c*d^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e)))/(2*e^{(9/2)})$

sympy [A] time = 1.08, size = 189, normalized size = 1.40

$$\frac{cx^5}{5e^2} + x^3 \left(\frac{b}{3e^2} - \frac{2cd}{3e^3} \right) + x \left(\frac{a}{e^2} - \frac{2bd}{e^3} + \frac{3cd^2}{e^4} \right) + \frac{x(ade^2 - bd^2e + cd^3)}{2de^4 + 2e^5x^2} + \frac{\sqrt{-\frac{d}{e^9}}(3ae^2 - 5bde + 7cd^2) \log\left(-e^4 \sqrt{-\frac{d}{e^9}} + x\right)}{4} - \frac{\sqrt{-\frac{d}{e^9}}(3ae^2 - 5bde + 7cd^2) \log\left(e^4 \sqrt{-\frac{d}{e^9}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)

[Out] $c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e**3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x**2) + \sqrt{-d/e**9}*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*\log(-e**4*\sqrt{-d/e**9} + x)/4 - \sqrt{-d/e**9}*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*\log(e**4*\sqrt{-d/e**9} + x)/4$

$$3.204 \quad \int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2 - e(3bd - ae))}{2\sqrt{d}e^{7/2}} - \frac{x(ae^2 - bde + cd^2)}{2e^3(d + ex^2)} - \frac{x(2cd - be)}{e^3} + \frac{cx^3}{3e^2}$$

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1257, 1153, 205}

$$-\frac{x(ae^2 - bde + cd^2)}{2e^3(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2 - e(3bd - ae))}{2\sqrt{d}e^{7/2}} - \frac{x(2cd - be)}{e^3} + \frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(7/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}

, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx &= -\frac{(cd^2 - bde + ae^2)x}{2e^3 (d + ex^2)} - \frac{\int \frac{-cd^2 + bde - ae^2 + 2e(cd - be)x^2 - 2ce^2x^4}{d + ex^2} dx}{2e^3} \\
 &= -\frac{(cd^2 - bde + ae^2)x}{2e^3 (d + ex^2)} - \frac{\int \left(2(2cd - be) - 2cex^2 + \frac{-5cd^2 + 3bde - ae^2}{d + ex^2} \right) dx}{2e^3} \\
 &= -\frac{(2cd - be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{(cd^2 - bde + ae^2)x}{2e^3 (d + ex^2)} - \frac{(-5cd^2 + e(3bd - ae)) \int \frac{1}{d + ex^2} dx}{2e^3} \\
 &= -\frac{(2cd - be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{(cd^2 - bde + ae^2)x}{2e^3 (d + ex^2)} + \frac{(5cd^2 - e(3bd - ae)) \tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right)}{2\sqrt{d} e^{7/2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 102, normalized size = 0.96

$$\frac{\tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right) (ae^2 - 3bde + 5cd^2)}{2\sqrt{d} e^{7/2}} - \frac{x (ae^2 - bde + cd^2)}{2e^3 (d + ex^2)} + \frac{x(be - 2cd)}{e^3} + \frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(7/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx^2 + cx^4)}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]

[Out] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2, x]

fricas [A] time = 1.03, size = 302, normalized size = 2.85

$$\frac{4cd^2x^5 - 4(5cd^2 - 3bde)x^3 - 3(5cd^3 - 3bde + ade^2 + (5cd^2 - 3bde + ae^2)x^2)\sqrt{-de}\log\left(\frac{e^{2x}\sqrt{-de} - 6(5cd^2 - 3bde + ade^2)x - 2cd^2x^3 - 2(5cd^2 - 3bde)x^2 + 3(5cd^3 - 3bde + ade^2 + (5cd^2 - 3bde + ae^2)x^2)\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right) - 3(5cd^2 - 3bde + ade^2)x}{12(d^2x^2 + d^2e^2)}\right) - 6(5cd^3 - 3bde + ade^2)x - 2cd^2x^3 - 2(5cd^2 - 3bde)x^2 + 3(5cd^3 - 3bde + ade^2 + (5cd^2 - 3bde + ae^2)x^2)\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right) - 3(5cd^2 - 3bde + ade^2)x}{6(d^2x^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)*x)/(d*e^5*x^2 + d^2*e^4)]

giac [A] time = 0.34, size = 91, normalized size = 0.86

$$\frac{(5cd^2 - 3bde + ae^2)\arctan\left(\frac{x\sqrt{d}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)} + \frac{1}{3}(cx^3e^4 - 6cdxe^3 + 3bx^4)e^{(-6)} - \frac{(cd^2x - bdx + axe^2)e^{(-3)}}{2(x^2e + d)}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-3)/(x^2*e + d)

maple [A] time = 0.01, size = 141, normalized size = 1.33

$$\frac{cx^3}{3e^2} - \frac{ax}{2(e^2x^2 + d)e} + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e} + \frac{bdx}{2(e^2x^2 + d)e^2} - \frac{3bd\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} - \frac{cd^2x}{2(e^2x^2 + d)e^3} + \frac{5cd^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^3} + \frac{bx}{e^2} - \frac{2cdx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] 1/3*c*x^3/e^2+1/e^2*b*x-2/e^3*c*d*x-1/2/e*x/(e*x^2+d)*a+1/2/e^2*x/(e*x^2+d)*d*b-1/2/e^3*x/(e*x^2+d)*c*d^2+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a-3/2/e^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*d*b+5/2/e^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c*d^2

maxima [A] time = 2.52, size = 95, normalized size = 0.90

$$-\frac{(cd^2 - bde + ae^2)x}{2(e^4x^2 + de^3)} + \frac{(5cd^2 - 3bde + ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^3} + \frac{cex^3 - 3(2cd - be)x}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2*(c*d^2 - b*d*e + a*e^2)*x/(e^4*x^2 + d*e^3) + 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) + 1/3*(c*e*x^3 - 3*(2*c*d - b*e)*x)/e^3

mupad [B] time = 0.34, size = 95, normalized size = 0.90

$$x \left(\frac{b}{e^2} - \frac{2cd}{e^3} \right) - \frac{x \left(\frac{cd^2}{2} - \frac{bde}{2} + \frac{ae^2}{2} \right)}{e^4x^2 + de^3} + \frac{cx^3}{3e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (5cd^2 - 3bde + ae^2)}{2\sqrt{d}e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)

[Out] x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 + e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(2*d^(1/2)*e^(7/2))

sympy [A] time = 0.97, size = 162, normalized size = 1.53

$$\frac{cx^3}{3e^2} + x \left(\frac{b}{e^2} - \frac{2cd}{e^3} \right) + \frac{x(-ae^2 + bde - cd^2)}{2de^3 + 2e^4x^2} - \frac{\sqrt{-\frac{1}{de^7}}(ae^2 - 3bde + 5cd^2) \log\left(-de^3\sqrt{-\frac{1}{de^7}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{de^7}}(ae^2 - 3bde + 5cd^2) \log\left(de^3\sqrt{-\frac{1}{de^7}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)

[Out] c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(2*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*log(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4

$$3.205 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3cd^2 - e(ae + bd))}{2d^{3/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]

[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)x}{2de^2(d + ex^2)} - \int \frac{\frac{cd^2 - e(bd + ae)}{e^2} - \frac{2cdx^2}{e}}{d + ex^2} dx \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \int \frac{1}{d + ex^2} dx}{2de^2} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{2de^2(d + ex^2)} - \frac{(3cd^2 - e(bd + ae)) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.06

$$\frac{x(ae^2 - bde + cd^2)}{2de^2(d + ex^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-ae^2 - bde + 3cd^2)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2, x]

[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/((2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^2, x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^2, x]

fricas [A] time = 1.08, size = 268, normalized size = 3.23

$$\left[\frac{4cd^2e^2x^3 + (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{-de} \log\left(\frac{e^2x^2 - \sqrt{-de}x - d}{e^2x^2 + d}\right) + 2(3cd^3e - bd^2e^2 + ade^3)x - 2cd^2e^2x^3 - (3cd^3 - bd^2e - ade^2 + (3cd^2e - bde^2 - ae^3)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right) + (3cd^3e - bd^2e^2 + ade^3)x}{4(d^2e^4x^2 + d^3e^3)}, \frac{2(d^2e^4x^2 + d^3e^3)}{2(d^2e^4x^2 + d^3e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]

giac [A] time = 0.41, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{2d^{\frac{3}{2}}} + \frac{(cd^2x - bdx + axe^2)e^{(-2)}}{2(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")

[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)

maple [A] time = 0.01, size = 118, normalized size = 1.42

$$\frac{ax}{2(e^2x^2 + d)d} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} - \frac{bx}{2(e^2x^2 + d)e} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e} + \frac{cdx}{2(e^2x^2 + d)e^2} - \frac{3cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^2} + \frac{cx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)

[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b-3/2/e^2*d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c

maxima [A] time = 2.34, size = 84, normalized size = 1.01

$$\frac{(cd^2 - bde + ae^2)x}{2(de^3x^2 + d^2e^2)} + \frac{cx}{e^2} - \frac{(3cd^2 - bde - ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^2 + d^2*e^2) + c*x/e^2 - \frac{1}{2}*(3*c*d^2 - b*d*e - a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d*e^2)$

mupad [B] time = 0.36, size = 77, normalized size = 0.93

$$\frac{c x}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e} x}{\sqrt{d}}\right) (-3 c d^2 + b d e + a e^2)}{2 d^{3/2} e^{5/2}} + \frac{x (c d^2 - b d e + a e^2)}{2 d (e^3 x^2 + d e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)`

[Out] $(c*x)/e^2 + (\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^{3/2}*e^{5/2}) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))$

sympy [B] time = 0.77, size = 153, normalized size = 1.84

$$\frac{c x}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2 + bde - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)`

[Out] $c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - \operatorname{sqrt}(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*\log(-d**2*e**2*\operatorname{sqrt}(-1/(d**3*e**5)) + x)/4 + \operatorname{sqrt}(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*\log(d**2*e**2*\operatorname{sqrt}(-1/(d**3*e**5)) + x)/4$

$$3.206 \quad \int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx$$

Optimal. Leaf size=89

$$-\frac{x(ae^2 - bde + cd^2)}{2d^2e(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(bd - 3ae) + cd^2)}{2d^{5/2}e^{3/2}} - \frac{a}{d^2x}$$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1259, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(bd - 3ae) + cd^2)}{2d^{5/2}e^{3/2}} - \frac{x\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)}{2(d + ex^2)} - \frac{a}{d^2x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2), x]

[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(m/2 - 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e

$\int \frac{a + bx^2 + cx^4}{x^2(d + ex^2)^2} dx$; Fr eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^2(d + ex^2)^2} dx &= \frac{\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)x}{2(d + ex^2)} - \frac{\int \frac{-2ade^2 - e(cd^2 + e(bd-ae))x^2}{x^2(d+ex^2)} dx}{2d^2e^2} \\ &= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)x}{2(d + ex^2)} + \frac{1}{2} \left(\frac{c}{e} + \frac{bd - 3ae}{d^2}\right) \int \frac{1}{d + ex^2} dx \\ &= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)x}{2(d + ex^2)} + \frac{(cd^2 + e(bd - 3ae)) \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$-\frac{x(ae^2 - bde + cd^2)}{2d^2e(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(-3ae^2 + bde + cd^2)}{2d^{5/2}e^{3/2}} - \frac{a}{d^2x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2), x]

[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2 + b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^2(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2), x]

fricas [A] time = 0.90, size = 267, normalized size = 3.00

$$\left[\frac{4ad^2e^2 + 2(cd^3e - bd^2e^2 + 3ade^3)x^2 - ((cd^2e + bde^2 - 3ae^3)x^3 + (cd^3 + bd^2e - 3ade^2)x)\sqrt{-de} \log\left(\frac{cx^2 + 2\sqrt{-de}x - d}{cx^2 + d}\right) - 2ad^2e^2 + (cd^3e - bd^2e^2 + 3ade^3)x^2 - ((cd^2e + bde^2 - 3ae^3)x^3 + (cd^3 + bd^2e - 3ade^2)x)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{4(d^3e^3x^3 + d^4e^2x)}, - \frac{2ad^2e^2 + (cd^3e - bd^2e^2 + 3ade^3)x^2 - ((cd^2e + bde^2 - 3ae^3)x^3 + (cd^3 + bd^2e - 3ade^2)x)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{2(d^3e^3x^3 + d^4e^2x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2*(2*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]

giac [A] time = 0.29, size = 83, normalized size = 0.93

$$\frac{(cd^2 + bde - 3ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{3}{2}\right)}}{2d^{\frac{5}{2}}} - \frac{(cd^2x^2 - bdx^2e + 3ax^2e^2 + 2ade)e^{(-1)}}{2(x^3e + dx)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) - 1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(-1)/((x^3*e + d*x)*d^2)

maple [A] time = 0.01, size = 121, normalized size = 1.36

$$-\frac{aex}{2(e x^2 + d) d^2} - \frac{3ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de} d^2} + \frac{bx}{2(e x^2 + d) d} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de} d} - \frac{cx}{2(e x^2 + d) e} + \frac{c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de} e} - \frac{a}{d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x)

[Out] -a/d^2/x-1/2/d^2*e*x/(e*x^2+d)*a+1/2/d*x/(e*x^2+d)*b-1/2/e*x/(e*x^2+d)*c-3/2/d^2*e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+1/2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*b+1/2/e/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c

maxima [A] time = 2.46, size = 87, normalized size = 0.98

$$-\frac{2ade + (cd^2 - bde + 3ae^2)x^2}{2(d^2e^2x^3 + d^3ex)} + \frac{(cd^2 + bde - 3ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $-1/2*(2*a*d*e + (c*d^2 - b*d*e + 3*a*e^2)*x^2)/(d^2*e^2*x^3 + d^3*e*x) + 1/2*(c*d^2 + b*d*e - 3*a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^2*e)$

mupad [B] time = 0.37, size = 81, normalized size = 0.91

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 + bde - 3ae^2)}{2d^{5/2}e^{3/2}} - \frac{\frac{a}{d} + \frac{x^2(c d^2 - b d e + 3 a e^2)}{2 d^2 e}}{e x^3 + d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)

[Out] $(\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^{5/2}*e^{3/2}) - (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)$

sympy [A] time = 1.12, size = 155, normalized size = 1.74

$$\frac{\sqrt{-\frac{1}{d^5 e^3}}(3ae^2 - bde - cd^2)\log\left(-d^3 e \sqrt{-\frac{1}{d^5 e^3}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5 e^3}}(3ae^2 - bde - cd^2)\log\left(d^3 e \sqrt{-\frac{1}{d^5 e^3}} + x\right)}{4} + \frac{-2ade + x^2(-3ae^2 + bde - cd^2)}{2d^3 ex + 2d^2 e^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)

[Out] $\sqrt{-1/(d**5*e**3)}*(3*a*e**2 - b*d*e - c*d**2)*\log(-d**3*e*\sqrt{-1/(d**5*e**3)} + x)/4 - \sqrt{-1/(d**5*e**3)}*(3*a*e**2 - b*d*e - c*d**2)*\log(d**3*e*\sqrt{-1/(d**5*e**3)} + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d**2))/(2*d**3*e*x + 2*d**2*e**2*x**3)$

$$3.207 \quad \int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2} dx$$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 - e(3bd - 5ae))}{2d^{7/2}\sqrt{e}} + \frac{x(ae^2 - bde + cd^2)}{2d^3(d + ex^2)} - \frac{bd - 2ae}{d^3x} - \frac{a}{3d^2x^3}$$

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1259, 1261, 205}

$$\frac{x(ae^2 - bde + cd^2)}{2d^3(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 - e(3bd - 5ae))}{2d^{7/2}\sqrt{e}} - \frac{bd - 2ae}{d^3x} - \frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x]

[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(7/2)*Sqrt[e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*

$(a + b*x^2 + c*x^4)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^4(d + ex^2)^2} dx &= \frac{(cd^2 - bde + ae^2)x}{2d^3(d + ex^2)} + \frac{\int \frac{2ad^2e^2 + 2de^2(bd - ae)x^2 + e^2(cd^2 - bde + ae^2)x^4}{x^4(d + ex^2)} dx}{2d^3e^2} \\ &= \frac{(cd^2 - bde + ae^2)x}{2d^3(d + ex^2)} + \frac{\int \left(\frac{2ade^2}{x^4} - \frac{2e^2(-bd + 2ae)}{x^2} + \frac{e^2(cd^2 - e(3bd - 5ae))}{d + ex^2} \right) dx}{2d^3e^2} \\ &= -\frac{a}{3d^2x^3} - \frac{bd - 2ae}{d^3x} + \frac{(cd^2 - bde + ae^2)x}{2d^3(d + ex^2)} + \frac{(cd^2 - e(3bd - 5ae)) \int \frac{1}{d + ex^2} dx}{2d^3} \\ &= -\frac{a}{3d^2x^3} - \frac{bd - 2ae}{d^3x} + \frac{(cd^2 - bde + ae^2)x}{2d^3(d + ex^2)} + \frac{(cd^2 - e(3bd - 5ae)) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 105, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5ae^2 - 3bde + cd^2)}{2d^{7/2}\sqrt{e}} + \frac{x(ae^2 - bde + cd^2)}{2d^3(d + ex^2)} + \frac{2ae - bd}{d^3x} - \frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x]

[Out] -1/3*a/(d^2*x^3) + (-b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(7/2)*Sqrt[e])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^4(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x]

fricas [A] time = 0.88, size = 316, normalized size = 2.98

$$\left[\frac{4ad^2e - 6(cd^2e - 3bd^2e + 5ade^2)x^4 + 4(3bd^2e - 5ad^2e^2)x^2 + 3((cd^2e - 3bd^2e + 5ade^2)^2 + (cd^2 - 3bd^2e + 5ade^2)x^2)\sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x + d}{x^2 + d}\right) - 2ad^2e - 3(cd^2e - 3bd^2e + 5ade^2)x^4 + 2(3bd^2e - 5ad^2e^2)x^2 - 3((cd^2e - 3bd^2e + 5ade^2)^2 + (cd^2 - 3bd^2e + 5ade^2)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{12(d^2e^2x^3 + d^2ex^2)}, \frac{2ad^2e - 3(cd^2e - 3bd^2e + 5ade^2)x^4 + 2(3bd^2e - 5ad^2e^2)x^2 - 3((cd^2e - 3bd^2e + 5ade^2)^2 + (cd^2 - 3bd^2e + 5ade^2)x^2)\sqrt{de} \arctan\left(\frac{\sqrt{de}x}{d}\right)}{6(d^2e^2x^3 + d^2ex^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)/(d^4*e^2*x^5 + d^5*e*x^3)]

giac [A] time = 0.26, size = 94, normalized size = 0.89

$$\frac{(cd^2 - 3bde + 5ae^2) \arctan\left(\frac{x\sqrt{d}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{2d^{\frac{7}{2}}} + \frac{cd^2x - bdx + axe^2}{2(x^2e + d)d^3} - \frac{3bdx^2 - 6ax^2e + ad}{3d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) - 1/3*(3*b*d*x^2 - 6*a*x^2*e + a*d)/(d^3*x^3)

maple [A] time = 0.01, size = 146, normalized size = 1.38

$$\frac{ae^2x}{2(ex^2 + d)d^3} + \frac{5ae^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^3} - \frac{bex}{2(ex^2 + d)d^2} - \frac{3be \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2} + \frac{cx}{2(ex^2 + d)d} + \frac{c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d} + \frac{2ae}{d^3x} - \frac{b}{d^2x} - \frac{a}{3d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x)

[Out] -1/3*a/d^2/x^3+2/d^3/x*a*e-1/d^2/x*b+1/2/d^3*x/(e*x^2+d)*a*e^2-1/2/d^2*x/(e*x^2+d)*e*b+1/2/d*x/(e*x^2+d)*c+5/2/d^3/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a*e^2-3/2/d^2/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*e*b+1/2/d/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*c

maxima [A] time = 2.39, size = 103, normalized size = 0.97

$$\frac{3(cd^2 - 3bde + 5ae^2)x^4 - 2ad^2 - 2(3bd^2 - 5ade)x^2}{6(d^3ex^5 + d^4x^3)} + \frac{(cd^2 - 3bde + 5ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")

[Out] 1/6*(3*(c*d^2 - 3*b*d*e + 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*d^2 - 5*a*d*e)*x^2)/(d^3*e*x^5 + d^4*x^3) + 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3)

mupad [B] time = 0.36, size = 98, normalized size = 0.92

$$\frac{\frac{x^2(5ae-3bd)}{3d^2} - \frac{a}{3d} + \frac{x^4(cd^2-3bde+5ae^2)}{2d^3}}{ex^5 + dx^3} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 - 3bde + 5ae^2)}{2d^{7/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2), x)

[Out] ((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e))/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 - 3*b*d*e))/(2*d^(7/2)*e^(1/2))

sympy [A] time = 1.53, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d'e}}(5ae^2 - 3bde + cd^2) \log\left(-d^4\sqrt{-\frac{1}{d'e}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d'e}}(5ae^2 - 3bde + cd^2) \log\left(d^4\sqrt{-\frac{1}{d'e}} + x\right)}{4} + \frac{-2ad^2 + x^4(15ae^2 - 9bde + 3cd^2) + x^2(10ade - 6bd^2)}{6d^4x^3 + 6d^3ex^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)

[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x**2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)

$$3.208 \quad \int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2} dx$$

Optimal. Leaf size=136

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (3cd^2 - e(5bd - 7ae))}{2d^{9/2}} - \frac{ex(ae^2 - bde + cd^2)}{2d^4(d+ex^2)} - \frac{cd^2 - e(2bd - 3ae)}{d^4x} - \frac{bd - 2ae}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Rubi [A] time = 0.25, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1259, 1802, 205}

$$\frac{ex(ae^2 - bde + cd^2)}{2d^4(d+ex^2)} - \frac{cd^2 - e(2bd - 3ae)}{d^4x} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (3cd^2 - e(5bd - 7ae))}{2d^{9/2}} - \frac{bd - 2ae}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x]

[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]

&& PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{x^6 (d + ex^2)^2} dx &= -\frac{e (cd^2 - bde + ae^2) x}{2d^4 (d + ex^2)} - \frac{\int \frac{-2ad^3e^2 - 2d^2e^2(bd - ae)x^2 - 2de^2(cd^2 - bde + ae^2)x^4 + e^3(cd^2 - bde + ae^2)x^6}{x^6(d + ex^2)} dx}{2d^4e^2} \\
 &= -\frac{e (cd^2 - bde + ae^2) x}{2d^4 (d + ex^2)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd - 2ae)}{x^4} + \frac{2e^2(-cd^2 + e(2bd - 3ae))}{x^2} + \frac{e^3(3cd^2 - e(5bd - 7ae))}{d + ex^2} \right) dx}{2d^4e^2} \\
 &= -\frac{a}{5d^2x^5} - \frac{bd - 2ae}{3d^3x^3} - \frac{cd^2 - e(2bd - 3ae)}{d^4x} - \frac{e (cd^2 - bde + ae^2) x}{2d^4 (d + ex^2)} - \frac{e (3cd^2 - e(5bd - 7ae))}{2d^4} \\
 &= -\frac{a}{5d^2x^5} - \frac{bd - 2ae}{3d^3x^3} - \frac{cd^2 - e(2bd - 3ae)}{d^4x} - \frac{e (cd^2 - bde + ae^2) x}{2d^4 (d + ex^2)} - \frac{\sqrt{e} (3cd^2 - e(5bd - 7ae))}{2d^{9/2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 0.99

$$-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) (7ae^2 - 5bde + 3cd^2)}{2d^{9/2}} - \frac{ex (ae^2 - bde + cd^2)}{2d^4 (d + ex^2)} + \frac{-3ae^2 + 2bde - cd^2}{d^4x} + \frac{2ae - bd}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x]

[Out] -1/5*a/(d^2*x^5) + (-b*d) + 2*a*e)/(3*d^3*x^3) + (-c*d^2) + 2*b*d*e - 3*a*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e] * (3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^6 (d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x]

fricas [A] time = 0.86, size = 360, normalized size = 2.65

$$\frac{30(3cd^2e - 5bde^2 + 7ae^3) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 20(3cd^3 - 5bde^2 + 7ae^3)x^4 + 12ad^4(5bd^2 - 7ade^2)^2 - 15((3cd^2e - 5bde^2 + 7ae^3)x^2 + (3cd^3 - 5bde^2 + 7ae^3)x^2)\sqrt{\frac{e^2 - 2de + d^2}{d^2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 15(3cd^2e - 5bde^2 + 7ae^3)x^6 + 10(3cd^3 - 5bde^2 + 7ae^3)x^4 + 6ad^5 + 2(5bd^2 - 7ade^2)^2 + 15((3cd^2e - 5bde^2 + 7ae^3)x^2 + (3cd^3 - 5bde^2 + 7ae^3)x^2)\sqrt{\frac{e^2 - 2de + d^2}{d^2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{60(d^2ex^2 + d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5), -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]

giac [A] time = 0.33, size = 131, normalized size = 0.96

$$\frac{(3cd^2e - 5bde^2 + 7ae^3) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{-\frac{1}{2}}}{2d^{\frac{9}{2}}} - \frac{cd^2xe - bdx^2e + axe^3}{2(x^2e + d)d^4} - \frac{15cd^2x^4 - 30bdx^4e + 45ax^4e^2 + 5bd^2x^2 - 10adx^2e + 3ad^2}{15d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")

[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e + d)*d^4) - 1/15*(15*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*b*d^2*x^2 - 10*a*d*x^2*e + 3*a*d^2)/(d^4*x^5)

maple [A] time = 0.02, size = 183, normalized size = 1.35

$$\frac{ae^3x}{2(e^2x + d)d^4} - \frac{7ae^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^4} + \frac{be^2x}{2(e^2x + d)d^3} + \frac{5be^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^3} - \frac{cex}{2(e^2x + d)d^2} - \frac{3ce \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2} - \frac{3ae^2}{d^4x} + \frac{2be}{d^3x} - \frac{c}{d^2x} + \frac{2ae}{3d^2x^3} - \frac{b}{3d^2x^3} - \frac{a}{5d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x)

[Out] -1/5*a/d^2/x^5+2/3/d^3/x^3*a*e-1/3/d^2/x^3*b-3/d^4/x*a*e^2+2/d^3/x*e*b-1/d^2/x*c-1/2*e^3/d^4*x/(e*x^2+d)*a+1/2*e^2/d^3*x/(e*x^2+d)*b-1/2*e/d^2*x/(e*x^2+d)*c-7/2*e^3/d^4/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)*a+5/2*e^2/d^3/(d*e

$\sqrt{e} \arctan\left(\frac{1}{\sqrt{d} \sqrt{e}}\right) e^{3/2} x^2 + \frac{1}{\sqrt{d} \sqrt{e}} \arctan\left(\frac{1}{\sqrt{d} \sqrt{e}}\right) e^{1/2} x^2 + c$

maxima [A] time = 2.46, size = 139, normalized size = 1.02

$$\frac{15(3cd^2e - 5bde^2 + 7ae^3)x^6 + 10(3cd^3 - 5bd^2e + 7ade^2)x^4 + 6ad^3 + 2(5bd^3 - 7ad^2e)x^2}{30(d^4ex^7 + d^5x^5)} - \frac{(3cd^2e - 5bde^2 + 7ae^3) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $-\frac{1}{30} \cdot \frac{15(3cd^2e - 5b*d*d^2e + 7*a*e^3)x^6 + 10(3cd^3 - 5b*d^2e + 7*a*d^2e^2)x^4 + 6*a*d^3 + 2(5b*d^3 - 7*a*d^2e)x^2}{(d^4e*x^7 + d^5*x^5)} - \frac{1}{2} \cdot \frac{(3cd^2e - 5b*d*d^2e + 7*a*e^3) \arctan(e*x/\sqrt{d*e})}{(\sqrt{d*e}) \cdot d^4}$

mupad [B] time = 0.38, size = 128, normalized size = 0.94

$$\frac{\frac{a}{5d} - \frac{x^2(7ae-5bd)}{15d^2} + \frac{x^4(3cd^2-5bde+7ae^2)}{3d^3} + \frac{ex^6(3cd^2-5bde+7ae^2)}{2d^4}}{ex^7 + dx^5} - \frac{\sqrt{e} \operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (3cd^2 - 5bde + 7ae^2)}{2d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2), x)

[Out] $-\frac{a}{5d} - \frac{x^2(7ae - 5bd)}{15d^2} + \frac{x^4(7ae^2 + 3cd^2 - 5bd^2e)}{3d^3} + \frac{e x^6(7ae^2 + 3cd^2 - 5bd^2e)}{2d^4} - \frac{e^{1/2} \operatorname{atan}\left(\frac{e^{1/2}x}{d^{1/2}}\right) (7ae^2 + 3cd^2 - 5bd^2e)}{2d^{9/2}}$

sympy [B] time = 2.13, size = 284, normalized size = 2.09

$$\frac{\sqrt{\frac{e}{d}} (7ae^2 - 5bde + 3cd^2) \log\left(\frac{d^2 \sqrt{\frac{e}{d}} (7ae^2 - 5bde + 3cd^2)}{7ae^3 - 5bd^2e + 3cd^2e} + x\right)}{4} - \frac{\sqrt{\frac{e}{d}} (7ae^2 - 5bde + 3cd^2) \log\left(\frac{d^2 \sqrt{\frac{e}{d}} (7ae^2 - 5bde + 3cd^2)}{7ae^3 - 5bd^2e + 3cd^2e} + x\right)}{4} + \frac{-6ad^3 + x^6(-105ae^3 + 75bd^2e - 45cd^2e) + x^4(-70ade^2 + 50bd^2e - 30cd^3) + x^2(14ad^2e - 10bd^3)}{30d^5x^5 + 30d^4ex^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)

[Out] $\sqrt{-e/d} (7ae^2 - 5bd^2e + 3cd^2) \log(-d \sqrt{-e/d} (7ae^2 - 5bd^2e + 3cd^2) / (7ae^3 - 5bd^2e + 3cd^2e) + x) / 4 - \sqrt{-e/d} (7ae^2 - 5bd^2e + 3cd^2) \log(d \sqrt{-e/d} (7ae^2 - 5bd^2e + 3cd^2) / (7ae^3 - 5bd^2e + 3cd^2e) + x) / 4 + (-6ad^3 + x^6(-105ae^3 + 75bd^2e - 45cd^2e) + x^4(-70ade^2 + 50bd^2e - 30cd^3) + x^2(14ad^2e - 10bd^3)) / (30d^5x^5 + 30d^4ex^7)$

$$3.209 \quad \int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2} dx$$

Optimal. Leaf size=167

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(5cd^2 - e(7bd - 9ae))}{2d^{11/2}} + \frac{e^2x(ae^2 - bde + cd^2)}{2d^5(d + ex^2)} + \frac{e(2cd^2 - e(3bd - 4ae))}{d^5x} - \frac{cd^2 - e(2bd - 3ae)}{3d^4x^3} - \frac{bd - ae}{5d^3}$$

Rubi [A] time = 0.33, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1259, 1802, 205}

$$\frac{e^2x(ae^2 - bde + cd^2)}{2d^5(d + ex^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(5cd^2 - e(7bd - 9ae))}{2d^{11/2}} - \frac{cd^2 - e(2bd - 3ae)}{3d^4x^3} + \frac{e(2cd^2 - e(3bd - 4ae))}{d^5x} - \frac{bd - 2ae}{5d^3x^5} - \frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x]

[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/(3*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]

&& PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{x^8(d + ex^2)^2} dx &= \frac{e^2(cd^2 - bde + ae^2)x}{2d^5(d + ex^2)} + \frac{\int \frac{2ad^4e^2 + 2d^3e^2(bd - ae)x^2 + 2d^2e^2(cd^2 - bde + ae^2)x^4 - 2de^3(cd^2 - bde + ae^2)x^6 + e^4(cd^2 - bde + ae^2)x^8}{x^8(d + ex^2)} dx}{2d^5e^2} \\
 &= \frac{e^2(cd^2 - bde + ae^2)x}{2d^5(d + ex^2)} + \frac{\int \left(\frac{2ad^3e^2}{x^8} + \frac{2d^2e^2(bd - 2ae)}{x^6} + \frac{2de^2(cd^2 - e(2bd - 3ae))}{x^4} + \frac{2e^3(-2cd^2 + e(3bd - 4ae))}{x^2} \right) dx}{2d^5e^2} \\
 &= -\frac{a}{7d^2x^7} - \frac{bd - 2ae}{5d^3x^5} - \frac{cd^2 - e(2bd - 3ae)}{3d^4x^3} + \frac{e(2cd^2 - e(3bd - 4ae))}{d^5x} + \frac{e^2(cd^2 - bde + ae^2)}{2d^5(d + ex^2)} \\
 &= -\frac{a}{7d^2x^7} - \frac{bd - 2ae}{5d^3x^5} - \frac{cd^2 - e(2bd - 3ae)}{3d^4x^3} + \frac{e(2cd^2 - e(3bd - 4ae))}{d^5x} + \frac{e^2(cd^2 - bde + ae^2)}{2d^5(d + ex^2)}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 166, normalized size = 0.99

$$\frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(9ae^2 - 7bde + 5cd^2)}{2d^{11/2}} + \frac{e^2x(ae^2 - bde + cd^2)}{2d^5(d + ex^2)} + \frac{e(4ae^2 - 3bde + 2cd^2)}{d^5x} + \frac{-3ae^2 + 2bde - cd^2}{3d^4x^3} + \frac{2ae - bd}{5d^3x^5} - \frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x]

[Out] -1/7*a/(d^2*x^7) + (-b*d) + 2*a*e)/(5*d^3*x^5) + (-c*d^2) + 2*b*d*e - 3*a*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*a*e^2)*ArcTan[Sqrt[e]*x/Sqrt[d]])/(2*d^(11/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^8(d + ex^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x]

fricas [A] time = 0.92, size = 436, normalized size = 2.61

$$\frac{140(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + \frac{140(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 140(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 140(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{420(d^2 + e^2)} + \frac{105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{210(d^2 + e^2)} + \frac{105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{210(d^2 + e^2)} + \frac{105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) + 105(5cd^2 - 7bd^2 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{210(d^2 + e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d))/(d^5*e*x^9 + d^6*x^7)]

giac [A] time = 0.42, size = 164, normalized size = 0.98

$$\frac{(5cd^2e^2 - 7bde^3 + 9ae^4) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{2d^{\frac{11}{2}}} + \frac{cd^2xe^2 - bdx^3e + axe^4}{2(x^2e + d)d^5} + \frac{210cd^2x^6e - 315bdx^6e^2 - 35cd^3x^4 + 420ax^6e^3 + 70bd^2x^4e - 105adx^4e^2 - 21bd^3x^2 + 42ad^2x^2e - 15ad^3}{105d^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")

[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^2*e + d)*d^5) + 1/105*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d^3*x^4 + 420*a*x^6*e^3 + 70*b*d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^5*x^7)

maple [A] time = 0.02, size = 221, normalized size = 1.32

$$\frac{ae^4x}{2(e^2x^2 + d)d^5} + \frac{9ae^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^5} - \frac{be^3x}{2(e^2x^2 + d)d^4} - \frac{7be^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^4} + \frac{ce^2x}{2(e^2x^2 + d)d^3} + \frac{5ce^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^3} + \frac{4ae^3}{d^5x} - \frac{3be^2}{d^4x} + \frac{2ce}{d^3x} - \frac{ae^2}{d^4x^3} + \frac{2be}{3d^3x^3} - \frac{c}{3d^2x^3} + \frac{2ae}{5d^3x^5} - \frac{b}{5d^2x^5} - \frac{a}{7d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x)

[Out] -1/7*a/d^2/x^7+2/5/d^3/x^5*a*e-1/5/d^2/x^5*b-1/d^4/x^3*a*e^2+2/3/d^3/x^3*e*b-1/3/d^2/x^3*c+4*e^3/d^5/x*a-3*e^2/d^4/x*b+2*e/d^3/x*c+1/2*e^4/d^5*x/(e*x^

$2+d)*a-1/2*e^3/d^4*x/(e*x^2+d)*b+1/2*e^2/d^3*x/(e*x^2+d)*c+9/2*e^4/d^5/(d*e)^{(1/2)*\arctan(1/(d*e)^{(1/2)*e*x)*a-7/2*e^3/d^4/(d*e)^{(1/2)*\arctan(1/(d*e)^{(1/2)*e*x)*b+5/2*e^2/d^3/(d*e)^{(1/2)*\arctan(1/(d*e)^{(1/2)*e*x)*c}$

maxima [A] time = 2.57, size = 174, normalized size = 1.04

$$\frac{105(5cd^2e^2 - 7bde^3 + 9ae^4)x^8 + 70(5cd^3e - 7bd^2e^2 + 9ade^3)x^6 - 30ad^4 - 14(5cd^4 - 7bd^3e + 9ad^2e^2)x^4 - 6(7bd^4 - 9ad^3e)x^2}{210(d^5ex^9 + d^6x^7)} + \frac{(5cd^2e^2 - 7bde^3 + 9ae^4)\arctan\left(\frac{ex}{\sqrt{d}}\right)}{2\sqrt{d}e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")

[Out] $1/210*(105*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e*x^9 + d^6*x^7) + 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^5)$

mupad [B] time = 0.40, size = 156, normalized size = 0.93

$$\frac{x^2(9ae-7bd)}{35d^2} - \frac{a}{7d} - \frac{x^4(5cd^2-7bde+9ae^2)}{15d^3} + \frac{ex^6(5cd^2-7bde+9ae^2)}{3d^4} + \frac{e^2x^8(5cd^2-7bde+9ae^2)}{2d^5} + \frac{e^{3/2}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-7bde+9ae^2)}{2d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2), x)

[Out] $((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^{(3/2)*\operatorname{atan}((e^{(1/2)*x}/d^{(1/2)})*(9*a*e^2 + 5*c*d^2 - 7*b*d*e)))/(2*d^{(11/2)})}$

sympy [B] time = 2.68, size = 328, normalized size = 1.96

$$-\frac{\sqrt{\frac{d}{d^2}}(9ae^2-7bde+5cd^2)\log\left(\frac{d^2\sqrt{\frac{d}{d^2}}(9ae^2-7bde+5cd^2)}{9ad^4-7bd^3+5cd^2e}+x\right)}{4} + \frac{\sqrt{\frac{d^3}{d^2}}(9ae^2-7bde+5cd^2)\log\left(\frac{d^2\sqrt{\frac{d}{d^2}}(9ae^2-7bde+5cd^2)}{9ad^4-7bd^3+5cd^2e}+x\right)}{4} + \frac{-30ad^4+x^8(945ae^4-735bd^3+525cd^2e^2)+x^6(630ad^3-490bd^2e^2+350cd^3e)+x^4(-126ad^2e^2+98bd^3e-70cd^4)+x^2(54ad^3e-42bd^4)}{210d^6x^7+210d^5e*x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)

[Out] $-\sqrt{-e**3/d**11}*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*\log(-d**6*\sqrt{-e**3/d**11}*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2) + x)/4 + \sqrt{-e**3/d**11}*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*\log(d**6*\sqrt{-e**3/d**11}*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**2*e**2) + x**6*(630*a*d**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(210*d**6*x**7 + 210*d**5*e*x**9)$

$$3.210 \quad \int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3} dx$$

Optimal. Leaf size=173

$$\frac{dx(17cd^2 - e(13bd - 9ae))}{8e^5(d+ex^2)} + \frac{x(6cd^2 - e(3bd - ae))}{e^5} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(15ae^2 - 35bde + 63cd^2)}{8e^{11/2}} - \frac{d^2x(ae^2 - bde + cd^2)}{4e^5(d+ex^2)^2}$$

Rubi [A] time = 0.32, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1257, 1814, 1810, 205}

$$\frac{dx(17cd^2 - e(13bd - 9ae))}{8e^5(d+ex^2)} - \frac{d^2x(ae^2 - bde + cd^2)}{4e^5(d+ex^2)^2} + \frac{x(6cd^2 - e(3bd - ae))}{e^5} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(15ae^2 - 35bde + 63cd^2)}{8e^{11/2}} - \frac{x^3(3cd - be)}{3e^4} + \frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^6 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx &= -\frac{d^2 (cd^2 - bde + ae^2) x}{4e^5 (d + ex^2)^2} - \frac{\int \frac{-d^2(cd^2 - bde + ae^2) + 4de(cd^2 - bde + ae^2)x^2 - 4e^2(cd^2 - bde + ae^2)x^4 + 4e^3(cd - bde + ae^2)x^6}{(d + ex^2)^2} dx}{4e^5} \\ &= -\frac{d^2 (cd^2 - bde + ae^2) x}{4e^5 (d + ex^2)^2} + \frac{d (17cd^2 - e(13bd - 9ae)) x}{8e^5 (d + ex^2)} + \frac{\int \frac{-d^2(15cd^2 - e(11bd - 7ae)) + 8de(3cd - bde + ae^2)x^2 - 8e^2(3cd - bde + ae^2)x^4 + 8e^3(3cd - bde + ae^2)x^6}{(d + ex^2)^2} dx}{8e^5} \\ &= -\frac{d^2 (cd^2 - bde + ae^2) x}{4e^5 (d + ex^2)^2} + \frac{d (17cd^2 - e(13bd - 9ae)) x}{8e^5 (d + ex^2)} + \frac{\int (8d (6cd^2 - e(3bd - ae))}{8e^5} \\ &= \frac{(6cd^2 - e(3bd - ae)) x}{e^5} - \frac{(3cd - be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2 (cd^2 - bde + ae^2) x}{4e^5 (d + ex^2)^2} + \frac{d (17cd^2 - e(13bd - 9ae)) x}{8e^5} \\ &= \frac{(6cd^2 - e(3bd - ae)) x}{e^5} - \frac{(3cd - be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2 (cd^2 - bde + ae^2) x}{4e^5 (d + ex^2)^2} + \frac{d (17cd^2 - e(13bd - 9ae)) x}{8e^5} \end{aligned}$$

Mathematica [A] time = 0.11, size = 170, normalized size = 0.98

$$\frac{x (de(9ae - 13bd) + 17cd^3)}{8e^5 (d + ex^2)} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) (5e(3ae - 7bd) + 63cd^2)}{8e^{11/2}} + \frac{x (e(ae - 3bd) + 6cd^2)}{e^5} - \frac{x (d^2e(ae - bd) + cd^4)}{4e^5 (d + ex^2)^2} + \frac{x^3 (be - 3cd)}{3e^4} + \frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((17*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] IntegrateAlgebraic[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]

fricas [A] time = 0.73, size = 504, normalized size = 2.91

$$\frac{(e^6 x^9 - 14(e^4 a^2 - 5a^2 b^2) e^3 + 14(3a^2 c^2 - 20a^2 b^2 + 15a^2 d^2) e^2 + 20a^2 b^2 c - 20a^2 b^2 d + 15a^2 d^2) \sqrt{\frac{d}{e}} \arctan\left(\frac{x \sqrt{\frac{d}{e}}}{d}\right) + 30(63c^2 d^4 - 35b^2 d^3 e + 15a^2 d^2 e^2) x / (e^7 x^4 + 2d^2 e^6 x^2 + d^2 e^5) + 1/120(24c^2 e^4 x^9 - 8(9c^2 d e^3 - 5b^2 e^4) x^7 + 8(63c^2 d^2 e^2 - 35b^2 d e^3 + 15a^2 e^4) x^5 + 25(63c^2 d^3 e - 35b^2 d^2 e^2 + 15a^2 d e^3) x^3 - 15(63c^2 d^4 - 35b^2 d^3 e + 15a^2 d^2 e^2 + (63c^2 d^2 e^2 - 35b^2 d e^3 + 15a^2 e^4) x^4 + 2(63c^2 d^3 e - 35b^2 d^2 e^2 + 15a^2 d e^3) x^2) \sqrt{d/e} \arctan(e x \sqrt{d/e} / d) + 15(63c^2 d^4 - 35b^2 d^3 e + 15a^2 d^2 e^2) x / (e^7 x^4 + 2d^2 e^6 x^2 + d^2 e^5)]}{8 \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 - 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5)]

giac [A] time = 0.36, size = 160, normalized size = 0.92

$$\frac{(63cd^3 - 35bd^2e + 15ade^2) \arctan\left(\frac{x\sqrt{\frac{d}{e}}}{\sqrt{d}}\right) e^{\frac{11}{2}}}{8\sqrt{d}} + \frac{1}{15} (3cx^5e^{12} - 15cdx^3e^{11} + 5bx^3e^{12} + 90cd^2xe^{10} - 45bdxe^{11} + 15axe^{12}) e^{(-15)} + \frac{(17cd^3x^3e - 13bd^2x^3e^2 + 15cd^4x + 9adx^3e^3 - 11bd^3xe + 7ad^2xe^2) e^{(-5)}}{8(x^2e + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b*x^3*e^12 + 90*c*d^2*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15) + 1/8*(17*c*d^3*x^3*e - 13*b*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e^(-5)/(x^2*e + d)^2

maple [A] time = 0.02, size = 239, normalized size = 1.38

$$\frac{9ad^3x^3}{8(e^2x^2+d)^2e^2} - \frac{13bd^2x^3}{8(e^2x^2+d)^2e^3} + \frac{17cd^3x^3}{8(e^2x^2+d)^2e^4} + \frac{cx^5}{5e^5} + \frac{7ad^2x}{8(e^2x^2+d)^2e^3} - \frac{11bd^3x}{8(e^2x^2+d)^2e^4} + \frac{bx^3}{3e^3} + \frac{15cd^4x}{8(e^2x^2+d)^2e^5} - \frac{cdx^3}{e^4} - \frac{15ad \arctan\left(\frac{cx}{\sqrt{de}}\right)}{8\sqrt{de}e^3} + \frac{35bd^2 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{8\sqrt{de}e^4} - \frac{63cd^3 \arctan\left(\frac{cx}{\sqrt{de}}\right)}{8\sqrt{de}e^5} + \frac{ax}{e^3} - \frac{3bdx}{e^4} + \frac{6cd^2x}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)

[Out] $\frac{1}{5}c^2x^5/e^3 + \frac{1}{3}e^3x^3b - \frac{1}{e^4}x^3cd + \frac{1}{e^3}ax - \frac{3}{e^4}dbx + \frac{6}{e^5}cd^2x^2 + \frac{9}{8}d/e^2/(e^2x^2+d)^2x^3a - \frac{13}{8}d^2/e^3/(e^2x^2+d)^2x^3b + \frac{17}{8}d^3/e^4/(e^2x^2+d)^2x^3c + \frac{7}{8}d^2/e^3/(e^2x^2+d)^2ax - \frac{11}{8}d^3/e^4/(e^2x^2+d)^2bx + \frac{15}{8}d^4/e^5/(e^2x^2+d)^2cx - \frac{15}{8}d/e^3/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) * a + \frac{35}{8}d^2/e^4/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) * b - \frac{63}{8}d^3/e^5/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) * c$

maxima [A] time = 2.47, size = 175, normalized size = 1.01

$$\frac{(17cd^3e - 13bd^2e^2 + 9ade^3)x^3 + (15cd^4 - 11bd^3e + 7ad^2e^2)x}{8(e^7x^4 + 2de^6x^2 + d^2e^5)} - \frac{(63cd^3 - 35bd^2e + 15ade^2) \arctan\left(\frac{cx}{\sqrt{de}}\right)}{8\sqrt{de}e^5} + \frac{3ce^2x^5 - 5(3cde - be^2)x^3 + 15(6cd^2 - 3bde + ae^2)x}{15e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} * ((17*c*d^3*e - 13*b*d^2*e^2 + 9*a*d*e^3)*x^3 + (15*c*d^4 - 11*b*d^3*e + 7*a*d^2*e^2)*x) / (e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5) - \frac{1}{8} * (63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2) * \arctan(e*x/\sqrt{d*e}) / (\sqrt{d*e}*e^5) + \frac{1}{15} * (3*c*e^2*x^5 - 5*(3*c*d*e - b*e^2)*x^3 + 15*(6*c*d^2 - 3*b*d*e + a*e^2)*x) / e^5$

mupad [B] time = 0.35, size = 223, normalized size = 1.29

$$x^3 \left(\frac{b}{3e^3} - \frac{cd}{e^4} \right) - x \left(\frac{3cd^2}{e^5} - \frac{a}{e^3} + \frac{3d \left(\frac{b}{e^3} - \frac{3cd}{e^4} \right)}{e} \right) + \frac{\left(\frac{17cd^3e}{8} - \frac{13bd^2e^2}{8} + \frac{9ad^2e^3}{8} \right) x^3 + \left(\frac{15cd^4}{8} - \frac{11bd^3e}{8} + \frac{7ad^2e^2}{8} \right) x}{d^2e^5 + 2de^6x^2 + e^7x^4} + \frac{cx^5}{5e^3} - \frac{\sqrt{d} \operatorname{atan}\left(\frac{\sqrt{d} \sqrt{e} x (63cd^3 - 35bd^2e + 15ade^2)}{63cd^3 - 35bd^2e + 15ade^2}\right) (63cd^3 - 35bd^2e + 15ade^2)}{8e^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)

[Out] $x^3 * (b/(3e^3) - (c*d)/e^4) - x * ((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*c*d)/e^4))/e) + (x^3 * ((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) + x * ((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8)) / (d^2*e^5 + e^7*x^4 + 2*d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^{1/2} * \operatorname{atan}((d^{1/2} * e^{1/2} * x * (15*a*e^2 + 63*c*d^2 - 35*b*d*e)) / (63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))) * (15*a*e^2 + 63*c*d^2 - 35*b*d*e)) / (8*e^{11/2})$

sympy [A] time = 3.58, size = 235, normalized size = 1.36

$$\frac{cx^5}{5e^3} + x^3 \left(\frac{b}{3e^3} - \frac{cd}{e^4} \right) + x \left(\frac{a}{e^3} - \frac{3bd}{e^4} + \frac{6cd^2}{e^5} \right) + \frac{\sqrt{-\frac{d}{e}} (15ae^2 - 35bde + 63cd^2) \log\left(-e^2 \sqrt{-\frac{d}{e}} + x\right)}{16} - \frac{\sqrt{-\frac{d}{e}} (15ae^2 - 35bde + 63cd^2) \log\left(e^2 \sqrt{-\frac{d}{e}} + x\right)}{16} + \frac{x^3 (9ade^3 - 13bd^2e^2 + 17cd^3e) + x (7ad^2e^2 - 11bd^3e + 15cd^4)}{8d^2e^5 + 16de^6x^2 + 8e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)`

[Out] $c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 + 6*c*d**2/e**5) + \sqrt{-d/e**11}*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*\log(-e**5*\sqrt{-d/e**11} + x)/16 - \sqrt{-d/e**11}*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*\log(e**5*\sqrt{-d/e**11} + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 17*c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*d*e**6*x**2 + 8*e**7*x**4)$

$$3.211 \quad \int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx$$

Optimal. Leaf size=143

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2 - 3e(5bd - ae))}{8\sqrt{d}e^{9/2}} - \frac{x(13cd^2 - e(9bd - 5ae))}{8e^4(d+ex^2)} + \frac{dx(ae^2 - bde + cd^2)}{4e^4(d+ex^2)^2} - \frac{x(3cd - be)}{e^4} + \frac{cx^3}{3e^3}$$

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1257, 1814, 1153, 205}

$$-\frac{x(13cd^2 - e(9bd - 5ae))}{8e^4(d+ex^2)} + \frac{dx(ae^2 - bde + cd^2)}{4e^4(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2 - 3e(5bd - ae))}{8\sqrt{d}e^{9/2}} - \frac{x(3cd - be)}{e^4} + \frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(9/2))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1257

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p

$p*(d + e*(2*q + 3)*x^2))/(d + e*x^2)], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{ILtQ}[q, -1] \ \&\& \ \text{IGtQ}[m/2, 0]$

Rule 1814

$\text{Int}[(\text{Pq}_-)*((a_-) + (b_-)*(x_-)^2)^(p_-), x_Symbol] \text{ :> With}[\{Q = \text{PolynomialQuotient}[\text{Pq}, a + b*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2, x], x, 1]\}, \text{Simp}[(a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + \text{Dist}[1/(2*a*(p + 1)), \text{Int}[(a + b*x^2)^(p + 1)*\text{ExpandToSum}[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{LtQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{x^4 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx &= \frac{d (cd^2 - bde + ae^2) x}{4e^4 (d + ex^2)^2} - \frac{\int \frac{d(cd^2 - bde + ae^2) - 4e(cd^2 - bde + ae^2)x^2 + 4e^2(cd - be)x^4 - 4ce^3x^6}{(d + ex^2)^2} dx}{4e^4} \\ &= \frac{d (cd^2 - bde + ae^2) x}{4e^4 (d + ex^2)^2} - \frac{(13cd^2 - e(9bd - 5ae)) x}{8e^4 (d + ex^2)} + \frac{\int \frac{d(11cd^2 - e(7bd - 3ae)) - 8de(2cd - be)x^2 + 8cd^3}{d + ex^2} dx}{8de^4} \\ &= \frac{d (cd^2 - bde + ae^2) x}{4e^4 (d + ex^2)^2} - \frac{(13cd^2 - e(9bd - 5ae)) x}{8e^4 (d + ex^2)} + \frac{\int (-8d(3cd - be) + 8cdex^2 + \frac{35cd^3}{d + ex^2}) dx}{8de^4} \\ &= -\frac{(3cd - be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d (cd^2 - bde + ae^2) x}{4e^4 (d + ex^2)^2} - \frac{(13cd^2 - e(9bd - 5ae)) x}{8e^4 (d + ex^2)} + \frac{(35cd^2 - 3e^2d^2)}{8de^4} \\ &= -\frac{(3cd - be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d (cd^2 - bde + ae^2) x}{4e^4 (d + ex^2)^2} - \frac{(13cd^2 - e(9bd - 5ae)) x}{8e^4 (d + ex^2)} + \frac{(35cd^2 - 3e^2d^2)}{8de^4} \end{aligned}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3ae^2 - 15bde + 35cd^2)}{8\sqrt{d}e^{9/2}} - \frac{x(5ae^2 - 9bde + 13cd^2)}{8e^4(d + ex^2)} + \frac{x(ade^2 - bd^2e + cd^3)}{4e^4(d + ex^2)^2} + \frac{x(be - 3cd)}{e^4} + \frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] $((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*\text{ArcTan}[\text{Sqrt}[e]*x/\text{Sqrt}[d]])/(8*\text{Sqrt}[d]*e^{(9/2)})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] IntegrateAlgebraic[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]

fricas [A] time = 0.61, size = 462, normalized size = 3.23

$$\frac{(64a^2d^2 - 16(7a^2d^2 - 3ab^2d^2 - 15a^2d^2 + 3ab^2d^2 - 3(5a^2d^2 - 15a^2d^2 + 3ab^2d^2) \sqrt{-d}) \log\left(\frac{d^2 + 2d^2x^2}{\sqrt{d}}\right) - 8(5a^2d^2 - 15a^2d^2 + 3ab^2d^2) - 8(7a^2d^2 - 3ab^2d^2 - 5(5a^2d^2 - 15a^2d^2 + 3ab^2d^2) + 3(5a^2d^2 - 15a^2d^2 + 3ab^2d^2) \sqrt{-d}) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) - 3(5a^2d^2 - 15a^2d^2 + 3ab^2d^2)}{24(a^2d^2 + 2d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] $[1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e^3)*x^2)*\text{sqrt}(-d*e)*\log((e*x^2 - 2*\text{sqrt}(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5*(35*c*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e^3)*x^2)*\text{sqrt}(d*e)*\text{arctan}(\text{sqrt}(d*e)*x/d) - 3*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5)]$

giac [A] time = 0.47, size = 125, normalized size = 0.87

$$\frac{(35cd^2 - 15bde + 3ae^2) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{\left(\frac{9}{2}\right)}}{8\sqrt{d}} + \frac{1}{3}(cx^3e^6 - 9cdxe^5 + 3bx^2e^6)e^{(-9)} - \frac{(13cd^2x^3e - 9bdx^3e^2 + 11cd^3x + 5ax^3e^3 - 7bd^2xe + 3adx^2e^2)e^{(-4)}}{8(x^2e + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] $1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*\text{arctan}(x*e^{(1/2)}/\text{sqrt}(d))*e^{(-9/2)}/\text{sqrt}(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^{(-9)} - 1/8*(13*c*d^2*x^3*$

$$e^{-4} \frac{-9bd^2x^3 + 11cd^3x + 5a^2x^3 - 7bd^2xe + 3ad^2xe^2}{(x^2e + d)^2}$$

maple [A] time = 0.01, size = 202, normalized size = 1.41

$$-\frac{5ax^3}{8(e^2x^2+d)^2} + \frac{9bdx^3}{8(e^2x^2+d)^2e^2} - \frac{13cd^2x^3}{8(e^2x^2+d)^2e^3} - \frac{3adx}{8(e^2x^2+d)^2e^2} + \frac{7bd^2x}{8(e^2x^2+d)^2e^3} - \frac{11cd^3x}{8(e^2x^2+d)^2e^4} + \frac{cx^3}{3e^3} + \frac{3a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^2} - \frac{15bd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^3} + \frac{35cd^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^4} + \frac{bx}{e^3} - \frac{3cdx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)

[Out] $\frac{1}{3}cx^3/e^3 + 1/e^3bx - 3/e^4cdx - 5/8e/(e^2x^2+d)^2x^3a + 9/8e^2/(e^2x^2+d)^2x^3bd - 13/8e^3/(e^2x^2+d)^2x^3cd^2 - 3/8e^2/(e^2x^2+d)^2ad^2x + 7/8e^3/(e^2x^2+d)^2d^2bx - 11/8e^4/(e^2x^2+d)^2cd^3x + 3/8e^2/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) - 15/8e^3/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) + 35/8e^4/(d^2e)^{1/2} \arctan(1/(d^2e)^{1/2}ex) + cd^2$

maxima [A] time = 2.51, size = 139, normalized size = 0.97

$$-\frac{(13cd^2e - 9bde^2 + 5ae^3)x^3 + (11cd^3 - 7bd^2e + 3ade^2)x}{8(e^6x^4 + 2de^5x^2 + d^2e^4)} + \frac{(35cd^2 - 15bde + 3ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^4} + \frac{cex^3 - 3(3cd - be)x}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] $-1/8*((13cd^2e - 9bde^2 + 5ae^3)x^3 + (11cd^3 - 7bd^2e + 3ade^2)x)/(e^6x^4 + 2d^2e^5x^2 + d^2e^4) + 1/8*(35cd^2 - 15bde + 3ae^2) \arctan(ex/\sqrt{de})/(\sqrt{de}e^4) + 1/3*(cex^3 - 3(3cd - be)x)/e^4$

mupad [B] time = 0.34, size = 137, normalized size = 0.96

$$x \left(\frac{b}{e^3} - \frac{3cd}{e^4} \right) - \frac{\left(\frac{13cd^2e}{8} - \frac{9bde^2}{8} + \frac{5ae^3}{8} \right) x^3 + \left(\frac{11cd^3}{8} - \frac{7bd^2e}{8} + \frac{3ade^2}{8} \right) x}{d^2e^4 + 2de^5x^2 + e^6x^4} + \frac{cx^3}{3e^3} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (35cd^2 - 15bde + 3ae^2)}{8\sqrt{d}e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)

[Out] $x \left(\frac{b}{e^3} - \frac{3cd}{e^4} \right) - \frac{x \left(\frac{11cd^3}{8} + \frac{3ad^2e^2}{8} - \frac{7bd^2e}{8} \right) + x^3 \left(\frac{5ae^3}{8} - \frac{9bde^2}{8} + \frac{13cd^2e}{8} \right)}{(d^2e^4 + e^6x^4 + 2d^2e^5x^2) + (cx^3)/(3e^3) + \operatorname{atan}\left(\frac{e^{1/2}x}{d^{1/2}}\right) (3ae^2 + 35cd^2 - 15bde)} / (8d^{1/2}e^{9/2})$

sympy [A] time = 3.37, size = 212, normalized size = 1.48

$$\frac{cx^3}{3e^3} + x\left(\frac{b}{e^3} - \frac{3cd}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}(3ae^2 - 15bde + 35cd^2)\log\left(-de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{de^9}}(3ae^2 - 15bde + 35cd^2)\log\left(de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} + \frac{x^3(-5ae^3 + 9bde^2 - 13cd^2e) + x(-3ade^2 + 7bd^2e - 11cd^3)}{8d^2e^4 + 16de^5x^2 + 8e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)

[Out] c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16 + (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d**2*e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)

$$3.212 \quad \int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx$$

Optimal. Leaf size=124

$$\frac{x(9cd^2 - e(5bd - ae))}{8de^3(d + ex^2)} - \frac{x(ae^2 - bde + cd^2)}{4e^3(d + ex^2)^2} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(15cd^2 - e(ae + 3bd))}{8d^{3/2}e^{7/2}} + \frac{cx}{e^3}$$

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1257, 1157, 388, 205}

$$\frac{x(9cd^2 - e(5bd - ae))}{8de^3(d + ex^2)} - \frac{x(ae^2 - bde + cd^2)}{4e^3(d + ex^2)^2} - \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(15cd^2 - e(ae + 3bd))}{8d^{3/2}e^{7/2}} + \frac{cx}{e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 - e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],

$x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{LtQ}[q, -1]$

Rule 1257

$\text{Int}[(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(q_.)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \text{:> Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q + 1)})/(2*e^{(2*p + m/2)}*(q + 1)), x] + \text{Dist}[1/(2*e^{(2*p + m/2)}*(q + 1)), \text{Int}[(d + e*x^2)^{(q + 1)}*\text{ExpandToSum}[\text{Together}[(1*(2*e^{(2*p + m/2)}*(q + 1)*x^m*(a + b*x^2 + c*x^4))^p - (-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{ILtQ}[q, -1] \ \&\& \ \text{IGtQ}[m/2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^2 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx &= -\frac{(cd^2 - bde + ae^2)x}{4e^3 (d + ex^2)^2} - \frac{\int \frac{-cd^2 + bde - ae^2 + 4e(cd - be)x^2 - 4ce^2x^4}{(d + ex^2)^2} dx}{4e^3} \\ &= -\frac{(cd^2 - bde + ae^2)x}{4e^3 (d + ex^2)^2} + \frac{(9cd^2 - e(5bd - ae))x}{8de^3 (d + ex^2)} + \frac{\int \frac{-7cd^2 + e(3bd + ae) + 8cdex^2}{d + ex^2} dx}{8de^3} \\ &= \frac{cx}{e^3} - \frac{(cd^2 - bde + ae^2)x}{4e^3 (d + ex^2)^2} + \frac{(9cd^2 - e(5bd - ae))x}{8de^3 (d + ex^2)} - \frac{(15cd^2 - e(3bd + ae))}{8de^3} \int \frac{1}{d + ex^2} dx \\ &= \frac{cx}{e^3} - \frac{(cd^2 - bde + ae^2)x}{4e^3 (d + ex^2)^2} + \frac{(9cd^2 - e(5bd - ae))x}{8de^3 (d + ex^2)} - \frac{(15cd^2 - e(3bd + ae)) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.98

$$\frac{x(ae^2 - 5bde + 9cd^2)}{8de^3 (d + ex^2)} - \frac{x(ae^2 - bde + cd^2)}{4e^3 (d + ex^2)^2} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-ae^2 - 3bde + 15cd^2)}{8d^{3/2}e^{7/2}} + \frac{cx}{e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 - 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx^2 + cx^4)}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]

[Out] IntegrateAlgebraic[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3, x]

fricas [A] time = 0.58, size = 421, normalized size = 3.40

$$\frac{16cd^2e^2 + 2(25cd^2 - 5bd^2 + ad^2)e^2 + (15cd^2 - 3bd^2 - ad^2)e^4 + 2(15cd^2 - 3bd^2 - ad^2)e^2 \sqrt{-de} \log\left(\frac{e^2x^2 - d}{2d}\right) + 2(15cd^2 - 3bd^2 - ad^2)e^2 \sqrt{de} \arctan\left(\frac{\sqrt{de}}{d}\right) + (15cd^2 - 3bd^2 - ad^2)e^2}{8(d^2e^2 + 2d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]

giac [A] time = 0.26, size = 107, normalized size = 0.86

$$cxe^{(-3)} - \frac{(15cd^2 - 3bde - ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{7}{2}\right)}}{8d^{\frac{3}{2}}} + \frac{(9cd^2x^3e - 5bdx^3e^2 + 7cd^3x + ax^3e^3 - 3bd^2xe - adxe^2)e^{(-3)}}{8(x^2e + d)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*c*d^3*x + a*x^3*e^3 - 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d)^2*d)

maple [A] time = 0.01, size = 179, normalized size = 1.44

$$\frac{ax^3}{8(e^2x^2 + d)^2d} - \frac{5bx^3}{8(e^2x^2 + d)^2e} + \frac{9cdx^3}{8(e^2x^2 + d)^2e^2} - \frac{ax}{8(e^2x^2 + d)^2e} - \frac{3bdx}{8(e^2x^2 + d)^2e^2} + \frac{7cd^2x}{8(e^2x^2 + d)^2e^3} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}de} + \frac{3b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^2} - \frac{15cd \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^3} + \frac{cx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)`

[Out] $c*x/e^3+1/8/(e*x^2+d)^2/d*x^3*a-5/8/e/(e*x^2+d)^2*x^3*b+9/8/e^2/(e*x^2+d)^2*x^3*c*d-1/8/e/(e*x^2+d)^2*a*x-3/8/e^2/(e*x^2+d)^2*d*b*x+7/8/e^3/(e*x^2+d)^2*c*d^2*x+1/8/e/d/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a+3/8/e^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b-15/8/e^3*d/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c$

maxima [A] time = 2.46, size = 126, normalized size = 1.02

$$\frac{(9cd^2e - 5bde^2 + ae^3)x^3 + (7cd^3 - 3bd^2e - ade^2)x}{8(de^5x^4 + 2d^2e^4x^2 + d^3e^3)} + \frac{cx}{e^3} - \frac{(15cd^2 - 3bde - ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")`

[Out] $1/8*((9*c*d^2*e - 5*b*d*e^2 + a*e^3)*x^3 + (7*c*d^3 - 3*b*d^2*e - a*d*e^2)*x)/(d*e^5*x^4 + 2*d^2*e^4*x^2 + d^3*e^3) + c*x/e^3 - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d*e^3)$

mupad [B] time = 0.39, size = 118, normalized size = 0.95

$$\frac{cx}{e^3} - \frac{x\left(-\frac{7cd^2}{8} + \frac{3bde}{8} + \frac{ae^2}{8}\right) - \frac{x^3(9cd^2e - 5bde^2 + ae^3)}{8d}}{d^2e^3 + 2de^4x^2 + e^5x^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-15cd^2 + 3bde + ae^2)}{8d^{3/2}e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)`

[Out] $(c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b*d*e^2 + 9*c*d^2*e))/(8*d))/((d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (\operatorname{atan}((e^{1/2})*x)/d^{1/2})*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^{3/2}*e^{7/2}))$

sympy [A] time = 2.62, size = 201, normalized size = 1.62

$$\frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^2}}(ae^2 + 3bde - 15cd^2)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^2}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^2}}(ae^2 + 3bde - 15cd^2)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^2}} + x\right)}{16} + \frac{x^3(ae^3 - 5bde^2 + 9cd^2e) + x(-ade^2 - 3bd^2e + 7cd^3)}{8d^3e^3 + 16d^2e^4x^2 + 8de^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)`

[Out] $c*x/e**3 - \sqrt{-1/(d**3*e**7)}*(a*e**2 + 3*b*d*e - 15*c*d**2)*\log(-d**2*e**3*\sqrt{-1/(d**3*e**7)} + x)/16 + \sqrt{-1/(d**3*e**7)}*(a*e**2 + 3*b*d*e -$

$$15*c*d**2)*\log(d**2*e**3*\sqrt{-1/(d**3*e**7)} + x)/16 + (x**3*(a*e**3 - 5*b*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)$$

$$3.213 \quad \int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$$

Optimal. Leaf size=115

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1157, 385, 205}

$$-\frac{x(5cd^2 - e(3ae + bd))}{8d^2e^2(d + ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae + bd) + 3cd^2)}{8d^{5/2}e^{5/2}} + \frac{x\left(a + \frac{d(cd-be)}{e^2}\right)}{4d(d + ex^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]

[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],

x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{(d + ex^2)^3} dx &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{\int \frac{-3a + \frac{d(cd-be)}{e^2} - \frac{4cdx^2}{e}}{(d+ex^2)^2} dx}{4d} \\ &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} - \frac{\left(-\frac{4cd^2}{e} + e\left(-3a + \frac{d(cd-be)}{e^2}\right)\right) \int \frac{1}{d+ex^2} dx}{8d^2e} \\ &= \frac{(cd^2 - bde + ae^2)x}{4de^2(d + ex^2)^2} - \frac{(5cd^2 - e(bd + 3ae))x}{8d^2e^2(d + ex^2)} + \frac{(3cd^2 + e(bd + 3ae)) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.96

$$\frac{x \left(e \left(ae \left(5d + 3ex^2 \right) + bd \left(ex^2 - d \right) \right) - cd^2 \left(3d + 5ex^2 \right) \right)}{8d^2e^2 \left(d + ex^2 \right)^2} + \frac{\tan^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d}} \right) \left(e \left(3ae + bd \right) + 3cd^2 \right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]

[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(d + e*x^2)^3, x]

fricas [A] time = 0.65, size = 391, normalized size = 3.40

$$\frac{2(5cd^2 - bd^2 - 3ade)^2 + (3cd^4 + bd^4 + 3ad^2 + bde + 3ae^2)x^4 + 2(3cd^3e + bd^3e + 3ade^2)x^2 + 2(3cd^2e + bd^2e - 5ade^2)x}{16(d^2e^2 + 2d^2e^2 + d^2e^2)} \sqrt{-de} \log\left(\frac{x^2 - 2\sqrt{-de}x - d}{(e^2x^2 + d)^2}\right) + 2(3cd^2e + bd^2e - 5ade^2)x^3 - (3cd^4 + bd^4 + 3ad^2 + bde + 3ae^2)x^4 + 2(3cd^3e + bd^3e + 3ade^2)x^2 + 2(3cd^2e + bd^2e - 5ade^2)x}{8(d^2e^2 + 2d^2e^2 + d^2e^2)} \sqrt{de} \arctan\left(\frac{xe^2}{\sqrt{d}}\right) + (3cd^2e + bd^2e - 5ade^2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] $[-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*\sqrt{-d*e}*\log((e*x^2 - 2*\sqrt{-d*e}*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*\sqrt{d*e}*\arctan(\sqrt{d*e}*x/d) + (3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]$

giac [A] time = 0.31, size = 101, normalized size = 0.88

$$\frac{(3cd^2 + bde + 3ae^2) \arctan\left(\frac{xe^2}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{8d^{\frac{5}{2}}} - \frac{(5cd^2x^3e - bdx^3e^2 + 3cd^3x - 3ax^3e^3 + bd^2xe - 5adxe^2)e^{(-2)}}{8(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")

[Out] $1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-5/2)}/d^{(5/2)} - 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e - 5*a*d*x*e^2)*e^{(-2)}/((x^2*e + d)^2*d^2)$

maple [A] time = 0.01, size = 131, normalized size = 1.14

$$\frac{3a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} d^2} + \frac{b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} de} + \frac{3c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de} e^2} + \frac{\frac{(3ae^2 + deb - 5cd^2)x^3}{8d^2e} + \frac{(5ae^2 - deb - 3cd^2)x}{8de^2}}{(ex^2 + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)

[Out] $(1/8*(3*a*e^2 + b*d*e - 5*c*d^2)/d^2/e*x^3 + 1/8*(5*a*e^2 - b*d*e - 3*c*d^2)/e^2/d*x)/(e*x^2 + d)^2 + 3/8/d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a + 1/8/d/e/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b + 3/8/e^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c$

maxima [A] time = 2.46, size = 121, normalized size = 1.05

$$\frac{(5cd^2e - bde^2 - 3ae^3)x^3 + (3cd^3 + bd^2e - 5ade^2)x}{8(d^2e^4x^4 + 2d^3e^3x^2 + d^4e^2)} + \frac{(3cd^2 + bde + 3ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] -1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)*x)/(d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2) + 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)

mupad [B] time = 0.38, size = 112, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2 + bde + 3ae^2)}{8d^{5/2}e^{5/2}} - \frac{x(3cd^2 + bde - 5ae^2)}{8de^2} - \frac{x^3(-5cd^2 + bde + 3ae^2)}{8d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)

[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2)) - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)

sympy [A] time = 1.50, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}(3ae^2 + bde + 3cd^2) \log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{x^3(3ae^3 + bde^2 - 5cd^2e) + x(5ade^2 - bd^2e - 3cd^3)}{8d^4e^2 + 16d^3e^3x^2 + 8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)

[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)

$$3.214 \quad \int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3} dx$$

Optimal. Leaf size=127

$$-\frac{x(ae^2 - bde + cd^2)}{4d^2e(d+ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3e(bd - 5ae) + cd^2)}{8d^{7/2}e^{3/2}} + \frac{x(e(3bd - 7ae) + cd^2)}{8d^3e(d+ex^2)} - \frac{a}{d^3x}$$

Rubi [A] time = 0.20, antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1259, 456, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(3e(bd - 5ae) + cd^2)}{8d^{7/2}e^{3/2}} + \frac{x(e(3bd - 7ae) + cd^2)}{8d^3e(d+ex^2)} - \frac{x\left(\frac{c}{e} - \frac{bd-ae}{d^2}\right)}{4(d+ex^2)^2} - \frac{a}{d^3x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3), x]

[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p+1))/(2*b^(m/2 + 1)*(p+1)), x] + Dist[1/(2*b^(m/2 + 1)*(p+1)), Int[x^m*(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -

$a*d)*x^{(-m + 2))/(a + b*x^2)] - ((-a)^{(m/2 - 1)}*(b*c - a*d))/x^m, x], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m/2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{EqQ}[m + 2*p + 1, 0])$

Rule 1259

$\text{Int}[(x_)^{(m_)*((d_)+(e_)*(x_)^2)^{(q_)*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_ \text{Symbol}] :> \text{Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q + 1)}/(2*e^{(2*p + m/2)}*(q + 1)), x] + \text{Dist}[(-d)^{(m/2 - 1)}/(2*e^{(2*p)}*(q + 1)), \text{Int}[x^m*(d + e*x^2)^{(q + 1)*\text{ExpandToSum}[\text{Together}[(1*(2*(-d)^{-(m/2) + 1})*e^{(2*p)}*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^{(m/2)}*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{ILtQ}[q, -1] \ \&\& \ \text{ILtQ}[m/2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{a + bx^2 + cx^4}{x^2(d + ex^2)^3} dx &= -\left(\frac{c}{e} - \frac{bd - ae}{d^2}\right)x \frac{\int \frac{-4ade^2 - e(cd^2 + 3e(bd - ae))x^2}{x^2(d + ex^2)^2} dx}{4d^2e^2} \\ &= -\left(\frac{c}{e} - \frac{bd - ae}{d^2}\right)x \frac{(cd^2 + e(3bd - 7ae))x}{8d^3e(d + ex^2)} + \frac{\int \frac{8ae^2 + e(cd + e(3b - \frac{7ae}{d}))x^2}{x^2(d + ex^2)} dx}{8d^2e^2} \\ &= -\frac{a}{d^3x} - \left(\frac{c}{e} - \frac{bd - ae}{d^2}\right)x \frac{(cd^2 + e(3bd - 7ae))x}{8d^3e(d + ex^2)} + \frac{(cd^2 + 3e(bd - 5ae)) \int \frac{1}{d + ex^2} dx}{8d^3e} \\ &= -\frac{a}{d^3x} - \left(\frac{c}{e} - \frac{bd - ae}{d^2}\right)x \frac{(cd^2 + e(3bd - 7ae))x}{8d^3e(d + ex^2)} + \frac{(cd^2 + 3e(bd - 5ae)) \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3e(bd - 5ae) + cd^2)}{e^{3/2}} + \frac{\sqrt{d}(dx^2(be(5d + 3ex^2) + cd(ex^2 - d)) - ae(8d^2 + 25dex^2 + 15e^2x^4))}{ex(d + ex^2)^2}$$

$$8d^{7/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3), x]

[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^2(d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3), x]

fricas [A] time = 0.67, size = 421, normalized size = 3.31

$$\frac{16ad^2 - 2(ad^2 + 3bd^2 - 15ade)^2 + 2(ad^2 - 5bd^2 + 25ad^2)^2 - ((d^2 + 3bd^2 - 15ad^2)^2 + (d^2 + 3bd^2 - 15ad^2)^2) \sqrt{-d} \log\left(\frac{d^2 + 3bd^2 - 15ad^2}{2d^2}\right) + 8ad^2 - (d^2 + 3bd^2 - 15ade)^2 + (d^2 - 5bd^2 + 25ad^2)^2 - ((d^2 + 3bd^2 - 15ad^2)^2 + 2(ad^2 + 3bd^2 - 15ade)^2 + (d^2 + 3bd^2 - 15ad^2)^2) \sqrt{d} \arctan\left(\frac{d}{2d}\right)}{16(d^2e^2 + 2d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]

giac [A] time = 0.32, size = 110, normalized size = 0.87

$$\frac{(cd^2 + 3bde - 15ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{3}{2}\right)}}{8d^{\frac{7}{2}}} - \frac{a}{d^3x} + \frac{(cd^2x^3e + 3bdx^3e^2 - cd^3x - 7ax^3e^3 + 5bd^2xe - 9adx^2e^2)e^{(-1)}}{8(x^2e + d)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")

[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2) - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3*x - 7*a*x^3*e^3 + 5*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)^2*d^3)

maple [A] time = 0.01, size = 182, normalized size = 1.43

$$-\frac{7ae^2x^3}{8(e^2x^2+d)^2d^3} + \frac{3be^2x^3}{8(e^2x^2+d)^2d^2} + \frac{cx^3}{8(e^2x^2+d)^2d} - \frac{9aex}{8(e^2x^2+d)^2d^2} + \frac{5bx}{8(e^2x^2+d)^2d} - \frac{cx}{8(e^2x^2+d)^2e} - \frac{15ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^3} + \frac{3b \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2} + \frac{c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}de} - \frac{a}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x)

[Out] $-a/d^3/x - 7/8/d^3/(e*x^2+d)^2*x^3*a*e^2 + 3/8/d^2/(e*x^2+d)^2*x^3*b*e + 1/8/d/(e*x^2+d)^2*x^3*c - 9/8/d^2/(e*x^2+d)^2*e*a*x + 5/8/d/(e*x^2+d)^2*b*x - 1/8/(e*x^2+d)^2/e*x*c - 15/8/d^3*e/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a + 3/8/d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b + 1/8/d/e/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c$

maxima [A] time = 2.65, size = 129, normalized size = 1.02

$$\frac{(cd^2e + 3bde^2 - 15ae^3)x^4 - 8ad^2e - (cd^3 - 5bd^2e + 25ade^2)x^2}{8(d^3e^3x^5 + 2d^4e^2x^3 + d^5ex)} + \frac{(cd^2 + 3bde - 15ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")

[Out] $1/8*((cd^2e + 3b*d*e^2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (cd^3 - 5*b*d^2*e + 25*a*d*e^2)*x^2)/(d^3*e^3*x^5 + 2*d^4*e^2*x^3 + d^5*e*x) + 1/8*(cd^2 + 3*b*d*e - 15*a*e^2)*\arctan(ex/\sqrt{d*e})/(\sqrt{d*e}*d^3*e)$

mupad [B] time = 0.39, size = 118, normalized size = 0.93

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2 + 3bde - 15ae^2)}{8d^{7/2}e^{3/2}} - \frac{a}{d} - \frac{x^4(cd^2 + 3bde - 15ae^2)}{8d^3} + \frac{x^2(cd^2 - 5bde + 25ae^2)}{8d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)

[Out] $(\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(cd^2 - 15*a*e^2 + 3*b*d*e))/(8*d^{7/2}*e^{3/2}) - (a/d - (x^4*(cd^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 + cd^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)$

sympy [A] time = 2.14, size = 202, normalized size = 1.59

$$\frac{\sqrt{-\frac{1}{d^3e^3}}(15ae^2 - 3bde - cd^2)\log\left(-d^4e\sqrt{-\frac{1}{d^3e^3}} + x\right)}{16} - \frac{\sqrt{-\frac{1}{d^3e^3}}(15ae^2 - 3bde - cd^2)\log\left(d^4e\sqrt{-\frac{1}{d^3e^3}} + x\right)}{16} + \frac{-8ad^2e + x^4(-15ae^3 + 3bde^2 + cd^2e) + x^2(-25ade^2 + 5bd^2e - cd^3)}{8d^5ex + 16d^4e^2x^3 + 8d^3e^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 + 3*
b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d**5*e
*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

$$3.215 \quad \int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3} dx$$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2 - 15bde + 3cd^2)}{8d^{9/2}\sqrt{e}} + \frac{x(3cd^2 - e(7bd - 11ae))}{8d^4(d + ex^2)} + \frac{x(ae^2 - bde + cd^2)}{4d^3(d + ex^2)^2} - \frac{bd - 3ae}{d^4x} - \frac{a}{3d^3x^3}$$

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1259, 1261, 205}

$$\frac{x(ae^2 - bde + cd^2)}{4d^3(d + ex^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2 - 15bde + 3cd^2)}{8d^{9/2}\sqrt{e}} + \frac{x(3cd^2 - e(7bd - 11ae))}{8d^4(d + ex^2)} - \frac{bd - 3ae}{d^4x} - \frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3), x]

[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sqrt[e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*

$(a + b*x^2 + c*x^4)^p, x]$, $x]$ /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2 + cx^4}{x^4(d + ex^2)^3} dx &= \frac{(cd^2 - bde + ae^2)x}{4d^3(d + ex^2)^2} + \frac{\int \frac{4ad^2e^2 + 4de^2(bd - ae)x^2 + 3e^2(cd^2 - bde + ae^2)x^4}{x^4(d + ex^2)^2} dx}{4d^3e^2} \\
 &= \frac{(cd^2 - bde + ae^2)x}{4d^3(d + ex^2)^2} + \frac{(3cd^2 - e(7bd - 11ae))x}{8d^4(d + ex^2)} + \frac{\int \frac{8ad^4e^4 + 8d^3e^4(bd - 2ae)x^2 + d^2e^4(3cd^2 - e(7bd - 11ae))x^4}{x^4(d + ex^2)} dx}{8d^6e^4} \\
 &= \frac{(cd^2 - bde + ae^2)x}{4d^3(d + ex^2)^2} + \frac{(3cd^2 - e(7bd - 11ae))x}{8d^4(d + ex^2)} + \frac{\int \left(\frac{8ad^3e^4}{x^4} + \frac{8d^2e^4(bd - 3ae)}{x^2} + \frac{d^2e^4(3cd^2 - 15bde + 8ae^2)}{d + ex^2} \right) dx}{8d^6e^4} \\
 &= -\frac{a}{3d^3x^3} - \frac{bd - 3ae}{d^4x} + \frac{(cd^2 - bde + ae^2)x}{4d^3(d + ex^2)^2} + \frac{(3cd^2 - e(7bd - 11ae))x}{8d^4(d + ex^2)} + \frac{(3cd^2 - 15bde + 8ae^2)}{8d^5} \\
 &= -\frac{a}{3d^3x^3} - \frac{bd - 3ae}{d^4x} + \frac{(cd^2 - bde + ae^2)x}{4d^3(d + ex^2)^2} + \frac{(3cd^2 - e(7bd - 11ae))x}{8d^4(d + ex^2)} + \frac{(3cd^2 - 15bde + 8ae^2)}{8d^5}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)(35ae^2 - 15bde + 3cd^2)}{8d^{9/2}\sqrt{e}} + \frac{x(11ae^2 - 7bde + 3cd^2)}{8d^4(d + ex^2)} + \frac{x(ae^2 - bde + cd^2)}{4d^3(d + ex^2)^2} + \frac{3ae - bd}{d^4x} - \frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3), x]

[Out] $-1/3*a/(d^3*x^3) + (-b*d + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(8*d^(9/2)*\text{Sqrt}[e])$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^4 (d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3), x]

fricas [A] time = 0.78, size = 476, normalized size = 3.35

$$\frac{6(3a^2e^2 - 15abd^2 + 35a^2e^2 - 16a^2e^2 + 10(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 - 16(3a^2e^2 - 7ad^2e^2)^2 - 3(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 + 2(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 + (3a^2e^2 - 15abd^2 + 35a^2e^2)^2)^2 \sqrt{\frac{d+ex^2}{d}} \log\left(\frac{d+ex^2}{d}\right) + 3(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 - 8a^2e^2 + 5(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 - 8(3a^2e^2 - 7ad^2e^2 + 3)(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 + 2(3a^2e^2 - 15abd^2 + 35a^2e^2)^2 + (3a^2e^2 - 15abd^2 + 35a^2e^2)^2 \sqrt{\frac{d+ex^2}{d}} \arctan\left(\frac{d+ex^2}{d}\right)}{8(d^2e^2 + 2d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2)*x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d))/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*b*d^4*e - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d^7*e*x^3)]

giac [A] time = 0.34, size = 128, normalized size = 0.90

$$\frac{(3cd^2 - 15bde + 35ae^2) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{8d^{\frac{9}{2}}} + \frac{3cd^2x^3e - 7bdx^3e^2 + 5cd^3x + 11ax^3e^3 - 9bd^2xe + 13adxe^2}{8(x^2e + d)^2d^4} - \frac{3bdx^2 - 9ax^2e + ad}{3d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")

[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x + 11*a*x^3*e^3 - 9*b*d^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*x^2*e + a*d)/(d^4*x^3)

maple [A] time = 0.02, size = 207, normalized size = 1.46

$$\frac{11ae^3x^3}{8(e^2x^2+d)^2d^4} - \frac{7be^2x^3}{8(e^2x^2+d)^2d^3} + \frac{3ce^2x^3}{8(e^2x^2+d)^2d^2} + \frac{13ae^2x}{8(e^2x^2+d)^2d^3} - \frac{9bex}{8(e^2x^2+d)^2d^2} + \frac{5cx}{8(e^2x^2+d)^2d} + \frac{35ae^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^4} - \frac{15be \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^3} + \frac{3c \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2} + \frac{3ae}{d^4x} - \frac{b}{d^3x} - \frac{a}{3d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x)

[Out] $-1/3*a/d^3/x^3+3/d^4/x*a*e-1/d^3/x*b+11/8/d^4/(e*x^2+d)^2*x^3*a*e^3-7/8/d^3/(e*x^2+d)^2*x^3*b*e^2+3/8/d^2/(e*x^2+d)^2*x^3*c*e+13/8/d^3/(e*x^2+d)^2*a*e^2*x-9/8/d^2/(e*x^2+d)^2*b*e*x+5/8/d/(e*x^2+d)^2*c*x+35/8/d^4/(d*e)^{(1/2)}*a \arctan(1/(d*e)^{(1/2)}*e*x)*a*e^2-15/8/d^3/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*e*b+3/8/d^2/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c$

maxima [A] time = 2.59, size = 147, normalized size = 1.04

$$\frac{3(3cd^2e - 15bde^2 + 35ae^3)x^6 + 5(3cd^3 - 15bd^2e + 35ade^2)x^4 - 8ad^3 - 8(3bd^3 - 7ad^2e)x^2}{24(d^4e^2x^7 + 2d^5ex^5 + d^6x^3)} + \frac{(3cd^2 - 15bde + 35ae^2) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")

[Out] $1/24*(3*(3*c*d^2*e - 15*b*d*e^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 15*b*d^2*e + 35*a*d*e^2)*x^4 - 8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*e^2*x^7 + 2*d^5*e*x^5 + d^6*x^3) + 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^4)$

mupad [B] time = 0.40, size = 138, normalized size = 0.97

$$\frac{x^2(7ae-3bd)}{3d^2} - \frac{a}{3d} + \frac{5x^4(3cd^2-15bde+35ae^2)}{24d^3} + \frac{ex^6(3cd^2-15bde+35ae^2)}{8d^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-15bde+35ae^2)}{8d^{9/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)

[Out] $((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*x^3 + e^2*x^7) + (\operatorname{atan}((e^{1/2}*x)/d^{1/2})*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^{9/2}*e^{1/2})$

sympy [A] time = 2.92, size = 214, normalized size = 1.51

$$\frac{\sqrt{\frac{1}{de}}(35ae^2 - 15bde + 3cd^2) \log\left(-d^5\sqrt{\frac{1}{de}} + x\right)}{16} + \frac{\sqrt{\frac{1}{de}}(35ae^2 - 15bde + 3cd^2) \log\left(d^5\sqrt{\frac{1}{de}} + x\right)}{16} + \frac{-8ad^3 + x^6(105ae^3 - 45bde^2 + 9cd^2e) + x^4(175ade^2 - 75bd^2e + 15cd^3) + x^2(56ad^2e - 24bd^3)}{24d^6x^3 + 48d^5ex^5 + 24d^4e^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)`

[Out]
$$-\sqrt{-1/(d^9e)}(35ae^2 - 15bd^2e + 3cd^2) \log(-d^5\sqrt{-1/(d^9e)} + x)/16 + \sqrt{-1/(d^9e)}(35ae^2 - 15bd^2e + 3cd^2) \log(d^5\sqrt{-1/(d^9e)} + x)/16 + (-8ad^3 + x^6(105ae^3 - 45bd^2e^2 + 9cd^2e) + x^4(175ad^2e^2 - 75bd^2e + 15cd^3) + x^2(56ad^2e - 24bd^3))/(24d^6x^3 + 48d^5ex^5 + 24d^4e^2x^7)$$

$$3.216 \quad \int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3} dx$$

Optimal. Leaf size=171

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (63ae^2 - 35bde + 15cd^2)}{8d^{11/2}} - \frac{6ae^2 - 3bde + cd^2}{d^5x} - \frac{ex(7cd^2 - e(11bd - 15ae))}{8d^5(d+ex^2)} - \frac{ex(ae^2 - bde + cd^2)}{4d^4(d+ex^2)^2}$$

Rubi [A] time = 0.37, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1259, 1805, 1802, 205}

$$\frac{ex(ae^2 - bde + cd^2)}{4d^4(d+ex^2)^2} - \frac{6ae^2 - 3bde + cd^2}{d^5x} - \frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) (63ae^2 - 35bde + 15cd^2)}{8d^{11/2}} - \frac{ex(7cd^2 - e(11bd - 15ae))}{8d^5(d+ex^2)} - \frac{bd - 3ae}{3d^4x^3} - \frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x]

[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2 - e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p)/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]

&& PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1805

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2 + cx^4}{x^6 (d + ex^2)^3} dx &= -\frac{e (cd^2 - bde + ae^2) x}{4d^4 (d + ex^2)^2} - \frac{\int \frac{-4ad^3e^2 - 4d^2e^2(bd - ae)x^2 - 4de^2(cd^2 - bde + ae^2)x^4 + 3e^3(cd^2 - bde + ae^2)x^6}{x^6(d + ex^2)^2} dx}{4d^4e^2} \\
&= -\frac{e (cd^2 - bde + ae^2) x}{4d^4 (d + ex^2)^2} - \frac{e (7cd^2 - e(11bd - 15ae)) x}{8d^5 (d + ex^2)} + \frac{\int \frac{8ad^3e^2 + 8d^2e^2(bd - 2ae)x^2 + 8de^2(cd^2 - e(2bd - 3de + ae^2))x^4 - 3e^3(cd^2 - bde + ae^2)x^6}{x^6(d + ex^2)^2} dx}{8d^5e^2} \\
&= -\frac{e (cd^2 - bde + ae^2) x}{4d^4 (d + ex^2)^2} - \frac{e (7cd^2 - e(11bd - 15ae)) x}{8d^5 (d + ex^2)} + \frac{\int \left(\frac{8ad^2e^2}{x^6} + \frac{8de^2(bd - 3ae)}{x^4} + \frac{8e^2(cd^2 - 3bde + ae^2)}{x^2} \right) dx}{8d^5e^2} \\
&= -\frac{a}{5d^3x^5} - \frac{bd - 3ae}{3d^4x^3} - \frac{cd^2 - 3bde + 6ae^2}{d^5x} - \frac{e (cd^2 - bde + ae^2) x}{4d^4 (d + ex^2)^2} - \frac{e (7cd^2 - e(11bd - 15ae)) x}{8d^5 (d + ex^2)} \\
&= -\frac{a}{5d^3x^5} - \frac{bd - 3ae}{3d^4x^3} - \frac{cd^2 - 3bde + 6ae^2}{d^5x} - \frac{e (cd^2 - bde + ae^2) x}{4d^4 (d + ex^2)^2} - \frac{e (7cd^2 - e(11bd - 15ae)) x}{8d^5 (d + ex^2)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 173, normalized size = 1.01

$$-\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) (63ae^2 - 35bde + 15cd^2)}{8d^{11/2}} + \frac{-6ae^2 + 3bde - cd^2}{d^5x} - \frac{x(15ae^3 - 11bde^2 + 7cd^2e)}{8d^5(d + ex^2)} - \frac{ex(ae^2 - bde + cd^2)}{4d^4(d + ex^2)^2} + \frac{3ae - bd}{3d^4x^3} - \frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x]

[Out] $-1/5*a/(d^3*x^5) + (-b*d) + 3*a*e)/(3*d^4*x^3) + (-c*d^2) + 3*b*d*e - 6*a$
 $*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c$
 $*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (sqrt[e]*(15*c*d^2$
 $- 35*b*d*e + 63*a*e^2)*ArcTan[(sqrt[e]*x)/sqrt[d]]/(8*d^(11/2))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^2 + cx^4}{x^6 (d + ex^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x]

[Out] IntegrateAlgebraic[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x]

fricas [A] time = 0.70, size = 514, normalized size = 3.01

$$\frac{(15cd^2e - 35bde^2 + 63ae^3) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{\frac{-1}{2}}}{8d^{\frac{11}{2}}} - \frac{7cd^2x^3e^2 - 11bdx^3e^3 + 9cd^3xe + 15ax^3e^4 - 13bd^2xe^2 + 17adx^3e}{8(x^2e + d)^2d^5} - \frac{15cd^2x^4 - 45bdx^4e + 90ax^4e^2 + 5bd^2x^2 - 15adx^2e + 3ad^2}{15d^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")

[Out] $[-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e - 3$
 $5*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63*a$
 $*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*d*e$
 $^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c$
 $*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5]*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-$
 $e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*($
 $15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 +$
 $63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4$
 $+ 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*$
 $x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3$
 $*e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d))/(d^5*e^2*x^9 + 2*d^6$
 $*e*x^7 + d^7*x^5)]$

giac [A] time = 0.35, size = 164, normalized size = 0.96

$$\frac{(15cd^2e - 35bde^2 + 63ae^3) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right) e^{\frac{-1}{2}}}{8d^{\frac{11}{2}}} - \frac{7cd^2x^3e^2 - 11bdx^3e^3 + 9cd^3xe + 15ax^3e^4 - 13bd^2xe^2 + 17adx^3e}{8(x^2e + d)^2d^5} - \frac{15cd^2x^4 - 45bdx^4e + 90ax^4e^2 + 5bd^2x^2 - 15adx^2e + 3ad^2}{15d^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")

[Out] $-1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*\arctan(x*e^{(1/2)}/\sqrt{d})*e^{(-1/2)}/d^{(11/2)} - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*c*d^3*x*e + 15*a*x^3*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e + d)^2*d^5) - 1/15*(15*c*d^2*x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 + 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(d^5*x^5)$

maple [A] time = 0.02, size = 245, normalized size = 1.43

$$\frac{15ae^4x^3}{8(e^2x+d)^2d^5} + \frac{11be^3x^3}{8(e^2x+d)^2d^4} - \frac{7ce^2x^3}{8(e^2x+d)^2d^3} - \frac{17ae^2x}{8(e^2x+d)^2d^4} + \frac{13be^2x}{8(e^2x+d)^2d^3} - \frac{9cex}{8(e^2x+d)^2d^2} - \frac{63ae^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^5} + \frac{35be^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^4} - \frac{15ce\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^3} - \frac{6ae^2}{d^5x} + \frac{3be}{d^4x} - \frac{c}{d^3x} + \frac{ae}{d^4x^3} - \frac{b}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3, x)$

[Out] $-1/5*a/d^3/x^5+1/d^4/x^3*a*e-1/3/d^3/x^3*b-6/d^5/x*a*e^2+3/d^4/x*e*b-1/d^3/x*c-15/8*e^4/d^5/(e*x^2+d)^2*x^3*a+11/8*e^3/d^4/(e*x^2+d)^2*x^3*b-7/8*e^2/d^3/(e*x^2+d)^2*x^3*c-17/8*e^3/d^4/(e*x^2+d)^2*a*x+13/8*e^2/d^3/(e*x^2+d)^2*b*x-9/8*e/d^2/(e*x^2+d)^2*c*x-63/8*e^3/d^5/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*a+35/8*e^2/d^4/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*b-15/8*e/d^3/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)*c$

maxima [A] time = 2.45, size = 183, normalized size = 1.07

$$\frac{15(15cd^2e^2 - 35bde^3 + 63ae^4)x^8 + 25(15cd^3e - 35bd^2e^2 + 63ade^3)x^6 + 24ad^4 + 8(15cd^4 - 35bd^3e + 63ad^2e^2)x^4 + 8(5bd^4 - 9ad^3e)x^2}{120(d^5e^2x^9 + 2d^6ex^7 + d^7x^5)} - \frac{(15cd^2e - 35bde^2 + 63ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3, x, \text{algorithm}="maxima")$

[Out] $-1/120*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5) - 1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*\arctan(e*x/\sqrt{d*e})/(\sqrt{d*e}*d^5)$

mupad [B] time = 0.41, size = 168, normalized size = 0.98

$$\frac{\frac{a}{5d} - \frac{x^2(9ae-5bd)}{15d^2} + \frac{x^4(15cd^2-35bde+63ae^2)}{15d^3} + \frac{5ex^6(15cd^2-35bde+63ae^2)}{24d^4} + \frac{e^2x^8(15cd^2-35bde+63ae^2)}{8d^5}}{d^2x^5 + 2dex^7 + e^2x^9} - \frac{\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15cd^2 - 35bde + 63ae^2)}{8d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3), x)$

[Out] $-(a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4) +$

$$(e^{2x^8}(63ae^2 + 15cd^2 - 35bde))/(8d^5)/(d^2x^5 + e^{2x^9} + 2de^x)^7 - (e^{1/2})\operatorname{atan}((e^{1/2})x/d^{1/2})*(63ae^2 + 15cd^2 - 35bde))/(8d^{11/2})$$

sympy [B] time = 3.76, size = 330, normalized size = 1.93

$$\frac{\sqrt{\frac{-x}{d}} (63ad^2 - 35bde + 15cd^2) \log\left(\frac{\sqrt{\frac{-x}{d}} (63ad^2 - 35bde + 15cd^2)}{63ad^2 - 35bde + 15cd^2} + x\right)}{16} - \frac{\sqrt{\frac{-x}{d}} (63ad^2 - 35bde + 15cd^2) \log\left(\frac{\sqrt{\frac{-x}{d}} (63ad^2 - 35bde + 15cd^2)}{63ad^2 - 35bde + 15cd^2} + x\right)}{16} + \frac{-24ad^4 + x^8(-945ad^4 + 525bd^3 - 225cd^2) + x^6(-1575ad^3 + 875bd^2e - 375cd^3) + x^4(-504ad^2e + 280bd^3e - 120cd^4) + x^2(72ad^3e - 40bd^4)}{120d^7x^5 + 240d^6e^xx^7 + 120d^5e^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)

[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e) + x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 225*c*d**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) + x**4*(-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e - 40*b*d**4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)

$$3.217 \quad \int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=230

$$\frac{(a^2ce - ab^2e - 2abcd + b^3d) \log(a + bx^2 + cx^4)}{4c^3(ae^2 - bde + cd^2)} - \frac{(3a^2bce + 2a^2c^2d - ab^3e - 4ab^2cd + b^4d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \dots$$

Rubi [A] time = 0.49, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{(a^2ce - ab^2e - 2abcd + b^3d) \log(a + bx^2 + cx^4)}{4c^3(ae^2 - bde + cd^2)} - \frac{(3a^2bce + 2a^2c^2d - ab^3e + b^4d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{d^4 \log(d + ex^2)}{2e^3(ae^2 - bde + cd^2)} - \frac{x^2(be + cd)}{2c^2e^2} + \frac{x^4}{4ce}$$

Antiderivative was successfully verified.

[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Log[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^9}{(d + ex^2)(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(d + ex)(a + bx + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{-cd - be}{c^2e^2} + \frac{x}{ce} + \frac{d^4}{e^2(cd^2 - bde + ae^2)(d + ex)} + \frac{-a(b^2d - acd - abe)}{c^2(cd^2 - bde + ae^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{(cd + be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{-a(b^2d - acd - abe) - (b^3d - 2abcd - ab^2e + a^2ce)}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2(cd^2 - bde + ae^2)} \\
 &= -\frac{(cd + be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 - bde + ae^2)} - \frac{(b^3d - 2abcd - ab^2e + a^2ce) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3(cd^2 - bde + ae^2)} \\
 &= -\frac{(cd + be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4 \log(d + ex^2)}{2e^3(cd^2 - bde + ae^2)} - \frac{(b^3d - 2abcd - ab^2e + a^2ce) \log\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{4c^3(cd^2 - bde + ae^2)} \\
 &= -\frac{(cd + be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{(b^4d - 4ab^2cd + 2a^2c^2d - ab^3e + 3a^2bce) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{2c^3\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 228, normalized size = 0.99

$$\frac{1}{4} \left(\frac{(-a^2ce + ab^2e + 2abcd + b^3(-d)) \log(a + bx^2 + cx^4)}{c^3(eae - bd) + cd^2} - \frac{2(3a^2bce + 2a^2c^2d - ab^3e - 4ab^2cd + b^4d) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}(e(bd-ae)-cd^2)} + \frac{2d^4 \log(d + ex^2)}{e^3(eae - bd) + cd^2} - \frac{2x^2(be + cd)}{c^2e^2} + \frac{x^4}{ce} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.73, size = 236, normalized size = 1.03

$$\frac{d^4 \log(|x^2e + d|)}{2(cd^2e^3 - bde^4 + ae^5)} - \frac{(b^3d - 2abcd - ab^2e + a^2ce) \log(cx^4 + bx^2 + a)}{4(c^4d^2 - bc^3de + ac^3e^2)} + \frac{(b^4d - 4ab^2cd + 2a^2c^2d - ab^3e + 3a^2bce) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(c^4d^2 - bc^3de + ac^3e^2)\sqrt{-b^2+4ac}} + \frac{(cx^4e - 2cdx^2 - 2bx^2e)e^{(-2)}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +

$$a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e) * \arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c}) / ((c^4*d^2 - b*c^3*d*e + a*c^3*e^2) * \sqrt{-b^2 + 4*a*c}) + 1/4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e) * e^{-2} / c^2$$

maple [B] time = 0.02, size = 538, normalized size = 2.34

$$\frac{3a^2be \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^2-dcb+cb^2)\sqrt{-b^2+4ac}} + \frac{e^2d \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(a^2-dcb+cb^2)\sqrt{-b^2+4ac}} - \frac{db^3e \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^2-dcb+cb^2)\sqrt{-b^2+4ac}} - \frac{2ab^2d \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(a^2-dcb+cb^2)\sqrt{-b^2+4ac}} + \frac{b^4d \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^2-dcb+cb^2)\sqrt{-b^2+4ac}} - \frac{e^2c \ln(cx^2+bx+a)}{4(a^2-dcb+cb^2)c^2} + \frac{db^2e \ln(cx^2+bx+a)}{4(a^2-dcb+cb^2)c^2} - \frac{abd \ln(cx^2+bx+a)}{2(a^2-dcb+cb^2)c^2} + \frac{b^3d \ln(cx^2+bx+a)}{4(a^2-dcb+cb^2)c^2} + \frac{x^4}{4cx} + \frac{d^4 \ln(x^2+d)}{2(a^2-dcb+cb^2)c^2} - \frac{bx^2}{2c^2} - \frac{dx^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a), x)

[Out] 1/4/c/e*x^4-1/2/c^2/e*x^2*b-1/2/c*d/e^2*x^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a^2*e+1/4/(a*e^2-b*d*e+c*d^2)/c^3*ln(c*x^4+b*x^2+a)*a*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*d-1/4/(a*e^2-b*d*e+c*d^2)/c^3*ln(c*x^4+b*x^2+a)*b^3*d+3/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*e+1/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*d-2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*d-1/2/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*a*e+1/2/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*d+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 69.94, size = 7024, normalized size = 30.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)), x)

[Out] (d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*

$$\begin{aligned}
& b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3)/(c^4*e^4) + \\
& (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 + 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) - (((x^2*(32*a*b^6*c^3*e^10 - 352*a^4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) - (8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 -
\end{aligned}$$

$$\begin{aligned}
& 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7)/(c^4*e^4)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4)/(c^4*e^4)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^5*d^2 + 4*a^2*c^4*e^2 - b^2*c^4*d^2 - a*b^2*c^3*e^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)) - (log((x^2*(a^7*e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) + (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d^2*e)^2)/(c^4*e^4) - (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) - (((x^2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e
\end{aligned}$$

$$\begin{aligned}
&^8 - 428*a^2*b^2*c^5*d^4*e^5 + 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + \\
&88*a*b^2*c^6*d^6*e^3 - 116*a*b^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3*e^6 - 4*a^3*b^3*c^3*d*e^8)/(c^4*e^4) + ((x \\
&^2*(32*a*b^6*c^3*e^10 - 352*a^4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7 \\
&*e^3 + 32*b^7*c^3*d*e^9 - 256*a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464 \\
&*a^2*c^8*d^4*e^6 + 592*a^3*c^7*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5 \\
&*e^5 - 48*b^4*c^6*d^4*e^6 + 56*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688* \\
&a^2*b^2*c^6*d^2*e^8 - 192*a*b*c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b* \\
&c^6*d*e^9 + 272*a*b^2*c^7*d^4*e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2 \\
&*e^8 + 136*a^2*b*c^7*d^3*e^7 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a* \\
&d*(2*b^6*e^6 + 2*c^6*d^6 - 15*a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2 \\
&*e^6 + 17*a^2*c^4*d^2*e^4 + 3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2 \\
&*e^4 - 14*a*b^4*c*e^6 - 2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3 \\
&*e^3 + 6*a*b^3*c^2*d*e^5 + a^2*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) + \\
&(8*e^2*(b^2*e^2 + c^2*d^2 - 3*a*c*e^2 - b*c*d*e)*(b^4*d*(b^2 - 4*a*c)^(1/2) \\
&) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) \\
&) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a* \\
&b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 \\
&+ a*b^2*e^3*x^2 + b*c^2*d^3*x^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2 \\
&*d*e^2 - 6*a^2*c*d*e^2 + 4*a*c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2 \\
&*e - 3*a*b*c*d*e^2*x^2))/(c*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e))*(b^4*d* \\
&(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3 \\
&*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4 \\
&*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c) \\
&^(1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 \\
&+ 4*c^8*d^8 + 37*a^4*c^4*e^8 - 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a \\
&^3*b^2*c^3*e^8 + 40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 \\
&- 4*b^3*c^5*d^5*e^3 + 13*b^4*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2 \\
&*e^6 - 32*a*b^6*c*e^8 + 98*a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3 \\
&*b*c^4*d*e^7 - 52*a*b^2*c^5*d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3 \\
&*d^2*e^6 - 16*a^2*b*c^5*d^3*e^5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4)*(b^4*d* \\
&(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d - a*b^3*e \\
&*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4 \\
&*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(\\
&1/2)))/(4*c^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 \\
&+ 5*a^5*c^2*e^7 + 2*b^3*c^4*d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6 \\
&*d^2*e^5 + 2*a^2*b^5*d*e^6 - 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3 \\
&*d^2*e^5 + b^5*c^2*d^5*e^2 - 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2 \\
&*b^3*c^2*d^3*e^4 + 12*a^3*b^2*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c \\
&*d^3*e^4 - 8*a^3*b^3*c*d*e^6 + 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a \\
&b^4*c^2*d^4*e^3 + 10*a^2*b*c^4*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3 \\
&*d^3*e^4))/(c^4*e^4)*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a \\
&*b^4*e + 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*
\end{aligned}$$

$$\frac{b^2 c e + 2 a^2 c^2 d (b^2 - 4 a c)^{1/2} - 4 a b^2 c d (b^2 - 4 a c)^{1/2} + 3 a^2 b c e (b^2 - 4 a c)^{1/2}}{4 c^3 (4 a c - b^2) (a e^2 + c d^2 - b d e)} \cdot \frac{b^4 d (b^2 - 4 a c)^{1/2} - b^5 d + 4 a^3 c^2 e + a b^4 e + 6 a b^3 c d - a b^3 e (b^2 - 4 a c)^{1/2} - 8 a^2 b c^2 d - 5 a^2 b^2 c e + 2 a^2 c^2 d (b^2 - 4 a c)^{1/2} - 4 a b^2 c d (b^2 - 4 a c)^{1/2} + 3 a^2 b c e (b^2 - 4 a c)^{1/2}}{4 (4 a c^5 d^2 + 4 a^2 c^4 e^2 - b^2 c^4 d^2 - a b^2 c^3 e^2 + b^3 c^3 d e - 4 a b c^4 d e)} + \frac{x^4}{4 c e} - \frac{x^2 (b e + c d)}{2 c^2 e^2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.218 \quad \int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=189

$$\frac{(2a^2ce - ab^2e - 3abcd + b^3d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{(-abe - acd + b^2d) \log(a + bx^2 + cx^4)}{4c^2(ae^2 - bde + cd^2)} - \frac{d^3 \log(d + ex^2)}{2e^2(ae^2 - bde + cd^2)} +$$

Rubi [A] time = 0.33, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{(2a^2ce - ab^2e - 3abcd + b^3d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{(-abe - acd + b^2d) \log(a + bx^2 + cx^4)}{4c^2(ae^2 - bde + cd^2)} - \frac{d^3 \log(d + ex^2)}{2e^2(ae^2 - bde + cd^2)} + \frac{x^2}{2ce}$$

Antiderivative was successfully verified.

[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)
)^4^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(d + ex^2)(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(d + ex)(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2 - bde + ae^2)(d + ex)} + \frac{a(bd - ae) + (b^2d - acd - abe)}{c(cd^2 - bde + ae^2)(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= \frac{x^2}{2ce} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{a(bd - ae) + (b^2d - acd - abe)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c(cd^2 - bde + ae^2)} \\
&= \frac{x^2}{2ce} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 - bde + ae^2)} + \frac{(b^2d - acd - abe) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^2(cd^2 - bde + ae^2)} \\
&= \frac{x^2}{2ce} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 - bde + ae^2)} + \frac{(b^2d - acd - abe) \log(a + bx^2 + cx^4)}{4c^2(cd^2 - bde + ae^2)} + \frac{(b^3d - 3abcd - ab^2e + 2a^2ce) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2c^2 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} - \frac{d^3 \log(d + ex^2)}{2e^2(cd^2 - bde + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 186, normalized size = 0.98

$$\frac{2e^2(2a^2ce - ab^2e - 3abcd + b^3d) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right) + \sqrt{4ac-b^2} \left(e \left(abe + acd + b^2(-d) \right) \log(a + bx^2 + cx^4) - 2cx^2(ae^2 - bde + cd^2) + 2c^2d^3 \log(d + ex^2) \right)}{4c^2e^2\sqrt{4ac-b^2} (e(bd - ae) - cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x^4]))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.19, size = 194, normalized size = 1.03

$$-\frac{d^3 \log(|x^2e + d|)}{2(cd^2e^2 - bde^3 + ae^4)} + \frac{x^2e^{(-1)}}{2c} + \frac{(b^2d - acd - abe) \log(cx^4 + bx^2 + a)}{4(c^3d^2 - bc^2de + ac^2e^2)} - \frac{(b^3d - 3abcd - ab^2e + 2a^2ce) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(c^3d^2 - bc^2de + ac^2e^2)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x^2*e^(-1)/c + 1/4*(b^2*d - a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*

$x^2 + b)/\sqrt{-b^2 + 4ac})/((c^3d^2 - b^2c^2d + ac^2e^2)\sqrt{-b^2 + 4ac})$

maple [B] time = 0.01, size = 408, normalized size = 2.16

$$\frac{a^2e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(a^2-deb+cd^2)\sqrt{4ac-b^2}} + \frac{ab^2e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-deb+cd^2)\sqrt{4ac-b^2}} + \frac{3abd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-deb+cd^2)\sqrt{4ac-b^2}} - \frac{b^3d \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-deb+cd^2)\sqrt{4ac-b^2}} - \frac{abe \ln(cx^4+bx^2+a)}{4(a^2-deb+cd^2)^2} - \frac{ad \ln(cx^4+bx^2+a)}{4(a^2-deb+cd^2)^2} + \frac{b^2d \ln(cx^4+bx^2+a)}{4(a^2-deb+cd^2)^2} - \frac{d^3 \ln(ex^2+d)}{2(a^2-deb+cd^2)^2} + \frac{x^2}{2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a), x)

[Out] $\frac{1}{2} \frac{1}{c} \frac{1}{e} x^2 - \frac{1}{4} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c^2} \ln(c^2x^4 + b^2x^2 + a) * a * b * e - \frac{1}{4} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c} \ln(c^2x^4 + b^2x^2 + a) * a * d + \frac{1}{4} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c^2} \ln(c^2x^4 + b^2x^2 + a) * b^2 * d - \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c} \frac{1}{(4ac - b^2)^{1/2}} * \arctan\left(\frac{2cx^2 + b}{(4ac - b^2)^{1/2}}\right) * a^2 * e + \frac{3}{2} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c} \frac{1}{(4ac - b^2)^{1/2}} * \arctan\left(\frac{2cx^2 + b}{(4ac - b^2)^{1/2}}\right) * a * b * d + \frac{1}{2} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c^2} \frac{1}{(4ac - b^2)^{1/2}} * \arctan\left(\frac{2cx^2 + b}{(4ac - b^2)^{1/2}}\right) * b^2 * a * e - \frac{1}{2} \frac{1}{(a^2e - b^2d + cd^2)} \frac{1}{c^2} \frac{1}{(4ac - b^2)^{1/2}} * \arctan\left(\frac{2cx^2 + b}{(4ac - b^2)^{1/2}}\right) * b^3 * d - \frac{1}{2} * d^3 * \ln(e * x^2 + d) / e^2 / (a^2e - b^2d + cd^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 15.21, size = 2304, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)), x)

[Out] $\frac{x^2}{2 * c * e} - \frac{(d^3 * \log(d + e * x^2))}{2 * (a^2 * e^4 + c * d^2 * e^2 - b * d * e^3)} - (\log(a^2 * e^5 * x^2 * (b^2 - 4 * a * c)^{7/2} - 128 * a^5 * c^3 * e^5 - 8 * c^3 * d^5 * (b^2 - 4 * a * c)^{5/2} - 512 * a^3 * c^5 * d^4 * e + 8 * b^2 * c^3 * d^5 * (b^2 - 4 * a * c)^{3/2} + 6 * b^3 * d^2 * e^3 * (b^2 - 4 * a * c)^{5/2} - 3 * b^5 * d^2 * e^3 * (b^2 - 4 * a * c)^{3/2} + 32 * a^4 * b^2 * c^2 * e^5 + 384 * a^4 * c^4 * d^2 * e^3 + 256 * a^2 * c^6 * d^5 * x^2 + 16 * b^4 * c^4 * d^5 * x^2 + 3 * a * d * e^4 * (b^2 - 4 * a * c)^{7/2} - 3 * b * d^2 * e^3 * (b^2 - 4 * a * c)^{7/2} - 3 * c * d^3 * e^2 * (b^2 - 4 * a * c)^{7/2} - 16 * a^2 * b^3 * c^3 * d^3 * e^2 + 48 * a^2 * b^4 * c^2 * d^2 * e^3 - 288$

$$\begin{aligned}
& a^3 b^2 c^3 d^2 e^3 + 16 a^3 b^3 c^2 e^5 x^2 - 384 a^3 c^5 d^3 e^2 x^2 + 16 b^6 c^2 d^3 e^2 x^2 - 6 a b^2 d e^4 (b^2 - 4 a c)^{(5/2)} + 3 a b^4 d e^4 (b^2 - 4 a c)^{(3/2)} + 8 b^2 c^2 d^4 e (b^2 - 4 a c)^{(5/2)} - 32 a b^4 c^3 d^4 e \\
& + 192 a^4 b^2 c^3 d e^4 - 2 b^2 c^2 d^3 e^2 (b^2 - 4 a c)^{(5/2)} - 8 b^3 c^2 d^4 e (b^2 - 4 a c)^{(3/2)} + 5 b^4 c^2 d^3 e^2 (b^2 - 4 a c)^{(3/2)} - 2 a b^2 e^5 x^2 (b^2 - 4 a c)^{(5/2)} + a b^4 e^5 x^2 (b^2 - 4 a c)^{(3/2)} + 16 b^2 c^4 d^5 x^2 (b^2 - 4 a c)^{(3/2)} - 3 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(7/2)} - 16 c^3 d^4 e x^2 (b^2 - 4 a c)^{(5/2)} + 256 a^2 b^2 c^4 d^4 e + 64 a^3 b^2 c^4 d^3 e^2 - 48 a^3 b^3 c^2 d e^4 - 128 a b^2 c^5 d^5 x^2 - 64 a^4 b^2 c^3 e^5 x^2 + 384 a^4 c^4 d e^4 x^2 - 32 b^5 c^3 d^4 e x^2 + 480 a^2 b^2 c^4 d^3 e^2 x^2 + 48 a^2 b^3 c^3 d^2 e^3 x^2 + 256 a b^3 c^4 d^4 e x^2 - 512 a^2 b^2 c^5 d^4 e x^2 - 8 b^2 c^2 d^3 e^2 x^2 (b^2 - 4 a c)^{(5/2)} + 6 b^2 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(5/2)} - 16 b^2 c^3 d^4 e x^2 (b^2 - 4 a c)^{(3/2)} - 3 b^4 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(3/2)} - 160 a b^4 c^3 d^3 e^2 x^2 - 192 a^3 b^2 c^4 d^2 e^3 x^2 - 96 a^3 b^2 c^3 d e^4 x^2 + 8 b^3 c^2 d^3 e^2 x^2 (b^2 - 4 a c)^{(3/2)} * (b^4 d - b^3 d (b^2 - 4 a c)^{(1/2)} + 4 a^2 c^2 d - a b^3 e - 5 a b^2 c d + 4 a^2 b c e + a b^2 e (b^2 - 4 a c)^{(1/2)} - 2 a^2 c e (b^2 - 4 a c)^{(1/2)} + 3 a b c d (b^2 - 4 a c)^{(1/2)})) / (4 * (4 a^2 c^4 d^2 + 4 a^2 c^3 e^2 - b^2 c^3 d^2 - a b^2 c^2 e^2 + b^3 c^2 d e - 4 a b c^3 d e)) - (\log(8 c^3 d^5 (b^2 - 4 a c)^{(5/2)} - 128 a^5 c^3 e^5 - a e^5 x^2 (b^2 - 4 a c)^{(7/2)} - 512 a^3 c^5 d^4 e - 8 b^2 c^3 d^5 (b^2 - 4 a c)^{(3/2)} - 6 b^3 d^2 e^3 (b^2 - 4 a c)^{(5/2)} + 3 b^5 d^2 e^3 (b^2 - 4 a c)^{(3/2)} + 32 a^4 b^2 c^2 e^5 + 384 a^4 c^4 d^2 e^3 + 256 a^2 c^6 d^5 x^2 + 16 b^4 c^4 d^5 x^2 - 3 a d e^4 (b^2 - 4 a c)^{(7/2)} + 3 b d^2 e^3 (b^2 - 4 a c)^{(7/2)} + 3 c^2 d^3 e^2 (b^2 - 4 a c)^{(7/2)}) - 16 a^2 b^3 c^3 d^3 e^2 + 48 a^2 b^4 c^2 d^2 e^3 - 288 a^3 b^2 c^3 d^2 e^3 + 16 a^3 b^3 c^2 e^5 x^2 - 384 a^3 c^5 d^3 e^2 x^2 + 16 b^6 c^2 d^3 e^2 x^2 + 6 a b^2 d e^4 (b^2 - 4 a c)^{(5/2)} - 3 a b^4 d e^4 (b^2 - 4 a c)^{(3/2)} - 8 b^2 c^2 d^4 e (b^2 - 4 a c)^{(5/2)} - 32 a b^4 c^3 d^4 e + 192 a^4 b^2 c^3 d e^4 + 2 b^2 c^2 d^3 e^2 (b^2 - 4 a c)^{(5/2)} + 8 b^3 c^2 d^4 e (b^2 - 4 a c)^{(3/2)} - 5 b^4 c^2 d^3 e^2 (b^2 - 4 a c)^{(3/2)} + 2 a b^2 e^5 x^2 (b^2 - 4 a c)^{(5/2)} - a b^4 e^5 x^2 (b^2 - 4 a c)^{(3/2)} - 16 b^2 c^4 d^5 x^2 (b^2 - 4 a c)^{(3/2)} + 3 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(7/2)} + 16 c^3 d^4 e x^2 (b^2 - 4 a c)^{(5/2)} + 256 a^2 b^2 c^4 d^4 e + 64 a^3 b^2 c^4 d^3 e^2 - 48 a^3 b^3 c^2 d e^4 - 128 a b^2 c^5 d^5 x^2 - 64 a^4 b^2 c^3 e^5 x^2 + 384 a^4 c^4 d e^4 x^2 - 32 b^5 c^3 d^4 e x^2 + 480 a^2 b^2 c^4 d^3 e^2 x^2 + 48 a^2 b^3 c^3 d^2 e^3 x^2 + 256 a b^3 c^4 d^4 e x^2 - 512 a^2 b^2 c^5 d^4 e x^2 - 8 b^2 c^2 d^3 e^2 x^2 (b^2 - 4 a c)^{(5/2)} - 6 b^2 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(5/2)} + 16 b^2 c^3 d^4 e x^2 (b^2 - 4 a c)^{(3/2)} + 3 b^4 c^2 d^2 e^3 x^2 (b^2 - 4 a c)^{(3/2)} - 160 a b^4 c^3 d^3 e^2 x^2 - 192 a^3 b^2 c^4 d^2 e^3 x^2 - 96 a^3 b^2 c^3 d e^4 x^2 - 8 b^3 c^2 d^3 e^2 x^2 (b^2 - 4 a c)^{(3/2)} * (b^4 d + b^3 d (b^2 - 4 a c)^{(1/2)} + 4 a^2 c^2 d - a b^3 e - 5 a b^2 c d + 4 a^2 b c e - a b^2 e (b^2 - 4 a c)^{(1/2)} + 2 a^2 c e (b^2 - 4 a c)^{(1/2)} - 3 a b c d (b^2 - 4 a c)^{(1/2)})) / (4 * (4 a^2 c^4 d^2 + 4 a^2 c^3 e^2 - b^2 c^3 d^2 - a b^2 c^2 e^2 + b^3 c^2 d e - 4 a b c^3 d e))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.219 \quad \int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=158

$$\frac{(-abe - 2acd + b^2d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{d^2 \log(d + ex^2)}{2e(ae^2 - bde + cd^2)} - \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4c(ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{(-abe - 2acd + b^2d) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{d^2 \log(d + ex^2)}{2e(ae^2 - bde + cd^2)} - \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4c(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 - b*d*e + a*e^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(d + ex^2)(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(d + ex)(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{d^2}{(cd^2 - bde + ae^2)(d + ex)} + \frac{-ad - (bd - ae)x}{(cd^2 - bde + ae^2)(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= \frac{d^2 \log(d + ex^2)}{2e(cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{-ad - (bd - ae)x}{a + bx + cx^2} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
&= \frac{d^2 \log(d + ex^2)}{2e(cd^2 - bde + ae^2)} - \frac{(bd - ae) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c(cd^2 - bde + ae^2)} + \frac{(b^2d - 2acd - abe)}{4c(cd^2 - bde + ae^2)} \\
&= \frac{d^2 \log(d + ex^2)}{2e(cd^2 - bde + ae^2)} - \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4c(cd^2 - bde + ae^2)} - \frac{(b^2d - 2acd - abe) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2c(cd^2 - bde + ae^2)} \\
&= -\frac{(b^2d - 2acd - abe) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2c\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)} + \frac{d^2 \log(d + ex^2)}{2e(cd^2 - bde + ae^2)} - \frac{(bd - ae) \log(a + bx^2 + cx^4)}{4c(cd^2 - bde + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 0.88

$$\frac{\sqrt{4ac - b^2} \left(e(bd - ae) \log(a + bx^2 + cx^4) - 2cd^2 \log(d + ex^2) \right) + 2e \left(abe + 2acd + b^2(-d) \right) \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{4ce\sqrt{4ac - b^2} (e(ae - bd) + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] $-1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a + b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 128.56, size = 421, normalized size = 2.66

$$\frac{2 \left((b^2c - 4ac^2)d^2 \log(ex^2 + d) + (ab^2 - 2ac)d \sqrt{b^2 - 4ac} \log \left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac}{4c^2d^2 + 4cd^2e + a^2} \right) - ((b^3 - 4abc)d - (ab^2 - 4a^2c^2)d^2) \log(cx^4 + bx^2 + a) \right) + 2 \left((b^2c - 4ac^2)d^2 \log(ex^2 + d) + 2(ab^2 - (b^2 - 2ac)d) \sqrt{b^2 - 4ac} \arctan \left(\frac{2cx^2 + b}{\sqrt{b^2 - 4ac}} \right) - ((b^3 - 4abc)d - (ab^2 - 4a^2c^2)d^2) \log(cx^4 + bx^2 + a) \right)}{4 \left((b^2c - 4ac^2)d^2e - (b^3c - 4abc^2)d^2 + (ab^2c - 4a^2c^2)e^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] $[1/4*(2*(b^2*c - 4*a*c^2)*d^2*\log(e*x^2 + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(b^2 - 4*a*c)*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^2)*\log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c^2)*d^2*\log(e*x^2 + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^2)*\log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]$

giac [A] time = 1.84, size = 157, normalized size = 0.99

$$\frac{d^2 \log(|x^2e + d|)}{2(c^2d^2e - bde^2 + ae^3)} - \frac{(bd - ae) \log(cx^4 + bx^2 + a)}{4(c^2d^2 - bcde + ace^2)} + \frac{(b^2d - 2acd - abe) \arctan \left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}} \right)}{2(c^2d^2 - bcde + ace^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{2}d^2 \log(\text{abs}(x^2e + d))/(cd^2e - bde^2 + ae^3) - \frac{1}{4}(bd - ae) \log(c^2x^4 + b^2x^2 + a)/(c^2d^2 - bcd^2e + ac^2e^2) + \frac{1}{2}(b^2d - 2abc^2d - ab^2e) \arctan((2cx^2 + b)/\sqrt{-b^2 + 4ac}) / ((c^2d^2 - bcd^2e + ac^2e^2) \sqrt{-b^2 + 4ac})$

maple [A] time = 0.01, size = 289, normalized size = 1.83

$$-\frac{abe \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2-deb+cd^2)\sqrt{4ac-b^2}c} - \frac{ad \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(ae^2-deb+cd^2)\sqrt{4ac-b^2}} + \frac{b^2d \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2-deb+cd^2)\sqrt{4ac-b^2}c} + \frac{ae \ln(cx^4+bx^2+a)}{4(ae^2-deb+cd^2)c} - \frac{bd \ln(cx^4+bx^2+a)}{4(ae^2-deb+cd^2)c} + \frac{d^2 \ln(ex^2+d)}{2(ae^2-deb+cd^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] $\frac{1}{4} / (ae^2 - bde + cd^2) / c \ln(cx^4 + bx^2 + a) + \frac{1}{4} / (ae^2 - bde + cd^2) / c \ln(cx^4 + bx^2 + a) + \frac{bd - 1}{(ae^2 - bde + cd^2) \sqrt{4ac - b^2}} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + \frac{ad - 1}{(ae^2 - bde + cd^2) \sqrt{4ac - b^2}} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + \frac{b^2d - 1}{(ae^2 - bde + cd^2) \sqrt{4ac - b^2}} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + \frac{bd^2}{c^2d} \ln(ex^2 + d) / (ae^2 - bde + cd^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 11.05, size = 1853, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] $\frac{d^2 \log(d + ex^2)}{(2ae^3 - 2bde^2 + 2cd^2e)} + \frac{\log(4a^2e^4(b^2 - 4ac)^{5/2} + 8c^2d^4(b^2 - 4ac)^{5/2} + 5d^2e^2(b^2 - 4ac)^{7/2} + 3de^3x^2(b^2 - 4ac)^{7/2} - 16a^3b^3ce^4 + 64a^4b^2c^2e^2)}{(2ae^3 - 2bde^2 + 2cd^2e)}$

$$\begin{aligned}
&^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^{(3/2)} - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^{(3/2)} - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^{(5/2)} \\
&+ b^4*d^2*e^2*(b^2 - 4*a*c)^{(3/2)} - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^3*e^4*x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e^4*x^2 \\
&+ 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^{(3/2)} + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^{(5/2)} + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 - 4*a*b^3* \\
&e^4*x^2*(b^2 - 4*a*c)^{(3/2)} - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^{(3/2)} - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^{(5/2)} + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^{(3/2)} + 20*c^2 \\
&d^3*e*x^2*(b^2 - 4*a*c)^{(5/2)} - 352*a^2*b^2*c^3*d^3*e - 64*a^3*b*c^3*d^2*e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^4*c*e^4*x^2 + \\
&32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^{(5/2)} - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 - 4*a*c)^{(5/2)} \\
&- 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^{(3/2)} - 4*b^3*c*d^2*e^2*x^2*(b^2 - \\
&4*a*c)^{(3/2)} + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2*d*e^3*x^2)*((b^3*d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^2*d*(b^2 - \\
&4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*d)/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e) - (1 \\
&og(4*a^2*e^4*(b^2 - 4*a*c)^{(5/2)} + 8*c^2*d^4*(b^2 - 4*a*c)^{(5/2)} + 5*d^2*e^2*(b^2 - 4*a*c)^{(7/2)} + 3*d*e^3*x^2*(b^2 - 4*a*c)^{(7/2)} + 16*a^3*b^3*c*e^4 \\
&- 64*a^4*b*c^2*e^4 - 640*a^3*c^4*d^3*e + 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^{(3/2)} - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^{(3/2)} - 6*b^2*d^2*e^2*(b^2 \\
&- 4*a*c)^{(5/2)} + b^4*d^2*e^2*(b^2 - 4*a*c)^{(3/2)} + 256*a^2*c^5*d^4*x^2 + 128*a^4*c^3*e^4*x^2 + 16*b^4*c^3*d^4*x^2 - 80*a^2*b^3*c^2*d^2*e^2 - 96*a^3*b^2*c^2 \\
&e^4*x^2 - 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^{(3/2)} + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^{(5/2)} - 48*a*b^4*c^2*d^3*e + 16*a*b^5*c*d^2 \\
&e^2 - 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^{(3/2)} - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^{(3/2)} - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^{(5/2)} + 3*b^4*d*e^3*x^2*(b^2 - 4*a* \\
&c)^{(3/2)} + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^{(5/2)} + 352*a^2*b^2*c^3*d^3*e + 64*a^3*b*c^3*d^2*e^2 - 96*a^3*b^2*c^2*d*e^3 - 128*a*b^2*c^4*d^4*x^2 + 16*a^2 \\
&*b^4*c*e^4*x^2 - 32*b^5*c^2*d^3*e*x^2 + 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^{(5/2)} + 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 \\
&- 4*a*c)^{(5/2)} + 240*a*b^3*c^3*d^3*e*x^2 - 448*a^2*b*c^4*d^3*e*x^2 + 192*a^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^{(3/2)} - 4*b^3*c*d^2 \\
&*e^2*x^2*(b^2 - 4*a*c)^{(3/2)} - 144*a*b^4*c^2*d^2*e^2*x^2 - 48*a^2*b^3*c^2*d*e^3*x^2)*(e*((a*b^2)/4 - a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^3*d)/4 \\
&- (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*d)/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a \\
&*b*c^2*d*e)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

$$3.220 \quad \int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=132

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac} (ae^2 - bde + cd^2)} - \frac{d \log(d + ex^2)}{2(ae^2 - bde + cd^2)} + \frac{d \log(a + bx^2 + cx^4)}{4(ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac} (ae^2 - bde + cd^2)} - \frac{d \log(d + ex^2)}{2(ae^2 - bde + cd^2)} + \frac{d \log(a + bx^2 + cx^4)}{4(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{(d + ex^2)(a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(d + ex)(a + bx + cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{de}{(cd^2 - bde + ae^2)(d + ex)} + \frac{ae + cdx}{(cd^2 - bde + ae^2)(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
 &= -\frac{d \log(d + ex^2)}{2(cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{ae + cdx}{a + bx + cx^2} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
 &= -\frac{d \log(d + ex^2)}{2(cd^2 - bde + ae^2)} + \frac{d \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4(cd^2 - bde + ae^2)} - \frac{(bd - 2ae) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4(cd^2 - bde + ae^2)} \\
 &= -\frac{d \log(d + ex^2)}{2(cd^2 - bde + ae^2)} + \frac{d \log(a + bx^2 + cx^4)}{4(cd^2 - bde + ae^2)} + \frac{(bd - 2ae) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
 &= \frac{(bd - 2ae) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2\sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} - \frac{d \log(d + ex^2)}{2(cd^2 - bde + ae^2)} + \frac{d \log(a + bx^2 + cx^4)}{4(cd^2 - bde + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 114, normalized size = 0.86

$$\frac{2(bd - 2ae) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right) + d\sqrt{4ac-b^2} \left(2 \log(d+ex^2) - \log(a+bx^2+cx^4)\right)}{4\sqrt{4ac-b^2} (e(bd-ae) - cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 37.37, size = 321, normalized size = 2.43

$$\frac{\left(\frac{(b^2-4ac)d \log(cx^4+bx^2+a) - 2(b^2-4ac)d \log(cx^2+d) - \sqrt{b^2-4ac}(bd-2ae) \log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4((b^2c-4ac^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}\right) \cdot \left(\frac{(b^2-4ac)d \log(cx^4+bx^2+a) - 2(b^2-4ac)d \log(cx^2+d) + 2\sqrt{-b^2+4ac}(bd-2ae) \arctan\left(\frac{-(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{4((b^2c-4ac^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}\right)}{4((b^2c-4ac^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)]

giac [A] time = 1.72, size = 133, normalized size = 1.01

$$-\frac{de \log(|x^2e + d|)}{2(cd^2e - bde^2 + ae^3)} + \frac{d \log(cx^4 + bx^2 + a)}{4(cd^2 - bde + ae^2)} - \frac{(bd - 2ae) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(cd^2 - bde + ae^2)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")`

[Out]
$$-1/2*d*e*\log(\text{abs}(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*\log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((c*d^2 - b*d*e + a*e^2)*\sqrt{-b^2 + 4*a*c})$$

maple [A] time = 0.01, size = 176, normalized size = 1.33

$$\frac{ae \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(ae^2 - deb + cd^2)\sqrt{4ac-b^2}} - \frac{bd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2 - deb + cd^2)\sqrt{4ac-b^2}} - \frac{d \ln(ex^2 + d)}{2(ae^2 - deb + cd^2)} + \frac{d \ln(cx^4 + bx^2 + a)}{4ae^2 - 4deb + 4cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)`

[Out]
$$1/4*d*\ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^{(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*a*e-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^{(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*d-1/2*d*\ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 9.75, size = 3704, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)`

[Out]
$$(\log(76*d^3*e^3*(b^2 - 4*a*c)^{(9/2)} - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 + 512*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4 - 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^{(3/2)} - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^{(3/2)} - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^{(7/2)} - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^{(5/2)} + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^{(3/2)} + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^{(3/2)})/((d + e*x^2)*(a + b*x^2 + c*x^4))$$

$$\begin{aligned}
& - 4*a*c)^{(7/2)} + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^{(5/2)} + 39*d^2*e^4*x^2*(b^2 \\
& - 4*a*c)^{(9/2)} + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448*a^4*c^ \\
& 5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4*d^6*x^2 \\
& - 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^{(5/2)} + 48*b*c^3*d^6*(b^ \\
& 2 - 4*a*c)^{(5/2)} + 40*a^2*d*e^5*(b^2 - 4*a*c)^{(7/2)} + 168*c^2*d^5*e*(b^2 - \\
& 4*a*c)^{(7/2)} + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^{(5/2)} + 20*a^2*b^4*e^6*x^2* \\
& (b^2 - 4*a*c)^{(3/2)} - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^{(3/2)} + 155*b^2*d^2* \\
& e^4*x^2*(b^2 - 4*a*c)^{(7/2)} - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^{(5/2)} + 25* \\
& b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^{(3/2)} + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^{(7/ \\
& 2)} + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 24448*a^3*b^2*c^ \\
& 4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^2*e^4 + 26240*a^ \\
& 4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c^5*d^3*e^3*x^2 - \\
& 384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^{(7/2)} - 176*b*c*d^4 \\
& *e^2*(b^2 - 4*a*c)^{(7/2)} + 256*a*b^5*c^3*d^5*e + 256*a*b^7*c*d^3*e^3 + 2560 \\
& *a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b*c^3*d*e^5 - 128*a*b^8* \\
& d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^{(5/2)} + 20*a*b^5*d^2*e^4*(b^2 - \\
& 4*a*c)^{(3/2)} + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^{(5/2)} - 56*a^2*b^4*d*e^5*(b \\
& ^2 - 4*a*c)^{(3/2)} - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^{(5/2)} + 256*b^3*c*d^4*e \\
& ^2*(b^2 - 4*a*c)^{(5/2)} + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^{(3/2)} - 80*b^5*c*d \\
& ^4*e^2*(b^2 - 4*a*c)^{(3/2)} - 384*a*b^6*c^2*d^4*e^2 - 1664*a^2*b^3*c^4*d^5*e \\
& + 1408*a^2*b^6*c*d^2*e^4 - 37888*a^4*b*c^4*d^3*e^3 - 6784*a^4*b^3*c^2*d*e^ \\
& 5 + 448*a*b^3*c^5*d^6*x^2 - 768*a^2*b*c^6*d^6*x^2 + 576*a^3*b^5*c*e^6*x^2 + \\
& 1280*a^5*b*c^3*e^6*x^2 - 21504*a^3*c^6*d^5*e*x^2 - 5120*a^5*c^4*d*e^5*x^2 \\
& + 256*b^6*c^3*d^5*e*x^2 + 256*b^8*c*d^3*e^3*x^2 - 26560*a^2*b^3*c^4*d^4*e^2 \\
& *x^2 + 25600*a^2*b^4*c^3*d^3*e^3*x^2 - 11264*a^2*b^5*c^2*d^2*e^4*x^2 - 5888 \\
& 0*a^3*b^2*c^4*d^3*e^3*x^2 + 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5* \\
& x^2*(b^2 - 4*a*c)^{(5/2)} - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^{(3/2)} - 448*b*c* \\
& d^3*e^3*x^2*(b^2 - 4*a*c)^{(7/2)} - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^{(5/2)} - \\
& 3200*a*b^4*c^4*d^5*e*x^2 + 1472*a*b^7*c*d^2*e^4*x^2 + 1792*a^2*b^6*c*d*e^5 \\
& *x^2 + 192*b^3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^{(5/2)} + 160*b^3*c^3*d^5*e*x^2*(b \\
& ^2 - 4*a*c)^{(3/2)} + 5504*a*b^5*c^3*d^4*e^2*x^2 - 4352*a*b^6*c^2*d^3*e^3*x^2 \\
& + 14080*a^2*b^2*c^5*d^5*e*x^2 + 42752*a^3*b*c^5*d^4*e^2*x^2 - 8320*a^3*b^4 \\
& *c^2*d*e^5*x^2 - 37120*a^4*b*c^4*d^2*e^4*x^2 + 14080*a^4*b^2*c^3*d*e^5*x^2 \\
& + 88*a*b*d*e^5*x^2*(b^2 - 4*a*c)^{(7/2)} + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a \\
& *c)^{(5/2)} - 100*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^{(3/2)}*(d*((b*(b^2 - 4*a* \\
& c)^{(1/2)))/4 - a*c + b^2/4) - (a*e*(b^2 - 4*a*c)^{(1/2)))/2))/(a*b^2*e^2 - 4*a \\
& *c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (log(76*d^3*e \\
& ^3*(b^2 - 4*a*c)^{(9/2)} + 64*a^3*b^6*e^6 + 4608*a^3*c^6*d^6 - 512*a^6*c^3*e^ \\
& 6 + 320*a*b^4*c^4*d^6 - 512*a^4*b^4*c*e^6 + 64*a*b^8*d^2*e^4 + 128*a^2*b^7* \\
& d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^{(3/2)} - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^{(\\
& 3/2)} - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^{(7/2)} - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^{(5 \\
& /2)} + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^{(3/2)} + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^{(7/2 \\
&)} + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^{(5/2)} + 39*d^2*e^4*x^2*(b^2 - 4*a*c)^{(9/2 \\
&)} - 2432*a^2*b^2*c^5*d^6 + 1152*a^5*b^2*c^2*e^6 - 40448*a^4*c^5*d^4*e^2 + 1 \\
& 9968*a^5*c^4*d^2*e^4 + 64*a^2*b^7*e^6*x^2 + 64*b^5*c^4*d^6*x^2 + 64*b^9*d^2
\end{aligned}$$

$$\begin{aligned}
& e^4 x^2 + 32 a^3 b e^6 (b^2 - 4 a^* c)^{(5/2)} + 48 b^* c^3 d^6 (b^2 - 4 a^* c)^{(5/2)} + 40 a^2 d^* e^5 (b^2 - 4 a^* c)^{(7/2)} + 168 c^2 d^5 e^* (b^2 - 4 a^* c)^{(7/2)} \\
& + 40 a^2 b^2 e^6 x^2 (b^2 - 4 a^* c)^{(5/2)} + 20 a^2 b^4 e^6 x^2 (b^2 - 4 a^* c)^{(3/2)} - 80 b^2 c^4 d^6 x^2 (b^2 - 4 a^* c)^{(3/2)} + 155 b^2 d^2 e^4 x^2 (b^2 - 4 a^* c)^{(7/2)} \\
& - 155 b^4 d^2 e^4 x^2 (b^2 - 4 a^* c)^{(5/2)} + 25 b^6 d^2 e^4 x^2 (b^2 - 4 a^* c)^{(3/2)} + 316 c^2 d^4 e^2 x^2 (b^2 - 4 a^* c)^{(7/2)} - 5120 a^2 b^4 c^3 d^4 e^2 \\
& + 4096 a^2 b^5 c^2 d^3 e^3 + 24448 a^3 b^2 c^4 d^4 e^2 - 21760 a^3 b^3 c^3 d^3 e^3 + 9920 a^3 b^4 c^2 d^2 e^4 - 26240 a^4 b^2 c^3 d^2 e^4 \\
& + 1600 a^4 b^3 c^2 e^6 x^2 - 38912 a^4 c^5 d^3 e^3 x^2 + 384 b^7 c^2 d^4 e^2 x^2 + 212 a^* b d^2 e^4 (b^2 - 4 a^* c)^{(7/2)} - 176 b^* c d^4 e^2 (b^2 - 4 a^* c)^{(7/2)} \\
& - 256 a^* b^5 c^3 d^5 e - 256 a^* b^7 c^* d^3 e^3 - 2560 a^3 b^* c^5 d^5 e - 1664 a^3 b^5 c^* d e^5 - 8704 a^5 b^* c^3 d e^5 + 128 a^* b^8 d^* e^5 x^2 - 168 a^* b^3 d^2 e^4 (b^2 - 4 a^* c)^{(5/2)} \\
& + 20 a^* b^5 d^2 e^4 (b^2 - 4 a^* c)^{(3/2)} + 144 a^2 b^2 d^* e^5 (b^2 - 4 a^* c)^{(5/2)} - 56 a^2 b^4 d^* e^5 (b^2 - 4 a^* c)^{(3/2)} - 272 b^2 c^2 d^5 e^* (b^2 - 4 a^* c)^{(5/2)} \\
& + 256 b^3 c^* d^4 e^2 (b^2 - 4 a^* c)^{(5/2)} + 104 b^4 c^2 d^5 e^* (b^2 - 4 a^* c)^{(3/2)} - 80 b^5 c^* d^4 e^2 (b^2 - 4 a^* c)^{(3/2)} + 384 a^* b^6 c^2 d^4 e^2 \\
& + 1664 a^2 b^3 c^4 d^5 e - 1408 a^2 b^6 c^* d^2 e^4 + 37888 a^4 b^* c^4 d^3 e^3 + 6784 a^4 b^3 c^2 d^* e^5 - 448 a^* b^3 c^5 d^6 x^2 + 768 a^2 b^* c^6 d^6 x^2 - 576 a^3 b^5 c^* e^6 x^2 - 1280 a^5 b^* c^3 e^6 x^2 \\
& + 21504 a^3 c^6 d^5 e^* x^2 + 5120 a^5 c^4 d^* e^5 x^2 - 256 b^6 c^3 d^5 e^* x^2 - 256 b^8 c^* d^3 e^3 x^2 + 26560 a^2 b^3 c^4 d^4 e^2 x^2 - 25600 a^2 b^4 c^3 d^3 e^3 x^2 \\
& + 11264 a^2 b^5 c^2 d^2 e^4 x^2 + 58880 a^3 b^2 c^4 d^3 e^3 x^2 - 34880 a^3 b^3 c^3 d^2 e^4 x^2 + 80 a^* b^3 d^* e^5 x^2 (b^2 - 4 a^* c)^{(5/2)} - 40 a^* b^5 d^* e^5 x^2 (b^2 - 4 a^* c)^{(3/2)} \\
& - 448 b^* c d^3 e^3 x^2 (b^2 - 4 a^* c)^{(7/2)} - 416 b^* c^3 d^5 e^* x^2 (b^2 - 4 a^* c)^{(5/2)} + 3200 a^* b^4 c^4 d^5 e^* x^2 - 1472 a^* b^7 c^* d^2 e^4 x^2 - 1792 a^2 b^6 c^* d^* e^5 x^2 + 192 b^3 c^* d^3 e^3 x^2 (b^2 - 4 a^* c)^{(5/2)} \\
& + 160 b^3 c^3 d^5 e^* x^2 (b^2 - 4 a^* c)^{(3/2)} - 5504 a^* b^5 c^3 d^4 e^2 x^2 + 4352 a^* b^6 c^2 d^3 e^3 x^2 - 14080 a^2 b^2 c^5 d^5 e^* x^2 - 42752 a^3 b^* c^5 d^4 e^2 x^2 + 8320 a^3 b^4 c^2 d^* e^5 x^2 \\
& + 37120 a^4 b^* c^4 d^2 e^4 x^2 - 14080 a^4 b^2 c^3 d^* e^5 x^2 + 88 a^* b d^* e^5 x^2 (b^2 - 4 a^* c)^{(7/2)} + 168 b^2 c^2 d^4 e^2 x^2 (b^2 - 4 a^* c)^{(5/2)} - 100 b^4 c^2 d^4 e^2 x^2 (b^2 - 4 a^* c)^{(3/2)} \\
& * (d^* (a^* c + (b^* (b^2 - 4 a^* c)^{(1/2)})) / 4 - b^2 / 4) - (a^* e^* (b^2 - 4 a^* c)^{(1/2)} / 2) / (a^* b^2 e^2 - 4 a^* c^2 d^2 - 4 a^2 c^* e^2 + b^2 c^* d^2 - b^3 d^* e + 4 a^* b^* c^* d^* e) - (d^* \log(d + e^* x^2)) / (2^* (a^* e^2 + c^* d^2 - b^* d^* e))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.221 \quad \int \frac{x}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=133

$$-\frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac} (ae^2 - bde + cd^2)} + \frac{e \log(d + ex^2)}{2(ae^2 - bde + cd^2)} - \frac{e \log(a + bx^2 + cx^4)}{4(ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1247, 705, 31, 634, 618, 206, 628}

$$-\frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac} (ae^2 - bde + cd^2)} + \frac{e \log(d + ex^2)}{2(ae^2 - bde + cd^2)} - \frac{e \log(a + bx^2 + cx^4)}{4(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1247

Int[(x_)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(d+ex^2)(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(d+ex)(a+bx+cx^2)} dx, x, x^2 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{cd-be-cex}{a+bx+cx^2} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} + \frac{e^2 \text{Subst} \left(\int \frac{1}{d+ex} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
 &= \frac{e \log(d+ex^2)}{2(cd^2 - bde + ae^2)} - \frac{e \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4(cd^2 - bde + ae^2)} + \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4(cd^2 - bde + ae^2)} \\
 &= \frac{e \log(d+ex^2)}{2(cd^2 - bde + ae^2)} - \frac{e \log(a+bx^2+cx^4)}{4(cd^2 - bde + ae^2)} - \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
 &= -\frac{(2cd - be) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{2\sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} + \frac{e \log(d+ex^2)}{2(cd^2 - bde + ae^2)} - \frac{e \log(a+bx^2+cx^4)}{4(cd^2 - bde + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.84

$$\frac{(2be - 4cd) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right) + e\sqrt{4ac-b^2} \left(\log(a+bx^2+cx^4) - 2\log(d+ex^2)\right)}{4\sqrt{4ac-b^2} (e(bd-ae) - cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-c*d^2) + e*(b*d - a*e))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [A] time = 26.75, size = 321, normalized size = 2.41

$$\frac{\left(\frac{(b^2-4ac)e\log(cx^4+bx^2+a) - 2(b^2-4ac)e\log(ex^2+d) + \sqrt{b^2-4ac}(2cd-be)\log\left(\frac{2c^2x^4+2bcx^2+2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{4((b^2c-4a^2c^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}\right) \cdot \left(\frac{(b^2-4ac)e\log(cx^4+bx^2+a) - 2(b^2-4ac)e\log(ex^2+d) + 2\sqrt{-b^2+4ac}(2cd-be)\arctan\left(\frac{-2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{4((b^2c-4a^2c^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}\right)}{4((b^2c-4a^2c^2)d^2 - (b^3-4abc)de + (ab^2-4a^2c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))]/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))]/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)]

giac [A] time = 1.91, size = 134, normalized size = 1.01

$$-\frac{e \log(cx^4 + bx^2 + a)}{4(cd^2 - bde + ae^2)} + \frac{e^2 \log(|x^2e + d|)}{2(cd^2e - bde^2 + ae^3)} + \frac{(2cd - be) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(cd^2 - bde + ae^2)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-1/4*e*\log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*\log(\text{abs}(x^2 * e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((c*d^2 - b*d*e + a*e^2)*\sqrt{-b^2 + 4*a*c})$

maple [A] time = 0.01, size = 176, normalized size = 1.32

$$-\frac{be \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2 - deb + cd^2)\sqrt{4ac-b^2}} + \frac{cd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(ae^2 - deb + cd^2)\sqrt{4ac-b^2}} + \frac{e \ln(e x^2 + d)}{2ae^2 - 2deb + 2cd^2} - \frac{e \ln(cx^4 + bx^2 + a)}{4(ae^2 - deb + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] $-1/4*e*\ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^{(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})}*b*e+1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^{(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})}*c*d+1/2*e*\ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 8.71, size = 2434, normalized size = 18.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] $(e*\log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (\log(36*a^4*c^3*e^5 - 4*a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^{(1/2)} - 73*a^3*b^2*c^2*e^5 - 184*a^3*c^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^{(1/2)} + 2*a*c^5*d^5*(b^2 - 4*a*c)^{(1/2)} + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)$

$$\begin{aligned}
& ^{(1/2)} + 18a^3c^3e^5x^2(b^2 - 4ac)^{(1/2)} + 146a^2b^2c^3d^2e^3 - \\
& 101a^2b^3c^2e^5x^2 + 120a^2c^5d^3e^2x^2 + 19b^4c^3d^3e^2x^2 \\
& - 25b^5c^2d^2e^3x^2 - 9ab^2c^4d^4e + 184a^3b^3c^3d^4e^4 + 36ab^5c^2e^5x^2 + 16b^6c^3d^4e^4x^2 + 24a^2b^3c^3e^5(b^2 - 4ac)^{(1/2)} - \\
& 33a^3b^3c^2e^5(b^2 - 4ac)^{(1/2)} + 66a^3c^3d^4e^4(b^2 - 4ac)^{(1/2)} \\
&) + b^5c^3d^5x^2(b^2 - 4ac)^{(1/2)} + 18ab^3c^3d^3e^2 - 25ab^4c^2 \\
& d^2e^3 - 72a^2b^3c^4d^3e^2 - 110a^2b^3c^2d^4e^4 + 84a^3b^3c^3e^5x^2 - 132a^3c^4d^4e^4x^2 - 7b^3c^4d^4e^4x^2 + 28ab^4c^3e^5x^2(b^2 \\
& - 4ac)^{(1/2)} + 18ac^5d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 16b^5c^3d^4e^4x^2 \\
& ^2(b^2 - 4ac)^{(1/2)} - 126ab^4c^2d^4e^4x^2 + 20ab^2c^3d^3e^2(b^2 \\
& - 4ac)^{(1/2)} - 25ab^3c^2d^2e^3(b^2 - 4ac)^{(1/2)} + 90a^2b^3c^3d^2e^3(b^2 - 4ac)^{(1/2)} - 78a^2b^2c^2d^4e^4(b^2 - 4ac)^{(1/2)} - 7b^2c^4d^4e^4x^2(b^2 - 4ac)^{(1/2)} - 106ab^2c^4d^3e^2x^2 + 168ab^3c^3d^2e^3x^2 - 272a^2b^3c^4d^2e^3x^2 + 281a^2b^2c^3d^4e^4x^2 - 5ab^3c^4d^4e^4(b^2 - 4ac)^{(1/2)} + 16ab^4c^3d^4e^4(b^2 - 4ac)^{(1/2)} - 53a^2b^2c^2e^5x^2(b^2 - 4ac)^{(1/2)} + 28ab^3c^5d^4e^4x^2 - 92a^2c^4d^2e^3x^2(b^2 - 4ac)^{(1/2)} + 19b^3c^3d^3e^2x^2(b^2 - 4ac)^{(1/2)} - 25b^4c^2d^2e^3x^2(b^2 - 4ac)^{(1/2)} + 118ab^2c^3d^2e^3x^2(b^2 - 4ac)^{(1/2)} - 66ab^3c^4d^3e^2x^2(b^2 - 4ac)^{(1/2)} - 94ab^3c^2d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 125a^2b^3c^3d^4e^4x^2(b^2 - 4ac)^{(1/2)} * (e((b(b^2 - 4ac)^{(1/2)))/4 - ac + b^2/4) - (c*d*(b^2 - 4ac)^{(1/2)))/2) / (ab^2e^2 - 4ac^2d^2 - 4a^2c^2e^2 + b^2cd^2 - b^3de + 4ab^3c^3d^2e^3x^2 + 118ab^2c^3d^2e^3x^2(b^2 - 4ac)^{(1/2)} - 66ab^3c^4d^3e^2x^2(b^2 - 4ac)^{(1/2)} - 94ab^3c^2d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 125a^2b^3c^3d^4e^4x^2(b^2 - 4ac)^{(1/2)} * (e((b(b^2 - 4ac)^{(1/2)))/4 - ac + b^2/4) - (c*d*(b^2 - 4ac)^{(1/2)))/2) / (ab^2e^2 - 4ac^2d^2 - 4a^2c^2e^2 + b^2cd^2 - b^3de + 4ab^3c^3d^2e^3x^2 - 32a^2b^4c^3e^5 - 36a^2c^5d^4e + 4aac^6d^5x^2 - 4b^6e^5x^2(b^2 - 4ac)^{(1/2)} + 73a^3b^2c^2e^5 + 184a^3c^4d^2e^3 - b^2c^5d^5x^2 - 4ab^5e^5(b^2 - 4ac)^{(1/2)} + 2ac^5d^5(b^2 - 4ac)^{(1/2)} - 16ab^5c^3d^4e^4 - 60a^2c^4d^3e^2(b^2 - 4ac)^{(1/2)} + 18a^3c^3e^5x^2(b^2 - 4ac)^{(1/2)} - 146a^2b^2c^3d^2e^3 + 101a^2b^3c^2e^5x^2 - 120a^2c^5d^3e^2x^2 - 19b^4c^3d^3e^2x^2 + 25b^5c^2d^2e^3x^2 + 9ab^2c^4d^4e - 184a^3b^3c^3d^4e^4 - 36ab^5c^3e^5x^2 - 16b^6c^3d^4e^4x^2 + 24a^2b^3c^3e^5(b^2 - 4ac)^{(1/2)} - 33a^3b^3c^2e^5(b^2 - 4ac)^{(1/2)} + 66a^3c^3d^4e^4(b^2 - 4ac)^{(1/2)} + b^5c^3d^5x^2(b^2 - 4ac)^{(1/2)} - 18ab^3c^3d^3e^2 + 25ab^4c^2d^2e^3 + 72a^2b^3c^4d^3e^2 + 110a^2b^3c^2d^4e^4 - 84a^3b^3c^3e^5x^2 + 132a^3c^4d^4e^4x^2 + 7b^3c^4d^4e^4x^2 + 28ab^4c^3e^5x^2(b^2 - 4ac)^{(1/2)} + 18ac^5d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 16b^5c^3d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 126ab^4c^2d^4e^4x^2 + 20ab^2c^3d^3e^2(b^2 - 4ac)^{(1/2)} - 25ab^3c^2d^2e^3(b^2 - 4ac)^{(1/2)} + 90a^2b^3c^3d^2e^3(b^2 - 4ac)^{(1/2)} - 78a^2b^2c^2d^4e^4(b^2 - 4ac)^{(1/2)} - 7b^2c^4d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 106ab^2c^4d^3e^2x^2 - 168ab^3c^3d^2e^3x^2 + 272a^2b^3c^4d^2e^3x^2 - 281a^2b^2c^3d^4e^4x^2 - 5ab^3c^4d^4e^4(b^2 - 4ac)^{(1/2)} + 16ab^4c^3d^4e^4(b^2 - 4ac)^{(1/2)} - 53a^2b^2c^2e^5x^2(b^2 - 4ac)^{(1/2)} - 28ab^3c^5d^4e^4x^2 - 92a^2c^4d^2e^3x^2(b^2 - 4ac)^{(1/2)} + 19b^3c^3d^3e^2x^2(b^2 - 4ac)^{(1/2)} - 25b^4c^2d^2e^3x^2(b^2 - 4ac)^{(1/2)} + 118ab^2c^3d^2e^3x^2(b^2 - 4ac)^{(1/2)} - 6
\end{aligned}$$

$$6*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^{(1/2)} - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^{(1/2)} + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^{(1/2)}*(e*(a*c + (b*(b^2 - 4*a*c)^{(1/2)))/4 - b^2/4) - (c*d*(b^2 - 4*a*c)^{(1/2)}/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.222 \quad \int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=167

$$\frac{(2ace + b^2(-e) + bcd) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} - \frac{e^2 \log(d + ex^2)}{2d(ae^2 - bde + cd^2)} - \frac{(cd - be) \log(a + bx^2 + cx^4)}{4a(ae^2 - bde + cd^2)} + \frac{\log(x)}{ad}$$

Rubi [A] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 893, 634, 618, 206, 628}

$$\frac{(2ace + b^2(-e) + bcd) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} - \frac{e^2 \log(d + ex^2)}{2d(ae^2 - bde + cd^2)} - \frac{(cd - be) \log(a + bx^2 + cx^4)}{4a(ae^2 - bde + cd^2)} + \frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(d+ex)(a+bx+cx^2)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx} - \frac{e^3}{d(cd^2 - bde + ae^2)(d+ex)} + \frac{-bcd + b^2e - ace - c(cd-be)x}{a(cd^2 - bde + ae^2)(a+bx+cx^2)} \right) dx, x, x^2 \right) \\
 &= \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{-bcd + b^2e - ace - c(cd-be)x}{a+bx+cx^2} dx, x, x^2 \right)}{2a(cd^2 - bde + ae^2)} \\
 &= \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2 - bde + ae^2)} - \frac{(cd-be) \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4a(cd^2 - bde + ae^2)} - \frac{(bcd - b^2e - ace)}{2a\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)} \\
 &= \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2 - bde + ae^2)} - \frac{(cd-be) \log(a+bx^2+cx^4)}{4a(cd^2 - bde + ae^2)} + \frac{(bcd - b^2e - ace)}{2a\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)} \\
 &= \frac{(bcd - b^2e + 2ace) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{2a\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)} + \frac{\log(x)}{ad} - \frac{e^2 \log(d+ex^2)}{2d(cd^2 - bde + ae^2)} - \frac{(cd-be) \log(a+bx^2+cx^4)}{4a(cd^2 - bde + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.32, size = 242, normalized size = 1.45

$$\frac{4 \log(x) \sqrt{b^2 - 4ac} (e(ac - bd) + cd^2) - 2ae^2 \sqrt{b^2 - 4ac} \log(d + ex^2) - d (cd \sqrt{b^2 - 4ac} - be \sqrt{b^2 - 4ac} + 2ace + b^2(-e) + bcd) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2) + d (-cd \sqrt{b^2 - 4ac} + be \sqrt{b^2 - 4ac} + 2ace + b^2(-e) + bcd) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{4ad \sqrt{b^2 - 4ac} (e(ac - bd) + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] (4*sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*sqrt[b^2 - 4*a*c]*e)*Log[b - sqrt[b^2 - 4*a*c] + 2*c*x^2] + d*(b*c*d - c*sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*sqrt[b^2 - 4*a*c]*e)*Log[b + sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*sqrt[b^2 - 4*a*c]*e^2*Log[d + e*x^2])/(4*a*sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) + a*e)))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.92, size = 172, normalized size = 1.03

$$-\frac{(cd - be) \log(cx^4 + bx^2 + a)}{4(acd^2 - abde + a^2e^2)} - \frac{e^3 \log(|x^2e + d|)}{2(cd^3e - bd^2e^2 + ade^3)} - \frac{(bcd - b^2e + 2ace) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(acd^2 - abde + a^2e^2)\sqrt{-b^2 + 4ac}} + \frac{\log(x^2)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/4*(c*d - b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/2*(b*c*d - b^2

$e + 2ac$)*arctan(($2cx^2 + b$)/sqrt($-b^2 + 4ac$))/(($a^2e^2 + a^2e^2$)*sqrt($-b^2 + 4ac$)) + 1/2*log(x^2)/(a^2d)

maple [A] time = 0.01, size = 298, normalized size = 1.78

$$\frac{b^2 e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2-deb+cd^2)\sqrt{4ac-b^2}} - \frac{bcd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(ae^2-deb+cd^2)\sqrt{4ac-b^2}} - \frac{ce \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(ae^2-deb+cd^2)\sqrt{4ac-b^2}} + \frac{be \ln(cx^4+bx^2+a)}{4(ae^2-deb+cd^2)a} - \frac{cd \ln(cx^4+bx^2+a)}{4(ae^2-deb+cd^2)a} - \frac{e^2 \ln(ex^2+d)}{2(ae^2-deb+cd^2)d} + \frac{\ln(x)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] 1/a/d*ln(x)+1/4/(a*e^2-b*d*e+c*d^2)/a*ln(c*x^4+b*x^2+a)*b*e-1/4/(a*e^2-b*d*e+c*d^2)/a*c*ln(c*x^4+b*x^2+a)*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 17.20, size = 6285, normalized size = 37.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] (log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4*(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(4*a*c - b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c - b^2)^5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)^4 - 152*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 200*c^2*d^6

$$\begin{aligned}
& *e^2*(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2 \\
& - 4*a*c)^{(7/2)} - 176*b*c^4*d^8*(b^2 - 4*a*c)^{(7/2)} - 26*a*d^3*e^5*(b^2 - 4* \\
& a*c)^{(11/2)} + 352*a^3*d*e^7*(b^2 - 4*a*c)^{(9/2)} - 319*b*d^4*e^4*(b^2 - 4*a* \\
& c)^{(11/2)} + 148*c*d^5*e^3*(b^2 - 4*a*c)^{(11/2)} + 168*c^3*d^7*e*(b^2 - 4*a*c \\
&)^{(9/2)} - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e^5*(4*a*c - b^ \\
& 2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2 \\
& - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^{(7/2)} - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a \\
& *c)^{(5/2)} + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^3*d^7*e*(4*a*c - \\
& b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*d^7*e*(4*a*c - b^2 \\
&)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c) \\
& ^{(7/2)} + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^{(5/2)} + 40*b^7*c^2*d^6*e^2*(b^2 \\
& - 4*a*c)^{(3/2)} + 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(\\
& b^2 - 4*a*c)^{(7/2)} - 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^ \\
& 2*(b^2 - 4*a*c)^{(5/2)} + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^{(3/2)} - 176*a^2*d* \\
& e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^{(9/2)} + 158*b^5 \\
& *d^3*e^5*x^2*(b^2 - 4*a*c)^{(7/2)} + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^{(5/2)} - \\
& b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^{(3/2)} - 336*c^4*d^7*e*x^2*(4*a*c - b^2)^4 + \\
& 400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^{(9/2)} - 608*a^2*b^2*d^2*e^6*(4*a*c - b^2) \\
& ^4 + 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 - 1096*b^2*c^2*d^6*e^2*(4*a*c - b^ \\
& 2)^4 - 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 + 424*b^6*c^2*d^6*e^2*(4*a*c - b \\
& ^2)^2 - 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 + 256*b^3*c^5*d^8*x^2*(4*a*c - \\
& b^2)^2 + 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 - 410*b^4*d^3*e^5*x^2*(4*a*c - \\
& b^2)^4 - 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 - 17*b^8*d^3*e^5*x^2*(4*a*c - \\
& b^2)^2 + 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 + 336*a*b*d^3*e^5*(4*a*c - b^ \\
& 2)^5 + 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^{(9 \\
& /2)} - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^{(7/2)} + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c \\
&)^{(5/2)} - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^{(3/2)} - 80*a^2*b*d^2*e^6*(b^2 - 4*a \\
& *c)^{(9/2)} - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^{(7/2)} + 96*a^3*b^4*d*e^7*(b^2 - \\
& 4*a*c)^{(5/2)} - 632*b*c*d^5*e^3*(4*a*c - b^2)^5 + 608*b*c^3*d^7*e*(4*a*c - \\
& b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^{(9/2)} + 920*b^2*c*d^5*e^3*(b^2 - 4 \\
& *a*c)^{(9/2)} + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^{(7/2)} - 384*b^4*c*d^5*e^3*(b^ \\
& 2 - 4*a*c)^{(7/2)} - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^{(5/2)} - 664*b^6*c*d^5*e^ \\
& 3*(b^2 - 4*a*c)^{(5/2)} - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^{(3/2)} - 20*b^8*c*d^5 \\
& *e^3*(b^2 - 4*a*c)^{(3/2)} + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^{(11/2)} - 181*b*d^ \\
& 3*e^5*x^2*(b^2 - 4*a*c)^{(11/2)} + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^{(11/2)} + 3 \\
& 68*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^{(9/2)} - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c) \\
& ^{(7/2)} - 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 - 4800*b^3*c^3*d^6*e^2*x^ \\
& 2*(4*a*c - b^2)^3 + 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 + 928*b^5*c^3* \\
& d^6*e^2*x^2*(4*a*c - b^2)^2 - 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 - 32* \\
& a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^{(9/2)} \\
& - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^{(7/2)} - 136*a*b^6*d^2*e^6*x^2*(b^2 - \\
& 4*a*c)^{(5/2)} - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^{(7/2)} + 48*a^2*b^5*d*e^ \\
& 7*x^2*(b^2 - 4*a*c)^{(5/2)} - 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^ \\
& 2*d^5*e^3*x^2*(b^2 - 4*a*c)^{(9/2)} + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^{(9 \\
& /2)} - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^{(5/2)} + 1452*b^4*c*d^4*e^4*x^2*(
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac)^{(7/2)} - 80b^5c^4d^7ex^2(b^2 - 4ac)^{(3/2)} + 408b^6c^4d^4e^4x^2(b^2 - 4ac)^{(5/2)} + 10b^8c^4d^4e^4x^2(b^2 - 4ac)^{(3/2)} - \\
& 640a^2b^3d^2e^6x^2(4ac - b^2)^4 + 96a^2b^2d^2e^7x^2(4ac - b^2)^4 + 416a^2b^5d^2e^6x^2(4ac - b^2)^3 + 16a^2b^4d^2e^7x^2(4ac - \\
& b^2)^3 + 1952b^3c^3d^6e^2x^2(4ac - b^2)^4 + 2216b^3c^3d^4e^4x^2(4 \\
& ac - b^2)^4 + 2720b^2c^4d^7ex^2(4ac - b^2)^3 - 712b^5c^4d^4e^4x^2(4ac - b^2)^3 - 784b^4c^4d^7ex^2(4ac - b^2)^2 + 152b^7c^4d^4 \\
& e^4x^2(4ac - b^2)^2 + 4144b^2c^3d^6e^2x^2(b^2 - 4ac)^{(7/2)} - 4 \\
& 216b^3c^2d^5e^3x^2(b^2 - 4ac)^{(7/2)} + 3056b^4c^3d^6e^2x^2(b^2 \\
& - 4ac)^{(5/2)} - 1864b^5c^2d^5e^3x^2(b^2 - 4ac)^{(5/2)} + 80b^6c^3 \\
& d^6e^2x^2(b^2 - 4ac)^{(3/2)} - 40b^7c^2d^5e^3x^2(b^2 - 4ac)^{(3/ \\
& 2)) * (d((b^2c)/4 - ac^2 + (bc(b^2 - 4ac)^{(1/2}))/4) - (b^3e)/4 - (b^2 \\
& *e(b^2 - 4ac)^{(1/2}))/4 + (ac*e(b^2 - 4ac)^{(1/2}))/2 + a*b*c*e))/(4a^ \\
& 3c^2e^2 - a^2b^2e^2 + 4a^2c^2d^2 + ab^3d^2e - ab^2c^2d^2 - 4a^2b^2c \\
& *d^2e) - (\log(80c^4d^8(4ac - b^2)^4 - 256a^4e^8(4ac - b^2)^4 + 61 \\
& d^4e^4(4ac - b^2)^6 + 160b^3c^4d^8(b^2 - 4ac)^{(5/2)} + 16b^5c^4d^8 \\
& (b^2 - 4ac)^{(3/2)} - 184b^3d^4e^4(b^2 - 4ac)^{(9/2)} + 370b^5d^4 \\
& e^4(b^2 - 4ac)^{(7/2)} + 128b^7d^4e^4(b^2 - 4ac)^{(5/2)} + 5b^9d^4e^4 \\
& (b^2 - 4ac)^{(3/2)} + 128a^3e^8x^2(b^2 - 4ac)^{(9/2)} + 160c^5d^8 \\
& x^2(b^2 - 4ac)^{(7/2)} + 256a^4b^2e^8(4ac - b^2)^3 - 32b^2c^4d^8 \\
& (4ac - b^2)^3 - 112b^4c^4d^8(4ac - b^2)^2 + 144a^2d^2e^6(4ac \\
& - b^2)^5 - 544b^2d^4e^4(4ac - b^2)^5 - 382b^4d^4e^4(4ac - b^2)^ \\
& ^4 + 152b^6d^4e^4(4ac - b^2)^3 - 71b^8d^4e^4(4ac - b^2)^2 - 200 \\
& c^2d^6e^2(4ac - b^2)^5 + 13d^3e^5x^2(4ac - b^2)^6 + 512a^4b^2e^8 \\
& (b^2 - 4ac)^{(7/2)} - 176b^2c^4d^8(b^2 - 4ac)^{(7/2)} - 26a^2d^3e^5(\\
& b^2 - 4ac)^{(11/2)} + 352a^3d^2e^7(b^2 - 4ac)^{(9/2)} - 319b^2d^4e^4(b^ \\
& ^2 - 4ac)^{(11/2)} + 148c^3d^5e^3(b^2 - 4ac)^{(11/2)} + 168c^3d^7e^3(b^2 \\
& - 4ac)^{(9/2)} + 768a^2b^3d^3e^5(4ac - b^2)^4 + 368a^2b^5d^3e^5(4 \\
& ac - b^2)^3 - 128a^3b^3d^2e^7(4ac - b^2)^3 + 32a^2b^7d^3e^5(4ac \\
& - b^2)^2 - 672a^2b^3d^2e^6(b^2 - 4ac)^{(7/2)} - 272a^2b^5d^2e^6(b \\
& ^2 - 4ac)^{(5/2)} - 408b^3c^4d^5e^3(4ac - b^2)^4 - 256b^3c^3d^7e^3(\\
& 4ac - b^2)^3 - 792b^5c^4d^5e^3(4ac - b^2)^3 + 352b^5c^3d^7e^3(4 \\
& ac - b^2)^2 + 248b^7c^4d^5e^3(4ac - b^2)^2 - 328b^3c^2d^6e^2(b^2 \\
& - 4ac)^{(7/2)} + 1064b^5c^2d^6e^2(b^2 - 4ac)^{(5/2)} + 40b^7c^2d^6e^2 \\
& (b^2 - 4ac)^{(3/2)} - 384a^3b^2e^8x^2(4ac - b^2)^4 + 384a^3b^2e^8 \\
& x^2(b^2 - 4ac)^{(7/2)} + 512b^2c^5d^8x^2(4ac - b^2)^3 + 576b^2c^5 \\
& d^8x^2(b^2 - 4ac)^{(5/2)} + 32b^4c^5d^8x^2(b^2 - 4ac)^{(3/2)} + 17 \\
& 6a^2d^2e^7x^2(4ac - b^2)^5 - 800b^3d^3e^5x^2(b^2 - 4ac)^{(9/2)} + \\
& 158b^5d^3e^5x^2(b^2 - 4ac)^{(7/2)} + 56b^7d^3e^5x^2(b^2 - 4ac) \\
& ^{(5/2)} - b^9d^3e^5x^2(b^2 - 4ac)^{(3/2)} + 336c^4d^7ex^2(4ac - b \\
& ^2)^4 + 400c^3d^6e^2x^2(b^2 - 4ac)^{(9/2)} + 608a^2b^2d^2e^6(4ac \\
& - b^2)^4 - 560a^2b^4d^2e^6(4ac - b^2)^3 + 1096b^2c^2d^6e^2(4 \\
& ac - b^2)^4 + 872b^4c^2d^6e^2(4ac - b^2)^3 - 424b^6c^2d^6e^2(4 \\
& ac - b^2)^2 + 128a^3b^3e^8x^2(4ac - b^2)^3 - 256b^3c^5d^8x^2(\\
& 4ac - b^2)^2 - 584b^2d^3e^5x^2(4ac - b^2)^5 + 410b^4d^3e^5x^2
\end{aligned}$$

$$\begin{aligned}
& (4ac - b^2)^4 + 256b^6d^3e^5x^2(4ac - b^2)^3 + 17b^8d^3e^5x^2 \\
& (4ac - b^2)^2 - 296c^2d^5e^3x^2(4ac - b^2)^5 - 336ab^6d^3e^5(4ac - b^2)^5 \\
& - 384a^3b^2de^7(4ac - b^2)^4 - 832ab^2d^3e^5(b^2 - 4ac)^{(9/2)} - 52ab^4d^3e^5(b^2 - 4ac)^{(7/2)} + 144ab^6d^3e^5(b^2 - 4ac)^{(5/2)} \\
& - 2ab^8d^3e^5(b^2 - 4ac)^{(3/2)} - 80a^2b^2d^2e^6(b^2 - 4ac)^{(9/2)} - 192a^3b^2de^7(b^2 - 4ac)^{(7/2)} + 96a^3b^4de^7(b^2 - 4ac)^{(5/2)} \\
& + 632b^2cd^5e^3(4ac - b^2)^5 - 608b^2c^3d^7e^6(4ac - b^2)^4 - 776b^2c^2d^6e^2(b^2 - 4ac)^{(9/2)} + 920b^2c^2d^5e^3(b^2 - 4ac)^{(9/2)} \\
& + 584b^2c^3d^7e^6(b^2 - 4ac)^{(7/2)} - 384b^4c^2d^5e^3(b^2 - 4ac)^{(7/2)} - 712b^4c^3d^7e^6(b^2 - 4ac)^{(5/2)} - 664b^6c^2d^5e^3(b^2 - 4ac)^{(5/2)} \\
& - 40b^6c^3d^7e^6(b^2 - 4ac)^{(3/2)} - 20b^8c^2d^5e^3(b^2 - 4ac)^{(3/2)} + 72a^2d^2e^6x^2(b^2 - 4ac)^{(11/2)} - 181b^2d^3e^5x^2(b^2 - 4ac)^{(11/2)} \\
& + 122c^2d^4e^4x^2(b^2 - 4ac)^{(1/2)} + 368a^2b^2de^7x^2(b^2 - 4ac)^{(9/2)} - 1552b^2c^4d^7e^6x^2(b^2 - 4ac)^{(7/2)} + 3400b^2c^2d^5e^3x^2(4ac - b^2)^4 \\
& + 4800b^3c^3d^6e^2x^2(4ac - b^2)^3 - 3448b^4c^2d^5e^3x^2(4ac - b^2)^3 - 928b^5c^3d^6e^2x^2(4ac - b^2)^2 + 536b^6c^2d^5e^3x^2(4ac - b^2)^2 \\
& + 32ab^2d^2e^6x^2(4ac - b^2)^5 - 344a^2b^2d^2e^6x^2(b^2 - 4ac)^{(9/2)} - 616ab^4d^2e^6x^2(b^2 - 4ac)^{(7/2)} - 136ab^6d^2e^6x^2(b^2 - 4ac)^{(5/2)} \\
& - 160a^2b^3d^2e^7x^2(b^2 - 4ac)^{(7/2)} + 48a^2b^5d^2e^7x^2(b^2 - 4ac)^{(5/2)} + 760b^2cd^4e^4x^2(4ac - b^2)^5 - 1560b^2c^2d^5e^3x^2(b^2 - 4ac)^{(9/2)} \\
& + 1848b^2c^2d^4e^4x^2(b^2 - 4ac)^{(9/2)} - 2208b^3c^4d^7e^6x^2(b^2 - 4ac)^{(5/2)} + 1452b^4c^4d^4e^4x^2(b^2 - 4ac)^{(7/2)} - 80b^5c^4d^7e^6x^2(b^2 - 4ac)^{(3/2)} \\
& + 408b^6c^4d^4e^4x^2(b^2 - 4ac)^{(5/2)} + 10b^8c^4d^4e^4x^2(b^2 - 4ac)^{(3/2)} + 640ab^3d^2e^6x^2(4ac - b^2)^4 - 96a^2b^2d^2e^7x^2(4ac - b^2)^4 \\
& - 416ab^5d^2e^6x^2(4ac - b^2)^3 - 16a^2b^4d^2e^7x^2(4ac - b^2)^3 - 1952b^2c^3d^6e^2x^2(4ac - b^2)^4 - 2216b^3c^2d^4e^4x^2(4ac - b^2)^4 \\
& - 2720b^2c^4d^7e^6x^2(4ac - b^2)^3 + 712b^5c^2d^4e^4x^2(4ac - b^2)^3 + 784b^4c^4d^7e^6x^2(4ac - b^2)^2 - 152b^7c^2d^4e^4x^2(4ac - b^2)^2 \\
& + 4144b^2c^3d^6e^2x^2(b^2 - 4ac)^{(7/2)} - 4216b^3c^2d^5e^3x^2(b^2 - 4ac)^{(7/2)} + 3056b^4c^3d^6e^2x^2(b^2 - 4ac)^{(5/2)} - 1864b^5c^2d^5e^3x^2(b^2 - 4ac)^{(5/2)} \\
& + 80b^6c^3d^6e^2x^2(b^2 - 4ac)^{(3/2)} - 40b^7c^2d^5e^3x^2(b^2 - 4ac)^{(3/2)} * ((b^3e)/4 + d*(ac^2 - (b^2c)/4 + (b*c*(b^2 - 4ac)^(1/2))/4) - (b^2e*(b^2 - 4ac)^(1/2))/4 + (a*c*e*(b^2 - 4ac)^(1/2))/2 - a*b*c*e) / (4a^3c^2e^2 - a^2b^2e^2 + 4a^2c^2d^2 + a*b^3d^2e - a*b^2c^2d^2 - 4a^2b^2c^2d^2e) - (e^2*log(d + e*x^2))/(2c^2d^3 + 2a*d^2e^2 - 2*b*d^2e) + log(x)/(a*d)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

$$3.223 \quad \int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=205

$$\frac{(ace + b^2(-e) + bcd) \log(a + bx^2 + cx^4)}{4a^2(ae^2 - bde + cd^2)} - \frac{(3abce - 2ac^2d + b^3(-e) + b^2cd) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} - \frac{\log(x)(ae + bd)}{a^2d^2} + \frac{1}{2ad^2}$$

Rubi [A] time = 0.47, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{(3abce - 2ac^2d + b^2cd + b^3(-e)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{(ace + b^2(-e) + bcd) \log(a + bx^2 + cx^4)}{4a^2(ae^2 - bde + cd^2)} - \frac{\log(x)(ae + bd)}{a^2d^2} + \frac{e^3 \log(d + ex^2)}{2d^2(ae^2 - bde + cd^2)} - \frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 - b*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (d + ex) (a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx^2} + \frac{-bd - ae}{a^2 d^2 x} + \frac{e^4}{d^2 (cd^2 - bde + ae^2) (d + ex)} + \frac{b^2 cd - ac^2}{a^2} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2adx^2} - \frac{(bd + ae) \log(x)}{a^2 d^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 - bde + ae^2)} + \frac{\text{Subst} \left(\int \frac{b^2 cd - ac^2 d - b^3 e}{2a^2} dx, x, x^2 \right)}{2a^2 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{2adx^2} - \frac{(bd + ae) \log(x)}{a^2 d^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 - bde + ae^2)} + \frac{(bcd - b^2 e + ace) \text{Subst} \left(\int \frac{1}{a} dx, x, x^2 \right)}{4a^2 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{2adx^2} - \frac{(bd + ae) \log(x)}{a^2 d^2} + \frac{e^3 \log(d + ex^2)}{2d^2 (cd^2 - bde + ae^2)} + \frac{(bcd - b^2 e + ace) \log(x)}{4a^2 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{2adx^2} - \frac{(b^2 cd - 2ac^2 d - b^3 e + 3abce) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2a^2 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} - \frac{(bd + ae) \log(x)}{a^2 d^2}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 331, normalized size = 1.61

$$\frac{1}{4} \left(\frac{(b^2 (e\sqrt{b^2 - 4ac} - cd) - bc(d\sqrt{b^2 - 4ac} + 3ac) + ac(2cd - e\sqrt{b^2 - 4ac}) + b^3 e) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{a^2 \sqrt{b^2 - 4ac} (e(bd - ae) - cd^2)} + \frac{(b^2 (e\sqrt{b^2 - 4ac} + cd) + bc(3ac - d\sqrt{b^2 - 4ac}) - ac(e\sqrt{b^2 - 4ac} + 2cd) + b^3 (-e)) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{a^2 \sqrt{b^2 - 4ac} (e(bd - ae) - cd^2)} + \frac{4 \log(x)(ae + bd)}{a^2 d^2} + \frac{2e^3 \log(d + ex^2)}{d^2 e(ac - bd) + cd^3} - \frac{2}{ad^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2 - 4*a*c])*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c])*d + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.96, size = 237, normalized size = 1.16

$$\frac{(bcd - b^2e + ace) \log(cx^4 + bx^2 + a)}{4(a^2cd^2 - a^2bde + a^3e^2)} + \frac{e^4 \log(|x^2e + d|)}{2(cd^4e - bd^3e^2 + ad^2e^3)} + \frac{(b^2cd - 2ac^2d - b^3e + 3abce) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^2cd^2 - a^2bde + a^3e^2)\sqrt{-b^2+4ac}} - \frac{(bd + ae) \log(x^2)}{2a^2d^2} + \frac{bdx^2 + ax^2e - ad}{2a^2d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{4}*(b*c*d - b^2*e + a*c*e)*\log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + \frac{1}{2}*e^4*\log(\text{abs}(x^2*e + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) + \frac{1}{2}*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*\arctan((2*c*x^2 + b)/\text{sqrt}(-b^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*\text{sqrt}(-b^2 + 4*a*c)) - \frac{1}{2}*(b*d + a*e)*\log(x^2)/(a^2*d^2) + \frac{1}{2}*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)$

maple [B] time = 0.02, size = 430, normalized size = 2.10

$$\frac{3bce \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-dcb+cd^2)\sqrt{4ac-b^2}} - \frac{c^2d \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(a^2-dcb+cd^2)\sqrt{4ac-b^2}} - \frac{b^3e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-dcb+cd^2)\sqrt{4ac-b^2}} + \frac{b^2cd \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(a^2-dcb+cd^2)\sqrt{4ac-b^2}} + \frac{ce \ln(cx^4+bx^2+a)}{4(a^2-dcb+cd^2)a} - \frac{b^2e \ln(cx^4+bx^2+a)}{4(a^2-dcb+cd^2)a^2} + \frac{bcd \ln(cx^4+bx^2+a)}{4(a^2-dcb+cd^2)a^2} + \frac{e^3 \ln(e^2x^2+d)}{2(a^2-dcb+cd^2)d^2} - \frac{e \ln(x)}{a^2d^2} - \frac{b \ln(x)}{a^2d} - \frac{1}{2ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] $-1/2/a/d/x^2 - 1/a/d^2*e*\ln(x) - 1/d/a^2*\ln(x)*b + 1/4/(a*e^2-b*d*e+c*d^2)/a*c*\ln(c*x^4+b*x^2+a)*e - 1/4/(a*e^2-b*d*e+c*d^2)/a^2*\ln(c*x^4+b*x^2+a)*b^2*e + 1/4/(a*e^2-b*d*e+c*d^2)/a^2*c*\ln(c*x^4+b*x^2+a)*b*d + 3/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e - 1/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d - 1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e + 1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*\arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*d + 1/2*e^3*\ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 62.95, size = 5368, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out]
$$\left(\log\left(\frac{\begin{aligned} & (4c^2e^2(a^6d^7 - 4a^2b^5e^7 - 4b^2c^5d^7 - 4b^7d^2e^5 + 28a^3b^3c^2e^7 - 48a^4b^2c^2e^7 + 8b^3c^4d^6e + 8b^6c^2d^3e^4 - 16a^2c^5d^5e^2 + 16a^3c^4d^3e^4 - 4b^4c^3d^5e^2 - 4b^5c^2d^4e^3 - 7ab^6d^2e^6 - 20ab^2c^5d^6e + 56a^2b^2c^3d^3e^4 - 76a^2b^3c^2d^2e^5 + 32ab^5c^2d^2e^5 + 46a^2b^4c^2d^2e^6 + 20ab^2c^4d^5e^2 + 6ab^3c^3d^4e^3 - 44ab^4c^2d^3e^4 + 22a^2b^2c^4d^4e^3 + 48a^3b^2c^3d^2e^5 - 75a^3b^2c^2d^2e^6) \end{aligned}}{a^2d^2} \right) + \frac{\begin{aligned} & ((16c^2e^2(a^3b^4e^7 + 16a^5c^2e^7 + b^3c^4d^7 + b^7d^3e^4 - 8a^4b^2c^2e^7 + 2ab^6d^2e^5 + 2a^2b^5d^2e^6 - 4a^2c^5d^6e - 4b^4c^3d^6e - 4b^6c^2d^4e^3 + 20a^3c^4d^4e^3 - 32a^4c^3d^2e^5 + 6b^5c^2d^5e^2 - ab^2c^5d^7 - 52a^2b^2c^3d^4e^3 + 45a^2b^3c^2d^3e^4 + 48a^3b^2c^2d^2e^5 + 11ab^2c^4d^6e - 12ab^5c^2d^3e^4 - 15a^3b^3c^2d^2e^6 + 28a^4b^2c^2d^2e^6 - 27ab^3c^3d^5e^2 + 27ab^4c^2d^4e^3 + 27a^2b^2c^4d^5e^2 - 18a^2b^4c^2d^2e^5 - 52a^3b^2c^3d^3e^4) \end{aligned}}{ad} \right) + \frac{\begin{aligned} & (8c^2e^2x^2(10ac^6d^7 + a^2b^5e^7 + b^2c^5d^7 + b^7d^2e^5 - 11a^3b^3c^2e^7 + 28a^4b^2c^2e^7 - 88a^4c^3d^2e^6 - 6b^3c^4d^6e - 6b^6c^2d^3e^4 + 26a^2c^5d^5e^2 + 88a^3c^4d^3e^4 + 5b^4c^3d^5e^2 + 5b^5c^2d^4e^3 + 12ab^6d^2e^6 - 3ab^2c^5d^6e - 110a^2b^2c^3d^3e^4 + 155a^2b^3c^2d^2e^5 - 28ab^5c^2d^2e^5 - 93a^2b^4c^2d^2e^6 - 10ab^2c^4d^5e^2 - 27ab^3c^3d^4e^3 + 46ab^4c^2d^3e^4 + 15a^2b^2c^4d^4e^3 - 236a^3b^2c^3d^2e^5 + 202a^3b^2c^2d^2e^6) \end{aligned}}{ad} \right) + \frac{\begin{aligned} & (4c^2e^2(ab^2e^3 + b^2c^2d^3 - 4a^2c^2e^3 + b^3d^2e^2 + 4ac^2d^2e - 2b^2c^2d^2e - 3ab^2c^2d^2e) \end{aligned}}{(b^4e + b^3e(b^2 - 4ac))^{1/2}} + 4 \frac{\begin{aligned} & (a^2c^2e - b^3cd + 4ab^2c^2d - 5ab^2c^2e + 2ac^2d(b^2 - 4ac)) \end{aligned}}{(b^2 - 4ac)^{1/2}} - 3 \frac{\begin{aligned} & (ab^2c^2e^2 + a^2b^2d^2e^3 + 4a^2c^2d^3e - 10ac^3d^4x^2 - 12a^3c^2e^2 \end{aligned}}{(b^2 - 4ac)^{1/2}} \right) \end{aligned}$$

$$\begin{aligned}
& 4x^2 + 3a^2b^2e^4x^2 + 3b^2c^2d^4x^2 + 3b^4d^2e^2x^2 + abc^2 \\
& *d^4 - 4a^3c*d*e^3 - 2a*b^2*c*d^3*e - 14a^2*c^2*d^2*e^2*x^2 - 3a^2*b*c \\
& *d^2*e^2 - 4a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + \\
& 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2)/(a^2*(4*a*c - b^2)*(a*e^2 + c \\
& *d^2 - b*d*e))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d \\
& + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2 \\
& - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^ \\
& 2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e \\
& ^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5 \\
& + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d \\
& ^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2 \\
& *c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e \\
& ^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 11 \\
& 7*a^2*b^3*c^2*d*e^6)/(a^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2 \\
& *c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) \\
&) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4 \\
& *a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 - \\
& 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 + \\
& 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a \\
& *b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^ \\
& 2*e^5 - 5*a^2*b^2*c^3*d*e^6)/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 + \\
& c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + \\
& 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c) \\
& ^{(1/2)} - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a \\
& ^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d \\
& ^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^ \\
& 2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - \\
& b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c \\
& e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c \\
& *d*e)) + (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4 \\
& *b^7*d^2*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^ \\
& 6*c*d^3*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - \\
& 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3 \\
& e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20 \\
& *a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b* \\
& c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6)))/(a^2*d^2) + (((\\
& 16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^ \\
& 4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c \\
& ^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^ \\
& 5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e \\
& ^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15* \\
& a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2* \\
& d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^ \\
& 4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7* \\
& d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^
\end{aligned}$$

$$\begin{aligned}
& 4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4 \\
& *c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a \\
& ^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2* \\
& b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^ \\
& 3*e^4 + 15*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^ \\
& 6)/(a*d) + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4 \\
& *a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e - b^3*e*(b^2 - 4*a*c))^ \\
& (1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 \\
& - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2 \\
&))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12 \\
& *a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 \\
& + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - \\
& 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e \\
& ^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(\\
& a*e^2 + c*d^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - \\
& b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) + b^2* \\
& c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b \\
& ^2)*(a*e^2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + \\
& 6*b^7*d*e^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c \\
& *d^2*e^5 + 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8* \\
& b^4*c^3*d^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 9 \\
& 6*a^2*b^2*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2* \\
& c^4*d^4*e^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3 \\
& *e^4 + 117*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2 \\
&) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4* \\
& a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/ \\
& (4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c \\
& ^7*d^7 - 6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c \\
& ^2*e^7 + 6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c* \\
& e^7 + 2*a*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2 \\
& *b*c^4*d^2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(\\
& b^3*e^3 + c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c) \\
& ^1/2) + 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 \\
& - 4*a*c)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/ \\
& 2)))/(4*a^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^ \\
& 3 + c^3*d^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + \\
& 4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e - 2*a*c^2*d*(b^2 - 4*a*c \\
&)^(1/2) + b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4* \\
& (4*a^4*c*e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - \\
& 4*a^3*b*c*d*e)) + (e^3*log(d + e*x^2))/(2*c*d^4 + 2*a*d^2*e^2 - 2*b*d^3*e) \\
& - 1/(2*a*d*x^2) - (log(x)*(a*e + b*d))/(a^2*d^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

$$3.224 \quad \int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=268

$$\frac{\log(x) (abde - a(cd^2 - ae^2) + b^2d^2)}{a^3d^3} - \frac{(2abce - ac^2d + b^3(-e) + b^2cd) \log(a + bx^2 + cx^4)}{4a^3(ae^2 - bde + cd^2)} + \frac{ae + bd}{2a^2d^2x^2} + \frac{(-2a^2c^2e + \dots)}{4ad^4}$$

Rubi [A] time = 0.60, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1251, 893, 634, 618, 206, 628}

$$\frac{(2abce - ac^2d + b^2cd + b^3(-e)) \log(a + bx^2 + cx^4)}{4a^3(ae^2 - bde + cd^2)} + \frac{(-2a^2c^2e + 4ab^2ce - 3abc^2d + b^3cd + b^4(-e)) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}(ae^2 - bde + cd^2)} + \frac{\log(x) (abde - a(cd^2 - ae^2) + b^2d^2)}{a^3d^3} + \frac{ae + bd}{2a^2d^2x^2} - \frac{e^4 \log(d + ex^2)}{2d^3(ae^2 - bde + cd^2)} - \frac{1}{4ad^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e - a*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 - b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 (d + ex^2) (a + bx^2 + cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 (d + ex) (a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{adx^3} + \frac{-bd - ae}{a^2 d^2 x^2} + \frac{b^2 d^2 + abde - a(cd^2 - ae^2)}{a^3 d^3 x} - \frac{1}{d^3 (cd^2 - bde + ae^2)} \right) dx, x, x^2 \right) \\
&= -\frac{1}{4adx^4} + \frac{bd + ae}{2a^2 d^2 x^2} + \frac{(b^2 d^2 + abde - a(cd^2 - ae^2)) \log(x)}{a^3 d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{4adx^4} + \frac{bd + ae}{2a^2 d^2 x^2} + \frac{(b^2 d^2 + abde - a(cd^2 - ae^2)) \log(x)}{a^3 d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{4adx^4} + \frac{bd + ae}{2a^2 d^2 x^2} + \frac{(b^2 d^2 + abde - a(cd^2 - ae^2)) \log(x)}{a^3 d^3} - \frac{e^4 \log(d + ex^2)}{2d^3 (cd^2 - bde + ae^2)} \\
&= -\frac{1}{4adx^4} + \frac{bd + ae}{2a^2 d^2 x^2} + \frac{(b^3 cd - 3abc^2 d - b^4 e + 4ab^2 ce - 2a^2 c^2 e) \tanh^{-1} \left(\frac{b+2ex}{\sqrt{b^2 - 4ac}} \right)}{2a^3 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 426, normalized size = 1.59

$$\frac{1}{2} \left(\frac{4 \log(x) (abd + a(a^2 - cd) + b^2 d^2)}{a^3 d^3} - \frac{(ac^2 (\sqrt{b^2 - 4ac} + 2a) - b^2 (d \sqrt{b^2 - 4ac} + 4a) + abc (2d - 2\sqrt{b^2 - 4ac}) + b^2 (\sqrt{b^2 - 4ac} - cd) + b^2 x) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{a^3 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} - \frac{(ac^2 (\sqrt{b^2 - 4ac} - 2a) + b^2 (4a - d \sqrt{b^2 - 4ac}) - abc (2\sqrt{b^2 - 4ac} + 3a) + b^2 (\sqrt{b^2 - 4ac} + cd) + b^2 (-c) \log(\sqrt{b^2 - 4ac} + b + 2cx^2))}{a^3 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} + \frac{2(ac + bd)}{a^2 b^2 d^2} - \frac{2e^4 \log(d + ex^2)}{d^3 \sqrt{b^2 - 4ac} (cd^2 - bde + ae^2)} - \frac{1}{2a^3 d^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] $(-1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a*(-(c*d^2) + a*e^2))*\text{Log}[x])/(a^3*d^3) - ((b^4*e + a*c^2*(\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(\text{Sqrt}[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*\text{Sqrt}[b^2 - 4*a*c]*e) + b^3*(-(c*d) + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{Log}[b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2])/(a^3*\text{Sqrt}[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - (((-b^4*e) + a*c^2*(\text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(\text{Sqrt}[b^2 - 4*a*c]*d + 4*a*e) + b^3*(c*d + \text{Sqrt}[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*\text{Sqrt}[b^2 - 4*a*c]*e))*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2])/(a^3*\text{Sqrt}[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - (2*e^4*\text{Log}[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*e)))/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (d + ex^2) (a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.46, size = 332, normalized size = 1.24

$$\frac{(b^2cd - ac^2d - b^2e + 2abce) \log(cx^4 + bx^2 + a)}{4(a^3cd^2 - a^2bde + a^2e^2)} - \frac{e^5 \log(|x^2e + d|)}{2(cd^2e - bd^2e^2 + ad^3e^2)} - \frac{(b^3cd - 3abc^2d - b^4e + 4ad^2ce - 2a^2c^2e) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(a^3cd^2 - a^2bde + a^2e^2)\sqrt{-b^2 + 4ac}} + \frac{(b^2d^2 - acd^2 + abde + a^2e^2) \log(x^2)}{2a^3d^3} - \frac{3b^2d^2x^4 - 3acd^2x^4 + 3abd^2x^4 + 3a^2x^4e^2 - 2abd^2x^2 - 2a^2dx^2e + a^2d^2}{4a^3d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*\log(c*x^4 + b*x^2 + a)/(a^3*c*d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*\log(\text{abs}(x^2*e + d))/(c*d^5*e - b*d^4*e^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*\sqrt{-b^2 + 4*a*c}) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*\log(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*a^2*x^4*e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)$$

maple [B] time = 0.02, size = 584, normalized size = 2.18

$$\frac{c^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(e^2 - db + c^2)\sqrt{4ac - b^2}e} - \frac{2b^2c \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(e^2 - db + c^2)\sqrt{4ac - b^2}e^2} - \frac{3b^2d \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(e^2 - db + c^2)\sqrt{4ac - b^2}e^3} - \frac{b^4 \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(e^2 - db + c^2)\sqrt{4ac - b^2}e^4} - \frac{b^2d \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(e^2 - db + c^2)\sqrt{4ac - b^2}e^5} - \frac{bc \ln(cx^4 + bx^2 + a)}{2(e^2 - db + c^2)e^2} - \frac{c^2d \ln(cx^4 + bx^2 + a)}{4(e^2 - db + c^2)e^3} - \frac{b^2e \ln(cx^4 + bx^2 + a)}{4(e^2 - db + c^2)e^4} - \frac{b^2cd \ln(cx^4 + bx^2 + a)}{4(e^2 - db + c^2)e^5} - \frac{e^4 \ln(e^2 + d)}{e^6d^2} - \frac{c^2 \ln(c)}{e^6d^2} - \frac{bc \ln(b)}{e^6d^2} - \frac{c \ln(d)}{e^6d^2} - \frac{b^2 \ln(d)}{e^6d^2} - \frac{c}{2e^6d^2} - \frac{b}{2e^6d^2} - \frac{1}{4e^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out]
$$-1/4/a/d/x^4 + 1/2/a/d^2*e/x^2 + 1/2/d/a^2/x^2*b + 1/a/d^3*e^2*\ln(x) + 1/d^2/a^2*\ln(x)*b*e - 1/a^2*c/d*\ln(x) + 1/d/a^3*\ln(x)*b^2 - 1/2/(a*e^2 - b*d*e + c*d^2)/a^2*c*\ln(c*x^4 + b*x^2 + a)*b*e + 1/4/(a*e^2 - b*d*e + c*d^2)/a^2*c^2*\ln(c*x^4 + b*x^2 + a)*d + 1/4/(a*e^2 - b*d*e + c*d^2)/a^3*\ln(c*x^4 + b*x^2 + a)*b^3*e - 1/4/(a*e^2 - b*d*e + c*d^2)/a^3*c*\ln(c*x^4 + b*x^2 + a)*b^2*d + 1/(a*e^2 - b*d*e + c*d^2)/a/(4*a*c - b^2)^(1/2)*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^(1/2))*e*c^2 - 2/(a*e^2 - b*d*e + c*d^2)/a^2/(4*a*c - b^2)^(1/2)*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^(1/2))*b^2*c*e + 3/2/(a*e^2 - b*d*e + c*d^2)$$

$$\frac{1}{a^2(4ac-b^2)^{1/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) * db * c^2 + \frac{1}{2} \frac{1}{(ae^2-bde+cd^2)^{1/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) * b^4 * e - \frac{1}{2} \frac{1}{(ae^2-bde+cd^2)^{1/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) * b^3 * c * d - \frac{1}{2} * e^4 * \ln(ex^2+d) / d^3 / (ae^2-bde+cd^2)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 144.76, size = 10300, normalized size = 38.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out]
$$\frac{\log\left(\frac{(c^8e^8(a^2e^2 + b^2d^2 - acd^2 + abde))}{(a^6d^6)} - \frac{(c^9e^9 * x^2)}{(a^5d^5)} - \left(\frac{(c^5e^5(4a^3b^3e^6 + 4b^3c^3d^6 + 4b^6d^3e^3 + 8ab^5d^2e^4 + 8a^2b^4de^5 + 4a^2c^4d^5e + 16a^4c^2de^5 - 19a^3c^3d^3e^3 - 4abc^4d^6 - 12a^4b^2c^2d^3e^3 - 24a^3b^4cd^3e^3 - 32a^3b^2c^2de^5 - 36a^2b^3c^2d^2e^4 + 28a^3b^3c^2d^2e^4))}{(a^6d^6)} - \left(\frac{(4a^4b^6c^2e^{12} - 24a^5b^4c^3e^{12} + 36a^6b^2c^4e^{12} - 4a^3c^9d^8e^4 + 64a^4c^8d^6e^6 - 144a^5c^7d^4e^8 + 96a^6c^6d^2e^{10} + 4b^4c^8d^{10}e^2 + 8b^7c^5d^7e^5 + 4b^{10}c^2d^4e^8 + 64a^2b^3c^7d^7e^5 - 8a^2b^4c^6d^6e^6 - 8a^2b^5c^5d^5e^7 + 172a^2b^6c^4d^4e^8 - 112a^2b^7c^3d^3e^9 + 16a^2b^8c^2d^2e^{10} - 72a^3b^2c^7d^6e^6 + 56a^3b^3c^6d^5e^7 - 312a^3b^4c^5d^4e^8 + 348a^3b^5c^4d^3e^9 - 132a^3b^6c^3d^2e^{10} + 324a^4b^2c^6d^4e^8 - 428a^4b^3c^5d^3e^9 + 344a^4b^4c^4d^2e^{10} - 300a^5b^2c^5d^2e^{10} - 96a^6b^2c^5d^2e^{11} - 4ab^2c^9d^{10}e^2 - 4ab^3c^8d^9e^3 - 48ab^5c^6d^7e^5 + 8ab^6c^5d^6e^6 - 44ab^8c^3d^4e^8 + 12ab^9c^2d^3e^9 + 8a^2b^3c^9d^9e^3 - 24a^3b^3c^8d^7e^5 + 12a^3b^7c^2d^5e^{11} - 88a^4b^3c^7d^5e^7 - 88a^4b^5c^3d^5e^{11} + 228a^5b^3c^6d^3e^9 + 188a^5b^3c^4d^5e^{11})}{(a^6d^6)} + (x^2 * (32a^6c^6d^5e^{11} - 24a^6b^3c^5e^{12} + 4a^3b^7c^2e^{12} - 28a^4b^5c^3e^{12} + 56a^5b^3c^4e^{12} + 2a^3c^9d^7e^5 + 104a^4c^8d^5e^7 - 156a^5c^7d^3e^9 + 4b^3c^9d^{10}e^2 + 4b^6c^6d^7e^5 + 4b^7c^5d^6e^6 + 4b^{10}c^2d^3e^9 + 8a^2b^2c^8d^7e^5 + 40a^2b^3c^7d^6e^6 - 12a$$

$$\begin{aligned}
& 2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^{10} - 9 \\
& 2*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 + \\
& 388*a^3*b^5*c^4*d^2*e^{10} + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^{10} \\
& - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4* \\
& a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^{10} + 8*a^2*b* \\
& c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^{11} - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^ \\
& 3*d*e^{11} - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^{11} + 264*a^5*b*c^6*d \\
& ^2*e^{10} - 224*a^5*b^2*c^5*d*e^{11}))/ (a^6*d^6) + (((192*a^6*b*c^4*e^{11} - 256* \\
& a^6*c^5*d*e^{10} + 16*a^4*b^5*c^2*e^{11} - 112*a^5*b^3*c^3*e^{11} + 60*a^3*c^8*d^ \\
& 7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32 \\
& *b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5 \\
& *e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^ \\
& 5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3 \\
& *e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5* \\
& d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2 \\
& *c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c \\
& ^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^ \\
& 5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 \\
& - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^{10} + 704*a^4*b*c^6*d^4*e^7 - 3 \\
& 28*a^4*b^4*c^3*d*e^{10} - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^{10}))/ (a^ \\
& 4*d^4) + (((256*a^6*c^4*e^{10} + 16*a^4*b^4*c^2*e^{10} - 128*a^5*b^2*c^3*e^{10} - \\
& 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c \\
& ^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + \\
& 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + \\
& 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 \\
& - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2 \\
& *e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 \\
& + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - \\
& 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848* \\
& a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4 \\
& *b^3*c^3*d*e^9))/ (a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8 \\
& *d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^ \\
& 6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d \\
& ^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b \\
& ^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^ \\
& 4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2 \\
& *d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 2 \\
& 7*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2* \\
& b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3 \\
& *d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/ (a^2*d^2) - (4*c^2*e^2*(a*b^2*e^3 + b*c^ \\
& 2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d \\
& *e^2)*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3* \\
& c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c \\
& ^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b \\
& ^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*
\end{aligned}$$

$$\begin{aligned}
& c^3d^4x^2 - 12a^3c^2e^4x^2 + 3a^2b^2e^4x^2 + 3b^2c^2d^4x^2 + 3b^4d^2e^2x^2 + abc^2d^4 - 4a^3c^2de^3 - 2ab^2c^2d^3e - 14a^2c^2d^2e^2x^2 - 3a^2b^2c^2d^2e^2 - 4a^2b^3d^2e^3x^2 - 6b^3c^2d^3e^2x^2 - 8a^2b^2c^2d^2e^2x^2 + 22abc^2d^3e^2x^2 + 16a^2b^2c^2d^3e^2x^2) / (a^3(4ac - b^2)(ae^2 + cd^2 - bde)) * (b^4e(b^2 - 4ac)^{1/2} - b^5e + 4a^2c^3d + b^4cd + 6ab^3ce - b^3cd(b^2 - 4ac)^{1/2} - 5ab^2c^2d - 8a^2b^2c^2e + 2a^2c^2e(b^2 - 4ac)^{1/2} + 3ab^2c^2d(b^2 - 4ac)^{1/2} - 4ab^2c^2e(b^2 - 4ac)^{1/2})) / (4a^3(4ac - b^2)(ae^2 + cd^2 - bde)) + (4c^2e^2x^2(6a^3b^6e^9 - 16a^6c^3e^9 + 6b^3c^6d^9 + 6b^9d^3e^6 - 44a^4b^4c^2e^9 + 13ab^8d^2e^7 + 13a^2b^7d^2e^8 + 5a^2c^7d^8e - 8b^4c^5d^8e - 8b^8c^2d^4e^5 + 84a^5b^2c^2e^9 + 2a^3c^6d^6e^3 - 160a^4c^5d^4e^5 + 124a^5c^4d^2e^7 - 2b^5c^4d^7e^2 + 8b^6c^3d^6e^3 - 2b^7c^2d^5e^4 - 5abc^7d^9 + 40a^2b^2c^5d^6e^3 - 45a^2b^3c^4d^5e^4 - 220a^2b^4c^3d^4e^5 + 316a^2b^5c^2d^3e^6 + 264a^3b^2c^4d^4e^5 - 546a^3b^3c^3d^3e^6 + 388a^3b^4c^2d^2e^7 - 447a^4b^2c^3d^2e^7 + 12ab^2c^6d^8e - 74ab^7c^2d^3e^6 - 111a^3b^5c^2d^2e^8 - 210a^5b^2c^3d^2e^8 + 18ab^3c^5d^7e^2 - 43ab^4c^4d^6e^3 + 19ab^5c^3d^5e^4 + 72ab^6c^2d^4e^5 - 20a^2b^2c^6d^7e^2 - 123a^2b^6c^2d^2e^7 + 31a^3b^2c^5d^5e^4 + 328a^4b^2c^4d^3e^6 + 290a^4b^3c^2d^2e^8)) / (a^4d^4) * (b^4e(b^2 - 4ac)^{1/2} - b^5e + 4a^2c^3d + b^4cd + 6ab^3ce - b^3cd(b^2 - 4ac)^{1/2} - 5ab^2c^2d - 8a^2b^2c^2e + 2a^2c^2e(b^2 - 4ac)^{1/2} + 3ab^2c^2d(b^2 - 4ac)^{1/2} - 4ab^2c^2e(b^2 - 4ac)^{1/2})) / (4a^3(4ac - b^2)(ae^2 + cd^2 - bde)) * (b^4e(b^2 - 4ac)^{1/2} - b^5e + 4a^2c^3d + b^4cd + 6ab^3ce - b^3cd(b^2 - 4ac)^{1/2} - 5ab^2c^2d - 8a^2b^2c^2e + 2a^2c^2e(b^2 - 4ac)^{1/2} + 3ab^2c^2d(b^2 - 4ac)^{1/2} - 4ab^2c^2e(b^2 - 4ac)^{1/2})) / (4a^3(4ac - b^2)(ae^2 + cd^2 - bde)) + (2c^5e^5x^2(a^2b^4e^6 + 2a^4c^2e^6 + b^2c^4d^6 + b^6d^2e^4 - 4a^3b^2c^2e^6 + 3a^2c^4d^4e^2 - 10a^3c^3d^2e^4 + 2ab^5d^5e - abc^4d^5e + 16a^2b^2c^2d^2e^4 - 7ab^4c^2d^2e^4 - 11a^2b^3c^2d^2e^5 + 13a^3b^2c^2d^2e^5)) / (a^6d^6) * (b^4e(b^2 - 4ac)^{1/2} - b^5e + 4a^2c^3d + b^4cd + 6ab^3ce - b^3cd(b^2 - 4ac)^{1/2} - 5ab^2c^2d - 8a^2b^2c^2e + 2a^2c^2e(b^2 - 4ac)^{1/2} + 3ab^2c^2d(b^2 - 4ac)^{1/2} - 4ab^2c^2e(b^2 - 4ac)^{1/2})) / (4(4a^5c^2e^2 - a^4b^2e^2 + 4a^4c^2d^2 - a^3b^2c^2d^2 + a^3b^3d^2e - 4a^4b^2c^2d^2e)) - (log((c^8e^8(a^2e^2 + b^2d^2 - acd^2 + abde)) / (a^6d^6) - (c^9e^9x^2) / (a^5d^5) + (((((4a^4b^6c^2e^12 - 24a^5b^4c^3e^12 + 36a^6b^2c^4e^12 - 4a^3c^9d^8e^4 + 64a^4c^8d^6e^6 - 144a^5c^7d^4e^8 + 96a^6c^6d^2e^10 + 4b^4c^8d^10e^2 + 8b^7c^5d^7e^5 + 4b^10c^2d^4e^8 + 64a^2b^3c^7d^7e^5 - 8a^2b^4c^6d^6e^6 - 8a^2b^5c^5d^5e^7 + 172a^2b^6c^4d^4e^8 - 112a^2b^7c^3
\end{aligned}$$

$$\begin{aligned}
& d^3e^9 + 16a^2b^8c^2d^2e^{10} - 72a^3b^2c^7d^6e^6 + 56a^3b^3c^6d^5e^7 - 312a^3b^4c^5d^4e^8 + 348a^3b^5c^4d^3e^9 - 132a^3b^6c^3d^2e^{10} + 324a^4b^2c^6d^4e^8 - 428a^4b^3c^5d^3e^9 + 344a^4b^4c^4d^2e^{10} - 300a^5b^2c^5d^2e^{10} - 96a^6b^6c^5d^6e^{11} - 4a^*b^2c^9d^{10}e^2 - 4a^*b^3c^8d^9e^3 - 48a^*b^5c^6d^7e^5 + 8a^*b^6c^5d^6e^6 - 44a^*b^8c^3d^4e^8 + 12a^*b^9c^2d^3e^9 + 8a^2b^*c^9d^9e^3 - 24a^3b^*c^8d^7e^5 + 12a^3b^7c^2d^*e^{11} - 88a^4b^*c^7d^5e^7 - 88a^4b^5c^3d^*e^{11} + 228a^5b^*c^6d^3e^9 + 188a^5b^3c^4d^*e^{11})/(a^6d^6) + (x^2(32a^6c^6d^*e^{11} - 24a^6b^*c^5e^{12} + 4a^3b^7c^2e^{12} - 28a^4b^5c^3e^{12} + 56a^5b^3c^4e^{12} + 2a^3c^9d^7e^5 + 104a^4c^8d^5e^7 - 156a^5c^7d^3e^9 + 4b^3c^9d^{10}e^2 + 4b^6c^6d^7e^5 + 4b^7c^5d^6e^6 + 4b^{10}c^2d^3e^9 + 8a^2b^2c^8d^7e^5 + 40a^2b^3c^7d^6e^6 - 12a^2b^5c^5d^4e^8 + 180a^2b^6c^4d^3e^9 - 116a^2b^7c^3d^2e^{10} - 92a^3b^2c^7d^5e^7 + 84a^3b^3c^6d^4e^8 - 350a^3b^4c^5d^3e^9 + 388a^3b^5c^4d^2e^{10} + 348a^4b^2c^6d^3e^9 - 524a^4b^3c^5d^2e^{10} - 4a^*b^2c^9d^9e^3 - 20a^*b^4c^7d^7e^5 - 20a^*b^5c^6d^6e^6 + 4a^*b^6c^5d^5e^7 - 44a^*b^8c^3d^3e^9 + 12a^*b^9c^2d^2e^{10} + 8a^2b^*c^9d^8e^4 + 12a^2b^8c^2d^*e^{11} - 36a^3b^*c^8d^6e^6 - 100a^3b^6c^3d^*e^{11} - 132a^4b^*c^7d^4e^8 + 264a^4b^4c^4d^*e^{11} + 264a^5b^*c^6d^2e^{10} - 224a^5b^2c^5d^*e^{11}))/((192a^6b^*c^4e^{11} - 256a^6c^5d^*e^{10} + 16a^4b^5c^2e^{11} - 112a^5b^3c^3e^{11} + 60a^3c^8d^7e^4 - 320a^4c^7d^5e^6 + 480a^5c^6d^3e^8 + 16b^4c^7d^9e^2 - 32b^5c^6d^8e^3 + 16b^6c^5d^7e^4 + 16b^7c^4d^6e^5 - 32b^8c^3d^5e^6 + 16b^9c^2d^4e^7 + 16a^2b^2c^7d^7e^4 + 120a^2b^3c^6d^6e^5 - 816a^2b^4c^5d^5e^6 + 880a^2b^5c^4d^4e^7 - 424a^2b^6c^3d^3e^8 + 56a^2b^7c^2d^2e^9 + 832a^3b^2c^6d^5e^6 - 1424a^3b^3c^5d^4e^7 + 1340a^3b^4c^4d^3e^8 - 464a^3b^5c^3d^2e^9 - 1512a^4b^2c^5d^3e^8 + 1144a^4b^3c^4d^2e^9 - 20a^*b^2c^8d^9e^2 + 96a^*b^3c^7d^8e^3 - 64a^*b^4c^6d^7e^4 - 88a^*b^5c^5d^6e^5 + 288a^*b^6c^4d^5e^6 - 208a^*b^7c^3d^4e^7 + 44a^*b^8c^2d^3e^8 - 40a^2b^*c^8d^8e^3 - 88a^3b^*c^7d^6e^5 + 44a^3b^6c^2d^*e^{10} + 704a^4b^*c^6d^4e^7 - 328a^4b^4c^3d^*e^{10} - 736a^5b^*c^5d^2e^9 + 684a^5b^2c^4d^*e^{10}))/((256a^6c^4e^{10} + 16a^4b^4c^2e^{10} - 128a^5b^2c^3e^{10} - 192a^3c^7d^6e^4 + 448a^4c^6d^4e^6 - 512a^5c^5d^2e^8 + 16b^4c^6d^8e^2 - 64b^5c^5d^7e^3 + 96b^6c^4d^6e^4 - 64b^7c^3d^5e^5 + 16b^8c^2d^4e^6 + 768a^2b^2c^6d^6e^4 - 1200a^2b^3c^5d^5e^5 + 896a^2b^4c^4d^4e^6 - 320a^2b^5c^3d^3e^7 + 32a^2b^6c^2d^2e^8 - 1392a^3b^2c^5d^4e^6 + 1024a^3b^3c^4d^3e^7 - 288a^3b^4c^3d^2e^8 + 768a^4b^2c^4d^2e^8 + 448a^5b^*c^4d^*e^9 - 32a^*b^2c^7d^8e^2 + 240a^*b^3c^6d^7e^3 - 528a^*b^4c^5d^6e^4 + 496a^*b^5c^4d^5e^5 - 208a^*b^6c^3d^4e^6 + 32a^*b^7c^2d^3e^7 - 176a^2b^*c^7d^7e^3 + 848a^3b^*c^6d^5e^5 + 32a^3b^5c^2d^*e^9 - 1024a^4b^*c^5d^3e^7 - 240a^4b^3c^3d^*e^9)/(a^2d^2) + (8c^2e^2x^2(a^3b^5e^8 + b^3c^5d^8 + b^8d^3e^5 - 11a^4b^3c^*e^8 + 28a^5b^*c^2e^8 + 8a^*b^7d^2e^6 + 8a^2b^6d^*e^7 - 30a^2c^6d^7e - 24a^5c^3d^*e^7 - 6b^4c^4d
\end{aligned}$$

$$\begin{aligned}
& ^7e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 8 \\
& 2*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3 \\
& *b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 \\
& - 352*a^4*b*c^3*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7)/(a^2*d^2) + (4*c^2*e^2*(\\
& a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4 \\
& *c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) \\
& - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 \\
& + 3*b^4*d^2*e^2*x^2 + a*b*c^2*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 + 22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(6*a^3*b^6*e^9 - 16*a^6*c^3*e^9 + 6*b^3*c^6*d^9 + 6*b^9*d^3*e^6 - 44*a^4*b^4*c*e^9 + 13*a*b^8*d^2*e^7 + 13*a^2*b^7*d*e^8 + 5*a^2*c^7*d^8*e - 8*b^4*c^5*d^8*e - 8*b^8*c*d^4*e^5 + 84*a^5*b^2*c^2*e^9 + 2*a^3*c^6*d^6*e^3 - 160*a^4*c^5*d^4*e^5 + 124*a^5*c^4*d^2*e^7 - 2*b^5*c^4*d^7*e^2 + 8*b^6*c^3*d^6*e^3 - 2*b^7*c^2*d^5*e^4 - 5*a*b*c^7*d^9 + 40*a^2*b^2*c^5*d^6*e^3 - 45*a^2*b^3*c^4*d^5*e^4 - 220*a^2*b^4*c^3*d^4*e^5 + 316*a^2*b^5*c^2*d^3*e^6 + 264*a^3*b^2*c^4*d^4*e^5 - 546*a^3*b^3*c^3*d^3*e^6 + 388*a^3*b^4*c^2*d^2*e^7 - 447*a^4*b^2*c^3*d^2*e^7 + 12*a*b^2*c^6*d^8*e - 74*a*b^7*c*d^3*e^6 - 111*a^3*b^5*c*d*e^8 - 210*a^5*b*c^3*d*e^8 + 18*a*b^3*c^5*d^7*e^2 - 43*a*b^4*c^4*d^6*e^3 + 19*a*b^5*c^3*d^5*e^4 + 72*a*b^6*c^2*d^4*e^5 - 20*a^2*b*c^6*d^7*e^2 - 123*a^2*b^6*c*d^2*e^7 + 31*a^3*b*c^5*d^5*e^4 + 328*a^4*b*c^4*d^3*e^6 + 290*a^4*b^3*c^2*d*e^8))/(a^4*d^4)*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 - 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^3*b*c^2*d^2*e^4))/(a^6*d^6) + (2*c^5*e^5*x^2*(a^2*b^4*e^6 + 2*a^4*c^2*e^6 + b^2*c^4*d^6 + b^6*d^2*e^4 - 4*a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 2*c*e^6 + 3*a^2*c^4*d^4*e^2 - 10*a^3*c^3*d^2*e^4 + 2*a*b^5*d*e^5 - a*b*c^4*d^5*e + 16*a^2*b^2*c^2*d^2*e^4 - 7*a*b^4*c*d^2*e^4 - 11*a^2*b^3*c*d*e^5 + 13*a^3*b*c^2*d*e^5)/(a^6*d^6))*(b^5*e + b^4*e*(b^2 - 4*a*c)^{(1/2)} - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c^2*d*(b^2 - 4*a*c)^{(1/2)} - 4*a*b^2*c*e*(b^2 - 4*a*c)^{(1/2)))/(4*a^3*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e))*(b^5*e + b^4*e*(b^2 - 4*a*c)^{(1/2)} - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e - b^3*c*d*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^2*c^2*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c^2*d*(b^2 - 4*a*c)^{(1/2)} - 4*a*b^2*c*e*(b^2 - 4*a*c)^{(1/2)))/(4*(4*a^5*c*e^2 - a^4*b^2*e^2 + 4*a^4*c^2*d^2 - a^3*b^2*c*d^2 + a^3*b^3*d*e - 4*a^4*b*c*d*e)) - (1/(4*a*d) - (x^2*(a*e + b*d))/(2*a^2*d^2))/x^4 - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2 - b*d^4*e)) + (log(x)*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^3*d^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.225 \quad \int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=387

$$\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}} + a^2ce - ab^2e - 2abcd + b^3d \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}} + a^2ce - ab^2e - 2abcd + b^3d \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}(ae^2 - bde + cd^2) + \sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}(ae^2 - bde + cd^2)}$$

Rubi [A] time = 4.03, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}} + a^2ce - ab^2e - 2abcd + b^3d \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}} + a^2ce - ab^2e - 2abcd + b^3d \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}} \right) + \frac{d^{7/2} \tan^{-1} \left(\frac{\sqrt{cx}}{\sqrt{a}} \right) - \frac{x(be+cd)}{c^2e^2} + \frac{x^3}{3ce}}{e^{5/2}(ae^2 - bde + cd^2)}}$$

Antiderivative was successfully verified.

[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287


```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)} dx &= \int \left(\frac{-cd-be}{c^2e^2} + \frac{x^2}{ce} + \frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex^2)} + \frac{-a(b^2d-acd-abe)}{c^2(cd^2-bde+ae^2)} \right) dx \\ &= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{\int \frac{-a(b^2d-acd-abe)+(-b^3d+2abcd+ab^2e-a^2ce)x^2}{a+bx^2+cx^4} dx}{c^2(cd^2-bde+ae^2)} + \frac{d^4 \int \frac{1}{d+ex^2} dx}{e^2(cd^2-bde+ae^2)} \\ &= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{d^{7/2} \tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{5/2}(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}})}{2c^2} \\ &= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} - \frac{(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}})}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.61, size = 463, normalized size = 1.20

$$\frac{\left(\sqrt{c}\left(\sqrt{b^2-4ac}-2cd\right)+ab^2\left(4cd-\sqrt{b^2-4ac}\right)-abc\left(2d\sqrt{b^2-4ac}+3ae\right)+b^3\left(d\sqrt{b^2-4ac}+ae\right)+b^4(-d)\right)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\left(\sqrt{c}\left(\sqrt{b^2-4ac}+2cd\right)-ab^2\left(\sqrt{b^2-4ac}+4cd\right)+abc\left(3ae-2d\sqrt{b^2-4ac}\right)+b^3\left(d\sqrt{b^2-4ac}-ae\right)+b^4d\right)\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}+\frac{x(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}})}{2c^2}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(\left(\alpha d-ac\right)-cd\right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2
- 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```


$$\begin{aligned}
& *c)*c)*a*b^5*c^7 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*b^6*c^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a \\
& ^3*b^2*c^8 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^ \\
& 2*b^3*c^8 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b \\
& ^4*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^2* \\
& c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c \\
&)*a^2*b^2*c^9)*d^4*e - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^6*c^4 \\
& - 9*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^4*c^5 - 2*\sqrt{2}*\sqrt{b \\
& *c - \sqrt{b^2 - 4*a*c}})*a*b^5*c^5 + 2*a*b^6*c^5 + 24*\sqrt{2}*\sqrt{b*c - s \\
& \sqrt{b^2 - 4*a*c}})*a^3*b^2*c^6 + 10*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*c \\
&)*a^2*b^3*c^6 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^4*c^6 - 18*a^2* \\
& b^4*c^6 - 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^4*c^7 - 8*\sqrt{2})*sq \\
& rt(b*c - \sqrt{b^2 - 4*a*c})*a^3*b*c^7 - 5*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4 \\
& *a*c}})*a^2*b^2*c^7 + 48*a^3*b^2*c^7 + 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a \\
& *c}})*a^3*c^8 - 32*a^4*c^8 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 10*(b^2 - 4*a*c)* \\
& a^2*b^2*c^6 - 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2 \\
& *e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^ \\
& 4*b*c^10 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^9*c^ \\
& 4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^7*c^5 + \\
& 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^8*c^5 + 12*s \\
& \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^5*c^6 - \sqrt{ \\
& 2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^7*c^6 - 52*\sqrt{2})* \\
& \sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^3*b^3*c^7 - 24*\sqrt{2})* \\
& \sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^4*c^7 + 16*\sqrt{2})* \\
& \sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^4*b*c^8 + 8*\sqrt{2})*sq \\
& rt(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^3*b^2*c^8 + 12*\sqrt{2})*sq \\
& rt(b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^3*c^8 - 4*\sqrt{2})*\sqrt{ \\
& (b^2 - 4*a*c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b \\
& ^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2 \\
& + 2*(\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^7*c^3 - 8*\sqrt{2})*\sqrt{b*c \\
& - \sqrt{b^2 - 4*a*c}})*a^2*b^5*c^4 - 2*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
&)*a*b^6*c^4 + 2*a*b^7*c^4 + 16*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a \\
& ^3*b^3*c^5 + 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^4*c^5 + \sqrt{2} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^5*c^5 - 16*a^2*b^5*c^5 - 4*\sqrt{2})*sq \\
& rt(b*c - \sqrt{b^2 - 4*a*c})*a^2*b^3*c^6 + 32*a^3*b^3*c^6 - 2*(b^2 - 4*a*c \\
&)*a*b^5*c^4 + 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a \\
& *c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - s \\
& \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^7 + 10*\sqrt{2})*s \\
& \sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^5*c + 2*\sqrt{2})*\sqrt{b \\
& ^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^6*c - 32*\sqrt{2})*\sqrt{b^2 - 4 \\
& *a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a^2*b^3*c^2 - 12*\sqrt{2})*\sqrt{b^2 - 4 \\
& *a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*a*b^4*c^2 - \sqrt{2})*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}})*b^5*c^2 + 32*\sqrt{2})*\sqrt{b^2 - 4*a*c})*sq \\
& rt(b*c - \sqrt{b^2 - 4*a*c})*a^3*b*c^3 + 16*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{ \\
& (b*c - \sqrt{b^2 - 4*a*c})*a^2*b^2*c^3 + 6*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b
\end{aligned}$$

$$\begin{aligned}
& *c - \sqrt{b^2 - 4ac} * c) * a * b^3 * c^3 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c} \\
& - \sqrt{b^2 - 4ac} * c) * a^2 * b * c^4 - 2 * (b^2 - 4ac) * b^5 * c^2 + 12 * (b^2 - 4ac) \\
& * a * b^3 * c^3 - 16 * (b^2 - 4ac) * a^2 * b * c^4) * (c^3 * d^2 - b * c^2 * d * e + a * c^2 * e^2) \\
& ^2 * d - (6 * a * b^8 * c^6 - 42 * a^2 * b^6 * c^7 + 68 * a^3 * b^4 * c^8 + 16 * a^4 * b^2 * c^9 - 3 \\
& * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^8 * c^4 + 21 * \sqrt{2} * \sqrt{b^2 - 4ac} \\
& * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 * c^5 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a * b^7 * c^5 - 34 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c^6 - 18 * \sqrt{2} \\
& * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^6 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a * b^6 * c^6 - 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^7 - 4 * \sqrt{2} * \sqrt{b^2 - 4ac} \\
& * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^7 + 9 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) \\
& * a^2 * b^4 * c^7 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^8 - 6 * (b^2 - 4ac) \\
& * a * b^6 * c^6 + 18 * (b^2 - 4ac) * a^2 * b^4 * c^7 + 4 * (b^2 - 4ac) * a^3 * b^2 * c^8) * d \\
& ^2 * e^3 - 2 * (2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 * c^3 - 17 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a^3 * b^4 * c^4 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^4 + 4 * a^2 * b^6 * c^4 + 40 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a^4 * b^2 * c^5 + 18 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^5 + 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a^2 * b^4 * c^5 - 34 * a^3 * b^4 * c^5 - 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * c^6 - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a^4 * b * c^6 - 9 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^6 + 80 * a^4 * b^2 * c^6 + 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4ac}} \\
& * c) * a^4 * c^7 - 32 * a^5 * c^7 - 4 * (b^2 - 4ac) * a^2 * b^4 * c^4 + 18 * (b^2 - 4ac) * a^3 * b^2 * c^5 - 8 * (b^2 - 4ac) * a^4 * c^6) * d * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * e^2 + (2 * a * b^6 * c^2 - 18 * a^2 * b^4 * c^3 + 48 * a^3 * b^2 * c^4 - 32 * a^4 * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^6 + 9 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^5 * c - 24 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^2 * c^2 - 10 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a * b^4 * c^2 + 16 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * c^3 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b * c^3 + 5 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^2 * c^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * c^4 - 2 * (b^2 - 4ac) * a * b^4 * c^2 + 10 * (b^2 - 4ac) * a^2 * b^2 * c^3 - 8 * (b^2 - 4ac) * a^3 * c^4) * (c^3 * d^2 - b * c^2 * d * e + a * c^2 * e^2) ^2 * e + (6 * a^2 * b^7 * c^6 - 44 * a^3 * b^5 * c^7 + 84 * a^4 * b^3 * c^8 - 16 * a^5 * b * c^9 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^7 * c^4 + 22 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^5 * c^5 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 * c^5 - 42 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c^6 - 20 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c^6 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^6 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{b * c - \sqrt{b^2 - 4ac}} * c) * a^5 * b * c^7 + 4 * s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^2 c^7 + 10 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^3 c^7 - 2 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^2 c^8 - 6 \cdot (b^2 - 4ac) \cdot a^2 b^5 c^6 + 20 \cdot (b^2 - 4ac) \cdot a^3 b^3 c^7 - 4 \cdot (b^2 - 4ac) \cdot a^4 b^2 c^8) \cdot d^4 + 2 \cdot (\text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^5 c^3 - 8 \cdot \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^3 c^4 - 2 \cdot \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^4 c^4 + 2 \cdot a^3 b^5 c^4 + 16 \cdot \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^5 b^2 c^5 + 8 \cdot \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^2 c^5 + \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^3 c^5 - 16 \cdot a^4 b^3 c^5 - 4 \cdot \text{qrt}(2) \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^2 c^6 + 32 \cdot a^5 b^2 c^6 - 2 \cdot (b^2 - 4ac) \cdot a^3 b^3 c^4 + 8 \cdot (b^2 - 4ac) \cdot a^4 b^2 c^5) \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot e^3 - (2 \cdot a^3 b^6 c^6 - 14 \cdot a^4 b^4 c^7 + 24 \cdot a^5 b^2 c^8 - \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^6 c^4 + 7 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^4 c^5 + 2 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^5 c^5 - 12 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^5 b^2 c^6 - 6 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^3 c^6 - \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^3 b^4 c^6 + 3 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc - \sqrt{b^2 - 4ac}} \cdot a^4 b^2 c^7 - 2 \cdot (b^2 - 4ac) \cdot a^3 b^4 c^6 + 6 \cdot (b^2 - 4ac) \cdot a^4 b^2 c^7) \cdot e^5) \cdot \arctan(2 \cdot \text{qrt}(1/2) \cdot x / \sqrt{((bc^3 d^2 - b^2 c^2 d e + a b^2 c^2 e^2 + \sqrt{(bc^3 d^2 - b^2 c^2 d e + a b^2 c^2 e^2)^2 - 4(a c^3 d^2 - a b^2 c^2 d e + a^2 c^2 e^2)} \cdot (c^4 d^2 - bc^3 d e + a^2 c^3 e^2)) / (c^4 d^2 - bc^3 d e + a^2 c^3 e^2)) / ((a b^4 c^7 - 8 a^2 b^2 c^8 - 2 a b^3 c^8 + 16 a^3 c^9 + 8 a^2 b^2 c^9 + a b^3 c^8 - 4 a^2 c^10) \cdot d^4 \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot \text{abs}(c) - 2 \cdot (a b^5 c^6 - 8 a^2 b^3 c^7 - 2 a b^4 c^7 + 16 a^3 b^2 c^8 + 8 a^2 b^2 c^8 + a b^3 c^8 - 4 a^2 b^2 c^9) \cdot d^3 \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot \text{abs}(c) \cdot e + (a b^6 c^5 - 6 a^2 b^4 c^6 - 2 a b^5 c^6 + 4 a^2 b^3 c^7 + a b^4 c^7 + 32 a^4 c^8 + 16 a^3 b^2 c^8 - 2 a^2 b^2 c^8 - 8 a^3 c^9) \cdot d^2 \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot \text{abs}(c) \cdot e^2 - 2 \cdot (a^2 b^5 c^5 - 8 a^3 b^3 c^6 - 2 a^2 b^4 c^6 + 16 a^4 b^2 c^7 + 8 a^3 b^2 c^7 + a^2 b^3 c^7 - 4 a^3 b^2 c^8) \cdot d \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot \text{abs}(c) \cdot e^3 + (a^3 b^4 c^5 - 8 a^4 b^2 c^6 - 2 a^3 b^3 c^6 + 16 a^5 c^7 + 8 a^4 b^2 c^7 + a^3 b^2 c^7 - 4 a^4 c^8) \cdot \text{abs}(-c^3 d^2 + bc^2 d e - a^2 c^2 e^2) \cdot \text{abs}(c) \cdot e^4) - 1/8 \cdot ((2 b^7 c^8 - 16 a b^5 c^9 + 36 a^2 b^3 c^10 - 16 a^3 b^2 c^11 - \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^2 b^7 c^6 + 8 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^3 b^6 c^7 + 2 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^4 b^5 c^8 - 18 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^2 b^3 c^8 - 8 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^3 b^4 c^8 - \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^4 b^5 c^8 + 8 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^3 b^2 c^9 + 4 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^2 b^2 c^9 + 4 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^3 b^3 c^9 - 2 \cdot \text{qrt}(2) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot a^2 b^2 c^10 - 2 \cdot (b^2 - 4ac) \cdot a^5 b^2 c^8 + 8 \cdot (b^2 - 4ac) \cdot a^3 b^3 c^9 - 4 \cdot (b^2 - 4ac) \cdot a^2 b^2 c^10) \cdot d^5 - (
\end{aligned}$$

$$\begin{aligned}
& 4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^{10} - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^8*c^5 + 15*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^6 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^7*c^6 - 29*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^7 - 14*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^7 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^6*c^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^8 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^8 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^8 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2*b^2*c^9)*d^4*e + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^4 - 9*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^5 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^5 - 2*a*b^6*c^5 + 24*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^6 + 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^6 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^6 + 18*a^2*b^4*c^6 - 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*c^7 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^7 - 5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^7 - 48*a^3*b^2*c^7 + 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*c^8 + 32*a^4*c^8 + 2*(b^2 - 4*a*c)*a*b^4*c^5 - 10*(b^2 - 4*a*c)*a^2*b^2*c^6 + 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^4*b*c^10 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^9*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^7*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^8*c^5 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^6 - 52*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^7 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^7 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^8 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^8 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^8 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^7*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^4 - 2*a*b^7*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^5 + 16*a^2*b^5*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^6 - 32*a^3*b^3*c^6 + 2*(b^2 - 4*a*c)*a*b^5*c^4 - 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6
\end{aligned}$$

$$\begin{aligned}
& *c - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^3*c \\
& ^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^2 \\
& - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^5*c^2 + 32*s \\
& \text{qrt}(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^3 + 16*sqr \\
& \text{t}(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^3 + 6*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^3 - 8*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^4 - 2*(b^2 - 4*a \\
& *c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3 \\
& *d^2 - b*c^2*d*e + a*c^2*e^2)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3* \\
& b^4*c^8 + 16*a^4*b^2*c^9 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - \\
& - 4*a*c}*c}*a*b^8*c^4 + 21*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c}*a^2*b^6*c^5 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4 \\
& *a*c}*c}*a*b^7*c^5 - 34*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a \\
& *c}*c}*a^3*b^4*c^6 - 18*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a \\
& *c}*c}*a^2*b^5*c^6 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a* \\
& c}*c}*a*b^6*c^6 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}* \\
& c}*a^4*b^2*c^7 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
&)*a^3*b^3*c^7 + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^4*c^7 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}* \\
& a^3*b^2*c^8 - 6*(b^2 - 4*a*c)*a*b^6*c^6 + 18*(b^2 - 4*a*c)*a^2*b^4*c^7 + 4* \\
& (b^2 - 4*a*c)*a^3*b^2*c^8)*d^2*e^3 + 2*(2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a \\
& *c}*c}*a^2*b^6*c^3 - 17*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^4*c^4 \\
& - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^5*c^4 - 4*a^2*b^6*c^4 + \\
& 40*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*b^2*c^5 + 18*\sqrt{2}*\sqrt{b* \\
& c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^3*c^5 + 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a* \\
& c}*c}*a^2*b^4*c^5 + 34*a^3*b^4*c^5 - 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c} \\
&)*c}*a^5*c^6 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*b*c^6 - 9*\sqrt{2} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^2*c^6 - 80*a^4*b^2*c^6 + 4*\sqrt{2} \\
&)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*c^7 + 32*a^5*c^7 + 4*(b^2 - 4*a*c)*a^ \\
& 2*b^4*c^4 - 18*(b^2 - 4*a*c)*a^3*b^2*c^5 + 8*(b^2 - 4*a*c)*a^4*c^6)*d*abs(- \\
& c^3*d^2 + b*c^2*d*e - a*c^2*e^2)*e^2 + (2*a*b^6*c^2 - 18*a^2*b^4*c^3 + 48*a \\
& ^3*b^2*c^4 - 32*a^4*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4 \\
& *a*c}*c}*a*b^6 + 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a \\
& *b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b \\
& ^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b \\
& ^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^ \\
& 2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*c^3 + \\
& 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b*c^3 + 5*s \\
& \text{qrt}(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^3 - 4*sqr \\
& \text{rt}(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*c^4 - 2*(b^2 - \\
& 4*a*c)*a*b^4*c^2 + 10*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)* \\
& (c^3*d^2 - b*c^2*d*e + a*c^2*e^2)^2*e + (6*a^2*b^7*c^6 - 44*a^3*b^5*c^7 + 8 \\
& 4*a^4*b^3*c^8 - 16*a^5*b*c^9 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - \\
& b^2 - 4*a*c}*c}*a^2*b^7*c^4 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}
\end{aligned}$$

$$\begin{aligned}
& b^2 - 4ac) * c) * a^3 * b^5 * c^5 + 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^2 * b^6 * c^5 - 42 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c^6 - 20 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c^6 - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^2 * b^5 * c^6 + 8 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^5 * b * c^7 + 4 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^7 + 10 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^7 - 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^8 - 6 * (b^2 - 4ac) * a^2 * b^5 * c^6 + 20 * (b^2 - 4ac) * a^3 * b^3 * c^7 - 4 * (b^2 - 4ac) * a^4 * b * c^8) * d * e^4 - 2 * (\sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^5 * c^3 - 8 * \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c^4 - 2 * \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c^4 - 2 * a^3 * b^5 * c^4 + 16 * \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^5 * b * c^5 + 8 * \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^5 + \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^3 * c^5 + 16 * a^4 * b^3 * c^5 - 4 * \sqrt{2} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b * c^6 - 32 * a^5 * b * c^6 + 2 * (b^2 - 4ac) * a^3 * b^3 * c^4 - 8 * (b^2 - 4ac) * a^4 * b * c^5) * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * e^3 - (2 * a^3 * b^6 * c^6 - 14 * a^4 * b^4 * c^7 + 24 * a^5 * b^2 * c^8 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^6 * c^4 + 7 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^4 * c^5 + 2 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^5 * c^5 - 12 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^5 * b^2 * c^6 - 6 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^3 * c^6 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^3 * b^4 * c^6 + 3 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}} * c) * a^4 * b^2 * c^7 - 2 * (b^2 - 4ac) * a^3 * b^4 * c^6 + 6 * (b^2 - 4ac) * a^4 * b^2 * c^7) * e^5 * \arctan(2 * \sqrt{1/2} * x / \sqrt{(b * c^3 * d^2 - b^2 * c^2 * d * e + a * b * c^2 * e^2 - \sqrt{(b * c^3 * d^2 - b^2 * c^2 * d * e + a * b * c^2 * e^2)^2 - 4 * (a * c^3 * d^2 - a * b * c^2 * d * e + a^2 * c^2 * e^2) * (c^4 * d^2 - b * c^3 * d * e + a * c^3 * e^2))}) / ((c^4 * d^2 - b * c^3 * d * e + a * c^3 * e^2)) / ((a * b^4 * c^7 - 8 * a^2 * b^2 * c^8 - 2 * a * b^3 * c^8 + 16 * a^3 * c^9 + 8 * a^2 * b * c^9 + a * b^2 * c^9 - 4 * a^2 * c^10) * d^4 * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * \text{abs}(c) - 2 * (a * b^5 * c^6 - 8 * a^2 * b^3 * c^7 - 2 * a * b^4 * c^7 + 16 * a^3 * b * c^8 + 8 * a^2 * b^2 * c^8 + a * b^3 * c^8 - 4 * a^2 * b * c^9) * d^3 * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * \text{abs}(c) * e + (a * b^6 * c^5 - 6 * a^2 * b^4 * c^6 - 2 * a * b^5 * c^6 + 4 * a^2 * b^3 * c^7 + a * b^4 * c^7 + 32 * a^4 * c^8 + 16 * a^3 * b * c^8 - 2 * a^2 * b^2 * c^8 - 8 * a^3 * c^9) * d^2 * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * \text{abs}(c) * e^2 - 2 * (a^2 * b^5 * c^5 - 8 * a^3 * b^3 * c^6 - 2 * a^2 * b^4 * c^6 + 16 * a^4 * b * c^7 + 8 * a^3 * b^2 * c^7 + a^2 * b^3 * c^7 - 4 * a^3 * b * c^8) * d * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * \text{abs}(c) * e^3 + (a^3 * b^4 * c^5 - 8 * a^4 * b^2 * c^6 - 2 * a^3 * b^3 * c^6 + 16 * a^5 * c^7 + 8 * a^4 * b * c^7 + a^3 * b^2 * c^7 - 4 * a^4 * c^8) * \text{abs}(-c^3 * d^2 + b * c^2 * d * e - a * c^2 * e^2) * \text{abs}(c) * e^4) + 1/3 * (c^2 * x^3 * e^2 - 3 * c^2 * d * x * e - 3 * b * c * x * e^2) * e^(-3) / c^3
\end{aligned}$$

maple [B] time = 0.04, size = 1449, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] $\frac{1}{3} \frac{1}{c} \frac{e x^3 - 1}{c^2} \frac{e b x - 1}{c} \frac{d}{e^2} \frac{x + 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 e - 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 e - 1}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 d + 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{b^3 d - 3/2}{(a e^2 - b d e + c d^2)} \frac{1}{c} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 b e - 1}{(a e^2 - b d e + c d^2)} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 d + 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^3 e + 2}{(a e^2 - b d e + c d^2)} \frac{1}{c} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 d - 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctanh}(2^{1/2})}{((-b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{b^4 d - 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 e + 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 e + 1}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 d - 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{b^3 d - 3/2}{(a e^2 - b d e + c d^2)} \frac{1}{c} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 b e - 1}{(a e^2 - b d e + c d^2)} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a^2 d + 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^3 e + 2}{(a e^2 - b d e + c d^2)} \frac{1}{c} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{a b^2 d - 1/2}{(a e^2 - b d e + c d^2)} \frac{1}{c^2} \frac{1}{(-4 a c + b^2)^{1/2}} \frac{1}{2} \frac{1}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{\operatorname{arctan}(2^{1/2})}{((b + (-4 a c + b^2)^{1/2}) c)^{1/2}} \frac{1}{c x} \frac{b^4 d + 1}{e^2 d^4} \frac{1}{(a e^2 - b d e + c d^2)} \frac{1}{(d e)^{1/2}} \frac{\operatorname{arctan}(1/(d e)^{1/2})}{e x}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e^2 - bde^3 + ae^4)\sqrt{de}} - \int \frac{a^2be - ((b^3 - 2abc)d - (ab^2 - a^2c)e)x^2 - (ab^2 - a^2c)d}{cx^4 + bx^2 + a} dx + \frac{cex^3 - 3(cd + be)x}{3c^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $d^4 \arctan(e x / \sqrt{d e}) / ((c d^2 e^2 - b d e^3 + a e^4) \sqrt{d e}) - \int \frac{-(a^2 b e - ((b^3 - 2 a b c) d - (a b^2 - a^2 c) e) x^2 - (a b^2 - a^2 c) d)}{(c x^4 + b x^2 + a), x} / (c^3 d^2 - b c^2 d e + a c^2 e^2) + 1/3 (c e x^3 - 3 (c d + b e) x) / (c^2 e^2)$

mupad [B] time = 7.13, size = 41755, normalized size = 107.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] $\operatorname{atan}\left(\frac{\left(\left(\left(\left(\left(192 a^3 c^8 d^6 e^5 + 384 a^4 c^7 d^4 e^7 + 192 a^5 c^6 d^2 e^9 - 48 a^2 b^2 c^7 d^6 e^5 + 96 a^2 b^3 c^6 d^5 e^6 - 48 a^2 b^4 c^5 d^4 e^7 + 96 a^3 b^2 c^6 d^4 e^7 + 96 a^3 b^3 c^5 d^3 e^8 - 48 a^4 b^2 c^5 d^2 e^9 - 384 a^3 b c^7 d^5 e^6 - 384 a^4 b c^6 d^3 e^8\right)\right)\right)\right)\right) / (c^3 e^3) - (2 x * (- (b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 * (- (4 a c - b^2)^3)^{1/2}) + 28 a^4 b c^4 d^2 - 9 a^3 b^5 c e^2 - 20 a^5 b c^3 e^2 - 2 a b^8 d e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 + a^2 b^4 e^2 * (- (4 a c - b^2)^3)^{1/2} - a^3 c^3 d^2 * (- (4 a c - b^2)^3)^{1/2} + 25 a^4 b^3 c^2 e^2 + a^4 c^2 e^2 * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c d^2 - 16 a^5 c^4 d e - 2 a b^5 d e * (- (4 a c - b^2)^3)^{1/2} + 20 a^2 b^6 c d e + 6 a^2 b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{1/2} - 5 a b^4 c d^2 * (- (4 a c - b^2)^3)^{1/2} - 66 a^3 b^4 c^2 d e + 76 a^4 b^2 c^3 d e - 3 a^3 b^2 c e^2 * (- (4 a c - b^2)^3)^{1/2} + 8 a^2 b^3 c d e * (- (4 a c - b^2)^3)^{1/2} - 6 a^3 b c^2 d e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 d^4 + 16 a^4 c^7 e^4 + b^4 c^7 d^4 - 8 a b^2 c^8 d^4 - 2 b^5 c^6 d^3 e + a^2 b^4 c^5 e^4 - 8 a^3 b^2 c^6 e^4 + 32 a^3 c^8 d^2 e^2 + b^6 c^5 d^2 e^2 + 16 a b^3 c^7 d^3 e - 2 a b^5 c^5 d e^3 - 32 a^2 b c^8 d^3 e - 32 a^3 b c^7 d e^3 - 6 a b^4 c^6 d^2 e^2 + 16 a^2 b^3 c^6 d e^3))^{1/2} * (128 a^4 b^2 c^6 e^{12} - 16 a^3 b^4 c^5 e^{12} - 256 a^5 c^7 e^{12} + 256 a^2 c^{10} d^6 e^6 + 256 a^3 c^9 d^4 e^8 - 256 a^4 c^8 d^2 e^{10} - 16 b^3 c^9 d^7 e^5 + 64 b^4 c^8 d^6 e^6 - 96 b^5 c^7 d^5 e^7 + 64 b^6 c^6 d^4 e^8 - 16 b^7 c^5 d^3 e^9 + 256 a^2 b^2 c^8 d^4 e^8 + 144 a^2 b^3 c^7 d^3 e^9 - 96 a^2 b^4 c^6 d^2 e^{10} + 192 a^3 b^2 c^7 d^2 e^{10} + 64 a b c^{10} d^7 e^5 + 320 a^4 b c^7 d e^{11} - 320 a b^2 c^9 d^6 e^6 + 528 a b^3 c^8 d^5 e^7 - 336 a b^4 c^7 d^4 e^8 + 48 a b^5 c^6 d^3 e^9 + 16 a b^6 c^5 d^2 e^{10} - 576 a^2 b c^9 d^5 e^7 + 16 a^2 b^5 c^5 d e^{11} - 320 a^3 b c^8 d^3 e^9 - 144 a^3 b^3 c^6 d e^{11}) / (c^3 e^3) * (- (b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 * (- (4 a c - b^2)^3)^{1/2}) + 28 a^4 b c^4 d^2 - 9 a^3 b^5 c e^2 - 20 a^5 b c^3 e^2 - 2 a b^8 d e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 + a^2 b^4 e^2 * (- (4 a c - b^2)^3)^{1/2} - a^3 c^3 d^2 * (- (4 a c - b^2)^3)^{1/2} + 25 a^4 b^3 c^2 e^2 + a^4 c^2 e^2 * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c d^2 - 16 a^5 c^4 d e - 2 a b^5 d e * (- (4 a c - b^2)^3)^{1/2} + 20 a^2 b^6 c d e + 6 a^2 b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{1/2} - 5 a b^4 c d^2 * (- (4 a c - b^2)^3)^{1/2} - 66 a^3 b^4 c^2 d e + 76 a^4 b^2 c^3 d e - 3$

$$\begin{aligned}
& a^3 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} + 8a^2 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} \\
& - 6a^3 b^2 c^2 d e (-4ac - b^2)^3)^{1/2} / (8(16a^2 c^9 d^4 + 16a^4 c^7 e^4 + b^4 c^7 d^4 - 8a^2 b^2 c^8 d^4 - 2b^5 c^6 d^3 e + a^2 b^4 c^5 e^4 - 8a^3 b^2 c^6 e^4 + 32a^3 c^8 d^2 e^2 + b^6 c^5 d^2 e^2 + 16a^2 b^3 c^7 d^3 e - 2a^2 b^5 c^5 d e^3 - 32a^2 b^2 c^8 d^3 e - 32a^3 b^2 c^7 d e^3 - 6a^2 b^4 c^6 d^2 e^2 + 16a^2 b^3 c^6 d e^3)))^{1/2} + (2x(4a^3 b^7 e^{10} + 4b^3 c^7 d^{10} + 4b^{10} d^3 e^7 - 36a^4 b^5 c^2 e^{10} - 80a^6 b^2 c^3 e^{10} - 4a^2 b^9 d^2 e^8 - 4a^2 b^8 d e^9 - 64a^2 c^8 d^9 e - 56a^6 c^4 d e^9 - 8b^4 c^6 d^9 e - 8b^9 c^4 d e^6 + 100a^5 b^3 c^2 e^{10} + 8a^4 c^6 d^5 e^5 + 16a^5 c^5 d^3 e^7 + 4b^5 c^5 d^8 e^2 + 4b^8 c^2 d^5 e^5 - 16a^2 b^3 c^8 d^{10} + 80a^2 b^4 c^4 d^5 e^5 - 160a^2 b^5 c^3 d^4 e^6 + 16a^2 b^6 c^2 d^3 e^7 - 64a^3 b^2 c^5 d^5 e^5 + 128a^3 b^3 c^4 d^4 e^6 + 96a^3 b^4 c^3 d^3 e^7 + 8a^3 b^5 c^2 d^2 e^8 - 120a^4 b^2 c^4 d^3 e^7 - 124a^4 b^3 c^3 d^2 e^8 + 48a^2 b^2 c^7 d^9 e - 24a^2 b^8 c^4 d^3 e^7 + 48a^3 b^6 c^2 d e^9 - 28a^2 b^3 c^6 d^8 e^2 - 32a^2 b^6 c^3 d^5 e^5 + 64a^2 b^7 c^2 d^4 e^6 + 48a^2 b^2 c^7 d^8 e^2 + 20a^2 b^7 c^4 d^2 e^8 - 16a^4 b^2 c^5 d^4 e^6 - 184a^4 b^4 c^2 d e^9 + 96a^5 b^2 c^4 d^2 e^8 + 240a^5 b^2 c^3 d e^9)) / (c^3 e^3)) * (-b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 (-4ac - b^2)^3)^{1/2} + 28a^4 b^2 c^4 d^2 - 9a^3 b^5 c^2 e^2 - 20a^5 b^2 c^3 e^2 - 2a^2 b^8 d e + 42a^2 b^5 c^2 d^2 - 63a^3 b^3 c^3 d^2 + a^2 b^4 e^2 (-4ac - b^2)^3)^{1/2} - a^3 c^3 d^2 (-4ac - b^2)^3)^{1/2} + 25a^4 b^3 c^2 e^2 + a^4 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 11a^2 b^7 c^2 d^2 - 16a^5 c^4 d e - 2a^2 b^5 d e (-4ac - b^2)^3)^{1/2} + 20a^2 b^6 c^2 d e + 6a^2 b^2 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 5a^2 b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 66a^3 b^4 c^2 d e + 76a^4 b^2 c^3 d e - 3a^3 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} + 8a^2 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} - 6a^3 b^2 c^2 d e (-4ac - b^2)^3)^{1/2} / (8(16a^2 c^9 d^4 + 16a^4 c^7 e^4 + b^4 c^7 d^4 - 8a^2 b^2 c^8 d^4 - 2b^5 c^6 d^3 e + a^2 b^4 c^5 e^4 - 8a^3 b^2 c^6 e^4 + 32a^3 c^8 d^2 e^2 + b^6 c^5 d^2 e^2 + 16a^2 b^3 c^7 d^3 e - 2a^2 b^5 c^5 d e^3 - 32a^2 b^2 c^8 d^3 e - 32a^3 b^2 c^7 d e^3 - 6a^2 b^4 c^6 d^2 e^2 + 16a^2 b^3 c^6 d e^3)))^{1/2} - (16a^3 c^6 d^9 + 4a^2 b^4 c^4 d^9 + 4a^2 b^8 d^5 e^4 + 4a^5 b^4 d e^8 + 4a^7 c^2 d e^8 - 20a^2 b^2 c^5 d^9 - 4a^2 b^7 d^4 e^5 - 4a^4 b^5 d^2 e^7 - 64a^4 c^5 d^7 e^2 + 64a^5 c^4 d^5 e^4 + 4a^6 c^3 d^3 e^6 - 36a^2 b^4 c^3 d^7 e^2 - 40a^2 b^5 c^2 d^6 e^3 + 96a^3 b^2 c^4 d^7 e^2 + 128a^3 b^3 c^3 d^6 e^3 + 164a^3 b^4 c^2 d^5 e^4 - 224a^4 b^2 c^3 d^5 e^4 - 104a^4 b^3 c^2 d^4 e^5 - 20a^5 b^2 c^2 d^3 e^6 + 4a^2 b^5 c^3 d^8 e + 4a^2 b^7 c^4 d^6 e^3 + 64a^3 b^2 c^5 d^8 e - 12a^6 b^2 c^2 d e^8 + 4a^2 b^6 c^2 d^7 e^2 - 32a^2 b^3 c^4 d^8 e - 44a^2 b^6 c^2 d^5 e^4 + 36a^3 b^5 c^2 d^4 e^5 - 128a^4 b^2 c^4 d^6 e^3 + 8a^4 b^4 c^2 d^3 e^6 + 88a^5 b^2 c^3 d^4 e^5 + 8a^5 b^3 c^2 d^2 e^7 + 4a^6 b^2 c^2 d^2 e^7) / (c^3 e^3)) * (-b^9 d^2 + a^2 b^7 e^2 + b^6 d^2 (-4ac - b^2)^3)^{1/2} + 28a^4 b^2 c^4 d^2 - 9a^3 b^5 c^2 e^2 - 20a^5 b^2 c^3 e^2 - 2a^2 b^8 d e + 42a^2 b^5 c^2 d^2 - 63a^3 b^3 c^3 d^2 + a^2 b^4 e^2 (-4ac - b^2)^3)^{1/2} - a^3 c^3 d^2 (-4ac - b^2)^3)^{1/2} + 25a^4 b^3 c^2 e^2 + a^4 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 11a^2 b^7 c^2 d^2 - 16a^5 c^4 d e - 2a^2 b^5 d e (-4ac - b^2)^3)^{1/2} + 20a^2 b^6 c^2 d e + 6a^2 b^2 c^2 d^2 (-4ac -
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d* \\
& e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b \\
& ^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2* \\
& b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + \\
& b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3* \\
& e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/ \\
& 2)} + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3* \\
& b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c \\
& ^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4* \\
& b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^ \\
& 5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e + 6*a^2 \\
& *b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e* \\
& (- (4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 \\
& - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 \\
& + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d \\
& *e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a \\
& ^2*b^3*c^6*d*e^3)))^{(1/2)}*1i - ((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e \\
& ^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 \\
& - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 \\
& - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(\\
& c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42 \\
& *a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(- \\
& -(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e \\
& + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3 \\
& *c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)) \\
& / (8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^ \\
& 5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^ \\
& 6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e \\
& - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/2)} \\
& *(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c \\
& ^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e \\
& ^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7* \\
& c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^ \\
& 4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b \\
& *c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7 \\
& *d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5
\end{aligned}$$

$$\begin{aligned}
& e^7 + 16a^2b^5c^5d^5e^{11} - 320a^3b^3c^8d^3e^9 - 144a^3b^3c^6d^5e^{11}) / (c^3e^3) * (- (b^9d^2 + a^2b^7e^2 + b^6d^2 * (- (4ac - b^2)^3)^{1/2}) \\
& + 28a^4b^3c^4d^2 - 9a^3b^5c^5e^2 - 20a^5b^3c^3e^2 - 2a^2b^8d^5e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2 * (- (4ac - b^2)^3)^{1/2} \\
&) - a^3c^3d^2 * (- (4ac - b^2)^3)^{1/2} + 25a^4b^3c^2e^2 + a^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 11a^2b^7c^4d^2 - 16a^5c^4d^5e - 2a^2b^5d^5e * \\
& (- (4ac - b^2)^3)^{1/2} + 20a^2b^6c^4d^2 * (- (4ac - b^2)^3)^{1/2} - 5a^2b^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 5a^2b^4c^4d^2 * (- (4ac - b^2)^3)^{1/2} - 66a^3b^4c^2d^5e \\
& + 76a^4b^2c^3d^5e - 3a^3b^2c^5e^2 * (- (4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^5e * (- (4ac - b^2)^3)^{1/2} - 6a^3b^3c^2d^5e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8 * (16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16a^2b^3c^7d^3e - 2a^2b^5c^5d^5e^3 - 32a^2b^3c^8d^3e \\
& - 32a^3b^3c^7d^5e^3 - 6a^2b^4c^6d^2e^2 + 16a^2b^3c^6d^5e^3))^{1/2} - (2 * x * (4a^3b^7e^{10} + 4b^3c^7d^{10} + 4b^{10}d^3e^7 - 36a^4b^5c^5e^{10} - 80a^6b^3c^3e^{10} - 4a^2b^9d^2e^8 - 4a^2b^8d^5e^9 - 64a^2c^8d^9e \\
& - 56a^6c^4d^5e^9 - 8b^4c^6d^9e - 8b^9c^4d^4e^6 + 100a^5b^3c^2e^{10} + 8a^4c^6d^5e^5 + 16a^5c^5d^3e^7 + 4b^5c^5d^8e^2 + 4b^8c^2d^5e^5 - 16a^2b^3c^8d^{10} + 80a^2b^4c^4d^5e^5 - 160a^2b^5c^3d^4e^6 + 16a^2b^6c^2d^3e^7 - 64a^3b^2c^5d^5e^5 + 128a^3b^3c^4d^4e^6 + 96a^3b^4c^3d^3e^7 + 8a^3b^5c^2d^2e^8 - 120a^4b^2c^4d^3e^7 - 124a^4b^3c^3d^2e^8 + 48a^2b^2c^7d^9e - 24a^2b^8c^3d^3e^7 + 48a^3b^6c^4d^5e^9 - 28a^2b^3c^6d^8e^2 - 32a^2b^6c^3d^5e^5 + 64a^2b^7c^2d^4e^6 + 48a^2b^3c^7d^8e^2 + 20a^2b^7c^4d^2e^8 - 16a^4b^3c^5d^4e^6 - 184a^4b^4c^2d^5e^9 + 96a^5b^3c^4d^2e^8 + 240a^5b^2c^3d^5e^9) / (c^3e^3) * (- (b^9d^2 + a^2b^7e^2 + b^6d^2 * (- (4ac - b^2)^3)^{1/2}) \\
& + 28a^4b^3c^4d^2 - 9a^3b^5c^5e^2 - 20a^5b^3c^3e^2 - 2a^2b^8d^5e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2 * (- (4ac - b^2)^3)^{1/2} - a^3c^3d^2 * (- (4ac - b^2)^3)^{1/2} + 25a^4b^3c^2e^2 + a^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 11a^2b^7c^4d^2 - 16a^5c^4d^5e - 2a^2b^5d^5e * \\
& (- (4ac - b^2)^3)^{1/2} + 20a^2b^6c^4d^2 * (- (4ac - b^2)^3)^{1/2} - 5a^2b^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 5a^2b^4c^4d^2 * (- (4ac - b^2)^3)^{1/2} - 66a^3b^4c^2d^5e \\
& + 76a^4b^2c^3d^5e - 3a^3b^2c^5e^2 * (- (4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^5e * (- (4ac - b^2)^3)^{1/2} - 6a^3b^3c^2d^5e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8 * (16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16a^2b^3c^7d^3e - 2a^2b^5c^5d^5e^3 - 32a^2b^3c^8d^3e \\
& - 32a^3b^3c^7d^5e^3 - 6a^2b^4c^6d^2e^2 + 16a^2b^3c^6d^5e^3))^{1/2} - (16a^3c^6d^9 + 4a^2b^4c^4d^9 + 4a^2b^8d^5e^4 + 4a^5b^4d^5e^8 + 4a^7c^2d^5e^8 - 20a^2b^2c^5d^9 - 4a^2b^7d^4e^5 - 4a^4b^5d^2e^7 - 64a^4c^5d^7e^2 + 64a^5c^4d^5e^4 + 4a^6c^3d^3e^6 - 36a^2b^4c^3d^7e^2 - 40a^2b^5c^2d^6e^3 + 96a^3b^2c^4d^7e^2 + 128a^3b^3c^3d^6e^3 + 164a^3b^4c^2d^5e^4 - 224a^4b^2c^3d^5e^4 - 104a^4b^3c^2d^4e^5 - 20a^5b^2c^2d^3e^6 + 4a^2b^5c^3d^8e + 4a^2b^7c^4d^4e^5 - 12a^6b^2c^4d^8e + 4a^2b^6c^2d^7e^2 -
\end{aligned}$$

$$\begin{aligned}
& *d^3e - 32a^2b^3c^8d^3e - 32a^3b^3c^7d^3e - 6a^4b^4c^6d^2e^2 + 16 \\
& *a^2b^3c^6d^3e))^{(1/2)} - (16a^3c^6d^9 + 4a^4b^4c^4d^9 + 4a^5b^8d \\
& ^5e^4 + 4a^5b^4d^8e + 4a^7c^2d^8e - 20a^2b^2c^5d^9 - 4a^2b^7 \\
& *d^4e^5 - 4a^4b^5d^2e^7 - 64a^4c^5d^7e^2 + 64a^5c^4d^5e^4 + 4a \\
& ^6c^3d^3e^6 - 36a^2b^4c^3d^7e^2 - 40a^2b^5c^2d^6e^3 + 96a^3b^2 \\
& *c^4d^7e^2 + 128a^3b^3c^3d^6e^3 + 164a^3b^4c^2d^5e^4 - 224a^4 \\
& *b^2c^3d^5e^4 - 104a^4b^3c^2d^4e^5 - 20a^5b^2c^2d^3e^6 + 4a^5 \\
& *b^5c^3d^8e + 4a^7c^3d^6e^3 + 64a^3b^3c^5d^8e - 12a^6b^2c^3d^8e \\
& + 4a^8b^6c^2d^7e^2 - 32a^2b^3c^4d^8e - 44a^2b^6c^3d^5e^4 + 36a^3 \\
& *b^5c^4d^4e^5 - 128a^4b^3c^4d^6e^3 + 8a^4b^4c^3d^3e^6 + 88a^5b^3 \\
& *c^3d^4e^5 + 8a^5b^3c^3d^2e^7 + 4a^6b^3c^2d^2e^7)/(c^3e^3))*(-(b^9 \\
& d^2 + a^2b^7e^2 + b^6d^2*(-(4ac - b^2)^3)^{(1/2)} + 28a^4b^3c^4d^2 - 9 \\
& *a^3b^5c^3e^2 - 20a^5b^3c^3e^2 - 2a^8b^3d^3e + 42a^2b^5c^2d^2 - 63a^3 \\
& *b^3c^3d^2 + a^2b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - a^3c^3d^2*(-(4ac \\
& - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 + a^4c^2e^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 11a^5b^7c^3d^2 - 16a^5c^4d^3e - 2a^8b^5d^3e*(-(4ac - b^2)^3)^{(1/2)} \\
& + 20a^2b^6c^3d^3e + 6a^2b^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 5a^4b^4 \\
& *c^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^3e + 76a^4b^2c^3d^3e - \\
& 3a^3b^2c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + 8a^2b^3c^3d^3e*(-(4ac - b^2)^3)^{(1/2)} \\
& - 6a^3b^3c^2d^3e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^9d^4 + 1 \\
& 6a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5 \\
& e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16a^2b^3 \\
& c^7d^3e - 2a^5b^5c^5d^3e - 32a^2b^3c^8d^3e - 32a^3b^3c^7d^3e - \\
& 6a^4b^4c^6d^2e^2 + 16a^2b^3c^6d^3e))^{(1/2)} + (2*x*(a^8e^8 + b^8 \\
& d^8 + 2a^4c^4d^8 + 20a^2b^4c^2d^8 - 16a^3b^2c^3d^8 - 8a^5b^6c^8 \\
& ^8))/(c^3e^3))*(-(b^9d^2 + a^2b^7e^2 + b^6d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& + 28a^4b^3c^4d^2 - 9a^3b^5c^3e^2 - 20a^5b^3c^3e^2 - 2a^8b^3d^3e + 42 \\
& *a^2b^5c^2d^2 - 63a^3b^3c^3d^2 + a^2b^4e^2*(-(4ac - b^2)^3)^{(1/2)} \\
&) - a^3c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 + a^4c^2e^2 \\
& *(-(4ac - b^2)^3)^{(1/2)} - 11a^5b^7c^3d^2 - 16a^5c^4d^3e - 2a^8b^5d^3e*(- \\
& -(4ac - b^2)^3)^{(1/2)} + 20a^2b^6c^3d^3e + 6a^2b^2c^2d^2*(-(4ac - b \\
& ^2)^3)^{(1/2)} - 5a^4b^4c^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^3e \\
& + 76a^4b^2c^3d^3e - 3a^3b^2c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + 8a^2b^3 \\
& *c^3d^3e*(-(4ac - b^2)^3)^{(1/2)} - 6a^3b^3c^2d^3e*(-(4ac - b^2)^3)^{(1/2)}) \\
& /(8*(16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5 \\
& c^6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6 \\
& c^5d^2e^2 + 16a^2b^3c^7d^3e - 2a^5b^5c^5d^3e - 32a^2b^3c^8d^3e \\
& - 32a^3b^3c^7d^3e - 6a^4b^4c^6d^2e^2 + 16a^2b^3c^6d^3e))^{(1/2)} \\
& + ((((((192a^3c^8d^6e^5 + 384a^4c^7d^4e^7 + 192a^5c^6d^2e^9 - 4 \\
& 8a^2b^2c^7d^6e^5 + 96a^2b^3c^6d^5e^6 - 48a^2b^4c^5d^4e^7 + 9 \\
& 6a^3b^2c^6d^4e^7 + 96a^3b^3c^5d^3e^8 - 48a^4b^2c^5d^2e^9 - 3 \\
& 84a^3b^3c^7d^5e^6 - 384a^4b^3c^6d^3e^8)/(c^3e^3) + (2*x*(-(b^9d^2 + \\
& a^2b^7e^2 + b^6d^2*(-(4ac - b^2)^3)^{(1/2)} + 28a^4b^3c^4d^2 - 9a^3 \\
& b^5c^3e^2 - 20a^5b^3c^3e^2 - 2a^8b^3d^3e + 42a^2b^5c^2d^2 - 63a^3b^3 \\
& c^3d^2 + a^2b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - a^3c^3d^2*(-(4ac - b
\end{aligned}$$

$$\begin{aligned}
& ^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20 \\
& *a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3 \\
& *b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^9*d^4 + 16*a^4 \\
& *c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^ \\
& 4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7 \\
& *d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a* \\
& b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/2)}*(128*a^4*b^2*c^6*e^12 - 16* \\
& a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^ \\
& 4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96 \\
& *b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^ \\
& 8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2 \\
& *c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9* \\
& d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3* \\
& e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 \\
& - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + \\
& a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3* \\
& b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^ \\
& 3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20 \\
& *a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3 \\
& *b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^9*d^4 + 16*a^4 \\
& *c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^ \\
& 4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7 \\
& *d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a* \\
& b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/2)} - (2*x*(4*a^3*b^7*e^10 + 4* \\
& b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a \\
& *b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^ \\
& 4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + \\
& 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^1 \\
& 0 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e \\
& ^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3* \\
& e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2 \\
& *e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a* \\
& b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^ \\
& 7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d \\
& *e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 \\
& + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^ \\
& 3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3* \\
& b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a \\
& ^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{ \\
& (1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2))}/(8*(16*a^2*c^9*d^4 + 16*a \\
& ^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5* \\
& e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c \\
& ^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6* \\
& a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/2)} - (16*a^3*c^6*d^9 + 4*a*b \\
& ^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b \\
& ^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 6 \\
& 4*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5 \\
& *c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b \\
& ^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5 \\
& *b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8 \\
& *e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a \\
& ^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4 \\
& *c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e \\
& ^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42* \\
& a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + \\
& 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3* \\
& c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2))}/ \\
& (8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5 \\
& *c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6 \\
& *c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e \\
& - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^{(1/2)} \\
& - (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2 \\
& *c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3* \\
& e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3 \\
& *c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c \\
& ^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e + 6*a^2*b^ \\
& 2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(- \\
& (4*a*c - b^2)^3)^{(1/2))}/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - \\
& 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 3 \\
& 2*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^
\end{aligned}$$

$$\begin{aligned}
& 3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^{(1/2)} + (2*(a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 - a^6*c*d^6*e))/(c^3*e^3)) * (- (b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^{(1/2)} * 2i + \text{atan}(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^{(1/2)} * (128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3)) * (- (b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4
\end{aligned}$$

$$\begin{aligned}
& 4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b^2 \\
& *c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8 \\
& *a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32 \\
& *a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 \\
& - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b \\
& ^3*c^6*d*e^3)))^{(1/2)} + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3* \\
& e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d \\
& *e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4* \\
& e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5 \\
& *c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 \\
& - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^ \\
& 5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^ \\
& 8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e \\
& - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6 \\
& *c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d \\
& ^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^ \\
& 8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3 \\
& *e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2* \\
& (-(4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^ \\
& 3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5* \\
& c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b \\
& ^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(- \\
& (4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - \\
& 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + \\
& 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e \\
& ^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2 \\
& *b^3*c^6*d*e^3)))^{(1/2)} - (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e \\
& ^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4 \\
& *e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6* \\
& c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2* \\
& c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b \\
& ^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5 \\
& *c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + \\
& 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3* \\
& b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3* \\
& d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 \\
& + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3 \\
& *b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b \\
& ^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 - a^4c^2e^2(-4ac - b^2)^3)^{(1/2)} - \\
& 11ab^7cd^2 - 16a^5c^4d^2e + 2ab^5d^2e(-4ac - b^2)^3)^{(1/2)} + 2 \\
& 0a^2b^6cd^2e - 6a^2b^2c^2d^2(-4ac - b^2)^3)^{(1/2)} + 5ab^4cd^2 \\
& 2(-4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3 \\
& 3b^2c^2e^2(-4ac - b^2)^3)^{(1/2)} - 8a^2b^3cd^2e(-4ac - b^2)^3)^{(1/2)} + 6a^3 \\
& 3b^2c^2d^2e(-4ac - b^2)^3)^{(1/2))}/(8(16a^2c^9d^4 + 16a^4 \\
& 4c^7e^4 + b^4c^7d^4 - 8ab^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e \\
& ^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16ab^3c^7 \\
& 7d^3e - 2ab^5c^5d^2e^3 - 32a^2b^3c^8d^3e - 32a^3b^3c^7d^2e^3 - 6a \\
& *b^4c^6d^2e^2 + 16a^2b^3c^6d^2e^3)))^{(1/2)} + (2x*(a^8e^8 + b^8d^8 \\
& + 2a^4c^4d^8 + 20a^2b^4c^2d^8 - 16a^3b^2c^3d^8 - 8ab^6cd^8)) \\
& / (c^3e^3))*(-b^9d^2 + a^2b^7e^2 - b^6d^2(-4ac - b^2)^3)^{(1/2)} + 2 \\
& 8a^4b^3c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^3c^3e^2 - 2ab^8d^2e + 42a^2 \\
& *b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(-4ac - b^2)^3)^{(1/2)} + \\
& a^3c^3d^2(-4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 - a^4c^2e^2(-4 \\
& 4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 - 16a^5c^4d^2e + 2ab^5d^2e(-4 \\
& ac - b^2)^3)^{(1/2)} + 20a^2b^6cd^2e - 6a^2b^2c^2d^2(-4ac - b^2)^ \\
& 3)^{(1/2)} + 5ab^4cd^2(-4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76 \\
& *a^4b^2c^3d^2e + 3a^3b^2c^2e^2(-4ac - b^2)^3)^{(1/2)} - 8a^2b^3cd \\
& *e(-4ac - b^2)^3)^{(1/2)} + 6a^3b^3c^2d^2e(-4ac - b^2)^3)^{(1/2))}/(8 \\
& (16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8ab^2c^8d^4 - 2b^5c^6 \\
& 6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5 \\
& 5d^2e^2 + 16ab^3c^7d^3e - 2ab^5c^5d^2e^3 - 32a^2b^3c^8d^3e - 3 \\
& 2a^3b^3c^7d^2e^3 - 6ab^4c^6d^2e^2 + 16a^2b^3c^6d^2e^3)))^{(1/2)}*1i \\
& - (((((192a^3c^8d^6e^5 + 384a^4c^7d^4e^7 + 192a^5c^6d^2e^9 - 48 \\
& *a^2b^2c^7d^6e^5 + 96a^2b^3c^6d^5e^6 - 48a^2b^4c^5d^4e^7 + 96 \\
& *a^3b^2c^6d^4e^7 + 96a^3b^3c^5d^3e^8 - 48a^4b^2c^5d^2e^9 - 38 \\
& 4a^3b^3c^7d^5e^6 - 384a^4b^3c^6d^3e^8))/(c^3e^3) + (2x*(-b^9d^2 + \\
& a^2b^7e^2 - b^6d^2(-4ac - b^2)^3)^{(1/2)} + 28a^4b^3c^4d^2 - 9a^3b \\
& ^5c^2e^2 - 20a^5b^3c^3e^2 - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3 \\
& *c^3d^2 - a^2b^4e^2(-4ac - b^2)^3)^{(1/2)} + a^3c^3d^2(-4ac - b^ \\
& 2)^3)^{(1/2)} + 25a^4b^3c^2e^2 - a^4c^2e^2(-4ac - b^2)^3)^{(1/2)} - 1 \\
& 1ab^7cd^2 - 16a^5c^4d^2e + 2ab^5d^2e(-4ac - b^2)^3)^{(1/2)} + 20 \\
& a^2b^6cd^2e - 6a^2b^2c^2d^2(-4ac - b^2)^3)^{(1/2)} + 5ab^4cd^2 \\
& (-4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3 \\
& b^2c^2e^2(-4ac - b^2)^3)^{(1/2)} - 8a^2b^3cd^2e(-4ac - b^2)^3)^{(1/ \\
& 2)} + 6a^3b^3c^2d^2e(-4ac - b^2)^3)^{(1/2))}/(8(16a^2c^9d^4 + 16a^4 \\
& c^7e^4 + b^4c^7d^4 - 8ab^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 \\
& - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16ab^3c^7 \\
& d^3e - 2ab^5c^5d^2e^3 - 32a^2b^3c^8d^3e - 32a^3b^3c^7d^2e^3 - 6ab \\
& ^4c^6d^2e^2 + 16a^2b^3c^6d^2e^3)))^{(1/2)}*(128a^4b^2c^6e^12 - 16a \\
& ^3b^4c^5e^12 - 256a^5c^7e^12 + 256a^2c^10d^6e^6 + 256a^3c^9d^4 \\
& *e^8 - 256a^4c^8d^2e^10 - 16b^3c^9d^7e^5 + 64b^4c^8d^6e^6 - 96 \\
& b^5c^7d^5e^7 + 64b^6c^6d^4e^8 - 16b^7c^5d^3e^9 + 256a^2b^2c^8 \\
& *d^4e^8 + 144a^2b^3c^7d^3e^9 - 96a^2b^4c^6d^2e^10 + 192a^3b^2*
\end{aligned}$$

$$\begin{aligned}
& c^7 d^2 e^{10} + 64 a^4 b^3 c^8 d^5 e^7 + 320 a^4 b^3 c^7 d^4 e^8 - 320 a^4 b^2 c^9 d^3 e^9 \\
& + 528 a^4 b^3 c^8 d^5 e^7 - 336 a^4 b^4 c^7 d^4 e^8 + 48 a^4 b^5 c^6 d^3 e^9 \\
& + 16 a^4 b^6 c^5 d^2 e^{10} - 576 a^4 b^2 c^9 d^5 e^7 + 16 a^4 b^5 c^5 d^4 e^{11} \\
& - 320 a^4 b^3 c^8 d^3 e^9 - 144 a^4 b^3 c^6 d^4 e^{11}) / (c^3 e^3) * (- (b^9 d^2 + \\
& a^2 b^7 e^2 - b^6 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^3 e^2 \\
& - 20 a^5 b^3 c^3 e^2 - 2 a^4 b^8 d^4 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 \\
& - a^2 b^4 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} + a^3 c^3 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& + 25 a^4 b^3 c^2 e^2 - a^4 c^2 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} - 11 a^4 b^7 c^3 d^2 \\
& - 16 a^5 c^4 d^4 e + 2 a^4 b^5 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} + 20 a^2 b^6 c^3 d^2 e \\
& - 6 a^2 b^2 c^2 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} + 5 a^4 b^4 c^3 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& - 66 a^3 b^4 c^2 d^4 e + 76 a^4 b^2 c^3 d^4 e + 3 a^3 b^2 c^3 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& - 8 a^2 b^3 c^3 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} + 6 a^3 b^3 c^2 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} \\
&) / (8 * (16 a^2 c^9 d^4 + 16 a^4 c^7 e^4 + b^4 c^7 d^4 - 8 a^4 b^2 c^8 d^4 - 2 b^5 c^6 d^3 e + a^2 b^4 c^5 e^4 \\
& - 8 a^3 b^2 c^6 e^4 + 32 a^3 c^8 d^2 e^2 + b^6 c^5 d^2 e^2 + 16 a^4 b^3 c^7 d^3 e - 2 a^4 b^5 c^5 d^4 e^3 \\
& - 32 a^2 b^3 c^8 d^3 e - 32 a^3 b^3 c^7 d^4 e^3 - 6 a^4 b^4 c^6 d^2 e^2 + 16 a^2 b^3 c^6 d^4 e^3))^{1/2} - (2 * x * (4 a^3 b^7 e^{10} + 4 b^3 c^7 d^{10} \\
& + 4 b^{10} d^3 e^7 - 36 a^4 b^5 c^3 e^{10} - 80 a^6 b^3 c^3 e^{10} - 4 a^4 b^9 d^2 e^8 - 4 a^2 b^8 d^4 e^9 \\
& - 64 a^2 c^8 d^9 e - 56 a^6 c^4 d^4 e^9 - 8 b^4 c^6 d^9 e - 8 b^9 c^4 d^4 e^6 + 100 a^5 b^3 c^2 e^{10} + 8 a^4 c^6 d^5 e^5 + 1 \\
& 6 a^5 c^5 d^3 e^7 + 4 b^5 c^5 d^8 e^2 + 4 b^8 c^2 d^5 e^5 - 16 a^4 b^3 c^8 d^{10} + 80 a^2 b^4 c^4 d^5 e^5 \\
& - 160 a^2 b^5 c^3 d^4 e^6 + 16 a^2 b^6 c^2 d^3 e^7 - 64 a^3 b^2 c^5 d^5 e^5 + 128 a^3 b^3 c^4 d^4 e^6 \\
& + 96 a^3 b^4 c^3 d^3 e^7 + 8 a^3 b^5 c^2 d^2 e^8 - 120 a^4 b^2 c^4 d^3 e^7 - 124 a^4 b^3 c^3 d^2 e^8 \\
& + 48 a^4 b^2 c^7 d^9 e - 24 a^4 b^8 c^3 d^3 e^7 + 48 a^3 b^6 c^3 d^4 e^9 - 28 a^4 b^3 c^6 d^8 e^2 \\
& - 32 a^4 b^6 c^3 d^5 e^5 + 64 a^4 b^7 c^2 d^4 e^6 + 48 a^2 b^3 c^7 d^8 e^2 + 20 a^2 b^7 c^3 d^2 e^8 \\
& - 16 a^4 b^3 c^5 d^4 e^6 - 184 a^4 b^4 c^2 d^4 e^9 + 96 a^5 b^3 c^4 d^2 e^8 + 240 a^5 b^2 c^3 d^4 e^9) / (c^3 e^3) * (- (b^9 d^2 \\
& + a^2 b^7 e^2 - b^6 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} + 28 a^4 b^3 c^4 d^2 - 9 a^3 b^5 c^3 e^2 \\
& - 20 a^5 b^3 c^3 e^2 - 2 a^4 b^8 d^4 e + 42 a^2 b^5 c^2 d^2 - 63 a^3 b^3 c^3 d^2 - a^2 b^4 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& + a^3 c^3 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} + 25 a^4 b^3 c^2 e^2 - a^4 c^2 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} - \\
& 11 a^4 b^7 c^3 d^2 - 16 a^5 c^4 d^4 e + 2 a^4 b^5 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} + 20 a^2 b^6 c^3 d^2 e \\
& - 6 a^2 b^2 c^2 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} + 5 a^4 b^4 c^3 d^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& - 66 a^3 b^4 c^2 d^4 e + 76 a^4 b^2 c^3 d^4 e + 3 a^3 b^2 c^3 e^2 * (- (4 a^4 c - b^2)^3)^{1/2} \\
& - 8 a^2 b^3 c^3 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} + 6 a^3 b^3 c^2 d^4 e * (- (4 a^4 c - b^2)^3)^{1/2} \\
&) / (8 * (16 a^2 c^9 d^4 + 16 a^4 c^7 e^4 + b^4 c^7 d^4 - 8 a^4 b^2 c^8 d^4 - 2 b^5 c^6 d^3 e + a^2 b^4 c^5 e^4 \\
& - 8 a^3 b^2 c^6 e^4 + 32 a^3 c^8 d^2 e^2 + b^6 c^5 d^2 e^2 + 16 a^4 b^3 c^7 d^3 e - 2 a^4 b^5 c^5 d^4 e^3 \\
& - 32 a^2 b^3 c^8 d^3 e - 32 a^3 b^3 c^7 d^4 e^3 - 6 a^4 b^4 c^6 d^2 e^2 + 16 a^2 b^3 c^6 d^4 e^3))^{1/2} - (16 a^3 c^6 d^9 + 4 a^4 b^4 c^4 d^9 \\
& + 4 a^4 b^8 d^5 e^4 + 4 a^5 b^4 d^4 e^8 + 4 a^7 c^2 d^4 e^8 - 20 a^2 b^2 c^5 d^9 - 4 a^2 b^7 d^4 e^5 \\
& - 4 a^4 b^5 d^2 e^7 - 64 a^4 c^5 d^7 e^2 + 64 a^5 c^4 d^5 e^4 + 4 a^6 c^3 d^3 e^6 - 36 a^2 b^4 c^3 d^7 e^2 - 40 a^2 b^5 c^2 d^6 e^3 \\
& + 96 a^3 b^2 c^4 d^7 e^2 + 128 a^3 b^3 c^3 d^6 e^3 + 164 a^3 b^
\end{aligned}$$

$$\begin{aligned}
& 4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5* \\
& b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8* \\
& e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^ \\
& 2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4* \\
& c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^ \\
& 7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + \\
& 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^ \\
& 2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) \\
& + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(- \\
& -(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(\\
& 4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2 \\
&)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + \\
& 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c \\
& *d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(\\
& 8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5* \\
& c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6* \\
& c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - \\
& 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - \\
& (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2* \\
& c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(\\
& 4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e \\
& ^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(- \\
& (4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3* \\
& c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^ \\
& 4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2 \\
& *c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) \\
& - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2) \\
& ^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4 \\
& *a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8 \\
& *a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32 \\
& *a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 \\
& - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b \\
& ^3*c^6*d*e^3)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + \\
& 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48 \\
& *a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48 \\
& *a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3* \\
& e^3) - (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 2 \\
& 8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2 \\
& *b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + \\
& a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(\\
& 4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4 \\
& a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^ \\
& 3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76 \\
& *a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d \\
& *e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
\end{aligned}$$

$$\begin{aligned}
& (16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8ab^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16ab^3c^7d^3e - 2ab^5c^5d^3e^3 - 32a^2b^2c^8d^3e - 32a^3b^2c^7d^3e^3 - 6ab^4c^6d^2e^2 + 16a^2b^3c^6d^3e^3))^{(1/2)} \cdot (128a^4b^2c^6e^{12} - 16a^3b^4c^5e^{12} - 256a^5c^7e^{12} + 256a^2c^{10}d^6e^6 + 256a^3c^9d^4e^8 - 256a^4c^8d^2e^{10} - 16b^3c^9d^7e^5 + 64b^4c^8d^6e^6 - 96b^5c^7d^5e^7 + 64b^6c^6d^4e^8 - 16b^7c^5d^3e^9 + 256a^2b^2c^8d^4e^8 + 144a^2b^3c^7d^3e^9 - 96a^2b^4c^6d^2e^{10} + 192a^3b^2c^7d^2e^{10} + 64a^2b^3c^7d^3e^9 - 96a^2b^4c^6d^2e^{10} + 192a^3b^2c^7d^2e^{10} + 64a^2b^3c^7d^3e^9 + 320a^4b^2c^6d^2e^{11} - 320a^3b^2c^9d^6e^6 + 528a^2b^3c^8d^5e^7 - 336a^2b^4c^7d^4e^8 + 48a^2b^5c^6d^3e^9 + 16a^2b^6c^5d^2e^{10} - 576a^2b^2c^9d^5e^7 + 16a^2b^5c^5d^3e^{11} - 320a^3b^2c^8d^3e^9 - 144a^3b^3c^6d^3e^{11})) \\
& / (c^3e^3) \cdot (-b^9d^2 + a^2b^7e^2 - b^6d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 28a^4b^2c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^2c^3e^2 - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2 \cdot (-4ac - b^2)^3)^{(1/2)} + a^3c^3d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 - a^4c^2e^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 - 16a^5c^4d^2e + 2ab^5d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 20a^2b^6cd^2e - 6a^2b^2c^2d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 5ab^4cd^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3b^2c^2e^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 8a^2b^3cd^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 6a^3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} / (8 \cdot (16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8ab^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16ab^3c^7d^3e - 2ab^5c^5d^3e^3 - 32a^2b^2c^8d^3e - 32a^3b^2c^7d^3e^3 - 6ab^4c^6d^2e^2 + 16a^2b^3c^6d^3e^3))^{(1/2)} + (2 \cdot x \cdot (4a^3b^7e^{10} + 4b^3c^7d^{10} + 4b^{10}d^3e^7 - 36a^4b^5c^2e^{10} - 80a^6b^3c^3e^{10} - 4ab^9d^2e^8 - 4a^2b^8d^2e^9 - 64a^2c^8d^9e - 56a^6c^4d^9e - 8b^4c^6d^9e - 8b^9cd^4e^6 + 100a^5b^3c^2e^{10} + 8a^4c^6d^5e^5 + 16a^5c^5d^3e^7 + 4b^5c^5d^8e^2 + 4b^8c^2d^5e^5 - 16ab^2c^8d^{10} + 80a^2b^4c^4d^5e^5 - 160a^2b^5c^3d^4e^6 + 16a^2b^6c^2d^3e^7 - 64a^3b^2c^5d^5e^5 + 128a^3b^3c^4d^4e^6 + 96a^3b^4c^3d^3e^7 + 8a^3b^5c^2d^2e^8 - 120a^4b^2c^4d^3e^7 - 124a^4b^3c^3d^2e^8 + 48ab^2c^7d^9e - 24ab^8cd^3e^7 + 48a^3b^6cd^9e - 28ab^3c^6d^8e^2 - 32ab^6c^3d^5e^5 + 64ab^7c^2d^4e^6 + 48a^2b^2c^7d^8e^2 + 20a^2b^7cd^2e^8 - 16a^4b^2c^5d^4e^6 - 184a^4b^4c^2d^2e^9 + 96a^5b^2c^4d^2e^8 + 240a^5b^2c^3d^2e^9)) / (c^3e^3) \cdot (-b^9d^2 + a^2b^7e^2 - b^6d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 28a^4b^2c^4d^2 - 9a^3b^5c^2e^2 - 20a^5b^2c^3e^2 - 2ab^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2 \cdot (-4ac - b^2)^3)^{(1/2)} + a^3c^3d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 - a^4c^2e^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 11ab^7cd^2 - 16a^5c^4d^2e + 2ab^5d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 20a^2b^6cd^2e - 6a^2b^2c^2d^2 \cdot (-4ac - b^2)^3)^{(1/2)} + 5ab^4cd^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3b^2c^2e^2 \cdot (-4ac - b^2)^3)^{(1/2)} - 8a^2b^3cd^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 6a^3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} / (
\end{aligned}$$

$$\begin{aligned}
& 8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^(1/2) - \\
& (16*a^3*c^6*d^9 + 4*a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 104*a^4*b^3*c^2*d^4*e^5 - 20*a^5*b^2*c^2*d^3*e^6 + 4*a*b^5*c^3*d^8*e + 4*a*b^7*c*d^6*e^3 + 64*a^3*b*c^5*d^8*e - 12*a^6*b^2*c*d*e^8 + 4*a*b^6*c^2*d^7*e^2 - 32*a^2*b^3*c^4*d^8*e - 44*a^2*b^6*c*d^5*e^4 + 36*a^3*b^5*c*d^4*e^5 - 128*a^4*b*c^4*d^6*e^3 + 8*a^4*b^4*c*d^3*e^6 + 88*a^5*b*c^3*d^4*e^5 + 8*a^5*b^3*c*d^2*e^7 + 4*a^6*b*c^2*d^2*e^7)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3))^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3))^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3))^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3))^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3))^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3))^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3))^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3))^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3))^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3))^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^(1/2) + (2*x*(a^8*e^8 + b^8*d^8 + 2*a^4*c^4*d^8 + 20*a^2*b^4*c^2*d^8 - 16*a^3*b^2*c^3*d^8 - 8*a*b^6*c*d^8))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3))^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3))^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3))^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3))^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3))^(1/2) + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3))^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3))^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3))^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3))^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3))^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^(1/2) + ((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) + (2*x*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 \\
& - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2 \\
& *e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d \\
& *e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^ \\
& 2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6 \\
& 6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4*a* \\
& c - b^2)^3)^{(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a* \\
& b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^ \\
& 3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - \\
& 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3* \\
& c^6*d*e^3)))^{(1/2)}*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^ \\
& 7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 \\
& - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6 \\
& *d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d \\
& ^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d \\
& ^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e \\
& ^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - \\
& 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 14 \\
& 4*a^3*b^3*c^6*d*e^11)/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 \\
& - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2 \\
& *e^2 - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d \\
& *e + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^ \\
& 2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6 \\
& 6*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e*(-(4*a* \\
& c - b^2)^3)^{(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a* \\
& b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^ \\
& 3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - \\
& 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3* \\
& c^6*d*e^3)))^{(1/2)} - (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 \\
& - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^ \\
& 9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 \\
& + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^ \\
& 5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - \\
& 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + \\
& 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - \\
& 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 2 \\
& 4*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^ \\
& 3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2* \\
& e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + \\
& 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4
\end{aligned}$$

$$\begin{aligned}
& a^2c - b^2)^3)^{(1/2)} + 28a^4b^4c^4d^2 - 9a^3b^5c^5e^2 - 20a^5b^4c^3e^2 \\
& - 2a^2b^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(- \\
& (4a^2c - b^2)^3)^{(1/2)} + a^3c^3d^2(-4a^2c - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 \\
& - a^4c^2e^2(-4a^2c - b^2)^3)^{(1/2)} - 11a^2b^7c^4d^2 - 16a^5c^4 \\
& *d^2e + 2a^2b^5d^2e(-4a^2c - b^2)^3)^{(1/2)} + 20a^2b^6c^4d^2e - 6a^2b^2c^2d^2 \\
& (-4a^2c - b^2)^3)^{(1/2)} + 5a^2b^4c^4d^2(-4a^2c - b^2)^3)^{(1/2)} - \\
& 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3b^2c^5e^2(-4a^2c - b^2)^3)^{(1/2)} \\
& - 8a^2b^3c^4d^2e(-4a^2c - b^2)^3)^{(1/2)} + 6a^3b^3c^2d^2e(-4a^2c - b^2)^3)^{(1/2)} \\
&) / (8(16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5c^6d^3e + a^2b^4c^5e^4 \\
& - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16a^2b^3c^7d^3e - 2a^2b^5c^5d^2e^3 \\
& - 32a^2b^2c^8d^3e - 32a^3b^2c^7d^2e^3 - 6a^2b^4c^6d^2e^2 + 16a^2b^3c^6d^2e^3)) \\
&)^{(1/2)} - (16a^3c^6d^9 + 4a^2b^4c^4d^9 + 4a^2b^8d^5e^4 + 4a^5b^4d^8e^8 + 4a^7c^2d^8e^8 \\
& - 20a^2b^2c^5d^9 - 4a^2b^7d^4e^5 - 4a^4b^5d^2e^7 - 64a^4c^5d^7e^2 + 64a^5c^4d^5e^4 + 4a^6c^3 \\
& *d^3e^6 - 36a^2b^4c^3d^7e^2 - 40a^2b^5c^2d^6e^3 + 96a^3b^2c^4d^7e^2 + 128a^3b^3c^3d^6e^3 \\
& + 164a^3b^4c^2d^5e^4 - 224a^4b^2c^3d^5e^4 - 104a^4b^3c^2d^4e^5 - 20a^5b^2c^2d^3e^6 \\
& + 4a^2b^5c^3d^8e + 4a^2b^7c^4d^6e^3 + 64a^3b^2c^5d^8e - 12a^6b^2c^4d^8e + 4a^2b^6c^2d^7e^2 \\
& - 32a^2b^3c^4d^8e - 44a^2b^6c^4d^5e^4 + 36a^3b^5c^4d^4e^5 - 128a^4b^2c^4d^6e^3 \\
& + 8a^4b^4c^4d^3e^6 + 88a^5b^2c^3d^4e^5 + 8a^5b^3c^4d^2e^7 + 4a^6b^2c^2d^2e^7) / (c^3e^3) * \\
& (-b^9d^2 + a^2b^7e^2 - b^6d^2(-4a^2c - b^2)^3)^{(1/2)} + 28a^4b^4c^4d^2 - 9a^3b^5c^5e^2 \\
& - 20a^5b^4c^3e^2 - 2a^2b^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(- \\
& (4a^2c - b^2)^3)^{(1/2)} + a^3c^3d^2(-4a^2c - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 \\
& - a^4c^2e^2(-4a^2c - b^2)^3)^{(1/2)} - 11a^2b^7c^4d^2 - 16a^5c^4d^2e + 2a^2b^5d^2e \\
& (-4a^2c - b^2)^3)^{(1/2)} + 20a^2b^6c^4d^2e - 6a^2b^2c^2d^2(-4a^2c - b^2)^3)^{(1/2)} \\
& + 5a^2b^4c^4d^2(-4a^2c - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3b^2c^5e^2 \\
& (-4a^2c - b^2)^3)^{(1/2)} - 8a^2b^3c^4d^2e(-4a^2c - b^2)^3)^{(1/2)} + 6a^3b^3c^2d^2e \\
& (-4a^2c - b^2)^3)^{(1/2)} / (8(16a^2c^9d^4 + 16a^4c^7e^4 + b^4c^7d^4 - 8a^2b^2c^8d^4 - 2b^5c^6d^3e \\
& + a^2b^4c^5e^4 - 8a^3b^2c^6e^4 + 32a^3c^8d^2e^2 + b^6c^5d^2e^2 + 16a^2b^3c^7d^3e - 2a^2b^5c^5d^2e^3 \\
& - 32a^2b^2c^8d^3e - 32a^3b^2c^7d^2e^3 - 6a^2b^4c^6d^2e^2 + 16a^2b^3c^6d^2e^3)) \\
&)^{(1/2)} - (2x(a^8e^8 + b^8d^8 + 2a^4c^4d^8 + 20a^2b^4c^2d^8 - 16a^3b^2c^3d^8 - 8a^2b^6c^4d^8)) / (c^3e^3) \\
&) * (-b^9d^2 + a^2b^7e^2 - b^6d^2(-4a^2c - b^2)^3)^{(1/2)} + 28a^4b^4c^4d^2 - 9a^3b^5c^5e^2 \\
& - 20a^5b^4c^3e^2 - 2a^2b^8d^2e + 42a^2b^5c^2d^2 - 63a^3b^3c^3d^2 - a^2b^4e^2(- \\
& (4a^2c - b^2)^3)^{(1/2)} + a^3c^3d^2(-4a^2c - b^2)^3)^{(1/2)} + 25a^4b^3c^2e^2 \\
& - a^4c^2e^2(-4a^2c - b^2)^3)^{(1/2)} - 11a^2b^7c^4d^2 - 16a^5c^4d^2e + 2a^2b^5d^2e \\
& (-4a^2c - b^2)^3)^{(1/2)} + 20a^2b^6c^4d^2e - 6a^2b^2c^2d^2(-4a^2c - b^2)^3)^{(1/2)} \\
& + 5a^2b^4c^4d^2(-4a^2c - b^2)^3)^{(1/2)} - 66a^3b^4c^2d^2e + 76a^4b^2c^3d^2e + 3a^3b^2c^5e^2 \\
& (-4a^2c - b^2)^3)^{(1/2)} - 8a^2b^3c^4d^2e(-4a^2c - b^2)^3)^{(1/2)} + 6a^3b^3c^2d^2e \\
& (-4a^2c - b^2)^3)^{(1/2)} / (8(16
\end{aligned}$$

$$\begin{aligned}
& *a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^{(1/2)} + (2*(a^4*b^3*d^7 + a^7*d^4*e^3 + a^5*b^2*d^6*e + a^6*b*d^5*e^2 - 2*a^5*b*c*d^7 - a^6*c*d^6*e))/(c^3*e^3)) * (- (b^9*d^2 + a^2*b^7*e^2 - b^6*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + a^3*c^3*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 25*a^4*b^3*c^2*e^2 - a^4*c^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e + 2*a*b^5*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 20*a^2*b^6*c*d*e - 6*a^2*b^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 3*a^3*b^2*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 6*a^3*b*c^2*d*e * (- (4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3))^{(1/2)} * 2i - (\log(a^9*d^4*e^26 - b^9*d^13*e^17 + 2*a*b^8*d^12*e^18 - 2*a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^19 + a^7*b^2*d^6*e^24 + 16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c^2*d^8*e^22 + b^4*c^5*d^18*e^12 + 16*a^2*c^7*x*(-d^7*e^5)^{(5/2)} + b^4*c^5*x*(-d^7*e^5)^{(5/2)} + a^9*e^24*x*(-d^7*e^5)^{(1/2)} - 8*a*b^2*c^6*x*(-d^7*e^5)^{(5/2)} - 42*a^2*b^5*c^2*d^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^2*d^12*e^18 - 76*a^4*b^2*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 + a^2*b^7*e^12*x*(-d^7*e^5)^{(3/2)} + b^9*d^2*e^10*x*(-d^7*e^5)^{(3/2)} + 11*a*b^7*c*d^13*e^17 - 2*a^7*b*c*d^7*e^23 - 8*a*b^2*c^6*d^18*e^12 - 20*a^2*b^6*c*d^12*e^18 + 9*a^3*b^5*c*d^11*e^19 - 28*a^4*b*c^4*d^13*e^17 + 20*a^5*b*c^3*d^11*e^19 + 25*a^4*b^3*c^2*e^12*x*(-d^7*e^5)^{(3/2)} + a^7*b^2*d^2*e^22*x*(-d^7*e^5)^{(1/2)} + a^7*c^2*d^4*e^20*x*(-d^7*e^5)^{(1/2)} - 2*a*b^8*d*e^11*x*(-d^7*e^5)^{(3/2)} - 2*a^8*b*d*e^23*x*(-d^7*e^5)^{(1/2)} - 9*a^3*b^5*c*e^12*x*(-d^7*e^5)^{(3/2)} - 20*a^5*b*c^3*e^12*x*(-d^7*e^5)^{(3/2)} - 16*a^5*c^4*d*e^11*x*(-d^7*e^5)^{(3/2)} + 2*a^8*c*d^2*e^22*x*(-d^7*e^5)^{(1/2)} - 11*a*b^7*c*d^2*e^10*x*(-d^7*e^5)^{(3/2)} + 20*a^2*b^6*c*d*e^11*x*(-d^7*e^5)^{(3/2)} - 2*a^7*b*c*d^3*e^21*x*(-d^7*e^5)^{(1/2)} - 66*a^3*b^4*c^2*d*e^11*x*(-d^7*e^5)^{(3/2)} + 28*a^4*b*c^4*d^2*e^10*x*(-d^7*e^5)^{(3/2)} + 76*a^4*b^2*c^3*d*e^11*x*(-d^7*e^5)^{(3/2)} + 42*a^2*b^5*c^2*d^2*e^10*x*(-d^7*e^5)^{(3/2)} - 63*a^3*b^3*c^3*d^2*e^10*x*(-d^7*e^5)^{(3/2)})) * (-d^7*e^5)^{(1/2)}) / (2*(a*e^7 + c*d^2*e^5 - b*d*e^6)) + (\log(a^9*d^4*e^26 - b^9*d^13*e^17 + 2*a*b^8*d^12*e^18 - 2*a^8*b*d^5*e^25 + 2*a^8*c*d^6*e^24 - a^2*b^7*d^11*e^19 + a^7*b^2*d^6*e^24 + 16*a^2*c^7*d^18*e^12 + 16*a^5*c^4*d^12*e^18 + a^7*c^2*d^8*e^22 + b^4*c^5*d^18*e^12 - 16*a^2*c^7*x*(-d^7*e^5)^{(5/2)} - b^4*c^5*x*(-d^7*e^5)^{(5/2)} - a^9*e^24*x*(-d^7*e^5)^{(1/2)} + 8*a*b^2*c^6*x*(-d^7*e^5)^{(5/2)} - 42*a^2*b^5*c^2*d^13*e^17 + 63*a^3*b^3*c^3*d^13*e^17 + 66*a^3*b^4*c^2*d^12*e^18 - 76*a^4*b^2*c^3*d^12*e^18 - 25*a^4*b^3*c^2*d^11*e^19 - a^2*b^7*e^12*x*(-d^7*e^5)^{(3/2)} - b^9*d^2*e^10*x*(-d^7*e^5)^{(3/2)} + 11*a*b^7*c*d^13
\end{aligned}$$

$$\begin{aligned}
& 3e^{17} - 2a^7 b c d^7 e^{23} - 8a^2 b^2 c^6 d^{18} e^{12} - 20a^2 b^6 c d^{12} e^{18} \\
& + 9a^3 b^5 c d^{11} e^{19} - 28a^4 b^2 c^4 d^{13} e^{17} + 20a^5 b^3 c^3 d^{11} e^{19} \\
& - 25a^4 b^3 c^2 e^{12} x (-d^7 e^5)^{3/2} - a^7 b^2 d^2 e^{22} x (-d^7 e^5)^{1/2} \\
& - a^7 c^2 d^4 e^{20} x (-d^7 e^5)^{1/2} + 2a^2 b^8 d e^{11} x (-d^7 e^5)^{3/2} \\
& + 2a^8 b d e^{23} x (-d^7 e^5)^{1/2} + 9a^3 b^5 c e^{12} x (-d^7 e^5)^{3/2} \\
& + 20a^5 b^3 c^3 e^{12} x (-d^7 e^5)^{3/2} + 16a^5 c^4 d e^{11} x (-d^7 e^5)^{3/2} \\
& - 2a^8 c d^2 e^{22} x (-d^7 e^5)^{1/2} + 11a^2 b^7 c d^2 e^{10} x (-d^7 e^5)^{3/2} \\
& - 20a^2 b^6 c d e^{11} x (-d^7 e^5)^{3/2} + 2a^7 b^3 c d^3 e^{21} x (-d^7 e^5)^{1/2} \\
& + 66a^3 b^4 c^2 d e^{11} x (-d^7 e^5)^{3/2} - 28a^4 b^2 c^4 d^2 e^{10} x (-d^7 e^5)^{3/2} \\
& - 76a^4 b^2 c^3 d e^{11} x (-d^7 e^5)^{3/2} - 42a^2 b^5 c^2 d^2 e^{10} x (-d^7 e^5)^{3/2} \\
& + 63a^3 b^3 c^3 d^2 e^{10} x (-d^7 e^5)^{3/2} (-d^7 e^5)^{1/2} / (2a^7 e^7 + 2c d^2 e^5 - 2b d e^6) + x^3 / (3c e) \\
& - (x(b e + c d)) / (c^2 e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.226 \quad \int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=323

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(ae^2 - bde + cd^2) + \sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(ae^2 - bde + cd^2)}$$

Rubi [A] time = 1.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right) - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right)}{e^{3/2}(ae^2 - bde + cd^2)} + \frac{x}{ce}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(ae^2 - bde + cd^2) + \sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} dx &= \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex^2)} + \frac{a(bd-ae) + (b^2d-acd-abe)x^2}{c(cd^2-bde+ae^2)(a+bx^2+cx^4)} \right) dx \\ &= \frac{x}{ce} + \frac{\int \frac{a(bd-ae) + (b^2d-acd-abe)x^2}{a+bx^2+cx^4} dx}{c(cd^2-bde+ae^2)} - \frac{d^3 \int \frac{1}{d+ex^2} dx}{e(cd^2-bde+ae^2)} \\ &= \frac{x}{ce} - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right) \int \frac{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}{a+bx^2+cx^4} dx}{2c(cd^2-bde+ae^2)} \\ &= \frac{x}{ce} + \frac{\left(b^2d-acd-abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)}{c(cd^2-bde+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.52, size = 385, normalized size = 1.19

$$\frac{(-b^2(d\sqrt{b^2-4ac}+ae) + ab(e\sqrt{b^2-4ac}-3cd) + ac(d\sqrt{b^2-4ac}+2ae) + b^3d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + (b^2(d\sqrt{b^2-4ac}-ae) - ab(e\sqrt{b^2-4ac}+3cd) + ac(2ae-d\sqrt{b^2-4ac}) + b^3d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right) - \frac{d^{5/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2-bde+cd^2)} + \frac{x}{ce}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(e(bd-ae)-cd^2) + \sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}} + b(e(ae-bd)+cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]
)*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e
*(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 13.87, size = 11030, normalized size = 34.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-d^{5/2} \arctan(xe^{1/2}/\sqrt{d})e^{-1/2}/(cd^2e - bde^2 + ae^3) - 1/8((2b^6c^6 - 14ab^4c^7 + 24a^2b^2c^8 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^6c^4 + 7\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^5c^5 - 12\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^6 - 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^6 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^4c^6 + 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^7 - 2(b^2 - 4ac)b^4c^6 + 6(b^2 - 4ac)a^2b^2c^7)d^5 - (4b^7c^5 - 26ab^5c^6 + 36a^2b^3c^7 + 16a^3bc^8 - 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^7c^3 + 13\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^5c^4 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^6c^4 - 18\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^5 - 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^5 - 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^5c^5 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3bc^6 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^6 + 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)$

$$\begin{aligned}
& b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^3*c^6 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c} \\
& - \sqrt{b^2 - 4*a*c}*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a \\
& *c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}*c)*a*b^5*c^3 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^ \\
& 2*b^3*c^4 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^4*c^4 + 2*a*b^5*c \\
& ^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^3*b*c^5 + 8*\sqrt{2}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^2*c^5 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^ \\
& 2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*c^4 - 6*a*b^6*c^5 - \\
& 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}*c)*b^8*c^2 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 \\
& - 4*a*c}*c)*a*b^6*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}*c)*b^7*c^3 + 14*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}*c)*a^2*b^4*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& }*c)*a*b^5*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)* \\
& b^6*c^4 - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^3* \\
& b^2*c^5 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2* \\
& b^3*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^4*c \\
& ^5 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^2*c \\
& ^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c) \\
& *a^2*b^2*c^6)*d^3*e^2 + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^6*c^ \\
& 2 - 7*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^4*c^3 - 2*\sqrt{2}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}*c)*a^3*b^2*c^4 + 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*b^3*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^4*c^4 - 14*a^2*b \\
& ^4*c^4 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a^4*c^5 + 8*\sqrt{2}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c)*a^3*b*c^5 - 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4* \\
& a*c}*c)*a^2*b^2*c^5 + 16*a^3*b^2*c^5 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}*c)*a^3*c^6 + 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 6*(b^2 - 4*a*c)*a^ \\
& 2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)* \\
& e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \sqrt{2}*\sqrt{ \\
& b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*b^6 + 9*\sqrt{2}*\sqrt{b^2 - 4*a \\
& *c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}* \\
& \sqrt{b*c - \sqrt{b^2 - 4*a*c}*c)*b^5*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b* \\
& c - \sqrt{b^2 - 4*a*c}*c)*a^2*b^2*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b* \\
& c - \sqrt{b^2 - 4*a*c}*c)*a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}*c)*b^4*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}*c)*a^3*c^3 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}*c)*a^2*b*c^3 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4* \\
& a*c}*c)*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& }*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^ \\
& 2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a \\
& ^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}*c)*a*b^7*c^2 + 18*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{
\end{aligned}$$

$$\begin{aligned}
& - 4*a*c)*c)*a^5*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^4 + 4*(b^2 - 4*a*c)*a^4*b*c^5)*e^5)*\arctan(2*\sqrt{1/2}*x/\sqrt{((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2 + \sqrt{((b*c^2*d^2 - b^2*c*d*e + a*b*c*e^2))^2 - 4*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)))/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2))})/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*d^4*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c) - 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 - 2*a*b^4*c^5 + 16*a^3*b*c^6 + 8*a^2*b^2*c^6 + a*b^3*c^6 - 4*a^2*b*c^7)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e + (a*b^6*c^3 - 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32*a^4*c^6 + 16*a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4 + 16*a^4*b*c^5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3*c^4 + 16*a^5*c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^4) + 1/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - \sqrt{2})*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c^4 + 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^5 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^5 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^6 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^7*c^3 + 13*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6*c^4 - 18*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^5 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^5 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^5 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^6 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 + 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^6 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^4 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^5 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^5 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*c)*a^2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*c^4 - 6*a*b^6*c^5 - 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}
\end{aligned}$$

$$\begin{aligned}
& a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^8*c^2 + 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^6*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^7*c^3 + 14*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c^4 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^5*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*b^6*c^4 - 40*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^3*b^2*c^5 - 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^2*b^3*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a*b^4*c^5 + 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 \\
& + 20*(b^2 - 4*a*c)*a^2*b^2*c^6)*d^3*e^2 - 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a*b^6*c^2 - 7*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c^3 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a*b^5*c^3 - 2*a*b^6*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^2*c^4 + 6*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^2*b^3*c^4 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c^4 + 14*a^2*b^4*c^4 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^4*c^5 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b*c^5 - 3*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b^2 - 4*a*c))*a^2*b^2*c^5 - 16*a^3*b^2*c^5 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*c^6 - 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 - \\
& 6*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*\text{abs}(-c^2*d^2 + b*c*d*e - a*c*e^2)*e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^6 + 9*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^4*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^5*c - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^2*c^2 - 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^3*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^4*c^2 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b*c^3 + 5*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^2*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c) \\
&)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^7*c^2 + 18*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^5*c^3 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^6*c^3 - 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^3*c^4 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^4*c^4 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^5*c^4 - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^4*b*c^5 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b^2*c^5 + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^3*c^5 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^3*b*c^6 - 6*(b^2 - 4*a*c)*a*b^5*c^4 + \\
& 12*(b^2 - 4*a*c)*a^2*b^3*c^5 + 8*(b^2 - 4*a*c)*a^3*b*c^6)*d^2*e^3 + 4*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*b^5*c^2 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))* \\
\end{aligned}$$

$$\begin{aligned}
& t(b^2 - 4ac)c a^3 b^3 c^3 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^4 c^3 - 2a^2 b^5 c^3 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 \\
& * b^c^4 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^2 c^4 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^3 c^4 + 16a^3 b^3 c^4 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c \\
& a^3 b^c^5 - 32a^4 b^c^5 + 2(b^2 - 4ac)a^2 b^3 c^3 - 8(b^2 - 4ac)a^3 b^c^4) * d^2 * \text{abs}(-c^2 d^2 + b^c d^e - a^c e^2) * e^2 \\
& + (2ab^5 c^2 - 16a^2 b^3 c^3 + 32a^3 b^c^4 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^5 + 8\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c \\
& a^2 b^3 c^c + 2\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^4 c^c - 16\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c \\
& a^2 b^2 c^2 - \sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^3 c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^c^3 - 2(b^2 - 4ac)a^2 b^3 c^2 + 8(b^2 - 4ac)a^2 b^c^3 \\
&) * (c^2 d^2 - b^c d^e + a^c e^2)^2 * e + (6a^2 b^6 c^4 - 38a^3 b^4 c^5 + 56a^4 b^2 c^6 - 3\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^6 c^2 + 19\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^4 c^3 + 6\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^5 c^3 - 28\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^2 c^4 - 14\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^3 c^4 - 3\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^2 b^4 c^4 + 7\sqrt{2}\sqrt{b^2 - 4ac} * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^2 c^5 - 6(b^2 - 4ac)a^2 b^4 c^4 + 14(b^2 - 4ac)a^3 b^2 c^5) * d^e^4 - 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^4 c^2 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^2 c^3 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^3 c^3 - 2a^3 b^4 c^3 + 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^5 c^4 + 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^c^4 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^2 c^4 + 16a^4 b^2 c^4 - 4\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 c^5 - 32a^5 c^5 + 2(b^2 - 4ac)a^3 b^2 c^3 - 8(b^2 - 4ac)a^4 c^4) * \text{abs}(-c^2 d^2 + b^c d^e - a^c e^2) * e^3 - (2a^3 b^5 c^4 - 12a^4 b^3 c^5 + 16a^5 b^c^6 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^5 c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^3 c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^4 c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^5 b^c^4 - 4\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^2 c^4 - \sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^3 b^3 c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac}) * \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4 b^c^5 - 2(b^2 - 4ac)a^3 b^3 c^4 + 4(b^2 - 4ac)a^4 b^c^5) * e^5) * \arctan(2\sqrt{2}\sqrt{x/\sqrt{(b^c d^2 - b^2 c^d e + a^c e^2 - \sqrt{(b^c d^2 - b^2 c^d e + a^c e^2)^2 - 4(a^c d^2 - a^b c^d e + a^2 c^e^2)(c^3 d^2 - b^c d^e + a^c e^2))})}) / (c^3 d^2 - b^c d^e + a^c e^2))) / ((a^b^4 c^5 - 8a^2 b^2 c^6 - 2a^b^3 c^6 + 16a^3 c^7 + 8a^2 b^c^7 + a^b^2 c^7 - 4a^2 c^8) * d^4 * \text{abs}(-c^2 d^2 + b^c d^e - a^c e^2) * \text{abs}(c) - 2(a^b^5 c^4 - 8a^2 b^3 c^5 - 2a^b^4 c^5 + 16a^3 b^c^6 + 8a^2 b^2 c^6 + a^b^3 c^6 - 4a^2 b^c^7) * d^3 * \text{abs}(-c^2 d^2 + b^c d^e - a^c e^2) * \text{abs}(
\end{aligned}$$

$$c)*e + (a*b^6*c^3 - 6*a^2*b^4*c^4 - 2*a*b^5*c^4 + 4*a^2*b^3*c^5 + a*b^4*c^5 + 32*a^4*c^6 + 16*a^3*b*c^6 - 2*a^2*b^2*c^6 - 8*a^3*c^7)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^2 - 2*(a^2*b^5*c^3 - 8*a^3*b^3*c^4 - 2*a^2*b^4*c^4 + 16*a^4*b*c^5 + 8*a^3*b^2*c^5 + a^2*b^3*c^5 - 4*a^3*b*c^6)*d*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^3 + (a^3*b^4*c^3 - 8*a^4*b^2*c^4 - 2*a^3*b^3*c^4 + 16*a^5*c^5 + 8*a^4*b*c^5 + a^3*b^2*c^5 - 4*a^4*c^6)*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*abs(c)*e^4 + x*e^(-1)/c$$

maple [B] time = 0.04, size = 1098, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6/(e*x^2+d)/(c*x^4+b*x^2+a), x)$

[Out] $\frac{1}{c} \frac{1}{e} x^{1/2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c^2} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b e^{1/2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a d - \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c^2} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) b^2 d + \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a^2 e - \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b^2 e - \frac{3}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b d + \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctanh\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) b^3 d - \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c^2} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b e - \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a d + \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c^2} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) b^2 d + \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a^2 e - \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b^2 e - \frac{3}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) a b d + \frac{1}{2} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{c} \frac{1}{(-4 a^2 c + b^2)^{1/2}} \frac{1}{c} \frac{1}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} \arctan\left(\frac{2^{1/2}}{((b + (-4 a^2 c + b^2))^{1/2})} \frac{1}{c} x\right) b^3 d - \frac{1}{e} \frac{1}{d^3} \frac{1}{(a e^2 - b d e + c d^2)^{1/2}} \frac{1}{(d e)^{1/2}} \arctan\left(\frac{1}{(d e)^{1/2}} e x\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e - bde^2 + ae^3)\sqrt{de}} - \frac{\int \frac{abd - a^2e - (abe - (b^2 - ac)d)x^2}{cx^4 + bx^2 + a} dx}{c^2d^2 - bcde + ace^2} + \frac{x}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] -d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e - b*d*e^2 + a*e^3)*sqrt(d*e)) - integrate(-(a*b*d - a^2*e - (a*b*e - (b^2 - a*c)*d)*x^2)/(c*x^4 + b*x^2 + a), x)/(c^2*d^2 - b*c*d*e + a*c*e^2) + x/(c*e)

mupad [B] time = 6.45, size = 33892, normalized size = 104.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] x/(c*e) - atan((((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)

$$\begin{aligned}
& - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2* \\
& a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3) \\
& ^{(1/2)})/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 \\
& - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e \\
& ^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^ \\
& 6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)) \\
&)^{(1/2)} + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3* \\
& c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d \\
& ^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5 \\
& *e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b* \\
& c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2* \\
& d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^ \\
& 7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a \\
& *b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5 \\
& *c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2* \\
& d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 1 \\
& 6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3 \\
& *a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e* \\
& (-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 \\
& - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 \\
& + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d \\
& *e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a \\
& ^2*b^3*c^4*d*e^3)))^{(1/2)} - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d \\
& ^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4 \\
& *d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^ \\
& 2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c* \\
& d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^ \\
& 2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e \\
&))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c \\
& ^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{ \\
& (1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{ \\
& (1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d* \\
& e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)})/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c \\
& ^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6 \\
& *d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^ \\
& 2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d \\
& *e^3)))^{(1/2)} + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 \\
& - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2
\end{aligned}$$

$$\begin{aligned}
&)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c* \\
& e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5* \\
& c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4* \\
& c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a \\
& ^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2))}/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + \\
& b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b \\
& ^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2* \\
& a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2 \\
& *e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)}*i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^ \\
& 6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4* \\
& d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3* \\
& d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^ \\
& 5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c* \\
& e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20* \\
& a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4 \\
& *c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^ \\
& 2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4* \\
& a*c - b^2)^3)^{(1/2))}/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8* \\
& a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32* \\
& a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 \\
& - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^ \\
& 3*c^4*d*e^3)))^{(1/2)}*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5* \\
& c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 \\
& - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4 \\
& *d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d \\
& ^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7* \\
& e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - \\
& 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576* \\
& a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3* \\
& b^3*c^4*d*e^9)))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c \\
& *d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c \\
& *d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2* \\
& b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2))}/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4 \\
& *c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2* \\
& c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b \\
& ^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^ \\
& 2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} - (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4 \\
& *b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^ \\
& 2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7
\end{aligned}$$

$$\begin{aligned}
& *c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4 \\
& *b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3 \\
& *d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2 \\
& *d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 2 \\
& 8*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2* \\
& b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3* \\
& d^2*e^6 - 108*a^4*b^2*c^2*d*e^7)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2 \\
& *(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c \\
& ^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2* \\
& d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1 \\
& /2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^ \\
& 2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16 \\
& *a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^ \\
& 3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3 \\
& *c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - \\
& 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16 \\
& *a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a \\
& ^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 \\
& - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a* \\
& b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4 \\
& *e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + \\
& 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - \\
& b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3 \\
& *c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^ \\
& 2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a \\
& *b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2 \\
& *b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - \\
& 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 \\
& + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^ \\
& 3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - \\
& 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4* \\
& d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2) - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c \\
& ^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^ \\
& 2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + \\
& 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a \\
& ^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a* \\
& c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - \\
& b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) \\
& - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c \\
& ^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + \\
& a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 \\
& + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c \\
& ^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*i)/((((((64 \\
& *a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5
\end{aligned}$$

$$\begin{aligned}
& d^5 e^4 + 64 a^2 b^3 c^4 d^4 e^5 + 16 a^2 b^4 c^3 d^3 e^6 - 96 a^3 b^2 c^4 \\
& d^3 e^6 + 16 a^3 b^3 c^3 d^2 e^7 - 16 a b^3 c^5 d^6 e^3 + 32 a b^4 c^4 d^5 \\
& e^4 - 16 a b^5 c^3 d^4 e^5 + 64 a^2 b c^6 d^6 e^3 - 64 a^4 b c^4 d^2 e^7 - \\
& 16 a^4 b^2 c^3 d e^8) / (c e) - (2 x x (-b^7 d^2 + a^2 b^5 e^2 - b^4 d^2 (-4 \\
& a c - b^2)^3)^{(1/2)} - 20 a^3 b c^3 d^2 - 7 a^3 b^3 c e^2 + 12 a^4 b c^2 e^ \\
& 2 + a^3 c e^2 (-4 a c - b^2)^3)^{(1/2)} - 2 a b^6 d e + 25 a^2 b^3 c^2 d^2 - \\
& a^2 b^2 e^2 (-4 a c - b^2)^3)^{(1/2)} - a^2 c^2 d^2 (-4 a c - b^2)^3)^{(1/2)} \\
&) - 9 a b^5 c d^2 + 16 a^4 c^3 d e + 2 a b^3 d e (-4 a c - b^2)^3)^{(1/2)} + \\
& 16 a^2 b^4 c d e + 3 a b^2 c d^2 (-4 a c - b^2)^3)^{(1/2)} - 36 a^3 b^2 c^2 \\
& d e - 4 a^2 b c d e (-4 a c - b^2)^3)^{(1/2)}) / (8 (16 a^2 c^7 d^4 + 16 a^4 c \\
& c^5 e^4 + b^4 c^5 d^4 - 8 a b^2 c^6 d^4 - 2 b^5 c^4 d^3 e + a^2 b^4 c^3 e^4 \\
& - 8 a^3 b^2 c^4 e^4 + 32 a^3 c^6 d^2 e^2 + b^6 c^3 d^2 e^2 + 16 a b^3 c^5 d \\
& d^3 e - 2 a b^5 c^3 d e^3 - 32 a^2 b c^6 d^3 e - 32 a^3 b c^5 d e^3 - 6 a b \\
& ^4 c^4 d^2 e^2 + 16 a^2 b^3 c^4 d e^3)))^{(1/2)} * (128 a^4 b^2 c^4 e^{10} - 16 a \\
& ^3 b^4 c^3 e^{10} - 256 a^5 c^5 e^{10} + 256 a^2 c^8 d^6 e^4 + 256 a^3 c^7 d^4 e \\
& e^6 - 256 a^4 c^6 d^2 e^8 - 16 b^3 c^7 d^7 e^3 + 64 b^4 c^6 d^6 e^4 - 96 b^ \\
& 5 c^5 d^5 e^5 + 64 b^6 c^4 d^4 e^6 - 16 b^7 c^3 d^3 e^7 + 256 a^2 b^2 c^6 d \\
& ^4 e^6 + 144 a^2 b^3 c^5 d^3 e^7 - 96 a^2 b^4 c^4 d^2 e^8 + 192 a^3 b^2 c^5 \\
& d^2 e^8 + 64 a b c^8 d^7 e^3 + 320 a^4 b c^5 d e^9 - 320 a b^2 c^7 d^6 e^4 \\
& + 528 a b^3 c^6 d^5 e^5 - 336 a b^4 c^5 d^4 e^6 + 48 a b^5 c^4 d^3 e^7 + 1 \\
& 6 a b^6 c^3 d^2 e^8 - 576 a^2 b c^7 d^5 e^5 + 16 a^2 b^5 c^3 d e^9 - 320 a^ \\
& 3 b c^6 d^3 e^7 - 144 a^3 b^3 c^4 d e^9)) / (c e)) * (-b^7 d^2 + a^2 b^5 e^2 - \\
& b^4 d^2 (-4 a c - b^2)^3)^{(1/2)} - 20 a^3 b c^3 d^2 - 7 a^3 b^3 c e^2 + 12 \\
& a^4 b c^2 e^2 + a^3 c e^2 (-4 a c - b^2)^3)^{(1/2)} - 2 a b^6 d e + 25 a^2 b \\
& b^3 c^2 d^2 - a^2 b^2 e^2 (-4 a c - b^2)^3)^{(1/2)} - a^2 c^2 d^2 (-4 a c - \\
& b^2)^3)^{(1/2)} - 9 a b^5 c d^2 + 16 a^4 c^3 d e + 2 a b^3 d e (-4 a c - b^ \\
& 2)^3)^{(1/2)} + 16 a^2 b^4 c d e + 3 a b^2 c d^2 (-4 a c - b^2)^3)^{(1/2)} - 3 \\
& 6 a^3 b^2 c^2 d e - 4 a^2 b c d e (-4 a c - b^2)^3)^{(1/2)}) / (8 (16 a^2 c^7 d \\
& d^4 + 16 a^4 c^5 e^4 + b^4 c^5 d^4 - 8 a b^2 c^6 d^4 - 2 b^5 c^4 d^3 e + a^ \\
& 2 b^4 c^3 e^4 - 8 a^3 b^2 c^4 e^4 + 32 a^3 c^6 d^2 e^2 + b^6 c^3 d^2 e^2 + \\
& 16 a b^3 c^5 d^3 e - 2 a b^5 c^3 d e^3 - 32 a^2 b c^6 d^3 e - 32 a^3 b c^5 d \\
& e^3 - 6 a b^4 c^4 d^2 e^2 + 16 a^2 b^3 c^4 d e^3)))^{(1/2)} + (2 x x (4 a^3 b \\
& ^5 e^8 + 4 b^3 c^5 d^8 + 4 b^8 d^3 e^5 - 28 a^4 b^3 c e^8 + 48 a^5 b c^2 e^ \\
& 8 - 4 a b^7 d^2 e^6 - 4 a^2 b^6 d e^7 - 64 a^2 c^6 d^7 e + 56 a^5 c^3 d e^7 \\
& - 8 b^4 c^4 d^7 e - 8 b^7 c d^4 e^4 - 8 a^3 c^5 d^5 e^3 - 16 a^4 c^4 d^3 e \\
& ^5 + 4 b^5 c^3 d^6 e^2 + 4 b^6 c^2 d^5 e^3 - 16 a b c^6 d^8 + 36 a^2 b^2 c^ \\
& 4 d^5 e^3 - 72 a^2 b^3 c^3 d^4 e^4 - 12 a^2 b^4 c^2 d^3 e^5 + 64 a^3 b^2 c^ \\
& 3 d^3 e^5 + 28 a^3 b^3 c^2 d^2 e^6 + 48 a b^2 c^5 d^7 e - 16 a b^6 c^3 d^3 e^ \\
& 5 + 40 a^3 b^4 c d e^7 - 28 a b^3 c^4 d^6 e^2 - 24 a b^4 c^3 d^5 e^3 + 48 a \\
& b^5 c^2 d^4 e^4 + 48 a^2 b c^5 d^6 e^2 + 12 a^2 b^5 c d^2 e^6 + 16 a^3 b c \\
& ^4 d^4 e^4 - 64 a^4 b c^3 d^2 e^6 - 108 a^4 b^2 c^2 d e^7)) / (c e)) * (-b^7 d \\
& ^2 + a^2 b^5 e^2 - b^4 d^2 (-4 a c - b^2)^3)^{(1/2)} - 20 a^3 b c^3 d^2 - 7 a \\
& ^3 b^3 c e^2 + 12 a^4 b c^2 e^2 + a^3 c e^2 (-4 a c - b^2)^3)^{(1/2)} - 2 a \\
& b^6 d e + 25 a^2 b^3 c^2 d^2 - a^2 b^2 e^2 (-4 a c - b^2)^3)^{(1/2)} - a^2 c \\
& ^2 d^2 (-4 a c - b^2)^3)^{(1/2)} - 9 a b^5 c d^2 + 16 a^4 c^3 d e + 2 a b^3
\end{aligned}$$

$$\begin{aligned}
& *d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2* \\
& b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + \\
& b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3* \\
& b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 \\
& + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92* \\
& a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 \\
& - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5 \\
& *e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 \\
& + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 2 \\
& 5*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^ \\
& 2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3* \\
& e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2* \\
& e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3* \\
& b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} + (2*x*(a \\
& ^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e \\
&))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c \\
& ^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d* \\
& e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)})/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3* \\
& e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3* \\
& b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^ \\
& 3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^ \\
& ^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6 \\
& *e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - \\
& 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) + (2*x*(-(b^7*d^2 + a^2 \\
& *b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3* \\
& c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e \\
& + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*c^2*d^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(1 \\
& 6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
\end{aligned}$$

$$\begin{aligned}
& d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^3e - 32a^2b^6c^6d^3e - 32a^3b^3c^5d^3e - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^3e^3))^{(1/2)} * (128a^4b^2c^4e^{10} - 16a^3b^4c^3e^{10} - 256a^5c^5e^{10} + 256a^2c^8d^6e^4 + 256a^3c^7d^4e^6 - 256a^4c^6d^2e^8 - 16b^3c^7d^7e^3 + 64b^4c^6d^6e^4 - 96b^5c^5d^5e^5 + 64b^6c^4d^4e^6 - 16b^7c^3d^3e^7 + 256a^2b^2c^6d^4e^6 + 144a^2b^3c^5d^3e^7 - 96a^2b^4c^4d^2e^8 + 192a^3b^2c^5d^2e^8 + 64a^4b^3c^8d^7e^3 + 320a^4b^3c^5d^8e^9 - 320a^4b^2c^7d^6e^4 + 528a^4b^3c^6d^5e^5 - 336a^4b^4c^5d^4e^6 + 48a^4b^5c^4d^3e^7 + 16a^4b^6c^3d^2e^8 - 576a^2b^6c^7d^5e^5 + 16a^2b^5c^3d^3e^9 - 320a^3b^6c^6d^3e^7 - 144a^3b^3c^4d^3e^9) / (c^2e) * (- (b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 + a^3c^3e^2 * (-4ac - b^2)^3)^{(1/2)} - 2ab^6d^3e + 25a^2b^3c^2d^2 - a^2b^2e^2 * (-4ac - b^2)^3)^{(1/2)} - a^2c^2d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3d^2 + 16a^4c^3d^3e + 2ab^3d^3e * (-4ac - b^2)^3)^{(1/2)} + 16a^2b^4c^3d^3e + 3ab^2c^3d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^3e - 4a^2b^3c^2d^3e * (-4ac - b^2)^3)^{(1/2)}) / (8 * (16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^3e - 32a^2b^6c^6d^3e - 32a^3b^3c^5d^3e - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^3e^3))^{(1/2)} - (2x * (4a^3b^5e^8 + 4b^3c^5d^8 + 4b^8d^3e^5 - 28a^4b^3c^3e^8 + 48a^5b^3c^2e^8 - 4ab^7d^2e^6 - 4a^2b^6d^3e^7 - 64a^2c^6d^7e + 56a^5c^3d^3e^7 - 8b^4c^4d^7e - 8b^7c^4d^4e^4 - 8a^3c^5d^5e^3 - 16a^4c^4d^3e^5 + 4b^5c^3d^6e^2 + 4b^6c^2d^5e^3 - 16ab^6c^6d^8 + 36a^2b^2c^4d^5e^3 - 72a^2b^3c^3d^4e^4 - 12a^2b^4c^2d^3e^5 + 64a^3b^2c^3d^3e^5 + 28a^3b^3c^2d^2e^6 + 48ab^2c^5d^7e - 16ab^6c^3d^3e^5 + 40a^3b^4c^3d^3e^7 - 28ab^3c^4d^6e^2 - 24a^4b^4c^3d^5e^3 + 48ab^5c^2d^4e^4 + 48a^2b^6c^5d^6e^2 + 12a^2b^5c^4d^2e^6 + 16a^3b^6c^4d^4e^4 - 64a^4b^3c^3d^2e^6 - 108a^4b^2c^2d^3e^7)) / (c^2e) * (- (b^7d^2 + a^2b^5e^2 - b^4d^2 * (-4ac - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 + a^3c^3e^2 * (-4ac - b^2)^3)^{(1/2)} - 2ab^6d^3e + 25a^2b^3c^2d^2 - a^2b^2e^2 * (-4ac - b^2)^3)^{(1/2)} - a^2c^2d^2 * (-4ac - b^2)^3)^{(1/2)} - 9ab^5c^3d^2 + 16a^4c^3d^3e + 2ab^3d^3e * (-4ac - b^2)^3)^{(1/2)} + 16a^2b^4c^3d^3e + 3ab^2c^3d^2 * (-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^3e - 4a^2b^3c^2d^3e * (-4ac - b^2)^3)^{(1/2)}) / (8 * (16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^3e - 32a^2b^6c^6d^3e - 32a^3b^3c^5d^3e - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^3e^3))^{(1/2)} - (4ab^3c^3d^7 - 16a^2b^6c^4d^7 + 4ab^6d^4e^3 + 4a^4b^3d^6e + 48a^3c^4d^6e - 4a^2b^5d^3e^4 - 4a^3b^4d^2e^5 - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5 - 8a^5b^3c^3d^6e - 32a^2b^3c^2d^5e^2 + 92a^3b^2c^2d^4e^3 + 4ab^4c^2d^6e + 4ab^5c^2d^5e^2 - 28a^2b^2c^3d^6e - 36a^2b^4c^3d^4e^3 + 64a^3b^3c^3d^5e^
\end{aligned}$$

$$\begin{aligned}
& 2 + 36a^3b^3c^2d^3e^4 - 60a^4b^2c^2d^3e^4 + 4a^4b^2c^2d^2e^5)/(c^2e^2) \\
& \cdot (-b^7d^2 + a^2b^5e^2 - b^4d^2(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3d^2 \\
& - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 + a^3c^2e^2(-4ac - b^2)^3)^{1/2} \\
& - 2a^2b^6d^2e + 25a^2b^3c^2d^2 - a^2b^2e^2(-4ac - b^2)^3)^{1/2} \\
& - a^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e \\
& + 2a^2b^3d^2e(-4ac - b^2)^3)^{1/2} + 16a^2b^4c^2d^2e + 3a^2b^2c^2d^2 \\
& \cdot (-4ac - b^2)^3)^{1/2} - 36a^3b^2c^2d^2e - 4a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2} \\
&)/(8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e \\
& + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e \\
& - 2a^2b^5c^3d^2e^3 - 32a^2b^2c^6d^3e - 32a^3b^2c^5d^2e^3 - 6a^2b^4c^4d^2e^2 \\
& + 16a^2b^3c^4d^2e^3)))^{1/2} - (2x(a^6e^6 + b^6d^6 - 2a^3c^3d^6 + 9a^2b^2c^2d^6 \\
& - 6a^2b^4c^2d^6))/(c^2e^2) \cdot (-b^7d^2 + a^2b^5e^2 - b^4d^2(-4ac - b^2)^3)^{1/2} \\
& - 20a^3b^3c^3d^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 + a^3c^2e^2(-4ac - b^2)^3)^{1/2} \\
& - 2a^2b^6d^2e + 25a^2b^3c^2d^2 - a^2b^2e^2(-4ac - b^2)^3)^{1/2} \\
& - a^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e + 2a^2b^3d^2e \\
& \cdot (-4ac - b^2)^3)^{1/2} + 16a^2b^4c^2d^2e + 3a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2} \\
& - 36a^3b^2c^2d^2e - 4a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7d^4 + 16a^4c^5e^4 \\
& + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 \\
& + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e - 2a^2b^5c^3d^2e^3 \\
& - 32a^2b^2c^6d^3e - 32a^3b^2c^5d^2e^3 - 6a^2b^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3)))^{1/2} \\
& + (2(a^3b^2d^5 - a^4c^2d^5 + a^5d^3e^2 + a^4b^2d^4e))/(c^2e^2) \cdot (-b^7d^2 + a^2b^5e^2 \\
& - b^4d^2(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3d^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 \\
& + a^3c^2e^2(-4ac - b^2)^3)^{1/2} - 2a^2b^6d^2e + 25a^2b^3c^2d^2 - a^2b^2e^2(-4ac \\
& - b^2)^3)^{1/2} - a^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e \\
& + 2a^2b^3d^2e(-4ac - b^2)^3)^{1/2} + 16a^2b^4c^2d^2e + 3a^2b^2c^2d^2e(-4ac \\
& - b^2)^3)^{1/2} - 36a^3b^2c^2d^2e - 4a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7d^4 \\
& + 16a^4c^5e^4 + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 \\
& + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e - 2a^2b^5c^3d^2e^3 - 32a^2b^2c^6d^3e \\
& - 32a^3b^2c^5d^2e^3 - 6a^2b^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3)))^{1/2} * 2i - \operatorname{atan}(\frac{(((((64a^5c^4d^2e^8 + 64a^4c^3d^5e^4 + 128a^4c^5d^3e^6 - 144a^2b^2c^5d^5e^4 + 64a^2b^3c^4d^4e^5 + 16a^2b^4c^3d^3e^6 - 96a^3b^2c^4d^3e^6 + 16a^3b^3c^3d^2e^7 - 16a^2b^3c^5d^6e^3 + 32a^2b^4c^4d^5e^4 - 16a^2b^5c^3d^4e^5 + 64a^2b^2c^6d^6e^3 - 64a^4b^2c^4d^2e^7 - 16a^4b^2c^3d^2e^8))/(c^2e^2) - (2x(-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3d^2 - 7a^3b^3c^2e^2 + 12a^4b^2c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{1/2} - 2a^2b^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{1/2} + a^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e - 2a^2b^3d^2e(-4ac - b^2)^3)^{1/2} + 16a^2b^4c^2d^2e - 3a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2} - 36a^3b^2c^2d^2e + 4a^2b^2c^2d^2e(-4ac - b^2)^3)^{1/2})/(8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e - 2a^2b^5c^3d^2e^3 - 32a^2b^2c^6d^3e - 32a^3b^2c^5d^2e^3 - 6a^2b^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3)))^{1/2}
\end{aligned}$$

$$\begin{aligned}
&5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4 \\
&*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5* \\
&c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + \\
&16*a^2*b^3*c^4*d*e^3))^{(1/2)}*(128*a^4*b^2*c^4*e^{10} - 16*a^3*b^4*c^3*e^{10} \\
&- 256*a^5*c^5*e^{10} + 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^ \\
&6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + \\
&64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2 \\
&*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a* \\
&b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6 \\
&*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2* \\
&e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 \\
&- 144*a^3*b^3*c^4*d*e^9)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a* \\
&c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - \\
&a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^ \\
&2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16 \\
&a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d* \\
&e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5 \\
&*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - \\
&8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3 \\
&*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4* \\
&c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))^{(1/2)} + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c \\
&^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2* \\
&e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7 \\
&*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d \\
&^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a \\
&^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a \\
&^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c \\
&*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 \\
&+ 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64* \\
&a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 \\
&+ b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + \\
&12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^ \\
&2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2*(-(4*a*c \\
&- b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - \\
&b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^ \\
&7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + \\
&a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 \\
&+ 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^ \\
&5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))^{(1/2)} - (4*a*b^3*c^ \\
&3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d \\
&^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c \\
&^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4* \\
&e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2 \\
& *d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2 \\
& *e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^ \\
& 2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2) \\
&) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2* \\
& c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7*d^4 + 16*a \\
& ^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3* \\
& e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c \\
& ^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6* \\
& a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} + (2*x*(a^6*e^6 + b^6*d^6 \\
& - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6))/(c*e))*(-(b^7*d^2 + \\
& a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b \\
& ^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d* \\
& e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8 \\
& *(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c \\
& ^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c \\
& ^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - \\
& 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)}*1i \\
& - ((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6 - 144*a \\
& ^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a \\
& ^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6*e^3 + 32*a*b \\
& ^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4 \\
& *d^2*e^7 - 16*a^4*b^2*c^3*d*e^8))/(c*e) + (2*x*(-(b^7*d^2 + a^2*b^5*e^2 + b^ \\
& 4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^ \\
& 4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3 \\
& *c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a \\
& ^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^7*d^4 \\
& + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b \\
& ^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16* \\
& a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e \\
& ^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)}*(128*a^4*b^2*c^4*e \\
& ^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6*e^4 + 256*a^ \\
& 3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*b^4*c^6*d^6*e \\
& ^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*e^7 + 256*a^2 \\
& *b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^2*e^8 + 192*a \\
& ^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9 - 320*a*b^2*c \\
& ^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 48*a*b^5*c^4*d \\
& ^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2*b^5*c^3*d*e^
\end{aligned}$$

$$\begin{aligned}
& 9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9)/(c*e))*(-(b^7*d^2 + a^2 \\
& *b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3* \\
& c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e \\
& + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2* \\
& (-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(1 \\
& 6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4* \\
& d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3* \\
& d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32* \\
& a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} - (2* \\
& x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*c*e^8 + 48*a^ \\
& 5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d^7*e + 56*a^5 \\
& *c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5*e^3 - 16*a^4 \\
& *c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*c^6*d^8 + 36* \\
& a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*d^3*e^5 + 64* \\
& a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^7*e - 16*a*b^ \\
& 6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5* \\
& e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5*c*d^2*e^6 + \\
& 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*d*e^7))/(c*e) \\
&)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^ \\
& 3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e \\
& - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2* \\
& (-(4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^ \\
& 6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6* \\
& d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2 \\
& *b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d* \\
& e^3)))^{(1/2)} - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d^4*e^3 + 4*a^ \\
& 4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4*d^2*e^5 - 60 \\
& *a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^2*b^3*c^2*d^5 \\
& *e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*d^5*e^2 - 28* \\
& a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^2 + 36*a^3*b^ \\
& 3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e))*(-(b^7*d^2 \\
& + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^ \\
& 3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b \\
& ^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*c^ \\
& 2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d \\
& *e*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)) \\
& /((8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^ \\
& 5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^ \\
& 6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e
\end{aligned}$$

$$\begin{aligned}
& - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} \\
& - (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^2*b^2*c^2*d^6 - 6*a*b^4*c*d^6)) / (c*e)) * (- (b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - \\
& 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2 * (- (4*a*c - \\
& b^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a \\
& *b^2*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - \\
& 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e \\
& ^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} * 1i) / ((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 1 \\
& 28*a^4*c^5*d^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16* \\
& a*b^3*c^5*d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8) / (c*e) - (2*x*(- (\\
& b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*d^2 \\
& - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
& - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2* \\
& a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2 * (- (4* \\
& a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 \\
& - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e \\
& ^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6 \\
& *d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)) \\
&)^{(1/2)} * (128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 25 \\
& 6*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7 \\
& *e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16 \\
& *b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a \\
& ^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4 \\
& *b*c^5*d*e^9 - 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5 \\
& *d^4*e^6 + 48*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5 \\
& *e^5 + 16*a^2*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9 \\
&)) / (c*e)) * (- (b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 20* \\
& a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2 * (- (4*a*c - b \\
& ^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2 * (- (4*a*c - b \\
& ^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + 16*a^4 \\
& *c^3*d*e - 2*a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - 3*a*b^ \\
& 2*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e * (- (4* \\
& a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8* \\
& a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32* \\
& a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 \\
& - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 3c^4d^3e^3))^{(1/2)} + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 \\
& - 28*a^4*b^3*c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - \\
& 64*a^2*c^6*d^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - \\
& 8*a^3*c^5*d^5*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5* \\
& e^3 - 16*a*b*c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12 \\
& *a^2*b^4*c^2*d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48 \\
& *a*b^2*c^5*d^7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d \\
& ^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 \\
& + 12*a^2*b^5*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108 \\
& *a^4*b^2*c^2*d*e^7)) / (c*e)) * (- (b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (- (4*a*c - b \\
& ^2)^3))^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3* \\
& c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2 \\
& *e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a* \\
& b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2* \\
& b^4*c*d*e - 3*a*b^2*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4 \\
& *a^2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 \\
& + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3 \\
& *b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - \\
& 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d \\
& ^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^{(1/2)} - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^ \\
& 7 + 4*a*b^6*d^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^ \\
& 4 - 4*a^3*b^4*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c* \\
& d*e^6 - 32*a^2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e \\
& + 4*a*b^5*c*d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3 \\
& *b*c^3*d^5*e^2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c* \\
& d^2*e^5) / (c*e)) * (- (b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (- (4*a*c - b^2)^3))^{(1/2)} \\
& - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2 * (- (4*a \\
& *c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2 * (- (4*a* \\
& c - b^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*d^2 + \\
& 16*a^4*c^3*d*e - 2*a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b^4*c*d*e - \\
& 3*a*b^2*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2*c^2*d*e + 4*a^2*b*c*d*e \\
& * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^ \\
& 4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 \\
& + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3* \\
& d*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16* \\
& a^2*b^3*c^4*d*e^3)))^{(1/2)} + (2*x*(a^6*e^6 + b^6*d^6 - 2*a^3*c^3*d^6 + 9*a^ \\
& 2*b^2*c^2*d^6 - 6*a*b^4*c*d^6)) / (c*e)) * (- (b^7*d^2 + a^2*b^5*e^2 + b^4*d^2 * (\\
& - (4*a*c - b^2)^3))^{(1/2)} - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2 \\
& *e^2 - a^3*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^ \\
& 2 + a^2*b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + a^2*c^2*d^2 * (- (4*a*c - b^2)^3)^{(\\
& 1/2)} - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e - 2*a*b^3*d*e * (- (4*a*c - b^2)^3)^{(1/2)} \\
&) + 16*a^2*b^4*c*d*e - 3*a*b^2*c*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 36*a^3*b^2* \\
& c^2*d*e + 4*a^2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^7*d^4 + 16*a \\
& ^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3* \\
& e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c
\end{aligned}$$

$$\begin{aligned}
& ^5d^3e - 2ab^5c^3d^2e^3 - 32a^2b^6c^4d^3e - 32a^3b^7c^5d^4e^3 - 6* \\
& ab^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)} + (((((64a^5c^4d^2e^8 + \\
& 64a^3c^6d^5e^4 + 128a^4c^5d^3e^6 - 144a^2b^2c^5d^5e^4 + 64a^ \\
& 2b^3c^4d^4e^5 + 16a^2b^4c^3d^3e^6 - 96a^3b^2c^4d^3e^6 + 16a^ \\
& 3b^3c^3d^2e^7 - 16a^2b^3c^5d^6e^3 + 32a^2b^4c^4d^5e^4 - 16a^2b^5* \\
& c^3d^4e^5 + 64a^2b^6c^6d^6e^3 - 64a^4b^7c^4d^2e^7 - 16a^4b^2c^3* \\
& d^2e^8)/(c^2e) + (2*x*(-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4a*c - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 - a^3c^3e^2*(-(4a*c - b^2)^3)^{(1/2)} - 2a^2b^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2*(-(4a*c - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4a*c - b^2)^3)^{(1/2)} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e - 2a^2b^3d^2e*(-(4a*c - b^2)^3)^{(1/2)} + 16a^2b^4c^2d^2e - 3a^2b^2c^2d^2*(-(4a*c - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2c^2d^2e*(-(4a*c - b^2)^3)^{(1/2)}))/(8*(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e - 2a^2b^5c^3d^2e^3 - 32a^2b^6c^6d^3e - 32a^3b^7c^5d^4e^3 - 6a^2b^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)}*(128a^4b^2c^4e^10 - 16a^3b^4c^3e^10 - 256a^5c^5e^10 + 256a^2c^8d^6e^4 + 256a^3c^7d^4e^6 - 256a^4c^6d^2e^8 - 16b^3c^7d^7e^3 + 64b^4c^6d^6e^4 - 96b^5c^5d^5e^5 + 64b^6c^4d^4e^6 - 16b^7c^3d^3e^7 + 256a^2b^2c^6d^4e^6 + 144a^2b^3c^5d^3e^7 - 96a^2b^4c^4d^2e^8 + 192a^3b^2c^5d^2e^8 + 64a^2b^3c^8d^7e^3 + 320a^4b^2c^5d^2e^9 - 320a^2b^2c^7d^6e^4 + 528a^2b^3c^6d^5e^5 - 336a^2b^4c^5d^4e^6 + 48a^2b^5c^4d^3e^7 + 16a^2b^6c^3d^2e^8 - 576a^2b^7c^2d^5e^5 + 16a^2b^5c^3d^2e^9 - 320a^3b^2c^6d^3e^7 - 144a^3b^3c^4d^2e^9))/(c^2e)*(-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4a*c - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 + 12a^4b^3c^2e^2 - a^3c^3e^2*(-(4a*c - b^2)^3)^{(1/2)} - 2a^2b^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2*(-(4a*c - b^2)^3)^{(1/2)} + a^2c^2d^2*(-(4a*c - b^2)^3)^{(1/2)} - 9a^2b^5c^2d^2 + 16a^4c^3d^2e - 2a^2b^3d^2e*(-(4a*c - b^2)^3)^{(1/2)} + 16a^2b^4c^2d^2e - 3a^2b^2c^2d^2*(-(4a*c - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2c^2d^2e*(-(4a*c - b^2)^3)^{(1/2)}))/(8*(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8a^2b^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16a^2b^3c^5d^3e - 2a^2b^5c^3d^2e^3 - 32a^2b^6c^6d^3e - 32a^3b^7c^5d^4e^3 - 6a^2b^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)} - (2*x*(4a^3b^5e^8 + 4b^3c^5d^8 + 4b^8d^3e^5 - 28a^4b^3c^3e^8 + 48a^5b^2c^2e^8 - 4a^2b^7d^2e^6 - 4a^2b^6d^7e^7 - 64a^2c^6d^7e^7 + 56a^5c^3d^2e^7 - 8b^4c^4d^7e^7 - 8b^7c^4d^4e^4 - 8a^3c^5d^5e^3 - 16a^4c^4d^3e^5 + 4b^5c^3d^6e^2 + 4b^6c^2d^5e^3 - 16a^2b^6c^6d^8 + 36a^2b^2c^4d^5e^3 - 72a^2b^3c^3d^4e^4 - 12a^2b^4c^2d^3e^5 + 64a^3b^2c^3d^3e^5 + 28a^3b^3c^2d^2e^6 + 48a^2b^2c^5d^7e^7 - 16a^2b^6c^6d^3e^5 + 40a^3b^4c^2d^2e^7 - 28a^2b^3c^4d^6e^2 - 24a^2b^4c^3d^5e^3 + 48a^2b^5c^2d^4e^4 + 48a^2b^6c^5d^6e^2 + 12a^2b^5c^3d^2e^6 + 16a^3b^2c^4d^4e^4 - 64a^4b^2c^3d^2e^6 - 108a^4b^2c^2d^2e^7))/(c^2e)*(-(b^7d^2 + a^2b^5e^2 + b^4d^2*(-(4a*c - b^2)^3)^{(1/2)} - 20a^3b^3c^3d^2 - 7a^3b^3c^3e^2 +
\end{aligned}$$

$$\begin{aligned}
& 12a^4b^3c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 + 16a^4c^3d^2e - 2ab^3d^2e(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} \\
& / (8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^2e^3 - 32a^2b^3c^6d^3e - 32a^3b^3c^5d^2e^3 - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)} - (4ab^3c^3d^7 - 16a^2b^3c^4d^7 + 4ab^6d^4e^3 + 4a^4b^3d^2e^6 + 48a^3c^4d^6e - 4a^2b^5d^3e^4 - 4a^3b^4d^2e^5 - 60a^4c^3d^4e^3 + 4a^5c^2d^2e^5 - 8a^5b^3cd^2e^6 - 32a^2b^3c^2d^5e^2 + 92a^3b^2c^2d^4e^3 + 4ab^4c^2d^6e + 4ab^5c^2d^5e^2 - 28a^2b^2c^3d^6e - 36a^2b^4c^2d^4e^3 + 64a^3b^3c^3d^5e^2 + 36a^3b^3c^3d^3e^4 - 60a^4b^3c^2d^3e^4 + 4a^4b^2c^2d^2e^5)/(c^2e)) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^3c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 + 16a^4c^3d^2e - 2ab^3d^2e(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^2e^3 - 32a^2b^3c^6d^3e - 32a^3b^3c^5d^2e^3 - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)} - (2x^2(a^6e^6 + b^6d^6 - 2a^3c^3d^6 + 9a^2b^2c^2d^6 - 6ab^4cd^6)/(c^2e)) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^3c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 + 16a^4c^3d^2e - 2ab^3d^2e(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3d^2e^3 - 32a^2b^3c^6d^3e - 32a^3b^3c^5d^2e^3 - 6ab^4c^4d^2e^2 + 16a^2b^3c^4d^2e^3))^{(1/2)} + (2(a^3b^2d^5 - a^4cd^5 + a^5d^3e^2 + a^4bd^4e))/(c^2e)) * (-b^7d^2 + a^2b^5e^2 + b^4d^2(-4ac - b^2)^3)^{(1/2)} - 20a^3b^3cd^2 - 7a^3b^3c^2e^2 + 12a^4b^3c^2e^2 - a^3c^2e^2(-4ac - b^2)^3)^{(1/2)} - 2ab^6d^2e + 25a^2b^3c^2d^2 + a^2b^2e^2(-4ac - b^2)^3)^{(1/2)} + a^2c^2d^2(-4ac - b^2)^3)^{(1/2)} - 9ab^5cd^2 + 16a^4c^3d^2e - 2ab^3d^2e(-4ac - b^2)^3)^{(1/2)} + 16a^2b^4cd^2e - 3ab^2cd^2(-4ac - b^2)^3)^{(1/2)} - 36a^3b^2c^2d^2e + 4a^2b^2cd^2e(-4ac - b^2)^3)^{(1/2)} / (8(16a^2c^7d^4 + 16a^4c^5e^4 + b^4c^5d^4 - 8ab^2c^6d^4 - 2b^5c^4d^3e + a^2b^4c^3e^4 - 8a^3b^2c^4e^4 + 32a^3c^6d^2e^2 + b^6c^3d^2e^2 + b
\end{aligned}$$

$$\begin{aligned} & \left(6c^3d^2e^2 + 16ab^3c^5d^3e - 2ab^5c^3de^3 - 32a^2b^6c^4d^3e \right. \\ & \left. - 32a^3b^5c^4de^3 - 6ab^4c^4d^2e^2 + 16a^2b^3c^4de^3 \right)^{(1/2)} \\ & \cdot 2i - \left(\log(b^7d^{10}e^{10} - a^7d^3e^{17} - 2ab^6d^9e^{11} + 2a^6b^4d^8e^{16} \right. \\ & \left. - 2a^6c^5d^5e^{15} + a^2b^5d^8e^{12} - a^5b^2d^5e^{15} - 16a^2c^5d^{13}e^7 \right. \\ & \left. + 16a^4c^3d^9e^{11} - a^5c^2d^7e^{13} - b^4c^3d^{13}e^7 + 16a^2c^5 \right. \\ & \left. \cdot x \cdot (-d^5e^3)^{(5/2)} + b^4c^3 \cdot x \cdot (-d^5e^3)^{(5/2)} + a^7e^{16} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 8ab^2c^4 \cdot x \cdot (-d^5e^3)^{(5/2)} + 25a^2b^3c^2d^{10}e^{10} - 36a^3b^2c^2d^9e^{11} \right. \\ & \left. + a^2b^5e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} + b^7d^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. - 9ab^5cd^{10}e^{10} + 2a^5b^3cd^6e^{14} + 8ab^2c^4d^{13}e^7 \right. \\ & \left. + 16a^2b^4cd^9e^{11} - 20a^3b^3cd^{10}e^{10} - 7a^3b^3cd^8e^{12} + \right. \\ & \left. 12a^4b^2cd^8e^{12} + a^5b^2d^2e^{14} \cdot x \cdot (-d^5e^3)^{(1/2)} + a^5c^2d^4e^{12} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 2ab^6d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} - 2a^6b^2d^5e^{15} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 7a^3b^3c^2e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} + 12a^4b^2c^2e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 16a^4c^3d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} + 2a^6cd^2e^14 \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 9ab^5cd^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} + 16a^2b^4cd^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. - 2a^5b^3cd^3e^{13} \cdot x \cdot (-d^5e^3)^{(1/2)} - 20a^3b^3cd^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. - 36a^3b^2c^2d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} + 25a^2b^3c^2d^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right) \\ & \cdot (-d^5e^3)^{(1/2)} / (2(a^5e^5 + cd^2e^3 - b^2d^4e^4)) + \left(\log(a^7d^3e^{17} - b^7d^{10}e^{10} + 2ab^6d^9e^{11} \right. \\ & \left. - 2a^6b^4d^8e^{16} + 2a^6c^5d^5e^{15} - a^2b^5d^8e^{12} + a^5b^2d^5e^{15} \right. \\ & \left. + 16a^2c^5d^{13}e^7 - 16a^4c^3d^9e^{11} + a^5c^2d^7e^{13} + b^4c^3d^{13}e^7 \right. \\ & \left. + 16a^2c^5 \cdot x \cdot (-d^5e^3)^{(5/2)} + b^4c^3 \cdot x \cdot (-d^5e^3)^{(5/2)} + a^7e^{16} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 8ab^2c^4 \cdot x \cdot (-d^5e^3)^{(5/2)} - 25a^2b^3c^2d^{10}e^{10} + 36a^3b^2c^2d^9e^{11} \right. \\ & \left. + a^2b^5e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} + b^7d^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 9ab^5cd^{10}e^{10} - 2a^5b^3cd^6e^{14} - 8ab^2c^4d^{13}e^7 \right. \\ & \left. - 16a^2b^4cd^9e^{11} + 20a^3b^3cd^{10}e^{10} + 7a^3b^3cd^8e^{12} \right. \\ & \left. - 12a^4b^2cd^8e^{12} + a^5b^2d^2e^{14} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. + a^5c^2d^4e^{12} \cdot x \cdot (-d^5e^3)^{(1/2)} - 2ab^6d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. - 2a^6b^2d^5e^{15} \cdot x \cdot (-d^5e^3)^{(1/2)} - 7a^3b^3c^2e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 12a^4b^2c^2e^8 \cdot x \cdot (-d^5e^3)^{(3/2)} + 16a^4c^3d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 2a^6cd^2e^{14} \cdot x \cdot (-d^5e^3)^{(1/2)} - 9ab^5cd^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 16a^2b^4cd^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} - 2a^5b^3cd^3e^{13} \cdot x \cdot (-d^5e^3)^{(1/2)} \right. \\ & \left. - 20a^3b^3cd^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} - 36a^3b^2c^2d^7e^7 \cdot x \cdot (-d^5e^3)^{(3/2)} \right. \\ & \left. + 25a^2b^3c^2d^2e^6 \cdot x \cdot (-d^5e^3)^{(3/2)} \right) \cdot (-d^5e^3)^{(1/2)} / (2a^5e^5 + 2cd^2e^3 - 2b^2d^4e^4) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.227 \quad \int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=280

$$\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + d^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}(ae^2 - bde + cd^2) - \sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}(ae^2 - bde + cd^2) + \sqrt{e}(ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.89, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + d^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}(ae^2 - bde + cd^2) - \sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}(ae^2 - bde + cd^2) + \sqrt{e}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287

```
Int[(((f_)*(x_))^(m_)*((d_)+(e_)*(x_)^2)^(q_))/((a_)+(b_)*(x_)^2+(c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d+e*x^2)^q)/(a+b*x^2+c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2-4*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)} dx &= \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)} \right) dx \\ &= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4} dx}{cd^2-bde+ae^2} + \frac{d^2 \int \frac{1}{d+ex^2} dx}{cd^2-bde+ae^2} \\ &= \frac{d^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2(cd^2-bde+ae^2)} - \frac{(bd-ae)}{\sqrt{e}(cd^2-bde+ae^2)} \\ &= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.33, size = 323, normalized size = 1.15

$$\frac{(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d))\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(-ae^2+bde-cd^2)} + \frac{(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}-abe-2acd+b^2d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}(-ae^2+bde-cd^2)} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{e}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d+e*x^2)*(a+b*x^2+c*x^4)),x]
```

```
[Out] ((-(b^2*d)+2*a*c*d+b*Sqrt[b^2-4*a*c]*d+a*b*e-a*Sqrt[b^2-4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b-Sqrt[b^2-4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2-4*a*c]*Sqrt[b-Sqrt[b^2-4*a*c]]*(-(c*d^2)+b*d*e-a*e^2))+((b^2*d-2*a*c*d+b*Sqrt[b^2-4*a*c]*d-a*b*e-a*Sqrt[b^2-4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b+Sqrt[b^2-4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2-4*a*c]*Sqrt[b+Sqrt[b^2-4*a*c]]*(-(c*d^2)+b*d*e-a*e^2))+d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[e]*(c*d^2-b*d*e+a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [B] time = 9.25, size = 15553, normalized size = 55.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\frac{1}{2} \left(\sqrt{\frac{1}{2}} \left(c d^2 - b d e + a e^2 \right) \sqrt{-(a^2 b e^2 + (b^3 - 3 a b c) d^2 - 2 (a b^2 - 2 a^2 c) d e + ((b^2 c^3 - 4 a c^4) d^4 - 2 (b^3 c^2 - 4 a b c^3) d^3 e + (b^4 c - 2 a b^2 c^2 - 8 a^2 c^3) d^2 e^2 - 2 (a b^3 c - 4 a^2 b c^2) d e^3 + (a^2 b^2 c - 4 a^3 c^2) e^4)} \right) \sqrt{-(4 a^3 b d e^3 - a^4 e^4 - (b^4 - 2 a b^2 c + a^2 c^2) d^4 + 4 (a b^3 - a^2 b c) d^3 e - 2 (3 a^2 b^2 - a^3 c) d^2 e^2)} \right) / \left((b^2 c^6 - 4 a c^7) d^8 - 4 (b^3 c^5 - 4 a b c^6) d^7 e + 2 (3 b^4 c^4 - 10 a b^2 c^5 - 8 a^2 c^6) d^6 e^2 - 4 (b^5 c^3 - a b^3 c^4 - 12 a^2 b c^5) d^5 e^3 + (b^6 c^2 + 8 a b^4 c^3 - 42 a^2 b^2 c^4 - 24 a^3 c^5) d^4 e^4 - 4 (a b^5 c^2 - a^2 b^3 c^3 - 12 a^3 b c^4) d^3 e^5 + 2 (3 a^2 b^4 c^2 - 10 a^3 b^2 c^3 - 8 a^4 c^4) d^2 e^6 - 4 (a^3 b^3 c^2 - 4 a^4 b c^3) d e^7 + (a^4 b^2 c^2 - 4 a^5 c^3) e^8 \right) / \left((b^2 c^3 - 4 a c^4) d^4 - 2 (b^3 c^2 - 4 a b c^3) d^3 e + (b^4 c - 2 a b^2 c^2 - 8 a^2 c^3) d^2 e^2 - 2 (a b^3 c - 4 a^2 b c^2) d e^3 + (a^2 b^2 c - 4 a^3 c^2) e^4 \right) \log \left(-2 (2 a^2 b d e - a^3 e^2 - (a b^2 - a^2 c) d^2) x + \sqrt{\frac{1}{2}} \left((b^4 - 5 a b^2 c + 4 a^2 c^2) d^3 - 2 (a b^3 - 4 a^2 b c) d^2 e + (a^2 b^2 - 4 a^3 c) d e^2 - ((b^3 c^3 - 4 a b c^4) d^5 - 2 (b^4 c^2 - 3 a b^2 c^3 - 4 a^2 c^4) d^4 e + (b^5 c + 2 a b^3 c^2 - 24 a^2 b c^3) d^3 e^2 - 4 (a b^4 c - 3 a^2 b^2 c^2 - 4 a^3 c^3) d^2 e^3 + 5 (a^2 b^3 c - 4 a^3 b c^2) d e^4 - 2 (a^3 b^2 c - 4 a^4 c^2) e^5 \right) \sqrt{-(4 a^3 b d e^3 - a^4 e^4 - (b^4 - 2 a b^2 c + a^2 c^2) d^4 + 4 (a b^3 - a^2 b c) d^3 e - 2 (3 a^2 b^2 - a^3 c) d^2 e^2)} \right) / \left((b^2 c^6 - 4 a c^7) d^8 - 4 (b^3 c^5 - 4 a b c^6) d^7 e + 2 (3 b^4 c^4 - 10 a b^2 c^5 - 8 a^2 c^6) d^6 e^2 - 4 (b^5 c^3 - a b^3 c^4 - 12 a^2 b c^5) d^5 e^3 + (b^6 c^2 + 8 a b^4 c^3 - 42 a^2 b^2 c^4 - 24 a^3 c^5) d^4 e^4 - 4 (a b^5 c^2 - a^2 b^3 c^3 - 12 a^3 b c^4) d^3 e^5 + 2 (3 a^2 b^4 c^2 - 10 a^3 b^2 c^3 - 8 a^4 c^4) d^2 e^6 - 4 (a^3 b^3 c^2 - 4 a^4 b c^3) d e^7 + (a^4 b^2 c^2 - 4 a^5 c^3) e^8 \right) \sqrt{-(a^2 b e^2 + (b^3 - 3 a b c) d^2 - 2 (a b^2 - 2 a^2 c) d e + ((b^2 c^3 - 4 a c^4) d^4 - 2 (b^3 c^2 - 4 a b c^3) d^3 e + (b^4 c - 2 a b^2 c^2 - 8 a^2 c^3) d^2 e^2 - 2 (a b^3 c - 4 a^2 b c^2) d e^3 + (a^2 b^2 c - 4 a^3 c^2) e^4)} \sqrt{-(4 a^3 b d e^3 - a^4 e^4 - (b^4 - 2 a b^2 c + a^2 c^2) d^4 + 4 (a b^3 - a^2 b c) d^3 e - 2 (3 a^2 b^2 - a^3 c) d^2 e^2)} \right) / \left((b^2 c^6 - 4 a c^7) d^8 - 4 (b^3 c^5 - 4 a b c^6) d^7 e + 2 (3 b^4 c^4 - 10 a b^2 c^5 - 8 a^2 c^6) d^6 e^2 - 4 (b^5 c^3 - a b^3 c^4 - 12 a^2 b c^5) d^5 e^3 + (b^6 c^2 + 8 a b^4 c^3 - 42 a^2 b^2 c^4 - 24 a^3 c^5) d^4 e^4 - 4 (a b^5 c^2 - a^2 b^3 c^3 - 12 a^3 b c^4) d^3 e^5 + 2 (3 a^2 b^4 c^2 - 10 a^3 b^2 c^3 - 8 a^4 c^4) d^2 e^6 - 4 (a^3 b^3 c^2 - 4 a^4 b c^3) d e^7 + (a^4 b^2 c^2 - 4 a^5 c^3) e^8 \right) \right)$$

$$\begin{aligned}
& 4e^4 - 4*(a^5b^2c^2 - a^2b^3c^3 - 12a^3b^2c^4)*d^3e^5 + 2*(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3b^3c^2 - 4a^4b^2c^3)*d^2e^7 + (a^4b^2c^2 - 4a^5c^3)*e^8)/((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4)) - \sqrt{1/2}*(c*d^2 - b*d*e + a*e^2)*\sqrt{-(a^2b^2e^2 + (b^3 - 3a^2b^2c)*d^2 - 2*(a^2b^2 - 2a^2c)*d^2e + ((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4)*\sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)*d^4 + 4*(a^2b^3 - a^2b^2c)*d^3e - 2*(3a^2b^2 - a^3c)*d^2e^2)/((b^2c^6 - 4a^2c^7)*d^8 - 4*(b^3c^5 - 4a^2b^2c^6)*d^7e + 2*(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)*d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)*d^3e^5 + 2*(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3b^3c^2 - 4a^4b^2c^3)*d^2e^7 + (a^4b^2c^2 - 4a^5c^3)*e^8))/((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4))*\log(-2*(2a^2b^2d^2e - a^3e^2 - (a^2b^2 - a^2c)*d^2)*x - \sqrt{1/2}*((b^4 - 5a^2b^2c + 4a^2c^2)*d^3 - 2*(a^2b^3 - 4a^2b^2c)*d^2e + (a^2b^2 - 4a^3c)*d^2e^2 - ((b^3c^3 - 4a^2b^2c^3)*d^5 - 2*(b^4c^2 - 3a^2b^2c^3 - 4a^2c^4)*d^4e + (b^5c + 2a^2b^3c^2 - 24a^2b^2c^3)*d^3e^2 - 4*(a^2b^4c - 3a^2b^2c^2 - 4a^3c^3)*d^2e^3 + 5*(a^2b^3c - 4a^3b^2c^2)*d^2e^4 - 2*(a^3b^2c - 4a^4c^2)*e^5)*\sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)*d^4 + 4*(a^2b^3 - a^2b^2c)*d^3e - 2*(3a^2b^2 - a^3c)*d^2e^2)/((b^2c^6 - 4a^2c^7)*d^8 - 4*(b^3c^5 - 4a^2b^2c^6)*d^7e + 2*(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)*d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)*d^3e^5 + 2*(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3b^3c^2 - 4a^4b^2c^3)*d^2e^7 + (a^4b^2c^2 - 4a^5c^3)*e^8)))*\sqrt{-(a^2b^2e^2 + (b^3 - 3a^2b^2c)*d^2 - 2*(a^2b^2 - 2a^2c)*d^2e + ((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4)*\sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)*d^4 + 4*(a^2b^3 - a^2b^2c)*d^3e - 2*(3a^2b^2 - a^3c)*d^2e^2)/((b^2c^6 - 4a^2c^7)*d^8 - 4*(b^3c^5 - 4a^2b^2c^6)*d^7e + 2*(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)*d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)*d^3e^5 + 2*(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3b^3c^2 - 4a^4b^2c^3)*d^2e^7 + (a^4b^2c^2 - 4a^5c^3)*e^8))/((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4)) + \sqrt{1/2}*(c*d^2 - b*d*e + a*e^2)*\sqrt{-(a^2b^2e^2 + (b^3 - 3a^2b^2c)*d^2 - 2*(a^2b^2 - 2a^2c)*d^2e - ((b^2c^3 - 4a^2c^4)*d^4 - 2*(b^3c^2 - 4a^2b^2c^3)*d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a^2b^3c - 4a^2b^2c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4))}
\end{aligned}$$

$$\begin{aligned}
& *c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e \\
& ^4)*\sqrt{-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a \\
& *b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7) \\
& *d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2* \\
& c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + \\
& 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3 \\
& *c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^ \\
& 4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3 \\
&)*e^8))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c \\
& - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2 \\
& *b^2*c - 4*a^3*c^2)*e^4))*\log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d \\
& ^2)*x + \sqrt{1/2}*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c \\
&)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c \\
& ^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)* \\
& d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - \\
& 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*\sqrt{-(4*a^3*b*d*e^3 - \\
& a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(\\
& 3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c \\
& ^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - \\
& a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^ \\
& 4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^ \\
& 5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 \\
& - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))*\sqrt{-(a^2*b*e^2 + \\
& (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - \\
& 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 \\
& - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*\sqrt{-(4*a \\
& ^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c \\
&)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c \\
& ^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - \\
& 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - \\
& 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3* \\
& b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4 \\
& *(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))/((b^2 \\
& *c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 \\
& - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3 \\
& *c^2)*e^4))) - \sqrt{1/2}*(c*d^2 - b*d*e + a*e^2)*\sqrt{-(a^2*b*e^2 + (b^3 - \\
& 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3* \\
& c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b \\
& ^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*\sqrt{-(4*a^3*b*d*e \\
& ^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e \\
& - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4* \\
& a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5* \\
& c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b \\
& ^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d \\
& ^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))/((b^2*c^3 - 4 \\
& *a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2* \\
& c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4) \\
& *log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 \\
& - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4 \\
& *a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a \\
& ^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - \\
& 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2 \\
& *(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b \\
& ^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2 \\
& *e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c \\
& ^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b* \\
& c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 \\
& - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - \\
& 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 \\
& + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - \\
& 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^ \\
& 3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b \\
& *c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - \\
& (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 \\
& - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e \\
& + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^ \\
& 4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^ \\
& 3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3* \\
& a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4* \\
& b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))/((b^2*c^3 - 4*a*c^4)*d^4 - \\
& 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - \\
& 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + d*sqrt(\\
& -d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d))/(c*d^2 - b*d*e + a*e \\
& ^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b \\
& *c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - \\
& 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c \\
& - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - \\
& a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(\\
& 3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^ \\
& 6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - \\
& a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^ \\
& 4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^ \\
& 5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 \\
& - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8))/((b^2*c^3 - 4*a*c^ \\
& 4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)* \\
& d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*1 \\
& og(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5 \\
& *a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3* \\
& c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^
\end{aligned}$$

$$\begin{aligned}
& 4)d^4e + (b^5c + 2a^2b^3c^2 - 24a^2b^3c^3)d^3e^2 - 4(a^2b^4c - 3a^2b^2c^2 - 4a^3c^3)d^2e^3 + 5(a^2b^3c - 4a^3b^2c^2)d^2e^4 - 2(a^3b^2c - 4a^4c^2)e^5) \sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)d^4 + 4(a^2b^3 - a^2b^2c)d^3e - 2(3a^2b^2 - a^3c)d^2e^2)} \\
& /((b^2c^6 - 4a^2c^7)d^8 - 4(b^3c^5 - 4a^2b^2c^6)d^7e + 2(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)d^6e^2 - 4(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)d^4e^4 - 4(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)d^3e^5 + 2(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)d^2e^6 - 4(a^3b^3c^2 - 4a^4b^2c^3)d^2e^7 + (a^4b^2c^2 - 4a^5c^3)e^8)) \sqrt{-(a^2b^2e^2 + (b^3 - 3a^2b^2c)d^2 - 2(a^2b^2 - 2a^2c)d^2e + ((b^2c^3 - 4a^2c^4)d^4 - 2(b^3c^2 - 4a^2b^2c^3)d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)d^2e^2 - 2(a^2b^3c - 4a^2b^2c^2)d^2e^3 + (a^2b^2c - 4a^3c^2)e^4)) \sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)d^4 + 4(a^2b^3 - a^2b^2c)d^3e - 2(3a^2b^2 - a^3c)d^2e^2)} \\
& /((b^2c^6 - 4a^2c^7)d^8 - 4(b^3c^5 - 4a^2b^2c^6)d^7e + 2(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)d^6e^2 - 4(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)d^4e^4 - 4(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)d^3e^5 + 2(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)d^2e^6 - 4(a^3b^3c^2 - 4a^4b^2c^3)d^2e^7 + (a^4b^2c^2 - 4a^5c^3)e^8)) /((b^2c^3 - 4a^2c^4)d^4 - 2(b^3c^2 - 4a^2b^2c^3)d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)d^2e^2 - 2(a^2b^3c - 4a^2b^2c^2)d^2e^3 + (a^2b^2c - 4a^3c^2)e^4)) - \sqrt{1/2} * (c^2d^2 - b^2d^2e + a^2e^2) \sqrt{-(a^2b^2e^2 + (b^3 - 3a^2b^2c)d^2 - 2(a^2b^2 - 2a^2c)d^2e + ((b^2c^3 - 4a^2c^4)d^4 - 2(b^3c^2 - 4a^2b^2c^3)d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)d^2e^2 - 2(a^2b^3c - 4a^2b^2c^2)d^2e^3 + (a^2b^2c - 4a^3c^2)e^4)) \sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)d^4 + 4(a^2b^3 - a^2b^2c)d^3e - 2(3a^2b^2 - a^3c)d^2e^2)} \\
& /((b^2c^6 - 4a^2c^7)d^8 - 4(b^3c^5 - 4a^2b^2c^6)d^7e + 2(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)d^6e^2 - 4(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)d^4e^4 - 4(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)d^3e^5 + 2(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)d^2e^6 - 4(a^3b^3c^2 - 4a^4b^2c^3)d^2e^7 + (a^4b^2c^2 - 4a^5c^3)e^8)) /((b^2c^3 - 4a^2c^4)d^4 - 2(b^3c^2 - 4a^2b^2c^3)d^3e + (b^4c - 2a^2b^2c^2 - 8a^2c^3)d^2e^2 - 2(a^2b^3c - 4a^2b^2c^2)d^2e^3 + (a^2b^2c - 4a^3c^2)e^4)) * \log(-2(2a^2b^2d^2e - a^3e^2 - (a^2b^2 - a^2c)d^2e) * x - \sqrt{1/2} * ((b^4 - 5a^2b^2c + 4a^2c^2)d^3 - 2(a^2b^3 - 4a^2b^2c)d^2e + (a^2b^2 - 4a^3c)d^2e^2 - ((b^3c^3 - 4a^2b^2c^4)d^5 - 2(b^4c^2 - 3a^2b^2c^3 - 4a^2c^4)d^4e + (b^5c + 2a^2b^3c^2 - 24a^2b^2c^3)d^3e^2 - 4(a^2b^4c - 3a^2b^2c^2 - 4a^3c^3)d^2e^3 + 5(a^2b^3c - 4a^3b^2c^2)d^2e^4 - 2(a^3b^2c - 4a^4c^2)e^5) \sqrt{-(4a^3b^2d^2e^3 - a^4e^4 - (b^4 - 2a^2b^2c + a^2c^2)d^4 + 4(a^2b^3 - a^2b^2c)d^3e - 2(3a^2b^2 - a^3c)d^2e^2)} \\
& /((b^2c^6 - 4a^2c^7)d^8 - 4(b^3c^5 - 4a^2b^2c^6)d^7e + 2(3b^4c^4 - 10a^2b^2c^5 - 8a^2c^6)d^6e^2 - 4(b^5c^3 - a^2b^3c^4 - 12a^2b^2c^5)d^5e^3 + (b^6c^2 + 8a^2b^4c^3 - 42a^2b^2c^4 - 24a^3c^5)d^4e^4 - 4(a^2b^5c^2 - a^2b^3c^3 - 12a^3b^2c^4)d^3e^5 + 2(3a^2b^4c^2 - 10a^3b^2c^3 - 8a^4c^4)d^2e^6 - 4(a^3b^3c^2 - 4a^4b^2c^3)d^2e^7 + (a^4b^2c^2 - 4a^5c^3)e^8))
\end{aligned}$$

$$\begin{aligned}
& b^3c^5 - 4a*bc^6)*d^7e + 2*(3b^4c^4 - 10a*b^2c^5 - 8a^2c^6)*d^6e \\
& ^2 - 4*(b^5c^3 - a*b^3c^4 - 12a^2*b*c^5)*d^5e^3 + (b^6c^2 + 8a*b^4c^3 - 42a^2*b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a*b^5c^2 - a^2*b^3c^3 - 12a^3*b*c^4)*d^3e^5 + 2*(3a^2*b^4c^2 - 10a^3*b^2c^3 - 8a^4c^4)*d^2e^6 \\
& - 4*(a^3*b^3c^2 - 4a^4*b*c^3)*d*e^7 + (a^4*b^2c^2 - 4a^5c^3)*e^8)))/((b^2c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 - 4a*b*c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2*b*c^2)*d*e^3 + (a^2*b^2c - 4a^3c^2)*e^4)) - \text{sqrt}(1/2)*(c*d^2 - b*d*e + a*e^2)*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3a*b*c)*d^2 - 2*(a*b^2 - 2a^2*c)*d*e - ((b^2*c^3 - 4a*c^4)*d^4 - 2*(b^3*c^2 - 4a*b*c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2*b*c^2)*d*e^3 + (a^2*b^2c - 4a^3c^2)*e^4))*\text{sqrt}(-(4a^3*b*d*e^3 - a^4e^4 - (b^4 - 2a*b^2c + a^2c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3e - 2*(3a^2*b^2 - a^3c)*d^2e^2))/((b^2*c^6 - 4a*c^7)*d^8 - 4*(b^3*c^5 - 4a*b*c^6)*d^7e + 2*(3b^4c^4 - 10a*b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a*b^3c^4 - 12a^2*b*c^5)*d^5e^3 + (b^6c^2 + 8a*b^4c^3 - 42a^2*b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a*b^5c^2 - a^2*b^3c^3 - 12a^3*b*c^4)*d^3e^5 + 2*(3a^2*b^4c^2 - 10a^3*b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3*b^3c^2 - 4a^4*b*c^3)*d*e^7 + (a^4*b^2c^2 - 4a^5c^3)*e^8)))/((b^2*c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 - 4a*b*c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2*b*c^2)*d*e^3 + (a^2*b^2c - 4a^3c^2)*e^4))*\log(-2*(2a^2*b*d*e - a^3e^2 - (a*b^2 - a^2*c)*d^2)*x - \text{sqrt}(1/2)*((b^4 - 5a*b^2c + 4a^2c^2)*d^3 - 2*(a*b^3 - 4a^2*b*c)*d^2e + (a^2*b^2 - 4a^3c)*d*e^2 + ((b^3c^3 - 4a*b*c^4)*d^5 - 2*(b^4c^2 - 3a*b^2c^3 - 4a^2c^4)*d^4e + (b^5c + 2a*b^3c^2 - 24a^2*b*c^3)*d^3e^2 - 4*(a*b^4c - 3a^2*b^2c^2 - 4a^3c^3)*d^2e^3 + 5*(a^2*b^3c - 4a^3*b*c^2)*d*e^4 - 2*(a^3*b^2c - 4a^4c^2)*e^5))*\text{sqrt}(-(4a^3*b*d*e^3 - a^4e^4 - (b^4 - 2a*b^2c + a^2c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3e - 2*(3a^2*b^2 - a^3c)*d^2e^2))/((b^2*c^6 - 4a*c^7)*d^8 - 4*(b^3*c^5 - 4a*b*c^6)*d^7e + 2*(3b^4c^4 - 10a*b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a*b^3c^4 - 12a^2*b*c^5)*d^5e^3 + (b^6c^2 + 8a*b^4c^3 - 42a^2*b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a*b^5c^2 - a^2*b^3c^3 - 12a^3*b*c^4)*d^3e^5 + 2*(3a^2*b^4c^2 - 10a^3*b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3*b^3c^2 - 4a^4*b*c^3)*d*e^7 + (a^4*b^2c^2 - 4a^5c^3)*e^8)))*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3a*b*c)*d^2 - 2*(a*b^2 - 2a^2*c)*d*e - ((b^2*c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 - 4a*b*c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2*b*c^2)*d*e^3 + (a^2*b^2c - 4a^3c^2)*e^4))*\text{sqrt}(-(4a^3*b*d*e^3 - a^4e^4 - (b^4 - 2a*b^2c + a^2c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3e - 2*(3a^2*b^2 - a^3c)*d^2e^2))/((b^2*c^6 - 4a*c^7)*d^8 - 4*(b^3*c^5 - 4a*b*c^6)*d^7e + 2*(3b^4c^4 - 10a*b^2c^5 - 8a^2c^6)*d^6e^2 - 4*(b^5c^3 - a*b^3c^4 - 12a^2*b*c^5)*d^5e^3 + (b^6c^2 + 8a*b^4c^3 - 42a^2*b^2c^4 - 24a^3c^5)*d^4e^4 - 4*(a*b^5c^2 - a^2*b^3c^3 - 12a^3*b*c^4)*d^3e^5 + 2*(3a^2*b^4c^2 - 10a^3*b^2c^3 - 8a^4c^4)*d^2e^6 - 4*(a^3*b^3c^2 - 4a^4*b*c^3)*d*e^7 + (a^4*b^2c^2 - 4a^5c^3)*e^8)))/((b^2*c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 - 4a*b*c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2*b*c^2)*d*e^3 + (a^2*b^2c - 4a^3c^2)*e^4)) + 2*d
\end{aligned}$$

*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d))/(c*d^2 - b*d*e + a*e^2)]

giac [B] time = 11.24, size = 8658, normalized size = 30.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $d^{3/2} \arctan(x e^{1/2} / \sqrt{d}) e^{-1/2} / (c d^2 - b d e + a e^2) + 1/8 * ((2 b^5 c^4 - 12 a b^3 c^5 + 16 a^2 b c^6 - \sqrt{2}) \sqrt{b^2 - 4 a c}) \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^5 c^2 + 6 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^3 c^3 + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^4 c^3 - 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b c^4 - 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^3 c^4 + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b c^5 - 2 (b^2 - 4 a c) b^3 c^4 + 4 (b^2 - 4 a c) a b c^5 d^5 - (4 b^6 c^3 - 22 a b^4 c^4 + 24 a^2 b^2 c^5 - 2 \sqrt{2}) \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^6 c + 11 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^4 c^2 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^5 c^2 - 12 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b^2 c^3 - 6 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^3 c^3 - 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^4 c^3 + 3 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^2 c^4 - 4 (b^2 - 4 a c) b^4 c^3 + 6 (b^2 - 4 a c) a b^2 c^4 d^4 e - 2 (\sqrt{2}) \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^4 c^2 - 8 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b^2 c^3 - 2 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^3 c^3 - 2 a b^4 c^3 + 16 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^3 c^4 + 8 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b c^4 + \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^2 c^4 + 16 a^2 b^2 c^4 - 4 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 c^5 - 32 a^3 c^5 + 2 (b^2 - 4 a c) a b^2 c^3 - 8 (b^2 - 4 a c) a^2 c^4 d^3 \operatorname{abs}(c d^2 - b d e + a e^2) + (2 b^7 c^2 - 4 a b^5 c^3 - 24 a^2 b^3 c^4 + 32 a^3 b c^5 - \sqrt{2}) \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^7 + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^5 c + 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^6 c + 12 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b^3 c^2 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^4 c^2 - \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c b^5 c^2 - 16 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^3 b c^3 - 8 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b^2 c^3 - 2 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^3 c^3 + 4 \sqrt{2} \sqrt{b^2 - 4 a c} \sqrt{b c + \sqrt{b^2 - 4 a c}} c a^2 b c^4 - 2 (b^2 - 4 a c) b^5 c^2 - 4 (b^2 - 4 a c) a b^3 c^3 + 8 (b^2 - 4 a c) a^2 b c^4 d^3 e^2 + 2 (\sqrt{2}) \sqrt{b c + \sqrt{b^2 - 4 a c}} c a b^5 c - 8 \sqrt{2} \sqrt{b c + \sqrt{b^2 - 4 a c}} c$

$$\begin{aligned}
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^2 - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& - 4*a*c))*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a* \\
& c))*c)*a^3*b*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^3 + s \\
& \text{qrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*\text{sqrt}(\\
& 2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a* \\
& c)*a*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*\text{abs}(c*d^2 - b*d*e + a*e^2)*e \\
& - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt} \\
& (b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^5 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + sq \\
& \text{rt}(b^2 - 4*a*c))*c)*a*b^3*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^ \\
& 2 - 4*a*c))*c)*b^4*c - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4* \\
& a*c))*c)*a^2*b*c^2 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c \\
&))*c)*a*b^2*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)* \\
& b^3*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c \\
& ^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a* \\
& e^2)^2*d - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*\text{sqrt}(2)*\text{sqrt}(\\
& b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^6 + 14*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a \\
& *c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^5*c - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*s \\
& \text{qrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*sq \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^2 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*sq \\
& \text{rt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c^2 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b \\
& *c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2 \\
& - 4*a*c)*a^2*b^2*c^3)*d^2*e^3 - 2*(\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)* \\
& a^2*b^4*c - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^2 - 2*\text{sqrt}(\\
& 2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*\text{sqrt}(2) \\
& *\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - \\
& 4*a*c))*c)*a^3*b*c^3 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c^3 + \\
& 16*a^3*b^2*c^3 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^4 - 32*a^ \\
& 4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*d*\text{abs}(c*d^2 \\
& - b*d*e + a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - \text{sqrt}(2) \\
& *\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4 + 8*\text{sqrt}(2)*\text{sqrt}(b \\
& ^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^2*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4* \\
& a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*c^2 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))* \\
& \text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(\\
& b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c \\
& + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a* \\
& c)*a^2*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + \\
& 16*a^4*b*c^4 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c) \\
& *a^2*b^5 + 14*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3 \\
& *b^3*c + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^ \\
& 4*c - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^2 \\
& - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^2*c^2 \\
& - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^3*c^2 + \\
& 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c))*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b*c^3 - 6*
\end{aligned}$$

$$\begin{aligned}
& (b^2 - 4ac) a^2 b^3 c^2 + 4(b^2 - 4ac) a^3 b^2 c^3) d e^4 - (2a^3 b^4 c^2 - 8a^4 b^2 c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^3 b^4 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^4 b^2 c + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^3 b^3 c - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^3 b^2 c^2 - 2(b^2 - 4ac) a^3 b^2 c^2) e^5) \arctan(2\sqrt{1/2} x / \sqrt{(b^2 c d^2 - b^2 d^2 e + a b^2 e^2 + \sqrt{(b^2 c d^2 - b^2 d^2 e + a b^2 e^2)^2 - 4(a^2 c d^2 - a b^2 d^2 e + a^2 e^2)} (c^2 d^2 - b^2 c d^2 e + a^2 c e^2)) / (c^2 d^2 - b^2 c d^2 e + a^2 c e^2)) / ((a^2 b^4 c^3 - 8a^2 b^2 c^4 - 2a^2 b^3 c^4 + 16a^3 c^5 + 8a^2 b^2 c^5 + a^2 b^2 c^5 - 4a^2 c^6) d^4 \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) \operatorname{abs}(c) - 2(a^2 b^5 c^2 - 8a^2 b^3 c^3 - 2a^2 b^4 c^3 + 16a^3 b^2 c^4 + 8a^2 b^2 c^4 + a^2 b^3 c^4 - 4a^2 b^2 c^5) d^3 \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) \operatorname{abs}(c) e + (a^2 b^6 c - 6a^2 b^4 c^2 - 2a^2 b^5 c^2 + 4a^2 b^3 c^3 + a^2 b^4 c^3 + 32a^4 c^4 + 16a^3 b^2 c^4 - 2a^2 b^2 c^4 - 8a^3 c^5) d^2 \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) \operatorname{abs}(c) e^2 - 2(a^2 b^5 c - 8a^3 b^3 c^2 - 2a^2 b^4 c^2 + 16a^4 b^2 c^3 + 8a^3 b^2 c^3 + a^2 b^3 c^3 - 4a^3 b^2 c^4) d \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) \operatorname{abs}(c) e^3 + (a^3 b^4 c - 8a^4 b^2 c^2 - 2a^3 b^3 c^2 + 16a^5 c^3 + 8a^4 b^2 c^3 + a^3 b^2 c^3 - 4a^4 c^4) \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) \operatorname{abs}(c) e^4) - 1/8((2b^5 c^4 - 12a^2 b^3 c^5 + 16a^2 b^2 c^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^5 c^2 + 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^3 c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^4 c^3 - 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^4 - 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^3 c^4 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^5 - 2(b^2 - 4ac) b^3 c^4 + 4(b^2 - 4ac) a^2 b^2 c^5) d^5 - (4b^6 c^3 - 22a^2 b^4 c^4 + 24a^2 b^2 c^5 - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^6 c + 11\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^4 c^2 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^5 c^2 - 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^3 - 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^3 c^3 - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^4 c^3 + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^4 - 4(b^2 - 4ac) b^4 c^3 + 6(b^2 - 4ac) a^2 b^2 c^4) d^4 e + 2(\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^3 - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^3 c^3 + 2a^2 b^4 c^3 + 16\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^3 c^4 + 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^4 + \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^2 c^4 - 16a^2 b^2 c^4 - 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 c^5 + 32a^3 c^5 - 2(b^2 - 4ac) a^2 b^2 c^3 + 8(b^2 - 4ac) a^2 c^4) d^3 \operatorname{abs}(c d^2 - b^2 d^2 e + a^2 e^2) + (2b^7 c^2 - 4a^2 b^5 c^3 - 24a^2 b^3 c^4 + 32a^3 b^2 c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^7 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^5 c + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) b^6 c + 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} c) a^2 b^3 c
\end{aligned}$$

$$\begin{aligned}
& c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4c^2 \\
& - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^5c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^3 - 2(b^2 - 4ac)b^5c^2 - 4(b^2 - 4ac)a^2b^3c^3 + 8(b^2 - 4ac)a^2b^4c^3)d^3e^2 \\
& - 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^5c - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4c^2 + 2ab^5c^2 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^3 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c^3 - 16a^2b^3c^3 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c^3 + 32a^3b^4c^3 - 2(b^2 - 4ac)a^2b^3c^2 + 8(b^2 - 4ac)a^2b^4c^3)d^2\text{abs}(cd^2 - bde + ae^2)e - (2b^5c^2 - 16ab^3c^3 + 32a^2b^4c^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^5 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)b^3c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c^3 - 2(b^2 - 4ac)b^3c^2 + 8(b^2 - 4ac)ab^3c^3)(cd^2 - bde + ae^2)^2d - (6ab^6c^2 - 28a^2b^4c^3 + 16a^3b^2c^4 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^6 + 14\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c + 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^5c - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 6(b^2 - 4ac)ab^4c^2 + 4(b^2 - 4ac)a^2b^2c^3)d^2e^3 + 2(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^4c - 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^2c^2 - 2\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^3c^2 + 2a^2b^4c^2 + 16\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^4c^3 + 8\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3b^3c^3 + \sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^3 - 16a^3b^2c^3 - 4\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3c^4 + 32a^4c^4 - 2(b^2 - 4ac)a^2b^2c^2 + 8(b^2 - 4ac)a^3c^3)d\text{abs}(cd^2 - bde + ae^2)e^2 + (2ab^4c^2 - 16a^2b^2c^3 + 32a^3c^4 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^3c - 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^3c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2b^2c^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)ab^2c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc - \sqrt{b^2 - 4ac}}c)a^2c^3 - 2(b^2 - 4ac)ab^2c^2 + 8(b^2 - 4ac)a^2c^3
\end{aligned}$$

$$\begin{aligned} &^3)(c*d^2 - b*d*e + a*e^2)^2*e + (6*a^2*b^5*c^2 - 28*a^3*b^3*c^3 + 16*a^4* \\ &b*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5 \\ &+ 14*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c + \\ &6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c - 8* \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^2 - 4*\sqrt{2} \\ &t(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^2 - 3*\sqrt{2} \\ &(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^2 + 2*\sqrt{2} \\ &(2)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^3 - 6*(b^2 - 4 \\ &*a*c)*a^2*b^3*c^2 + 4*(b^2 - 4*a*c)*a^3*b*c^3)*d*e^4 - (2*a^3*b^4*c^2 - 8*a \\ &^4*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3* \\ &b^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b^2*c \\ &+ 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c - \\ &\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^2 - 2*(\\ &b^2 - 4*a*c)*a^3*b^2*c^2)*e^5)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c*d^2 - b^2*d*e \\ &+ a*b*e^2 - \sqrt{(b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + \\ &a^2*e^2)}*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((\\ &a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2* \\ &c^5 - 4*a^2*c^6)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c^2 - 8*a \\ &^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2 \\ &*b*c^5)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6*c - 6*a^2*b^4*c^2 \\ &- 2*a*b^5*c^2 + 4*a^2*b^3*c^3 + a*b^4*c^3 + 32*a^4*c^4 + 16*a^3*b*c^4 - 2*a \\ &^2*b^2*c^4 - 8*a^3*c^5)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2* \\ &b^5*c - 8*a^3*b^3*c^2 - 2*a^2*b^4*c^2 + 16*a^4*b*c^3 + 8*a^3*b^2*c^3 + a^2* \\ &b^3*c^3 - 4*a^3*b*c^4)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4*c \\ &- 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - \\ &4*a^4*c^4)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4) \end{aligned}$$

maple [B] time = 0.03, size = 764, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4/(e*x^2+d)/(c*x^4+b*x^2+a), x)$

[Out]
$$\begin{aligned} &-1/2/(a*e^2-b*d*e+c*d^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(\\ &2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)* \\ &2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &^2)^{(1/2)}*c)^{(1/2)}*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/((-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ &)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)}) \\ &)*c)^{(1/2)}*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/((-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((- \\ &-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c) \\ &^2)^{(1/2)}*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/((-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4 \\ &*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &*c*x)*b^2*d+1/2/(a*e^2-b*d*e+c*d^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*\arctan(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e-1/2/(a*e^2-b*d*e \end{aligned}$$

$$+c*d^2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^{(1/2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^{(1/2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^{(1/2)*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*\arctan(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2)}*c*x)})*b^2*d+d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^{(1/2)}*\arctan(1/(d*e)^{(1/2)}*e*x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2 - bde + ae^2)\sqrt{de}} + \frac{-\int \frac{(bd-ae)x^2+ad}{cx^4+bx^2+a} dx}{cd^2 - bde + ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] d^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + integrate(-((b*d - a*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)

mupad [B] time = 5.80, size = 25202, normalized size = 90.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] atan(((((-b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))))^(1/2)

$$\begin{aligned}
& 2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 \\
& - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 \\
& - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3* \\
& d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)}*(64*a^2*c^6*d^6 \\
& *e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d \\
& ^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12 \\
& *a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 1 \\
& 6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d \\
& ^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d \\
& *e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b \\
& ^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^ \\
& 3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 \\
& - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5 \\
& *c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d \\
& ^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^ \\
& 4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e \\
& ^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 \\
& + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 64 \\
& 0*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4 \\
& *c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4 \\
& *c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5* \\
& d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^ \\
& 5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 \\
& + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12* \\
& a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a* \\
& b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c \\
& ^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2 \\
& *b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d* \\
& e^3)))^{(1/2)} + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24 \\
& *a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c* \\
& d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - \\
& 16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - \\
& 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - \\
& 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c \\
& ^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2* \\
& c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - \\
& 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - \\
& 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d* \\
& e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)}*i + ((-(b^5*d^2 \\
& + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*
\end{aligned}$$

$$\begin{aligned}
& (c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a \\
& *b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4* \\
& c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 \\
& - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3* \\
& d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16* \\
& a^2*b^3*c^2*d*e^3)))^{(1/2)}*((x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^ \\
& 4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32* \\
& a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e \\
& - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^ \\
& 2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d \\
& ^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6 \\
&) - (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - \\
& a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2* \\
& b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4 \\
& *c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e \\
& + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 \\
& + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^ \\
& 2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)}*(x*(-(b^5*d^2 + a^2*b^3*e^2 + a^2 \\
& *e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b \\
& *c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c \\
& ^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^ \\
& 4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^ \\
& 4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)) \\
&)^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a \\
& ^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7 \\
& *e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32 \\
& *b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192* \\
& a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a \\
& ^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4 \\
& *c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6 \\
& *d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d \\
& *e^8) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96* \\
& a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144 \\
& *a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3 \\
& *c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d \\
& ^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2* \\
& a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^ \\
& 2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)}) \\
& /((8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2* \\
& b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^ \\
& 4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*
\end{aligned}$$

$$\begin{aligned}
& a^3 b^3 c^3 d^3 e^3 - 6 a^2 b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d^2 e^3))^{(1/2)} - 16 a^2 c^4 d^5 e - 4 a^4 c^2 d^2 e^5 + 60 a^3 c^3 d^3 e^3 - 24 a^2 b^2 c^2 d^3 e^3 + 4 a^2 b^2 c^3 d^5 e + 4 a^2 b^4 c^2 d^3 e^3 + 4 a^3 b^2 c^2 d^5 e + 4 a^2 b^3 c^2 d^4 e^2 - 20 a^2 b^3 c^3 d^4 e^2 - 8 a^2 b^3 c^3 d^2 e^4 + 16 a^3 b^3 c^2 d^2 e^4) + x(2 a^4 c^3 e^5 + 4 a^2 c^3 d^4 e + 2 b^4 c^2 d^4 e - 8 a^2 b^2 c^2 d^4 e) \\
&) * (- (b^5 d^2 + a^2 b^3 e^2 + a^2 e^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + b^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + 12 a^2 b^3 c^2 d^2 - 2 a^2 b^4 d^2 e - 7 a^2 b^3 c^2 d^2 - a^2 c^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} - 4 a^3 b^3 c^2 d^2 e + 12 a^2 b^2 c^2 d^2 e - 2 a^2 b^3 c^2 d^2 e * (- (4 a^2 c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a^2 b^2 c^4 d^4 + a^2 b^4 c^2 e^4 - 2 b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a^2 b^5 c^2 d^3 e + 16 a^2 b^3 c^3 d^3 e - 32 a^2 b^3 c^4 d^3 e - 32 a^3 b^3 c^3 d^2 e^3 - 6 a^2 b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d^2 e^3))^{(1/2)} * i) / (((- (b^5 d^2 + a^2 b^3 e^2 + a^2 e^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + b^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + 12 a^2 b^3 c^2 d^2 - 2 a^2 b^4 d^2 e - 7 a^2 b^3 c^2 d^2 - a^2 c^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} - 4 a^3 b^3 c^2 d^2 e - 16 a^3 c^2 d^2 e + 12 a^2 b^2 c^2 d^2 e - 2 a^2 b^3 c^2 d^2 e * (- (4 a^2 c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a^2 b^2 c^4 d^4 + a^2 b^4 c^2 e^4 - 2 b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a^2 b^5 c^2 d^3 e + 16 a^2 b^3 c^3 d^3 e - 32 a^2 b^3 c^4 d^3 e - 32 a^3 b^3 c^3 d^2 e^3 - 6 a^2 b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d^2 e^3))^{(1/2)} * ((x * (8 a^3 b^3 c^3 e^7 - 32 a^4 b^3 c^2 e^7 - 112 a^4 c^3 d^3 e^6 + 8 b^3 c^4 d^6 e + 8 b^6 c^3 d^3 e^4 - 112 a^2 c^5 d^5 e^2 + 32 a^3 c^4 d^3 e^4 - 8 b^4 c^3 d^5 e^2 - 8 b^5 c^2 d^4 e^3 - 32 a^2 b^3 c^5 d^6 e - 48 a^2 b^2 c^3 d^3 e^4 + 8 a^2 b^3 c^2 d^2 e^5 - 8 a^2 b^5 c^2 d^2 e^5 - 8 a^2 b^4 c^2 d^2 e^6 + 64 a^2 b^2 c^4 d^5 e^2 + 8 a^2 b^3 c^3 d^4 e^3 - 16 a^2 b^4 c^2 d^3 e^4 + 64 a^2 b^2 c^4 d^4 e^3 + 64 a^3 b^3 c^3 d^2 e^5 + 64 a^3 b^2 c^2 d^2 e^6) + (- (b^5 d^2 + a^2 b^3 e^2 + a^2 e^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + b^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} + 12 a^2 b^3 c^2 d^2 - 2 a^2 b^4 d^2 e - 7 a^2 b^3 c^2 d^2 - a^2 c^2 d^2 * (- (4 a^2 c - b^2)^3)^{(1/2)} - 4 a^3 b^3 c^2 d^2 e - 16 a^3 c^2 d^2 e + 12 a^2 b^2 c^2 d^2 e - 2 a^2 b^3 c^2 d^2 e * (- (4 a^2 c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a^2 b^2 c^4 d^4 + a^2 b^4 c^2 e^4 - 2 b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a^2 b^5 c^2 d^3 e + 16 a^2 b^3 c^3 d^3 e - 32 a^2 b^3 c^4 d^3 e - 32 a^3 b^3 c^3 d^2 e^3 - 6 a^2 b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d^2 e^3))^{(1/2)} * (256 a^4 b^2 c^3 e^9 - 32 a^3 b^4 c^2 e^9 - 512 a^5 c^4 e^9 + 512 a^2 c^7 d^6 e^3 + 512 a^3 c^6 d^4 e^5 - 512 a^4 c^5 d^2 e^7 - 32 b^3 c^6 d^7 e^2 + 128 b^4 c^5 d^6 e^3 - 192 b^5 c^4 d^5 e^4 + 128 b^6 c^3 d^4 e^5 - 32 b^7 c^2 d^3 e^6 + 512 a^2 b^2 c^5 d^4 e^5 + 288 a^2 b^3 c^4 d^3 e^6 - 1
\end{aligned}$$

$$\begin{aligned}
& 92a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + 128ab^3c^7d^7e^2 + 64 \\
& 0a^4b^3c^4d^2e^8 - 640a^3b^2c^6d^6e^3 + 1056a^2b^3c^5d^5e^4 - 672a^2 \\
& b^4c^4d^4e^5 + 96a^2b^5c^3d^3e^6 + 32a^2b^6c^2d^2e^7 - 1152a^2b^3 \\
& c^6d^5e^4 + 32a^2b^5c^2d^2e^8 - 640a^3b^3c^5d^3e^6 - 288a^3b^3c^3 \\
& d^3e^8) + 128a^3c^5d^4e^4 + 64a^4c^4d^2e^6 - 96a^2b^2c^4d^4e^4 \\
& + 64a^2b^3c^3d^3e^5 + 32a^2b^4c^2d^2e^6 - 144a^3b^2c^3d^2e^6 \\
& + 64a^4b^3c^3d^2e^7 - 16a^2b^2c^5d^6e^2 + 16a^2b^3c^4d^5e^3 + 16a \\
& a^2b^4c^3d^4e^4 - 16a^2b^5c^2d^3e^5 - 64a^2b^3c^5d^5e^3 - 16a^3b^3 \\
& c^2d^2e^7) * (- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{1/2} + \\
& b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 - 2a^2b^4d^2e - 7a^2b^3 \\
& c^2d^2 - a^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e - 16a^3c^2d^2e \\
& + 12a^2b^2c^2d^2e - 2a^2b^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 \\
& + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e \\
& + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e \\
& + 16a^2b^3c^3d^3e - 32a^2b^2c^4d^3e - 32a^3b^3c^3d^2e^3 - 6a^2b^4c^2d^2e^2 \\
& + 16a^2b^3c^2d^2e^3))^{1/2} + 16a^2c^4d^5e + 4a^4c^2d^2e^5 - 60a^3c^3d^3e^3 \\
& + 24a^2b^2c^2d^3e^3 - 4a^2b^2c^3d^5e - 4a^2b^4c^2d^3e^3 - 4a^3b^2c^2d^4e^2 \\
& + 20a^2b^3c^3d^4e^2 + 8a^2b^3c^2d^2e^4 - 16a^3b^3c^2d^2e^4) + x * (2a^4c^2e^5 \\
& + 4a^2c^3d^4e + 2b^4c^2d^4e - 8a^2b^2c^2d^4e) * (- (b^5d^2 + a^2b^3e^2 \\
& + a^2e^2 * (- (4ac - b^2)^3)^{1/2} + b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 \\
& - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e \\
& + 12a^2b^2c^2d^2e - 2a^2b^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 \\
& + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e \\
& + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e \\
& - 32a^2b^2c^4d^3e - 32a^3b^3c^3d^2e^3 - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} - \\
& ((- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{1/2} + b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 \\
& - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e \\
& + 12a^2b^2c^2d^2e - 2a^2b^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 \\
& + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e \\
& + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e \\
& - 32a^2b^2c^4d^3e - 32a^3b^3c^3d^2e^3 - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} * ((x * (8a^3b^3 \\
& c^2e^7 - 32a^4b^3c^2e^7 - 112a^4c^3d^2e^6 + 8b^3c^4d^6e + 8b^6c^2d^3e^4 \\
& - 112a^2c^5d^5e^2 + 32a^3c^4d^3e^4 - 8b^4c^3d^5e^2 - 8b^5c^2d^4e^3 - 32a^2b^3c^5d^6e \\
& - 48a^2b^2c^3d^3e^4 + 8a^2b^3c^2d^2e^5 - 8a^2b^5c^2d^2e^5 - 8a^2b^4c^2d^2e^6 \\
& + 64a^2b^2c^4d^5e^2 + 8a^2b^3c^3d^4e^3 - 16a^2b^4c^2d^3e^4 + 64a^2b^3c^4d^4e^3 \\
& + 64a^3b^3c^3d^2e^5 + 64a^3b^2c^2d^2e^6) - (- (b^5d^2 + a^2b^3e^2 + a^2e^2 * \\
& (- (4ac - b^2)^3)^{1/2} + b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 \\
& - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e \\
& - 16a^3c^2d^2e + 12a^2b^2c^2d^2e - 2a^2b^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 \\
& + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4
\end{aligned}$$

$$\begin{aligned}
& 4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 3 \\
& 2a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e - 32a^2b^2c^4d^3 \\
& *e - 32a^3b^2c^3d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} \\
& *(x*(-(b^5d^2 + a^2b^3e^2 + a^2e^2*(-(4ac - b^2)^3)^{(1/2)} + b^2d^2 \\
& *(-(4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2a^2b^4d^2e - 7a^2b^3c^2d^2 \\
& - a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2d^2e + 12a^2 \\
& 2b^2c^2d^2e - 2a^2b^2d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^5d^4 + 16a^4 \\
& c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6c^2d^2 \\
& e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e \\
& - 32a^2b^2c^4d^3e - 32a^3b^2c^3d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} \\
& *(256a^4b^2c^3e^9 - 32a^3b^4c^2e^9 - 512a^5c^4e^9 + 512a^2c^7d^6e^3 + 512a^3c^6d^4e^5 - \\
& 512a^4c^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 + 128b^6c^3d^4e^5 \\
& - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 \\
& + 384a^3b^2c^4d^2e^7 + 128a^2b^3c^7d^7e^2 + 640a^4b^2c^4d^2e^8 - 640a^2b^2c^6d^6e^3 \\
& + 1056a^2b^3c^5d^5e^4 - 672a^2b^4c^4d^4e^5 + 96a^2b^5c^3d^3e^6 + 32a^2b^6c^2d^2e^7 \\
& - 1152a^2b^2c^6d^5e^4 + 32a^2b^5c^2d^2e^8 - 640a^3b^2c^5d^3e^6 - 288a^3b^3c^3d^3e^8) \\
& + 64a^2c^6d^6e^2 + 128a^3c^5d^4e^4 + 64a^4c^4d^2e^6 - 96a^2b^2c^4d^4e^4 + 64a^2b^3c^3d^3 \\
& e^5 + 32a^2b^4c^2d^2e^6 - 144a^3b^2c^3d^2e^6 + 64a^4b^2c^3d^2e^7 - 16a^2b^2c^5d^6e^2 \\
& + 16a^2b^3c^4d^5e^3 + 16a^2b^4c^3d^4e^4 - 16a^2b^5c^2d^3e^5 - 64a^2b^2c^5d^5e^3 - 16a^3b^3c^2d^2e^7) \\
& *(-(b^5d^2 + a^2b^3e^2 + a^2e^2*(-(4ac - b^2)^3)^{(1/2)} + b^2d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& + 12a^2b^2c^2d^2 - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& - 4a^3b^2c^2e^2 - 16a^3c^2d^2e + 12a^2b^2c^2d^2e - 2a^2b^2d^2e*(-(4ac - b^2)^3)^{(1/2)}) \\
& / (8*(16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6c^2d^2 \\
& e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e - 32a^2b^2c^4d^3e \\
& - 32a^3b^2c^3d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} - 16a^2c^4d^5e - 4a^4c^2d^2e^5 \\
& + 60a^3c^3d^3e^3 - 24a^2b^2c^2d^3e^3 + 4a^2b^2c^3d^5e + 4a^2b^4c^3d^3e^3 + 4a^3b^2c^2d^2e^5 \\
& + 4a^2b^3c^2d^4e^2 - 20a^2b^2c^3d^4e^2 - 8a^2b^3c^2d^2e^4 + 16a^3b^2c^2d^2e^4) + x*(2a^4c^2e^5 \\
& + 4a^2c^3d^4e + 2b^4c^2d^4e - 8a^2b^2c^2d^4e) *(-(b^5d^2 + a^2b^3e^2 + a^2e^2*(-(4ac - b^2)^3)^{(1/2)} \\
& + b^2d^2*(-(4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2 \\
& *(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2d^2e + 12a^2b^2c^2d^2e - 2a^2b^2d^2e \\
& *(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 \\
& - 2b^5c^2d^3e + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e \\
& - 32a^2b^2c^4d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} + 2a^3c^2d^2e^2 + 2a^2b^2c^2d^3e) \\
& *(-(b^5d^2 + a^2b^3e^2 + a^2e^2*(-(4ac - b^2)^3)^{(1/2)} + b^2d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& + 12a^2b^2c^2d^2 - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 \\
& - 16a^3c^2d^2e + 12a^2b^2c^2d^2e - 2a^2b^2d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^5d^4 + 16a^4c^3e^4 \\
& + b^4c^3d^4 - 8a^2b^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 \\
& - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e - 32a^2b^2c^4d^3e - 32a^3b^2c^3d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} \\
& + 2a^3c^2d^2e^2 + 2a^2b^2c^2d^3e) *(-(b^5d^2 + a^2b^3e^2 + a^2e^2*(-(4ac - b^2)^3)^{(1/2)} + b^2d^2*(-(4ac - b^2)^3)^{(1/2)} \\
& + 12a^2b^2c^2d^2 - 2a^2b^4d^2e - 7a^2b^3c^2d^2 - a^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2d^2e \\
& + 12a^2b^2c^2d^2e - 2a^2b^2d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8a^2b^2c^4d^4 \\
& + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6c^2d^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2a^2b^5c^2d^3e + 16a^2b^3c^3d^3e \\
& - 32a^2b^2c^4d^3e - 6a^2b^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{(1/2)} + 2a^3c^2d^2e^2 + 2a^2b^2c^2d^3e)
\end{aligned}$$

$$\begin{aligned}
& *b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 \\
& + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32 \\
& *a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3* \\
& e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)} \\
&) * 2i + \operatorname{atan}(\left(\left(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \right. \right. \\
& \left. \left. b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e \right. \right. \\
& \left. \left. + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \right) / (8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3) \right) \right)^{(1/2)} * (x*(8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3) \right) \right)^{(1/2)} * (64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3) \right) \right)^{(1/2)} * (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2
\end{aligned}$$

$$\begin{aligned}
& *d^3e^5 - 64a^2b^3c^5d^5e^3 - 16a^3b^3c^2d^5e^7)) * (- (b^5d^2 + a^2b^3e^2 - a^2e^2 * (- (4ac - b^2)^3)^{1/2} - b^2d^2 * (- (4ac - b^2)^3)^{1/2}) + 12a^2b^3c^2d^2 - 2ab^4d^2e - 7ab^3c^2d^2 + acd^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e + 12a^2b^2c^2d^2e + 2ab^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8ab^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6cd^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2ab^5c^2d^2e^3 + 16ab^3c^3d^3e - 32a^2b^3c^4d^3e - 32a^3b^3c^3d^2e^3 - 6ab^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} + 16a^2c^4d^5e + 4a^4c^2d^2e^5 - 60a^3c^3d^3e^3 + 24a^2b^2c^2d^3e^3 - 4ab^2c^3d^5e - 4ab^4c^2d^3e^3 - 4a^3b^2c^2d^2e^5 - 4ab^3c^2d^4e^2 + 20a^2b^3c^3d^4e^2 + 8a^2b^3c^2d^2e^4 - 16a^3b^3c^2d^2e^4) + x * (2a^4c^2e^5 + 4a^2c^3d^4e + 2b^4cd^4e - 8ab^2c^2d^4e)) * (- (b^5d^2 + a^2b^3e^2 - a^2e^2 * (- (4ac - b^2)^3)^{1/2} - b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 - 2ab^4d^2e - 7ab^3c^2d^2 + acd^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e - 16a^3c^2d^2e + 12a^2b^2c^2d^2e + 2ab^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8ab^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6cd^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2ab^5c^2d^2e^3 + 16ab^3c^3d^3e - 32a^2b^3c^4d^3e - 32a^3b^3c^3d^2e^3 - 6ab^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} * i + ((- (b^5d^2 + a^2b^3e^2 - a^2e^2 * (- (4ac - b^2)^3)^{1/2} - b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 - 2ab^4d^2e - 7ab^3c^2d^2 + acd^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e - 16a^3c^2d^2e + 12a^2b^2c^2d^2e + 2ab^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8ab^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6cd^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2ab^5c^2d^2e^3 + 16ab^3c^3d^3e - 32a^2b^3c^4d^3e - 32a^3b^3c^3d^2e^3 - 6ab^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} * ((x * (8a^3b^3c^2e^7 - 32a^4b^3c^2e^7 - 112a^4c^3d^2e^6 + 8b^3c^4d^6e + 8b^6cd^3e^4 - 112a^2c^5d^5e^2 + 32a^3c^4d^3e^4 - 8b^4c^3d^5e^2 - 8b^5c^2d^4e^3 - 32ab^3c^5d^6e - 48a^2b^2c^3d^3e^4 + 8a^2b^3c^2d^2e^5 - 8ab^5c^2d^2e^5 - 8a^2b^4c^2d^2e^6 + 64ab^2c^4d^5e^2 + 8ab^3c^3d^4e^3 - 16ab^4c^2d^3e^4 + 64a^2b^3c^4d^4e^3 + 64a^3b^3c^3d^2e^5 + 64a^3b^2c^2d^2e^6) - (- (b^5d^2 + a^2b^3e^2 - a^2e^2 * (- (4ac - b^2)^3)^{1/2} - b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 - 2ab^4d^2e - 7ab^3c^2d^2 + acd^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e - 16a^3c^2d^2e + 12a^2b^2c^2d^2e + 2ab^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8ab^2c^4d^4 + a^2b^4c^2e^4 - 2b^5c^2d^3e + b^6cd^2e^2 - 8a^3b^2c^2e^4 + 32a^3c^4d^2e^2 - 2ab^5c^2d^2e^3 + 16ab^3c^3d^3e - 32a^2b^3c^4d^3e - 32a^3b^3c^3d^2e^3 - 6ab^4c^2d^2e^2 + 16a^2b^3c^2d^2e^3))^{1/2} * (x * (- (b^5d^2 + a^2b^3e^2 - a^2e^2 * (- (4ac - b^2)^3)^{1/2} - b^2d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2d^2 - 2ab^4d^2e - 7ab^3c^2d^2 + acd^2 * (- (4ac - b^2)^3)^{1/2} - 4a^3b^3c^2d^2e - 16a^3c^2d^2e + 12a^2b^2c^2d^2e + 2ab^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8 * (16a^2c^5d^4 + 16a^4c^3e^4 + b^4c^3d^4 - 8
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^4 d^4 + a^2 b^4 c^4 e^4 - 2 b^5 c^2 d^3 e + b^6 c d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a b^5 c d e^3 + 16 a b^3 c^3 d^3 e - 32 a^2 b^2 c^4 d^3 e - 32 a^3 b^2 c^3 d e^3 - 6 a b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d e^3))^{(1/2)} * (256 a^4 b^2 c^3 e^9 - 32 a^3 b^4 c^2 e^9 - 512 a^5 c^4 e^9 + 512 a^2 c^7 d^6 e^3 + 512 a^3 c^6 d^4 e^5 - 512 a^4 c^5 d^2 e^7 - 32 b^3 c^6 d^7 e^2 + 128 b^4 c^5 d^6 e^3 - 192 b^5 c^4 d^5 e^4 + 128 b^6 c^3 d^4 e^5 - 32 b^7 c^2 d^3 e^6 + 512 a^2 b^2 c^5 d^4 e^5 + 288 a^2 b^3 c^4 d^3 e^6 - 192 a^2 b^4 c^3 d^2 e^7 + 384 a^3 b^2 c^4 d^2 e^7 + 128 a b^5 c^7 d^7 e^2 + 640 a^4 b^2 c^4 d e^8 - 640 a b^2 c^6 d^6 e^3 + 1056 a b^3 c^5 d^5 e^4 - 672 a b^4 c^4 d^4 e^5 + 96 a b^5 c^3 d^3 e^6 + 32 a b^6 c^2 d^2 e^7 - 1152 a^2 b^2 c^6 d^5 e^4 + 32 a^2 b^5 c^2 d e^8 - 640 a^3 b^2 c^5 d^3 e^6 - 288 a^3 b^3 c^3 d e^8) + 64 a^2 c^6 d^6 e^2 + 128 a^3 c^5 d^4 e^4 + 64 a^4 c^4 d^2 e^6 - 96 a^2 b^2 c^4 d^4 e^4 + 64 a^2 b^3 c^3 d^3 e^5 + 32 a^2 b^4 c^2 d^2 e^6 - 144 a^3 b^2 c^3 d^2 e^6 + 64 a^4 b^2 c^3 d e^7 - 16 a b^2 c^5 d^6 e^2 + 16 a b^3 c^4 d^5 e^3 + 16 a b^4 c^3 d^4 e^4 - 16 a b^5 c^2 d^3 e^5 - 64 a^2 b^2 c^5 d^5 e^3 - 16 a^3 b^3 c^2 d e^7) * (- (b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - b^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} + 12 a^2 b^2 c^2 d^2 - 2 a b^4 d e - 7 a b^3 c d^2 + a c d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 4 a^3 b^2 c e^2 - 16 a^3 c^2 d e + 12 a^2 b^2 c d e + 2 a b^2 d e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a b^2 c^4 d^4 + a^2 b^4 c e^4 - 2 b^5 c^2 d^3 e + b^6 c d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a b^5 c d e^3 + 16 a b^3 c^3 d^3 e - 32 a^2 b^2 c^4 d^3 e - 32 a^3 b^2 c^3 d e^3 - 6 a b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d e^3))^{(1/2)} - 16 a^2 c^4 d^5 e - 4 a^4 c^2 d e^5 + 60 a^3 c^3 d^3 e^3 - 24 a^2 b^2 c^2 d^3 e^3 + 4 a b^2 c^3 d^5 e + 4 a b^4 c d^3 e^3 + 4 a^3 b^2 c d e^5 + 4 a b^3 c^2 d^4 e^2 - 20 a^2 b^2 c^3 d^4 e^2 - 8 a^2 b^3 c d^2 e^4 + 16 a^3 b^2 c^2 d^2 e^4) + x * (2 a^4 c e^5 + 4 a^2 c^3 d^4 e + 2 b^4 c d^4 e - 8 a b^2 c^2 d^4 e) * (- (b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - b^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} + 12 a^2 b^2 c^2 d^2 - 2 a b^4 d e - 7 a b^3 c d^2 + a c d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 4 a^3 b^2 c e^2 - 16 a^3 c^2 d e + 12 a^2 b^2 c d e + 2 a b^2 d e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a b^2 c^4 d^4 + a^2 b^4 c e^4 - 2 b^5 c^2 d^3 e + b^6 c d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a b^5 c d e^3 + 16 a b^3 c^3 d^3 e - 32 a^2 b^2 c^4 d^3 e - 32 a^3 b^2 c^3 d e^3 - 6 a b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d e^3))^{(1/2)} * 1i) / (((- (b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - b^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} + 12 a^2 b^2 c^2 d^2 - 2 a b^4 d e - 7 a b^3 c d^2 + a c d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 4 a^3 b^2 c e^2 - 16 a^3 c^2 d e + 12 a^2 b^2 c d e + 2 a b^2 d e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 * (16 a^2 c^5 d^4 + 16 a^4 c^3 e^4 + b^4 c^3 d^4 - 8 a b^2 c^4 d^4 + a^2 b^4 c e^4 - 2 b^5 c^2 d^3 e + b^6 c d^2 e^2 - 8 a^3 b^2 c^2 e^4 + 32 a^3 c^4 d^2 e^2 - 2 a b^5 c d e^3 + 16 a b^3 c^3 d^3 e - 32 a^2 b^2 c^4 d^3 e - 32 a^3 b^2 c^3 d e^3 - 6 a b^4 c^2 d^2 e^2 + 16 a^2 b^3 c^2 d e^3))^{(1/2)} * ((x * (8 a^3 b^3 c e^7 - 32 a^4 b^2 c^2 e^7 - 112 a^4 c^3 d e^6 + 8 b^3 c^4 d^6 e + 8 b^6 c d^3 e^4 - 112 a^2 c^5 d^5 e^2 + 32 a^3 c^4 d^3 e^4 - 8 b^4 c^3 d^5 e^2 - 8 b^5 c^2 d^4 e^3 - 32 a b^2 c^5 d^6 e - 48 a^2 b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64 \\
& *a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + \\
& 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2 \\
& *e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + \\
& 16*a^2*b^3*c^2*d*e^3))^{(1/2)}*(64*a^2*c^6*d^6*e^2 - x*(-(b^5*d^2 + a^2*b^3 \\
& *e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^ \\
& ^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - \\
& 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 3 \\
& 2*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e \\
& ^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4 \\
& *e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e \\
& ^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152 \\
& *a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4 \\
& *d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16 \\
& *a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - \\
& 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2 \\
& *c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - \\
& 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d \\
& *e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3))^{(1/2)} + 16*a^2*c^4*d^5 \\
& *e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2 \\
& *c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*d*e^5 - 4*a*b^3*c^2*d^4*e^2 + \\
& 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 - 16*a^3*b*c^2*d^2*e^4) + x*(2* \\
& a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e - 8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 \\
& + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2
\end{aligned}$$

$$\begin{aligned}
& *a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e \\
& ^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 1 \\
& 6*a^2*b^3*c^2*d*e^3)))^{(1/2)} - (((-b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2* \\
& a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)}) \\
& / (8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4 \\
& *d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)} * ((x*(\\
& 8*a^3*b^3*c*e^7 - 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5* \\
& e^2 - 8*b^5*c^2*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2 \\
& *b^3*c^2*d^2*e^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5 \\
& *e^2 + 8*a*b^3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + \\
& 64*a^3*b*c^3*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) - (-b^5*d^2 + a^2*b^3*e^2 - a \\
& ^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2 \\
& *b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2 \\
& *c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2* \\
& e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c \\
& ^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3 \\
&)))^{(1/2)} * (x*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3 \\
& *c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e \\
& + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*d^4 \\
& + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c \\
& ^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5 \\
& *c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6 \\
& *a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^{(1/2)} * (256*a^4*b^2*c^3*e^9 - 3 \\
& 2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4 \\
& *e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192 \\
& *b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c \\
& ^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^ \\
& 2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d \\
& ^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3* \\
& e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 \\
& - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 64*a^2*c^6*d^6*e^2 + 128 \\
& *a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3 \\
& *c^3*d^3*e^5 + 32*a^2*b^4*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b* \\
& c^3*d*e^7 - 16*a*b^2*c^5*d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4* \\
& e^4 - 16*a*b^5*c^2*d^3*e^5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7)))*
\end{aligned}$$

$$\begin{aligned}
& \left(-(b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{1/2} - b^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 4a^3 b^3 c^2 d e + 12a^2 b^2 c^2 d e + 2a^2 b^3 c^2 d e + 2a^2 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} \right) / \left(8(16a^2 c^5 d^4 + 16a^4 c^3 e^4 + b^4 c^3 d^4 - 8a^2 b^2 c^4 d^4 + a^2 b^4 c^4 e^4 - 2b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8a^3 b^2 c^2 e^4 + 32a^3 c^4 d^2 e^2 - 2a^2 b^5 c^2 d e^3 + 16a^2 b^3 c^3 d^3 e - 32a^2 b^3 c^4 d^3 e - 32a^3 b^3 c^3 d e^3 - 6a^2 b^4 c^2 d^2 e^2 + 16a^2 b^3 c^2 d e^3) \right)^{1/2} - 16a^2 c^4 d^5 e - 4a^4 c^2 d e^5 + 60a^3 c^3 d^3 e^3 - 24a^2 b^2 c^2 d^3 e^3 + 4a^2 b^2 c^3 d^5 e + 4a^2 b^4 c^2 d^3 e^3 + 4a^3 b^2 c^2 d e^5 + 4a^2 b^3 c^2 d^4 e^2 - 20a^2 b^3 c^3 d^4 e^2 - 8a^2 b^3 c^3 d^2 e^4 + 16a^3 b^3 c^2 d^2 e^4) + x(2a^4 c^2 e^5 + 4a^2 c^3 d^4 e + 2b^4 c^2 d^4 e - 8a^2 b^2 c^2 d^4 e) * \left(-(b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{1/2} - b^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 4a^3 b^3 c^2 d e + 12a^2 b^2 c^2 d e + 2a^2 b^3 c^2 d e + 2a^2 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} \right) / \left(8(16a^2 c^5 d^4 + 16a^4 c^3 e^4 + b^4 c^3 d^4 - 8a^2 b^2 c^4 d^4 + a^2 b^4 c^4 e^4 - 2b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8a^3 b^2 c^2 e^4 + 32a^3 c^4 d^2 e^2 - 2a^2 b^5 c^2 d e^3 + 16a^2 b^3 c^3 d^3 e - 32a^2 b^3 c^4 d^3 e - 32a^3 b^3 c^3 d e^3 - 6a^2 b^4 c^2 d^2 e^2 + 16a^2 b^3 c^2 d e^3) \right)^{1/2} + 2a^3 c^2 d^2 e^2 + 2a^2 b^3 c^2 d^3 e) * \left(-(b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{1/2} - b^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 4a^3 b^3 c^2 d e + 12a^2 b^2 c^2 d e + 2a^2 b^3 c^2 d e + 2a^2 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} \right) / \left(8(16a^2 c^5 d^4 + 16a^4 c^3 e^4 + b^4 c^3 d^4 - 8a^2 b^2 c^4 d^4 + a^2 b^4 c^4 e^4 - 2b^5 c^2 d^3 e + b^6 c^2 d^2 e^2 - 8a^3 b^2 c^2 e^4 + 32a^3 c^4 d^2 e^2 - 2a^2 b^5 c^2 d e^3 + 16a^2 b^3 c^3 d^3 e - 32a^2 b^3 c^4 d^3 e - 32a^3 b^3 c^3 d e^3 - 6a^2 b^4 c^2 d^2 e^2 + 16a^2 b^3 c^2 d e^3) \right)^{1/2} * 2i - \left(\log(a^5 d^2 e^8 - b^5 d^7 e^3 + 2a^2 b^4 d^6 e^4 - 2a^4 b^3 d^3 e^7 + 2a^4 c^2 d^4 e^6 + b^4 c^2 d^8 e^2 + 16a^2 c^3 x (-d^3 e)^{5/2} + a^5 e^8 x (-d^3 e)^{1/2} - a^2 b^3 d^5 e^5 + a^3 b^2 d^4 e^6 + 16a^2 c^3 d^8 e^2 + 17a^3 c^2 d^6 e^4 + b^4 c^2 x (-d^3 e)^{5/2} + a^2 b^3 e^4 x (-d^3 e)^{3/2} + b^5 d^2 e^2 x (-d^3 e)^{3/2} + 7a^2 b^3 c^2 d^7 e^3 + 2a^3 b^3 c^2 d^5 e^5 - 8a^2 b^2 c^2 x (-d^3 e)^{5/2} - 8a^2 b^2 c^2 d^8 e^2 - 12a^2 b^3 c^2 d^7 e^3 - 12a^2 b^2 c^2 d^6 e^4 - 2a^3 b^3 c^2 e^4 x (-d^3 e)^{3/2} - 2a^2 b^4 d e^3 x (-d^3 e)^{3/2} - 2a^4 b^3 d^2 e^6 x (-d^3 e)^{1/2} + a^3 b^2 d^2 e^6 x (-d^3 e)^{1/2} + 12a^2 b^3 c^2 d^2 e^2 x (-d^3 e)^{3/2} - 7a^2 b^3 c^2 d^2 e^2 x (-d^3 e)^{3/2} + 12a^2 b^2 c^2 d e^3 x (-d^3 e)^{3/2} \right) * \left(-(d^3 e)^{1/2} \right) / \left(2(a^2 e^3 - b^2 d e^2 + c^2 d^2 e) \right) + \left(\log(a^5 d^2 e^8 - b^5 d^7 e^3 + 2a^2 b^4 d^6 e^4 - 2a^4 b^3 d^3 e^7 + 2a^4 c^2 d^4 e^6 + b^4 c^2 d^8 e^2 - 16a^2 c^3 x (-d^3 e)^{5/2} - a^5 e^8 x (-d^3 e)^{1/2} - a^2 b^3 d^5 e^5 + a^3 b^2 d^4 e^6 + 16a^2 c^3 d^8 e^2 + 17a^3 c^2 d^6 e^4 - b^4 c^2 x (-d^3 e)^{5/2} - a^2 b^3 e^4 x (-d^3 e)^{3/2} - b^5 d^2 e^2 x (-d^3 e)^{3/2} + 7a^2 b^3 c^2 d^7 e^3 + 2a^3 b^3 c^2 d^5 e^5 + 8a^2 b^2 c^2 x (-d^3 e)^{5/2} - 8a^2 b^2 c^2 d^8 e^2 - 12a^2 b^3 c^2 d^7 e^3 - 12a^2 b^2 c^2 d^6 e^4 + 2a^3 b^3 c^2 e^4 x (-d^3 e)^{3/2} + 2a^2 b^
\end{aligned}$$

$$4*d*e^3*x*(-d^3*e)^{(3/2)} + 2*a^4*b*d*e^7*x*(-d^3*e)^{(1/2)} + 17*a^3*c^2*d*e^3*x*(-d^3*e)^{(3/2)} - 2*a^4*c*d^2*e^6*x*(-d^3*e)^{(1/2)} - a^3*b^2*d^2*e^6*x*(-d^3*e)^{(1/2)} - 12*a^2*b*c^2*d^2*e^2*x*(-d^3*e)^{(3/2)} + 7*a*b^3*c*d^2*e^2*x*(-d^3*e)^{(3/2)} - 12*a^2*b^2*c*d*e^3*x*(-d^3*e)^{(3/2)}*(-d^3*e)^{(1/2)}/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.228 \quad \int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=251

$$\frac{\sqrt{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{2} \sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2)} + \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{2} \sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2)} - \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{ae^2 - bde + cd^2}}$$

Rubi [A] time = 0.45, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{2} \sqrt{b-\sqrt{b^2-4ac}} (ae^2 - bde + cd^2)} + \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{2} \sqrt{\sqrt{b^2-4ac}+b} (ae^2 - bde + cd^2)} - \frac{\sqrt{d} \sqrt{e} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{ae^2 - bde + cd^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx &= \int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)} \right) dx \\ &= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4} dx}{cd^2-bde+ae^2} - \frac{(de) \int \frac{1}{d+ex^2} dx}{cd^2-bde+ae^2} \\ &= -\frac{\sqrt{d} \sqrt{e} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2(cd^2-bde+ae^2)} + \frac{c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2(cd^2-bde+ae^2)} \\ &= \frac{\sqrt{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} \end{aligned}$$

Mathematica [A] time = 0.50, size = 277, normalized size = 1.10

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(-ae^2+bde-cd^2)} - \frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}(-ae^2+bde-cd^2)} - \frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])*(-(c*d^2) + b*d*e - a*e^2)) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])*(-(c*d^2) + b*d*e - a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [B] time = 3.76, size = 12269, normalized size = 48.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\frac{1}{2} \left(\sqrt{\frac{1}{2}} (c d^2 - b d e + a e^2) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 + ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4)} \sqrt{\frac{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4)}{(b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8} \right) \log(-2 (c^2 d^2 - a c e^2) x + \sqrt{\frac{1}{2}} ((b^2 c - 4 a c^2) d^2 e - (a b^2 - 4 a^2 c) e^3 - (2 (b^2 c^3 - 4 a c^4) d^5 - 5 (b^3 c^2 - 4 a b c^3) d^4 e + 4 (b^4 c - 3 a b^2 c^2 - 4 a^2 c^3) d^3 e^2 - (b^5 + 2 a b^3 c - 24 a^2 b c^2) d^2 e^3 + 2 (a b^4 - 3 a^2 b^2 c - 4 a^3 c^2) d e^4 - (a^2 b^3 - 4 a^3 b c) e^5) \sqrt{\frac{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4)}{(b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8}) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 + ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4)} \sqrt{\frac{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4)}{(b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8}) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 + ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4)}) - \sqrt{\frac{1}{2}} (c d^2 - b d e + a e^2) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 + ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4)}$$

$$\begin{aligned}
& d^3 e^3 + (a^2 b^2 - 4 a^3 c) e^4 \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)} / ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) * \log(-2 (c^2 d^2 - a c e^2) x - \sqrt{1/2} ((b^2 c - 4 a c^2) d^2 e - (a b^2 - 4 a^2 c) e^3 - (2 (b^2 c^3 - 4 a c^4) d^5 - 5 (b^3 c^2 - 4 a b c^3) d^4 e + 4 (b^4 c - 3 a b^2 c^2 - 4 a^2 c^3) d^3 e^2 - (b^5 + 2 a b^3 c - 24 a^2 b c^2) d^2 e^3 + 2 (a b^4 - 3 a^2 b^2 c - 4 a^3 c^2) d e^4 - (a^2 b^3 - 4 a^3 b c) e^5) \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)}) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 + ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)}) / ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) + \sqrt{1/2} (c d^2 - b d e + a e^2) \sqrt{-(b c d^2 - 4 a c d e + a b e^2 - ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)}) / ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) * \log(-2 (c^2 d^2 - a c e^2) x + \sqrt{1/2} ((b^2 c - 4 a c^2) d^2 e - (a b^2 - 4 a^2 c) e^3 + (2 (b^2 c^3 - 4 a c^4) d^5 - 5 (b^3 c^2 - 4 a b c^3) d^4 e + 4 (b^4 c - 3 a b^2 c^2 - 4 a^2 c^3) d^3 e^2 - (b^5 + 2 a b^3 c - 24 a^2 b c^2) d^2 e^3 + 2 (a b^4 - 3 a^2 b^2 c - 4 a^3 c^2) d e^4 - (a^2 b^3 - 4 a^3 b c) e^5) \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)}) / ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4) * \log(-2 (c^2 d^2 - a c e^2) x + \sqrt{1/2} ((b^2 c - 4 a c^2) d^2 e - (a b^2 - 4 a^2 c) e^3 + (2 (b^2 c^3 - 4 a c^4) d^5 - 5 (b^3 c^2 - 4 a b c^3) d^4 e + 4 (b^4 c - 3 a b^2 c^2 - 4 a^2 c^3) d^3 e^2 - (b^5 + 2 a b^3 c - 24 a^2 b c^2) d^2 e^3 + 2 (a b^4 - 3 a^2 b^2 c - 4 a^3 c^2) d e^4 - (a^2 b^3 - 4 a^3 b c) e^5) \sqrt{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) / ((b^2 c^4 - 4 a c^5) d^8 - 4 (b^3 c^3 - 4 a b c^4) d^7 e + 2 (3 b^4 c^2 - 10 a b^2 c^3 - 8 a^2 c^4) d^6 e^2 - 4 (b^5 c - a b^3 c^2 - 12 a^2 b c^3) d^5 e^3 + (b^6 + 8 a b^4 c - 42 a^2 b^2 c^2 - 24 a^3 c^3) d^4 e^4 - 4 (a b^5 - a^2 b^3 c - 12 a^3 b c^2) d^3 e^5 + 2 (3 a^2 b^4 - 10 a^3 b^2 c - 8 a^4 c^2) d^2 e^6 - 4 (a^3 b^3 - 4 a^4 b c) d e^7 + (a^4 b^2 - 4 a^5 c) e^8)}) / ((b^2 c^2 - 4 a c^3) d^4 - 2 (b^3 c - 4 a b c^2) d^3 e + (b^4 - 2 a b^2 c - 8 a^2 c^2) d^2 e^2 - 2 (a b^3 - 4 a^2 b c) d e^3 + (a^2 b^2 - 4 a^3 c) e^4)
\end{aligned}$$

$$\begin{aligned}
& e + a*e^2)*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 \\
& - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(\\
& a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*\sqrt{((c^2*d^4 - 2*a*c*d \\
& ^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e \\
& + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - \\
& 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^ \\
& 4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^ \\
& 3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4 \\
& *a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^ \\
& 4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 \\
& - 4*a^3*c)*e^4)*\log(-2*(c^2*d^2 - a*c*e^2)*x + \sqrt{1/2}*((b^2*c - 4*a*c^ \\
& 2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 \\
& - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2 \\
& *a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^ \\
& 4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*\sqrt{((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b \\
& ^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a \\
& *b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^ \\
& 3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^ \\
& 2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)* \\
& d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*\sqrt{-(\\
& (b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a* \\
& b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c \\
&)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*\sqrt{((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4) \\
& /((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - \\
& 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^ \\
& 5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 \\
& - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c \\
& ^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((\\
& b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8 \\
& *a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)) \\
&) - \sqrt{1/2}*(c*d^2 - b*d*e + a*e^2)*\sqrt{-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 \\
& + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c \\
& - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e \\
& ^4)*\sqrt{((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(\\
& b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e \\
& ^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a \\
& ^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3 \\
& *e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^ \\
& 4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3 \\
& *c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - \\
& 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*\log(-2*(c^2*d^2 - a*c*e^2)*x - \\
& \sqrt{1/2}*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - \\
& 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4* \\
& a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3* \\
& a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*\sqrt{((c^2*d^4 - 2
\end{aligned}$$

$$\begin{aligned}
& *a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)* \\
& d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3 \\
& *c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3) \\
& *d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - \\
& 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 \\
& - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a \\
& *c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2 \\
& *e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*sqrt((c^2*d^4 \\
& - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4) \\
& *d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a \\
& *b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3 \\
& *c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 \\
& - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 \\
& - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e \\
& + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 \\
& + (a^2*b^2 - 4*a^3*c)*e^4)) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c \\
& *d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2) \\
& *d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d* \\
& e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b \\
& ^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a \\
& *b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 \\
& + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2 \\
& *b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)* \\
& d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2* \\
& c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2 \\
& *c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log \\
& (-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4 \\
& *a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + \\
& 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2) \\
& *d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3* \\
& b*c)*e^5))*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 \\
& - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4) \\
& *d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c \\
& - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2) \\
& *d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 \\
& - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e \\
& + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - \\
& 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - \\
& 4*a^3*c)*e^4))*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5) \\
& *d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2* \\
& c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4 \\
& *c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3* \\
& b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3* \\
& b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 \\
& - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
\end{aligned}$$

$$\begin{aligned} & * (a*b^3 - 4*a^2*b*c) * d * e^3 + (a^2*b^2 - 4*a^3*c) * e^4)) - \text{sqrt}(1/2) * (c*d^2 \\ & - b*d*e + a*e^2) * \text{sqrt}(- (b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3) \\ &) * d^4 - 2*(b^3*c - 4*a*b*c^2) * d^3 * e + (b^4 - 2*a*b^2*c - 8*a^2*c^2) * d^2 * e^2 \\ & - 2*(a*b^3 - 4*a^2*b*c) * d * e^3 + (a^2*b^2 - 4*a^3*c) * e^4) * \text{sqrt}((c^2*d^4 - 2 \\ & * a*c*d^2 * e^2 + a^2 * e^4) / ((b^2*c^4 - 4*a*c^5) * d^8 - 4*(b^3*c^3 - 4*a*b*c^4) * \\ & d^7 * e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4) * d^6 * e^2 - 4*(b^5*c - a*b^3 \\ & * c^2 - 12*a^2*b*c^3) * d^5 * e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c \\ & ^3) * d^4 * e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2) * d^3 * e^5 + 2*(3*a^2*b^4 - \\ & 10*a^3*b^2*c - 8*a^4*c^2) * d^2 * e^6 - 4*(a^3*b^3 - 4*a^4*b*c) * d * e^7 + (a^4*b \\ & ^2 - 4*a^5*c) * e^8))) / ((b^2*c^2 - 4*a*c^3) * d^4 - 2*(b^3*c - 4*a*b*c^2) * d^3 * e \\ & + (b^4 - 2*a*b^2*c - 8*a^2*c^2) * d^2 * e^2 - 2*(a*b^3 - 4*a^2*b*c) * d * e^3 + (a \\ & ^2*b^2 - 4*a^3*c) * e^4) * \log(-2*(c^2*d^2 - a*c*e^2) * x - \text{sqrt}(1/2) * ((b^2*c - \\ & 4*a*c^2) * d^2 * e - (a*b^2 - 4*a^2*c) * e^3 + (2*(b^2*c^3 - 4*a*c^4) * d^5 - 5*(b^ \\ & 3*c^2 - 4*a*b*c^3) * d^4 * e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3) * d^3 * e^2 - (b \\ & ^5 + 2*a*b^3*c - 24*a^2*b*c^2) * d^2 * e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2) \\ &) * d * e^4 - (a^2*b^3 - 4*a^3*b*c) * e^5) * \text{sqrt}((c^2*d^4 - 2*a*c*d^2 * e^2 + a^2 * e^ \\ & 4) / ((b^2*c^4 - 4*a*c^5) * d^8 - 4*(b^3*c^3 - 4*a*b*c^4) * d^7 * e + 2*(3*b^4*c^2 \\ & - 10*a*b^2*c^3 - 8*a^2*c^4) * d^6 * e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3) * \\ & d^5 * e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3) * d^4 * e^4 - 4*(a*b^ \\ & 5 - a^2*b^3*c - 12*a^3*b*c^2) * d^3 * e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4 \\ & * c^2) * d^2 * e^6 - 4*(a^3*b^3 - 4*a^4*b*c) * d * e^7 + (a^4*b^2 - 4*a^5*c) * e^8))) * \\ & \text{sqrt}(- (b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3) * d^4 - 2*(b^3*c \\ & - 4*a*b*c^2) * d^3 * e + (b^4 - 2*a*b^2*c - 8*a^2*c^2) * d^2 * e^2 - 2*(a*b^3 - 4*a \\ & ^2*b*c) * d * e^3 + (a^2*b^2 - 4*a^3*c) * e^4) * \text{sqrt}((c^2*d^4 - 2*a*c*d^2 * e^2 + a^ \\ & 2 * e^4) / ((b^2*c^4 - 4*a*c^5) * d^8 - 4*(b^3*c^3 - 4*a*b*c^4) * d^7 * e + 2*(3*b^4*c \\ & ^2 - 10*a*b^2*c^3 - 8*a^2*c^4) * d^6 * e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3) * \\ & ^3) * d^5 * e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3) * d^4 * e^4 - 4*(\\ & a*b^5 - a^2*b^3*c - 12*a^3*b*c^2) * d^3 * e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8 \\ & * a^4*c^2) * d^2 * e^6 - 4*(a^3*b^3 - 4*a^4*b*c) * d * e^7 + (a^4*b^2 - 4*a^5*c) * e^8 \\ &)))) / ((b^2*c^2 - 4*a*c^3) * d^4 - 2*(b^3*c - 4*a*b*c^2) * d^3 * e + (b^4 - 2*a*b^2 \\ & * c - 8*a^2*c^2) * d^2 * e^2 - 2*(a*b^3 - 4*a^2*b*c) * d * e^3 + (a^2*b^2 - 4*a^3*c) \\ & * e^4))) - 2 * \text{sqrt}(d * e) * \arctan(\text{sqrt}(d * e) * x / d) / (c * d^2 - b * d * e + a * e^2)] \end{aligned}$$

giac [B] time = 8.45, size = 6921, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-\text{sqrt}(d) * \arctan(x * e^{(1/2)} / \text{sqrt}(d)) * e^{(1/2)} / (c * d^2 - b * d * e + a * e^2) - 1/8 * ((2 * b^4 * c^4 - 8 * a * b^2 * c^5 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * b^4 * c^2 + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * a * b^2 * c^3 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * b^3 * c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c + \text{sqrt}(b^2 - 4 * a * c)) * c) * b^2 * c^4$$

$$\begin{aligned}
& 4 - 2*(b^2 - 4*a*c)*b^2*c^4*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 \\
& - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c + 3*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 + 2*\sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4*c^2 + 4*\sqrt{2}*\sqrt{2} \\
& *(b^2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4 \\
& *a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4 \\
& *a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - \\
& 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - \sqrt{2} \\
&)*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^6 - 2*\sqrt{2}*\sqrt{b^2 - 4 \\
& *a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c + 2*\sqrt{2}*\sqrt{b^2 - 4 \\
& *a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^5*c + 24*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^2 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*b^4*c^2 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c \\
&)*a*b^2*c^3)*d^3*e^2 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c - \\
& 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c \\
& + \sqrt{b^2 - 4*a*c}*c})*a*b^3*c^2 - 2*a*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{2} \\
& *(b^2 - 4*a*c)*c})*a^3*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b \\
& *c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^3 + 16*a^2*b^2*c^3 - \\
& 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - \\
& 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)* \\
& e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*b^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*b^3*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*b^2*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - \\
& 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - \\
& 8*a^3*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a*b^5 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a^2*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a*b^4*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a^3*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a^2*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d \\
& ^2*e^3 + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5 - 8*\sqrt{2}*\sqrt{2} \\
& *(b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*b^3*c - 2*\sqrt{2}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a* \\
& c}*c)*a*b^4*c - 2*a*b^5*c + 16*\sqrt{2}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c) \\
& *a^3*b*c^2 + 8*\sqrt{2}*\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*b^2*c^2 + \sqrt{2} \\
& *\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*\sqrt{2} \\
& *\sqrt{2}*(b*c + \sqrt{b^2 - 4*a*c}*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c) \\
& *a*b^3*c
\end{aligned}$$

$$\begin{aligned}
& - 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^2*b \\
& ^4*c^2 - 8*a^3*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4* \\
& a*c})*c)*a^2*b^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}* \\
& c)*a^3*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)* \\
& a^2*b^3*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b \\
& ^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^ \\
& 2 - 4*a*c})*c)*a^2*b^4 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c \\
& - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*s \\
& \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c})*c)*a^3*b*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^2 \\
& *c^2 + 16*a^3*b^2*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*c^3 - \\
& 32*a^4*c^3 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c*d^ \\
& 2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - \sqrt{2}*\sqrt{b^2 \\
& - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a \\
& *c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*s \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b \\
& *c + \sqrt{b^2 - 4*a*c})*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*arcta \\
& n(2*\sqrt{1/2}*x/\sqrt{((b*c*d^2 - b^2*d*e + a*b*e^2 + \sqrt{((b*c*d^2 - b^2*d*e \\
& + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^ \\
& 2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3* \\
& c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d \\
& *e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^ \\
& 3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) \\
& *abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + \\
& 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d* \\
& e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c \\
& ^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2 \\
&)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b* \\
& c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4 + 1/8 \\
& *((2*b^4*c^4 - 8*a*b^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a \\
& *c})*c)*b^4*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a \\
& *c})*c)*a*b^2*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c} \\
& *c)*b^3*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^2 \\
& *c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b* \\
& c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^5*c + 3*s \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^3*c^2 + 2*\sqrt{ \\
& 2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^4*c^2 + 4*\sqrt{2})*s \\
& \sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b*c^3 + 2*\sqrt{2})*\sqrt{ \\
& b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^2*c^3 - \sqrt{2})*\sqrt{b^2 \\
& - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^3*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c} \\
&)*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^ \\
& 2 - 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - \sqrt{ \\
& 2})*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^6 - 2*\sqrt{2})*\sqrt{ \\
& b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*a*b^4*c + 2*\sqrt{2})*\sqrt{b^2 \\
& - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c})*c)*b^5*c + 24*\sqrt{2})*\sqrt{b^2 - 4*a*
\end{aligned}$$

$$\begin{aligned}
& *d^2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*e^5)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c*d^2 - b^2*d*e + a*b*e^2 - \sqrt{(b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)}*(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^4)
\end{aligned}$$

maple [B] time = 0.02, size = 478, normalized size = 1.90

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}{(a^2-d^2+cd)\sqrt{4ac+d^2}\sqrt{(b+\sqrt{4ac+d^2})}c} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}{(a^2-d^2+cd)\sqrt{4ac+d^2}\sqrt{(b-\sqrt{4ac+d^2})}c} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}{2(a^2-d^2+cd)\sqrt{4ac+d^2}\sqrt{(b+\sqrt{4ac+d^2})}c} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}{2(a^2-d^2+cd)\sqrt{4ac+d^2}\sqrt{(b-\sqrt{4ac+d^2})}c} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b+\sqrt{4ac+d^2})}}\right)}{2(a^2-d^2+cd)\sqrt{(b+\sqrt{4ac+d^2})}c} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b-\sqrt{4ac+d^2})}}\right)}{2(a^2-d^2+cd)\sqrt{(b-\sqrt{4ac+d^2})}c} + \frac{d \operatorname{arctan}\left(\frac{x}{d}\right)}{(a^2-d^2+cd)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a), x)

[Out]
$$\begin{aligned}
& -1/2/(a*e^2-b*d*e+c*d^2)*c*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/} \\
& (-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a} \\
& a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d+1/2/(a*e^2-b*d*e+c*d^2)*c*2^{(1/2)/} \\
& ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d-1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c} \\
& +b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*} \\
& a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*d-d*e} \\
& / (a*e^2-b*d*e+c*d^2)/(d*e)^{(1/2)}*\operatorname{arctan}(1/(d*e)^{(1/2)}*e*x)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] $-d*e*\arctan(e*x/\sqrt{d*e})/((c*d^2 - b*d*e + a*e^2)*\sqrt{d*e}) + \text{integrate}((c*d*x^2 + a*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)$

mupad [B] time = 4.96, size = 19401, normalized size = 77.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)), x)$

[Out] $(\log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^{(5/2)} + a*b^3*e^5*x*(-d*e)^{(3/2)} + a^3*c*e^7*x*(-d*e)^{(1/2)} + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^{(5/2)} - a*c^3*d*x*(-d*e)^{(7/2)} + b^3*c*e*x*(-d*e)^{(7/2)} - 2*a*b*c^2*e*x*(-d*e)^{(7/2)} - 3*a*b^2*c*e^3*x*(-d*e)^{(5/2)} - 2*a^2*b*c*e^5*x*(-d*e)^{(3/2)})*(-d*e)^{(1/2)})/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - \text{atan}(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)}*(x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)}*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2$

$$\begin{aligned}
& *b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3 \\
& *c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 \\
& - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - \\
& 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384* \\
& a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - \\
& b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 \\
& - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4*e^4 + 16*a^2* \\
& c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8* \\
& a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b \\
& ^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 \\
& - 32*a^3*b*c^2*d*e^3)))^{1/2} - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a* \\
& b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^ \\
& 4)) * (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4* \\
& a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a* \\
& b^2*c*d*e) / (8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 \\
& + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2* \\
& a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a \\
& ^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{1/2} * i + (x*(\\
& 2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (- (a*b^3*e^2 - a*e^2 \\
& * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4* \\
& a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4*e \\
& ^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^ \\
& 3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^ \\
& 3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2* \\
& b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{1/2} * ((x*(32*a^3*b*c^3*e^7 + 16*a*c^6* \\
& d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b \\
& ^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 \\
& - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a \\
& ^2*b^2*c^3*d*e^6) + (- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d \\
& ^2 + c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^ \\
& 2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^ \\
& 4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3* \\
& c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4* \\
& c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))) \\
& ^{1/2} * (x*(- (a*b^3*e^2 - a*e^2 * (- (4*a*c - b^2)^3)^{1/2} + b^3*c*d^2 + c*d^2 \\
& * (- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e \\
& - 4*a*b^2*c*d*e) / (8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^ \\
& 2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^ \\
& 2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 \\
& - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{1/2} * (25 \\
& 6*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6* \\
& e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128* \\
& b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^ \\
& 3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3 \\
& *d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*
\end{aligned}$$

$$\begin{aligned}
& e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 \\
& + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + \\
& 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192 \\
& *a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4 \\
& *d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4 \\
& *e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 \\
& - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b \\
& ^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + \\
& 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2 \\
& *e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 3 \\
& 2*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6* \\
& a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d* \\
& e^3)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4 \\
& *a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 \\
& - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/ \\
& 2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^ \\
& 2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8 \\
& *a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b \\
& ^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + \\
& 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*i)/((x*(2*a^2*c^3*e^5 - 4 \\
& *a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - ((a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^ \\
& 3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a \\
& ^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d \\
& ^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b \\
& ^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^ \\
& 2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32* \\
& a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3 \\
& *c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + \\
& 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^ \\
& 3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) \\
& + ((a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a \\
& *c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b \\
& ^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + \\
& b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a \\
& *b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^ \\
& 2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^ \\
& 3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^ \\
& 3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/ \\
& (8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e \\
& ^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 \\
& - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^ \\
& 3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 \\
& - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6 \\
& *d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - \\
& 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b
\end{aligned}$$

$$\begin{aligned}
&^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7)) * ((- (a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4)) * ((- (a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) - (x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (- (a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) * ((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (- (a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) * (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c
\end{aligned}$$

$$\begin{aligned}
& ^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 \\
& + 128b^6c^3d^4e^5 - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288 \\
& *a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + \\
& 128a*b*c^7d^7e^2 + 640a^4b*c^4d*e^8 - 640a*b^2*c^6*d^6e^3 + 1056a* \\
& b^3*c^5*d^5e^4 - 672a*b^4*c^4*d^4e^5 + 96a*b^5*c^3*d^3e^6 + 32a*b^6*c \\
& ^2*d^2e^7 - 1152a^2*b*c^6*d^5e^4 + 32a^2*b^5*c^2*d*e^8 - 640a^3*b*c^5* \\
& d^3e^6 - 288a^3*b^3*c^3*d*e^8) + 192a^4*c^4*d*e^7 + 192a^2*c^6*d^5e^3 \\
& + 384a^3*c^5*d^3e^5 + 96a^2*b^2*c^4*d^3e^5 + 96a^2*b^3*c^3*d^2e^6 - 4 \\
& 8a*b^2*c^5*d^5e^3 + 96a*b^3*c^4*d^4e^4 - 48a*b^4*c^3*d^3e^5 - 384a^2 \\
& *b*c^5*d^4e^4 - 384a^3*b*c^4*d^2e^6 - 48a^3*b^2*c^3*d*e^7)) * (- (a*b^3e^ \\
& 2 - a*e^2 * (- (4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^(\\
& 1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(\\
& a^2*b^4e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2e^4 + b^4*c^2*d^4 + b^6*d^2e^2 - \\
& 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2e^2 - 2*a*b^5*d*e^3 - 2 \\
& *b^5*c*d^3e + 16*a*b^3*c^2*d^3e - 6*a*b^4*c*d^2e^2 - 32*a^2*b*c^3*d^3e \\
& + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2) + 4*a*c^5*d^4e^2 + 52*a \\
& ^2*c^4*d^2e^4 - 8*a*b*c^4*d^3e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 \\
& - 8*a*b^2*c^3*d^2e^4)) * (- (a*b^3e^2 - a*e^2 * (- (4*a*c - b^2)^3)^(1/2) + b^ \\
& 3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + \\
& 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4e^4 + 16*a^2*c^4*d^4 + 16*a^4*c \\
& ^2e^4 + b^4*c^2*d^4 + b^6*d^2e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32 \\
& *a^3*c^3*d^2e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3e + 16*a*b^3*c^2*d^3e - 6*a \\
& *b^4*c*d^2e^2 - 32*a^2*b*c^3*d^3e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e \\
& ^3)))^(1/2) + 2*a*c^3*d*e^3)) * (- (a*b^3e^2 - a*e^2 * (- (4*a*c - b^2)^3)^(1/2) \\
& + b^3*c*d^2 + c*d^2 * (- (4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e \\
& ^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4e^4 + 16*a^2*c^4*d^4 + 16* \\
& a^4*c^2e^4 + b^4*c^2*d^4 + b^6*d^2e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 \\
& + 32*a^3*c^3*d^2e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3e + 16*a*b^3*c^2*d^3e \\
& - 6*a*b^4*c*d^2e^2 - 32*a^2*b*c^3*d^3e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^ \\
& 2*d*e^3)))^(1/2) * 2i - (\log(b^4*d^3e^5 - a*b^3*d^2e^6 + a*c^3*d^5e^3 - b^ \\
& 3*c*d^4e^4 + 2*a^2*c^2*d^3e^5 + a^3*c*d*e^7 - b^4e^3*x*(-d*e)^(5/2) - a \\
& b^3e^5*x*(-d*e)^(3/2) - a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4e^4 - 3*a \\
& *b^2*c*d^3e^5 + 2*a^2*b*c*d^2e^6 - 2*a^2*c^2e^3*x*(-d*e)^(5/2) + a*c^3*d \\
& *x*(-d*e)^(7/2) - b^3*c*e*x*(-d*e)^(7/2) + 2*a*b*c^2*e*x*(-d*e)^(7/2) + 3*a \\
& *b^2*c*e^3*x*(-d*e)^(5/2) + 2*a^2*b*c*e^5*x*(-d*e)^(3/2)) * (-d*e)^(1/2) / (2* \\
& (a*e^2 + c*d^2 - b*d*e)) - \operatorname{atan}(((x*(2*a^2*c^3e^5 - 4*a*c^4*d^2e^3 + 2*b^ \\
& 2*c^3*d^2e^3) - (- (a*b^3e^2 + a*e^2 * (- (4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 \\
& - c*d^2 * (- (4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c \\
& ^2*d*e - 4*a*b^2*c*d*e) / (8*(a^2*b^4e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2e^4 + \\
& b^4*c^2*d^4 + b^6*d^2e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3 \\
& *d^2e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3e + 16*a*b^3*c^2*d^3e - 6*a*b^4*c*d \\
& ^2e^2 - 32*a^2*b*c^3*d^3e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1 \\
& /2) * ((x*(32*a^3*b*c^3e^7 + 16*a*c^6*d^5e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^ \\
& 3*c^2e^7 + 160*a^2*c^5*d^3e^4 - 8*b^2*c^5*d^5e^2 + 8*b^3*c^4*d^4e^3 + 8 \\
& *b^4*c^3*d^3e^4 - 8*b^5*c^2*d^2e^5 - 96*a*b^2*c^4*d^3e^4 + 64*a*b^3*c^3*
\end{aligned}$$

$$\begin{aligned}
& d^2e^5 - 96a^2b^3c^4d^2e^5 + 24a^2b^2c^3d^2e^6) + (-(ab^3e^2 + ae^2(-4ac - b^2)^3)^{1/2} + b^3cd^2 - cd^2(-4ac - b^2)^3)^{1/2} - \\
& 4ab^2c^2d^2 - 4a^2b^2c^2e^2 + 16a^2c^2d^2e - 4ab^2cd^2e)/(8(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8ab^2c^3d^4 \\
& - 8a^3b^2c^2e^4 + 32a^3c^3d^2e^2 - 2ab^5d^3e - 2b^5cd^3e + 16ab^3c^2d^3e - 6ab^4cd^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3cd^3e^3 - 32a^3b^2c^2d^3e^3))^{1/2} * (x * (-(ab^3e^2 + ae^2(-4ac - b^2)^3)^{1/2} + b^3cd^2 - cd^2(-4ac - b^2)^3)^{1/2} - 4ab^2c^2d^2 - 4a^2b^2c^2e^2 + 16a^2c^2d^2e - 4ab^2cd^2e)/(8(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8ab^2c^3d^4 - 8a^3b^2c^2e^4 + 32a^3c^3d^2e^2 - 2ab^5d^3e - 2b^5cd^3e + 16ab^3c^2d^3e - 6ab^4cd^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3cd^3e^3 - 32a^3b^2c^2d^3e^3))^{1/2} * (256a^4b^2c^3e^9 - 32a^3b^4c^2e^9 - 512a^5c^4e^9 + 512a^2c^7d^6e^3 + 512a^3c^6d^4e^5 - 512a^4c^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 + 128b^6c^3d^4e^5 - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + 128ab^3c^7d^7e^2 + 640a^4b^3c^4d^2e^8 - 640ab^2c^6d^6e^3 + 1056ab^3c^5d^5e^4 - 672ab^4c^4d^4e^5 + 96ab^5c^3d^3e^6 + 32ab^6c^2d^2e^7 - 1152a^2b^3c^6d^5e^4 + 32a^2b^5c^2d^2e^8 - 640a^3b^3c^5d^3e^6 - 288a^3b^3c^3d^2e^8) - 192a^4c^4d^2e^7 - 192a^2c^6d^5e^3 - 384a^3c^5d^3e^5 - 96a^2b^2c^4d^3e^5 - 96a^2b^3c^3d^2e^6 + 48ab^2c^5d^5e^3 - 96ab^3c^4d^4e^4 + 48ab^4c^3d^3e^5 + 384a^2b^3c^5d^4e^4 + 384a^3b^3c^4d^2e^6 + 48a^3b^2c^3d^2e^7) * (-(ab^3e^2 + ae^2(-4ac - b^2)^3)^{1/2} + b^3cd^2 - cd^2(-4ac - b^2)^3)^{1/2} - 4ab^2c^2d^2 - 4a^2b^2c^2e^2 + 16a^2c^2d^2e - 4ab^2cd^2e)/(8(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8ab^2c^3d^4 - 8a^3b^2c^2e^4 + 32a^3c^3d^2e^2 - 2ab^5d^3e - 2b^5cd^3e + 16ab^3c^2d^3e - 6ab^4cd^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3cd^3e^3 - 32a^3b^2c^2d^3e^3))^{1/2} - 4ac^5d^4e^2 - 52a^2c^4d^2e^4 + 8ab^2c^4d^3e^3 - 4ab^3c^2d^2e^5 + 20a^2b^3c^3d^2e^5 + 8ab^2c^3d^2e^4) * (-(ab^3e^2 + ae^2(-4ac - b^2)^3)^{1/2} + b^3cd^2 - cd^2(-4ac - b^2)^3)^{1/2} - 4ab^2c^2d^2 - 4a^2b^2c^2e^2 + 16a^2c^2d^2e - 4ab^2cd^2e)/(8(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8ab^2c^3d^4 - 8a^3b^2c^2e^4 + 32a^3c^3d^2e^2 - 2ab^5d^3e - 2b^5cd^3e + 16ab^3c^2d^3e - 6ab^4cd^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3cd^3e^3 - 32a^3b^2c^2d^3e^3))^{1/2} * (11 + (x * (2a^2c^3e^5 - 4ac^4d^2e^3 + 2b^2c^3d^2e^3) - (-(ab^3e^2 + ae^2(-4ac - b^2)^3)^{1/2} + b^3cd^2 - cd^2(-4ac - b^2)^3)^{1/2} - 4ab^2c^2d^2 - 4a^2b^2c^2e^2 + 16a^2c^2d^2e - 4ab^2cd^2e)/(8(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8ab^2c^3d^4 - 8a^3b^2c^2e^4 + 32a^3c^3d^2e^2 - 2ab^5d^3e - 2b^5cd^3e + 16ab^3c^2d^3e - 6ab^4cd^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3cd^3e^3 - 32a^3b^2c^2d^3e^3))^{1/2} * ((x * (32a^3b^3c^3e^7 + 16ac^6d^5e^2 - 112a^3c^4d^2e^6 - 8a^2b^3c^2e^7 + 160a
\end{aligned}$$

$$\begin{aligned}
&^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 \\
&- 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2* \\
&b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d^2*e^6) + (- (a*b^3*e^2 + a*e^2*(- (4*a*c - b^2)^3)^{1/2}) \\
&+ b^3*c*d^2 - c*d^2*(- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4 \\
&*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4 \\
&*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3 \\
&*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3* \\
&c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 3 \\
&2*a^3*b*c^2*d*e^3)))^{1/2}*(x*(- (a*b^3*e^2 + a*e^2*(- (4*a*c - b^2)^3)^{1/2}) \\
&+ b^3*c*d^2 - c*d^2*(- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e \\
&^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16* \\
&a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 \\
&+ 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e \\
&- 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^ \\
&2*d*e^3)))^{1/2}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^ \\
&9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^ \\
&3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4 \\
&*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e \\
&^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^ \\
&2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - \\
&672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152* \\
&a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3* \\
&b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3* \\
&e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^ \\
&3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 3 \\
&84*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(- (a*b^3*e^2 + a*e^2*(- (4*a*c \\
&- b^2)^3)^{1/2}) + b^3*c*d^2 - c*d^2*(- (4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d \\
&^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a \\
&^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - \\
&8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16* \\
&a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e \\
&^3 - 32*a^3*b*c^2*d*e^3)))^{1/2} + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8 \\
&*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2 \\
&*e^4))*(- (a*b^3*e^2 + a*e^2*(- (4*a*c - b^2)^3)^{1/2}) + b^3*c*d^2 - c*d^2*(- \\
&(4*a*c - b^2)^3)^{1/2} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4 \\
&*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d \\
&^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - \\
&2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 3 \\
&2*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{1/2}*ii)/((\\
&x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (- (a*b^3*e^2 + a \\
&e^2*(- (4*a*c - b^2)^3)^{1/2}) + b^3*c*d^2 - c*d^2*(- (4*a*c - b^2)^3)^{1/2} - \\
&4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^ \\
&4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b \\
&^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c \\
&*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))^{(1/2)}*((x*(32*a^3*b*c^3*e^7 + 16*a*c \\
& ^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - \\
& 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e \\
& ^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 2 \\
& 4*a^2*b^2*c^3*d*e^6) + (- (a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c \\
& *d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16 \\
& *a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2 \\
& *e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a \\
& ^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b \\
& ^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) \\
&))^{(1/2)}*((x*(- (a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c \\
& *d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d \\
& *e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4 \\
& *c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2 \\
& *e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e \\
& ^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))^{(1/2)}* \\
& (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d \\
& ^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1 \\
& 28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2 \\
& *d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c \\
& ^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4 \\
& *d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4 \\
& *e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 \\
& + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - \\
& 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5 - 96*a^2*b^2* \\
& c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 - 96*a*b^3*c^4* \\
& d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*a^3*b*c^4*d^2* \\
& e^6 + 48*a^3*b^2*c^3*d*e^7))*(- (a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 \\
& + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a \\
& ^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 \\
& + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - \\
& 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2 \\
& *d*e^3))^{(1/2)} - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*b*c^4*d^3*e^3 \\
& - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^4))*(- (a*b^3*e \\
& ^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^ \\
& (1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8* \\
& (a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 \\
& - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - \\
& 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e \\
& + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))^{(1/2)} - (x*(2*a^2*c^3*e^5 - 4 \\
& *a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (- (a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a \\
& ^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d \\
& ^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b
\end{aligned}$$

$$\begin{aligned}
& ^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))^{(1/2)}*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) \\
& + (- (a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)}*(x*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 192*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5 + 96*a^2*b^2*c^4*d^3*e^5 + 96*a^2*b^3*c^3*d^2*e^6 - 48*a*b^2*c^5*d^5*e^3 + 96*a*b^3*c^4*d^4*e^4 - 48*a*b^4*c^3*d^3*e^5 - 384*a^2*b*c^5*d^4*e^4 - 384*a^3*b*c^4*d^2*e^6 - 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)} + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4 - 8*a*b*c^4*d^3*e^3 + 4*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 8*a*b^2*c^3*d^2*e^4))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^{(1/2)} + 2*a*c^3*d*e^3))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 +
\end{aligned}$$

$$16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3))^{(1/2)*2i}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.229 \quad \int \frac{1}{(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=254

$$\frac{\sqrt{c} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.52, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1170, 205, 1166}

$$\frac{\sqrt{c} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{e^{3/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{\sqrt{d} (ae^2 - bde + cd^2)}}{\sqrt{2} \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1170


```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)} dx &= \int \left(\frac{e^2}{(cd^2 - bde + ae^2)(d + ex^2)} + \frac{cd - be - cex^2}{(cd^2 - bde + ae^2)(a + bx^2 + cx^4)} \right) dx \\ &= \frac{\int \frac{cd - be - cex^2}{a + bx^2 + cx^4} dx}{cd^2 - bde + ae^2} + \frac{e^2 \int \frac{1}{d + ex^2} dx}{cd^2 - bde + ae^2} \\ &= \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2 - bde + ae^2)} - \frac{\left(c\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2(cd^2 - bde + ae^2)} - \frac{c\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right)}{2} \\ &= -\frac{\sqrt{c}\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b - \sqrt{b^2 - 4ac}}(cd^2 - bde + ae^2)} - \frac{\sqrt{c}\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b + \sqrt{b^2 - 4ac}}(cd^2 - bde + ae^2)} \end{aligned}$$

Mathematica [A] time = 0.25, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2 - 4ac} + be - 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}(-ae^2 + bde - cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2 - 4ac} - be + 2cd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{\sqrt{b^2 - 4ac} + b}(-ae^2 + bde - cd^2)} + \frac{e^{3/2} \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 9.13, size = 7650, normalized size = 30.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\frac{1}{8} \left(2 \left(2b^3c^5 - 8ab^2c^6 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} \right) b^3c^3 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} \right) c) ab^2c^4 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^2c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^2c^5 - 2(b^2 - 4ac) b^2c^5) d^5 - 5(2b^4c^4 - 8ab^2c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^4c^2 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^2c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^3c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^2c^4 - 2(b^2 - 4ac) b^2c^4) d^4 e + 2(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^4c^2 - 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^2c^3 - 2\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^3c^3 - 2b^4c^3 + 16\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) a^2c^4 + 8\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^2c^4 + 16ab^2c^4 - 4\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) a^2c^5 - 32a^2c^5 + 2(b^2 - 4ac) b^2c^3 - 8(b^2 - 4ac) a^2c^4) d^3 \text{abs}(cd^2 - bde + ae^2) + 4(2b^5c^3 - 6ab^3c^4 - 8a^2b^2c^5 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^5c + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^3c^2 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^4c^2 + 4\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) a^2b^2c^3 + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^2c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^3c^3 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}} c) ab^2c^4 - 2(b^2 - 4ac) b^3c^3 - 2(b^2 - 4ac) ab^2c^4) d^3 e^2 - 4(\sqrt{2} \sqrt{bc + \sqrt{b^2 - 4ac}} c) b^5c$$

$$\begin{aligned}
& c - 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - 2\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 - 2*b^5*c^2 + 16\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 + \sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^3 + 16*a*b^3*c^3 - 4\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6 - 2\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c + 2\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c + 24\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + 12\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 - 6\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^6 - 7\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 2\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c - 2*b^6*c + 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + 6\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + \sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^2 + 14*a*b^4*c^2 + 16\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 + 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 3\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4 + 8\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 2\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 16\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 - 8\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 4\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5 + 3\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c + 2\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c + 4\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 + 2\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 - \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 - 2*(\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5 - 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c - 2\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 2*a*b^5*c + 16\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 + 8\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + \sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*
\end{aligned}$$


```
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)
*a^2*b^2*c^2)*e^5)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 -
sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^
2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 -
8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*
c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c)*e + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^
2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c
^4)*d^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^2 - 2*(a^2*b^5 - 8*a^3*b^3*c -
2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*a
bs(c*d^2 - b*d*e + a*e^2)*abs(c)*e^3 + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
+ 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c*d^2 - b*d*e +
a*e^2)*abs(c)*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a
e^2)*sqrt(d))
```

maple [B] time = 0.03, size = 480, normalized size = 1.89

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{2(a^2-db+cd)\sqrt{4ac+P}\sqrt{(b+\sqrt{4ac+P})}}c + \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{2(a^2-db+cd)\sqrt{4ac+P}\sqrt{(b+\sqrt{4ac+P})}}c - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{(a^2-db+cd)\sqrt{4ac+P}\sqrt{(b+\sqrt{4ac+P})}}c - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{(a^2-db+cd)\sqrt{4ac+P}\sqrt{(b+\sqrt{4ac+P})}}c + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{2(a^2-db+cd)\sqrt{(b+\sqrt{4ac+P})}}c - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2}x}{\sqrt{(b+\sqrt{4ac+P})}}\right)}{2(a^2-db+cd)\sqrt{(b+\sqrt{4ac+P})}}c + \frac{e^2 \operatorname{arctan}\left(\frac{x}{\sqrt{d}}\right)}{(a^2-db+cd)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a), x)

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c^2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/(a*e^2-b*d*e+c*d^2)*c^2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
)*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+e^2/
(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(1/(d*e)^(1/2)*e*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a), x, algorithm="maxima")

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - integrate((
c*e*x^2 - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

mupad [B] time = 5.61, size = 23640, normalized size = 93.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((d + e*x^2)*(a + b*x^2 + c*x^4)), x)$

[Out]
$$\text{atan}\left(\frac{\left(\left(-b^5e^2 + b^3c^2d^2 + b^2e^2(-4ac - b^2)^3\right)^{1/2} + c^2d^2\left(-4ac - b^2\right)^3\right)^{1/2} + 12a^2bc^2e^2 - 2b^4cde - 4abc^3d^2 - 7ab^3ce^2 - ace^2(-4ac - b^2)^3)^{1/2} - 16a^2c^3de + 12ab^2c^2de - 2b^2c^2de(-4ac - b^2)^3)^{1/2}}{8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2ce^4 + ab^6d^2e^2 - 2a^2b^5d^3e - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5cd^3e - 32a^3bc^3d^3e + 16a^3b^3cde^3 - 32a^4bc^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4cd^2e^2)}\right)^{1/2} \cdot \left(\frac{x(16b^5c^2e^7 + 16c^7d^5e^2 - 112ab^3c^3e^7 + 192a^2b^4c^4e^7 + 32ac^6d^3e^4 - 240a^2c^5d^6e - 32b^2c^6d^4e^3 - 32b^4c^3d^6e + 16b^2c^5d^3e^4 + 16b^3c^4d^2e^5 - 96abc^5d^2e^5 + 192ab^2c^4d^6e) - (-b^5e^2 + b^3c^2d^2 + b^2e^2(-4ac - b^2)^3)^{1/2} + c^2d^2(-4ac - b^2)^3)^{1/2} + 12a^2bc^2e^2 - 2b^4cde - 4abc^3d^2 - 7ab^3ce^2 - ace^2(-4ac - b^2)^3)^{1/2} - 16a^2c^3de + 12ab^2c^2de - 2b^2c^2de(-4ac - b^2)^3)^{1/2}}{8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2ce^4 + ab^6d^2e^2 - 2a^2b^5d^3e - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5cd^3e - 32a^3bc^3d^3e + 16a^3b^3cde^3 - 32a^4bc^2d^3e - 6a^2b^4cd^2e^2)}\right)^{1/2} \cdot \left(\frac{x(-b^5e^2 + b^3c^2d^2 + b^2e^2(-4ac - b^2)^3)^{1/2} + c^2d^2(-4ac - b^2)^3)^{1/2} + 12a^2bc^2e^2 - 2b^4cde - 4abc^3d^2 - 7ab^3ce^2 - ace^2(-4ac - b^2)^3)^{1/2} - 16a^2c^3de + 12ab^2c^2de - 2b^2c^2de(-4ac - b^2)^3)^{1/2}}{8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2ce^4 + ab^6d^2e^2 - 2a^2b^5d^3e - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5cd^3e - 32a^3bc^3d^3e + 16a^3b^3cde^3 - 32a^4bc^2d^3e - 6a^2b^4cd^2e^2)}\right)^{1/2} \cdot (256a^4b^2c^3e^9 - 32a^3b^4c^2e^9 - 512a^5c^4e^9 + 512a^2c^7d^6e^3 + 512a^3c^6d^4e^5 - 512a^4c^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 + 128b^6c^3d^4e^5 - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + 128abc^7d^7e^2 + 640a^4bc^4d^8e - 640ab^2c^6d^6e^3 + 1056a^3bc^5d^5e^4 - 672ab^4c^4d^4e^5 + 96ab^5c^3d^3e^6 + 32ab^6c^2d^2e^7 - 1152a^2bc^6d^5e^4 + 32a^2b^5c^2d^8e - 640a^3bc^5d^3e^6 - 288a^3b^3c^3d^8e) - 256a^4c^4e^8 + 64aac^7d^6e^2 - 16a^2b^4c^2e^8 + 128a^3b^2c^3e^8 - 128a^2c^6d^4e^4 - 448a^3c^5d^2e^6 - 16b^2c^6d^6e^2 + 64b^3c^5d^5e^3 - 96b^4c^4d^4e^4 + 64b^5c^3d^3e^5 - 16b^6c^2d^2e^6 + 240a^2b^2c^4d^2e^6 - 256abc^6d^5e^3 + 32ab^5c^2d^7e + 384a^3bc^$$

$$\begin{aligned}
&^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2 \\
&*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7)) * (- (b^5*e^2 + b^3*c^2 \\
&*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
&+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2 * \\
&(- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e * (- \\
&(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + \\
&a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2* \\
&b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 1 \\
&6*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c \\
&*d^2*e^2)))^{(1/2)} - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2 \\
&*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x) * (- (b^5*e^2 + b \\
&^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
&+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a* \\
&c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c* \\
&d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2* \\
&*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - \\
&8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3* \\
&*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2 \\
&*b^4*c*d^2*e^2)))^{(1/2)} * i + ((- (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - \\
&b^2)^3)^{(1/2)} + c^2*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4 \\
&*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} \\
&- 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (\\
&8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2 \\
&*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3* \\
&d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4 \\
&*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} * ((x*(16 \\
&*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32* \\
&a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1 \\
&6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4 \\
&*d*e^6) - (- (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + c^2 \\
&*d^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3* \\
&d^2 - 7*a*b^3*c*e^2 - a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 1 \\
&2*a*b^2*c^2*d*e - 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16* \\
&a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2* \\
&*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c* \\
&d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a \\
&^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} * (256*a^4*c^4*e^8 + x * (- (b^5 \\
&*e^2 + b^3*c^2*d^2 + b^2*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} + c^2*d^2 * (- (4*a*c - \\
&b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c* \\
&*e^2 - a*c*e^2 * (- (4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e \\
&- 2*b*c*d*e * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16 \\
&*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5* \\
&d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b \\
&*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e \\
&- 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} * (256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 -
\end{aligned}$$

$$\begin{aligned}
& 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + \\
& 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128 \\
& *a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2* \\
& d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128 \\
& *a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16* \\
& b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3 \\
& *e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7) * (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(- \\
& (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7 \\
& *a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2 \\
& *c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4 \\
& *d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2 \\
& *a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - \\
& 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3* \\
& c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - \\
& 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^ \\
& 5*e^5*x) * (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(- \\
& (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 \\
& ^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12 \\
& *a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a \\
& ^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e \\
& ^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d \\
& ^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^ \\
& 2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) * ((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + \\
& 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 \\
& - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5* \\
& d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a* \\
& c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2 \\
& *b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/ \\
& 2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2) \\
&)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32 \\
& *a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*(x*(\\
& -(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b \\
& ^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2 \\
& *d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 \\
& + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2 \\
& *b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32* \\
& a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2* \\
& d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2* \\
& e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4 \\
& *c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e \\
& ^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2 \\
& 88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 \\
& + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056* \\
& a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6 \\
& *c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^ \\
& 5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1 \\
& 6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5 \\
& *d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6 \\
& 4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b* \\
& c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4* \\
& e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 \\
& - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c* \\
& d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16 \\
& *a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a \\
& ^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c* \\
& e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2* \\
& e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b* \\
& c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} - 4*b^3*c^3 \\
& *e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - \\
& 20*a*c^5*d*e^5) + 6*c^5*e^5*x))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2* \\
& b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}) \\
& / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c^ \\
& b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3* \\
& d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32* \\
& a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2))^{(1/2)} - ((- \\
& (b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + c^2*d^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^ \\
& 3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2* \\
& d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
\end{aligned}$$

$$\begin{aligned}
& 2 + b^2 e^2 (-4ac - b^2)^3)^{1/2} + c^2 d^2 (-4ac - b^2)^3)^{1/2} + 1 \\
& 2a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 - a^2 c^2 e^2 (- \\
& 4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e - 2b^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2} \\
&) / (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 \\
& * c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^2 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} \\
&) * (-b^5 e^2 + b^3 c^2 d^2 + b^2 e^2 (-4ac - b^2)^3)^{1/2} + c^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 \\
& - 7a^2 b^3 c^2 e^2 - a^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e - 2b^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2} / (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^2 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} * 2i + \operatorname{atan}(\frac{(-b^5 e^2 + b^3 c^2 d^2 - b^2 e^2 (-4ac - b^2)^3)^{1/2} - c^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2}}{(8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^2 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2}} * (x(16b^5 c^2 e^7 + 16c^7 d^5 e^2 - 112a^2 b^3 c^3 e^7 + 192a^2 b^2 c^4 e^7 + 32a^2 c^6 d^3 e^4 - 240a^2 c^5 d^2 e^6 - 32b^2 c^6 d^4 e^3 - 32b^4 c^3 d^2 e^6 + 16b^2 c^5 d^3 e^4 + 16b^3 c^4 d^2 e^5 - 96a^2 b^2 c^5 d^2 e^5 + 192a^2 b^2 c^4 d^2 e^6) - (-b^5 e^2 + b^3 c^2 d^2 - b^2 e^2 (-4ac - b^2)^3)^{1/2} - c^2 d^2 (-4ac - b^2)^3)^{1/2} + 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2} / (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^2 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} * (256a^4 b^2 c^3 e^9 - 32a^3 b^4 c^2 e^9 - 512a^5 c^4 e^9 + 512a^2 c^7 d^6 e^3 + 512a^3 c^6 d^4 e^5 - 512a^4 c^5 d^2 e^7 - 32b^3 c^6 d^7 e^2 + 128b^4 c^5 d^6 e^3 - 192b^5 c^4 d^5 e^4 + 128b^6 c^3 d^4 e^5 - 32b^7 c^2 d^3 e^6 + 512a^2 b^2 c^5 d^4 e^5 + 288a^2 b^3 c^4 d^3 e^6 - 192a^2 b^4 c^3 d^2 e^7 + 384a^3 b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 \\
& - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7) * (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^(1/2) - c^2*d^2 * (- (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^(1/2)) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x) * (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^(1/2) - c^2*d^2 * (- (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^(1/2)) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) * 1i + ((- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^(1/2) - c^2*d^2 * (- (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^(1/2)) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) * ((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^(1/2) - c^2*d^2 * (- (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^(1/2)) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) * (256*a^4*c^4*e^8 + x * (- (b^5*e^2 + b^3*c^2*d^2 - b^2*e^2 * (- (4*a*c - b^2)^3)^(1/2) - c^2*d^2 * (- (4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2 * (- (4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e * (- (4*a*c - b^2)^3)^(1/2)) / (8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
\end{aligned}$$

$$\begin{aligned}
& d^2 - b^2 e^2 * (- (4ac - b^2)^3)^{1/2} - c^2 d^2 * (- (4ac - b^2)^3)^{1/2} + \\
& 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- \\
& - (4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e * (- \\
& (4ac - b^2)^3)^{1/2} / (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + \\
& a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - \\
& 2a^2 b^5 c^3 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} * \\
& (256a^4 b^2 c^3 e^9 - 32a^3 b^4 c^2 e^9 - 512a^5 c^4 e^9 + 512a^2 c^7 d^6 e^3 + 512a^3 c^6 d^4 e^5 - 512a^4 c^5 d^2 e^7 - 32b^3 c^6 d^7 e^2 + \\
& 128b^4 c^5 d^6 e^3 - 192b^5 c^4 d^5 e^4 + 128b^6 c^3 d^4 e^5 - 32b^7 c^2 d^3 e^6 + 512a^2 b^2 c^5 d^4 e^5 + 288a^2 b^3 c^4 d^3 e^6 - \\
& 192a^2 b^4 c^3 d^2 e^7 + 384a^3 b^2 c^4 d^2 e^7 + 128a^2 b^3 c^7 d^7 e^2 + 640a^4 b^2 c^4 d^2 e^8 - 640a^2 b^2 c^6 d^6 e^3 + 1056a^2 b^3 c^5 d^5 e^4 - \\
& 672a^2 b^4 c^4 d^4 e^5 + 96a^2 b^5 c^3 d^3 e^6 + 32a^2 b^6 c^2 d^2 e^7 - 1152a^2 b^2 c^6 d^5 e^4 + 32a^2 b^5 c^2 d^2 e^8 - 640a^3 b^2 c^5 d^3 e^6 - 288a^3 b^3 c^3 d^2 e^8) - \\
& 64a^2 c^7 d^6 e^2 + 16a^2 b^4 c^2 e^8 - 128a^3 b^2 c^3 e^8 + 128a^2 c^6 d^4 e^4 + 448a^3 c^5 d^2 e^6 + 16b^2 c^6 d^6 e^2 - 64b^3 c^5 d^5 e^3 + \\
& 96b^4 c^4 d^4 e^4 - 64b^5 c^3 d^3 e^5 + 16b^6 c^2 d^2 e^6 - 240a^2 b^2 c^4 d^2 e^6 + 256a^2 b^3 c^6 d^5 e^3 - 32a^2 b^5 c^2 d^2 e^7 - 384a^3 b^2 c^4 d^2 e^7 - \\
& 416a^2 b^2 c^5 d^4 e^4 + 288a^2 b^3 c^4 d^3 e^5 - 32a^2 b^4 c^3 d^2 e^6 - 128a^2 b^2 c^5 d^3 e^5 + 224a^2 b^3 c^3 d^2 e^7) * (- (b^5 e^2 + \\
& b^3 c^2 d^2 - b^2 e^2 * (- (4ac - b^2)^3)^{1/2} - c^2 d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - \\
& 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- (4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e * (- (4ac - b^2)^3)^{1/2} / \\
& (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - \\
& 2a^2 b^5 c^3 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} + 4b^3 c^3 e^6 + \\
& 4c^6 d^3 e^3 - 4b^2 c^5 d^2 e^4 - 4b^2 c^4 d^2 e^5 - 16a^2 b^2 c^4 e^6 + 20a^2 c^5 d^2 e^5) + 6c^5 e^5 x) * (- (b^5 e^2 + b^3 c^2 d^2 - b^2 e^2 * (- (4ac - b^2)^3)^{1/2} - \\
& c^2 d^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- (4ac - b^2)^3)^{1/2} - \\
& 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e * (- (4ac - b^2)^3)^{1/2} / (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - \\
& 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^3 d^3 e - 32a^3 b^2 c^3 d^3 e + 16a^3 b^3 c^2 d^2 e^3 - \\
& 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2} * 1i) / (((- (b^5 e^2 + b^3 c^2 d^2 - b^2 e^2 * (- (4ac - b^2)^3)^{1/2} - c^2 d^2 * (- (4ac - b^2)^3)^{1/2} + \\
& 12a^2 b^2 c^2 e^2 - 2b^4 c^2 d^2 e - 4a^2 b^2 c^3 d^2 - 7a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- (4ac - b^2)^3)^{1/2} - 16a^2 c^3 d^2 e + 12a^2 b^2 c^2 d^2 e + 2b^2 c^2 d^2 e * (- (4ac - b^2)^3)^{1/2} / \\
& (8(a^3 b^4 e^4 + 16a^3 c^4 d^4 + 16a^5 c^2 e^4 + a^2 b^4 c^2 d^4 - 8a^4 b^2 c^2 e^4 + a^2 b^6 d^2 e^2 - 2a^2 b^5 d^2 e^3 - 8a^2 b^2 c^3 d^4 + 32a^4 c^3 d^2 e^2 - 2a^2 b^5 c^3 d^3 e - 32a^3 b^2 c^3 d^3 e + \\
& 16a^3 b^3 c^2 d^2 e^3 - 32a^4 b^2 c^2 d^2 e^3 + 16a^2 b^3 c^2 d^3 e - 6a^2 b^4 c^2 d^2 e^2))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& *((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - ((b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)}*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)}*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x))
\end{aligned}$$

$$\begin{aligned}
& 4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2c^2e^4 + ab^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5c^3d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e \\
& + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^3e - 6a^2b^4c^2d^3e - 6a^2b^4c^2d^3e))^{(1/2)} - ((-b^5e^2 + b^3c^2d^2 - b^2e^2(-4ac - b^2)^3)^{(1/2)} - c^2d^2(-4ac - b^2)^3)^{(1/2)} \\
& + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4ab^3c^2d^2 - 7ab^3c^2e^2 + ac^2e^2(-4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12ab^2c^2d^2e + 2b^2c^2d^2e \\
& *(-4ac - b^2)^3)^{(1/2)} / (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2c^2e^4 + ab^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5c^3d^3e - 32a^3b^2c^3d^3e \\
& + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^3e - 6a^2b^4c^2d^3e))^{(1/2)} * (x(16b^5c^2e^7 + 16c^7d^5e^2 - 112ab^3c^3e^7 + 192a^2b^2c^4e^7 + 32a^2c^6d^3e^4 - 240a^2c^5d^3e^6 - 32b^2c^6d^4e^3 - 32b^4c^3d^2e^6 + 16b^2c^5d^3e^4 + 16b^3c^4d^2e^5 - 96ab^2c^5d^2e^5 + 192ab^2c^4d^2e^6) - (-b^5e^2 + b^3c^2d^2 - b^2e^2(-4ac - b^2)^3)^{(1/2)} - c^2d^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4ab^3c^2d^2 - 7ab^3c^2e^2 + ac^2e^2(-4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12ab^2c^2d^2e + 2b^2c^2d^2e * (-4ac - b^2)^3)^{(1/2)} / (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2c^2e^4 + ab^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5c^3d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^3e - 6a^2b^4c^2d^3e))^{(1/2)} * (256a^4c^4e^8 + x(-b^5e^2 + b^3c^2d^2 - b^2e^2(-4ac - b^2)^3)^{(1/2)} - c^2d^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2e^2 - 2b^4c^2d^2e - 4ab^3c^2d^2 - 7ab^3c^2e^2 + ac^2e^2(-4ac - b^2)^3)^{(1/2)} - 16a^2c^3d^2e + 12ab^2c^2d^2e + 2b^2c^2d^2e * (-4ac - b^2)^3)^{(1/2)} / (8(a^3b^4e^4 + 16a^3c^4d^4 + 16a^5c^2e^4 + ab^4c^2d^4 - 8a^4b^2c^2e^4 + ab^6d^2e^2 - 2a^2b^5d^3e^3 - 8a^2b^2c^3d^4 + 32a^4c^3d^2e^2 - 2ab^5c^3d^3e - 32a^3b^2c^3d^3e + 16a^3b^3c^2d^3e - 32a^4b^2c^2d^3e + 16a^2b^3c^2d^3e - 6a^2b^4c^2d^3e - 6a^2b^4c^2d^3e))^{(1/2)} * (256a^4b^2c^3e^9 - 32a^3b^4c^2e^9 - 512a^5c^4e^9 + 512a^2c^7d^6e^3 + 512a^3c^6d^4e^5 - 512a^4c^5d^2e^7 - 32b^3c^6d^7e^2 + 128b^4c^5d^6e^3 - 192b^5c^4d^5e^4 + 128b^6c^3d^4e^5 - 32b^7c^2d^3e^6 + 512a^2b^2c^5d^4e^5 + 288a^2b^3c^4d^3e^6 - 192a^2b^4c^3d^2e^7 + 384a^3b^2c^4d^2e^7 + 128ab^2c^7d^7e^2 + 640a^4b^2c^4d^2e^8 - 640ab^2c^6d^6e^3 + 1056ab^3c^5d^5e^4 - 672ab^4c^4d^4e^5 + 96ab^5c^3d^3e^6 + 32ab^6c^2d^2e^7 - 1152a^2b^3c^6d^5e^4 + 32a^2b^5c^2d^2e^8 - 640a^3b^2c^5d^3e^6 - 288a^3b^3c^3d^2e^8) - 64ac^7d^6e^2 + 16a^2b^4c^2e^8 - 128a^3b^2c^3e^8 + 128a^2c^6d^4e^4 + 448a^3c^5d^2e^6 + 16b^2c^6d^6e^2 - 64b^3c^5d^5e^3 + 96b^4c^4d^4e^4 - 64b^5c^3d^3e^5 + 16b^6c^2d^2e^6 - 240a^2b^2c^4d^2e^6 + 256ab^2c^6d^5e^3 - 32ab^5c^2d^2e^7 - 384a^3b^2c^4d^2e^7 - 416ab^2c^5d^4e^4 + 288ab^3c^4d^3e^5 - 32ab^4c^3d^2e^6 - 128a^2b^2c^5d^3e^5 + 224a^2b^3c^3d^2e^7) * (-b^5e^2 + b^3c^2d^2 - b^2e^2(-4ac - b^2)^3)^{(1/2)} - c^2d^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2e^2 - 2b
\end{aligned}$$

$$\begin{aligned}
&^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&- 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/ \\
&(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b \\
&^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3 \\
&*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a \\
&^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)} + 4*b^ \\
&3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4* \\
&e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(\\
&4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 \\
&- 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3) \\
&^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(\\
&1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8 \\
&*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a \\
&^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 \\
&- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)) \\
&)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - c^2*d^2*(-(\\
&4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7* \\
&a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^2*c^3*d*e + 12*a*b^2* \\
&c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^4 + 16*a^3*c^4* \\
&d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2* \\
&a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - \\
&32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c \\
&^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^{(1/2)}*2i - (\log(b^5*d*(-d*e^3)^{(5/2)} - b^ \\
&5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^{(3/2)} - 16*a^3*c^2*e*(-d \\
&*e^3)^{(5/2)} - c^5*d^8*e*(-d*e^3)^{(1/2)} + b^2*c^3*d^5*(-d*e^3)^{(3/2)} - a*b^4 \\
&*e*(-d*e^3)^{(5/2)} - 7*a*b^3*c*d*(-d*e^3)^{(5/2)} + 17*a^2*c^3*d^3*e^2*(-d*e^3 \\
&)^{(3/2)} + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c \\
&*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^{(5/2)} + 8*a^2*b^2*c*e*(-d*e^3)^{(5/2)} + \\
&17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2* \\
&d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^{(3/2)} + 2*b^4*c*d^3*e^2*(-d*e^3)^{(3/2)} + \\
&2*b*c^4*d^7*e^2*(-d*e^3)^{(1/2)} - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3 \\
&*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^{(3/2)} + 2*a* \\
&b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^{(3/2)})*(-d \\
&*e^3)^{(1/2)})/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (\log(b^5*d*(-d*e^3)^{(5/2)} + \\
&b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^{(3/2)} - 16*a^3*c^2*e*(- \\
&-d*e^3)^{(5/2)} - c^5*d^8*e*(-d*e^3)^{(1/2)} + b^2*c^3*d^5*(-d*e^3)^{(3/2)} - a*b \\
&^4*e*(-d*e^3)^{(5/2)} - 7*a*b^3*c*d*(-d*e^3)^{(5/2)} + 17*a^2*c^3*d^3*e^2*(-d*e \\
&^3)^{(3/2)} - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4 \\
&*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^{(5/2)} + 8*a^2*b^2*c*e*(-d*e^3)^{(5/2)} \\
&- 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^ \\
&2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^{(3/2)} + 2*b^4*c*d^3*e^2*(-d*e^3)^{(3/2)} \\
&+ 2*b*c^4*d^7*e^2*(-d*e^3)^{(1/2)} + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d \\
&^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^{(3/2)} - 2* \\
&a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^{(3/2)})*(- \\
&-d*e^3)^{(1/2)})/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.230 \quad \int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=298

$$\frac{\sqrt{c} \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{e^{5/2}}{d^{3/2}}}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} a \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2) - \frac{e^{5/2}}{d^{3/2}}}$$

Rubi [A] time = 0.96, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c} \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) - \sqrt{c} \left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (ae^2 - bde + cd^2)} - \frac{1}{adx}}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) - \sqrt{2} a \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2) - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (ae^2 - bde + cd^2)} - \frac{1}{adx}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] $-(1/(a*d*x)) - (\text{Sqrt}[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) - (\text{Sqrt}[c]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) - (e^{5/2}*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]])/(d^{3/2}*(c*d^2 - b*d*e + a*e^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} dx &= \int \left(\frac{1}{adx^2} - \frac{e^3}{d (cd^2 - bde + ae^2) (d + ex^2)} + \frac{-bcd + b^2e - ace - c(cd - be)x}{a (cd^2 - bde + ae^2) (a + bx^2 + cx^4)} \right) dx \\ &= -\frac{1}{adx} + \frac{\int \frac{-bcd + b^2e - ace - c(cd - be)x^2}{a + bx^2 + cx^4} dx}{a (cd^2 - bde + ae^2)} - \frac{e^3 \int \frac{1}{d + ex^2} dx}{d (cd^2 - bde + ae^2)} \\ &= -\frac{1}{adx} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (cd^2 - bde + ae^2)} - \frac{\left(c \left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2}}{2a (cd^2 - bde + ae^2)} \\ &= -\frac{1}{adx} - \frac{\sqrt{c} \left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b - \sqrt{b^2 - 4ac}} (cd^2 - bde + ae^2)} - \frac{\sqrt{c} \left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b + \sqrt{b^2 - 4ac}} (cd^2 - bde + ae^2)} \end{aligned}$$

Mathematica [A] time = 0.40, size = 340, normalized size = 1.14

$$-\frac{\sqrt{c} \left(cd\sqrt{b^2 - 4ac} - be\sqrt{b^2 - 4ac} + 2ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} (e(ae - bd) + cd^2)} + \frac{\sqrt{c} \left(-cd\sqrt{b^2 - 4ac} + be\sqrt{b^2 - 4ac} + 2ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b^2 - 4ac} + b} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} + b (e(ae - bd) + cd^2)} - \frac{e^{5/2} \tan^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d}} \right)}{d^{3/2} (ae^2 - bde + cd^2)} - \frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]]/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e
(- (b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 12.82, size = 10058, normalized size = 33.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5*d^5 - (6*a^2*b^5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^5*c^2 + 14*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3*c^3 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*c^5*d^4*e + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2* \end{aligned}$$

$$\begin{aligned}
& (b^2 - 4ac)ab^3c^3 - 8(b^2 - 4ac)a^2b^4c^4)d^3\text{abs}(acd^2 - abde + a^2e^2) + (6a^2b^6c^3 - 28a^3b^4c^4 + 16a^4b^2c^5 - 3\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^6c + 14\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^4c^2 + 6\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^5c^2 - 8\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^2c^3 - 4\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^3c^3 - 3\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c^3 + 2\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^4 - 6(b^2 - 4ac) \\
& a^2b^4c^3 + 4(b^2 - 4ac)a^3b^2c^4)d^3e^2 - 2(2\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^6c - 17\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c^2 - 4\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c^2 - 4ab^6c^2 + 40\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^2c^3 + 18\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^3 + 2\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^4c^3 + 34a^2b^4c^3 - 16\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4c^4 - 8\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^4c^4 - 9\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^4 - 80a^3b^2c^4 + 4 \\
& \sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^5 + 32a^4c^5 + 4(b^2 - 4ac) \\
& ab^4c^2 - 18(b^2 - 4ac)a^2b^2c^3 + 8(b^2 - 4ac)a^3c^4)d^2 \\
& \text{abs}(acd^2 - abde + a^2e^2)e + (2b^4c^3 - 16ab^2c^4 + 32a^2c^5 - \\
& \sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^4c + 8\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^2 + 2\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)b^3c^2 - 16\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^3 - 8\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^2c^3 + 4\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)ac^4 - 2(b^2 - 4ac)b^2c^3 + 8(b^2 - \\
& 4ac)ac^4)(acd^2 - abde + a^2e^2)^2d - (2a^2b^7c^2 - 4a^3b^5c^3 - \\
& 24a^4b^3c^4 + 32a^5b^2c^5 - \sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^7 + 2\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^5c + 2\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^6c + 12\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^3c^2 + 4\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^4c^2 - \sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^5c^2 - 16\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^2c^3 - 8\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^2c^3 - 2\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^3c^3 + 4\sqrt{2}) \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^3c^4 - 2(b^2 - 4ac)a^2b^5c^2 - \\
& 4(b^2 - 4ac)a^3b^3c^3 + 8(b^2 - 4ac)a^4b^3c^4)d^2e^3 + 2(\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^7 - 8\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^5c - 2\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^6c - 2ab^7c + 16\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^3c^2 + 8\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^4c^2 + \sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)ab^5c^2 + 16a^2b^5c^2 - 4\sqrt{2}) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^3c^3 - 32a^
\end{aligned}$$

$$\begin{aligned}
& 3*b^3*c^3 + 2*(b^2 - 4*a*c)*a*b^5*c - 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a* \\
& c*d^2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^5 + 8*\sqrt{2})* \\
& \sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^3*c + 2*\sqrt{2}*\sqrt{ \\
& b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - \\
& 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a* \\
& c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*s \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b \\
& *c + \sqrt{b^2 - 4*a*c}*c}*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a* \\
& c)*a*b*c^3)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*e + (4*a^3*b^6*c^2 - 22*a^4*b^4 \\
& *c^3 + 24*a^5*b^2*c^4 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4 \\
& *a*c}*c}*a^3*b^6 + 11*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c} \\
& }*c)*a^4*b^4*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
&)*a^3*b^5*c - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}* \\
& a^5*b^2*c^2 - 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a \\
& ^4*b^3*c^2 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^ \\
& 3*b^4*c^2 + 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4 \\
& *b^2*c^3 - 4*(b^2 - 4*a*c)*a^3*b^4*c^2 + 6*(b^2 - 4*a*c)*a^4*b^2*c^3)*d*e^4 \\
& - 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^6 - 9*\sqrt{2}*\sqrt{b*c \\
& + \sqrt{b^2 - 4*a*c}*c}*a^3*b^4*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
&)*a^2*b^5*c - 2*a^2*b^6*c + 24*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4* \\
& b^2*c^2 + 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^3*c^2 + \sqrt{2})* \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^2*b^4*c^2 + 18*a^3*b^4*c^2 - 16*\sqrt{2})*s \\
& \sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^5*c^3 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4* \\
& a*c}*c}*a^4*b*c^3 - 5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^3*b^2*c^3 - \\
& 48*a^4*b^2*c^3 + 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4*c^4 + 32*a^ \\
& 5*c^4 + 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - \\
& 4*a*c)*a^4*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12 \\
& *a^5*b^3*c^3 + 16*a^6*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c}*a^4*b^5 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4* \\
& a*c}*c}*a^5*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c} \\
& }*c)*a^4*b^4*c - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
&)*a^6*b*c^2 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a \\
& ^5*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^4* \\
& b^3*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c}*a^5*b \\
& *c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*\arctan \\
& (2*\sqrt{1/2}*x/\sqrt{(a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 + \sqrt{(a*b*c*d^2 - \\
& a*b^2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - \\
& a*b*c*d*e + a^2*c*e^2)))/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c \\
& ^2 - 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 \\
& - 4*a^4*c^5)*d^4*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c) - 2*(a^3*b^5*c - \\
& 8*a^4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 \\
& - 4*a^4*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e + (a^3*b^6 - 6 \\
& *a^4*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^ \\
& 5*b*c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*a
\end{aligned}$$

$$\begin{aligned}
& bs(c)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2*c^2 - 4*a^6*c^3)*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*abs(c)*e^4 + 1/8 * ((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*c^5)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^6*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 + 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^5*c^2 + 4*a*b^6*c^2 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b^2*c^3 + 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^4*c^3 - 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*b*c^4 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^2*b^2*c^4 + 80*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a^3*c^5 - 32*a^4*c^5 - 4*(b^2 - 4*a*c)*a*b^4*c^2 + 18*(b^2 - 4*a*c)*a^2*b^2*c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*c)*a*b^2*c^2 + 2*sqrt(2)*s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot b^3c^2 - 16\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2c^3 - 8\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2c^3 - 8\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2c^3 - \text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot b^2c^3 + 4\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2c^4 - 2(b^2 - 4ac)b^2c^3 + 8(b^2 - 4ac)c \cdot a^2c^4) \cdot (acd^2 - abd^2 + a^2e^2)^2d - (2a^2b^7c^2 - 4a^3b^5c^3 - 24a^4b^3c^4 + 32a^5b^2c^5 - \text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^7 + 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^7 + 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^6c + 12\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^3c^2 + 4\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^4c^2 - \text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^5c^2 - 16\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^5b^2c^3 - 8\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^2c^3 - 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^3c^3 + 4\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^2c^3 - 2(b^2 - 4ac)a^2b^5c^2 - 4(b^2 - 4ac)a^3b^3c^3 + 8(b^2 - 4ac)a^4b^2c^3) \cdot d^2e^3 - 2(\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^7 - 8\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^5c - 2\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^6c + 2a^2b^7c + 16\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^3c^2 + 8\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^4c^2 + \text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^5c^2 - 16a^2b^5c^2 - 4\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^3c^3 + 32a^3b^3c^3 - 2(b^2 - 4ac)a^2b^5c + 8(b^2 - 4ac)a^2b^3c^2) \cdot d \cdot \text{abs}(acd^2 - abd^2 + a^2e^2)e^2 - (2b^5c^2 - 16a^2b^3c^3 + 32a^2b^2c^4 - \text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot b^5 + 8\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^3c + 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot b^4c - 16\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^2c^2 - 8\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^2c^2 - \text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot b^3c^2 + 4\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^3c^2 - 2(b^2 - 4ac)b^3c^2 + 8(b^2 - 4ac)a^2b^3c^3) \cdot (acd^2 - abd^2 + a^2e^2)^2e + (4a^3b^6c^2 - 22a^4b^4c^3 + 24a^5b^2c^4 - 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^6 + 11\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^4c + 4\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^5c - 12\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^5b^2c^2 - 6\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^3c^2 - 2\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^4c^2 + 3\text{sqrt}(2) \cdot \text{sqrt}(b^2 - 4ac) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^2c^3 - 4(b^2 - 4ac)a^3b^4c^2 + 6(b^2 - 4ac)a^4b^2c^3) \cdot d^2e^4 + 2(\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^6 - 9\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^4c - 2\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^2b^5c + 2a^2b^6c + 24\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^4b^2c^2 + 10\text{sqrt}(2) \cdot \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac)c) \cdot a^3b^3c^2 + \text{sqrt}(2) \cdot \text{sqrt}(
\end{aligned}$$

$$\begin{aligned}
& (b*c - \sqrt{b^2 - 4*a*c})*c)*a^2*b^4*c^2 - 18*a^3*b^4*c^2 - 16*\sqrt{2}*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*c^3 - 8*\sqrt{2}*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b*c^3 - 5*\sqrt{2}*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^3*b^2*c^3 + 48*a^4*b^2*c^3 + 4*\sqrt{2}*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*c^4 - 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*e^3 - (2*a^4*b^5*c^2 - 12*a^5*b^3*c^3 + 16*a^6*b*c^4 - \sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^5 + 6*\sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^3*c + 2*\sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^4*c - 8*\sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^6*b*c^2 - 4*\sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b^2*c^2 - \sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^4*b^3*c^2 + 2*\sqrt{2}*\sqrt{(b^2 - 4*a*c})*\sqrt{(b*c - \sqrt{b^2 - 4*a*c})*c)*a^5*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 4*(b^2 - 4*a*c)*a^5*b*c^3)*e^5)*\arctan(2*\sqrt{1/2}*x/\sqrt{(a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2 - \sqrt{(a*b*c*d^2 - a*b^2*d*e + a^2*b*e^2)^2 - 4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2))})/(a*c^2*d^2 - a*b*c*d*e + a^2*c*e^2)))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 - 2*a^3*b^3*c^3 + 16*a^5*c^4 + 8*a^4*b*c^4 + a^3*b^2*c^4 - 4*a^4*c^5)*d^4*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*\text{abs}(c) - 2*(a^3*b^5*c - 8*a^4*b^3*c^2 - 2*a^3*b^4*c^2 + 16*a^5*b*c^3 + 8*a^4*b^2*c^3 + a^3*b^3*c^3 - 4*a^4*b*c^4)*d^3*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*\text{abs}(c)*e + (a^3*b^6 - 6*a^4*b^4*c - 2*a^3*b^5*c + 4*a^4*b^3*c^2 + a^3*b^4*c^2 + 32*a^6*c^3 + 16*a^5*b*c^3 - 2*a^4*b^2*c^3 - 8*a^5*c^4)*d^2*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*\text{abs}(c)*e^2 - 2*(a^4*b^5 - 8*a^5*b^3*c - 2*a^4*b^4*c + 16*a^6*b*c^2 + 8*a^5*b^2*c^2 + a^4*b^3*c^2 - 4*a^5*b*c^3)*d*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*\text{abs}(c)*e^3 + (a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2*c^2 - 4*a^6*c^3)*\text{abs}(a*c*d^2 - a*b*d*e + a^2*e^2)*\text{abs}(c)*e^4) - \arctan(x*e^(1/2)/\sqrt{d})*e^(5/2)/((c*d^3 - b*d^2*e + a*d*e^2)*\sqrt{d}) - 1/(a*d*x)
\end{aligned}$$

maple [B] time = 0.03, size = 817, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a), x)$

[Out] $-1/a/d/x - 1/2/(a*e^2 - b*d*e + c*d^2)/a*c*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b*e^{1/2}/(a*e^2 - b*d*e + c*d^2)/a*c^2*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*c*x)*d + 1/(a*e^2 - b*d*e + c*d^2)*c^2/(-4*a*c + b^2)^{(1/2)}*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*c*x)*e^{-1/2}/(a*e^2 - b*d*e + c*d^2)/a*c/(-4*a*c + b^2)^{(1/2)}*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2*e^{1/2}/(a*e^2 - b*d*e + c*d^2)/a*c^2/(-4*a*c + b^2)^{(1/2)}$

$$\begin{aligned} & ^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d + 1/2 / (a * e^2 - b * d * e + c * d^2) / a * c * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * e - 1/2 / (a * e^2 - b * d * e + c * d^2) / a * c^2 * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * d + 1 / (a * e^2 - b * d * e + c * d^2) * c^2 / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * e - 1/2 / (a * e^2 - b * d * e + c * d^2) / a * c / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^2 * e + 1/2 / (a * e^2 - b * d * e + c * d^2) / a * c^2 / (-4*a*c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4*a*c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b * d - 1 / d * e^3 / (a * e^2 - b * d * e + c * d^2) / (d * e)^{(1/2)} * \operatorname{arctan}(1 / (d * e)^{(1/2)} * e * x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^3 - bd^2e + ade^2)\sqrt{de}} + \frac{-\int \frac{bcd+(c^2d-bce)x^2-(b^2-ac)e}{cx^4+bx^2+a} dx}{acd^2 - abde + a^2e^2} - \frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $-e^3 \arctan(e * x / \sqrt{d * e}) / ((c * d^3 - b * d^2 * e + a * d * e^2) * \sqrt{d * e}) + \int \frac{-(b * c * d + (c^2 * d - b * c * e) * x^2 - (b^2 - a * c) * e)}{(c * x^4 + b * x^2 + a), x} / (a * c * d^2 - a * b * d * e + a^2 * e^2) - 1 / (a * d * x)$

mupad [B] time = 5.89, size = 33644, normalized size = 112.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] $\operatorname{atan}\left(\frac{\left(\left(-b^7 * e^2 + b^5 * c^2 * d^2 + b^4 * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 7 * a * b^3 * c^3 * d^2 + 12 * a^2 * b * c^4 * d^2 - a * c^3 * d^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 20 * a^3 * b * c^3 * e^2 - 2 * b^6 * c * d * e + 25 * a^2 * b^3 * c^2 * e^2 + a^2 * c^2 * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} + b^2 * c^2 * d^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 9 * a * b^5 * c * e^2 + 16 * a^3 * c^4 * d * e + 16 * a * b^4 * c^2 * d * e - 2 * b^3 * c * d * e * (-4 * a * c - b^2)^3\right)^{1/2} - 3 * a * b^2 * c * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 36 * a^2 * b^2 * c^3 * d * e + 4 * a * b * c^2 * d * e * (-4 * a * c - b^2)^3\right)^{1/2}}{(8 * (a^5 * b^4 * e^4 + 16 * a^5 * c^4 * d^4 + 16 * a^7 * c^2 * e^4 - 8 * a^6 * b^2 * c * e^4 - 2 * a^4 * b^5 * d * e^3 + a^3 * b^4 * c^2 * d^4 - 8 * a^4 * b^2 * c^3 * d^4 + a^3 * b^6 * d^2 * e^2 + 32 * a^6 * c^3 * d^2 * e^2 - 2 * a^3 * b^5 * c * d^3 * e - 32 * a^5 * b * c^3 * d^3 * e + 16 * a^5 * b^3 * c * d * e^3 - 32 * a^6 * b * c^2 * d * e^3 + 16 * a^4 * b^3 * c^2 * d^3 * e - 6 * a^4 * b^4 * c * d^2 * e^2))^{1/2} * \left(\left(-b^7 * e^2 + b^5 * c^2 * d^2 + b^4 * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 7 * a * b^3 * c^3 * d^2 + 12 * a^2 * b * c^4 * d^2 - a * c^3 * d^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 20 * a^3 * b * c^3 * e^2 - 2 * b^6 * c * d * e + 25 * a^2 * b^3 * c^2 * e^2 + a^2 * c^2 * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} + b^2 * c^2 * d^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 9 * a * b^5 * c * e^2 + 16 * a^3 * c^4 * d * e + 16 * a * b^4 * c^2 * d * e - 2 * b^3 * c * d * e * (-4 * a * c - b^2)^3\right)^{1/2} - 3 * a * b^2 * c * e^2 * (-4 * a * c - b^2)^3\right)^{1/2} - 36 * a^2 * b^2 * c^3 * d * e + 4 * a * b * c^2 * d * e * (-4 * a * c - b^2)^3\right)^{1/2}}$

$$\begin{aligned}
& /2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 \\
& + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2 \\
& *d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2 \\
& *e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3 \\
& *d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e \\
& - 6*a^4*b^4*c*d^2*e^2))^{(1/2)}*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4 \\
& *d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e \\
& - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d \\
& *e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))^{(1/2)}*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14 \\
& *c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 64 \\
& 0*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4 \\
& *d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4 \\
& *d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2
\end{aligned}$$

$$\begin{aligned}
& *(- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2* \\
& e^2 + a^2*c^2*e^2*(- (4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(- (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^ \\
& 2*c^3*d*e + 4*a*b*c^2*d*e*(- (4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^ \\
& 5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^ \\
& 2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^ \\
& 5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + \\
& 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)} + 4*a^7*c^8*d^13*e^2 + \\
& 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^ \\
& 6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^ \\
& 2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e^3 - 32*a^9*b*c^5*d^8*e^7) + x*(2*a^7*c^7 \\
& *d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2 \\
& *d^2 + b^4*e^2*(- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^ \\
& 2 - a*c^3*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 2 \\
& 5*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(- (4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(- (4 \\
& *a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - \\
& 2*b^3*c*d*e*(- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(- (4*a*c - b^2)^3)^{(1/ \\
& 2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(- (4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b \\
& ^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^ \\
& 3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2* \\
& e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6* \\
& b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*i - ((- (\\
& b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(- (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 \\
& + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 \\
& - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(- (4*a*c - b^2)^3)^{(1/2)} + \\
& b^2*c^2*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16 \\
& *a*b^4*c^2*d*e - 2*b^3*c*d*e*(- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(- (4* \\
& a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(- (4*a*c - b^2)^3) \\
& ^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 \\
& - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 \\
& + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3* \\
& c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)) \\
&)^{(1/2)}*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(- (4*a*c - b^2)^3)^{(1/2)} - 7*a \\
& *b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 20*a \\
& ^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(- (4*a*c - b^ \\
& 2)^3)^{(1/2)} + b^2*c^2*d^2*(- (4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3 \\
& *c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(- (4*a*c - b^2)^3)^{(1/2)} - 3*a*b^ \\
& 2*c*e^2*(- (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(- (4* \\
& a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8* \\
& a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3 \\
& *b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e \\
& + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^ \\
& 4*c*d^2*e^2)))^{(1/2)}*(x*(- (b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(- (4*a*c - b^2)^
\end{aligned}$$

$$\begin{aligned}
& 3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 \\
& - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e \\
& + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))^{(1/2)}*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 \\
& - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 \\
& - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 \\
& + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 \\
& + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4*d^10*e^8) + 192*a^10*c^7*d^14*e^3 \\
& + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4*d^13*e^4 \\
& + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^9*b^5*c^3*d^11*e^6 \\
& + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 + 80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*e^8 + 336*a^11*b^2*c^4*d^10*e^7 \\
& - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 \\
& + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 \\
& - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 \\
& + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 \\
& + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9)*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 \\
& + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3
\end{aligned}$$

$$\begin{aligned}
& *c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))^{(1/2)} + 4*a^7*c^8*d^{13}*e^2 + 4*a^8*c^7* \\
& d^{11}*e^4 - 16*a^{10}*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7* \\
& e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7* \\
& e^8 - 4*a^7*b*c^7*d^{12}*e^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - \\
& 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4 \\
& *e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3* \\
& c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d* \\
& e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^ \\
& 2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 1 \\
& 6*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^ \\
& 4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^ \\
& 3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^ \\
& 3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))^{(1/2)}*1i)/(((-(b^7*e^2 + \\
& b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b \\
& *c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c* \\
& d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2 \\
& *d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8 \\
& *(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b \\
& ^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c \\
& ^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - \\
& 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))^{(1/2)}*((\\
& (-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d \\
& ^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e \\
& ^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2} \\
&) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + \\
& 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2) \\
& ^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c* \\
& e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e \\
& ^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b \\
& ^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^ \\
& 2))^{(1/2)}*(192*a^{10}*c^7*d^{14}*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 \\
& + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c \\
& ^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c \\
& ^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d \\
& ^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c \\
& ^3*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*
\end{aligned}$$

$$\begin{aligned}
& a^4 b^3 c^2 d^3 e - 6 a^4 b^4 c d^2 e^2))^{(1/2)} * (512 a^{11} c^7 d^{15} e^3 + 512 a^{12} c^6 d^{13} e^5 - 512 a^{13} c^5 d^{11} e^7 - 512 a^{14} c^4 d^9 e^9 - 32 a^9 b^3 c^6 d^{16} e^2 + 128 a^9 b^4 c^5 d^{15} e^3 - 192 a^9 b^5 c^4 d^{14} e^4 + 128 a^9 b^6 c^3 d^{13} e^5 - 32 a^9 b^7 c^2 d^{12} e^6 - 640 a^{10} b^2 c^6 d^{15} e^3 + 1056 a^{10} b^3 c^5 d^{14} e^4 - 672 a^{10} b^4 c^4 d^{13} e^5 + 96 a^{10} b^5 c^3 d^{12} e^6 + 32 a^{10} b^6 c^2 d^{11} e^7 + 512 a^{11} b^2 c^5 d^{13} e^5 + 288 a^{11} b^3 c^4 d^{12} e^6 - 192 a^{11} b^4 c^3 d^{11} e^7 + 32 a^{11} b^5 c^2 d^{10} e^8 + 384 a^{12} b^2 c^4 d^{11} e^7 - 288 a^{12} b^3 c^3 d^{10} e^8 - 32 a^{12} b^4 c^2 d^9 e^9 + 256 a^{13} b^2 c^3 d^9 e^9 + 128 a^{10} b^2 c^7 d^{16} e^2 - 1152 a^{11} b^2 c^6 d^{14} e^4 - 640 a^{12} b^2 c^5 d^{12} e^6 + 640 a^{13} b^2 c^4 d^{10} e^8) + 128 a^{11} c^6 d^{12} e^5 - 320 a^{12} c^5 d^{10} e^7 - 256 a^{13} c^4 d^8 e^9 - 16 a^8 b^3 c^6 d^{15} e^2 + 64 a^8 b^4 c^5 d^{14} e^3 - 96 a^8 b^5 c^4 d^{13} e^4 + 64 a^8 b^6 c^3 d^{12} e^5 - 16 a^8 b^7 c^2 d^{11} e^6 - 304 a^9 b^2 c^6 d^{14} e^3 + 512 a^9 b^3 c^5 d^{13} e^4 - 352 a^9 b^4 c^4 d^{12} e^5 + 64 a^9 b^5 c^3 d^{11} e^6 + 16 a^9 b^6 c^2 d^{10} e^7 + 352 a^{10} b^2 c^5 d^{12} e^5 + 80 a^{10} b^3 c^4 d^{11} e^6 - 128 a^{10} b^4 c^3 d^{10} e^7 + 16 a^{10} b^5 c^2 d^9 e^8 + 336 a^{11} b^2 c^4 d^{10} e^7 - 128 a^{11} b^3 c^3 d^9 e^8 - 16 a^{11} b^4 c^2 d^8 e^9 + 128 a^{12} b^2 c^3 d^8 e^9 + 64 a^9 b^2 c^7 d^{15} e^2 - 512 a^{10} b^2 c^6 d^{13} e^4 - 320 a^{11} b^2 c^5 d^{11} e^6 + 256 a^{12} b^2 c^4 d^9 e^8) + x * (112 a^{10} c^6 d^{10} e^6 - 32 a^9 c^7 d^{12} e^4 - 16 a^8 c^8 d^{14} e^2 - 128 a^{11} c^5 d^8 e^8 + 8 a^7 b^2 c^7 d^{14} e^2 - 16 a^7 b^3 c^6 d^{13} e^3 + 8 a^7 b^4 c^5 d^{12} e^4 + 8 a^7 b^5 c^4 d^{11} e^5 - 16 a^7 b^6 c^3 d^{10} e^6 + 8 a^7 b^7 c^2 d^9 e^7 - 72 a^8 b^3 c^5 d^{11} e^5 + 128 a^8 b^4 c^4 d^{10} e^6 - 72 a^8 b^5 c^3 d^9 e^7 - 280 a^9 b^2 c^5 d^{10} e^6 + 208 a^9 b^3 c^4 d^9 e^7 - 16 a^9 b^4 c^3 d^8 e^8 + 8 a^9 b^5 c^2 d^7 e^9 + 96 a^{10} b^2 c^4 d^8 e^8 - 56 a^{10} b^3 c^3 d^7 e^9 + 32 a^8 b^2 c^7 d^{13} e^3 + 128 a^9 b^2 c^6 d^{11} e^5 - 192 a^{10} b^2 c^5 d^9 e^7 + 96 a^{11} b^2 c^4 d^7 e^9) * (- (b^7 e^2 + b^5 c^2 d^2 + b^4 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 7 a b^3 c^3 d^2 + 12 a^2 b c^4 d^2 - a c^3 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 20 a^3 b c^3 e^2 - 2 b^6 c d e + 25 a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} + b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 9 a b^5 c e^2 + 16 a^3 c^4 d e + 16 a b^4 c^2 d e - 2 b^3 c d e * (- (4 a c - b^2)^3)^{(1/2)} - 3 a b^2 c e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 36 a^2 b^2 c^3 d e + 4 a b c^2 d e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 (a^5 b^4 e^4 + 16 a^5 c^4 d^4 + 16 a^7 c^2 e^4 - 8 a^6 b^2 c e^4 - 2 a^4 b^5 d e^3 + a^3 b^4 c^2 d^4 - 8 a^4 b^2 c^3 d^4 + a^3 b^6 d^2 e^2 + 32 a^6 c^3 d^2 e^2 - 2 a^3 b^5 c d^3 e - 32 a^5 b c^3 d^3 e + 16 a^5 b^3 c d e^3 - 32 a^6 b c^2 d e^3 + 16 a^4 b^3 c^2 d^3 e - 6 a^4 b^4 c d^2 e^2))^{(1/2)} + 4 a^7 c^8 d^{13} e^2 + 4 a^8 c^7 d^{11} e^4 - 16 a^{10} c^5 d^7 e^8 - 4 a^7 b^5 c^3 d^8 e^7 + 4 a^7 b^6 c^2 d^7 e^8 + 24 a^8 b^3 c^4 d^8 e^7 - 28 a^8 b^4 c^3 d^7 e^8 + 52 a^9 b^2 c^4 d^7 e^8 - 4 a^7 b^2 c^7 d^{12} e^3 - 32 a^9 b^2 c^5 d^8 e^7) + x * (2 a^7 c^7 d^9 e^5 - 4 a^8 c^6 d^7 e^7 + 2 a^7 b^2 c^5 d^7 e^7) * (- (b^7 e^2 + b^5 c^2 d^2 + b^4 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 7 a b^3 c^3 d^2 + 12 a^2 b c^4 d^2 - a c^3 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 20 a^3 b c^3 e^2 - 2 b^6 c d e + 25 a^2 b^3 c^2 e^2 + a^2 c^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} + b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 9 a b^5 c e^2 + 16 a^3 c^4 d e + 16 a b^4 c^2 d e - 2 b^3 c d e * (- (4 a c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3* \\
& d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4* \\
& d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^ \\
& 3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4 \\
& *b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)} + (((-b^7*e^2 + b^5*c^2*d^2 + \\
& b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a* \\
& c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2* \\
& b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3* \\
& c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3 \\
& 6*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^4 \\
& + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^ \\
& 3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - \\
& 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2* \\
& d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*(((-b^7*e^2 + \\
& b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b \\
& *c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c* \\
& d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2 \\
& *d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8 \\
& *(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b \\
& ^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c \\
& ^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - \\
& 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*(x \\
& *(-b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3* \\
& d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3* \\
& e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e \\
& + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2 \\
&)^3)^{(1/2)})/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c \\
& *e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2* \\
& e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5* \\
& b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e \\
& ^2)))^{(1/2)}*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d \\
& ^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5* \\
& d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7 \\
& *c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 67 \\
& 2*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e \\
& ^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c \\
& ^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^ \\
& 12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + \\
& 128*a^10*b*c^7*d^16*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^
\end{aligned}$$

$$\begin{aligned}
& 6 + 640*a^{13}*b*c^4*d^{10}*e^8) + 192*a^{10}*c^7*d^{14}*e^3 + 128*a^{11}*c^6*d^{12}*e^5 - 320*a^{12}*c^5*d^{10}*e^7 - 256*a^{13}*c^4*d^8*e^9 - 16*a^8*b^3*c^6*d^{15}*e^2 \\
& + 64*a^8*b^4*c^5*d^{14}*e^3 - 96*a^8*b^5*c^4*d^{13}*e^4 + 64*a^8*b^6*c^3*d^{12}*e^5 - 16*a^8*b^7*c^2*d^{11}*e^6 - 304*a^9*b^2*c^6*d^{14}*e^3 + 512*a^9*b^3*c^5*d^{13}*e^4 \\
& - 352*a^9*b^4*c^4*d^{12}*e^5 + 64*a^9*b^5*c^3*d^{11}*e^6 + 16*a^9*b^6*c^2*d^{10}*e^7 + 352*a^{10}*b^2*c^5*d^{12}*e^5 + 80*a^{10}*b^3*c^4*d^{11}*e^6 - 128*a^{10}*b^4*c^3*d^{10}*e^7 \\
& + 16*a^{10}*b^5*c^2*d^9*e^8 + 336*a^{11}*b^2*c^4*d^{10}*e^7 - 128*a^{11}*b^3*c^3*d^9*e^8 - 16*a^{11}*b^4*c^2*d^8*e^9 + 128*a^{12}*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^{15}*e^2 \\
& - 512*a^{10}*b*c^6*d^{13}*e^4 - 320*a^{11}*b*c^5*d^{11}*e^6 + 256*a^{12}*b*c^4*d^9*e^8) - x*(112*a^{10}*c^6*d^{10}*e^6 - 32*a^9*c^7*d^{12}*e^4 - 16*a^8*c^8*d^{14}*e^2 \\
& - 128*a^{11}*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^{14}*e^2 - 16*a^7*b^3*c^6*d^{13}*e^3 + 8*a^7*b^4*c^5*d^{12}*e^4 + 8*a^7*b^5*c^4*d^{11}*e^5 - 16*a^7*b^6*c^3*d^{10}*e^6 \\
& + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^{11}*e^5 + 128*a^8*b^4*c^4*d^{10}*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^{10}*e^6 + 208*a^9*b^3*c^4*d^9*e^7 \\
& - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^{10}*b^2*c^4*d^8*e^8 - 56*a^{10}*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^{13}*e^3 + 128*a^9*b*c^6*d^{11}*e^5 \\
& - 192*a^{10}*b*c^5*d^9*e^7 + 96*a^{11}*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 \\
& - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e \\
& + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 \\
& + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)} + 4*a^7*c^8*d^{13}*e^2 + 4*a^8*c^7*d^{11}*e^4 - 16*a^{10}*c^5*d^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 \\
& + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^{12}*e^3 - 32*a^9*b*c^5*d^8*e^7) \\
& - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 \\
& + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 \\
& + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 \\
& + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 \\
& - a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3* \\
& d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^5*b^4*e^4 + 16*a^5*c^4* \\
& d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 \\
& - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^ \\
& 3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4 \\
& *b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*2i + \operatorname{atan}((((-(b^7*e^2 + b^5* \\
& c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4 \\
& *d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e \\
& + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*d^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e \\
& + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{ \\
& (1/2)} - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^ \\
& 5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d \\
& *e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d \\
& ^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a \\
& ^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*((-(b \\
& ^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + \\
& 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - \\
& 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16* \\
& a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^2*c*e^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{ \\
& (1/2)}/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 \\
& - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + \\
& 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c \\
& *d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2))) \\
& ^{(1/2)}*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a \\
& c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a \\
& ^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 36*a^2*b^2*c^3*d \\
& *e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d \\
& ^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - \\
& 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3 \\
& *e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4* \\
& b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^{(1/2)}*(512*a^11*c^7*d^15*e^3 + 512*a \\
& ^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9 - 32*a^9*b^ \\
& 3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9*b^5*c^4*d^14*e^4 + 128* \\
& a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 640*a^10*b^2*c^6*d^15*e^3 \\
& + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^13*e^5 + 96*a^10*b^5*c^3* \\
& d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*c^5*d^13*e^5 + 288*a^11* \\
& b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a^11*b^5*c^2*d^10*e^8 + 3 \\
& 84*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^8 - 32*a^12*b^4*c^2*d^9*
\end{aligned}$$

$$\begin{aligned}
& e^9 + 256a^{13}b^2c^3d^9e^9 + 128a^{10}b^*c^7d^{16}e^2 - 1152a^{11}b^*c^6d^{14}e^4 - 640a^{12}b^*c^5d^{12}e^6 + 640a^{13}b^*c^4d^{10}e^8) + 128a^{11}c^6d^{12}e^5 - 320a^{12}c^5d^{10}e^7 - 256a^{13}c^4d^8e^9 - 16a^8b^3c^6d^{15}e^2 + 64a^8b^4c^5d^{14}e^3 - 96a^8b^5c^4d^{13}e^4 + 64a^8b^6c^3d^{12}e^5 - 16a^8b^7c^2d^{11}e^6 - 304a^9b^2c^6d^{14}e^3 + 512a^9b^3c^5d^{13}e^4 - 352a^9b^4c^4d^{12}e^5 + 64a^9b^5c^3d^{11}e^6 + 16a^9b^6c^2d^{10}e^7 + 352a^{10}b^2c^5d^{12}e^5 + 80a^{10}b^3c^4d^{11}e^6 - 128a^{10}b^4c^3d^{10}e^7 + 16a^{10}b^5c^2d^9e^8 + 336a^{11}b^2c^4d^{10}e^7 - 128a^{11}b^3c^3d^9e^8 - 16a^{11}b^4c^2d^8e^9 + 128a^{12}b^2c^3d^8e^9 + 64a^9b^*c^7d^{15}e^2 - 512a^{10}b^*c^6d^{13}e^4 - 320a^{11}b^*c^5d^{11}e^6 + 256a^{12}b^*c^4d^9e^8) + x((112a^{10}c^6d^{10}e^6 - 32a^9c^7d^{12}e^4 - 16a^8c^8d^{14}e^2 - 128a^{11}c^5d^8e^8 + 8a^7b^2c^7d^{14}e^2 - 16a^7b^3c^6d^{13}e^3 + 8a^7b^4c^5d^{12}e^4 + 8a^7b^5c^4d^{11}e^5 - 16a^7b^6c^3d^{10}e^6 + 8a^7b^7c^2d^9e^7 - 72a^8b^3c^5d^{11}e^5 + 128a^8b^4c^4d^{10}e^6 - 72a^8b^5c^3d^9e^7 - 280a^9b^2c^5d^{10}e^6 + 208a^9b^3c^4d^9e^7 - 16a^9b^4c^3d^8e^8 + 8a^9b^5c^2d^7e^9 + 96a^{10}b^2c^4d^8e^8 - 56a^{10}b^3c^3d^7e^9 + 32a^8b^*c^7d^{13}e^3 + 128a^9b^*c^6d^{11}e^5 - 192a^{10}b^*c^5d^9e^7 + 96a^{11}b^*c^4d^7e^9))*(-(b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2)^3)^{1/2}) - 7a^*b^3c^3d^2 + 12a^2b^*c^4d^2 + a^*c^3d^2*(-(4ac - b^2)^3)^{1/2}) - 20a^3b^*c^3e^2 - 2b^6c*d*e + 25a^2b^3c^2e^2 - a^2c^2e^2*(-(4ac - b^2)^3)^{1/2} - b^2c^2d^2*(-(4ac - b^2)^3)^{1/2} - 9a^*b^5c*e^2 + 16a^3c^4d*e + 16a^*b^4c^2d*e + 2b^3c*d*e*(-(4ac - b^2)^3)^{1/2} + 3a^*b^2c*e^2*(-(4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d*e - 4a^*b^*c^2d*e*(-(4ac - b^2)^3)^{1/2}))/ (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c*e^4 - 2a^4b^5d*e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c*d^3e - 32a^5b^*c^3d^3e + 16a^5b^3c*d*e^3 - 32a^6b^*c^2d*e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c*d^2e^2))^{1/2} + 4a^7c^8d^{13}e^2 + 4a^8c^7d^{11}e^4 - 16a^{10}c^5d^7e^8 - 4a^7b^5c^3d^8e^7 + 4a^7b^6c^2d^7e^8 + 24a^8b^3c^4d^8e^7 - 28a^8b^4c^3d^7e^8 + 52a^9b^2c^4d^7e^8 - 4a^7b^*c^7d^{12}e^3 - 32a^9b^*c^5d^8e^7) + x(2a^7c^7d^9e^5 - 4a^8c^6d^7e^7 + 2a^7b^2c^5d^7e^7))*(-(b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2)^3)^{1/2}) - 7a^*b^3c^3d^2 + 12a^2b^*c^4d^2 + a^*c^3d^2*(-(4ac - b^2)^3)^{1/2}) - 20a^3b^*c^3e^2 - 2b^6c*d*e + 25a^2b^3c^2e^2 - a^2c^2e^2*(-(4ac - b^2)^3)^{1/2} - b^2c^2d^2*(-(4ac - b^2)^3)^{1/2} - 9a^*b^5c*e^2 + 16a^3c^4d*e + 16a^*b^4c^2d*e + 2b^3c*d*e*(-(4ac - b^2)^3)^{1/2} + 3a^*b^2c*e^2*(-(4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d*e - 4a^*b^*c^2d*e*(-(4ac - b^2)^3)^{1/2}))/ (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c*e^4 - 2a^4b^5d*e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c*d^3e - 32a^5b^*c^3d^3e + 16a^5b^3c*d*e^3 - 32a^6b^*c^2d*e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c*d^2e^2))^{1/2} * i - ((-(b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2)^3)^{1/2}) - 7a^*b^3c^3d^2 + 12a^2b^*c^4d^2 + a^*c^3d^2*(-(4ac - b^2)^3)^{1/2}) - 20a^3b^*c^3e^2 - 2b^6c*d*e + 25a^2b^3c^2e^2 - a^2c^2e^2*(-(4ac - b^2)^3)^{1/2} - b^2c^2d^2*(-(4ac - b^2)^3)^{1/2} - 9a^*b^5c*e^2 + 16a^3c^4d*e + 16a^*b^4c^2d*e + 2b^3c*d*e*(-(4ac - b^2)^3)^{1/2} + 3a^*b^2c*e^2*(-(4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d*e - 4a^*b^*c^2d*e*(-(4ac - b^2)^3)^{1/2}))/ (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c*e^4 - 2a^4b^5d*e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c*d^3e - 32a^5b^*c^3d^3e + 16a^5b^3c*d*e^3 - 32a^6b^*c^2d*e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c*d^2e^2))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3} - \sqrt{b^2c^2d^2(-4ac - b^2)^3} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e \\
& \sqrt{(-4ac - b^2)^3} + 3ab^2c^2e^2(-4ac - b^2)^3 - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e(-4ac - b^2)^3) / (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^3c^3d^3e + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2))^{1/2} * (((-b^7e^2 + b^5c^2d^2 - b^4e^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^3d^2 + 12a^2b^2c^4d^2 + ac^3d^2(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3e^2 - 2b^6c^2d^2e + 25a^2b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 3ab^2c^2e^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e(-4ac - b^2)^3)^{1/2} / (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^3c^3d^3e + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2))^{1/2} * (x(-b^7e^2 + b^5c^2d^2 - b^4e^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^3d^2 + 12a^2b^2c^4d^2 + ac^3d^2(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3e^2 - 2b^6c^2d^2e + 25a^2b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 3ab^2c^2e^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e(-4ac - b^2)^3)^{1/2} / (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^3c^3d^3e + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2))^{1/2} * (512a^11c^7d^15e^3 + 512a^12c^6d^13e^5 - 512a^13c^5d^11e^7 - 512a^14c^4d^9e^9 - 32a^9b^3c^6d^16e^2 + 128a^9b^4c^5d^15e^3 - 192a^9b^5c^4d^14e^4 + 128a^9b^6c^3d^13e^5 - 32a^9b^7c^2d^12e^6 - 640a^10b^2c^6d^15e^3 + 1056a^10b^3c^5d^14e^4 - 672a^10b^4c^4d^13e^5 + 96a^10b^5c^3d^12e^6 + 32a^10b^6c^2d^11e^7 + 512a^11b^2c^5d^13e^5 + 288a^11b^3c^4d^12e^6 - 192a^11b^4c^3d^11e^7 + 32a^11b^5c^2d^10e^8 + 384a^12b^2c^4d^11e^7 - 288a^12b^3c^3d^10e^8 - 32a^12b^4c^2d^9e^9 + 256a^13b^2c^3d^9e^9 + 128a^10b^2c^7d^16e^2 - 1152a^11b^2c^6d^14e^4 - 640a^12b^2c^5d^12e^6 + 640a^13b^2c^4d^10e^8) + 192a^10c^7d^14e^3 + 128a^11c^6d^12e^5 - 320a^12c^5d^10e^7 - 256a^13c^4d^8e^9 - 16a^8b^3c^6d^15e^2 + 64a^8b^4c^5d^14e^3 - 96a^8b^5c^4d^13e^4 + 64a^8b^6c^3d^12e^5 - 16a^8b^7c^2d^11e^6 - 304a^9b^2c^6d^14e^3 + 512a^9b^3c^5d^13e^4 - 352a^9b^4c^4d^12e^5 + 64a^9b^5c^3d^11e^6 + 16a^9b^6c^2d^10e^7 + 352a^10b^2c^5d^12e^5 + 80a^10b^3c^4d^11e^6 - 128a^10b^4c^3d^10e^7 + 16a^10b^5c^2d^9e^8 + 336a^11b^2c^4d^10e^7 - 128a^11b^3c^3d^9e^8 - 16a^11b^4c^2d^8e^9 + 128a^12b^2c^3d^8e^9
\end{aligned}$$

$$\begin{aligned}
&^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c^6*d^13*e^4 - 320*a^11*b*c^5*d^11* \\
&e^6 + 256*a^12*b*c^4*d^9*e^8) - x*(112*a^10*c^6*d^10*e^6 - 32*a^9*c^7*d^12* \\
&e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - \\
&16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 \\
&- 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 \\
&+ 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10 \\
&*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7 \\
&*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^1 \\
&3*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7 \\
&*e^9)))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^ \\
&3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3* \\
&b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(-(4*a*c - b^2)^ \\
&3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^ \\
&4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c \\
&*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c \\
&- b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6 \\
&*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^ \\
&6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1 \\
&6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c \\
&*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d \\
&^7*e^8 - 4*a^7*b^5*c^3*d^8*e^7 + 4*a^7*b^6*c^2*d^7*e^8 + 24*a^8*b^3*c^4*d^8 \\
&*e^7 - 28*a^8*b^4*c^3*d^7*e^8 + 52*a^9*b^2*c^4*d^7*e^8 - 4*a^7*b*c^7*d^12*e \\
&^3 - 32*a^9*b*c^5*d^8*e^7) - x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7 + 2*a \\
&^7*b^2*c^5*d^7*e^7))*(-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(-(4*a*c - b^2)^3)^(\\
&1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(-(4*a*c - b^2)^3)^(\\
&1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 - a^2*c^2*e^2*(- \\
&(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e \\
&^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/ \\
&2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e - 4*a*b*c^ \\
&2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c \\
&^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^ \\
&3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b \\
&*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e \\
&- 6*a^4*b^4*c*d^2*e^2)))^(1/2)*i)/(((-(b^7*e^2 + b^5*c^2*d^2 - b^4*e^2*(- \\
&(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 + a*c^3*d^2*(- \\
&(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 \\
&- a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1 \\
&/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e + 2*b^3*c*d*e*(-(4* \\
&a*c - b^2)^3)^(1/2) + 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c \\
&^3*d*e - 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c \\
&^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d \\
&^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c \\
&*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16* \\
&a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 \\
&- b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 +
\end{aligned}$$

$$\begin{aligned}
& a^2c^3d^2(-4ac - b^2)^3)^{1/2} - 20a^3bc^3e^2 - 2b^6c^2d^2e + 25a^2b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 3ab^2c^2e^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e(-4ac - b^2)^3)^{1/2}) / (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^3e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^2c^3d^3e + 16a^5b^3c^2d^3e^3 - 32a^6b^2c^2d^3e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2))^{1/2} * (192a^{10}c^7d^{14}e^3 - x(-(b^7e^2 + b^5c^2d^2 - b^4e^2(-4ac - b^2)^3)^{1/2} - 7ab^3c^3d^2 + 12a^2b^2c^4d^2 + a^2c^3d^2(-4ac - b^2)^3)^{1/2} - 20a^3bc^3e^2 - 2b^6c^2d^2e + 25a^2b^3c^2e^2 - a^2c^2e^2(-4ac - b^2)^3)^{1/2} - b^2c^2d^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 3ab^2c^2e^2(-4ac - b^2)^3)^{1/2} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e(-4ac - b^2)^3)^{1/2}) / (8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^3e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^2c^3d^3e + 16a^5b^3c^2d^3e^3 - 32a^6b^2c^2d^3e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2))^{1/2} * (512a^{11}c^7d^{15}e^3 + 512a^{12}c^6d^{13}e^5 - 512a^{13}c^5d^{11}e^7 - 512a^{14}c^4d^9e^9 - 32a^9b^3c^6d^{16}e^2 + 128a^9b^4c^5d^{15}e^3 - 192a^9b^5c^4d^{14}e^4 + 128a^9b^6c^3d^{13}e^5 - 32a^9b^7c^2d^{12}e^6 - 640a^{10}b^2c^6d^{15}e^3 + 1056a^{10}b^3c^5d^{14}e^4 - 672a^{10}b^4c^4d^{13}e^5 + 96a^{10}b^5c^3d^{12}e^6 + 32a^{10}b^6c^2d^{11}e^7 + 512a^{11}b^2c^5d^{13}e^5 + 288a^{11}b^3c^4d^{12}e^6 - 192a^{11}b^4c^3d^{11}e^7 + 32a^{11}b^5c^2d^{10}e^8 + 384a^{12}b^2c^4d^{11}e^7 - 288a^{12}b^3c^3d^{10}e^8 - 32a^{12}b^4c^2d^9e^9 + 256a^{13}b^2c^3d^9e^9 + 128a^{10}b^2c^7d^{16}e^2 - 1152a^{11}b^2c^6d^{14}e^4 - 640a^{12}b^2c^5d^{12}e^6 + 640a^{13}b^2c^4d^{10}e^8) + 128a^{11}c^6d^{12}e^5 - 320a^{12}c^5d^{10}e^7 - 256a^{13}c^4d^8e^9 - 16a^8b^3c^6d^{15}e^2 + 64a^8b^4c^5d^{14}e^3 - 96a^8b^5c^4d^{13}e^4 + 64a^8b^6c^3d^{12}e^5 - 16a^8b^7c^2d^{11}e^6 - 304a^9b^2c^6d^{14}e^3 + 512a^9b^3c^5d^{13}e^4 - 352a^9b^4c^4d^{12}e^5 + 64a^9b^5c^3d^{11}e^6 + 16a^9b^6c^2d^{10}e^7 + 352a^{10}b^2c^5d^{12}e^5 + 80a^{10}b^3c^4d^{11}e^6 - 128a^{10}b^4c^3d^{10}e^7 + 16a^{10}b^5c^2d^9e^8 + 336a^{11}b^2c^4d^{10}e^7 - 128a^{11}b^3c^3d^9e^8 - 16a^{11}b^4c^2d^8e^9 + 128a^{12}b^2c^3d^8e^9 + 64a^9b^2c^7d^{15}e^2 - 512a^{10}b^2c^6d^{13}e^4 - 320a^{11}b^2c^5d^{11}e^6 + 256a^{12}b^2c^4d^9e^8) + x(112a^{10}c^6d^{10}e^6 - 32a^9c^7d^{12}e^4 - 16a^8c^8d^{14}e^2 - 128a^{11}c^5d^8e^8 + 8a^7b^2c^7d^{14}e^2 - 16a^7b^3c^6d^{13}e^3 + 8a^7b^4c^5d^{12}e^4 + 8a^7b^5c^4d^{11}e^5 - 16a^7b^6c^3d^{10}e^6 + 8a^7b^7c^2d^9e^7 - 72a^8b^3c^5d^{11}e^5 + 128a^8b^4c^4d^{10}e^6 - 72a^8b^5c^3d^9e^7 - 280a^9b^2c^5d^{10}e^6 + 208a^9b^3c^4d^9e^7 - 16a^9b^4c^3d^8e^8 + 8a^9b^5c^2d^7e^9 + 96a^{10}b^2c^4d^8e^8 - 56a^{10}b^3c^3d^7e^9 + 32a^8b^2c^7d^{13}e^3 + 128a^9b^2c^6d^{11}e^5 - 192a^{10}b^2c^5d^9e^7 + 96a^{11}b^2c^4d^7e^9)) * (-
\end{aligned}$$

$$\begin{aligned}
& b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - 7ab^3c^3d^2 \\
& + 12a^2b^2c^4d^2 + ac^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 20a^3b^2c^3e^2 \\
& - 2b^6c^2d^2 + 25a^2b^3c^2e^2 - a^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - \\
& b^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16 \\
& ab^4c^2d^2e + 2b^3c^2d^2e*(-(4ac - b^2)^3)^{(1/2)} + 3ab^2c^2e^2*(-(4ac \\
& - b^2)^3)^{(1/2)} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e*(-(4ac - b^2)^3) \\
& ^{(1/2)})/(8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 \\
& - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 \\
& + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^2c^3d^3e + 16a^5b^3c \\
& d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d^3e - 6a^4b^4c^2d^2e^2)) \\
&)^{(1/2)} + 4a^7c^8d^13e^2 + 4a^8c^7d^11e^4 - 16a^10c^5d^7e^8 - 4 \\
& a^7b^5c^3d^8e^7 + 4a^7b^6c^2d^7e^8 + 24a^8b^3c^4d^8e^7 - 28a \\
& ^8b^4c^3d^7e^8 + 52a^9b^2c^4d^7e^8 - 4a^7b^2c^7d^12e^3 - 32a^ \\
& 9b^2c^5d^8e^7) + x(2a^7c^7d^9e^5 - 4a^8c^6d^7e^7 + 2a^7b^2c^5 \\
& d^7e^7))*(-(b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2)^3)^{(1/2)} - 7 \\
& ab^3c^3d^2 + 12a^2b^2c^4d^2 + ac^3d^2*(-(4ac - b^2)^3)^{(1/2)} - 20a \\
& ^3b^2c^3e^2 - 2b^6c^2d^2 + 25a^2b^3c^2e^2 - a^2c^2e^2*(-(4ac - b \\
& ^2)^3)^{(1/2)} - b^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9ab^5c^2e^2 + 16a^ \\
& 3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e*(-(4ac - b^2)^3)^{(1/2)} + 3ab \\
& ^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^3d^2e - 4ab^2c^2d^2e*(-(4 \\
& ac - b^2)^3)^{(1/2)})/(8(a^5b^4e^4 + 16a^5c^4d^4 + 16a^7c^2e^4 - 8 \\
& a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^2c^3d^4 + a^ \\
& 3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a^5b^2c^3d^3e \\
& + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d^3e - 6a^4b \\
& ^4c^2d^2e^2))^{(1/2)} + (((-b^7e^2 + b^5c^2d^2 - b^4e^2*(-(4ac - b^2) \\
& ^3)^{(1/2)} - 7ab^3c^3d^2 + 12a^2b^2c^4d^2 + ac^3d^2*(-(4ac - b^2)^ \\
& 3)^{(1/2)} - 20a^3b^2c^3e^2 - 2b^6c^2d^2 + 25a^2b^3c^2e^2 - a^2c^2e^ \\
& 2*(-(4ac - b^2)^3)^{(1/2)} - b^2c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 9ab^5 \\
& c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e*(-(4ac - b^2)^3) \\
& ^{(1/2)} + 3ab^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^2b^2c^3d^2e - 4ab \\
& b^2c^2d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8(a^5b^4e^4 + 16a^5c^4d^4 + 16a \\
& ^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 - 8a^4b^ \\
& 2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a \\
& ^5b^2c^3d^3e + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d \\
& ^3e - 6a^4b^4c^2d^2e^2))^{(1/2)}*(((-b^7e^2 + b^5c^2d^2 - b^4e^2*(- \\
& (4ac - b^2)^3)^{(1/2)} - 7ab^3c^3d^2 + 12a^2b^2c^4d^2 + ac^3d^2*(-(\\
& 4ac - b^2)^3)^{(1/2)} - 20a^3b^2c^3e^2 - 2b^6c^2d^2 + 25a^2b^3c^2e^2 \\
& - a^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - b^2c^2d^2*(-(4ac - b^2)^3)^{(1 \\
& /2)} - 9ab^5c^2e^2 + 16a^3c^4d^2e + 16ab^4c^2d^2e + 2b^3c^2d^2e*(-(4 \\
& ac - b^2)^3)^{(1/2)} + 3ab^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 36a^2b^2c \\
& ^3d^2e - 4ab^2c^2d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8(a^5b^4e^4 + 16a^5c \\
& ^4d^4 + 16a^7c^2e^4 - 8a^6b^2c^2e^4 - 2a^4b^5d^2e^3 + a^3b^4c^2d^4 \\
& - 8a^4b^2c^3d^4 + a^3b^6d^2e^2 + 32a^6c^3d^2e^2 - 2a^3b^5c^2d^3e - 32a \\
& ^5b^2c^3d^3e + 16a^5b^3c^2d^2e^3 - 32a^6b^2c^2d^2e^3 + 16a^4b^3c^2d \\
& ^3e - 6a^4b^4c^2d^2e^2))^{(1/2)}*(x*(-b^7e^2 + b^5c^2d^2
\end{aligned}$$

$$\begin{aligned}
& 2 - b^4 e^2 (-4ac - b^2)^3)^{1/2} - 7ab^3 c^3 d^2 + 12a^2 b^4 c^4 d^2 + \\
& a^3 c^3 d^2 (-4ac - b^2)^3)^{1/2} - 20a^3 b^3 c^3 e^2 - 2b^6 c^3 d^2 e + 25a^2 b^3 c^2 e^2 - a^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} - b^2 c^2 d^2 (-4ac - \\
& c - b^2)^3)^{1/2} - 9ab^5 c^3 e^2 + 16a^3 c^4 d^2 e + 16ab^4 c^2 d^2 e + 2b^3 c^3 d^2 e (-4ac - b^2)^3)^{1/2} + 3ab^2 c^3 e^2 (-4ac - b^2)^3)^{1/2} \\
& - 36a^2 b^2 c^3 d^2 e - 4ab^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2} / (8(a^5 b^4 e^4 + 16a^5 c^4 d^4 + 16a^7 c^2 e^4 - 8a^6 b^2 c^2 e^4 - 2a^4 b^5 d^2 e^3 + \\
& a^3 b^4 c^2 d^4 - 8a^4 b^2 c^3 d^4 + a^3 b^6 d^2 e^2 + 32a^6 c^3 d^2 e^2 - 2a^3 b^5 c^3 d^3 e - 32a^5 b^3 c^3 d^3 e + 16a^5 b^3 c^3 d^3 e^3 - 32a^6 b^3 c^2 d^2 e^3 + \\
& 16a^4 b^3 c^2 d^3 e - 6a^4 b^4 c^2 d^2 e^2))^{1/2} (512a^{11} c^7 d^{15} e^3 + 512a^{12} c^6 d^{13} e^5 - 512a^{13} c^5 d^{11} e^7 - 512a^{14} c^4 d^9 e^9 - 32a^9 b^3 c^6 d^{16} e^2 + \\
& 128a^9 b^4 c^5 d^{15} e^3 - 192a^9 b^5 c^4 d^{14} e^4 + 128a^9 b^6 c^3 d^{13} e^5 - 32a^9 b^7 c^2 d^{12} e^6 - 640a^{10} b^2 c^6 d^{15} e^3 + 1056a^{10} b^3 c^5 d^{14} e^4 - 672a^{10} b^4 c^4 d^{13} e^5 + \\
& 96a^{10} b^5 c^3 d^{12} e^6 + 32a^{10} b^6 c^2 d^{11} e^7 + 512a^{11} b^2 c^5 d^{13} e^5 + 288a^{11} b^3 c^4 d^{12} e^6 - 192a^{11} b^4 c^3 d^{11} e^7 + 32a^{11} b^5 c^2 d^{10} e^8 + 384a^{12} b^2 c^4 d^{11} e^7 - \\
& 288a^{12} b^3 c^3 d^{10} e^8 - 32a^{12} b^4 c^2 d^9 e^9 + 256a^{13} b^2 c^3 d^9 e^9 + 128a^{10} b^3 c^7 d^{16} e^2 - 1152a^{11} b^3 c^6 d^{14} e^4 - 640a^{12} b^3 c^5 d^{12} e^6 + 640a^{13} b^3 c^4 d^{10} e^8) + \\
& 192a^{10} c^7 d^{14} e^3 + 128a^{11} c^6 d^{12} e^5 - 320a^{12} c^5 d^{10} e^7 - 256a^{13} c^4 d^8 e^9 - 16a^8 b^3 c^6 d^{15} e^2 + 64a^8 b^4 c^5 d^{14} e^3 - 96a^8 b^5 c^4 d^{13} e^4 + 64a^8 b^6 c^3 d^{12} e^5 - \\
& 16a^8 b^7 c^2 d^{11} e^6 - 304a^9 b^2 c^6 d^{14} e^3 + 512a^9 b^3 c^5 d^{13} e^4 - 352a^9 b^4 c^4 d^{12} e^5 + 64a^9 b^5 c^3 d^{11} e^6 + 16a^9 b^6 c^2 d^{10} e^7 + 352a^{10} b^2 c^5 d^{12} e^5 + \\
& 80a^{10} b^3 c^4 d^{11} e^6 - 128a^{10} b^4 c^3 d^{10} e^7 + 16a^{10} b^5 c^2 d^9 e^8 + 336a^{11} b^2 c^4 d^{10} e^7 - 128a^{11} b^3 c^3 d^9 e^8 - 16a^{11} b^4 c^2 d^8 e^9 + 128a^{12} b^2 c^3 d^8 e^9 + \\
& 64a^9 b^3 c^7 d^{15} e^2 - 512a^{10} b^3 c^6 d^{13} e^4 - 320a^{11} b^3 c^5 d^{11} e^6 + 256a^{12} b^3 c^4 d^9 e^8) - x(112a^{10} c^6 d^{10} e^6 - 32a^9 c^7 d^{12} e^4 - 16a^8 c^8 d^{14} e^2 - \\
& 128a^{11} c^5 d^8 e^8 + 8a^7 b^2 c^7 d^{14} e^2 - 16a^7 b^3 c^6 d^{13} e^3 + 8a^7 b^4 c^5 d^{12} e^4 + 8a^7 b^5 c^4 d^{11} e^5 - 16a^7 b^6 c^3 d^{10} e^6 + 8a^7 b^7 c^2 d^9 e^7 - \\
& 72a^8 b^3 c^5 d^{11} e^5 + 128a^8 b^4 c^4 d^{10} e^6 - 72a^8 b^5 c^3 d^9 e^7 - 280a^9 b^2 c^5 d^{10} e^6 + 208a^9 b^3 c^4 d^9 e^7 - 16a^9 b^4 c^3 d^8 e^8 + 8a^9 b^5 c^2 d^7 e^9 + \\
& 96a^{10} b^2 c^4 d^8 e^8 - 56a^{10} b^3 c^3 d^7 e^9 + 32a^8 b^3 c^7 d^{13} e^3 + 128a^9 b^3 c^6 d^{11} e^5 - 192a^{10} b^3 c^5 d^9 e^7 + 96a^{11} b^3 c^4 d^7 e^9) * (-b^7 e^2 + b^5 c^2 d^2 - \\
& b^4 e^2 (-4ac - b^2)^3)^{1/2} - 7ab^3 c^3 d^2 + 12a^2 b^4 c^4 d^2 + a^3 c^3 d^2 (-4ac - b^2)^3)^{1/2} - 20a^3 b^3 c^3 e^2 - 2b^6 c^3 d^2 e + 25a^2 b^3 c^2 e^2 - a^2 c^2 e^2 (-4ac - \\
& b^2)^3)^{1/2} - b^2 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 9ab^5 c^3 e^2 + 16a^3 c^4 d^2 e + 16ab^4 c^2 d^2 e + 2b^3 c^3 d^2 e (-4ac - b^2)^3)^{1/2} + 3ab^2 c^3 e^2 (-4ac - b^2)^3)^{1/2} \\
& - 36a^2 b^2 c^3 d^2 e - 4ab^2 c^2 d^2 e (-4ac - b^2)^3)^{1/2} / (8(a^5 b^4 e^4 + 16a^5 c^4 d^4 + 16a^7 c^2 e^4 - 8a^6 b^2 c^2 e^4 - 2a^4 b^5 d^2 e^3 + a^3 b^4 c^2 d^4 - 8a^4 b^2 c^3 d^4 + \\
& a^3 b^6 d^2 e^2 + 32a^6 c^3 d^2 e^2 - 2a^3 b^5 c^3 d^3 e - 32a^5 b^3 c^3 d^3 e + 16a^5 b^3 c^3 d^3 e^3 - 32a^6 b^3 c^2 d^2 e^3 - 16a^4 b^3 c^2 d^3 e - 6a^4 b^4 c^2 d^2 e^2))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& a^6 b^c d^2 e^3 + 16 a^4 b^3 c^2 d^3 e - 6 a^4 b^4 c d^2 e^2))^{(1/2)} + 4 a^7 c^8 d^{13} e^2 + 4 a^8 c^7 d^{11} e^4 - 16 a^{10} c^5 d^7 e^8 - 4 a^7 b^5 c^3 d^8 e^7 + 4 a^7 b^6 c^2 d^7 e^8 + 24 a^8 b^3 c^4 d^8 e^7 - 28 a^8 b^4 c^3 d^7 e^8 + 52 a^9 b^2 c^4 d^7 e^8 - 4 a^7 b^6 c^7 d^{12} e^3 - 32 a^9 b^5 c^5 d^8 e^7) - x(2 a^7 c^7 d^9 e^5 - 4 a^8 c^6 d^7 e^7 + 2 a^7 b^2 c^5 d^7 e^7) * (- (b^7 e^2 + b^5 c^2 d^2 - b^4 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 7 a b^3 c^3 d^2 + 12 a^2 b^c d^2 + a c^3 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 20 a^3 b^c d^3 e^2 - 2 b^6 c d e + 25 a^2 b^3 c^2 e^2 - a^2 c^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 9 a b^5 c e^2 + 16 a^3 c^4 d e + 16 a b^4 c^2 d e + 2 b^3 c d e * (- (4 a c - b^2)^3)^{(1/2)} + 3 a b^2 c e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 36 a^2 b^2 c^3 d e - 4 a b^c d^2 e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 (a^5 b^4 e^4 + 16 a^5 c^4 d^4 + 16 a^7 c^2 e^4 - 8 a^6 b^2 c e^4 - 2 a^4 b^5 d e^3 + a^3 b^4 c^2 d^4 - 8 a^4 b^2 c^3 d^4 + a^3 b^6 d^2 e^2 + 32 a^6 c^3 d^2 e^2 - 2 a^3 b^5 c d^3 e - 32 a^5 b^c d^3 e + 16 a^5 b^3 c d e^3 - 32 a^6 b^c d^2 e^3 + 16 a^4 b^3 c^2 d^3 e - 6 a^4 b^4 c d^2 e^2))^{(1/2)}) * (- (b^7 e^2 + b^5 c^2 d^2 - b^4 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 7 a b^3 c^3 d^2 + 12 a^2 b^c d^2 + a c^3 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 20 a^3 b^c d^3 e^2 - 2 b^6 c d e + 25 a^2 b^3 c^2 e^2 - a^2 c^2 e^2 * (- (4 a c - b^2)^3)^{(1/2)} - b^2 c^2 d^2 * (- (4 a c - b^2)^3)^{(1/2)} - 9 a b^5 c e^2 + 16 a^3 c^4 d e + 16 a b^4 c^2 d e + 2 b^3 c d e * (- (4 a c - b^2)^3)^{(1/2)} + 3 a b^2 c e^2 * (- (4 a c - b^2)^3)^{(1/2)} - 36 a^2 b^2 c^3 d e - 4 a b^c d^2 e * (- (4 a c - b^2)^3)^{(1/2)}) / (8 (a^5 b^4 e^4 + 16 a^5 c^4 d^4 + 16 a^7 c^2 e^4 - 8 a^6 b^2 c e^4 - 2 a^4 b^5 d e^3 + a^3 b^4 c^2 d^4 - 8 a^4 b^2 c^3 d^4 + a^3 b^6 d^2 e^2 + 32 a^6 c^3 d^2 e^2 - 2 a^3 b^5 c d^3 e - 32 a^5 b^c d^3 e + 16 a^5 b^3 c d e^3 - 32 a^6 b^c d^2 e^3 + 16 a^4 b^3 c^2 d^3 e - 6 a^4 b^4 c d^2 e^2))^{(1/2)}) * 2i - (\log(c^6 d^{11} (-d^3 e^5)^{(1/2)} + b^6 d^6 e^9 x + c^6 d^{12} e^3 x + b^5 c d^3 (-d^3 e^5)^{(3/2)} - b^6 d^2 e (-d^3 e^5)^{(3/2)} - a^2 b^4 e^3 (-d^3 e^5)^{(3/2)} - 16 a^4 c^2 e^3 (-d^3 e^5)^{(3/2)} - 7 a b^3 c^2 d^3 (-d^3 e^5)^{(3/2)} + 12 a^2 b^c d^3 (-d^3 e^5)^{(3/2)} + 8 a^3 b^2 c e^3 (-d^3 e^5)^{(3/2)} + 16 a^3 c^3 d^2 e (-d^3 e^5)^{(3/2)} + a c^5 d^9 e^2 (-d^3 e^5)^{(1/2)} + a b^5 d^5 e^{10} x + a c^5 d^{10} e^5 x - b^c d^5 e^{11} x - b^5 c d^7 e^8 x + a^2 b^4 d^4 e^{11} x - 16 a^3 c^3 d^6 e^9 x + 16 a^4 c^2 d^4 e^{11} x - a b^5 d e^2 (-d^3 e^5)^{(3/2)} - b^c d^5 e^{10} (-d^3 e^5)^{(1/2)} - 24 a^2 b^2 c^2 d^2 e (-d^3 e^5)^{(3/2)} + 7 a b^3 c^2 d^7 e^8 x - 12 a^2 b^c d^7 e^8 x - 8 a^2 b^3 c d^5 e^{10} x + 16 a^3 b^c d^5 e^{10} x - 8 a^3 b^2 c d^4 e^{11} x + 9 a b^4 c d^2 e (-d^3 e^5)^{(3/2)} + 24 a^2 b^2 c^2 d^6 e^9 x + 8 a^2 b^3 c d e^2 (-d^3 e^5)^{(3/2)} - 16 a^3 b^c d^2 e^2 (-d^3 e^5)^{(3/2)} - 9 a b^4 c d^6 e^9 x) * (-d^3 e^5)^{(1/2)}) / (2 (c d^5 + a d^3 e^2 - b d^4 e)) + (\log(b^6 d^6 e^9 x - c^6 d^{11} (-d^3 e^5)^{(1/2)} + c^6 d^{12} e^3 x - b^5 c d^3 (-d^3 e^5)^{(3/2)} + b^6 d^2 e (-d^3 e^5)^{(3/2)} + a^2 b^4 e^3 (-d^3 e^5)^{(3/2)} + 16 a^4 c^2 e^3 (-d^3 e^5)^{(3/2)} + 7 a b^3 c^2 d^3 (-d^3 e^5)^{(3/2)} - 12 a^2 b^c d^3 (-d^3 e^5)^{(3/2)} - 8 a^3 b^2 c e^3 (-d^3 e^5)^{(3/2)} - 16 a^3 c^3 d^2 e (-d^3 e^5)^{(3/2)} - a c^5 d^9 e^2 (-d^3 e^5)^{(1/2)} + a b^5 d^5 e^{10} x + a c^5 d^{10} e^5 x - b^c d^5 e^{11} x - b^5 c d^7 e^8 x + a^2 b^4 d^4 e^{11} x - 16 a^3 c^3 d^6 e^9 x + 16 a^4 c^2 d^4 e^{11} x + a b^5 d e^2 (-d^3 e^5)^{(3/2)}
\end{aligned}$$

$$\begin{aligned} & ^5)^{(3/2)} + b*c^5*d^{10}*e*(-d^3*e^5)^{(1/2)} + 24*a^2*b^2*c^2*d^2*e*(-d^3*e^5) \\ & ^{(3/2)} + 7*a*b^3*c^2*d^7*e^8*x - 12*a^2*b*c^3*d^7*e^8*x - 8*a^2*b^3*c*d^5*e \\ & ^{10}*x + 16*a^3*b*c^2*d^5*e^{10}*x - 8*a^3*b^2*c*d^4*e^{11}*x - 9*a*b^4*c*d^2*e* \\ & (-d^3*e^5)^{(3/2)} + 24*a^2*b^2*c^2*d^6*e^9*x - 8*a^2*b^3*c*d*e^2*(-d^3*e^5)^{ \\ & (3/2)} + 16*a^3*b*c^2*d*e^2*(-d^3*e^5)^{(3/2)} - 9*a*b^4*c*d^6*e^9*x)*(-d^3*e^ \\ & 5)^{(1/2)))/(2*c*d^5 + 2*a*d^3*e^2 - 2*b*d^4*e) - 1/(a*d*x) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.231 \quad \int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=348

$$\frac{\sqrt{c} \left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) \right)}{\sqrt{2} a^2 \sqrt{b - \sqrt{b^2 - 4ac}} (ae^2 - bde + cd^2) + \sqrt{2} a^2 \sqrt{\sqrt{b^2 - 4ac} + b} (ae^2 - bde + cd^2)}$$

Rubi [A] time = 1.55, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c} \left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right) + \frac{ae+bd}{a^2 d^2 x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{c} x}{\sqrt{d}} \right)}{d^{5/2} (ae^2 - bde + cd^2)} - \frac{1}{3adx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] $-1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (\text{Sqrt}[c]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) + (\text{Sqrt}[c]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])*(c*d^2 - b*d*e + a*e^2) + (e^{7/2})*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]]/(d^{5/2})*(c*d^2 - b*d*e + a*e^2)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx &= \int \left(\frac{1}{adx^4} + \frac{-bd - ae}{a^2 d^2 x^2} + \frac{e^4}{d^2 (cd^2 - bde + ae^2) (d + ex^2)} + \frac{b^2 cd - ac^2 d - b^3 e}{a^2 (cd^2 - bde + ae^2)} \right) dx \\ &= -\frac{1}{3adx^3} + \frac{bd + ae}{a^2 d^2 x} + \frac{\int \frac{b^2 cd - ac^2 d - b^3 e + 2abce + c(bcd - b^2 e + ace)x^2}{a + bx^2 + cx^4} dx}{a^2 (cd^2 - bde + ae^2)} + \frac{e^4 \int \frac{1}{d + ex^2} dx}{d^2 (cd^2 - bde + ae^2)} \\ &= -\frac{1}{3adx^3} + \frac{bd + ae}{a^2 d^2 x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{d^{5/2} (cd^2 - bde + ae^2)} + \frac{\left(c (bcd - b^2 e + ace - \frac{b^2 cd - 2ac^2 d - b^3 e + 3abce}{\sqrt{b^2 - 4ac}}) \right) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{2a^2 (cd^2 - bde + ae^2)} \\ &= -\frac{1}{3adx^3} + \frac{bd + ae}{a^2 d^2 x} + \frac{\sqrt{c} \left(bcd - b^2 e + ace + \frac{b^2 cd - 2ac^2 d - b^3 e + 3abce}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{\sqrt{2} a^2 \sqrt{b - \sqrt{b^2 - 4ac}} (cd^2 - bde + ae^2)} \end{aligned}$$

Mathematica [A] time = 0.52, size = 410, normalized size = 1.18

$$\frac{\sqrt{c} \left(cd - e\sqrt{b^2 - 4ac} \right) + bc \left(d\sqrt{b^2 - 4ac} + 3ae \right) + ac \left(e\sqrt{b^2 - 4ac} - 2cd \right) + b^3(-e)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}} (e(ac - bd) + cd^2)} + \frac{\sqrt{c} \left(-b^2 \left(e\sqrt{b^2 - 4ac} + cd \right) + bc \left(d\sqrt{b^2 - 4ac} - 3ae \right) + ac \left(e\sqrt{b^2 - 4ac} + 2cd \right) + b^3 c \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{ex}}{\sqrt{b^2 - 4ac} + b} \right) + \frac{ae + bd}{a^2 d^2 x} + \frac{e^{7/2} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right)}{d^{5/2} (ae^2 - bde + cd^2)} - \frac{1}{3ad^3 x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out]
$$-1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (\text{Sqrt}[c]*(-(b^3*e) + b*c*(\text{Sqrt}[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - \text{Sqrt}[b^2 - 4*a*c]*e) + a*c*(-2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (\text{Sqrt}[c]*(b^3*e + b*c*(\text{Sqrt}[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + a*c*(2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2))*\text{ArcTan}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d]]/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 12.00, size = 12268, normalized size = 35.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\frac{1}{8} \left((2a^4b^5c^5 - 12a^5b^3c^6 + 16a^6b^2c^7 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c \right) a^4b^5c^3 + 6\sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^3c^4 + 2\sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4b^4c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^6b^2c^5 - 4\sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^4b^3c^5 + 2\sqrt{2}\sqrt{b^2 - 4ac} \sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^2c^6 - 2(b^2 - 4ac)a^4b^3c^5 + 4(b^2 - 4ac)a^5b^2c^6) d^5 - (6a^4b^6c^4 - 38a^5b^4c^5 + 56a^6b^2c^6 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c) a^4b^6c^2 + 19\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^4c^3 + 6\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4b^5c^3 - 28\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^6b^2c^4 - 14\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^3c^4 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^4b^4c^4 + 7\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^5b^2c^5 - 6(b^2 - 4ac)a^4b^4c^4 + 14(b^2 - 4ac)a^5b^2c^5) d^4 e + 2(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^2b^6c^2 - 9\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^3b^4c^3 - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c a^2$$

$$\begin{aligned}
& b^5c^3 - 2a^2b^6c^3 + 24\sqrt{2}\sqrt{b^2 - 4ac}c)a^4b^2c^4 + 10\sqrt{2}\sqrt{b^2 - 4ac}c)a^3b^3c^4 + \sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^4c^4 + 18a^3b^4c^4 - 16\sqrt{2}\sqrt{b^2 - 4ac}c)a^5c^5 - 8\sqrt{2}\sqrt{b^2 - 4ac}c)a^4b^2c^5 - 5\sqrt{2}\sqrt{b^2 - 4ac}c)a^3b^2c^5 - 48a^4b^2c^5 + 4\sqrt{2}\sqrt{b^2 - 4ac}c)a^4c^6 + 32a^5c^6 + 2(b^2 - 4ac)a^2b^4c^3 - 10(b^2 - 4ac)a^3b^2c^4 + 8(b^2 - 4ac)a^4c^5)d^3\text{abs}(a^2cd^2 - a^2bde + a^3e^2) + (6a^4b^7c^3 - 36a^5b^5c^4 + 40a^6b^3c^5 + 32a^7b^2c^6 - 3\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}c)a^4b^7c + 18\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^5b^5c^2 + 6\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^4b^6c^2 - 20\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^6b^3c^3 - 12\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^5b^4c^3 - 3\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^4b^5c^3 - 16\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^7b^2c^4 - 8\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^6b^2c^4 + 6\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^5b^3c^4 + 4\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^6b^2c^5 - 6(b^2 - 4ac)a^4b^5c^3 + 12(b^2 - 4ac)a^5b^3c^4 + 8(b^2 - 4ac)a^6b^2c^5)d^3e^2 - 2(2\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^7c - 19\sqrt{2}\sqrt{b^2 - 4ac}c)a^3b^5c^2 - 4\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^6c^2 - 4a^2b^7c^2 + 56\sqrt{2}\sqrt{b^2 - 4ac}c)a^4b^3c^3 + 22\sqrt{2}\sqrt{b^2 - 4ac}c)a^3b^4c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac}c)a^2b^5c^3 + 38a^3b^5c^3 - 48\sqrt{2}\sqrt{b^2 - 4ac}c)a^5b^2c^4 - 24\sqrt{2}\sqrt{b^2 - 4ac}c)a^4b^2c^4 - 11\sqrt{2}\sqrt{b^2 - 4ac}c)a^3b^3c^4 - 112a^4b^3c^4 + 12\sqrt{2}\sqrt{b^2 - 4ac}c)a^4b^2c^5 + 96a^5b^2c^5 + 4(b^2 - 4ac)a^2b^5c^2 - 22(b^2 - 4ac)a^3b^3c^3 + 24(b^2 - 4ac)a^4b^2c^4)d^2\text{abs}(a^2cd^2 - a^2bde + a^3e^2)e + (2b^5c^3 - 16a^2b^3c^4 + 32a^2b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}c)b^5c + 8\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^2b^3c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)b^4c^2 - 16\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^2b^2c^3 - 8\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^2b^2c^3 - \sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)b^3c^3 + 4\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^2b^2c^4 - 2(b^2 - 4ac)b^3c^3 + 8(b^2 - 4ac)a^2b^2c^4)(a^2cd^2 - a^2bde + a^3e^2)^2d - (2a^4b^8c^2 - 6a^5b^6c^3 - 28a^6b^4c^4 + 80a^7b^2c^5 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{b^2 - 4ac}c)a^4b^8 + 3\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^5b^6c + 2\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^4b^7c + 14\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^6b^4c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^5b^5c^2 - \sqrt{2}\sqrt{b^2 - 4ac}c)\sqrt{b^2 - 4ac}c)a^4b^6c^2
\end{aligned}$$

$$\begin{aligned}
& - 40\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^2c^3 \\
& - 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^3c^3 \\
& - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^4c^3 + 1 \\
& 0\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^6b^2c^4 - 2 \\
& *(b^2 - 4ac)a^4b^6c^2 - 2*(b^2 - 4ac)a^5b^4c^3 + 20*(b^2 - 4ac) \\
& *a^6b^2c^4)d^2e^3 + 2*(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^8 \\
& - 9\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^6c - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^2b^7c - 2a^2b^8c + 23\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^4c^2 + 10\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a \\
& ^3b^5c^2 + \sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^6c^2 + 18a^3b \\
& ^6c^2 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^2c^3 - 6\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^3c^3 - 5\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^4c^3 - 46a^4b^4c^3 - 16\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^6c^4 - 8\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^3c^4 + \\
& 3\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^2c^4 + 16a^5b^2c^4 + 4 \\
& *\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5c^5 + 32a^6c^5 + 2*(b^2 - 4a \\
& ac)a^2b^6c - 10*(b^2 - 4ac)a^3b^4c^2 + 6*(b^2 - 4ac)a^4b^2c^3 \\
& + 8*(b^2 - 4ac)a^5c^4)d*abs(a^2c*d^2 - a^2b*d*e + a^3*e^2)*e^2 - (2 \\
& *b^6c^2 - 18a*b^4c^3 + 48a^2b^2c^4 - 32a^3c^5 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *b^6 + 9\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a*b^4c + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *b^5c - 24\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2b^2c^2 - 10\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a*b^3c^2 - \sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *b^4c^2 + 16\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3c^3 + 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^2b^2c^3 + 5\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a*b^2c^3 - 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^2c^4 - 2*(b^2 - 4ac)*b^4c^2 + 10*(b^2 - 4ac) \\
& *a*b^2c^3 - 8*(b^2 - 4ac)a^2c^4)*(a^2c*d^2 - a^2b*d*e + a^3*e^2)^2*e + (4a^5b^7c^2 - 26a \\
& ^6b^5c^3 + 36a^7b^3c^4 + 16a^8b^3c^5 - 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^7 + 13\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^6b^5c + 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^5b^6c - 18\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^7b^3c^2 - 10\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^6b^4c^2 - 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^5b^5c^2 - 8\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^8b^3c^3 - 4\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^7b^2c^3 + 5\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^6b^3c^3 + 2\sqrt{2}\sqrt{b^2 - 4ac})\sqrt{bc + \sqrt{b^2 - 4ac}}c) \\
& *a^7b^3c^4 - 4*(b^2 - 4ac)a^5b^5c^2 + 10*(b^2 - 4ac)a^6b^3c^3 + 4*(b^2 - 4ac)a^7b^3c^4)d*e^4 - 2*(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^7 - 10\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^4b^5c - 2\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^3b^6c - 2a^3b^7c + 32\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4ac}}c)a^5b^3c^2 + 12\sqrt{2}\sqrt{bc}
\end{aligned}$$

$$\begin{aligned}
& + \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^4 c^2 + \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot c \\
& \cdot a^3 b^5 c^2 + 20 a^4 b^5 c^2 - 32 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot c \\
& \cdot a^6 b^3 c^3 - 16 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^2 c^3 - 6 \sqrt{2} \\
& \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^3 c^3 - 64 a^5 b^3 c^3 + 8 \sqrt{2} \\
& \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^4 c^4 + 64 a^6 b^4 c^4 + 2(b^2 - 4ac) \\
& \cdot a^3 b^5 c - 12(b^2 - 4ac) \cdot a^4 b^3 c^2 + 16(b^2 - 4ac) \cdot a^5 b^3 c^3 \cdot a \\
& b^5 (a^2 c^2 d^2 - a^2 b^2 d^2 e + a^3 e^2) \cdot e^3 - (2 a^6 b^6 c^2 - 14 a^7 b^4 c^3 + \\
& 24 a^8 b^2 c^4 - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c) \\
& \cdot a^6 b^6 + 7 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^4 c^3 \\
& + 2 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^5 c^2 \\
& - 12 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^2 c^2 \\
& - 6 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^3 c^3 \\
& - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^4 c^2 + \\
& 3 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^2 c^3 - \\
& 2(b^2 - 4ac) \cdot a^6 b^4 c^2 + 6(b^2 - 4ac) \cdot a^7 b^2 c^3 \cdot e^5 \cdot \arctan(2 \sqrt{2} \\
& \cdot \sqrt{1/2} \cdot x / \sqrt{(a^2 b^2 c^2 d^2 - a^2 b^2 d^2 e + a^3 b^2 e^2 + \sqrt{(a^2 b^2 c^2 d^2 - \\
& a^2 b^2 d^2 e + a^3 b^2 e^2)^2 - 4(a^3 c^2 d^2 - a^3 b^2 d^2 e + a^4 e^2) \cdot (a^2 c^2 d^2 \\
& d^2 - a^2 b^2 c^2 d^2 e + a^3 c^2 e^2))}) / (a^2 c^2 d^2 - a^2 b^2 c^2 d^2 e + a^3 c^2 e^2)) / \\
& ((a^5 b^4 c^2 - 8 a^6 b^2 c^3 - 2 a^5 b^3 c^3 + 16 a^7 c^4 + 8 a^6 b^3 c^4 + \\
& a^5 b^2 c^4 - 4 a^6 c^5) \cdot d^4 \cdot \text{abs}(a^2 c^2 d^2 - a^2 b^2 d^2 e + a^3 e^2) \cdot \text{abs}(c) - \\
& 2(a^5 b^5 c - 8 a^6 b^3 c^2 - 2 a^5 b^4 c^2 + 16 a^7 b^3 c^3 + 8 a^6 b^2 c^3 \\
& + a^5 b^3 c^3 - 4 a^6 b^3 c^4) \cdot d^3 \cdot \text{abs}(a^2 c^2 d^2 - a^2 b^2 d^2 e + a^3 e^2) \cdot \text{abs}(\\
& c) \cdot e + (a^5 b^6 - 6 a^6 b^4 c - 2 a^5 b^5 c + 4 a^6 b^3 c^2 + a^5 b^4 c^2 + \\
& 32 a^8 c^3 + 16 a^7 b^3 c^3 - 2 a^6 b^2 c^3 - 8 a^7 c^4) \cdot d^2 \cdot \text{abs}(a^2 c^2 d^2 - \\
& a^2 b^2 d^2 e + a^3 e^2) \cdot \text{abs}(c) \cdot e^2 - 2(a^6 b^5 - 8 a^7 b^3 c - 2 a^6 b^4 c + \\
& 16 a^8 b^3 c^2 + 8 a^7 b^2 c^2 + a^6 b^3 c^2 - 4 a^7 b^3 c^3) \cdot d \cdot \text{abs}(a^2 c^2 d^2 - \\
& a^2 b^2 d^2 e + a^3 e^2) \cdot \text{abs}(c) \cdot e^3 + (a^7 b^4 - 8 a^8 b^2 c - 2 a^7 b^3 c + \\
& 16 a^9 c^2 + 8 a^8 b^3 c^2 + a^7 b^2 c^2 - 4 a^8 c^3) \cdot \text{abs}(a^2 c^2 d^2 - a^2 b^2 d^2 \\
& e + a^3 e^2) \cdot \text{abs}(c) \cdot e^4) - 1/8 \cdot ((2 a^4 b^5 c^5 - 12 a^5 b^3 c^6 + 16 a^6 b^3 \\
& c^7 - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c) \cdot a^4 b^5 c^3 \\
& + 6 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^3 c^4 \\
& + 2 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^4 c^4 \\
& - 8 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^3 c^5 - 4 \\
& \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^2 c^5 - \sqrt{2} \\
& \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^3 c^5 + 2 \sqrt{2} \\
& \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^3 c^6 - 2(b^2 - \\
& 4ac) \cdot a^4 b^3 c^5 + 4(b^2 - 4ac) \cdot a^5 b^3 c^6) \cdot d^5 - (6 a^4 b^6 c^4 - 38 a^5 \\
& b^4 c^5 + 56 a^6 b^2 c^6 - 3 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^4 b^6 c^2 + 19 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^5 b^4 c^3 + 6 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^4 b^5 c^3 - 28 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^6 b^2 c^4 - 14 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^5 b^3 c^4 - 3 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^4 b^4 c^4 + 7 \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{b^2 - 4ac} \cdot c \cdot \sqrt{b^2 - 4ac} \\
& \cdot c) \cdot a^5 b^2 c^5 - 6(b^2 - 4ac) \cdot a^4 b^4 c^4 + 14(b^2 - 4ac) \cdot a
\end{aligned}$$

$$\begin{aligned}
& ^5b^2c^5)d^4e - 2*(\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^6*c^2 \\
& - 9*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^4*c^3 - 2*\text{sqrt}(2)*\text{sqrt}(b* \\
& c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^3 + 2*a^2*b^6*c^3 + 24*\text{sqrt}(2)*\text{sqrt}(b*c \\
& - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^2*c^4 + 10*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c) \\
&)*c)*a^3*b^3*c^4 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^4*c^4 - 18 \\
& *a^3*b^4*c^4 - 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*c^5 - 8*\text{sqrt}(\\
& 2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b*c^5 - 5*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^ \\
& 2 - 4*a*c))*c)*a^3*b^2*c^5 + 48*a^4*b^2*c^5 + 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 \\
& - 4*a*c))*c)*a^4*c^6 - 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^2*b^4*c^3 + 10*(b^2 - \\
& 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e \\
& + a^3*e^2) + (6*a^4*b^7*c^3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c \\
& ^6 - 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^7*c \\
& + 18*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^5*c^2 \\
& + 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^6*c^2 - \\
& 20*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^3*c^3 - \\
& 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^4*c^3 - \\
& 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^5*c^3 - \\
& 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^7*b*c^4 - 8* \\
& \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b^2*c^4 + 6*s \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^3*c^4 + 4*s \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^6*b*c^5 - 6*(b^2 \\
& - 4*a*c)*a^4*b^5*c^3 + 12*(b^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b \\
& *c^5)*d^3*e^2 + 2*(2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^7*c - 19 \\
& *\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^3*b^5*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^6*c^2 + 4*a^2*b^7*c^2 + 56*\text{sqrt}(2)*\text{sqrt}(b*c - s \\
& \text{qrt}(b^2 - 4*a*c))*c)*a^4*b^3*c^3 + 22*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c \\
&)*a^3*b^4*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b^5*c^3 - 38* \\
& a^3*b^5*c^3 - 48*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b*c^4 - 24*s \\
& \text{qrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^2*c^4 - 11*\text{sqrt}(2)*\text{sqrt}(b*c - s \\
& \text{qrt}(b^2 - 4*a*c))*c)*a^3*b^3*c^4 + 112*a^4*b^3*c^4 + 12*\text{sqrt}(2)*\text{sqrt}(b*c - s \\
& \text{qrt}(b^2 - 4*a*c))*c)*a^4*b*c^5 - 96*a^5*b*c^5 - 4*(b^2 - 4*a*c)*a^2*b^5*c^2 + \\
& 22*(b^2 - 4*a*c)*a^3*b^3*c^3 - 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d \\
& ^2 - a^2*b*d*e + a^3*e^2)*e + (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - s \\
& \text{qrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^5*c + 8*\text{sqrt}(2)*s \\
& \text{qrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c^2 + 2*\text{sqrt}(2)*\text{sqrt} \\
& (b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^4*c^2 - 16*\text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^2*b*c^3 - 8*\text{sqrt}(2)*\text{sqrt}(b^2 - \\
& 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c \\
&)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^3*c^3 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4 \\
& *a*c)*a*b*c^4)*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a \\
& ^5*b^6*c^3 - 28*a^6*b^4*c^4 + 80*a^7*b^2*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*s \\
& \text{qrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^8 + 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b* \\
& c - \text{sqrt}(b^2 - 4*a*c))*c)*a^5*b^6*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c))*c)*a^4*b^7*c + 14*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(b^2 - 4ac) * c * a^6 b^4 c^2 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^5 c^2 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^4 b^6 c^2 - 40 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^7 b^2 c^3 - 20 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 b^3 c^3 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^4 c^3 + 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 b^2 c^4 - 2 * (b^2 - 4ac) * a^4 b^6 c^2 - 2 * (b^2 - 4ac) * a^5 b^4 c^3 + 20 * (b^2 - 4ac) * a^6 b^2 c^4) * d^2 e^3 - 2 * (\text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 b^8 - 9 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^3 b^6 c - 2 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 b^7 c + 2 * a^2 b^8 c + 23 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^4 b^4 c^2 + 10 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^3 b^5 c^2 + \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 b^6 c^2 - 18 * a^3 b^6 c^2 - 8 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^2 c^3 - 6 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^4 b^3 c^3 - 5 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^3 b^4 c^3 + 46 * a^4 b^4 c^3 - 16 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 c^4 - 8 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^2 c^4 + 3 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^4 b^2 c^4 - 16 * a^5 b^2 c^4 + 4 * \text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 c^5 - 32 * a^6 c^5 - 2 * (b^2 - 4ac) * a^2 b^6 c + 10 * (b^2 - 4ac) * a^3 b^4 c^2 - 6 * (b^2 - 4ac) * a^4 b^2 c^3 - 8 * (b^2 - 4ac) * a^5 c^4) * d * \text{abs}(a^2 c * d^2 - a^2 b * d * e + a^3 e^2) * e^2 - (2 * b^6 c^2 - 18 * a * b^4 c^3 + 48 * a^2 b^2 c^4 - 32 * a^3 c^5 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * b^6 + 9 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a * b^4 c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * b^5 c - 24 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 b^2 c^2 - 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a * b^3 c^2 - \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * b^4 c^2 + 16 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^3 c^3 + 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 b^2 c^3 + 5 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a * b^2 c^3 - 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^2 c^4 - 2 * (b^2 - 4ac) * b^4 c^2 + 10 * (b^2 - 4ac) * a * b^2 c^3 - 8 * (b^2 - 4ac) * a^2 c^4) * (a^2 c * d^2 - a^2 b * d * e + a^3 e^2) * e^2 + (4 * a^5 b^7 c^2 - 26 * a^6 b^5 c^3 + 36 * a^7 b^3 c^4 + 16 * a^8 b^2 c^5 - 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^7 + 13 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 b^5 c + 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^6 c - 18 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^7 b^3 c^2 - 10 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 b^4 c^2 - 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^5 b^5 c^2 - 8 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^8 b^2 c^3 - 4 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^7 b^2 c^3 + 5 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^6 b^3 c^3 + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4ac) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^7 b^2 c^3 - 4 * (b^2 - 4ac) * a^5 b^5 c^2 + 10 * (b^2 - 4ac) * a^6 b^3 c^3 + 4 * (b^2 - 4ac) * a^7 b^2 c^3) * d * e^4 + 2 * (\text{sqrt}(2) * \text{sqrt}(bc - \text{sqrt}(b^2 - 4ac) * c) * a^3 b^7 - 10 * \text{sqrt}(2) * \text{sqrt}(bc -
\end{aligned}$$

$$\begin{aligned} & \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^5 c - 2\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 b^6 c + 2a^3 b^7 c + 32\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^3 c^2 + 12\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^4 c^2 + \sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^3 b^5 c^2 - 20a^4 b^5 c^2 - 32\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^3 c^3 - 6\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^4 b^3 c^3 + 64a^5 b^3 c^3 + 8\sqrt{2} \sqrt{b^2 - 4ac} \cdot c \cdot a^5 b^3 c^4 - 64a^6 b^3 c^4 - 2(b^2 - 4ac) a^3 b^5 c + 12(b^2 - 4ac) a^4 b^3 c^2 - 16(b^2 - 4ac) a^5 b^3 c^3 \cdot \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) e^3 - (2a^6 b^6 c^2 - 14a^7 b^4 c^3 + 24a^8 b^2 c^4 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^6 + 7\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^4 c + 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^5 c - 12\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^8 b^2 c^2 - 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^3 c^2 - \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^6 b^4 c^2 + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b^2 - 4ac} \cdot c \cdot a^7 b^2 c^3 - 2(b^2 - 4ac) a^6 b^4 c^2 + 6(b^2 - 4ac) a^7 b^2 c^3) e^5 \cdot \arctan(2\sqrt{1/2} x / \sqrt{(a^2 b c d^2 - a^2 b^2 d e + a^3 b e^2)^2 - 4(a^3 c d^2 - a^3 b d e + a^4 e^2)(a^2 c^2 d^2 - a^2 b c d e + a^3 c e^2)}) / ((a^5 b^4 c^2 - 8a^6 b^2 c^3 - 2a^5 b^3 c^3 + 16a^7 c^4 + 8a^6 b c^4 + a^5 b^2 c^4 - 4a^6 c^5) d^4 \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) \text{abs}(c) - 2(a^5 b^5 c - 8a^6 b^3 c^2 - 2a^5 b^4 c^2 + 16a^7 b c^3 + 8a^6 b^2 c^3 + a^5 b^3 c^3 - 4a^6 b c^4) d^3 \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) \text{abs}(c) e + (a^5 b^6 - 6a^6 b^4 c - 2a^5 b^5 c + 4a^6 b^3 c^2 + a^5 b^4 c^2 + 32a^8 c^3 + 16a^7 b c^3 - 2a^6 b^2 c^3 - 8a^7 c^4) d^2 \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) \text{abs}(c) e^2 - 2(a^6 b^5 - 8a^7 b^3 c - 2a^6 b^4 c + 16a^8 b c^2 + 8a^7 b^2 c^2 + a^6 b^3 c^2 - 4a^7 b c^3) d \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) \text{abs}(c) e^3 + (a^7 b^4 - 8a^8 b^2 c - 2a^7 b^3 c + 16a^9 c^2 + 8a^8 b c^2 + a^7 b^2 c^2 - 4a^8 c^3) \text{abs}(a^2 c d^2 - a^2 b d e + a^3 e^2) \text{abs}(c) e^4) + \arctan(x e^{1/2} / \sqrt{d}) e^{7/2} / ((c d^4 - b d^3 e + a d^2 e^2) \sqrt{d}) + 1/3(3 b d x^2 + 3 a x^2 e - a d) / (a^2 d^2 x^3) \end{aligned}$$

maple [B] time = 0.04, size = 1160, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a), x)$

[Out]
$$\begin{aligned} & -1/3/a/d/x^3 + 1/a/d^2 e/x + 1/d/a^2/x b - 1/2/(a e^2 - b d e + c d^2)/a c^2 2^{1/2} / \\ & ((-b + (-4 a c + b^2)^{1/2}) c)^{1/2} \cdot \text{arctanh}(2^{1/2} / ((-b + (-4 a c + b^2)^{1/2}) c)^{1/2} c x) e + 1/2/(a e^2 - b d e + c d^2)/a^2 c^2 2^{1/2} / ((-b + (-4 a c + b^2)^{1/2}) c)^{1/2} \cdot \text{arctanh}(2^{1/2} / ((-b + (-4 a c + b^2)^{1/2}) c)^{1/2} c x) b^2 e - 1 \end{aligned}$$

$$\begin{aligned} & /2/(a^2e^2-bde+cd^2)/a^2c^22^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2d-3/2/(a^2e^2-bde+cd^2)/a^2c^2/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2e+1/(a^2e^2-bde+cd^2)/a^2c^3/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * d+1/2/(a^2e^2-bde+cd^2)/a^2c/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^3e-1/2/(a^2e^2-bde+cd^2)/a^2c^2/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2d+1/2/(a^2e^2-bde+cd^2)/a^2c^22^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * e-1/2/(a^2e^2-bde+cd^2)/a^2c^22^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2e+1/2/(a^2e^2-bde+cd^2)/a^2c^22^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2d-3/2/(a^2e^2-bde+cd^2)/a^2c^2/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2e+1/(a^2e^2-bde+cd^2)/a^2c^3/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * d+1/2/(a^2e^2-bde+cd^2)/a^2c/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^3e-1/2/(a^2e^2-bde+cd^2)/a^2c^2/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c^2x) * b^2d+1/d^2e^4/(a^2e^2-bde+cd^2)/(de)^{(1/2)}\operatorname{arctan}(1/(de)^{(1/2)}e^2x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] $e^4 \operatorname{arctan}(ex/\sqrt{de})/((cd^4 - bd^3e + ad^2e^2)\sqrt{de}) + \operatorname{integrate}(((b^2c^2d - (b^2c - ac^2)e)x^2 + (b^2c - ac^2)d - (b^3 - 2ab^2c)e)/(c^2x^4 + b^2x^2 + a), x)/(a^2cd^2 - a^2bde + a^3e^2) + 1/3(3(b^2d + ae)x^2 - ad)/(a^2d^2x^3)$

mupad [B] time = 6.73, size = 42882, normalized size = 123.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)

[Out] $(\log(c^9d^{27}e^6 - b^9d^{18}e^{15} + 2a^2c^8d^{25}e^8 - 2b^2c^8d^{26}e^7 + 2b^8c^8d^{19}e^{14} + a^5b^4d^{13}e^{20} + a^2c^7d^{23}e^{10} + 16a^4c^5d^{19}e^4)) / (c^9d^{27}e^6 - b^9d^{18}e^{15} + 2a^2c^8d^{25}e^8 - 2b^2c^8d^{26}e^7 + 2b^8c^8d^{19}e^{14} + a^5b^4d^{13}e^{20} + a^2c^7d^{23}e^{10} + 16a^4c^5d^{19}e^4)$

$$\begin{aligned}
& e^{14} + 16a^7c^2d^{13}e^{20} + b^2c^7d^{25}e^8 - b^7c^2d^{20}e^{13} - 25a^2 \\
& *b^3c^4d^{20}e^{13} + 66a^2b^4c^3d^{19}e^{14} - 42a^2b^5c^2d^{18}e^{15} - \\
& 76a^3b^2c^4d^{19}e^{14} + 63a^3b^3c^3d^{18}e^{15} - a^5b^4e^3x*(-d^5e^7)^{(5/2)} + a^2c^7d^{15}x*(-d^5e^7)^{(3/2)} - 16a^7c^2e^3x*(-d^5e^7)^{(5/2)} - \\
& b^9d^{10}e^5x*(-d^5e^7)^{(3/2)} - c^9d^{24}e^3x*(-d^5e^7)^{(1/2)} - \\
& 2a*b*c^7*d^{24}*e^9 + 11a*b^7*c*d^{18}*e^{15} + 9a*b^5*c^3*d^{20}*e^{13} - 20a*b^6*c^2*d^{19}*e^{14} + 20a^3*b*c^5*d^{20}*e^{13} - 28a^4*b*c^4*d^{18}*e^{15} - 8a^6*b^2*c*d^{13}*e^{20} + 16a^4*c^5*d^{11}*e^4x*(-d^5e^7)^{(3/2)} - b^7*c^2*d^{12}*e^3x*(-d^5e^7)^{(3/2)} - b^2*c^7*d^{22}*e^5x*(-d^5e^7)^{(1/2)} + 8a^6*b^2*c*e^3x*(-d^5e^7)^{(5/2)} - 2a*c^8*d^{22}*e^5x*(-d^5e^7)^{(1/2)} + 2b^8*c*d^{11}*e^4x*(-d^5e^7)^{(3/2)} + 2b*c^8*d^{23}*e^4x*(-d^5e^7)^{(1/2)} + 11a*b^7*c*d^{10}*e^5x*(-d^5e^7)^{(3/2)} + 2a*b*c^7*d^{21}*e^6x*(-d^5e^7)^{(1/2)} + 9a*b^5*c^3*d^{12}*e^3x*(-d^5e^7)^{(3/2)} - 20a*b^6*c^2*d^{11}*e^4x*(-d^5e^7)^{(3/2)} + 20a^3*b*c^5*d^{12}*e^3x*(-d^5e^7)^{(3/2)} - 28a^4*b*c^4*d^{10}*e^5x*(-d^5e^7)^{(3/2)} - 25a^2*b^3*c^4*d^{12}*e^3x*(-d^5e^7)^{(3/2)} + 66a^2*b^4*c^3*d^{11}*e^4x*(-d^5e^7)^{(3/2)} - 42a^2*b^5*c^2*d^{10}*e^5x*(-d^5e^7)^{(3/2)} - 76a^3*b^2*c^4*d^{11}*e^4x*(-d^5e^7)^{(3/2)} + 63a^3*b^3*c^3*d^{10}*e^5x*(-d^5e^7)^{(3/2))}*(-d^5e^7)^{(1/2)))/(2*c*d^7 + 2*a*d^5*e^2 - 2*b*d^6*e) - atan((((- (b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9a*b^5*c^3*d^2 - 20a^3*b*c^5*d^2 + 28a^4*b*c^4*e^2 - 2b^8*c*d*e + 25a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42a^2*b^5*c^2*e^2 - 63a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11a*b^7*c*e^2 - 16a^4*c^5*d*e + 20a*b^6*c^2*d*e + 2b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66a^2*b^4*c^3*d*e + 76a^3*b^2*c^4*d*e + 3a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^7*b^4*e^4 + 16a^7*c^4*d^4 + 16a^9*c^2*e^4 - 8a^8*b^2*c*e^4 - 2a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32a^8*c^3*d^2*e^2 - 2a^5*b^5*c*d^3*e - 32a^7*b*c^3*d^3*e + 16a^7*b^3*c*d*e^3 - 32a^8*b*c^2*d*e^3 + 16a^6*b^3*c^2*d^3*e - 6a^6*b^4*c*d^2*e^2)))^{(1/2)}*(((-b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9a*b^5*c^3*d^2 - 20a^3*b*c^5*d^2 + 28a^4*b*c^4*e^2 - 2b^8*c*d*e + 25a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42a^2*b^5*c^2*e^2 - 63a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11a*b^7*c*e^2 - 16a^4*c^5*d*e + 20a*b^6*c^2*d*e + 2b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66a^2*b^4*c^3*d*e + 76a^3*b^2*c^4*d*e + 3a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^7*b^4*e^4 + 16a^7*c^4*d^4 + 16a^9*c^2*e^4 - 8a^8*b^2*c*e^4 - 2a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32a^8*c^3*d^2*e^2 - 2a^5*b^5*c*d^3*e - 32a^7*b*c^3*d^3*e + 16a^7*b^3*c*d*e^3 - 32a^8*b*c^2*d*e^3 + 16a^6*b^3*c^2*d^3*e - 6a^6*b^4*c*d^2*e^2)))^{(1/2)}*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9a*b^5*c^3*d^2 - 20a^3*b*c^5*d^2 + 28
\end{aligned}$$

$$\begin{aligned}
& a^4 b^3 c^4 e^2 - 2 b^8 c^3 d e + 25 a^2 b^3 c^4 d^2 - a^2 c^4 d^2 (- (4 a^3 c - b^2)^3)^{1/2} + 42 a^2 b^5 c^2 e^2 - 63 a^3 b^3 c^3 e^2 + a^3 c^3 e^2 (- (4 a^3 c - b^2)^3)^{1/2} - b^4 c^2 d^2 (- (4 a^3 c - b^2)^3)^{1/2} - 11 a^7 b^3 c^3 e^2 \\
& - 16 a^4 c^5 d e + 20 a^2 b^6 c^2 d e + 2 b^5 c^3 d e (- (4 a^3 c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e^2 (- (4 a^3 c - b^2)^3)^{1/2} + 5 a^2 b^4 c^2 e^2 (- (4 a^3 c - b^2)^3)^{1/2} - 66 a^2 b^4 c^3 d e + 76 a^3 b^2 c^4 d e + 3 a^2 b^2 c^3 d^2 (- (4 a^3 c - b^2)^3)^{1/2} \\
& - 8 a^2 b^3 c^2 d e (- (4 a^3 c - b^2)^3)^{1/2} + 6 a^2 b^3 c^3 d e (- (4 a^3 c - b^2)^3)^{1/2} / (8 (a^7 b^4 e^4 + 16 a^7 c^4 d^4 + 16 a^9 c^2 e^4 - 8 a^8 b^2 c^2 e^4 - 2 a^6 b^5 d e^3 + a^5 b^4 c^2 d^4 - 8 a^6 b^2 c^3 d^4 + a^5 b^6 d^2 e^2 + 32 a^8 c^3 d^2 e^2 - 2 a^5 b^5 c^3 d^3 e - 32 a^7 b^3 c^3 d^3 e + 16 a^7 b^3 c^3 d^3 e^3 - 32 a^8 b^2 c^2 d^3 e^3 + 16 a^6 b^3 c^2 d^3 e^3 - 6 a^6 b^4 c^2 d^2 e^2))^{1/2} \\
& * (512 a^{20} c^7 d^{24} e^3 + 512 a^{21} c^6 d^{22} e^5 - 512 a^{22} c^5 d^{20} e^7 - 512 a^{23} c^4 d^{18} e^9 - 32 a^{18} b^3 c^6 d^{25} e^2 + 128 a^{18} b^4 c^5 d^{24} e^3 - 192 a^{18} b^5 c^4 d^{23} e^4 + 128 a^{18} b^6 c^3 d^{22} e^5 - 32 a^{18} b^7 c^2 d^{21} e^6 - 640 a^{19} b^2 c^6 d^{24} e^3 + 105 \\
& 6 a^{19} b^3 c^5 d^{23} e^4 - 672 a^{19} b^4 c^4 d^{22} e^5 + 96 a^{19} b^5 c^3 d^{21} e^6 + 32 a^{19} b^6 c^2 d^{20} e^7 + 512 a^{20} b^2 c^5 d^{22} e^5 + 288 a^{20} b^3 c^4 d^{21} e^6 - 192 a^{20} b^4 c^3 d^{20} e^7 + 32 a^{20} b^5 c^2 d^{19} e^8 + 384 a^{21} b^2 c^4 d^{20} e^7 - 288 a^{21} b^3 c^3 d^{19} e^8 - 32 a^{21} b^4 c^2 d^{18} e^9 \\
& + 256 a^{22} b^2 c^3 d^{18} e^9 + 128 a^{19} b^3 c^7 d^{25} e^2 - 1152 a^{20} b^3 c^6 d^{23} e^4 - 640 a^{21} b^3 c^5 d^{21} e^6 + 640 a^{22} b^3 c^4 d^{19} e^8) - 64 a^{18} c^8 d^{24} e^2 + 128 a^{19} c^7 d^{22} e^4 + 192 a^{20} c^6 d^{20} e^6 - 256 a^{21} c^5 d^{18} e^8 - 256 a^{22} c^4 d^{16} e^{10} - 16 a^{16} b^4 c^6 d^{24} e^2 + 64 a^{16} b^5 c^5 d^{23} e^3 - 96 a^{16} b^6 c^4 d^{22} e^4 + 64 a^{16} b^7 c^3 d^{21} e^5 - 16 a^{16} b^8 c^2 d^{20} e^6 + 80 a^{17} b^2 c^7 d^{24} e^2 - 368 a^{17} b^3 c^6 d^{23} e^3 + 608 a^{17} b^4 c^5 d^{22} e^4 - 416 a^{17} b^5 c^4 d^{21} e^5 + 80 a^{17} b^6 c^3 d^{20} e^6 + 16 a^{17} b^7 c^2 d^{19} e^7 - 928 a^{18} b^2 c^6 d^{22} e^4 + 640 a^{18} b^3 c^5 d^{21} e^5 + 32 a^{18} b^4 c^4 d^{20} e^6 - 128 a^{18} b^5 c^3 d^{19} e^7 - 432 a^{19} b^2 c^5 d^{20} e^6 + 304 a^{19} b^3 c^4 d^{19} e^7 - 16 a^{19} b^4 c^3 d^{18} e^8 + 16 a^{19} b^5 c^2 d^{17} e^9 + 128 a^{20} b^2 c^4 d^{18} e^8 - 128 a^{20} b^3 c^3 d^{17} e^9 - 16 a^{20} b^4 c^2 d^{16} e^{10} + 128 a^{21} b^2 c^3 d^{16} e^{10} + 448 a^{18} b^3 c^7 d^{23} e^3 - 192 a^{20} b^3 c^5 d^{19} e^7 + 256 a^{21} b^3 c^4 d^{17} e^9) - x (16 a^{16} c^9 d^{23} e^2 + 32 a^{17} c^8 d^{21} e^4 - 112 a^{18} c^7 d^{19} e^6 - 128 a^{20} c^5 d^{15} e^{10} + 8 a^{14} b^4 c^7 d^{23} e^2 - 16 a^{14} b^5 c^6 d^{22} e^3 + 8 a^{14} b^6 c^5 d^{21} e^4 + 8 a^{14} b^7 c^4 d^{20} e^5 - 16 a^{14} b^8 c^3 d^{19} e^6 + 8 a^{14} b^9 c^2 d^{18} e^7 - 32 a^{15} b^2 c^8 d^{23} e^2 + 64 a^{15} b^3 c^7 d^{22} e^3 - 16 a^{15} b^4 c^6 d^{21} e^4 - 88 a^{15} b^5 c^5 d^{20} e^5 + 160 a^{15} b^6 c^4 d^{19} e^6 - 88 a^{15} b^7 c^3 d^{18} e^7 - 48 a^{16} b^2 c^7 d^{21} e^4 + 264 a^{16} b^3 c^6 d^{20} e^5 - 520 a^{16} b^4 c^5 d^{19} e^6 + 336 a^{16} b^5 c^4 d^{18} e^7 + 576 a^{17} b^2 c^6 d^{19} e^6 - 504 a^{17} b^3 c^5 d^{18} e^7 + 8 a^{18} b^3 c^4 d^{16} e^9 - 16 a^{18} b^4 c^3 d^{15} e^{10} + 8 a^{18} b^5 c^2 d^{14} e^{11} + 96 a^{19} b^2 c^4 d^{15} e^{10} - 56 a^{19} b^3 c^3 d^{14} e^{11} - 32 a^{16} b^3 c^8 d^{22} e^3 - 192 a^{17} b^3 c^7 d^{20} e^5 + 224 a^{18} b^3 c^6 d^{18} e^7 - 32 a^{19} b^3 c^5 d^{16} e^9 + 96 a^{20} b^3 c^4 d^{14} e^{11})) * (- (b^9 e^2 + b^7 c^2 d^2 - b^6 e^2 (- (4 a^3 c - b^2)^3)^{1/2} - 9 a^2 b^5 c^3 d^2 - 20 a^3 b^3 c^5 d^2 + 28 a^4 b^3 c^4 e^2 - 2 b^8 c^3 d e
\end{aligned}$$

$$\begin{aligned}
&^3)^{(1/2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c \\
&*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b \\
&^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^ \\
&4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a \\
&*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e^2*(-(\\
&4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4 \\
&*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^ \\
&3)^{(1/2)))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e \\
&^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^ \\
&2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^ \\
&3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2 \\
&)))^{(1/2)}*(x*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^ \\
&2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - \\
&63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*d^2*(-(\\
&4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e \\
&+ 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2) \\
&^3)^{(1/2)} + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 7 \\
&6*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2* \\
&d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8 \\
&*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b \\
&^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c \\
&^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - \\
&32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1/2)}*(5 \\
&12*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^22*e^5 - 512*a^22*c^5*d^20*e^7 - 512* \\
&a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^25*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - \\
&192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^2 \\
&1*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 1056*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b \\
&^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512 \\
&*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20* \\
&e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c \\
&^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9 + 256*a^22*b^2*c^3*d^18*e^9 + 128*a^ \\
&19*b*c^7*d^25*e^2 - 1152*a^20*b*c^6*d^23*e^4 - 640*a^21*b*c^5*d^21*e^6 + 64 \\
&0*a^22*b*c^4*d^19*e^8) + 64*a^18*c^8*d^24*e^2 - 128*a^19*c^7*d^22*e^4 - 192 \\
&*a^20*c^6*d^20*e^6 + 256*a^21*c^5*d^18*e^8 + 256*a^22*c^4*d^16*e^10 + 16*a^ \\
&16*b^4*c^6*d^24*e^2 - 64*a^16*b^5*c^5*d^23*e^3 + 96*a^16*b^6*c^4*d^22*e^4 - \\
&64*a^16*b^7*c^3*d^21*e^5 + 16*a^16*b^8*c^2*d^20*e^6 - 80*a^17*b^2*c^7*d^24 \\
&*e^2 + 368*a^17*b^3*c^6*d^23*e^3 - 608*a^17*b^4*c^5*d^22*e^4 + 416*a^17*b^5 \\
&*c^4*d^21*e^5 - 80*a^17*b^6*c^3*d^20*e^6 - 16*a^17*b^7*c^2*d^19*e^7 + 928*a \\
&^18*b^2*c^6*d^22*e^4 - 640*a^18*b^3*c^5*d^21*e^5 - 32*a^18*b^4*c^4*d^20*e^6 \\
&+ 128*a^18*b^5*c^3*d^19*e^7 + 432*a^19*b^2*c^5*d^20*e^6 - 304*a^19*b^3*c^4 \\
&*d^19*e^7 + 16*a^19*b^4*c^3*d^18*e^8 - 16*a^19*b^5*c^2*d^17*e^9 - 128*a^20* \\
&b^2*c^4*d^18*e^8 + 128*a^20*b^3*c^3*d^17*e^9 + 16*a^20*b^4*c^2*d^16*e^10 - \\
&128*a^21*b^2*c^3*d^16*e^10 - 448*a^18*b*c^7*d^23*e^3 + 192*a^20*b*c^5*d^19*
\end{aligned}$$

$$\begin{aligned}
& e^7 - 256a^{21}b^4c^4d^{17}e^9) - x(16a^{16}c^9d^{23}e^2 + 32a^{17}c^8d^{21} \\
& *e^4 - 112a^{18}c^7d^{19}e^6 - 128a^{20}c^5d^{15}e^{10} + 8a^{14}b^4c^7d^{23} \\
& *e^2 - 16a^{14}b^5c^6d^{22}e^3 + 8a^{14}b^6c^5d^{21}e^4 + 8a^{14}b^7c^4 \\
& d^{20}e^5 - 16a^{14}b^8c^3d^{19}e^6 + 8a^{14}b^9c^2d^{18}e^7 - 32a^{15}b^2 \\
& *c^8d^{23}e^2 + 64a^{15}b^3c^7d^{22}e^3 - 16a^{15}b^4c^6d^{21}e^4 - 88a^{15} \\
& b^5c^5d^{20}e^5 + 160a^{15}b^6c^4d^{19}e^6 - 88a^{15}b^7c^3d^{18}e^7 \\
& - 48a^{16}b^2c^7d^{21}e^4 + 264a^{16}b^3c^6d^{20}e^5 - 520a^{16}b^4c^5d \\
& ^{19}e^6 + 336a^{16}b^5c^4d^{18}e^7 + 576a^{17}b^2c^6d^{19}e^6 - 504a^{17} \\
& b^3c^5d^{18}e^7 + 8a^{18}b^3c^4d^{16}e^9 - 16a^{18}b^4c^3d^{15}e^{10} + 8 \\
& a^{18}b^5c^2d^{14}e^{11} + 96a^{19}b^2c^4d^{15}e^{10} - 56a^{19}b^3c^3d^{14}e \\
& ^{11} - 32a^{16}b^4c^8d^{22}e^3 - 192a^{17}b^5c^7d^{20}e^5 + 224a^{18}b^6c^6d^{18} \\
& e^7 - 32a^{19}b^7c^5d^{16}e^9 + 96a^{20}b^8c^4d^{14}e^{11})) * (- (b^9e^2 + b^7 \\
& *c^2d^2 - b^6e^2 * (- (4ac - b^2)^3)^{(1/2)} - 9a^2b^5c^3d^2 - 20a^3b^4c^5 \\
& d^2 + 28a^4b^3c^4e^2 - 2b^8c^4d^2 + 25a^2b^3c^4d^2 - a^2c^4d^2 * (- \\
& (4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 + a^3c^3 \\
& *e^2 * (- (4ac - b^2)^3)^{(1/2)} - b^4c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} - 11a \\
& *b^7c^4e^2 - 16a^4c^5d^2 + 20a^2b^6c^2d^2 + 2b^5c^4d^2 * (- (4ac - b^2 \\
&)^3)^{(1/2)} - 6a^2b^2c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + 5a^2b^4c^2e^2 * (- \\
& (4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^2 + 76a^3b^2c^4d^2 + 3a^2b^2c^3 \\
& d^2 * (- (4ac - b^2)^3)^{(1/2)} - 8a^2b^3c^2d^2 * (- (4ac - b^2)^3)^{(1/2)} \\
& + 6a^2b^3c^3d^2 * (- (4ac - b^2)^3)^{(1/2))} / (8(a^7b^4e^4 + 16a^7c^4d^4 \\
& + 16a^9c^2e^4 - 8a^8b^2c^4e^4 - 2a^6b^5d^3e^3 + a^5b^4c^2d^4 - \\
& 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e \\
& - 32a^7b^3c^3d^3e + 16a^7b^3c^3d^3e^3 - 32a^8b^3c^2d^3e^3 + 16a^6b^3 \\
& c^2d^3e^3 - 6a^6b^4c^2d^2e^2))^{(1/2)} + 4a^{15}c^9d^{21}e^3 + 4a^{16} \\
& c^8d^{19}e^5 - 48a^{18}c^6d^{15}e^9 + 4a^{14}b^2c^8d^{21}e^3 + 4a^{14}b^7c^3 \\
& d^{16}e^8 - 4a^{14}b^8c^2d^{15}e^9 - 36a^{15}b^5c^4d^{16}e^8 + 44a^{15} \\
& b^6c^3d^{15}e^9 - 4a^{15}b^7c^2d^{14}e^{10} + 100a^{16}b^3c^5d^{16}e^8 - \\
& 160a^{16}b^4c^4d^{15}e^9 + 32a^{16}b^5c^3d^{14}e^{10} + 204a^{17}b^2c^5d^{15} \\
& e^9 - 76a^{17}b^3c^4d^{14}e^{10} - 4a^{14}b^6c^9d^{22}e^2 - 8a^{15}b^6c^8d^{20} \\
& e^4 - 80a^{17}b^6c^6d^{16}e^8 + 48a^{18}b^5c^5d^{14}e^{10}) - x(2a^{14}c^9 \\
& *d^{18}e^5 + 4a^{16}c^7d^{14}e^9 + 2a^{14}b^4c^5d^{14}e^9 - 8a^{15}b^2c^6d^{14} \\
& e^9)) * (- (b^9e^2 + b^7c^2d^2 - b^6e^2 * (- (4ac - b^2)^3)^{(1/2)} - 9 \\
& a^2b^5c^3d^2 - 20a^3b^4c^5d^2 + 28a^4b^3c^4e^2 - 2b^8c^4d^2 + 25a^2b^3 \\
& c^4d^2 - a^2c^4d^2 * (- (4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 6 \\
& 3a^3b^3c^3e^2 + a^3c^3e^2 * (- (4ac - b^2)^3)^{(1/2)} - b^4c^2d^2 * (- (4 \\
& ac - b^2)^3)^{(1/2)} - 11a^2b^7c^4e^2 - 16a^4c^5d^2 + 20a^2b^6c^2d^2 + \\
& 2b^5c^4d^2 * (- (4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^2e^2 * (- (4ac - b^2)^3 \\
&)^{(1/2)} + 5a^2b^4c^2e^2 * (- (4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^2 + 76 \\
& a^3b^2c^4d^2 + 3a^2b^2c^3d^2 * (- (4ac - b^2)^3)^{(1/2)} - 8a^2b^3c^2d^2 \\
& * (- (4ac - b^2)^3)^{(1/2)} + 6a^2b^3c^3d^2 * (- (4ac - b^2)^3)^{(1/2))} / (8(\\
& a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^4e^4 - 2a^6b^5d^3 \\
& *d^3e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3 \\
& *d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^3c^3d^3e + 16a^7b^3c^3d^3e^3 - 32 \\
& a^8b^3c^2d^3e^3 + 16a^6b^3c^2d^3e^3 - 6a^6b^4c^2d^2e^2))^{(1/2)} * i) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(- (b^9 e^2 + b^7 c^2 d^2 - b^6 e^2 (-4ac - b^2)^3)^{1/2} - 9a^3 b^5 c^3 d^2 - 20a^3 b^3 c^5 d^2 + 28a^4 b^3 c^4 e^2 - 2b^8 c^3 d e + 25a^2 b^3 c^4 d^2 - a^2 c^4 d^2 (-4ac - b^2)^3)^{1/2} + 42a^2 b^5 c^2 e^2 - 63a^3 b^3 c^3 e^2 + a^3 c^3 e^2 (-4ac - b^2)^3)^{1/2} - b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 11a^3 b^7 c e^2 - 16a^4 c^5 d e + 20a^3 b^6 c^2 d e + 2b^5 c^3 d e (-4ac - b^2)^3)^{1/2} - 6a^2 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} + 5a^3 b^4 c e^2 (-4ac - b^2)^3)^{1/2} - 66a^2 b^4 c^3 d e + 76a^3 b^2 c^4 d e + 3a^3 b^2 c^3 d^2 (-4ac - b^2)^3)^{1/2} - 8a^3 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} + 6a^2 b^3 c^3 d e (-4ac - b^2)^3)^{1/2} \right) / (8(a^7 b^4 e^4 + 16a^7 c^4 d^4 + 16a^9 c^2 e^4 - 8a^8 b^2 c e^4 - 2a^6 b^5 d e^3 + a^5 b^4 c^2 d^4 - 8a^6 b^2 c^3 d^4 + a^5 b^6 d^2 e^2 + 32a^8 c^3 d^2 e^2 - 2a^5 b^5 c d^3 e - 32a^7 b^3 c^3 d^3 e + 16a^7 b^3 c^3 d e^3 - 32a^8 b^3 c^2 d e^3 + 16a^6 b^3 c^2 d^3 e - 6a^6 b^4 c^2 d^2 e^2))^{1/2} * \left(- (b^9 e^2 + b^7 c^2 d^2 - b^6 e^2 (-4ac - b^2)^3)^{1/2} - 9a^3 b^5 c^3 d^2 - 20a^3 b^3 c^5 d^2 + 28a^4 b^3 c^4 e^2 - 2b^8 c^3 d e + 25a^2 b^3 c^4 d^2 - a^2 c^4 d^2 (-4ac - b^2)^3)^{1/2} + 42a^2 b^5 c^2 e^2 - 63a^3 b^3 c^3 e^2 + a^3 c^3 e^2 (-4ac - b^2)^3)^{1/2} - b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 11a^3 b^7 c e^2 - 16a^4 c^5 d e + 20a^3 b^6 c^2 d e + 2b^5 c^3 d e (-4ac - b^2)^3)^{1/2} - 6a^2 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} + 5a^3 b^4 c e^2 (-4ac - b^2)^3)^{1/2} - 66a^2 b^4 c^3 d e + 76a^3 b^2 c^4 d e + 3a^3 b^2 c^3 d^2 (-4ac - b^2)^3)^{1/2} - 8a^3 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} + 6a^2 b^3 c^3 d e (-4ac - b^2)^3)^{1/2} \right) / (8(a^7 b^4 e^4 + 16a^7 c^4 d^4 + 16a^9 c^2 e^4 - 8a^8 b^2 c e^4 - 2a^6 b^5 d e^3 + a^5 b^4 c^2 d^4 - 8a^6 b^2 c^3 d^4 + a^5 b^6 d^2 e^2 + 32a^8 c^3 d^2 e^2 - 2a^5 b^5 c d^3 e - 32a^7 b^3 c^3 d^3 e + 16a^7 b^3 c^3 d e^3 - 32a^8 b^3 c^2 d e^3 + 16a^6 b^3 c^2 d^3 e - 6a^6 b^4 c^2 d^2 e^2))^{1/2} * (x * (- (b^9 e^2 + b^7 c^2 d^2 - b^6 e^2 (-4ac - b^2)^3)^{1/2} - 9a^3 b^5 c^3 d^2 - 20a^3 b^3 c^5 d^2 + 28a^4 b^3 c^4 e^2 - 2b^8 c^3 d e + 25a^2 b^3 c^4 d^2 - a^2 c^4 d^2 (-4ac - b^2)^3)^{1/2} + 42a^2 b^5 c^2 e^2 - 63a^3 b^3 c^3 e^2 + a^3 c^3 e^2 (-4ac - b^2)^3)^{1/2} - b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 11a^3 b^7 c e^2 - 16a^4 c^5 d e + 20a^3 b^6 c^2 d e + 2b^5 c^3 d e (-4ac - b^2)^3)^{1/2} - 6a^2 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} + 5a^3 b^4 c e^2 (-4ac - b^2)^3)^{1/2} - 66a^2 b^4 c^3 d e + 76a^3 b^2 c^4 d e + 3a^3 b^2 c^3 d^2 (-4ac - b^2)^3)^{1/2} - 8a^3 b^3 c^2 d e (-4ac - b^2)^3)^{1/2} + 6a^2 b^3 c^3 d e (-4ac - b^2)^3)^{1/2} \right) / (8(a^7 b^4 e^4 + 16a^7 c^4 d^4 + 16a^9 c^2 e^4 - 8a^8 b^2 c e^4 - 2a^6 b^5 d e^3 + a^5 b^4 c^2 d^4 - 8a^6 b^2 c^3 d^4 + a^5 b^6 d^2 e^2 + 32a^8 c^3 d^2 e^2 - 2a^5 b^5 c d^3 e - 32a^7 b^3 c^3 d^3 e + 16a^7 b^3 c^3 d e^3 - 32a^8 b^3 c^2 d e^3 + 16a^6 b^3 c^2 d^3 e - 6a^6 b^4 c^2 d^2 e^2))^{1/2} * (512a^{20} c^7 d^{24} e^3 + 512a^{21} c^6 d^{22} e^5 - 512a^{22} c^5 d^{20} e^7 - 512a^{23} c^4 d^{18} e^9 - 32a^{18} b^3 c^6 d^{25} e^2 + 128a^{18} b^4 c^5 d^{24} e^3 - 192a^{18} b^5 c^4 d^{23} e^4 + 128a^{18} b^6 c^3 d^{22} e^5 - 32a^{18} b^7 c^2 d^{21} e^6 - 640a^{19} b^2 c^6 d^{24} e^3 + 1056a^{19} b^3 c^5 d^{23} e^4 - 672a^{19} b^4 c^4 d^{22} e^5 + 96a^{19} b^5 c^3 d^{21} e^6 + 32a^{19} b^6 c^2 d^{20} e^7 + 512a^{20} b^2 c^5 d^{22} e^5 + 288a^{20} b^3 c^4 d^{21} e^6 - 192a^{20} b^4 c^3 d^{20} e^7 + 32a^{20} b^5 c^2 d^{19} e^8 + 384
\end{aligned}$$

$$\begin{aligned}
& a^{21}b^2c^4d^{20}e^7 - 288a^{21}b^3c^3d^{19}e^8 - 32a^{21}b^4c^2d^{18}e^9 + 256a^{22}b^2c^3d^{18}e^9 + 128a^{19}b^3c^7d^{25}e^2 - 1152a^{20}b^3c^6d^{23}e^4 - 640a^{21}b^3c^5d^{21}e^6 + 640a^{22}b^3c^4d^{19}e^8) + 64a^{18}c^8d^{24}e^2 - 128a^{19}c^7d^{22}e^4 - 192a^{20}c^6d^{20}e^6 + 256a^{21}c^5d^{18}e^8 + 256a^{22}c^4d^{16}e^{10} + 16a^{16}b^4c^6d^{24}e^2 - 64a^{16}b^5c^5d^{23}e^3 + 96a^{16}b^6c^4d^{22}e^4 - 64a^{16}b^7c^3d^{21}e^5 + 16a^{16}b^8c^2d^{20}e^6 - 80a^{17}b^2c^7d^{24}e^2 + 368a^{17}b^3c^6d^{23}e^3 - 608a^{17}b^4c^5d^{22}e^4 + 416a^{17}b^5c^4d^{21}e^5 - 80a^{17}b^6c^3d^{20}e^6 - 16a^{17}b^7c^2d^{19}e^7 + 928a^{18}b^2c^6d^{22}e^4 - 640a^{18}b^3c^5d^{21}e^5 - 32a^{18}b^4c^4d^{20}e^6 + 128a^{18}b^5c^3d^{19}e^7 + 432a^{19}b^2c^5d^{20}e^6 - 304a^{19}b^3c^4d^{19}e^7 + 16a^{19}b^4c^3d^{18}e^8 - 16a^{19}b^5c^2d^{17}e^9 - 128a^{20}b^2c^4d^{18}e^8 + 128a^{20}b^3c^3d^{17}e^9 + 16a^{20}b^4c^2d^{16}e^{10} - 128a^{21}b^2c^3d^{16}e^{10} - 448a^{18}b^3c^7d^{23}e^3 + 192a^{20}b^3c^5d^{19}e^7 - 256a^{21}b^3c^4d^{17}e^9) - x*(16a^{16}c^9d^{23}e^2 + 32a^{17}c^8d^{21}e^4 - 112a^{18}c^7d^{19}e^6 - 128a^{20}c^5d^{15}e^{10} + 8a^{14}b^4c^7d^{23}e^2 - 16a^{14}b^5c^6d^{22}e^3 + 8a^{14}b^6c^5d^{21}e^4 + 8a^{14}b^7c^4d^{20}e^5 - 16a^{14}b^8c^3d^{19}e^6 + 8a^{14}b^9c^2d^{18}e^7 - 32a^{15}b^2c^8d^{23}e^2 + 64a^{15}b^3c^7d^{22}e^3 - 16a^{15}b^4c^6d^{21}e^4 - 88a^{15}b^5c^5d^{20}e^5 + 160a^{15}b^6c^4d^{19}e^6 - 88a^{15}b^7c^3d^{18}e^7 - 48a^{16}b^2c^7d^{21}e^4 + 264a^{16}b^3c^6d^{20}e^5 - 520a^{16}b^4c^5d^{19}e^6 + 336a^{16}b^5c^4d^{18}e^7 + 576a^{17}b^2c^6d^{19}e^6 - 504a^{17}b^3c^5d^{18}e^7 + 8a^{18}b^3c^4d^{16}e^9 - 16a^{18}b^4c^3d^{15}e^{10} + 8a^{18}b^5c^2d^{14}e^{11} + 96a^{19}b^2c^4d^{15}e^{10} - 56a^{19}b^3c^3d^{14}e^{11} - 32a^{16}b^3c^8d^{22}e^3 - 192a^{17}b^3c^7d^{20}e^5 + 224a^{18}b^3c^6d^{18}e^7 - 32a^{19}b^3c^5d^{16}e^9 + 96a^{20}b^3c^4d^{14}e^{11})*(-(b^9e^2 + b^7c^2d^2 - b^6e^2*(-(4ac - b^2)^3))^(1/2) - 9a^5b^5c^3d^2 - 20a^3b^3c^5d^2 + 28a^4b^3c^4e^2 - 2b^8c^2d^2e + 25a^2b^3c^4d^2 - a^2c^4d^2*(-(4ac - b^2)^3))^(1/2) + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 + a^3c^3e^2*(-(4ac - b^2)^3))^(1/2) - b^4c^2d^2*(-(4ac - b^2)^3))^(1/2) - 11a^7b^7c^2e^2 - 16a^4c^5d^2e + 20a^6b^6c^2d^2e + 2b^5c^2d^2e*(-(4ac - b^2)^3))^(1/2) - 6a^2b^2c^2e^2*(-(4ac - b^2)^3))^(1/2) + 5a^2b^4c^2e^2*(-(4ac - b^2)^3))^(1/2) - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e + 3a^2b^2c^3d^2*(-(4ac - b^2)^3))^(1/2) - 8a^2b^3c^2d^2e*(-(4ac - b^2)^3))^(1/2) + 6a^2b^3c^3d^2e*(-(4ac - b^2)^3))^(1/2))/(8*(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^3c^3d^3e + 16a^7b^3c^2d^3e - 32a^8b^3c^2d^3e + 16a^6b^3c^2d^3e - 6a^6b^4c^2d^2e^2))^(1/2) + 4a^{15}c^9d^{21}e^3 + 4a^{16}c^8d^{19}e^5 - 48a^{18}c^6d^{15}e^9 + 4a^{14}b^2c^8d^{21}e^3 + 4a^{14}b^7c^3d^{16}e^8 - 4a^{14}b^8c^2d^{15}e^9 - 36a^{15}b^5c^4d^{16}e^8 + 44a^{15}b^6c^3d^{15}e^9 - 4a^{15}b^7c^2d^{14}e^{10} + 100a^{16}b^3c^5d^{16}e^8 - 160a^{16}b^4c^4d^{15}e^9 + 32a^{16}b^5c^3d^{14}e^{10} + 204a^{17}b^2c^5d^{15}e^9 - 76a^{17}b^3c^4d^{14}e^{10} - 4a^{14}b^3c^9d^{22}e^2 - 8a^{15}b^3c^8d^{20}e^4 - 80a^{17}b^3c^6d^{16}e^8 + 48a^{18}b^3c^5d^{14}e^{10}) - x*(2a^{14}c^9d^{18}e^5 + 4a^{16}c^7d^{14}e^9 + 2*
\end{aligned}$$

$$\begin{aligned}
& a^{14}b^4c^5d^{14}e^9 - 8a^{15}b^2c^6d^{14}e^9) * (- (b^9e^2 + b^7c^2d^2 \\
& - b^6e^2 * (- (4ac - b^2)^3)^{1/2} - 9ab^5c^3d^2 - 20a^3b^2c^5d^2 + 2 \\
& 8a^4b^2c^4e^2 - 2b^8c^2d^2e + 25a^2b^3c^4d^2 - a^2c^4d^2 * (- (4ac - \\
& b^2)^3)^{1/2} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 + a^3c^3e^2 * (- (4 \\
& ac - b^2)^3)^{1/2} - b^4c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 \\
& - 16a^4c^5d^2e + 20ab^6c^2d^2e + 2b^5c^2d^2e * (- (4ac - b^2)^3)^{1/2} \\
&) - 6a^2b^2c^2e^2 * (- (4ac - b^2)^3)^{1/2} + 5ab^4c^2e^2 * (- (4ac - b \\
& ^2)^3)^{1/2} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e + 3ab^2c^3d^2 * (- \\
& (4ac - b^2)^3)^{1/2} - 8ab^3c^2d^2e * (- (4ac - b^2)^3)^{1/2} + 6a^2b \\
& *c^3d^2e * (- (4ac - b^2)^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^ \\
& 9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2 \\
& *c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^2d^3e - 32a^ \\
& 7b^2c^3d^3e + 16a^7b^3c^2d^3e - 32a^8b^2c^2d^3e + 16a^6b^3c^2d^ \\
& 3e - 6a^6b^4c^2d^2e^2))^{1/2} - ((- (b^9e^2 + b^7c^2d^2 - b^6e^2 * (- \\
& (4ac - b^2)^3)^{1/2} - 9ab^5c^3d^2 - 20a^3b^2c^5d^2 + 28a^4b^2c^4e^2 \\
& - 2b^8c^2d^2e + 25a^2b^3c^4d^2 - a^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} \\
& + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 + a^3c^3e^2 * (- (4ac - b^2)^ \\
& 3)^{1/2} - b^4c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c \\
& ^5d^2e + 20ab^6c^2d^2e + 2b^5c^2d^2e * (- (4ac - b^2)^3)^{1/2} - 6a^2b^ \\
& 2c^2e^2 * (- (4ac - b^2)^3)^{1/2} + 5ab^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} \\
& - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e + 3ab^2c^3d^2 * (- (4ac - b^2 \\
&)^3)^{1/2} - 8ab^3c^2d^2e * (- (4ac - b^2)^3)^{1/2} + 6a^2b^2c^3d^2e * (- \\
& (4ac - b^2)^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - \\
& 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a \\
& ^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^2d^3e - 32a^7b^2c^3d^3e \\
& + 16a^7b^3c^2d^3e - 32a^8b^2c^2d^3e + 16a^6b^3c^2d^3e - 6a^6b \\
& ^4c^2d^2e^2))^{1/2} * ((- (b^9e^2 + b^7c^2d^2 - b^6e^2 * (- (4ac - b^2) \\
& ^3)^{1/2} - 9ab^5c^3d^2 - 20a^3b^2c^5d^2 + 28a^4b^2c^4e^2 - 2b^8c^2 \\
& *d^2e + 25a^2b^3c^4d^2 - a^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} + 42a^2b \\
& ^5c^2e^2 - 63a^3b^3c^3e^2 + a^3c^3e^2 * (- (4ac - b^2)^3)^{1/2} - b^ \\
& 4c^2d^2 * (- (4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c^5d^2e + 20a \\
& *b^6c^2d^2e + 2b^5c^2d^2e * (- (4ac - b^2)^3)^{1/2} - 6a^2b^2c^2e^2 * (- \\
& (4ac - b^2)^3)^{1/2} + 5ab^4c^2e^2 * (- (4ac - b^2)^3)^{1/2} - 66a^2b^4 \\
& *c^3d^2e + 76a^3b^2c^4d^2e + 3ab^2c^3d^2 * (- (4ac - b^2)^3)^{1/2} - \\
& 8ab^3c^2d^2e * (- (4ac - b^2)^3)^{1/2} + 6a^2b^2c^3d^2e * (- (4ac - b^2) \\
& ^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^ \\
& ^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 \\
& + 32a^8c^3d^2e^2 - 2a^5b^5c^2d^3e - 32a^7b^2c^3d^3e + 16a^7b^ \\
& 3c^2d^3e - 32a^8b^2c^2d^3e + 16a^6b^3c^2d^3e - 6a^6b^4c^2d^2e^2 \\
&))^{1/2} * (x * (- (b^9e^2 + b^7c^2d^2 - b^6e^2 * (- (4ac - b^2)^3)^{1/2} - \\
& 9ab^5c^3d^2 - 20a^3b^2c^5d^2 + 28a^4b^2c^4e^2 - 2b^8c^2d^2e + 25a^ \\
& 2b^3c^4d^2 - a^2c^4d^2 * (- (4ac - b^2)^3)^{1/2} + 42a^2b^5c^2e^2 - \\
& 63a^3b^3c^3e^2 + a^3c^3e^2 * (- (4ac - b^2)^3)^{1/2} - b^4c^2d^2 * (- \\
& (4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e \\
& + 2b^5c^2d^2e * (- (4ac - b^2)^3)^{1/2} - 6a^2b^2c^2e^2 * (- (4ac - b^2)
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - \\
& 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1/2)} - 4*a^15*c^9*d^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21*e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 204*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1/2)} + 2*a^14*c^8*d^14*e^8))*(-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1/2)}*2i - (1/(3*a*d) - (x^2*(a*e + b*d))/(a^2*d^2))/x^3 - (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 - 76*a^3*b^2*c^4*d^
\end{aligned}$$

$$\begin{aligned}
& 19e^{14} + 63a^3b^3c^3d^{18}e^{15} + a^5b^4e^3x(-d^5e^7)^{(5/2)} - a^2c^7d^{15}x(-d^5e^7)^{(3/2)} + 16a^7c^2e^3x(-d^5e^7)^{(5/2)} + b^9d^{10}e^5x(-d^5e^7)^{(3/2)} + c^9d^{24}e^3x(-d^5e^7)^{(1/2)} - 2a^*b^*c^7d^{24}e^9 + 11a^*b^7c^*d^{18}e^{15} + 9a^*b^5c^3d^{20}e^{13} - 20a^*b^6c^2d^{19}e^{14} + 20a^3b^*c^5d^{20}e^{13} - 28a^4b^*c^4d^{18}e^{15} - 8a^6b^2c^*d^{13}e^{20} - 16a^4c^5d^{11}e^4x(-d^5e^7)^{(3/2)} + b^7c^2d^{12}e^3x(-d^5e^7)^{(3/2)} + b^2c^7d^{22}e^5x(-d^5e^7)^{(1/2)} - 8a^6b^2c^*e^3x(-d^5e^7)^{(5/2)} + 2a^*c^8d^{22}e^5x(-d^5e^7)^{(1/2)} - 2b^8c^*d^{11}e^4x(-d^5e^7)^{(3/2)} - 2b^*c^8d^{23}e^4x(-d^5e^7)^{(1/2)} - 11a^*b^7c^*d^{10}e^5x(-d^5e^7)^{(3/2)} - 2a^*b^*c^7d^{21}e^6x(-d^5e^7)^{(1/2)} - 9a^*b^5c^3d^{12}e^3x(-d^5e^7)^{(3/2)} + 20a^*b^6c^2d^{11}e^4x(-d^5e^7)^{(3/2)} - 20a^3b^*c^5d^{12}e^3x(-d^5e^7)^{(3/2)} + 28a^4b^*c^4d^{10}e^5x(-d^5e^7)^{(3/2)} + 25a^2b^3c^4d^{12}e^3x(-d^5e^7)^{(3/2)} - 66a^2b^4c^3d^{11}e^4x(-d^5e^7)^{(3/2)} + 42a^2b^5c^2d^{10}e^5x(-d^5e^7)^{(3/2)} + 76a^3b^2c^4d^{11}e^4x(-d^5e^7)^{(3/2)} - 63a^3b^3c^3d^{10}e^5x(-d^5e^7)^{(3/2)})(-d^5e^7)^{(1/2)}/(2(c*d^7 + a*d^5e^2 - b*d^6e)) - \operatorname{atan}\left(\frac{(-(b^9e^2 + b^7c^2d^2 + b^6e^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^*b^5c^3d^2 - 20a^3b^*c^5d^2 + 28a^4b^*c^4e^2 - 2b^8c^*d^*e + 25a^2b^3c^4d^2 + a^2c^4d^2*(-(4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + b^4c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 11a^*b^7c^*e^2 - 16a^4c^5d^*e + 20a^*b^6c^2d^*e - 2b^5c^*d^*e*(-(4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5a^*b^4c^*e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^*e + 76a^3b^2c^4d^*e - 3a^*b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 8a^*b^3c^2d^*e*(-(4ac - b^2)^3)^{(1/2)} - 6a^2b^*c^3d^*e*(-(4ac - b^2)^3)^{(1/2))}/(8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^*e^4 - 2a^6b^5d^*e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^*d^3e - 32a^7b^*c^3d^3e + 16a^7b^3c^*d^*e^3 - 32a^8b^*c^2d^*e^3 + 16a^6b^3c^2d^3e - 6a^6b^4c^*d^2e^2))\right)^{(1/2)} * (x*(-(b^9e^2 + b^7c^2d^2 + b^6e^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^*b^5c^3d^2 - 20a^3b^*c^5d^2 + 28a^4b^*c^4e^2 - 2b^8c^*d^*e + 25a^2b^3c^4d^2 + a^2c^4d^2*(-(4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + b^4c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 11a^*b^7c^*e^2 - 16a^4c^5d^*e + 20a^*b^6c^2d^*e - 2b^5c^*d^*e*(-(4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5a^*b^4c^*e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^*e + 76a^3b^2c^4d^*e - 3a^*b^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 8a^*b^3c^2d^*e*(-(4ac - b^2)^3)^{(1/2)} - 6a^2b^*c^3d^*e*(-(4ac - b^2)^3)^{(1/2))}/(8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^*e^4 - 2a^6b^5d^*e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^*d^3e - 32a^7b^*c^3d^3e + 16a^7b^3c^*d^*e^3 - 32a^8b^*c^2d^*e^3 + 16a^6b^3c^2d^3e - 6a^6b^4c^*d^2e^2))\right)^{(1/2)} * (x*(-(b^9e^2 + b^7c^2d^2 + b^6e^2*(-(4ac - b^2)^3)^{(1/2)} - 9a^*b^5c^3d^2 - 20a^3b^*c^5d^2 + 28a^4b^*c^4e^2 - 2b^8c^*d^*e + 25a^2b^3c^4d^2 + a^2c^4d^2*(-(4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& + b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} - 11ab^7 c^2 e^2 - 16a^4 c^5 d^2 e + \\
& 20ab^6 c^2 d^2 e - 2b^5 c^2 d^2 e (-4ac - b^2)^3)^{1/2} + 6a^2 b^2 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 5ab^4 c^2 e^2 (-4ac - b^2)^3)^{1/2} - 66a^2 b^4 c^3 d^2 e + 76a^3 b^2 c^4 d^2 e - 3ab^2 c^3 d^2 (-4ac - b^2)^3)^{1/2} + 8ab^3 c^2 d^2 e (-4ac - b^2)^3)^{1/2} - 6a^2 b^2 c^3 d^2 e (-4ac - b^2)^3)^{1/2} \\
& \left. \left(8(a^7 b^4 e^4 + 16a^7 c^4 d^4 + 16a^9 c^2 e^4 - 8a^8 b^2 c^2 e^4 - 2a^6 b^5 d^3 e + a^5 b^4 c^2 d^4 - 8a^6 b^2 c^3 d^4 + a^5 b^6 d^2 e^2 + 32a^8 c^3 d^2 e^2 - 2a^5 b^5 c^2 d^3 e - 32a^7 b^2 c^3 d^3 e + 16a^7 b^3 c^2 d^3 e - 32a^8 b^2 c^2 d^3 e + 16a^6 b^3 c^2 d^3 e - 6a^6 b^4 c^2 d^2 e^2) \right)^{1/2} \right. \\
& \left. (512a^{20} c^7 d^{24} e^3 + 512a^{21} c^6 d^{22} e^5 - 512a^{22} c^5 d^{20} e^7 - 512a^{23} c^4 d^{18} e^9 - 32a^{18} b^3 c^6 d^{25} e^2 + 128a^{18} b^4 c^5 d^{24} e^3 - 192a^{18} b^5 c^4 d^{23} e^4 + 128a^{18} b^6 c^3 d^{22} e^5 - 32a^{18} b^7 c^2 d^{21} e^6 - 640a^{19} b^2 c^6 d^{24} e^3 + 1056a^{19} b^3 c^5 d^{23} e^4 - 672a^{19} b^4 c^4 d^{22} e^5 + 96a^{19} b^5 c^3 d^{21} e^6 + 32a^{19} b^6 c^2 d^{20} e^7 + 512a^{20} b^2 c^5 d^{22} e^5 + 288a^{20} b^3 c^4 d^{21} e^6 - 192a^{20} b^4 c^3 d^{20} e^7 + 32a^{20} b^5 c^2 d^{19} e^8 + 384a^{21} b^2 c^4 d^{20} e^7 - 288a^{21} b^3 c^3 d^{19} e^8 - 32a^{21} b^4 c^2 d^{18} e^9 + 256a^{22} b^2 c^3 d^{18} e^9 + 128a^{19} b^2 c^7 d^{25} e^2 - 1152a^{20} b^2 c^6 d^{23} e^4 - 640a^{21} b^2 c^5 d^{21} e^6 + 640a^{22} b^2 c^4 d^{19} e^8) - 64a^{18} c^8 d^{24} e^2 + 128a^{19} c^7 d^{22} e^4 + 192a^{20} c^6 d^{20} e^6 - 256a^{21} c^5 d^{18} e^8 - 256a^{22} c^4 d^{16} e^{10} - 16a^{16} b^4 c^6 d^{24} e^2 + 64a^{16} b^5 c^5 d^{23} e^3 - 96a^{16} b^6 c^4 d^{22} e^4 + 64a^{16} b^7 c^3 d^{21} e^5 - 16a^{16} b^8 c^2 d^{20} e^6 + 80a^{17} b^2 c^7 d^{24} e^2 - 368a^{17} b^3 c^6 d^{23} e^3 + 608a^{17} b^4 c^5 d^{22} e^4 - 416a^{17} b^5 c^4 d^{21} e^5 + 80a^{17} b^6 c^3 d^{20} e^6 + 16a^{17} b^7 c^2 d^{19} e^7 - 928a^{18} b^2 c^6 d^{22} e^4 + 640a^{18} b^3 c^5 d^{21} e^5 + 32a^{18} b^4 c^4 d^{20} e^6 - 128a^{18} b^5 c^3 d^{19} e^7 - 432a^{19} b^2 c^5 d^{20} e^6 + 304a^{19} b^3 c^4 d^{19} e^7 - 16a^{19} b^4 c^3 d^{18} e^8 + 16a^{19} b^5 c^2 d^{17} e^9 + 128a^{20} b^2 c^4 d^{18} e^8 - 128a^{20} b^3 c^3 d^{17} e^9 - 16a^{20} b^4 c^2 d^{16} e^{10} + 128a^{21} b^2 c^3 d^{16} e^{10} + 448a^{18} b^2 c^7 d^{23} e^3 - 192a^{20} b^2 c^5 d^{19} e^7 + 256a^{21} b^2 c^4 d^{17} e^9) - x(16a^{16} c^9 d^{23} e^2 + 32a^{17} c^8 d^{21} e^4 - 112a^{18} c^7 d^{19} e^6 - 128a^{20} c^5 d^{15} e^{10} + 8a^{14} b^4 c^7 d^{23} e^2 - 16a^{14} b^5 c^6 d^{22} e^3 + 8a^{14} b^6 c^5 d^{21} e^4 + 8a^{14} b^7 c^4 d^{20} e^5 - 16a^{14} b^8 c^3 d^{19} e^6 + 8a^{14} b^9 c^2 d^{18} e^7 - 32a^{15} b^2 c^8 d^{23} e^2 + 64a^{15} b^3 c^7 d^{22} e^3 - 16a^{15} b^4 c^6 d^{21} e^4 - 88a^{15} b^5 c^5 d^{20} e^5 + 160a^{15} b^6 c^4 d^{19} e^6 - 88a^{15} b^7 c^3 d^{18} e^7 - 48a^{16} b^2 c^7 d^{21} e^4 + 264a^{16} b^3 c^6 d^{20} e^5 - 520a^{16} b^4 c^5 d^{19} e^6 + 336a^{16} b^5 c^4 d^{18} e^7 + 576a^{17} b^2 c^6 d^{19} e^6 - 504a^{17} b^3 c^5 d^{18} e^7 + 8a^{18} b^3 c^4 d^{16} e^9 - 16a^{18} b^4 c^3 d^{15} e^{10} + 8a^{18} b^5 c^2 d^{14} e^{11} + 96a^{19} b^2 c^4 d^{15} e^{10} - 56a^{19} b^3 c^3 d^{14} e^{11} - 32a^{16} b^2 c^8 d^{22} e^3 - 192a^{17} b^2 c^7 d^{20} e^5 + 224a^{18} b^2 c^6 d^{18} e^7 - 32a^{19} b^2 c^5 d^{16} e^9 + 96a^{20} b^2 c^4 d^{14} e^{11}) \right) \\
& \left. (-b^9 e^2 + b^7 c^2 d^2 + b^6 e^2 (-4ac - b^2)^3)^{1/2} - 9ab^5 c^3 d^2 - 20a^3 b^2 c^5 d^2 + 28a^4 b^2 c^4 e^2 - 2b^8 c^2 d^2 + 25a^2 b^3 c^4 d^2 + a^2 c^4 d^2 (-4ac - b^2)^3)^{1/2} + 42a^2 b^5 c^2 e^2 - 63a^3 b^3 c^3 e^2 - a^3 c^3 e^2 (-4ac - b^2)^3)^{1/2} + b^4 c^2 d^2 (-4ac - b^2)^3)^{1/2} \right)
\end{aligned}$$

$$\begin{aligned}
&)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d \\
& *e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4 \\
& *d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*d*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^7*b^4*e^4 \\
& + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a \\
& ^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - \\
& 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2 \\
& *d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1/2)} - 4*a^15*c^9*d \\
& ^21*e^3 - 4*a^16*c^8*d^19*e^5 + 48*a^18*c^6*d^15*e^9 - 4*a^14*b^2*c^8*d^21* \\
& e^3 - 4*a^14*b^7*c^3*d^16*e^8 + 4*a^14*b^8*c^2*d^15*e^9 + 36*a^15*b^5*c^4*d \\
& ^16*e^8 - 44*a^15*b^6*c^3*d^15*e^9 + 4*a^15*b^7*c^2*d^14*e^10 - 100*a^16*b^ \\
& 3*c^5*d^16*e^8 + 160*a^16*b^4*c^4*d^15*e^9 - 32*a^16*b^5*c^3*d^14*e^10 - 20 \\
& 4*a^17*b^2*c^5*d^15*e^9 + 76*a^17*b^3*c^4*d^14*e^10 + 4*a^14*b*c^9*d^22*e^2 \\
& + 8*a^15*b*c^8*d^20*e^4 + 80*a^17*b*c^6*d^16*e^8 - 48*a^18*b*c^5*d^14*e^10 \\
&) - x*(2*a^14*c^9*d^18*e^5 + 4*a^16*c^7*d^14*e^9 + 2*a^14*b^4*c^5*d^14*e^9 \\
& - 8*a^15*b^2*c^6*d^14*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^ \\
& 8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^ \\
& 2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 2 \\
& 0*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2* \\
& (-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2* \\
& b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2* \\
& c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2 \\
& *e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7 \\
& *b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2* \\
& e^2)))^{(1/2)}*i + (((- (b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + \\
& 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2 \\
& *e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c \\
& ^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d \\
& *e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^ \\
& 3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2 \\
& *a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32 \\
& *a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d* \\
& e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^{(1 \\
& /2)}*(((- (b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5 \\
& *c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c \\
& ^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - 63*a^3
\end{aligned}$$

$$\begin{aligned}
& b^3c^3e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + b^4c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 11ab^7c^3e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e - 2b^5c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab^4c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e - 3ab^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 8ab^3c^2d^2e*(-(4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^3d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^2c^3d^3e + 16a^7b^3c^3d^3e - 32a^8b^2c^2d^3e + 16a^6b^3c^2d^3e - 6a^6b^4c^2d^2e^2))^{(1/2)}*(x*(-(b^9e^2 + b^7c^2d^2 + b^6e^2*(-(4ac - b^2)^3)^{(1/2)} - 9ab^5c^3d^2 - 20a^3b^2c^5d^2 + 28a^4b^2c^4e^2 - 2b^8c^2d^2e + 25a^2b^3c^4d^2 + a^2c^4d^2*(-(4ac - b^2)^3)^{(1/2)} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2*(-(4ac - b^2)^3)^{(1/2)} + b^4c^2d^2*(-(4ac - b^2)^3)^{(1/2)} - 11ab^7c^3e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e - 2b^5c^3d^2e*(-(4ac - b^2)^3)^{(1/2)} + 6a^2b^2c^2e^2*(-(4ac - b^2)^3)^{(1/2)} - 5ab^4c^3e^2*(-(4ac - b^2)^3)^{(1/2)} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e - 3ab^2c^3d^2*(-(4ac - b^2)^3)^{(1/2)} + 8ab^3c^2d^2e*(-(4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^3d^2e*(-(4ac - b^2)^3)^{(1/2)})/(8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^2c^3d^3e + 16a^7b^3c^3d^3e - 32a^8b^2c^2d^3e + 16a^6b^3c^2d^3e - 6a^6b^4c^2d^2e^2))^{(1/2)}*(512a^20c^7d^24e^3 + 512a^21c^6d^22e^5 - 512a^22c^5d^20e^7 - 512a^23c^4d^18e^9 - 32a^18b^3c^6d^25e^2 + 128a^18b^4c^5d^24e^3 - 192a^18b^5c^4d^23e^4 + 128a^18b^6c^3d^22e^5 - 32a^18b^7c^2d^21e^6 - 640a^19b^2c^6d^24e^3 + 1056a^19b^3c^5d^23e^4 - 672a^19b^4c^4d^22e^5 + 96a^19b^5c^3d^21e^6 + 32a^19b^6c^2d^20e^7 + 512a^20b^2c^5d^22e^5 + 288a^20b^3c^4d^21e^6 - 192a^20b^4c^3d^20e^7 + 32a^20b^5c^2d^19e^8 + 384a^21b^2c^4d^20e^7 - 288a^21b^3c^3d^19e^8 - 32a^21b^4c^2d^18e^9 + 256a^22b^2c^3d^18e^9 + 128a^19b^2c^7d^25e^2 - 1152a^20b^2c^6d^23e^4 - 640a^21b^2c^5d^21e^6 + 640a^22b^2c^4d^19e^8) + 64a^18c^8d^24e^2 - 128a^19c^7d^22e^4 - 192a^20c^6d^20e^6 + 256a^21c^5d^18e^8 + 256a^22c^4d^16e^10 + 16a^16b^4c^6d^24e^2 - 64a^16b^5c^5d^23e^3 + 96a^16b^6c^4d^22e^4 - 64a^16b^7c^3d^21e^5 + 16a^16b^8c^2d^20e^6 - 80a^17b^2c^7d^24e^2 + 368a^17b^3c^6d^23e^3 - 608a^17b^4c^5d^22e^4 + 416a^17b^5c^4d^21e^5 - 80a^17b^6c^3d^20e^6 - 16a^17b^7c^2d^19e^7 + 928a^18b^2c^6d^22e^4 - 640a^18b^3c^5d^21e^5 - 32a^18b^4c^4d^20e^6 + 128a^18b^5c^3d^19e^7 + 432a^19b^2c^5d^20e^6 - 304a^19b^3c^4d^19e^7 + 16a^19b^4c^3d^18e^8 - 16a^19b^5c^2d^17e^9 - 128a^20b^2c^4d^18e^8 + 128a^20b^3c^3d^17e^9 + 16a^20b^4c^2d^16e^10 - 128a^21b^2c^3d^16e^10 - 448a^18b^2c^7d^23e^3 + 192a^20b^2c^5d^19e^7 - 256a^21b^2c^4d^17e^9) - x*(16a^16c^9d^23e^2 + 32a^17c^8d^21e^4 - 112a^18c^7d^19e^6 - 128a^20c^5d^15e^10 + 8a^14b^4c^7d^23e^2 - 16a^14b^5c^
\end{aligned}$$

$$\begin{aligned}
& (4ac - b^2)^3)^{1/2} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2(-4ac - b^2)^3)^{1/2} + b^4c^2d^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e - 2b^5c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 5ab^4c^2e^2(-4ac - b^2)^3)^{1/2} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e - 3ab^2c^3d^2(-4ac - b^2)^3)^{1/2} + 8ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 6a^2b^3c^3d^2e(-4ac - b^2)^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^3c^3d^3e + 16a^7b^3c^2d^3e^3 - 32a^8b^2c^2d^3e^3 + 16a^6b^3c^2d^3e^3 - 6a^6b^4c^2d^3e^2))^{1/2} * (((-b^9e^2 + b^7c^2d^2 + b^6e^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^3d^2 - 20a^3b^5c^5d^2 + 28a^4b^3c^4e^2 - 2b^8c^2d^2e + 25a^2b^3c^4d^2 + a^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2(-4ac - b^2)^3)^{1/2} + b^4c^2d^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e - 2b^5c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 5ab^4c^2e^2(-4ac - b^2)^3)^{1/2} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e - 3ab^2c^3d^2(-4ac - b^2)^3)^{1/2} + 8ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 6a^2b^3c^3d^2e(-4ac - b^2)^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^3c^3d^3e + 16a^7b^3c^2d^3e^3 - 32a^8b^2c^2d^3e^3 + 16a^6b^3c^2d^3e^3 - 6a^6b^4c^2d^3e^2))^{1/2} * (x(-b^9e^2 + b^7c^2d^2 + b^6e^2(-4ac - b^2)^3)^{1/2} - 9ab^5c^3d^2 - 20a^3b^5c^5d^2 + 28a^4b^3c^4e^2 - 2b^8c^2d^2e + 25a^2b^3c^4d^2 + a^2c^4d^2(-4ac - b^2)^3)^{1/2} + 42a^2b^5c^2e^2 - 63a^3b^3c^3e^2 - a^3c^3e^2(-4ac - b^2)^3)^{1/2} + b^4c^2d^2(-4ac - b^2)^3)^{1/2} - 11ab^7c^2e^2 - 16a^4c^5d^2e + 20ab^6c^2d^2e - 2b^5c^2d^2e(-4ac - b^2)^3)^{1/2} + 6a^2b^2c^2e^2(-4ac - b^2)^3)^{1/2} - 5ab^4c^2e^2(-4ac - b^2)^3)^{1/2} - 66a^2b^4c^3d^2e + 76a^3b^2c^4d^2e - 3ab^2c^3d^2(-4ac - b^2)^3)^{1/2} + 8ab^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 6a^2b^3c^3d^2e(-4ac - b^2)^3)^{1/2}) / (8(a^7b^4e^4 + 16a^7c^4d^4 + 16a^9c^2e^4 - 8a^8b^2c^2e^4 - 2a^6b^5d^2e^3 + a^5b^4c^2d^4 - 8a^6b^2c^3d^4 + a^5b^6d^2e^2 + 32a^8c^3d^2e^2 - 2a^5b^5c^3d^3e - 32a^7b^3c^3d^3e + 16a^7b^3c^2d^3e^3 - 32a^8b^2c^2d^3e^3 + 16a^6b^3c^2d^3e^3 - 6a^6b^4c^2d^3e^2))^{1/2} * (512a^20c^7d^24e^3 + 512a^21c^6d^22e^5 - 512a^22c^5d^20e^7 - 512a^23c^4d^18e^9 - 32a^18b^3c^6d^25e^2 + 128a^18b^4c^5d^24e^3 - 192a^18b^5c^4d^23e^4 + 128a^18b^6c^3d^22e^5 - 32a^18b^7c^2d^21e^6 - 640a^19b^2c^6d^24e^3 + 1056a^19b^3c^5d^23e^4 - 672a^19b^4c^4d^22e^5 + 96a^19b^5c^3d^21e^6 + 32a^19b^6c^2d^20e^7 + 512a^20b^2c^5d^22e^5 + 288a^20b^3c^4d^21e^6 - 192a^20b^4c^3d^20e^7 + 32a^20b^5c^2d^19e^8 + 384a^21b^2c^4d^20e^7 - 288a^21b^3c^3d^19e^8 - 32a^21b^4c^2d^18e^9 + 256a^22b^2c^3d^18e^9 + 128a^19b^3c^7d^25e^2 - 1152a^20b^3c^6d^23e^4 - 640a^21
\end{aligned}$$

$$\begin{aligned}
& *b*c^5*d^{21}*e^6 + 640*a^{22}*b*c^4*d^{19}*e^8) + 64*a^{18}*c^8*d^{24}*e^2 - 128*a^{19}*c^7*d^{22}*e^4 - 192*a^{20}*c^6*d^{20}*e^6 + 256*a^{21}*c^5*d^{18}*e^8 + 256*a^{22}*c^4*d^{16}*e^{10} + 16*a^{16}*b^4*c^6*d^{24}*e^2 - 64*a^{16}*b^5*c^5*d^{23}*e^3 + 96*a^{16}*b^6*c^4*d^{22}*e^4 - 64*a^{16}*b^7*c^3*d^{21}*e^5 + 16*a^{16}*b^8*c^2*d^{20}*e^6 - 80*a^{17}*b^2*c^7*d^{24}*e^2 + 368*a^{17}*b^3*c^6*d^{23}*e^3 - 608*a^{17}*b^4*c^5*d^{22}*e^4 + 416*a^{17}*b^5*c^4*d^{21}*e^5 - 80*a^{17}*b^6*c^3*d^{20}*e^6 - 16*a^{17}*b^7*c^2*d^{19}*e^7 + 928*a^{18}*b^2*c^6*d^{22}*e^4 - 640*a^{18}*b^3*c^5*d^{21}*e^5 - 32*a^{18}*b^4*c^4*d^{20}*e^6 + 128*a^{18}*b^5*c^3*d^{19}*e^7 + 432*a^{19}*b^2*c^5*d^{20}*e^6 - 304*a^{19}*b^3*c^4*d^{19}*e^7 + 16*a^{19}*b^4*c^3*d^{18}*e^8 - 16*a^{19}*b^5*c^2*d^{17}*e^9 - 128*a^{20}*b^2*c^4*d^{18}*e^8 + 128*a^{20}*b^3*c^3*d^{17}*e^9 + 16*a^{20}*b^4*c^2*d^{16}*e^{10} - 128*a^{21}*b^2*c^3*d^{16}*e^{10} - 448*a^{18}*b*c^7*d^{23}*e^3 + 192*a^{20}*b*c^5*d^{19}*e^7 - 256*a^{21}*b*c^4*d^{17}*e^9) - x*(16*a^{16}*c^9*d^{23}*e^2 + 32*a^{17}*c^8*d^{21}*e^4 - 112*a^{18}*c^7*d^{19}*e^6 - 128*a^{20}*c^5*d^{15}*e^{10} + 8*a^{14}*b^4*c^7*d^{23}*e^2 - 16*a^{14}*b^5*c^6*d^{22}*e^3 + 8*a^{14}*b^6*c^5*d^{21}*e^4 + 8*a^{14}*b^7*c^4*d^{20}*e^5 - 16*a^{14}*b^8*c^3*d^{19}*e^6 + 8*a^{14}*b^9*c^2*d^{18}*e^7 - 32*a^{15}*b^2*c^8*d^{23}*e^2 + 64*a^{15}*b^3*c^7*d^{22}*e^3 - 16*a^{15}*b^4*c^6*d^{21}*e^4 - 88*a^{15}*b^5*c^5*d^{20}*e^5 + 160*a^{15}*b^6*c^4*d^{19}*e^6 - 88*a^{15}*b^7*c^3*d^{18}*e^7 - 48*a^{16}*b^2*c^7*d^{21}*e^4 + 264*a^{16}*b^3*c^6*d^{20}*e^5 - 520*a^{16}*b^4*c^5*d^{19}*e^6 + 336*a^{16}*b^5*c^4*d^{18}*e^7 + 576*a^{17}*b^2*c^6*d^{19}*e^6 - 504*a^{17}*b^3*c^5*d^{18}*e^7 + 8*a^{18}*b^3*c^4*d^{16}*e^9 - 16*a^{18}*b^4*c^3*d^{15}*e^{10} + 8*a^{18}*b^5*c^2*d^{14}*e^{11} + 96*a^{19}*b^2*c^4*d^{15}*e^{10} - 56*a^{19}*b^3*c^3*d^{14}*e^{11} - 32*a^{16}*b*c^8*d^{22}*e^3 - 192*a^{17}*b*c^7*d^{20}*e^5 + 224*a^{18}*b*c^6*d^{18}*e^7 - 32*a^{19}*b*c^5*d^{16}*e^9 + 96*a^{20}*b*c^4*d^{14}*e^{11})*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2) + 4*a^{15}*c^9*d^{21}*e^3 + 4*a^{16}*c^8*d^{19}*e^5 - 48*a^{18}*c^6*d^{15}*e^9 + 4*a^{14}*b^2*c^8*d^{21}*e^3 + 4*a^{14}*b^7*c^3*d^{16}*e^8 - 4*a^{14}*b^8*c^2*d^{15}*e^9 - 36*a^{15}*b^5*c^4*d^{16}*e^8 + 44*a^{15}*b^6*c^3*d^{15}*e^9 - 4*a^{15}*b^7*c^2*d^{14}*e^{10} + 100*a^{16}*b^3*c^5*d^{16}*e^8 - 160*a^{16}*b^4*c^4*d^{15}*e^9 + 32*a^{16}*b^5*c^3*d^{14}*e^{10} + 204*a^{17}*b^2*c^5*d^{15}*e^9 - 76*a^{17}*b^3*c^4*d^{14}*e^{10} - 4*a^{14}*b*c^9*d^{22}*e^2 - 8*a^{15}*b*c^8*d^{20}*e^4 - 80*a^{17}*b*c^6*d^{16}*e^8 + 48*a^{18}*b*c^5*d^{14}*e^{10}) - x*(2*a^{14}*c^9*d^{18}*e^5 + 4*a^{16}*c^7*d^{14}*e^9 + 2*a^{14}*b^4*c^5*d^{14}*e^9 - 8*a^{15}*b^2*c^6*d^{14}*e^9))*(-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2
\end{aligned}$$

$$\begin{aligned}
& *b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42 \\
& *a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e \\
& + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a \\
& ^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b \\
& ^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6* \\
& d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16* \\
& a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d \\
& ^2*e^2))^{(1/2)} - ((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + \\
& 25*a^2*b^3*c^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2 \\
& *e^2 - 63*a^3*b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c \\
& ^2*d*e - 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d \\
& *e + 76*a^3*b^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^ \\
& 3*c^2*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2 \\
& *a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32 \\
& *a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d* \\
& e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2))^{(1 \\
& /2)}*(((-(b^9*e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5 \\
& *c^3*d^2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c \\
& ^4*d^2 + a^2*c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - 63*a^3 \\
& *b^3*c^3*e^2 - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^ \\
& 5*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} - 5*a*b^4*c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b \\
& ^2*c^4*d*e - 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*d*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^7*b \\
& ^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^ \\
& 3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2* \\
& e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8* \\
& b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2))^{(1/2)}*(x*(-(b^9 \\
& *e^2 + b^7*c^2*d^2 + b^6*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c^3*d^2 - 2 \\
& 0*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 + a^2 \\
& *c^4*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 \\
& - a^3*c^3*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c^2*d^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e - 2*b^5*c*d*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a*b^4 \\
& *c*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e - \\
& 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)
\end{aligned}$$

[Out] Timed out

$$3.232 \quad \int \frac{1}{\sqrt{fx}(d+ex^2)(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=866

$$\frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{e} \sqrt{fx}}{\sqrt[4]{d} \sqrt{f}}\right) e^{7/4}}{\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{e} \sqrt{fx}}{\sqrt[4]{d} \sqrt{f}} + 1\right) e^{7/4}}{\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}} - \frac{\log\left(\sqrt{e} \sqrt{f} x + \sqrt{d} \sqrt{f} - \sqrt{2} \sqrt[4]{d} \sqrt[4]{e} \sqrt{fx}\right) e^{7/4}}{2\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}}$$

Rubi [A] time = 2.51, antiderivative size = 866, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {1269, 1424, 211, 1165, 628, 1162, 617, 204, 1422, 212, 208, 205}

$$\frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{e} \sqrt{fx}}{\sqrt[4]{d} \sqrt{f}}\right) e^{7/4}}{\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{e} \sqrt{fx}}{\sqrt[4]{d} \sqrt{f}} + 1\right) e^{7/4}}{\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}} - \frac{\log\left(\sqrt{e} \sqrt{f} x + \sqrt{d} \sqrt{f} - \sqrt{2} \sqrt[4]{d} \sqrt[4]{e} \sqrt{fx}\right) e^{7/4}}{2\sqrt{2} d^{3/4} (cd^2 - bed + ae^2) \sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])]/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])]/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &

& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1269

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[Int[x^(k*(m + 1) - 1)*(d + (e*x^(2*k))/f^2)^q*(a + (b*x^(2*k))/f^k + (c*x^(4*k))/f^4)^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]

Rule 1422

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rule 1424

Int[((d_) + (e_)*(x_)^(n_))^(q_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{fx} (d + ex^2) (a + bx^2 + cx^4)} dx &= \frac{2 \operatorname{Subst} \left(\int \frac{1}{\left(d + \frac{ex^4}{f^2}\right) \left(a + \frac{bx^4}{f^2} + \frac{cx^8}{f^4}\right)} dx, x, \sqrt{fx} \right)}{f} \\
&= \frac{2 \operatorname{Subst} \left(\int \left(\frac{e^2 f^2}{(cd^2 - bde + ae^2)(df^2 + ex^4)} + \frac{cdf^4 - bef^4 - cef^2 x^4}{(cd^2 - bde + ae^2)(af^4 + bf^2 x^4 + cx^8)} \right) dx, x, \sqrt{fx} \right)}{f} \\
&= \frac{2 \operatorname{Subst} \left(\int \frac{cdf^4 - bef^4 - cef^2 x^4}{af^4 + bf^2 x^4 + cx^8} dx, x, \sqrt{fx} \right)}{(cd^2 - bde + ae^2) f} + \frac{(2e^2 f) \operatorname{Subst} \left(\int \frac{1}{df^2 + ex^4} dx, x, \sqrt{fx} \right)}{cd^2 - bde + ae^2} \\
&= \frac{e^2 \operatorname{Subst} \left(\int \frac{\sqrt{d} f - \sqrt{e} x^2}{df^2 + ex^4} dx, x, \sqrt{fx} \right)}{\sqrt{d} (cd^2 - bde + ae^2)} + \frac{e^2 \operatorname{Subst} \left(\int \frac{\sqrt{d} f + \sqrt{e} x^2}{df^2 + ex^4} dx, x, \sqrt{fx} \right)}{\sqrt{d} (cd^2 - bde + ae^2)} \\
&= \frac{e^{3/2} \operatorname{Subst} \left(\int \frac{1}{\frac{\sqrt{d} f}{\sqrt{e}} - \frac{\sqrt{2} \sqrt[4]{d} \sqrt{fx}}{\sqrt[4]{e}} + x^2} dx, x, \sqrt{fx} \right)}{2\sqrt{d} (cd^2 - bde + ae^2)} + \frac{e^{3/2} \operatorname{Subst} \left(\int \frac{1}{\frac{\sqrt{d} f}{\sqrt{e}} + \frac{\sqrt{2} \sqrt[4]{d} \sqrt{fx}}{\sqrt[4]{e}}} dx, x, \sqrt{fx} \right)}{2\sqrt{d} (cd^2 - bde + ae^2)} \\
&= \frac{c^{3/4} \left(2cd - \left(b - \sqrt{b^2 - 4ac} \right) e \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} \sqrt{fx}}{\sqrt{-b - \sqrt{b^2 - 4ac}} \sqrt{f}} \right)}{\sqrt[4]{2} \sqrt{b^2 - 4ac} \left(-b - \sqrt{b^2 - 4ac} \right)^{3/4} (cd^2 - bde + ae^2) \sqrt{f}} - \frac{c^{3/4} \left(2cd - \left(b + \sqrt{b^2 - 4ac} \right) e \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} \sqrt{fx}}{\sqrt{-b + \sqrt{b^2 - 4ac}} \sqrt{f}} \right)}{\sqrt[4]{2} \sqrt{b^2 - 4ac} \left(-b + \sqrt{b^2 - 4ac} \right)^{3/4} (cd^2 - bde + ae^2) \sqrt{f}} \\
&= \frac{c^{3/4} \left(2cd - \left(b - \sqrt{b^2 - 4ac} \right) e \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} \sqrt{fx}}{\sqrt{-b - \sqrt{b^2 - 4ac}} \sqrt{f}} \right)}{\sqrt[4]{2} \sqrt{b^2 - 4ac} \left(-b - \sqrt{b^2 - 4ac} \right)^{3/4} (cd^2 - bde + ae^2) \sqrt{f}} - \frac{c^{3/4} \left(2cd - \left(b + \sqrt{b^2 - 4ac} \right) e \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} \sqrt{fx}}{\sqrt{-b + \sqrt{b^2 - 4ac}} \sqrt{f}} \right)}{\sqrt[4]{2} \sqrt{b^2 - 4ac} \left(-b + \sqrt{b^2 - 4ac} \right)^{3/4} (cd^2 - bde + ae^2) \sqrt{f}}
\end{aligned}$$

Mathematica [C] time = 0.38, size = 267, normalized size = 0.31

$$\frac{\sqrt{x} \left(\sqrt{2} e^{7/4} \left(-\log \left(-\sqrt{2} \sqrt[4]{d} \sqrt[4]{e} \sqrt{x} + \sqrt{d} + \sqrt{e} x \right) + \log \left(\sqrt{2} \sqrt[4]{d} \sqrt[4]{e} \sqrt{x} + \sqrt{d} + \sqrt{e} x \right) - 2 \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{d}} \right) + 2 \tan^{-1} \left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt{x}}{\sqrt{d}} + 1 \right) \right) - 2 d^{3/4} \operatorname{RootSum} \left[\#1^5 c + \#1^4 b + a \&, \frac{\#1^4 c \log(\sqrt{x} - \#1) + b c \log(\sqrt{x} - \#1) - c d \log(\sqrt{x} - \#1)}{2 \#1^3 c + \#1^2 b} \right]}{4 d^{3/4} \sqrt{fx} (e(ae - bd) + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[x]*(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - Log[Sqrt[d] - Sqrt[2]*

$d^{1/4}e^{1/4}\sqrt{x} + \sqrt{e}x + \text{Log}[\sqrt{d} + \sqrt{2}d^{1/4}e^{1/4}]\sqrt{x} + \sqrt{e}x) - 2d^{3/4}\text{RootSum}[a + b\#1^4 + c\#1^8 \& , (-c*d*\text{Log}[\sqrt{x} - \#1]) + b*e*\text{Log}[\sqrt{x} - \#1] + c*e*\text{Log}[\sqrt{x} - \#1]*\#1^4)/(b*\#1^3 + 2*c*\#1^7) \&])/(4*d^{3/4}*(c*d^2 + e*(-(b*d) + a*e))*\sqrt{f*x})$

IntegrateAlgebraic [C] time = 0.40, size = 308, normalized size = 0.36

$$\frac{\text{RootSum}\left[\#1^8c + \#1^4bf^2 + af^4\&, \frac{\#1^4cef\log(\sqrt{f*x-\#1})+bcf^3\log(\sqrt{f*x-\#1})-cdf^3\log(\sqrt{f*x-\#1})\&}{2\#1^7c+\#1^3bf^2}\& \right]}{2(ae^2 - bde + cd^2)} - \frac{e^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{f} - \sqrt[4]{e}\sqrt{f*x}}{\sqrt{2}\sqrt[4]{e}\sqrt{2}\sqrt[4]{d}}\right)}{\sqrt{2}d^{3/4}\sqrt{f}(ae^2 - bde + cd^2)} + \frac{e^{7/4} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{f}\sqrt{f*x}}{\sqrt{d}f + \sqrt{e}f*x}\right)}{\sqrt{2}d^{3/4}\sqrt{f}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] $-\left(\frac{e^{7/4}\text{ArcTan}\left[\frac{d^{1/4}\sqrt{f}}{\sqrt{2}e^{1/4}}\right] - e^{1/4}\sqrt{f}x}{\sqrt{2}d^{1/4}}\right)/\sqrt{f*x})/\left(\sqrt{2}d^{3/4}(c*d^2 - b*d*e + a*e^2)*\sqrt{f}\right) + \left(\frac{e^{7/4}\text{ArcTanh}\left[\frac{\sqrt{2}d^{1/4}e^{1/4}\sqrt{f}\sqrt{f*x}}{\sqrt{d}f + \sqrt{e}f*x}\right]}{\sqrt{2}d^{3/4}(c*d^2 - b*d*e + a*e^2)*\sqrt{f}}\right) - \text{RootSum}[a*f^4 + b*f^2*\#1^4 + c*\#1^8 \& , (-c*d*f^3*\text{Log}[\sqrt{f*x} - \#1]) + b*e*f^3*\text{Log}[\sqrt{f*x} - \#1] + c*e*f*\text{Log}[\sqrt{f*x} - \#1]*\#1^4)/(b*f^2*\#1^3 + 2*c*\#1^7) \&]/(2*(c*d^2 - b*d*e + a*e^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")

[Out] sage2

maple [C] time = 0.10, size = 336, normalized size = 0.39

$$\frac{f\left(-\text{RootOf}(c_Z^8 + b f^2_Z^4 + a f^4)ce - be f^2 + cd f^2\right)\ln\left(-\text{RootOf}(c_Z^8 + b f^2_Z^4 + a f^4) + \sqrt{f x}\right)}{2(ae^2 - deb + cd^2)\left(2\text{RootOf}(c_Z^8 + b f^2_Z^4 + a f^4)^7c + \text{RootOf}(c_Z^8 + b f^2_Z^4 + a f^4)^3b f^2\right)} + \frac{\left(\frac{df}{c}\right)^{\frac{1}{4}}\sqrt{2}e^2\arctan\left(\frac{\sqrt{2}\sqrt{f x}-1}{\left(\frac{df}{c}\right)^{\frac{1}{4}}}\right)}{2(ae^2 - deb + cd^2)df} + \frac{\left(\frac{df}{c}\right)^{\frac{1}{4}}\sqrt{2}e^2\arctan\left(\frac{\sqrt{2}\sqrt{f x}+1}{\left(\frac{df}{c}\right)^{\frac{1}{4}}}\right)}{2(ae^2 - deb + cd^2)df} + \frac{\left(\frac{df}{c}\right)^{\frac{1}{4}}\sqrt{2}e^2\ln\left(\frac{f x+\left(\frac{df}{c}\right)^{\frac{1}{4}}\sqrt{f x}\sqrt{2}+\sqrt{\frac{df}{c}}}{f x-\left(\frac{df}{c}\right)^{\frac{1}{4}}\sqrt{f x}\sqrt{2}+\sqrt{\frac{df}{c}}}\right)}{4(ae^2 - deb + cd^2)df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x)`

[Out] $\frac{1}{2} \frac{f}{(a e^2 - b d e + c d^2)} \sum \left(-R^4 c e - b e f^2 + c d f^2 \right) / \left(2 R^7 c + R^3 b f^2 \right) \ln \left((f x)^{1/2} - R \right), R = \text{RootOf} \left(Z^8 c + Z^4 b f^2 + a f^4 \right) + \frac{1}{4} \frac{f e^2}{(a e^2 - b d e + c d^2)} \left(\frac{d f^2 / e}{d^2} \right)^{1/4} / d^{1/2} \ln \left(\frac{(f x + (d f^2 / e)^{1/4}) (f x)^{1/2}}{2^{1/2} + (d f^2 / e)^{1/4}} \right) + \frac{1}{2} \frac{f e^2}{(a e^2 - b d e + c d^2)} \left(\frac{d f^2 / e}{d^2} \right)^{1/4} / d^{1/2} \arctan \left(\frac{2^{1/2}}{(d f^2 / e)^{1/4} (f x)^{1/2} + 1} \right) + \frac{1}{2} \frac{f e^2}{(a e^2 - b d e + c d^2)} \left(\frac{d f^2 / e}{d^2} \right)^{1/4} / d^{1/2} \arctan \left(\frac{2^{1/2}}{(d f^2 / e)^{1/4} (f x)^{1/2} - 1} \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \sqrt{d} \arctan \left(\frac{\sqrt{d} \sqrt{a^2 + c d}}{2 \sqrt{d} \sqrt{e}} \right)}{c d^3 \sqrt{f} - b d^2 e \sqrt{f} + a d e^2 \sqrt{f}} + \frac{2 \sqrt{d} \arctan \left(\frac{\sqrt{d} \sqrt{a^2 + c d}}{2 \sqrt{d} \sqrt{e}} \right)}{4 \sqrt{d} \sqrt{e}} + \frac{\sqrt{d} \sqrt{a^2 + c d} \left(\sqrt{2} d^{1/4} \sqrt{e} + \sqrt{e} \sqrt{d} \right)}{d^2} - \frac{\sqrt{d} \sqrt{a^2 + c d} \left(-\sqrt{2} d^{1/4} \sqrt{e} + \sqrt{e} \sqrt{d} \right)}{d^2} + \frac{2 \sqrt{d}}{a d \sqrt{f}} + \int \frac{(c^2 d - b c e)^2 + (b c d - b^2 e + a c e)^2}{a^2 d^2 \sqrt{f} + (a^2 c d^2 \sqrt{f} + (c^2 d^2 \sqrt{f} - b c d e \sqrt{f})^2) + (c d^2 \sqrt{f} - b d e \sqrt{f})^2 + (a^2 b e^2 \sqrt{f} + (b a d^2 \sqrt{f} - b^2 d e \sqrt{f})^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")`

[Out] $-2 e^2 \sqrt{x} / (c d^3 \sqrt{f} - b d^2 e \sqrt{f} + a d e^2 \sqrt{f}) + \frac{1}{4} \left(2 \sqrt{2} e^2 \arctan \left(\frac{1}{2} \sqrt{2} \right) \left(\sqrt{2} d^{1/4} e^{1/4} + 2 \sqrt{e} \sqrt{x} \right) / \sqrt{d \sqrt{e}} + 2 \sqrt{2} e^2 \arctan \left(-\frac{1}{2} \sqrt{2} \right) \left(\sqrt{2} d^{1/4} e^{1/4} - 2 \sqrt{e} \sqrt{x} \right) / \sqrt{d \sqrt{e}} \right) / \left(\sqrt{d} \sqrt{d \sqrt{e}} \right) + \frac{\sqrt{2} e^{7/4} \log \left(\sqrt{2} d^{1/4} e^{1/4} \sqrt{x} + \sqrt{e} x + \sqrt{d} \right) / d^{3/4} - \sqrt{2} e^{7/4} \log \left(-\sqrt{2} d^{1/4} e^{1/4} \sqrt{x} + \sqrt{e} x + \sqrt{d} \right) / d^{3/4}}{c d^2 \sqrt{f} - b d e \sqrt{f} + a e^2 \sqrt{f}} + \frac{2 \sqrt{x}}{a d \sqrt{f}} + \int \frac{-((c^2 d - b c e) x^{7/2} + (b c d - b^2 e + a c e) x^{3/2}) / (a^3 e^2 \sqrt{f} + (a^2 c e^2 \sqrt{f} + (c^2 d^2 \sqrt{f} - b c d e \sqrt{f})^2) a) x^4 + (c d^2 \sqrt{f} - b d e \sqrt{f}) a^2 + (a^2 b e^2 \sqrt{f} + (b c d^2 \sqrt{f} - b^2 d e \sqrt{f})^2) a) x^2}{(c d^3 \sqrt{f} - b d^2 e \sqrt{f} + a d e^2 \sqrt{f})^2} dx$

mupad [B] time = 6.84, size = 43112, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)`

[Out] $\text{symsum} \left(\log \left(-\text{root} \left(8388608 a^7 b^2 c^{11} d^{18} e^6 f^6 h^{12} - 513802240 a^{10} b^2 c^7 d^{11} e^8 f^6 h^{12} - 381681664 a^{11} b^2 c^6 d^9 e^{10} f^6 h^{12} - 381681664 a^9 b^2 c^8 d^{13} e^6 f^6 h^{12} - 300941312 a^9 b^5 c^5 d^{10} e^9 f^6 h^{12} - 300941312 a^8 b^5 c^6 d^{12} e^7 f^6 h^{12} + 293601280 a^{10} b^3 c^6 d^{10} e^9 f^6 h^{12} + 293601280 a^9 b^3 c^7 d^{12} e^7 f^6 h^{12} - 168820736 a^{10} b^5 c^4 d^8 e^{11} f^6 h^{12} - 168820736 a^7 b^5 c^7 d^{14} e^5 f^6 h^{12} + 166068224 a^8 \right) \right)$

$$\begin{aligned}
& b^6c^5d^{11}e^8f^6h^{12} - 146800640a^{12}b^2c^5d^7e^{12}f^6h^{12} - 146800640a^8b^2c^9d^{15}e^4f^6h^{12} + 124780544a^{10}b^4c^5d^9e^{10}f^6h^{12} \\
& + 124780544a^8b^4c^7d^{13}e^6f^6h^{12} + 119275520a^9b^4c^6d^{11}e^8f^6h^{12} + 117440512a^{11}b^3c^5d^8e^{11}f^6h^{12} + 117440512a^8b^3 \\
& *c^8d^{14}e^5f^6h^{12} + 102760448a^9b^6c^4d^9e^{10}f^6h^{12} + 102760448a^7b^6c^6d^{13}e^6f^6h^{12} + 91750400a^{11}b^4c^4d^7e^{12}f^6h^{12} + \\
& 91750400a^7b^4c^8d^{15}e^4f^6h^{12} - 71065600a^7b^8c^4d^{11}e^8f^6h^{12} - 53444608a^8b^8c^3d^9e^{10}f^6h^{12} - 53444608a^6b^8c^5d^{13} \\
& e^6f^6h^{12} + 40370176a^9b^7c^3d^8e^{11}f^6h^{12} + 40370176a^6b^7c^6d^{14}e^5f^6h^{12} - 36700160a^{11}b^5c^3d^6e^{13}f^6h^{12} - 36700160a^6 \\
& b^5c^8d^{16}e^3f^6h^{12} + 34078720a^8b^7c^4d^{10}e^9f^6h^{12} + 34078720a^7b^7c^5d^{12}e^7f^6h^{12} + 26214400a^{12}b^4c^3d^5e^{14}f^6h^{12} \\
& + 26214400a^6b^4c^9d^{17}e^2f^6h^{12} + 22118400a^7b^9c^3d^{10}e^9f^6h^{12} + 22118400a^6b^9c^4d^{12}e^7f^6h^{12} - 20971520a^{13}b^2c^4d^5 \\
& e^{14}f^6h^{12} - 20971520a^7b^2c^{10}d^{17}e^2f^6h^{12} + 18350080a^{10}b^7c^2d^6e^{13}f^6h^{12} + 18350080a^5b^7c^7d^{16}e^3f^6h^{12} - 16629760a^9 \\
& b^8c^2d^7e^{12}f^6h^{12} - 16629760a^5b^8c^6d^{15}e^4f^6h^{12} - 10485760a^{11}b^6c^2d^5e^{14}f^6h^{12} - 10485760a^5b^6c^8d^{17}e^2f^6 \\
& h^{12} + 9175040a^{10}b^6c^3d^7e^{12}f^6h^{12} + 9175040a^6b^6c^7d^{15}e^4f^6h^{12} - 8388608a^{13}b^3c^3d^4e^{15}f^6h^{12} + 5619712a^7b^{10}c^2 \\
& d^9e^{10}f^6h^{12} + 5619712a^5b^{10}c^4d^{13}e^6f^6h^{12} - 5570560a^6b^{11}c^2d^{10}e^9f^6h^{12} - 5570560a^5b^{11}c^3d^{12}e^7f^6h^{12} + 4358144 \\
& a^8b^9c^2d^8e^{11}f^6h^{12} + 4358144a^5b^9c^5d^{14}e^5f^6h^{12} + 4259840a^6b^{10}c^3d^{11}e^8f^6h^{12} + 3899392a^4b^{10}c^5d^{15}e^4f^6h^{12} \\
& - 3440640a^4b^9c^6d^{16}e^3f^6h^{12} + 3145728a^{12}b^5c^2d^4e^15f^6h^{12} - 2523136a^4b^{11}c^4d^{14}e^5f^6h^{12} + 1802240a^4b^8c^7d^{17} \\
& e^2f^6h^{12} + 1556480a^5b^{12}c^2d^{11}e^8f^6h^{12} + 1048576a^{14}b^2c^3d^3e^{16}f^6h^{12} + 688128a^4b^{12}c^3d^{13}e^6f^6h^{12} - 393216a^{13} \\
& b^4c^2d^3e^{16}f^6h^{12} - 286720a^3b^{12}c^4d^{15}e^4f^6h^{12} + 229376a^3b^{13}c^3d^{14}e^5f^6h^{12} + 229376a^3b^{11}c^5d^{16}e^3f^6h^{12} + \\
& 163840a^4b^{13}c^2d^{12}e^7f^6h^{12} - 114688a^3b^{14}c^2d^{13}e^6f^6h^{12} - 114688a^3b^{10}c^6d^{17}e^2f^6h^{12} + 293601280a^{11}b^c^7d^{10}e^9 \\
& f^6h^{12} + 293601280a^{10}b^c^8d^{12}e^7f^6h^{12} + 176160768a^{12}b^c^6d^8e^{11}f^6h^{12} + 176160768a^9b^c^9d^{14}e^5f^6h^{12} + 58720256a^{13}b^c^5 \\
& d^6e^{13}f^6h^{12} + 58720256a^8b^c^{10}d^{16}e^3f^6h^{12} + 8388608a^{14}b^c^4d^4e^{15}f^6h^{12} - 8388608a^6b^3c^{10}d^{18}e^5f^6h^{12} + 3899392a^8 \\
& b^{10}c^d^7e^{12}f^6h^{12} - 3440640a^9b^9c^d^6e^{13}f^6h^{12} + 3145728a^5b^5c^9d^{18}e^5f^6h^{12} - 2523136a^7b^{11}c^d^8e^{11}f^6h^{12} + 1802240 \\
& a^{10}b^8c^d^5e^{14}f^6h^{12} + 688128a^6b^{12}c^d^9e^{10}f^6h^{12} - 524288a^{11}b^7c^d^4e^{15}f^6h^{12} - 524288a^4b^7c^8d^{18}e^5f^6h^{12} + 163840 \\
& a^5b^{13}c^d^{10}e^9f^6h^{12} - 163840a^4b^{14}c^d^{11}e^8f^6h^{12} + 65536a^{12}b^6c^d^3e^{16}f^6h^{12} + 32768a^3b^{15}c^d^{12}e^7f^6h^{12} + 32768 \\
& a^3b^9c^7d^{18}e^5f^6h^{12} - 73400320a^{11}c^8d^{11}e^8f^6h^{12} - 58720256a^{12}c^7d^9e^{10}f^6h^{12} - 58720256a^{10}c^9d^{13}e^6f^6h^{12} - 29360128 \\
& a^{13}c^6d^7e^{12}f^6h^{12} - 29360128a^9c^{10}d^{15}e^4f^6h^{12} - 8
\end{aligned}$$

$$\begin{aligned}
& 388608*a^{14}*c^5*d^5*e^{14}*f^6*h^{12} - 8388608*a^8*c^{11}*d^{17}*e^{2}*f^6*h^{12} - 10 \\
& 48576*a^{15}*c^4*d^3*e^{16}*f^6*h^{12} - 286720*a^7*b^{12}*d^7*e^{12}*f^6*h^{12} + 2293 \\
& 76*a^8*b^{11}*d^6*e^{13}*f^6*h^{12} + 229376*a^6*b^{13}*d^8*e^{11}*f^6*h^{12} - 114688* \\
& a^9*b^{10}*d^5*e^{14}*f^6*h^{12} - 114688*a^5*b^{14}*d^9*e^{10}*f^6*h^{12} + 32768*a^{10} \\
& *b^9*d^4*e^{15}*f^6*h^{12} + 32768*a^4*b^{15}*d^{10}*e^9*f^6*h^{12} - 4096*a^{11}*b^8*d \\
& ^3*e^{16}*f^6*h^{12} - 4096*a^3*b^{16}*d^{11}*e^8*f^6*h^{12} + 1048576*a^6*b^2*c^{11}*d \\
& ^{19}*f^6*h^{12} - 393216*a^5*b^4*c^{10}*d^{19}*f^6*h^{12} + 65536*a^4*b^6*c^9*d^{19}*f \\
& ^6*h^{12} - 4096*a^3*b^8*c^8*d^{19}*f^6*h^{12} - 1048576*a^7*c^{12}*d^{19}*f^6*h^{12} + \\
& 262144*a^{10}*b*c^4*d*e^{14}*f^4*h^8 - 23552*a*b^6*c^8*d^{14}*e*f^4*h^8 - 16384* \\
& a^7*b^7*c*d*e^{14}*f^4*h^8 - 3328*a*b^{13}*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5* \\
& c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4 \\
& *c^7*d^{10}*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b \\
& ^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^{10}*e^5*f^4*h^8 - 1433600*a^5 \\
& *b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^ \\
& 3*b^4*c^8*d^{12}*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^{10}*f^4*h^8 + 1106176 \\
& *a^3*b^5*c^7*d^{11}*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4*h^8 - 838656 \\
& *a^2*b^7*c^6*d^{11}*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4*h^8 + 730880 \\
& *a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^{12}*e^3*f^4*h^8 + 686080 \\
& *a^7*b^4*c^4*d^4*e^{11}*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4*h^8 - 595968 \\
& *a^8*b^3*c^4*d^3*e^{12}*f^4*h^8 + 544768*a^3*b^3*c^9*d^{13}*e^2*f^4*h^8 + 51609 \\
& 6*a^2*b^8*c^5*d^{10}*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^{10}*f^4*h^8 + 3932 \\
& 16*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^{12}*e^3*f^4*h^8 - 3665 \\
& 92*a^6*b^6*c^3*d^4*e^{11}*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4*h^8 - 3563 \\
& 52*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^{13}*e^2*f^4*h^8 - 3440 \\
& 64*a^8*b^2*c^5*d^4*e^{11}*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^{13}*f^4*h^8 + 210 \\
& 944*a^4*b^3*c^8*d^{11}*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4*h^8 - 144 \\
& 640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^{13}*f^4*h^8 - 131 \\
& 072*a^7*b^6*c^2*d^2*e^{13}*f^4*h^8 - 129024*a^3*b^6*c^6*d^{10}*e^5*f^4*h^8 - 10 \\
& 4448*a^2*b^{10}*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^{11}*f^4*h^8 + 91 \\
& 904*a^7*b^5*c^3*d^3*e^{12}*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^{10}*f^4*h^8 - 716 \\
& 80*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^{11}*c^2*d^7*e^8*f^4*h^8 + 36864 \\
& *a^5*b^7*c^3*d^5*e^{10}*f^4*h^8 - 35328*a^3*b^{10}*c^2*d^6*e^9*f^4*h^8 + 27136* \\
& a^6*b^7*c^2*d^3*e^{12}*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^{10}*f^4*h^8 + 815104*a \\
& ^9*b*c^5*d^3*e^{12}*f^4*h^8 - 651264*a^5*b*c^9*d^{11}*e^4*f^4*h^8 - 573440*a^6* \\
& b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^{14}*f^4*h^8 + 217088*a^7*b*c^ \\
& 7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^{11}*e^4*f^4*h^8 - 204800*a^4*b*c^{10}*d \\
& ^{13}*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^{12}*e^3*f^4*h^8 - 157696*a*b^{10}*c^4*d^{1} \\
& 0*e^5*f^4*h^8 - 131072*a^3*b^2*c^{10}*d^{14}*e*f^4*h^8 + 98304*a^8*b^5*c^2*d*e^ \\
& 14*f^4*h^8 + 92160*a^2*b^4*c^9*d^{14}*e*f^4*h^8 + 84992*a*b^7*c^7*d^{13}*e^2*f^ \\
& 4*h^8 + 64512*a*b^{11}*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2*e^{13}*f^4*h^8 \\
& + 18944*a^3*b^{11}*c*d^5*e^{10}*f^4*h^8 - 13312*a^4*b^{10}*c*d^4*e^{11}*f^4*h^8 - \\
& 9472*a^5*b^9*c*d^3*e^{12}*f^4*h^8 - 8192*a*b^{12}*c^2*d^8*e^7*f^4*h^8 - 6144*a^ \\
& 2*b^{12}*c*d^6*e^9*f^4*h^8 - 17920*b^{11}*c^4*d^{11}*e^4*f^4*h^8 + 14336*b^{12}*c^3 \\
& *d^{10}*e^5*f^4*h^8 + 14336*b^{10}*c^5*d^{12}*e^3*f^4*h^8 - 7168*b^{13}*c^2*d^9*e^6 \\
& *f^4*h^8 - 7168*b^9*c^6*d^{13}*e^2*f^4*h^8 - 425984*a^9*c^6*d^4*e^{11}*f^4*h^8
\end{aligned}$$

$$\begin{aligned}
& - 360448a^8c^7d^6e^9f^4h^8 - 262144a^{10}c^5d^2e^{13}f^4h^8 - 131072a^7c^8d^8e^7f^4h^8 + 98304a^5c^{10}d^{12}e^3f^4h^8 + 65536a^6c^9d^{10}e^5f^4h^8 - 1536a^5b^{10}d^2e^{13}f^4h^8 - 1536a^2b^{13}d^5e^{10}f^4h^8 + 768a^4b^{11}d^3e^{12}f^4h^8 + 768a^3b^{12}d^4e^{11}f^4h^8 + 65536a^{10}b^2c^3e^{15}f^4h^8 - 24576a^9b^4c^2e^{15}f^4h^8 - 10240a^2b^3c^{10}d^{15}f^4h^8 + 2048b^{14}c^8d^8e^7f^4h^8 + 2048b^8c^7d^{14}e^7f^4h^8 + 32768a^4c^{11}d^{14}e^7f^4h^8 + 1024a^6b^9d^8e^{14}f^4h^8 + 1024a^6b^{14}d^6e^9f^4h^8 + 4096a^8b^6c^2e^{15}f^4h^8 + 12288a^3b^6c^{11}d^{15}f^4h^8 + 2816a^6b^5c^9d^{15}f^4h^8 - 256b^{15}d^7e^8f^4h^8 - 65536a^{11}c^4e^{15}f^4h^8 - 256b^7c^8d^{15}f^4h^8 - 256a^7b^8e^{15}f^4h^8 - 896a^6b^8c^2d^8e^{10}f^2h^4 + 192a^6b^8c^9d^8e^3f^2h^4 + 11520a^3b^3c^5d^2e^9f^2h^4 - 5856a^2b^5c^4d^2e^9f^2h^4 - 5120a^3b^2c^6d^3e^8f^2h^4 + 3200a^2b^4c^5d^3e^8f^2h^4 - 640a^2b^3c^6d^4e^7f^2h^4 - 96a^2b^2c^7d^5e^6f^2h^4 - 10880a^3b^4c^4d^8e^{10}f^2h^4 + 10240a^4b^2c^5d^8e^{10}f^2h^4 - 7680a^4b^6c^6d^2e^9f^2h^4 + 4672a^2b^6c^3d^8e^{10}f^2h^4 + 1248a^6b^7c^3d^2e^9f^2h^4 + 832a^3b^6c^7d^4e^7f^2h^4 - 768a^6b^6c^4d^3e^8f^2h^4 + 192a^2b^6c^8d^6e^5f^2h^4 - 192a^6b^2c^8d^7e^4f^2h^4 + 176a^6b^5c^5d^4e^7f^2h^4 + 64a^6b^3c^7d^6e^5f^2h^4 - 96b^9c^2d^2e^9f^2h^4 - 96b^2c^9d^9e^2f^2h^4 + 64b^8c^3d^3e^8f^2h^4 + 64b^3c^8d^8e^3f^2h^4 - 16b^7c^4d^4e^7f^2h^4 - 16b^4c^7d^7e^4f^2h^4 + 2032a^4c^7d^3e^8f^2h^4 - 96a^2c^9d^7e^4f^2h^4 - 64a^3c^8d^5e^6f^2h^4 - 4480a^4b^3c^4e^{11}f^2h^4 + 3696a^3b^5c^3e^{11}f^2h^4 - 1376a^2b^7c^2e^{11}f^2h^4 - 2048a^5c^6d^8e^{10}f^2h^4 - 64a^6c^{10}d^9e^2f^2h^4 + 1792a^5b^6c^5e^{11}f^2h^4 + 64b^{10}c^8d^8e^{10}f^2h^4 + 64b^6c^{10}d^{10}e^7f^2h^4 + 240a^6b^9c^8e^{11}f^2h^4 - 16c^{11}d^{11}f^2h^4 - 16b^{11}e^{11}f^2h^4 - c^7e^7, h, k) \cdot (\text{root}(8388608a^7b^6c^{11}d^{18}e^6f^6h^{12} - 513802240a^{10}b^2c^7d^{11}e^8f^6h^{12} - 381681664a^{11}b^2c^6d^9e^{10}f^6h^{12} - 381681664a^9b^2c^8d^{13}e^6f^6h^{12} - 300941312a^9b^5c^5d^{10}e^9f^6h^{12} - 300941312a^8b^5c^6d^{12}e^7f^6h^{12} + 293601280a^{10}b^3c^6d^{10}e^9f^6h^{12} + 293601280a^9b^3c^7d^{12}e^7f^6h^{12} - 168820736a^{10}b^5c^4d^8e^{11}f^6h^{12} - 168820736a^7b^5c^7d^{14}e^5f^6h^{12} + 166068224a^8b^6c^5d^{11}e^8f^6h^{12} - 146800640a^{12}b^2c^5d^7e^{12}f^6h^{12} - 146800640a^8b^2c^9d^{15}e^4f^6h^{12} + 124780544a^{10}b^4c^5d^9e^{10}f^6h^{12} + 124780544a^8b^4c^7d^{13}e^6f^6h^{12} + 119275520a^9b^4c^6d^{11}e^8f^6h^{12} + 117440512a^{11}b^3c^5d^8e^{11}f^6h^{12} + 117440512a^8b^3c^8d^{14}e^5f^6h^{12} + 102760448a^9b^6c^4d^9e^{10}f^6h^{12} + 102760448a^7b^6c^6d^{13}e^6f^6h^{12} + 91750400a^{11}b^4c^4d^7e^{12}f^6h^{12} + 91750400a^7b^4c^8d^{15}e^4f^6h^{12} - 71065600a^7b^8c^4d^{11}e^8f^6h^{12} - 53444608a^8b^8c^3d^9e^{10}f^6h^{12} - 53444608a^6b^8c^5d^{13}e^6f^6h^{12} + 40370176a^9b^7c^3d^8e^{11}f^6h^{12} + 40370176a^6b^7c^6d^{14}e^5f^6h^{12} - 36700160a^{11}b^5c^3d^6e^{13}f^6h^{12} - 36700160a^6b^5c^8d^{16}e^3f^6h^{12} + 34078720a^8b^7c^4d^{10}e^9f^6h^{12} + 34078720a^7b^7c^5d^{12}e^7f^6h^{12} + 26214400a^{12}b^4c^3d^5e^{14}f^6h^{12} + 26214400a^6b^4c^9d^{17}e^2f^6h^{12} + 22118400a^7b^9
\end{aligned}$$

$$\begin{aligned}
& *c^3*d^{10}*e^9*f^6*h^{12} + 22118400*a^6*b^9*c^4*d^{12}*e^7*f^6*h^{12} - 20971520* \\
& a^{13}*b^2*c^4*d^5*e^{14}*f^6*h^{12} - 20971520*a^7*b^2*c^{10}*d^{17}*e^2*f^6*h^{12} + \\
& 18350080*a^{10}*b^7*c^2*d^6*e^{13}*f^6*h^{12} + 18350080*a^5*b^7*c^7*d^{16}*e^3*f^6 \\
& *h^{12} - 16629760*a^9*b^8*c^2*d^7*e^{12}*f^6*h^{12} - 16629760*a^5*b^8*c^6*d^{15} \\
& e^4*f^6*h^{12} - 10485760*a^{11}*b^6*c^2*d^5*e^{14}*f^6*h^{12} - 10485760*a^5*b^6*c \\
& ^8*d^{17}*e^2*f^6*h^{12} + 9175040*a^{10}*b^6*c^3*d^7*e^{12}*f^6*h^{12} + 9175040*a^6 \\
& *b^6*c^7*d^{15}*e^4*f^6*h^{12} - 8388608*a^{13}*b^3*c^3*d^4*e^{15}*f^6*h^{12} + 56197 \\
& 12*a^7*b^{10}*c^2*d^9*e^{10}*f^6*h^{12} + 5619712*a^5*b^{10}*c^4*d^{13}*e^6*f^6*h^{12} \\
& - 5570560*a^6*b^{11}*c^2*d^{10}*e^9*f^6*h^{12} - 5570560*a^5*b^{11}*c^3*d^{12}*e^7*f^ \\
& 6*h^{12} + 4358144*a^8*b^9*c^2*d^8*e^{11}*f^6*h^{12} + 4358144*a^5*b^9*c^5*d^{14}*e \\
& ^5*f^6*h^{12} + 4259840*a^6*b^{10}*c^3*d^{11}*e^8*f^6*h^{12} + 3899392*a^4*b^{10}*c^5 \\
& *d^{15}*e^4*f^6*h^{12} - 3440640*a^4*b^9*c^6*d^{16}*e^3*f^6*h^{12} + 3145728*a^{12}*b \\
& ^5*c^2*d^4*e^{15}*f^6*h^{12} - 2523136*a^4*b^{11}*c^4*d^{14}*e^5*f^6*h^{12} + 1802240 \\
& *a^4*b^8*c^7*d^{17}*e^2*f^6*h^{12} + 1556480*a^5*b^{12}*c^2*d^{11}*e^8*f^6*h^{12} + 1 \\
& 048576*a^{14}*b^2*c^3*d^3*e^{16}*f^6*h^{12} + 688128*a^4*b^{12}*c^3*d^{13}*e^6*f^6*h^ \\
& 12 - 393216*a^{13}*b^4*c^2*d^3*e^{16}*f^6*h^{12} - 286720*a^3*b^{12}*c^4*d^{15}*e^4*f \\
& ^6*h^{12} + 229376*a^3*b^{13}*c^3*d^{14}*e^5*f^6*h^{12} + 229376*a^3*b^{11}*c^5*d^{16} \\
& e^3*f^6*h^{12} + 163840*a^4*b^{13}*c^2*d^{12}*e^7*f^6*h^{12} - 114688*a^3*b^{14}*c^2* \\
& d^{13}*e^6*f^6*h^{12} - 114688*a^3*b^{10}*c^6*d^{17}*e^2*f^6*h^{12} + 293601280*a^{11} \\
& *b*c^7*d^{10}*e^9*f^6*h^{12} + 293601280*a^{10}*b*c^8*d^{12}*e^7*f^6*h^{12} + 17616076 \\
& 8*a^{12}*b*c^6*d^8*e^{11}*f^6*h^{12} + 176160768*a^9*b*c^9*d^{14}*e^5*f^6*h^{12} + 58 \\
& 720256*a^{13}*b*c^5*d^6*e^{13}*f^6*h^{12} + 58720256*a^8*b*c^{10}*d^{16}*e^3*f^6*h^{12} \\
& + 8388608*a^{14}*b*c^4*d^4*e^{15}*f^6*h^{12} - 8388608*a^6*b^3*c^{10}*d^{18}*e*f^6*h \\
& ^{12} + 3899392*a^8*b^{10}*c*d^7*e^{12}*f^6*h^{12} - 3440640*a^9*b^9*c*d^6*e^{13}*f^6 \\
& *h^{12} + 3145728*a^5*b^5*c^9*d^{18}*e*f^6*h^{12} - 2523136*a^7*b^{11}*c*d^8*e^{11}*f \\
& ^6*h^{12} + 1802240*a^{10}*b^8*c*d^5*e^{14}*f^6*h^{12} + 688128*a^6*b^{12}*c*d^9*e^{10} \\
& *f^6*h^{12} - 524288*a^{11}*b^7*c*d^4*e^{15}*f^6*h^{12} - 524288*a^4*b^7*c^8*d^{18}*e \\
& *f^6*h^{12} + 163840*a^5*b^{13}*c*d^{10}*e^9*f^6*h^{12} - 163840*a^4*b^{14}*c*d^{11}*e^ \\
& 8*f^6*h^{12} + 65536*a^{12}*b^6*c*d^3*e^{16}*f^6*h^{12} + 32768*a^3*b^{15}*c*d^{12}*e^7 \\
& *f^6*h^{12} + 32768*a^3*b^9*c^7*d^{18}*e*f^6*h^{12} - 73400320*a^{11}*c^8*d^{11}*e^8* \\
& f^6*h^{12} - 58720256*a^{12}*c^7*d^9*e^{10}*f^6*h^{12} - 58720256*a^{10}*c^9*d^{13}*e^6 \\
& *f^6*h^{12} - 29360128*a^{13}*c^6*d^7*e^{12}*f^6*h^{12} - 29360128*a^9*c^{10}*d^{15}*e^ \\
& 4*f^6*h^{12} - 8388608*a^{14}*c^5*d^5*e^{14}*f^6*h^{12} - 8388608*a^8*c^{11}*d^{17}*e^2 \\
& *f^6*h^{12} - 1048576*a^{15}*c^4*d^3*e^{16}*f^6*h^{12} - 286720*a^7*b^{12}*d^7*e^{12}*f \\
& ^6*h^{12} + 229376*a^8*b^{11}*d^6*e^{13}*f^6*h^{12} + 229376*a^6*b^{13}*d^8*e^{11}*f^6* \\
& h^{12} - 114688*a^9*b^{10}*d^5*e^{14}*f^6*h^{12} - 114688*a^5*b^{14}*d^9*e^{10}*f^6*h^{12} \\
& + 32768*a^{10}*b^9*d^4*e^{15}*f^6*h^{12} + 32768*a^4*b^{15}*d^{10}*e^9*f^6*h^{12} - 4 \\
& 096*a^{11}*b^8*d^3*e^{16}*f^6*h^{12} - 4096*a^3*b^{16}*d^{11}*e^8*f^6*h^{12} + 1048576* \\
& a^6*b^2*c^{11}*d^{19}*f^6*h^{12} - 393216*a^5*b^4*c^{10}*d^{19}*f^6*h^{12} + 65536*a^4* \\
& b^6*c^9*d^{19}*f^6*h^{12} - 4096*a^3*b^8*c^8*d^{19}*f^6*h^{12} - 1048576*a^7*c^{12}*d \\
& ^{19}*f^6*h^{12} + 262144*a^{10}*b*c^4*d*e^{14}*f^4*h^8 - 23552*a*b^6*c^8*d^{14}*e*f^ \\
& 4*h^8 - 16384*a^7*b^7*c*d*e^{14}*f^4*h^8 - 3328*a*b^{13}*c*d^7*e^8*f^4*h^8 + 24 \\
& 29952*a^4*b^5*c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1 \\
& 716224*a^4*b^4*c^7*d^{10}*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + \\
& 1584384*a^5*b^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^{10}*e^5*f^4*h^8
\end{aligned}$$

$$\begin{aligned}
& - 1433600*a^5*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 \\
& - 1124352*a^3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4 \\
& *h^8 + 1106176*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*c^4*d^6*e^9*f^4 \\
& *h^8 - 838656*a^2*b^7*c^6*d^11*e^4*f^4*h^8 - 795648*a^3*b^7*c^5*d^9*e^6*f^4 \\
& *h^8 + 730880*a^3*b^8*c^4*d^8*e^7*f^4*h^8 + 714752*a^2*b^6*c^7*d^12*e^3*f^4 \\
& *h^8 + 686080*a^7*b^4*c^4*d^4*e^11*f^4*h^8 + 641024*a^6*b^4*c^5*d^6*e^9*f^4 \\
& *h^8 - 595968*a^8*b^3*c^4*d^3*e^12*f^4*h^8 + 544768*a^3*b^3*c^9*d^13*e^2*f^4 \\
& *h^8 + 516096*a^2*b^8*c^5*d^10*e^5*f^4*h^8 + 441856*a^6*b^5*c^4*d^5*e^10*f^4 \\
& *h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^2*c^9*d^12*e^3*f^4 \\
& *h^8 - 366592*a^6*b^6*c^3*d^4*e^11*f^4*h^8 + 363520*a^4*b^8*c^3*d^6*e^9*f^4 \\
& *h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^5*c^8*d^13*e^2*f^4 \\
& *h^8 - 344064*a^8*b^2*c^5*d^4*e^11*f^4*h^8 + 294912*a^8*b^4*c^3*d^2*e^13*f^4 \\
& *h^8 + 210944*a^4*b^3*c^8*d^11*e^4*f^4*h^8 - 198400*a^3*b^9*c^3*d^7*e^8*f^4 \\
& *h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2*c^4*d^2*e^13*f^4 \\
& *h^8 - 131072*a^7*b^6*c^2*d^2*e^13*f^4*h^8 - 129024*a^3*b^6*c^6*d^10*e^5 \\
& *f^4*h^8 - 104448*a^2*b^10*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8*c^2*d^4*e^11 \\
& *f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^12*f^4*h^8 - 74240*a^4*b^9*c^2*d^5*e^10 \\
& *f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11*c^2*d^7*e^8*f^4 \\
& *h^8 + 36864*a^5*b^7*c^3*d^5*e^10*f^4*h^8 - 35328*a^3*b^10*c^2*d^6*e^9*f^4 \\
& *h^8 + 27136*a^6*b^7*c^2*d^3*e^12*f^4*h^8 + 909312*a^8*b*c^6*d^5*e^10*f^4 \\
& *h^8 + 815104*a^9*b*c^5*d^3*e^12*f^4*h^8 - 651264*a^5*b*c^9*d^11*e^4*f^4*h^8 \\
& - 573440*a^6*b*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d*e^14*f^4*h^8 + 2 \\
& 17088*a^7*b*c^7*d^7*e^8*f^4*h^8 + 211456*a*b^9*c^5*d^11*e^4*f^4*h^8 - 20480 \\
& 0*a^4*b*c^10*d^13*e^2*f^4*h^8 - 172032*a*b^8*c^6*d^12*e^3*f^4*h^8 - 157696*a \\
& *b^10*c^4*d^10*e^5*f^4*h^8 - 131072*a^3*b^2*c^10*d^14*e*f^4*h^8 + 98304*a^8 \\
& *b^5*c^2*d*e^14*f^4*h^8 + 92160*a^2*b^4*c^9*d^14*e*f^4*h^8 + 84992*a*b^7*c^7 \\
& *d^13*e^2*f^4*h^8 + 64512*a*b^11*c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c*d^2 \\
& *e^13*f^4*h^8 + 18944*a^3*b^11*c*d^5*e^10*f^4*h^8 - 13312*a^4*b^10*c*d^4*e^11 \\
& *f^4*h^8 - 9472*a^5*b^9*c*d^3*e^12*f^4*h^8 - 8192*a*b^12*c^2*d^8*e^7*f^4 \\
& *h^8 - 6144*a^2*b^12*c*d^6*e^9*f^4*h^8 - 17920*b^11*c^4*d^11*e^4*f^4*h^8 + \\
& 14336*b^12*c^3*d^10*e^5*f^4*h^8 + 14336*b^10*c^5*d^12*e^3*f^4*h^8 - 7168*b^13 \\
& *c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^13*e^2*f^4*h^8 - 425984*a^9*c^6*d^4 \\
& *e^11*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^10*c^5*d^2*e^13*f^4 \\
& *h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^10*d^12*e^3*f^4*h^8 + \\
& 65536*a^6*c^9*d^10*e^5*f^4*h^8 - 1536*a^5*b^10*d^2*e^13*f^4*h^8 - 1536*a^2 \\
& *b^13*d^5*e^10*f^4*h^8 + 768*a^4*b^11*d^3*e^12*f^4*h^8 + 768*a^3*b^12*d^4*e^11 \\
& *f^4*h^8 + 65536*a^10*b^2*c^3*e^15*f^4*h^8 - 24576*a^9*b^4*c^2*e^15*f^4 \\
& *h^8 - 10240*a^2*b^3*c^10*d^15*f^4*h^8 + 2048*b^14*c*d^8*e^7*f^4*h^8 + 2048* \\
& b^8*c^7*d^14*e*f^4*h^8 + 32768*a^4*c^11*d^14*e*f^4*h^8 + 1024*a^6*b^9*d*e^14 \\
& *f^4*h^8 + 1024*a*b^14*d^6*e^9*f^4*h^8 + 4096*a^8*b^6*c*e^15*f^4*h^8 + 122 \\
& 88*a^3*b*c^11*d^15*f^4*h^8 + 2816*a*b^5*c^9*d^15*f^4*h^8 - 256*b^15*d^7*e^8 \\
& *f^4*h^8 - 65536*a^11*c^4*e^15*f^4*h^8 - 256*b^7*c^8*d^15*f^4*h^8 - 256*a^7 \\
& *b^8*e^15*f^4*h^8 - 896*a*b^8*c^2*d*e^10*f^2*h^4 + 192*a*b*c^9*d^8*e^3*f^2* \\
& h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856*a^2*b^5*c^4*d^2*e^9*f^2*h^4 \\
& - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200*a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640
\end{aligned}$$

$$\begin{aligned}
& *a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96*a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d*e^10*f^2*h^4 + 10240*a^4*b^2*c^5*d*e^10*f^2*h^4 - 7680*a^4*b*c^6*d^2*e^9*f^2*h^4 + 4672*a^2*b^6*c^3*d*e^10*f^2*h^4 + 1248*a*b^7*c^3*d^2*e^9*f^2*h^4 + 832*a^3*b*c^7*d^4*e^7*f^2*h^4 - 768*a*b^6*c^4*d^3*e^8*f^2*h^4 + 192*a^2*b*c^8*d^6*e^5*f^2*h^4 - 192*a*b^2*c^8*d^7*e^4*f^2*h^4 + 176*a*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a*b^3*c^7*d^6*e^5*f^2*h^4 - 96*b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3*d^3*e^8*f^2*h^4 + 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k)*(root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 4358144*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 + 4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^15*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*
\end{aligned}$$

$$\begin{aligned}
& h^{12} + 1802240a^4b^8c^7d^{17}e^2f^6h^{12} + 1556480a^5b^{12}c^2d^{11}e^8f^6h^{12} + 1048576a^{14}b^2c^3d^3e^{16}f^6h^{12} + 688128a^4b^{12}c^3d^{13}e^6f^6h^{12} - 393216a^{13}b^4c^2d^3e^{16}f^6h^{12} - 286720a^3b^{12}c^4d^{15}e^4f^6h^{12} + 229376a^3b^{13}c^3d^{14}e^5f^6h^{12} + 229376a^3b^{11}c^5d^{16}e^3f^6h^{12} + 163840a^4b^{13}c^2d^{12}e^7f^6h^{12} - 114688a^3b^{14}c^2d^{13}e^6f^6h^{12} - 114688a^3b^{10}c^6d^{17}e^2f^6h^{12} + 293601280a^{11}b^7c^7d^{10}e^9f^6h^{12} + 293601280a^{10}b^7c^8d^{12}e^7f^6h^{12} + 176160768a^{12}b^7c^6d^8e^{11}f^6h^{12} + 176160768a^9b^7c^9d^{14}e^5f^6h^{12} + 58720256a^{13}b^7c^5d^6e^{13}f^6h^{12} + 58720256a^8b^7c^{10}d^16e^3f^6h^{12} + 8388608a^{14}b^7c^4d^4e^{15}f^6h^{12} - 8388608a^6b^3c^10d^{18}e^3f^6h^{12} + 3899392a^8b^{10}c^7d^7e^{12}f^6h^{12} - 3440640a^9b^9c^6d^6e^{13}f^6h^{12} + 3145728a^5b^5c^9d^{18}e^8f^6h^{12} - 2523136a^7b^11c^6d^8e^{11}f^6h^{12} + 1802240a^{10}b^8c^5d^5e^{14}f^6h^{12} + 688128a^6b^{12}c^6d^9e^{10}f^6h^{12} - 524288a^{11}b^7c^4d^4e^{15}f^6h^{12} - 524288a^4b^7c^8d^{18}e^8f^6h^{12} + 163840a^5b^{13}c^3d^{10}e^9f^6h^{12} - 163840a^4b^{14}c^3d^{11}e^8f^6h^{12} + 65536a^{12}b^6c^3d^3e^{16}f^6h^{12} + 32768a^3b^{15}c^3d^{12}e^7f^6h^{12} + 32768a^3b^9c^7d^{18}e^8f^6h^{12} - 73400320a^{11}c^8d^{11}e^8f^6h^{12} - 58720256a^{12}c^7d^9e^{10}f^6h^{12} - 58720256a^{10}c^9d^{13}e^6f^6h^{12} - 29360128a^{13}c^6d^7e^{12}f^6h^{12} - 29360128a^9c^{10}d^{15}e^4f^6h^{12} - 8388608a^{14}c^5d^5e^{14}f^6h^{12} - 8388608a^8c^{11}d^{17}e^2f^6h^{12} - 1048576a^{15}c^4d^3e^{16}f^6h^{12} - 286720a^7b^{12}d^7e^{12}f^6h^{12} + 229376a^8b^{11}d^6e^{13}f^6h^{12} + 229376a^6b^{13}d^8e^{11}f^6h^{12} - 114688a^9b^{10}d^5e^{14}f^6h^{12} - 114688a^5b^{14}d^9e^{10}f^6h^{12} + 32768a^{10}b^9d^4e^{15}f^6h^{12} + 32768a^4b^{15}d^{10}e^9f^6h^{12} - 4096a^{11}b^8d^3e^{16}f^6h^{12} - 4096a^3b^{16}d^{11}e^8f^6h^{12} + 1048576a^6b^2c^{11}d^{19}f^6h^{12} - 393216a^5b^4c^{10}d^{19}f^6h^{12} + 65536a^4b^6c^9d^{19}f^6h^{12} - 4096a^3b^8c^8d^{19}f^6h^{12} - 1048576a^7c^{12}d^{19}f^6h^{12} + 262144a^{10}b^7c^4d^4e^{14}f^4h^8 - 23552a^6b^6c^8d^{14}e^8f^4h^8 - 16384a^7b^7c^4d^4e^{14}f^4h^8 - 3328a^6b^{13}c^4d^7e^8f^4h^8 + 2429952a^4b^5c^6d^9e^6f^4h^8 - 1865728a^6b^3c^6d^7e^8f^4h^8 - 1716224a^4b^4c^7d^{10}e^5f^4h^8 + 1605632a^6b^2c^7d^8e^7f^4h^8 + 1584384a^5b^5c^5d^7e^8f^4h^8 + 1572864a^5b^2c^8d^{10}e^5f^4h^8 - 1433600a^5b^3c^7d^9e^6f^4h^8 - 1261568a^4b^6c^5d^8e^7f^4h^8 - 1124352a^3b^4c^8d^{12}e^3f^4h^8 - 1110016a^7b^3c^5d^5e^{10}f^4h^8 + 1106176a^3b^5c^7d^{11}e^4f^4h^8 - 936960a^5b^6c^4d^6e^9f^4h^8 - 838656a^2b^7c^6d^{11}e^4f^4h^8 - 795648a^3b^7c^5d^9e^6f^4h^8 + 730880a^3b^8c^4d^8e^7f^4h^8 + 714752a^2b^6c^7d^{12}e^3f^4h^8 + 686080a^7b^4c^4d^4e^{11}f^4h^8 + 641024a^6b^4c^5d^6e^9f^4h^8 - 595968a^8b^3c^4d^3e^{12}f^4h^8 + 544768a^3b^3c^9d^{13}e^2f^4h^8 + 516096a^2b^8c^5d^{10}e^5f^4h^8 + 441856a^6b^5c^4d^5e^{10}f^4h^8 + 393216a^7b^2c^6d^6e^9f^4h^8 + 376832a^4b^2c^9d^{12}e^3f^4h^8 - 366592a^6b^6c^3d^4e^{11}f^4h^8 + 363520a^4b^8c^3d^6e^9f^4h^8 - 356352a^5b^4c^6d^8e^7f^4h^8 - 348672a^2b^5c^8d^{13}e^2f^4h^8 - 344064a^8b^2c^5d^4e^{11}f^4h^8 + 294912a^8b^4c^3d^2e^{13}f^4h^8 + 210944a^4b^3c^8d^{11}e^4f^4h^8 - 198400a^3b
\end{aligned}$$

$$\begin{aligned}
& ^9c^3d^7e^8f^4h^8 - 144640a^4b^7c^4d^7e^8f^4h^8 - 131072a^9b^7 \\
& 2c^4d^2e^{13}f^4h^8 - 131072a^7b^6c^2d^2e^{13}f^4h^8 - 129024a^3b^6 \\
& ^6c^6d^{10}e^5f^4h^8 - 104448a^2b^{10}c^3d^8e^7f^4h^8 + 96768a^5b^8 \\
& ^8c^2d^4e^{11}f^4h^8 + 91904a^7b^5c^3d^3e^{12}f^4h^8 - 74240a^4b^9 \\
& ^9c^2d^5e^{10}f^4h^8 - 71680a^2b^9c^4d^9e^6f^4h^8 + 58368a^2b^{11} \\
& ^8c^2d^7e^8f^4h^8 + 36864a^5b^7c^3d^5e^{10}f^4h^8 - 35328a^3b^{10} \\
& ^8c^2d^6e^9f^4h^8 + 27136a^6b^7c^2d^3e^{12}f^4h^8 + 909312a^8b^6c^6 \\
& ^8d^5e^{10}f^4h^8 + 815104a^9b^6c^5d^3e^{12}f^4h^8 - 651264a^5b^6c^9d^7 \\
& ^8e^4f^4h^8 - 573440a^6b^6c^8d^9e^6f^4h^8 - 262144a^9b^3c^3d^8e^7 \\
& ^8f^4h^8 + 217088a^7b^6c^7d^7e^8f^4h^8 + 211456a^8b^9c^5d^{11}e^4f^4 \\
& ^4h^8 - 204800a^4b^6c^{10}d^{13}e^2f^4h^8 - 172032a^8b^8c^6d^{12}e^3f^4 \\
& ^4h^8 - 157696a^8b^{10}c^4d^{10}e^5f^4h^8 - 131072a^3b^2c^{10}d^{14}e^4f^4 \\
& ^4h^8 + 98304a^8b^5c^2d^8e^{14}f^4h^8 + 92160a^2b^4c^9d^{14}e^4f^4h^8 + \\
& 84992a^8b^7c^7d^{13}e^2f^4h^8 + 64512a^8b^{11}c^3d^9e^6f^4h^8 + 2355 \\
& 2a^6b^8c^8d^2e^{13}f^4h^8 + 18944a^3b^{11}c^5d^5e^{10}f^4h^8 - 13312a^4 \\
& ^4b^{10}c^8d^4e^{11}f^4h^8 - 9472a^5b^9c^8d^3e^{12}f^4h^8 - 8192a^8b^{12}c^2 \\
& ^2d^8e^7f^4h^8 - 6144a^2b^{12}c^8d^6e^9f^4h^8 - 17920b^{11}c^4d^{11}e^4 \\
& ^4f^4h^8 + 14336b^{12}c^3d^{10}e^5f^4h^8 + 14336b^{10}c^5d^{12}e^3f^4 \\
& ^4h^8 - 7168b^{13}c^2d^9e^6f^4h^8 - 7168b^9c^6d^{13}e^2f^4h^8 - 4259 \\
& 84a^9c^6d^4e^{11}f^4h^8 - 360448a^8c^7d^6e^9f^4h^8 - 262144a^{10}c^5 \\
& ^5d^2e^{13}f^4h^8 - 131072a^7c^8d^8e^7f^4h^8 + 98304a^5c^{10}d^{12}e^3 \\
& ^3f^4h^8 + 65536a^6c^9d^{10}e^5f^4h^8 - 1536a^5b^{10}d^2e^{13}f^4h^8 - \\
& 1536a^2b^{13}d^5e^{10}f^4h^8 + 768a^4b^{11}d^3e^{12}f^4h^8 + 768a^3b^{12} \\
& ^4d^4e^{11}f^4h^8 + 65536a^{10}b^2c^3e^{15}f^4h^8 - 24576a^9b^4c^2e^{15} \\
& ^5f^4h^8 - 10240a^2b^3c^{10}d^{15}f^4h^8 + 2048b^{14}c^8d^8e^7f^4h^8 + \\
& 2048b^8c^7d^{14}e^4f^4h^8 + 32768a^4c^{11}d^{14}e^4f^4h^8 + 1024a^6b^9 \\
& ^9d^8e^{14}f^4h^8 + 1024a^8b^{14}d^6e^9f^4h^8 + 4096a^8b^6c^8e^{15}f^4 \\
& ^4h^8 + 12288a^3b^6c^{11}d^{15}f^4h^8 + 2816a^8b^5c^9d^{15}f^4h^8 - 25 \\
& 6b^{15}d^7e^8f^4h^8 - 65536a^{11}c^4e^{15}f^4h^8 - 256b^7c^8d^{15}f^4h^8 - \\
& 256a^7b^8e^{15}f^4h^8 - 896a^8b^8c^2d^8e^{10}f^2h^4 + 192a^8b^6c^9 \\
& ^9d^8e^3f^2h^4 + 11520a^3b^3c^5d^2e^9f^2h^4 - 5856a^2b^5c^4d^2e^9 \\
& ^9f^2h^4 - 5120a^3b^2c^6d^3e^8f^2h^4 + 3200a^2b^4c^5d^3e^8f^2h^4 - \\
& 640a^2b^3c^6d^4e^7f^2h^4 - 96a^2b^2c^7d^5e^6f^2h^4 - 10880a^3b^4c^4 \\
& ^4d^8e^{10}f^2h^4 + 10240a^4b^2c^5d^8e^{10}f^2h^4 - 7680a^4b^6c^6d^2e^9 \\
& ^9f^2h^4 + 4672a^2b^6c^3d^8e^{10}f^2h^4 + 1248a^8b^7c^3d^2e^9f^2h^4 \\
& + 832a^3b^6c^7d^4e^7f^2h^4 - 768a^8b^6c^4d^3e^8f^2h^4 + 192a^2b^6c^8 \\
& ^8d^6e^5f^2h^4 - 192a^8b^2c^8d^7e^4f^2h^4 + 176a^8b^5c^5d^4e^7f^2h^4 \\
& + 64a^8b^3c^7d^6e^5f^2h^4 - 96b^9c^2d^2e^9f^2h^4 - 96b^2c^9d^9e^2f^2h^4 \\
& + 64b^8c^3d^3e^8f^2h^4 + 64b^3c^8d^8e^3f^2h^4 - 16b^7c^4d^4e^7f^2h^4 - \\
& 16b^4c^7d^7e^4f^2h^4 + 2032a^4c^7d^3e^8f^2h^4 - 96a^2c^9d^7e^4f^2h^4 - \\
& 64a^3c^8d^5e^6f^2h^4 - 4480a^4b^3c^4e^{11}f^2h^4 + 3696a^3b^5c^3e^{11} \\
& ^11f^2h^4 - 1376a^2b^7c^2e^{11}f^2h^4 - 2048a^5c^6d^8e^{10}f^2h^4 - \\
& 64a^8c^{10}d^9e^2f^2h^4 + 1792a^5b^6c^5e^{11}f^2h^4 + 64b^{10}c^8d^8e^{10} \\
& ^10f^2h^4 + 64b^6c^{10}d^{10}e^4f^2h^4 + 240a^8b^9c^8e^{11}f^2h^4 - 16c^{11}
\end{aligned}$$

$$\begin{aligned}
& d^{11}f^2h^4 - 16b^{11}e^{11}f^2h^4 - c^7e^7, h, k)^3(\text{root}(8388608a^7b^* \\
& c^{11}d^{18}e^*f^6h^{12} - 513802240a^{10}b^2c^7d^{11}e^8f^6h^{12} - 381681664 \\
& *a^{11}b^2c^6d^9e^{10}f^6h^{12} - 381681664a^9b^2c^8d^{13}e^6f^6h^{12} - \\
& 300941312a^9b^5c^5d^{10}e^9f^6h^{12} - 300941312a^8b^5c^6d^{12}e^7f^* \\
& ^6h^{12} + 293601280a^{10}b^3c^6d^{10}e^9f^6h^{12} + 293601280a^9b^3c^7* \\
& d^{12}e^7f^6h^{12} - 168820736a^{10}b^5c^4d^8e^{11}f^6h^{12} - 168820736a^* \\
& 7b^5c^7d^{14}e^5f^6h^{12} + 166068224a^8b^6c^5d^{11}e^8f^6h^{12} - 146 \\
& 800640a^{12}b^2c^5d^7e^{12}f^6h^{12} - 146800640a^8b^2c^9d^{15}e^4f^6* \\
& h^{12} + 124780544a^{10}b^4c^5d^9e^{10}f^6h^{12} + 124780544a^8b^4c^7d^1 \\
& 3e^6f^6h^{12} + 119275520a^9b^4c^6d^{11}e^8f^6h^{12} + 117440512a^{11}b^* \\
& ^3c^5d^8e^{11}f^6h^{12} + 117440512a^8b^3c^8d^{14}e^5f^6h^{12} + 102760 \\
& 448a^9b^6c^4d^9e^{10}f^6h^{12} + 102760448a^7b^6c^6d^{13}e^6f^6h^{12} \\
& + 91750400a^{11}b^4c^4d^7e^{12}f^6h^{12} + 91750400a^7b^4c^8d^{15}e^4* \\
& f^6h^{12} - 71065600a^7b^8c^4d^{11}e^8f^6h^{12} - 53444608a^8b^8c^3d^ \\
& 9e^{10}f^6h^{12} - 53444608a^6b^8c^5d^{13}e^6f^6h^{12} + 40370176a^9b^7 \\
& *c^3d^8e^{11}f^6h^{12} + 40370176a^6b^7c^6d^{14}e^5f^6h^{12} - 36700160* \\
& a^{11}b^5c^3d^6e^{13}f^6h^{12} - 36700160a^6b^5c^8d^{16}e^3f^6h^{12} + 3 \\
& 4078720a^8b^7c^4d^{10}e^9f^6h^{12} + 34078720a^7b^7c^5d^{12}e^7f^6* \\
& h^{12} + 26214400a^{12}b^4c^3d^5e^{14}f^6h^{12} + 26214400a^6b^4c^9d^{17}e^ \\
& ^2f^6h^{12} + 22118400a^7b^9c^3d^{10}e^9f^6h^{12} + 22118400a^6b^9c^4 \\
& *d^{12}e^7f^6h^{12} - 20971520a^{13}b^2c^4d^5e^{14}f^6h^{12} - 20971520a^7 \\
& *b^2c^{10}d^{17}e^2f^6h^{12} + 18350080a^{10}b^7c^2d^6e^{13}f^6h^{12} + 183 \\
& 50080a^5b^7c^7d^{16}e^3f^6h^{12} - 16629760a^9b^8c^2d^7e^{12}f^6h^1 \\
& 2 - 16629760a^5b^8c^6d^{15}e^4f^6h^{12} - 10485760a^{11}b^6c^2d^5e^{14} \\
& *f^6h^{12} - 10485760a^5b^6c^8d^{17}e^2f^6h^{12} + 9175040a^{10}b^6c^3d^ \\
& ^7e^{12}f^6h^{12} + 9175040a^6b^6c^7d^{15}e^4f^6h^{12} - 8388608a^{13}b^3 \\
& *c^3d^4e^{15}f^6h^{12} + 5619712a^7b^{10}c^2d^9e^{10}f^6h^{12} + 5619712a^ \\
& ^5b^{10}c^4d^{13}e^6f^6h^{12} - 5570560a^6b^{11}c^2d^{10}e^9f^6h^{12} - 55 \\
& 70560a^5b^{11}c^3d^{12}e^7f^6h^{12} + 4358144a^8b^9c^2d^8e^{11}f^6h^1 \\
& 2 + 4358144a^5b^9c^5d^{14}e^5f^6h^{12} + 4259840a^6b^{10}c^3d^{11}e^8f^ \\
& ^6h^{12} + 3899392a^4b^{10}c^5d^{15}e^4f^6h^{12} - 3440640a^4b^9c^6d^{16} \\
& *e^3f^6h^{12} + 3145728a^{12}b^5c^2d^4e^{15}f^6h^{12} - 2523136a^4b^{11}c^ \\
& ^4d^{14}e^5f^6h^{12} + 1802240a^4b^8c^7d^{17}e^2f^6h^{12} + 1556480a^5* \\
& b^{12}c^2d^{11}e^8f^6h^{12} + 1048576a^{14}b^2c^3d^3e^{16}f^6h^{12} + 68812 \\
& 8a^4b^{12}c^3d^{13}e^6f^6h^{12} - 393216a^{13}b^4c^2d^3e^{16}f^6h^{12} - \\
& 286720a^3b^{12}c^4d^{15}e^4f^6h^{12} + 229376a^3b^{13}c^3d^{14}e^5f^6h^ \\
& 12 + 229376a^3b^{11}c^5d^{16}e^3f^6h^{12} + 163840a^4b^{13}c^2d^{12}e^7f^ \\
& ^6h^{12} - 114688a^3b^{14}c^2d^{13}e^6f^6h^{12} - 114688a^3b^{10}c^6d^{17}* \\
& e^2f^6h^{12} + 293601280a^{11}b^*c^7d^{10}e^9f^6h^{12} + 293601280a^{10}b^*c^ \\
& 8d^{12}e^7f^6h^{12} + 176160768a^{12}b^*c^6d^8e^{11}f^6h^{12} + 176160768a^ \\
& 9b^*c^9d^{14}e^5f^6h^{12} + 58720256a^{13}b^*c^5d^6e^{13}f^6h^{12} + 5872025 \\
& 6a^8b^*c^{10}d^{16}e^3f^6h^{12} + 8388608a^{14}b^*c^4d^4e^{15}f^6h^{12} - 838 \\
& 8608a^6b^3c^{10}d^{18}e^*f^6h^{12} + 3899392a^8b^{10}c^*d^7e^{12}f^6h^{12} - \\
& 3440640a^9b^9c^*d^6e^{13}f^6h^{12} + 3145728a^5b^5c^9d^{18}e^*f^6h^{12} - \\
& 2523136a^7b^{11}c^*d^8e^{11}f^6h^{12} + 1802240a^{10}b^8c^*d^5e^{14}f^6h^1
\end{aligned}$$

$$\begin{aligned}
& 2 + 688128a^6b^{12}c^8d^9e^{10}f^6h^{12} - 524288a^{11}b^7c^8d^4e^{15}f^6h^{12} \\
& - 524288a^4b^7c^8d^{18}e^8f^6h^{12} + 163840a^5b^{13}c^8d^{10}e^9f^6h^{12} \\
& - 163840a^4b^{14}c^8d^{11}e^8f^6h^{12} + 65536a^{12}b^6c^8d^3e^{16}f^6h^{12} \\
& + 32768a^3b^{15}c^8d^{12}e^7f^6h^{12} + 32768a^3b^9c^7d^{18}e^8f^6h^{12} \\
& - 73400320a^{11}c^8d^{11}e^8f^6h^{12} - 58720256a^{12}c^7d^9e^{10}f^6h^{12} \\
& - 58720256a^{10}c^9d^{13}e^6f^6h^{12} - 29360128a^{13}c^6d^7e^{12}f^6h^{12} \\
& - 29360128a^9c^{10}d^{15}e^4f^6h^{12} - 8388608a^{14}c^5d^5e^{14}f^6h^{12} \\
& - 8388608a^8c^{11}d^{17}e^2f^6h^{12} - 1048576a^{15}c^4d^3e^{16}f^6h^{12} \\
& - 286720a^7b^{12}d^7e^{12}f^6h^{12} + 229376a^8b^{11}d^6e^{13}f^6h^{12} + \\
& 229376a^6b^{13}d^8e^{11}f^6h^{12} - 114688a^9b^{10}d^5e^{14}f^6h^{12} - 11 \\
& 4688a^5b^{14}d^9e^{10}f^6h^{12} + 32768a^{10}b^9d^4e^{15}f^6h^{12} + 32768a^4b^{15}d^{10}e^9f^6h^{12} \\
& - 4096a^{11}b^8d^3e^{16}f^6h^{12} - 4096a^3b^{16}d^{11}e^8f^6h^{12} + 1048576a^6b^2c^{11}d^{19}f^6h^{12} \\
& - 393216a^5b^4c^{10}d^{19}f^6h^{12} + 65536a^4b^6c^9d^{19}f^6h^{12} - 4096a^3b^8c^8d^{19}f^6h^{12} \\
& - 1048576a^7c^{12}d^{19}f^6h^{12} + 262144a^{10}b^3c^4d^5e^{14}f^4h^8 - 23552a^2b^6c^8d^{14}e^8f^4h^8 \\
& - 16384a^7b^7c^8d^{14}e^8f^4h^8 - 3328a^2b^13c^8d^7e^8f^4h^8 + 2429952a^4b^5c^6d^9e^6f^4h^8 \\
& - 1865728a^6b^3c^6d^7e^8f^4h^8 - 1716224a^4b^4c^7d^{10}e^5f^4h^8 + 1605632a^6b^2c^7d^8e^7f^4h^8 \\
& + 1584384a^5b^5c^5d^7e^8f^4h^8 + 1572864a^5b^2c^8d^{10}e^5f^4h^8 - 1433600a^5b^3c^7d^9e^6f^4h^8 \\
& - 1261568a^4b^6c^5d^8e^7f^4h^8 - 1124352a^3b^4c^8d^{12}e^3f^4h^8 - 110016a^7b^3c^5d^5e^{10}f^4h^8 \\
& + 1106176a^3b^5c^7d^{11}e^4f^4h^8 - 936960a^5b^6c^4d^6e^9f^4h^8 - 838656a^2b^7c^6d^{11}e^4f^4h^8 \\
& - 795648a^3b^7c^5d^9e^6f^4h^8 + 730880a^3b^8c^4d^8e^7f^4h^8 + 714752a^2b^6c^7d^{12}e^3f^4h^8 \\
& + 686080a^7b^4c^4d^4e^{11}f^4h^8 + 641024a^6b^4c^5d^6e^9f^4h^8 - 595968a^8b^3c^4d^3e^{12}f^4h^8 \\
& + 544768a^3b^3c^9d^{13}e^2f^4h^8 + 516096a^2b^8c^5d^{10}e^5f^4h^8 + 441856a^6b^5c^4d^5e^{10}f^4h^8 \\
& + 393216a^7b^2c^6d^6e^9f^4h^8 + 376832a^4b^2c^9d^{12}e^3f^4h^8 - 366592a^6b^6c^3d^4e^{11}f^4h^8 \\
& + 363520a^4b^8c^3d^6e^9f^4h^8 - 356352a^5b^4c^6d^8e^7f^4h^8 - 348672a^2b^5c^8d^{13}e^2f^4h^8 \\
& - 344064a^8b^2c^5d^4e^{11}f^4h^8 + 294912a^8b^4c^3d^2e^{13}f^4h^8 + 210944a^4b^3c^8d^{11}e^4f^4h^8 \\
& - 198400a^3b^9c^3d^7e^8f^4h^8 - 144640a^4b^7c^4d^7e^8f^4h^8 - 131072a^9b^2c^4d^2e^{13}f^4h^8 \\
& - 131072a^7b^6c^2d^2e^{13}f^4h^8 - 129024a^3b^6c^6d^{10}e^5f^4h^8 - 104448a^2b^{10}c^3d^8e^7f^4h^8 \\
& + 96768a^5b^8c^2d^4e^{11}f^4h^8 + 91904a^7b^5c^3d^3e^{12}f^4h^8 - 74240a^4b^9c^2d^5e^{10}f^4h^8 \\
& - 71680a^2b^9c^4d^9e^6f^4h^8 + 58368a^2b^{11}c^2d^7e^8f^4h^8 + 36864a^5b^7c^3d^5e^{10}f^4h^8 \\
& - 35328a^3b^{10}c^2d^6e^9f^4h^8 + 27136a^6b^7c^2d^3e^{12}f^4h^8 + 909312a^8b^3c^6d^5e^{10}f^4h^8 \\
& + 815104a^9b^3c^5d^3e^{12}f^4h^8 - 651264a^5b^3c^9d^{11}e^4f^4h^8 - 573440a^6b^3c^8d^9e^6f^4h^8 - 262144a^9b^3c^3d^5e^{14}f^4h^8 \\
& + 217088a^7b^3c^7d^7e^8f^4h^8 + 211456a^9b^9c^5d^{11}e^4f^4h^8 - 204800a^4b^3c^{10}d^{13}e^2f^4h^8 \\
& - 172032a^8b^8c^6d^{12}e^3f^4h^8 - 157696a^2b^{10}c^4d^{10}e^5f^4h^8 - 131072a^3b^2c^{10}d^{14}e^8f^4h^8 \\
& + 98304a^8b^5c^2d^5e^{14}f^4h^8 + 92160a^2b^4c^9
\end{aligned}$$

$$\begin{aligned}
& d^{14}e^4f^4h^8 + 84992a^7b^7c^7d^{13}e^2f^4h^8 + 64512a^7b^7c^7d^{13}e^2f^4h^8 + 64512a^7b^7c^7d^{13}e^2f^4h^8 + 64512a^7b^7c^7d^{13}e^2f^4h^8 + 23552a^6b^8c^8d^2e^{13}f^4h^8 + 18944a^3b^{11}c^3d^9e^6f^4h^8 \\
& - 13312a^4b^{10}c^4d^4e^{11}f^4h^8 - 9472a^5b^9c^4d^3e^{12}f^4h^8 - 8192a^7b^7c^7d^{13}e^2f^4h^8 - 6144a^2b^{12}c^6d^6e^9f^4h^8 - 17920b^{11}c^4d^{11}e^4f^4h^8 \\
& + 14336b^{12}c^3d^{10}e^5f^4h^8 + 14336b^{10}c^5d^{12}e^3f^4h^8 - 7168b^{13}c^2d^9e^6f^4h^8 - 7168b^9c^6d^{13}e^2f^4h^8 - 425984a^9c^6d^4e^{11}f^4h^8 \\
& - 360448a^8c^7d^6e^9f^4h^8 - 262144a^{10}c^5d^2e^{13}f^4h^8 - 131072a^7c^8d^8e^7f^4h^8 + 98304a^5c^{10}d^{12}e^3f^4h^8 \\
& + 65536a^6c^9d^{10}e^5f^4h^8 - 1536a^5b^{10}d^2e^{13}f^4h^8 - 1536a^2b^{13}d^5e^{10}f^4h^8 + 768a^4b^{11}d^3e^{12}f^4h^8 \\
& + 768a^3b^{12}d^4e^{11}f^4h^8 + 65536a^{10}b^2c^3e^{15}f^4h^8 - 24576a^9b^4c^2e^{15}f^4h^8 - 10240a^2b^3c^{10}d^{15}f^4h^8 \\
& + 2048b^{14}c^8d^8e^7f^4h^8 + 2048b^8c^7d^{14}e^4f^4h^8 + 32768a^4c^{11}d^{14}e^4f^4h^8 + 1024a^6b^9d^4e^{14}f^4h^8 \\
& + 1024a^7b^9d^4e^{14}f^4h^8 + 1024a^7b^9d^4e^{14}f^4h^8 + 1024a^7b^9d^4e^{14}f^4h^8 + 4096a^8b^6c^6e^{15}f^4h^8 \\
& + 12288a^3b^6c^{11}d^{15}f^4h^8 + 2816a^5b^5c^9d^{15}f^4h^8 - 256b^{15}d^7e^8f^4h^8 - 65536a^{11}c^4e^{15}f^4h^8 - 256b^7c^8d^{15}f^4h^8 \\
& - 256a^7b^8e^{15}f^4h^8 - 896a^8b^8c^2d^8e^{10}f^2h^4 + 192a^8b^8c^2d^8e^{10}f^2h^4 + 11520a^3b^3c^5d^2e^9f^2h^4 \\
& - 5856a^2b^5c^4d^2e^9f^2h^4 - 5120a^3b^2c^6d^3e^8f^2h^4 + 3200a^2b^4c^5d^3e^8f^2h^4 - 640a^2b^3c^6d^4e^7f^2h^4 \\
& - 96a^2b^2c^7d^5e^6f^2h^4 - 10880a^3b^4c^4d^4e^{10}f^2h^4 + 10240a^4b^2c^5d^4e^{10}f^2h^4 - 7680a^4b^2c^6d^2e^9f^2h^4 \\
& + 4672a^2b^6c^3d^4e^{10}f^2h^4 + 1248a^7b^7c^3d^2e^9f^2h^4 + 832a^3b^3c^7d^4e^7f^2h^4 - 768a^6b^6c^4d^3e^8f^2h^4 \\
& + 192a^2b^6c^8d^6e^5f^2h^4 - 192a^2b^2c^8d^7e^4f^2h^4 + 176a^5b^5c^5d^4e^7f^2h^4 + 64a^3b^3c^7d^6e^5f^2h^4 \\
& - 96b^9c^2d^2e^9f^2h^4 - 96b^2c^9d^9e^2f^2h^4 + 64b^8c^3d^3e^8f^2h^4 + 64b^3c^8d^8e^3f^2h^4 - 16b^7c^4d^4e^7f^2h^4 \\
& - 16b^4c^7d^7e^4f^2h^4 + 2032a^4c^7d^3e^8f^2h^4 - 96a^2c^9d^7e^4f^2h^4 - 64a^3c^8d^5e^6f^2h^4 - 4480a^4b^3c^4e^{11}f^2h^4 \\
& + 3696a^3b^5c^3e^{11}f^2h^4 - 1376a^2b^7c^2e^{11}f^2h^4 - 2048a^5c^6d^4e^{10}f^2h^4 - 64a^2c^10d^9e^2f^2h^4 \\
& + 1792a^5b^5c^5e^{11}f^2h^4 + 64b^{10}c^4d^4e^{10}f^2h^4 + 64b^6c^{10}d^{10}e^4f^2h^4 + 240a^9b^9c^9e^{11}f^2h^4 \\
& - 16c^{11}d^{11}f^2h^4 - 16b^{11}e^{11}f^2h^4 - c^7e^7, h, k) \cdot (\text{root}(8388608a^7b^6c^{11}d^{18}e^6f^6h^{12} - 513802240a^{10}b^2c^7d^{11}e^8f^6h^{12} - 381681664a^{11}b^2c^6d^9e^{10}f^6h^{12} \\
& - 381681664a^9b^2c^8d^{13}e^6f^6h^{12} - 300941312a^9b^5c^5d^{10}e^9f^6h^{12} - 300941312a^8b^5c^6d^{12}e^7f^6h^{12} + 293601280a^{10}b^3c^6d^{10}e^9f^6h^{12} \\
& + 293601280a^9b^3c^7d^{12}e^7f^6h^{12} - 168820736a^{10}b^5c^4d^8e^{11}f^6h^{12} - 168820736a^7b^5c^7d^{14}e^5f^6h^{12} + 166068224a^8b^6c^5d^{11}e^8f^6h^{12} \\
& - 146800640a^{12}b^2c^5d^7e^{12}f^6h^{12} - 146800640a^8b^2c^9d^{15}e^4f^6h^{12} + 124780544a^{10}b^4c^5d^9e^{10}f^6h^{12} + 124780544a^8b^4c^7d^{13}e^6f^6h^{12} \\
& + 119275520a^9b^4c^6d^{11}e^8f^6h^{12} + 117440512a^{11}b^3c^5d^8e^{11}f^6h^{12} + 117440512a^8b^3c^8d^{14}e^5f^6h^{12} + 102760448a^9b^6c^4d^9e^{10}f^6h^{12} \\
& + 102760448a^7b^6c^6d^{13}e^6f^6h^{12} + 91750400a^{11}b^4c^4d^7e^{12}f^6h^{12} + 91750400a^7b^4
\end{aligned}$$

$$\begin{aligned}
& *c^8*d^{15}*e^4*f^6*h^{12} - 71065600*a^7*b^8*c^4*d^{11}*e^8*f^6*h^{12} - 53444608* \\
& a^8*b^8*c^3*d^9*e^{10}*f^6*h^{12} - 53444608*a^6*b^8*c^5*d^{13}*e^6*f^6*h^{12} + 40 \\
& 370176*a^9*b^7*c^3*d^8*e^{11}*f^6*h^{12} + 40370176*a^6*b^7*c^6*d^{14}*e^5*f^6*h^{12} \\
& - 36700160*a^{11}*b^5*c^3*d^6*e^{13}*f^6*h^{12} - 36700160*a^6*b^5*c^8*d^{16}*e^3* \\
& f^6*h^{12} + 34078720*a^8*b^7*c^4*d^{10}*e^9*f^6*h^{12} + 34078720*a^7*b^7*c^5* \\
& d^{12}*e^7*f^6*h^{12} + 26214400*a^{12}*b^4*c^3*d^5*e^{14}*f^6*h^{12} + 26214400*a^6* \\
& b^4*c^9*d^{17}*e^2*f^6*h^{12} + 22118400*a^7*b^9*c^3*d^{10}*e^9*f^6*h^{12} + 221184 \\
& 00*a^6*b^9*c^4*d^{12}*e^7*f^6*h^{12} - 20971520*a^{13}*b^2*c^4*d^5*e^{14}*f^6*h^{12} \\
& - 20971520*a^7*b^2*c^{10}*d^{17}*e^2*f^6*h^{12} + 18350080*a^{10}*b^7*c^2*d^6*e^{13}* \\
& f^6*h^{12} + 18350080*a^5*b^7*c^7*d^{16}*e^3*f^6*h^{12} - 16629760*a^9*b^8*c^2*d^7* \\
& e^{12}*f^6*h^{12} - 16629760*a^5*b^8*c^6*d^{15}*e^4*f^6*h^{12} - 10485760*a^{11}*b^6* \\
& c^2*d^5*e^{14}*f^6*h^{12} - 10485760*a^5*b^6*c^8*d^{17}*e^2*f^6*h^{12} + 9175040* \\
& a^{10}*b^6*c^3*d^7*e^{12}*f^6*h^{12} + 9175040*a^6*b^6*c^7*d^{15}*e^4*f^6*h^{12} - 83 \\
& 88608*a^{13}*b^3*c^3*d^4*e^{15}*f^6*h^{12} + 5619712*a^7*b^{10}*c^2*d^9*e^{10}*f^6*h^{12} \\
& + 5619712*a^5*b^{10}*c^4*d^{13}*e^6*f^6*h^{12} - 5570560*a^6*b^{11}*c^2*d^{10}*e^9* \\
& *f^6*h^{12} - 5570560*a^5*b^{11}*c^3*d^{12}*e^7*f^6*h^{12} + 4358144*a^8*b^9*c^2*d^8* \\
& e^{11}*f^6*h^{12} + 4358144*a^5*b^9*c^5*d^{14}*e^5*f^6*h^{12} + 4259840*a^6*b^{10}* \\
& c^3*d^{11}*e^8*f^6*h^{12} + 3899392*a^4*b^{10}*c^5*d^{15}*e^4*f^6*h^{12} - 3440640*a^4* \\
& b^9*c^6*d^{16}*e^3*f^6*h^{12} + 3145728*a^{12}*b^5*c^2*d^4*e^{15}*f^6*h^{12} - 2523 \\
& 136*a^4*b^{11}*c^4*d^{14}*e^5*f^6*h^{12} + 1802240*a^4*b^8*c^7*d^{17}*e^2*f^6*h^{12} \\
& + 1556480*a^5*b^{12}*c^2*d^{11}*e^8*f^6*h^{12} + 1048576*a^{14}*b^2*c^3*d^3*e^{16}*f^6* \\
& h^{12} + 688128*a^4*b^{12}*c^3*d^{13}*e^6*f^6*h^{12} - 393216*a^{13}*b^4*c^2*d^3*e^ \\
& 16*f^6*h^{12} - 286720*a^3*b^{12}*c^4*d^{15}*e^4*f^6*h^{12} + 229376*a^3*b^{13}*c^3*d^ \\
& ^{14}*e^5*f^6*h^{12} + 229376*a^3*b^{11}*c^5*d^{16}*e^3*f^6*h^{12} + 163840*a^4*b^{13}* \\
& c^2*d^{12}*e^7*f^6*h^{12} - 114688*a^3*b^{14}*c^2*d^{13}*e^6*f^6*h^{12} - 114688*a^3* \\
& b^{10}*c^6*d^{17}*e^2*f^6*h^{12} + 293601280*a^{11}*b*c^7*d^{10}*e^9*f^6*h^{12} + 29360 \\
& 1280*a^{10}*b*c^8*d^{12}*e^7*f^6*h^{12} + 176160768*a^{12}*b*c^6*d^8*e^{11}*f^6*h^{12} \\
& + 176160768*a^9*b*c^9*d^{14}*e^5*f^6*h^{12} + 58720256*a^{13}*b*c^5*d^6*e^{13}*f^6* \\
& h^{12} + 58720256*a^8*b*c^{10}*d^{16}*e^3*f^6*h^{12} + 8388608*a^{14}*b*c^4*d^4*e^{15}* \\
& f^6*h^{12} - 8388608*a^6*b^3*c^{10}*d^{18}*e*f^6*h^{12} + 3899392*a^8*b^{10}*c*d^7*e^ \\
& 12*f^6*h^{12} - 3440640*a^9*b^9*c*d^6*e^{13}*f^6*h^{12} + 3145728*a^5*b^5*c^9*d^1 \\
& 8*e*f^6*h^{12} - 2523136*a^7*b^{11}*c*d^8*e^{11}*f^6*h^{12} + 1802240*a^{10}*b^8*c*d^ \\
& 5*e^{14}*f^6*h^{12} + 688128*a^6*b^{12}*c*d^9*e^{10}*f^6*h^{12} - 524288*a^{11}*b^7*c*d^ \\
& ^4*e^{15}*f^6*h^{12} - 524288*a^4*b^7*c^8*d^{18}*e*f^6*h^{12} + 163840*a^5*b^{13}*c*d^ \\
& ^{10}*e^9*f^6*h^{12} - 163840*a^4*b^{14}*c*d^{11}*e^8*f^6*h^{12} + 65536*a^{12}*b^6*c*d^ \\
& ^3*e^{16}*f^6*h^{12} + 32768*a^3*b^{15}*c*d^{12}*e^7*f^6*h^{12} + 32768*a^3*b^9*c^7*d^ \\
& ^{18}*e*f^6*h^{12} - 73400320*a^{11}*c^8*d^{11}*e^8*f^6*h^{12} - 58720256*a^{12}*c^7*d^ \\
& ^9*e^{10}*f^6*h^{12} - 58720256*a^{10}*c^9*d^{13}*e^6*f^6*h^{12} - 29360128*a^{13}*c^6*d^ \\
& ^7*e^{12}*f^6*h^{12} - 29360128*a^9*c^{10}*d^{15}*e^4*f^6*h^{12} - 8388608*a^{14}*c^5*d^ \\
& ^5*e^{14}*f^6*h^{12} - 8388608*a^8*c^{11}*d^{17}*e^2*f^6*h^{12} - 1048576*a^{15}*c^4*d^ \\
& ^3*e^{16}*f^6*h^{12} - 286720*a^7*b^{12}*d^7*e^{12}*f^6*h^{12} + 229376*a^8*b^{11}*d^6*e^ \\
& ^{13}*f^6*h^{12} + 229376*a^6*b^{13}*d^8*e^{11}*f^6*h^{12} - 114688*a^9*b^{10}*d^5*e^{14} \\
& *f^6*h^{12} - 114688*a^5*b^{14}*d^9*e^{10}*f^6*h^{12} + 32768*a^{10}*b^9*d^4*e^{15}*f^6* \\
& *h^{12} + 32768*a^4*b^{15}*d^{10}*e^9*f^6*h^{12} - 4096*a^{11}*b^8*d^3*e^{16}*f^6*h^{12} \\
& - 4096*a^3*b^{16}*d^{11}*e^8*f^6*h^{12} + 1048576*a^6*b^2*c^{11}*d^{19}*f^6*h^{12} - 39
\end{aligned}$$

$$\begin{aligned}
& 3216a^5b^4c^{10}d^{19}f^6h^{12} + 65536a^4b^6c^9d^{19}f^6h^{12} - 4096a^3b^8c^8d^{19}f^6h^{12} - 1048576a^7c^{12}d^{19}f^6h^{12} + 262144a^{10}b^6c^4d^8e^{14}f^4h^8 - 23552a^6b^6c^8d^{14}e^8f^4h^8 - 16384a^7b^7c^8d^8e^{14}f^4h^8 - 3328a^6b^{13}c^8d^7e^8f^4h^8 + 2429952a^4b^5c^6d^9e^6f^4h^8 - 1865728a^6b^3c^6d^7e^8f^4h^8 - 1716224a^4b^4c^7d^{10}e^5f^4h^8 + 1605632a^6b^2c^7d^8e^7f^4h^8 + 1584384a^5b^5c^5d^7e^8f^4h^8 + 1572864a^5b^2c^8d^{10}e^5f^4h^8 - 1433600a^5b^3c^7d^9e^6f^4h^8 - 1261568a^4b^6c^5d^8e^7f^4h^8 - 1124352a^3b^4c^8d^{12}e^3f^4h^8 - 1110016a^7b^3c^5d^5e^{10}f^4h^8 + 1106176a^3b^5c^7d^{11}e^4f^4h^8 - 936960a^5b^6c^4d^6e^9f^4h^8 - 838656a^2b^7c^6d^{11}e^4f^4h^8 - 795648a^3b^7c^5d^9e^6f^4h^8 + 730880a^3b^8c^4d^8e^7f^4h^8 + 714752a^2b^6c^7d^{12}e^3f^4h^8 + 686080a^7b^4c^4d^4e^{11}f^4h^8 + 641024a^6b^4c^5d^6e^9f^4h^8 - 595968a^8b^3c^4d^3e^{12}f^4h^8 + 544768a^3b^3c^9d^{13}e^2f^4h^8 + 516096a^2b^8c^5d^{10}e^5f^4h^8 + 441856a^6b^5c^4d^5e^{10}f^4h^8 + 393216a^7b^2c^6d^6e^9f^4h^8 + 376832a^4b^2c^9d^{12}e^3f^4h^8 - 366592a^6b^6c^3d^4e^{11}f^4h^8 + 363520a^4b^8c^3d^6e^9f^4h^8 - 356352a^5b^4c^6d^8e^7f^4h^8 - 348672a^2b^5c^8d^{13}e^2f^4h^8 - 344064a^8b^2c^5d^4e^{11}f^4h^8 + 294912a^8b^4c^3d^2e^{13}f^4h^8 + 210944a^4b^3c^8d^{11}e^4f^4h^8 - 198400a^3b^9c^3d^7e^8f^4h^8 - 144640a^4b^7c^4d^7e^8f^4h^8 - 131072a^9b^2c^4d^2e^{13}f^4h^8 - 131072a^7b^6c^2d^2e^{13}f^4h^8 - 129024a^3b^6c^6d^{10}e^5f^4h^8 - 104448a^2b^{10}c^3d^8e^7f^4h^8 + 96768a^5b^8c^2d^4e^{11}f^4h^8 + 91904a^7b^5c^3d^3e^{12}f^4h^8 - 74240a^4b^9c^2d^5e^{10}f^4h^8 - 71680a^2b^9c^4d^9e^6f^4h^8 + 58368a^2b^{11}c^2d^7e^8f^4h^8 + 36864a^5b^7c^3d^5e^{10}f^4h^8 - 35328a^3b^{10}c^2d^6e^9f^4h^8 + 27136a^6b^7c^2d^3e^{12}f^4h^8 + 909312a^8b^6c^6d^5e^{10}f^4h^8 + 815104a^9b^6c^5d^3e^{12}f^4h^8 - 651264a^5b^6c^9d^{11}e^4f^4h^8 - 573440a^6b^6c^8d^9e^6f^4h^8 - 262144a^9b^3c^3d^8e^{14}f^4h^8 + 217088a^7b^6c^7d^7e^8f^4h^8 + 211456a^6b^9c^5d^{11}e^4f^4h^8 - 204800a^4b^6c^{10}d^{13}e^2f^4h^8 - 172032a^6b^8c^6d^{12}e^3f^4h^8 - 157696a^6b^{10}c^4d^{10}e^5f^4h^8 - 131072a^3b^2c^{10}d^{14}e^8f^4h^8 + 98304a^8b^5c^2d^8e^{14}f^4h^8 + 92160a^2b^4c^9d^{14}e^8f^4h^8 + 84992a^6b^7c^7d^{13}e^2f^4h^8 + 64512a^6b^{11}c^3d^9e^6f^4h^8 + 23552a^6b^8c^8d^2e^{13}f^4h^8 + 18944a^3b^{11}c^8d^5e^{10}f^4h^8 - 13312a^4b^{10}c^8d^4e^{11}f^4h^8 - 9472a^5b^9c^8d^3e^{12}f^4h^8 - 8192a^6b^{12}c^2d^8e^7f^4h^8 - 6144a^2b^{12}c^6d^6e^9f^4h^8 - 17920b^{11}c^4d^{11}e^4f^4h^8 + 14336b^{12}c^3d^{10}e^5f^4h^8 + 14336b^{10}c^5d^{12}e^3f^4h^8 - 7168b^{13}c^2d^9e^6f^4h^8 - 7168b^9c^6d^{13}e^2f^4h^8 - 425984a^9c^6d^4e^{11}f^4h^8 - 360448a^8c^7d^6e^9f^4h^8 - 262144a^{10}c^5d^2e^{13}f^4h^8 - 131072a^7c^8d^8e^7f^4h^8 + 98304a^5c^{10}d^{12}e^3f^4h^8 + 65536a^6c^9d^{10}e^5f^4h^8 - 1536a^5b^{10}d^2e^{13}f^4h^8 - 1536a^2b^{13}d^5e^{10}f^4h^8 + 768a^4b^{11}d^3e^{12}f^4h^8 + 768a^3b^{12}d^4e^{11}f^4h^8 + 65536a^{10}b^2c^3e^{15}f^4h^8 - 24576a^9b^4c^2e^{15}f^4h^8 - 10240a^2b^3c^{10}d^{15}f^4h^8 + 2048b^{14}c^8d^8e^7f^4h^8 + 2048b^8c^7d^{14}e^8f^4h^8 + 32768*
\end{aligned}$$

$$\begin{aligned}
& a^4c^{11}d^{14}ef^4h^8 + 1024a^6b^9d^{14}f^4h^8 + 1024a^8b^{14}d^6e^9 \\
& f^4h^8 + 4096a^8b^6c^6e^{15}f^4h^8 + 12288a^3b^3c^{11}d^{15}f^4h^8 + 28 \\
& 16a^8b^5c^9d^{15}f^4h^8 - 256b^{15}d^7e^8f^4h^8 - 65536a^{11}c^4e^{15} \\
& f^4h^8 - 256b^7c^8d^{15}f^4h^8 - 256a^7b^8e^{15}f^4h^8 - 896a^8b^8c \\
& ^2d^6e^{10}f^2h^4 + 192a^8b^3c^9d^8e^3f^2h^4 + 11520a^3b^3c^5d^2e^9 \\
& f^2h^4 - 5856a^2b^5c^4d^2e^9f^2h^4 - 5120a^3b^2c^6d^3e^8f^2h \\
& ^4 + 3200a^2b^4c^5d^3e^8f^2h^4 - 640a^2b^3c^6d^4e^7f^2h^4 - \\
& 96a^2b^2c^7d^5e^6f^2h^4 - 10880a^3b^4c^4d^2e^{10}f^2h^4 + 10240a \\
& ^4b^2c^5d^2e^{10}f^2h^4 - 7680a^4b^3c^6d^2e^9f^2h^4 + 4672a^2b^6c \\
& ^3d^2e^{10}f^2h^4 + 1248a^8b^7c^3d^2e^9f^2h^4 + 832a^3b^3c^7d^4e^7f \\
& ^2h^4 - 768a^8b^6c^4d^3e^8f^2h^4 + 192a^2b^3c^8d^6e^5f^2h^4 - 1 \\
& 92a^8b^2c^8d^7e^4f^2h^4 + 176a^8b^5c^5d^4e^7f^2h^4 + 64a^8b^3c^7 \\
& d^6e^5f^2h^4 - 96b^9c^2d^2e^9f^2h^4 - 96b^2c^9d^9e^2f^2h^4 \\
& + 64b^8c^3d^3e^8f^2h^4 + 64b^3c^8d^8e^3f^2h^4 - 16b^7c^4d^4e \\
& ^7f^2h^4 - 16b^4c^7d^7e^4f^2h^4 + 2032a^4c^7d^3e^8f^2h^4 - 9 \\
& 6a^2c^9d^7e^4f^2h^4 - 64a^3c^8d^5e^6f^2h^4 - 4480a^4b^3c^4e \\
& ^{11}f^2h^4 + 3696a^3b^5c^3e^{11}f^2h^4 - 1376a^2b^7c^2e^{11}f^2h^4 \\
& - 2048a^5c^6d^2e^{10}f^2h^4 - 64a^2c^10d^9e^2f^2h^4 + 1792a^5b^3c^5 \\
& e^{11}f^2h^4 + 64b^{10}c^2d^2e^{10}f^2h^4 + 64b^3c^10d^10e^2f^2h^4 + 240a \\
& ^8b^9c^2e^{11}f^2h^4 - 16c^{11}d^{11}f^2h^4 - 16b^{11}e^{11}f^2h^4 - c^7e^7 \\
& , h, k)^3(\text{root}(8388608a^7b^3c^{11}d^{18}e^6f^6h^{12} - 513802240a^{10}b^2c^7 \\
& d^{11}e^8f^6h^{12} - 381681664a^{11}b^2c^6d^9e^{10}f^6h^{12} - 381681664a \\
& ^9b^2c^8d^{13}e^6f^6h^{12} - 300941312a^9b^5c^5d^{10}e^9f^6h^{12} - 30 \\
& 0941312a^8b^5c^6d^{12}e^7f^6h^{12} + 293601280a^{10}b^3c^6d^{10}e^9f^6 \\
& h^{12} + 293601280a^9b^3c^7d^{12}e^7f^6h^{12} - 168820736a^{10}b^5c^4d^ \\
& 8e^{11}f^6h^{12} - 168820736a^7b^5c^7d^{14}e^5f^6h^{12} + 166068224a^8b \\
& ^6c^5d^{11}e^8f^6h^{12} - 146800640a^{12}b^2c^5d^7e^{12}f^6h^{12} - 14680 \\
& 0640a^8b^2c^9d^{15}e^4f^6h^{12} + 124780544a^{10}b^4c^5d^9e^{10}f^6h^ \\
& ^{12} + 124780544a^8b^4c^7d^{13}e^6f^6h^{12} + 119275520a^9b^4c^6d^{11}e \\
& ^8f^6h^{12} + 117440512a^{11}b^3c^5d^8e^{11}f^6h^{12} + 117440512a^8b^3c \\
& ^8d^{14}e^5f^6h^{12} + 102760448a^9b^6c^4d^9e^{10}f^6h^{12} + 102760448 \\
& a^7b^6c^6d^{13}e^6f^6h^{12} + 91750400a^{11}b^4c^4d^7e^{12}f^6h^{12} + \\
& 91750400a^7b^4c^8d^{15}e^4f^6h^{12} - 71065600a^7b^8c^4d^{11}e^8f^6h \\
& ^{12} - 53444608a^8b^8c^3d^9e^{10}f^6h^{12} - 53444608a^6b^8c^5d^{13}e \\
& ^6f^6h^{12} + 40370176a^9b^7c^3d^8e^{11}f^6h^{12} + 40370176a^6b^7c^6 \\
& d^{14}e^5f^6h^{12} - 36700160a^{11}b^5c^3d^6e^{13}f^6h^{12} - 36700160a^6 \\
& b^5c^8d^{16}e^3f^6h^{12} + 34078720a^8b^7c^4d^{10}e^9f^6h^{12} + 34078 \\
& 720a^7b^7c^5d^{12}e^7f^6h^{12} + 26214400a^{12}b^4c^3d^5e^{14}f^6h^{12} \\
& + 26214400a^6b^4c^9d^{17}e^2f^6h^{12} + 22118400a^7b^9c^3d^{10}e^9f \\
& ^6h^{12} + 22118400a^6b^9c^4d^{12}e^7f^6h^{12} - 20971520a^{13}b^2c^4d^ \\
& 5e^{14}f^6h^{12} - 20971520a^7b^2c^{10}d^{17}e^2f^6h^{12} + 18350080a^{10}b \\
& ^7c^2d^6e^{13}f^6h^{12} + 18350080a^5b^7c^7d^{16}e^3f^6h^{12} - 1662976 \\
& 0a^9b^8c^2d^7e^{12}f^6h^{12} - 16629760a^5b^8c^6d^{15}e^4f^6h^{12} - \\
& 10485760a^{11}b^6c^2d^5e^{14}f^6h^{12} - 10485760a^5b^6c^8d^{17}e^2f^6 \\
& h^{12} + 9175040a^{10}b^6c^3d^7e^{12}f^6h^{12} + 9175040a^6b^6c^7d^{15}e
\end{aligned}$$

$$\begin{aligned}
&^4f^6h^{12} - 8388608a^{13}b^3c^3d^4e^{15}f^6h^{12} + 5619712a^7b^{10}c^2 \\
&d^9e^{10}f^6h^{12} + 5619712a^5b^{10}c^4d^{13}e^6f^6h^{12} - 5570560a^6b \\
&^{11}c^2d^{10}e^9f^6h^{12} - 5570560a^5b^{11}c^3d^{12}e^7f^6h^{12} + 435814 \\
&4a^8b^9c^2d^8e^{11}f^6h^{12} + 4358144a^5b^9c^5d^{14}e^5f^6h^{12} + 4 \\
&259840a^6b^{10}c^3d^{11}e^8f^6h^{12} + 3899392a^4b^{10}c^5d^{15}e^4f^6h \\
&^{12} - 3440640a^4b^9c^6d^{16}e^3f^6h^{12} + 3145728a^{12}b^5c^2d^4e^{15} \\
&f^6h^{12} - 2523136a^4b^{11}c^4d^{14}e^5f^6h^{12} + 1802240a^4b^8c^7d^ \\
&17e^2f^6h^{12} + 1556480a^5b^{12}c^2d^{11}e^8f^6h^{12} + 1048576a^{14}b^2 \\
&c^3d^3e^{16}f^6h^{12} + 688128a^4b^{12}c^3d^{13}e^6f^6h^{12} - 393216a^1 \\
&3b^4c^2d^3e^{16}f^6h^{12} - 286720a^3b^{12}c^4d^{15}e^4f^6h^{12} + 22937 \\
&6a^3b^{13}c^3d^{14}e^5f^6h^{12} + 229376a^3b^{11}c^5d^{16}e^3f^6h^{12} + \\
&163840a^4b^{13}c^2d^{12}e^7f^6h^{12} - 114688a^3b^{14}c^2d^{13}e^6f^6h^ \\
&12 - 114688a^3b^{10}c^6d^{17}e^2f^6h^{12} + 293601280a^{11}b^3c^7d^{10}e^9 \\
&f^6h^{12} + 293601280a^{10}b^3c^8d^{12}e^7f^6h^{12} + 176160768a^{12}b^3c^6d^ \\
&8e^{11}f^6h^{12} + 176160768a^9b^3c^9d^{14}e^5f^6h^{12} + 58720256a^{13}b^3c \\
&^5d^6e^{13}f^6h^{12} + 58720256a^8b^3c^{10}d^{16}e^3f^6h^{12} + 8388608a^{14} \\
&b^3c^4d^4e^{15}f^6h^{12} - 8388608a^6b^3c^{10}d^{18}e^6f^6h^{12} + 3899392a \\
&^8b^{10}c^d^7e^{12}f^6h^{12} - 3440640a^9b^9c^d^6e^{13}f^6h^{12} + 3145728 \\
&a^5b^5c^9d^{18}e^6f^6h^{12} - 2523136a^7b^{11}c^d^8e^{11}f^6h^{12} + 18022 \\
&40a^{10}b^8c^d^5e^{14}f^6h^{12} + 688128a^6b^{12}c^d^9e^{10}f^6h^{12} - 524 \\
&288a^{11}b^7c^d^4e^{15}f^6h^{12} - 524288a^4b^7c^8d^{18}e^6f^6h^{12} + 163 \\
&840a^5b^{13}c^d^{10}e^9f^6h^{12} - 163840a^4b^{14}c^d^{11}e^8f^6h^{12} + 65 \\
&536a^{12}b^6c^d^3e^{16}f^6h^{12} + 32768a^3b^{15}c^d^{12}e^7f^6h^{12} + 327 \\
&68a^3b^9c^7d^{18}e^6f^6h^{12} - 73400320a^{11}c^8d^{11}e^8f^6h^{12} - 5872 \\
&0256a^{12}c^7d^9e^{10}f^6h^{12} - 58720256a^{10}c^9d^{13}e^6f^6h^{12} - 293 \\
&60128a^{13}c^6d^7e^{12}f^6h^{12} - 29360128a^9c^{10}d^{15}e^4f^6h^{12} - 83 \\
&88608a^{14}c^5d^5e^{14}f^6h^{12} - 8388608a^8c^{11}d^{17}e^2f^6h^{12} - 104 \\
&8576a^{15}c^4d^3e^{16}f^6h^{12} - 286720a^7b^{12}d^7e^{12}f^6h^{12} + 22937 \\
&6a^8b^{11}d^6e^{13}f^6h^{12} + 229376a^6b^{13}d^8e^{11}f^6h^{12} - 114688a \\
&^9b^{10}d^5e^{14}f^6h^{12} - 114688a^5b^{14}d^9e^{10}f^6h^{12} + 32768a^{10} \\
&b^9d^4e^{15}f^6h^{12} + 32768a^4b^{15}d^{10}e^9f^6h^{12} - 4096a^{11}b^8d^ \\
&3e^{16}f^6h^{12} - 4096a^3b^{16}d^{11}e^8f^6h^{12} + 1048576a^6b^2c^{11}d^ \\
&19f^6h^{12} - 393216a^5b^4c^{10}d^{19}f^6h^{12} + 65536a^4b^6c^9d^{19}f^ \\
&6h^{12} - 4096a^3b^8c^8d^{19}f^6h^{12} - 1048576a^7c^{12}d^{19}f^6h^{12} + \\
&262144a^{10}b^3c^4d^e^{14}f^4h^8 - 23552a^b^6c^8d^{14}e^f^4h^8 - 16384a \\
&^7b^7c^d^e^{14}f^4h^8 - 3328a^b^{13}c^d^7e^8f^4h^8 + 2429952a^4b^5c \\
&^6d^9e^6f^4h^8 - 1865728a^6b^3c^6d^7e^8f^4h^8 - 1716224a^4b^4c \\
&^7d^{10}e^5f^4h^8 + 1605632a^6b^2c^7d^8e^7f^4h^8 + 1584384a^5b^ \\
&5c^5d^7e^8f^4h^8 + 1572864a^5b^2c^8d^{10}e^5f^4h^8 - 1433600a^5b \\
&^3c^7d^9e^6f^4h^8 - 1261568a^4b^6c^5d^8e^7f^4h^8 - 1124352a^3 \\
&b^4c^8d^{12}e^3f^4h^8 - 1110016a^7b^3c^5d^5e^{10}f^4h^8 + 1106176* \\
&a^3b^5c^7d^{11}e^4f^4h^8 - 936960a^5b^6c^4d^6e^9f^4h^8 - 838656* \\
&a^2b^7c^6d^{11}e^4f^4h^8 - 795648a^3b^7c^5d^9e^6f^4h^8 + 730880* \\
&a^3b^8c^4d^8e^7f^4h^8 + 714752a^2b^6c^7d^{12}e^3f^4h^8 + 686080* \\
&a^7b^4c^4d^4e^{11}f^4h^8 + 641024a^6b^4c^5d^6e^9f^4h^8 - 595968*
\end{aligned}$$

$$\begin{aligned}
& a^8 b^3 c^4 d^3 e^{12} f^4 h^8 + 544768 a^3 b^3 c^9 d^{13} e^2 f^4 h^8 + 516096 \\
& a^2 b^8 c^5 d^{10} e^5 f^4 h^8 + 441856 a^6 b^5 c^4 d^5 e^{10} f^4 h^8 + 39321 \\
& 6 a^7 b^2 c^6 d^6 e^9 f^4 h^8 + 376832 a^4 b^2 c^9 d^{12} e^3 f^4 h^8 - 36659 \\
& 2 a^6 b^6 c^3 d^4 e^{11} f^4 h^8 + 363520 a^4 b^8 c^3 d^6 e^9 f^4 h^8 - 35635 \\
& 2 a^5 b^4 c^6 d^8 e^7 f^4 h^8 - 348672 a^2 b^5 c^8 d^{13} e^2 f^4 h^8 - 34406 \\
& 4 a^8 b^2 c^5 d^4 e^{11} f^4 h^8 + 294912 a^8 b^4 c^3 d^2 e^{13} f^4 h^8 + 2109 \\
& 44 a^4 b^3 c^8 d^{11} e^4 f^4 h^8 - 198400 a^3 b^9 c^3 d^7 e^8 f^4 h^8 - 1446 \\
& 40 a^4 b^7 c^4 d^7 e^8 f^4 h^8 - 131072 a^9 b^2 c^4 d^2 e^{13} f^4 h^8 - 1310 \\
& 72 a^7 b^6 c^2 d^2 e^{13} f^4 h^8 - 129024 a^3 b^6 c^6 d^{10} e^5 f^4 h^8 - 104 \\
& 448 a^2 b^{10} c^3 d^8 e^7 f^4 h^8 + 96768 a^5 b^8 c^2 d^4 e^{11} f^4 h^8 + 919 \\
& 04 a^7 b^5 c^3 d^3 e^{12} f^4 h^8 - 74240 a^4 b^9 c^2 d^5 e^{10} f^4 h^8 - 7168 \\
& 0 a^2 b^9 c^4 d^9 e^6 f^4 h^8 + 58368 a^2 b^{11} c^2 d^7 e^8 f^4 h^8 + 36864 a \\
& a^5 b^7 c^3 d^5 e^{10} f^4 h^8 - 35328 a^3 b^{10} c^2 d^6 e^9 f^4 h^8 + 27136 a \\
& ^6 b^7 c^2 d^3 e^{12} f^4 h^8 + 909312 a^8 b^6 c^6 d^5 e^{10} f^4 h^8 + 815104 a^ \\
& 9 b^6 c^5 d^3 e^{12} f^4 h^8 - 651264 a^5 b^6 c^9 d^{11} e^4 f^4 h^8 - 573440 a^6 b \\
& ^8 c^8 d^9 e^6 f^4 h^8 - 262144 a^9 b^3 c^3 d^5 e^{14} f^4 h^8 + 217088 a^7 b^6 c^7 \\
& ^7 d^7 e^8 f^4 h^8 + 211456 a^8 b^9 c^5 d^{11} e^4 f^4 h^8 - 204800 a^4 b^6 c^{10} d^ \\
& 13 e^2 f^4 h^8 - 172032 a^8 b^8 c^6 d^{12} e^3 f^4 h^8 - 157696 a^8 b^{10} c^4 d^{10} \\
& e^5 f^4 h^8 - 131072 a^3 b^2 c^{10} d^{14} e^4 f^4 h^8 + 98304 a^8 b^5 c^2 d^5 e^{11} \\
& 4 f^4 h^8 + 92160 a^2 b^4 c^9 d^{14} e^4 f^4 h^8 + 84992 a^8 b^7 c^7 d^{13} e^2 f^4 \\
& ^4 h^8 + 64512 a^8 b^{11} c^3 d^9 e^6 f^4 h^8 + 23552 a^6 b^8 c^6 d^2 e^{13} f^4 h^8 \\
& + 18944 a^3 b^{11} c^5 d^5 e^{10} f^4 h^8 - 13312 a^4 b^{10} c^4 d^4 e^{11} f^4 h^8 - 9 \\
& 472 a^5 b^9 c^4 d^3 e^{12} f^4 h^8 - 8192 a^8 b^{12} c^2 d^8 e^7 f^4 h^8 - 6144 a^2 \\
& ^2 b^{12} c^6 d^6 e^9 f^4 h^8 - 17920 a^8 b^{11} c^4 d^{11} e^4 f^4 h^8 + 14336 b^{12} c^3 \\
& ^3 d^{10} e^5 f^4 h^8 + 14336 b^{10} c^5 d^{12} e^3 f^4 h^8 - 7168 b^{13} c^2 d^9 e^6 \\
& ^6 f^4 h^8 - 7168 b^9 c^6 d^{13} e^2 f^4 h^8 - 425984 a^9 c^6 d^4 e^{11} f^4 h^8 - \\
& 360448 a^8 c^7 d^6 e^9 f^4 h^8 - 262144 a^{10} c^5 d^2 e^{13} f^4 h^8 - 131072 \\
& ^7 a^7 c^8 d^8 e^7 f^4 h^8 + 98304 a^5 c^{10} d^{12} e^3 f^4 h^8 + 65536 a^6 c^9 \\
& ^9 d^{10} e^5 f^4 h^8 - 1536 a^5 b^{10} d^2 e^{13} f^4 h^8 - 1536 a^2 b^{13} d^5 e^{10} \\
& ^10 f^4 h^8 + 768 a^4 b^{11} d^3 e^{12} f^4 h^8 + 768 a^3 b^{12} d^4 e^{11} f^4 h^8 + 6 \\
& 5536 a^{10} b^2 c^3 e^{15} f^4 h^8 - 24576 a^9 b^4 c^2 e^{15} f^4 h^8 - 10240 a^2 \\
& ^2 b^3 c^{10} d^{15} f^4 h^8 + 2048 b^{14} c^4 d^8 e^7 f^4 h^8 + 2048 b^8 c^7 d^{14} e^ \\
& ^4 f^4 h^8 + 32768 a^4 c^{11} d^{14} e^4 f^4 h^8 + 1024 a^6 b^9 d^4 e^{14} f^4 h^8 + 102 \\
& 4 a^8 b^{14} d^6 e^9 f^4 h^8 + 4096 a^8 b^6 c^6 e^{15} f^4 h^8 + 12288 a^3 b^6 c^{11} d \\
& ^{15} f^4 h^8 + 2816 a^8 b^5 c^9 d^{15} f^4 h^8 - 256 b^{15} d^7 e^8 f^4 h^8 - 6553 \\
& 6 a^{11} c^4 e^{15} f^4 h^8 - 256 b^7 c^8 d^{15} f^4 h^8 - 256 a^7 b^8 e^{15} f^4 h \\
& ^8 - 896 a^8 b^8 c^2 d^2 e^{10} f^2 h^4 + 192 a^8 b^8 c^9 d^8 e^3 f^2 h^4 + 11520 a^3 \\
& ^3 b^3 c^5 d^2 e^9 f^2 h^4 - 5856 a^2 b^5 c^4 d^2 e^9 f^2 h^4 - 5120 a^3 b^2 c^6 \\
& ^6 d^3 e^8 f^2 h^4 + 3200 a^2 b^4 c^5 d^3 e^8 f^2 h^4 - 640 a^2 b^3 c^6 d^4 \\
& ^4 e^7 f^2 h^4 - 96 a^2 b^2 c^7 d^5 e^6 f^2 h^4 - 10880 a^3 b^4 c^4 d^4 e^{10} f \\
& ^2 h^4 + 10240 a^4 b^2 c^5 d^4 e^{10} f^2 h^4 - 7680 a^4 b^6 c^6 d^2 e^9 f^2 h^4 \\
& + 4672 a^2 b^6 c^3 d^4 e^{10} f^2 h^4 + 1248 a^8 b^7 c^3 d^2 e^9 f^2 h^4 + 832 a^ \\
& ^3 b^6 c^7 d^4 e^7 f^2 h^4 - 768 a^8 b^6 c^4 d^3 e^8 f^2 h^4 + 192 a^2 b^6 c^8 d^6 \\
& ^6 e^5 f^2 h^4 - 192 a^8 b^2 c^8 d^7 e^4 f^2 h^4 + 176 a^8 b^5 c^5 d^4 e^7 f^2 h^ \\
& 4 + 64 a^8 b^3 c^7 d^6 e^5 f^2 h^4 - 96 b^9 c^2 d^2 e^9 f^2 h^4 - 96 b^2 c^9
\end{aligned}$$

$$\begin{aligned}
& d^9 e^2 f^2 h^4 + 64 b^8 c^3 d^3 e^8 f^2 h^4 + 64 b^3 c^8 d^8 e^3 f^2 h^4 - \\
& 16 b^7 c^4 d^4 e^7 f^2 h^4 - 16 b^4 c^7 d^7 e^4 f^2 h^4 + 2032 a^4 c^7 d^3 \\
& e^8 f^2 h^4 - 96 a^2 c^9 d^7 e^4 f^2 h^4 - 64 a^3 c^8 d^5 e^6 f^2 h^4 - 44 \\
& 80 a^4 b^3 c^4 e^{11} f^2 h^4 + 3696 a^3 b^5 c^3 e^{11} f^2 h^4 - 1376 a^2 b^7 c^2 \\
& e^{11} f^2 h^4 - 2048 a^5 c^6 d e^{10} f^2 h^4 - 64 a c^{10} d^9 e^2 f^2 h^4 \\
& + 1792 a^5 b c^5 e^{11} f^2 h^4 + 64 b^{10} c d e^{10} f^2 h^4 + 64 b c^{10} d^{10} e \\
& f^2 h^4 + 240 a b^9 c e^{11} f^2 h^4 - 16 c^{11} d^{11} f^2 h^4 - 16 b^{11} e^{11} f \\
& ^2 h^4 - c^7 e^7, h, k) (4697620480 a^9 c^{11} d^7 e^{13} f^{55} - 1879048192 a^6 \\
& c^{14} d^{13} e^7 f^{55} - 2818572288 a^7 c^{13} d^{11} e^9 f^{55} - 402653184 a^5 c^1 \\
& 5 d^{15} e^5 f^{55} + 5637144576 a^{10} c^{10} d^5 e^{15} f^{55} + 2818572288 a^{11} c^9 \\
& d^3 e^{17} f^{55} + 536870912 a^{12} c^8 d e^{19} f^{55} + 2097152 a b^7 c^{12} d^{16} e^ \\
& 4 f^{55} - 16777216 a b^8 c^{11} d^{15} e^5 f^{55} + 58720256 a b^9 c^{10} d^{14} e^6 f \\
& ^{55} - 117440512 a b^{10} c^9 d^{13} e^7 f^{55} + 146800640 a b^{11} c^8 d^{12} e^8 f^ \\
& 55 - 117440512 a b^{12} c^7 d^{11} e^9 f^{55} + 58720256 a b^{13} c^6 d^{10} e^{10} f^5 \\
& 5 - 16777216 a b^{14} c^5 d^9 e^{11} f^{55} + 2097152 a b^{15} c^4 d^8 e^{12} f^{55} - \\
& 134217728 a^4 b c^{15} d^{16} e^4 f^{55} + 2147483648 a^5 b c^{14} d^{14} e^6 f^{55} + \\
& 10066329600 a^6 b c^{13} d^{12} e^8 f^{55} + 13421772800 a^7 b c^{12} d^{10} e^{10} f^5 \\
& 5 + 671088640 a^8 b c^{11} d^8 e^{12} f^{55} + 2097152 a^8 b^8 c^4 d e^{19} f^{55} - \\
& 12884901888 a^9 b c^{10} d^6 e^{14} f^{55} - 33554432 a^9 b^6 c^5 d e^{19} f^{55} - 1 \\
& 0603200512 a^{10} b c^9 d^4 e^{16} f^{55} + 201326592 a^{10} b^4 c^6 d e^{19} f^{55} - \\
& 2684354560 a^{11} b c^8 d^2 e^{18} f^{55} - 536870912 a^{11} b^2 c^7 d e^{19} f^{55} - \\
& 25165824 a^2 b^5 c^{13} d^{16} e^4 f^{55} + 207618048 a^2 b^6 c^{12} d^{15} e^5 f^{55} \\
& - 738197504 a^2 b^7 c^{11} d^{14} e^6 f^{55} + 1468006400 a^2 b^8 c^{10} d^{13} e^7 f^ \\
& ^{55} - 1761607680 a^2 b^9 c^9 d^{12} e^8 f^{55} + 1262485504 a^2 b^{10} c^8 d^{11} e \\
& ^9 f^{55} - 469762048 a^2 b^{11} c^7 d^{10} e^{10} f^{55} + 25165824 a^2 b^{12} c^6 d^9 \\
& e^{11} f^{55} + 41943040 a^2 b^{13} c^5 d^8 e^{12} f^{55} - 10485760 a^2 b^{14} c^4 d^ \\
& 7 e^{13} f^{55} + 100663296 a^3 b^3 c^{14} d^{16} e^4 f^{55} - 880803840 a^3 b^4 c^{13} \\
& d^{15} e^5 f^{55} + 3221225472 a^3 b^5 c^{12} d^{14} e^6 f^{55} - 6312427520 a^3 b^6 \\
& c^{11} d^{13} e^7 f^{55} + 6889144320 a^3 b^7 c^{10} d^{12} e^8 f^{55} - 3548381184 a^ \\
& 3 b^8 c^9 d^{11} e^9 f^{55} - 304087040 a^3 b^9 c^8 d^{10} e^{10} f^{55} + 1371537408 \\
& a^3 b^{10} c^7 d^9 e^{11} f^{55} - 597688320 a^3 b^{11} c^6 d^8 e^{12} f^{55} + 419430 \\
& 40 a^3 b^{12} c^5 d^7 e^{13} f^{55} + 18874368 a^3 b^{13} c^4 d^6 e^{14} f^{55} + 13757 \\
& 31712 a^4 b^2 c^{14} d^{15} e^5 f^{55} - 5368709120 a^4 b^3 c^{13} d^{14} e^6 f^{55} + \\
& 9982443520 a^4 b^4 c^{12} d^{13} e^7 f^{55} - 7507804160 a^4 b^5 c^{11} d^{12} e^8 f^ \\
& 55 - 3412066304 a^4 b^6 c^{10} d^{11} e^9 f^{55} + 10955522048 a^4 b^7 c^9 d^{10} e \\
& ^{10} f^{55} - 7748976640 a^4 b^8 c^8 d^9 e^{11} f^{55} + 1468006400 a^4 b^9 c^7 d^ \\
& 8 e^{12} f^{55} + 618659840 a^4 b^{10} c^6 d^7 e^{13} f^{55} - 218103808 a^4 b^{11} c^5 \\
& d^6 e^{14} f^{55} - 10485760 a^4 b^{12} c^4 d^5 e^{15} f^{55} - 2348810240 a^5 b^2 c \\
& ^{13} d^{13} e^7 f^{55} - 7549747200 a^5 b^3 c^{12} d^{12} e^8 f^{55} + 24570232832 a^5 \\
& b^4 c^{11} d^{11} e^9 f^{55} - 27111981056 a^5 b^5 c^{10} d^{10} e^{10} f^{55} + 9638510 \\
& 592 a^5 b^6 c^9 d^9 e^{11} f^{55} + 4854906880 a^5 b^7 c^8 d^8 e^{12} f^{55} - 4697 \\
& 620480 a^5 b^8 c^7 d^7 e^{13} f^{55} + 742391808 a^5 b^9 c^6 d^6 e^{14} f^{55} + 16 \\
& 7772160 a^5 b^{10} c^5 d^5 e^{15} f^{55} - 10485760 a^5 b^{11} c^4 d^4 e^{16} f^{55} - \\
& 18824036352 a^6 b^2 c^{12} d^{11} e^9 f^{55} + 9395240960 a^6 b^3 c^{11} d^{10} e^{10} \\
& f^{55} + 14596177920 a^6 b^4 c^{10} d^9 e^{11} f^{55} - 22825402368 a^6 b^5 c^9 d^8
\end{aligned}$$

$$\begin{aligned}
& *e^{12}f^{55} + 10328473600*a^6*b^6*c^8*d^7*e^{13}f^{55} + 150994944*a^6*b^7*c^7*d^6*e^{14}f^{55} - 1170210816*a^6*b^8*c^6*d^5*e^{15}f^{55} + 142606336*a^6*b^9*c^5*d^4*e^{16}f^{55} + 18874368*a^6*b^10*c^4*d^3*e^{17}f^{55} - 24830279680*a^7*b^2*c^11*d^9*e^{11}f^{55} + 20971520000*a^7*b^3*c^10*d^8*e^{12}f^{55} - 4487905280*a^7*b^4*c^9*d^7*e^{13}f^{55} - 5972688896*a^7*b^5*c^8*d^6*e^{14}f^{55} + 4559208448*a^7*b^6*c^7*d^5*e^{15}f^{55} - 538968064*a^7*b^7*c^6*d^4*e^{16}f^{55} - 293601280*a^7*b^8*c^5*d^3*e^{17}f^{55} - 10485760*a^7*b^9*c^4*d^2*e^{18}f^{55} - 6207569920*a^8*b^2*c^10*d^7*e^{13}f^{55} + 13690208256*a^8*b^3*c^9*d^6*e^{14}f^{55} - 9479127040*a^8*b^4*c^8*d^5*e^{15}f^{55} - 511705088*a^8*b^5*c^7*d^4*e^{16}f^{55} + 1667235840*a^8*b^6*c^6*d^3*e^{17}f^{55} + 167772160*a^8*b^7*c^5*d^2*e^{18}f^{55} + 6241124352*a^9*b^2*c^9*d^5*e^{15}f^{55} + 6878658560*a^9*b^3*c^8*d^4*e^{16}f^{55} - 3900702720*a^9*b^4*c^7*d^3*e^{17}f^{55} - 1006632960*a^9*b^5*c^6*d^2*e^{18}f^{55} + 2181038080*a^10*b^2*c^8*d^3*e^{17}f^{55} + 2684354560*a^10*b^3*c^7*d^2*e^{18}f^{55} + (f*x)^{(1/2)}*(268435456*a^11*c^8*e^{19}f^{54} + 1048576*a^7*b^8*c^4*e^{19}f^{54} - 16777216*a^8*b^6*c^5*e^{19}f^{54} + 100663296*a^9*b^4*c^6*e^{19}f^{54} - 268435456*a^10*b^2*c^7*e^{19}f^{54} - 134217728*a^4*c^15*d^14*e^5*f^{54} - 402653184*a^5*c^14*d^12*e^7*f^{54} - 268435456*a^6*c^13*d^10*e^9*f^{54} + 536870912*a^7*c^12*d^8*e^11*f^{54} + 1476395008*a^8*c^11*d^6*e^13*f^{54} + 1744830464*a^9*c^10*d^4*e^15*f^{54} + 1073741824*a^10*c^9*d^2*e^17*f^{54} + 1048576*b^7*c^12*d^15*e^4*f^{54} - 8388608*b^8*c^11*d^14*e^5*f^{54} + 29360128*b^9*c^10*d^13*e^6*f^{54} - 58720256*b^10*c^9*d^12*e^7*f^{54} + 73400320*b^11*c^8*d^11*e^8*f^{54} - 58720256*b^12*c^7*d^10*e^9*f^{54} + 29360128*b^13*c^6*d^9*e^10*f^{54} - 8388608*b^14*c^5*d^8*e^11*f^{54} + 1048576*b^15*c^4*d^7*e^12*f^{54} - 1073741824*a^10*b*c^8*d*e^18*f^{54} - 11534336*a*b^5*c^13*d^15*e^4*f^{54} + 96468992*a*b^6*c^12*d^14*e^5*f^{54} - 348127232*a*b^7*c^11*d^13*e^6*f^{54} + 704643072*a*b^8*c^10*d^12*e^7*f^{54} - 866123776*a*b^9*c^9*d^11*e^8*f^{54} + 645922816*a*b^10*c^8*d^10*e^9*f^{54} - 264241152*a*b^11*c^7*d^9*e^10*f^{54} + 33554432*a*b^12*c^6*d^8*e^11*f^{54} + 13631488*a*b^13*c^5*d^7*e^12*f^{54} - 4194304*a*b^14*c^4*d^6*e^13*f^{54} - 50331648*a^3*b*c^15*d^15*e^4*f^{54} + 838860800*a^4*b*c^14*d^13*e^6*f^{54} + 2667577344*a^5*b*c^13*d^11*e^8*f^{54} + 2348810240*a^6*b*c^12*d^9*e^10*f^{54} - 4194304*a^6*b^9*c^4*d*e^18*f^{54} - 889192448*a^7*b*c^11*d^7*e^12*f^{54} + 67108864*a^7*b^7*c^5*d*e^18*f^{54} - 3724541952*a^8*b*c^10*d^5*e^14*f^{54} - 402653184*a^8*b^5*c^6*d*e^18*f^{54} - 3338665984*a^9*b*c^9*d^3*e^16*f^{54} + 1073741824*a^9*b^3*c^7*d*e^18*f^{54} + 41943040*a^2*b^3*c^14*d^15*e^4*f^{54} - 377487360*a^2*b^4*c^13*d^14*e^5*f^{54} + 1428160512*a^2*b^5*c^12*d^13*e^6*f^{54} - 2927624192*a^2*b^6*c^11*d^12*e^7*f^{54} + 3435134976*a^2*b^7*c^10*d^11*e^8*f^{54} - 2113929216*a^2*b^8*c^9*d^10*e^9*f^{54} + 293601280*a^2*b^9*c^8*d^9*e^10*f^{54} + 427819008*a^2*b^10*c^7*d^8*e^11*f^{54} - 239075328*a^2*b^11*c^6*d^7*e^12*f^{54} + 25165824*a^2*b^12*c^5*d^6*e^13*f^{54} + 6291456*a^2*b^13*c^4*d^5*e^14*f^{54} + 536870912*a^3*b^2*c^14*d^14*e^5*f^{54} - 2231369728*a^3*b^3*c^13*d^13*e^6*f^{54} + 4605345792*a^3*b^4*c^12*d^12*e^7*f^{54} - 4530896896*a^3*b^5*c^11*d^11*e^8*f^{54} + 528482304*a^3*b^6*c^10*d^10*e^9*f^{54} + 3258974208*a^3*b^7*c^9*d^9*e^10*f^{54} - 2993684480*a^3*b^8*c^8*d^8*e^11*f^{54} + 812646400*a^3*b^9*c^7*d^7*e^12*f^{54} + 144703488*a^3*b^10*c^6*d^6*e^13*f^{54} - 77594624*a^3*b^11*c^5*d^5*e^14*f^{54} - 3145728*a^3*b^12*c^4*d^4*e^15*f^{54} - 1
\end{aligned}$$

$$\begin{aligned}
& 543503872a^4b^2c^{13}d^{12}e^7f^{54} - 864026624a^4b^3c^{12}d^{11}e^8f^{54} \\
& + 7029653504a^4b^4c^{11}d^{10}e^9f^{54} - 9953083392a^4b^5c^{10}d^9e^{10} \\
& *f^{54} + 5167382528a^4b^6c^9d^8e^{11}f^{54} + 592445440a^4b^7c^8d^7e^{12} \\
& *f^{54} - 1488977920a^4b^8c^7d^6e^{13}f^{54} + 304087040a^4b^9c^6d^5e^{14} \\
& *f^{54} + 54525952a^4b^{10}c^5d^4e^{15}f^{54} - 3145728a^4b^{11}c^4d^3e^{16} \\
& *f^{54} - 6442450944a^5b^2c^{12}d^{10}e^9f^{54} + 5872025600a^5b^3c^{11} \\
& *d^9e^{10}f^{54} + 1459617792a^5b^4c^{10}d^8e^{11}f^{54} - 6489636864a^5b^5 \\
& *c^9d^7e^{12}f^{54} + 3837788160a^5b^6c^8d^6e^{13}f^{54} - 150994944a^5b^7 \\
& *c^7d^5e^{14}f^{54} - 396361728a^5b^8c^6d^4e^{15}f^{54} + 38797312a^5b^9 \\
& *c^5d^3e^{16}f^{54} + 6291456a^5b^{10}c^4d^2e^{17}f^{54} - 6576668672a^6b^2 \\
& *c^{11}d^8e^{11}f^{54} + 7642021888a^6b^3c^{10}d^7e^{12}f^{54} - 2625634304 \\
& *a^6b^4c^9d^6e^{13}f^{54} - 1809842176a^6b^5c^8d^5e^{14}f^{54} + 1501560 \\
& 832a^6b^6c^7d^4e^{15}f^{54} - 111149056a^6b^7c^6d^3e^{16}f^{54} - 96468 \\
& 992a^6b^8c^5d^2e^{17}f^{54} - 1610612736a^7b^2c^{10}d^6e^{13}f^{54} + 454 \\
& 6625536a^7b^3c^9d^5e^{14}f^{54} - 2810183680a^7b^4c^8d^4e^{15}f^{54} - \\
& 376438784a^7b^5c^7d^3e^{16}f^{54} + 536870912a^7b^6c^6d^2e^{17}f^{54} + \\
& 1409286144a^8b^2c^9d^4e^{15}f^{54} + 2441084928a^8b^3c^8d^3e^{16}f^{54} \\
& 4 - 1207959552a^8b^4c^7d^2e^{17}f^{54} + 536870912a^9b^2c^8d^2e^{17}f^{54} \\
&) + 8388608a^7c^9e^{16}f^{53} - 131072a^2b^{10}c^4e^{16}f^{53} + 1966080 \\
& *a^3b^8c^5e^{16}f^{53} - 11141120a^4b^6c^6e^{16}f^{53} + 28835840a^5b^4c^7 \\
& *e^{16}f^{53} - 31457280a^6b^2c^8e^{16}f^{53} + 2097152a^2c^{14}d^{10}e^6f^{53} \\
& + 3145728a^3c^{13}d^8e^8f^{53} - 14680064a^4c^{12}d^6e^{10}f^{53} - 24 \\
& 641536a^5c^{11}d^4e^{12}f^{53} - 131072b^2c^{14}d^{12}e^4f^{53} + 655360b^3c^{13} \\
& *d^{11}e^5f^{53} - 1310720b^4c^{12}d^{10}e^6f^{53} + 1310720b^5c^{11}d^9e^7 \\
& *f^{53} - 655360b^6c^{10}d^8e^8f^{53} + 262144b^7c^9d^7e^9f^{53} - 655 \\
& 360b^8c^8d^6e^{10}f^{53} + 1310720b^9c^7d^5e^{11}f^{53} - 1310720b^{10}c^6 \\
& *d^4e^{12}f^{53} + 655360b^{11}c^5d^3e^{13}f^{53} - 131072b^{12}c^4d^2e^{14} \\
& *f^{53} + 524288a^c^{15}d^{12}e^4f^{53} - 2621440a^*b^*c^{14}d^{11}e^5f^{53} + 26214 \\
& 4a^*b^{11}c^4d^*e^{15}f^{53} + 27262976a^6b^*c^9d^*e^{15}f^{53} + 4718592a^*b^2c^ \\
& ^{13}d^{10}e^6f^{53} - 3145728a^*b^3c^{12}d^9e^7f^{53} - 524288a^*b^4c^{11}d^8 \\
& *e^8f^{53} + 131072a^*b^5c^{10}d^7e^9f^{53} + 7208960a^*b^6c^9d^6e^{10}f^{53} \\
& 3 - 16252928a^*b^7c^8d^5e^{11}f^{53} + 16515072a^*b^8c^7d^4e^{12}f^{53} - 7 \\
& 733248a^*b^9c^6d^3e^{13}f^{53} + 917504a^*b^{10}c^5d^2e^{14}f^{53} - 8388608 \\
& *a^2b^*c^{13}d^9e^7f^{53} - 3538944a^2b^9c^5d^*e^{15}f^{53} - 15728640a^3b^*c^ \\
& ^{12}d^7e^9f^{53} + 16908288a^3b^7c^6d^*e^{15}f^{53} + 60817408a^4b^*c^{11} \\
& *d^5e^{11}f^{53} - 30801920a^4b^5c^7d^*e^{15}f^{53} + 98041856a^5b^*c^{10}d^3 \\
& *e^{13}f^{53} + 5242880a^5b^3c^8d^*e^{15}f^{53} + 11796480a^2b^2c^{12}d^8e^8 \\
& *f^{53} - 786432a^2b^3c^{11}d^7e^9f^{53} - 31719424a^2b^4c^{10}d^6e^{10}f^ \\
& ^{53} + 71958528a^2b^5c^9d^5e^{11}f^{53} - 73269248a^2b^6c^8d^4e^{12}f^ \\
& ^{53} + 28835840a^2b^7c^7d^3e^{13}f^{53} + 3145728a^2b^8c^6d^2e^{14}f^ \\
& ^{53} + 57147392a^3b^2c^{11}d^6e^{10}f^{53} - 126877696a^3b^3c^{10}d^5e^{11}f^ \\
& ^{53} + 126877696a^3b^4c^9d^4e^{12}f^{53} - 21102592a^3b^5c^8d^3e^{13}f^ \\
& ^{53} - 42336256a^3b^6c^7d^2e^{14}f^{53} - 50462720a^4b^2c^{10}d^4e^{12}f^ \\
& ^{53} - 74317824a^4b^3c^9d^3e^{13}f^{53} + 120586240a^4b^4c^8d^2e^{14}f^ \\
& ^{53} - 106954752a^5b^2c^9d^2e^{14}f^{53} + (f*x)^{(1/2)}*(131072b^{11}c^4e^
\end{aligned}$$

$$\begin{aligned}
& 15*f^52 + 131072*c^15*d^11*e^4*f^52 + 11272192*a^2*b^7*c^6*e^15*f^52 - 3027 \\
& 7632*a^3*b^5*c^7*e^15*f^52 + 36700160*a^4*b^3*c^8*e^15*f^52 + 786432*a^2*c^ \\
& 13*d^7*e^8*f^52 + 524288*a^3*c^12*d^5*e^10*f^52 - 16646144*a^4*c^11*d^3*e^1 \\
& 2*f^52 + 786432*b^2*c^13*d^9*e^6*f^52 - 524288*b^3*c^12*d^8*e^7*f^52 + 1310 \\
& 72*b^4*c^11*d^7*e^8*f^52 + 131072*b^7*c^8*d^4*e^11*f^52 - 524288*b^8*c^7*d^ \\
& 3*e^12*f^52 + 786432*b^9*c^6*d^2*e^13*f^52 - 1966080*a*b^9*c^5*e^15*f^52 - \\
& 14680064*a^5*b*c^9*e^15*f^52 + 524288*a*c^14*d^9*e^6*f^52 + 16777216*a^5*c^ \\
& 10*d*e^14*f^52 - 524288*b*c^14*d^10*e^5*f^52 - 524288*b^10*c^5*d*e^14*f^52 \\
& - 1572864*a*b*c^13*d^8*e^7*f^52 + 7340032*a*b^8*c^6*d*e^14*f^52 + 1572864*a \\
& *b^2*c^12*d^7*e^8*f^52 - 524288*a*b^3*c^11*d^6*e^9*f^52 - 1441792*a*b^5*c^9 \\
& *d^4*e^11*f^52 + 6291456*a*b^6*c^8*d^3*e^12*f^52 - 10223616*a*b^7*c^7*d^2*e \\
& ^13*f^52 - 1572864*a^2*b*c^12*d^6*e^9*f^52 - 38273024*a^2*b^6*c^7*d*e^14*f^ \\
& 52 - 6815744*a^3*b*c^11*d^4*e^11*f^52 + 89128960*a^3*b^4*c^8*d*e^14*f^52 + \\
& 62914560*a^4*b*c^10*d^2*e^13*f^52 - 83886080*a^4*b^2*c^9*d*e^14*f^52 + 7864 \\
& 32*a^2*b^2*c^11*d^5*e^10*f^52 + 5242880*a^2*b^3*c^10*d^4*e^11*f^52 - 262144 \\
& 00*a^2*b^4*c^9*d^3*e^12*f^52 + 47972352*a^2*b^5*c^8*d^2*e^13*f^52 + 4194304 \\
& 0*a^3*b^2*c^10*d^3*e^12*f^52 - 94371840*a^3*b^3*c^9*d^2*e^13*f^52)) + 8192* \\
& b^3*c^9*e^12*f^51 + 8192*c^12*d^3*e^9*f^51 - 32768*a*b*c^10*e^12*f^51 + 409 \\
& 60*a*c^11*d*e^11*f^51 - 8192*b*c^11*d^2*e^10*f^51 - 8192*b^2*c^10*d*e^11*f^ \\
& 51) + 12288*c^11*e^11*f^50*(f*x)^(1/2))*root(8388608*a^7*b*c^11*d^18*e*f^6 \\
& *h^12 - 513802240*a^10*b^2*c^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d \\
& ^9*e^10*f^6*h^12 - 381681664*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9* \\
& b^5*c^5*d^10*e^9*f^6*h^12 - 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 29360 \\
& 1280*a^10*b^3*c^6*d^10*e^9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^ \\
& 12 - 168820736*a^10*b^5*c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14* \\
& e^5*f^6*h^12 + 166068224*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2 \\
& *c^5*d^7*e^12*f^6*h^12 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 12478054 \\
& 4*a^10*b^4*c^5*d^9*e^10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 \\
& + 119275520*a^9*b^4*c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11 \\
& *f^6*h^12 + 117440512*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4 \\
& *d^9*e^10*f^6*h^12 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^1 \\
& 1*b^4*c^4*d^7*e^12*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 7106 \\
& 5600*a^7*b^8*c^4*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 \\
& - 53444608*a^6*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f \\
& ^6*h^12 + 40370176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^ \\
& 6*e^13*f^6*h^12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7 \\
& *c^4*d^10*e^9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400* \\
& a^12*b^4*c^3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 2 \\
& 2118400*a^7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h \\
& ^12 - 20971520*a^13*b^2*c^4*d^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17* \\
& e^2*f^6*h^12 + 18350080*a^10*b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c \\
& ^7*d^16*e^3*f^6*h^12 - 16629760*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^ \\
& 5*b^8*c^6*d^15*e^4*f^6*h^12 - 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 104 \\
& 85760*a^5*b^6*c^8*d^17*e^2*f^6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^1 \\
& 2 + 9175040*a^6*b^6*c^7*d^15*e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f
\end{aligned}$$

$$\begin{aligned}
&^6h^{12} + 5619712a^7b^{10}c^2d^9e^{10}f^6h^{12} + 5619712a^5b^{10}c^4d^{13}e^6f^6h^{12} - 5570560a^6b^{11}c^2d^{10}e^9f^6h^{12} - 5570560a^5b^{11}c^3d^{12}e^7f^6h^{12} + 4358144a^8b^9c^2d^8e^{11}f^6h^{12} + 4358144a^5b^9c^5d^{14}e^5f^6h^{12} + 4259840a^6b^{10}c^3d^{11}e^8f^6h^{12} + 3899392a^4b^{10}c^5d^{15}e^4f^6h^{12} - 3440640a^4b^9c^6d^{16}e^3f^6h^{12} + 3145728a^{12}b^5c^2d^4e^{15}f^6h^{12} - 2523136a^4b^{11}c^4d^{14}e^5f^6h^{12} + 1802240a^4b^8c^7d^{17}e^2f^6h^{12} + 1556480a^5b^{12}c^2d^{11}e^8f^6h^{12} + 1048576a^{14}b^2c^3d^3e^{16}f^6h^{12} + 688128a^4b^{12}c^3d^{13}e^6f^6h^{12} - 393216a^{13}b^4c^2d^3e^{16}f^6h^{12} - 286720a^3b^{12}c^4d^{15}e^4f^6h^{12} + 229376a^3b^{13}c^3d^{14}e^5f^6h^{12} + 229376a^3b^{11}c^5d^{16}e^3f^6h^{12} + 163840a^4b^{13}c^2d^{12}e^7f^6h^{12} - 114688a^3b^{14}c^2d^{13}e^6f^6h^{12} - 114688a^3b^{10}c^6d^{17}e^2f^6h^{12} + 293601280a^{11}b^3c^7d^{10}e^9f^6h^{12} + 293601280a^{10}b^3c^8d^{12}e^7f^6h^{12} + 176160768a^{12}b^3c^6d^8e^{11}f^6h^{12} + 176160768a^9b^3c^9d^{14}e^5f^6h^{12} + 58720256a^{13}b^3c^5d^6e^{13}f^6h^{12} + 58720256a^8b^3c^{10}d^{16}e^3f^6h^{12} + 8388608a^{14}b^3c^4d^4e^{15}f^6h^{12} - 8388608a^6b^3c^{10}d^{18}e^3f^6h^{12} + 3899392a^8b^{10}c^4d^7e^{12}f^6h^{12} - 3440640a^9b^9c^4d^6e^{13}f^6h^{12} + 3145728a^5b^5c^9d^{18}e^3f^6h^{12} - 2523136a^7b^{11}c^4d^8e^{11}f^6h^{12} + 1802240a^{10}b^8c^4d^5e^{14}f^6h^{12} + 688128a^6b^{12}c^4d^9e^{10}f^6h^{12} - 524288a^{11}b^7c^4d^4e^{15}f^6h^{12} - 524288a^4b^7c^8d^{18}e^3f^6h^{12} + 163840a^5b^{13}c^4d^{10}e^9f^6h^{12} - 163840a^4b^{14}c^4d^{11}e^8f^6h^{12} + 65536a^{12}b^6c^4d^3e^{16}f^6h^{12} + 32768a^3b^{15}c^4d^{12}e^7f^6h^{12} + 32768a^3b^9c^7d^{18}e^3f^6h^{12} - 73400320a^{11}c^8d^{11}e^8f^6h^{12} - 58720256a^{12}c^7d^9e^{10}f^6h^{12} - 58720256a^{10}c^9d^{13}e^6f^6h^{12} - 29360128a^{13}c^6d^7e^{12}f^6h^{12} - 29360128a^9c^{10}d^{15}e^4f^6h^{12} - 8388608a^{14}c^5d^5e^{14}f^6h^{12} - 8388608a^8c^{11}d^{17}e^2f^6h^{12} - 1048576a^{15}c^4d^3e^{16}f^6h^{12} - 286720a^7b^{12}d^7e^{12}f^6h^{12} + 229376a^8b^{11}d^6e^{13}f^6h^{12} + 229376a^6b^{13}d^8e^{11}f^6h^{12} - 114688a^9b^{10}d^5e^{14}f^6h^{12} - 114688a^5b^{14}d^9e^{10}f^6h^{12} + 32768a^{10}b^9d^4e^{15}f^6h^{12} + 32768a^4b^{15}d^{10}e^9f^6h^{12} - 4096a^{11}b^8d^3e^{16}f^6h^{12} - 4096a^3b^{16}d^{11}e^8f^6h^{12} + 1048576a^6b^2c^{11}d^{19}f^6h^{12} - 393216a^5b^4c^{10}d^{19}f^6h^{12} + 65536a^4b^6c^9d^{19}f^6h^{12} - 4096a^3b^8c^8d^{19}f^6h^{12} - 1048576a^7c^{12}d^{19}f^6h^{12} + 262144a^{10}b^3c^4d^4e^{14}f^4h^8 - 23552a^6b^6c^8d^{14}e^4f^4h^8 - 16384a^7b^7c^4d^4e^{14}f^4h^8 - 3328a^6b^{13}c^4d^7e^8f^4h^8 + 2429952a^4b^5c^6d^9e^6f^4h^8 - 1865728a^6b^3c^6d^7e^8f^4h^8 - 1716224a^4b^4c^7d^{10}e^5f^4h^8 + 1605632a^6b^2c^7d^8e^7f^4h^8 + 1584384a^5b^5c^5d^7e^8f^4h^8 + 1572864a^5b^2c^8d^{10}e^5f^4h^8 - 1433600a^5b^3c^7d^9e^6f^4h^8 - 1261568a^4b^6c^5d^8e^7f^4h^8 - 1124352a^3b^4c^8d^{12}e^3f^4h^8 - 1110016a^7b^3c^5d^5e^{10}f^4h^8 + 1106176a^3b^5c^7d^{11}e^4f^4h^8 - 936960a^5b^6c^4d^6e^9f^4h^8 - 838656a^2b^7c^6d^{11}e^4f^4h^8 - 795648a^3b^7c^5d^9e^6f^4h^8 + 730880a^3b^8c^4d^8e^7f^4h^8 + 714752a^2b^6c^7d^{12}e^3f^4h^8 + 686080a^7b^4c^4d^4e^{11}f^4h^8 + 641024a^6b^4c^5d^6e^9f^4h^8 - 595968a^8b^3c^4d^3e^{12}f^4h^8 + 544768a^3b^3
\end{aligned}$$

$$\begin{aligned}
& *c^9*d^{13}*e^2*f^4*h^8 + 516096*a^2*b^8*c^5*d^{10}*e^5*f^4*h^8 + 441856*a^6*b^8 \\
& 5*c^4*d^5*e^{10}*f^4*h^8 + 393216*a^7*b^2*c^6*d^6*e^9*f^4*h^8 + 376832*a^4*b^8 \\
& 2*c^9*d^{12}*e^3*f^4*h^8 - 366592*a^6*b^6*c^3*d^4*e^{11}*f^4*h^8 + 363520*a^4*b^8 \\
& 8*c^3*d^6*e^9*f^4*h^8 - 356352*a^5*b^4*c^6*d^8*e^7*f^4*h^8 - 348672*a^2*b^8 \\
& 5*c^8*d^{13}*e^2*f^4*h^8 - 344064*a^8*b^2*c^5*d^4*e^{11}*f^4*h^8 + 294912*a^8*b^4 \\
& 4*c^3*d^2*e^{13}*f^4*h^8 + 210944*a^4*b^3*c^8*d^{11}*e^4*f^4*h^8 - 198400*a^3*b^9 \\
& c^3*d^7*e^8*f^4*h^8 - 144640*a^4*b^7*c^4*d^7*e^8*f^4*h^8 - 131072*a^9*b^2 \\
& c^4*d^2*e^{13}*f^4*h^8 - 131072*a^7*b^6*c^2*d^2*e^{13}*f^4*h^8 - 129024*a^3*b^6 \\
& c^6*d^{10}*e^5*f^4*h^8 - 104448*a^2*b^{10}*c^3*d^8*e^7*f^4*h^8 + 96768*a^5*b^8 \\
& c^2*d^4*e^{11}*f^4*h^8 + 91904*a^7*b^5*c^3*d^3*e^{12}*f^4*h^8 - 74240*a^4*b^9 \\
& c^2*d^5*e^{10}*f^4*h^8 - 71680*a^2*b^9*c^4*d^9*e^6*f^4*h^8 + 58368*a^2*b^11 \\
& c^2*d^7*e^8*f^4*h^8 + 36864*a^5*b^7*c^3*d^5*e^{10}*f^4*h^8 - 35328*a^3*b^{10} \\
& c^2*d^6*e^9*f^4*h^8 + 27136*a^6*b^7*c^2*d^3*e^{12}*f^4*h^8 + 909312*a^8*b^6*c^6 \\
& d^5*e^{10}*f^4*h^8 + 815104*a^9*b^6*c^5*d^3*e^{12}*f^4*h^8 - 651264*a^5*b^6*c^9*d^{11} \\
& e^4*f^4*h^8 - 573440*a^6*b^6*c^8*d^9*e^6*f^4*h^8 - 262144*a^9*b^3*c^3*d^6 \\
& e^{14}*f^4*h^8 + 217088*a^7*b^6*c^7*d^7*e^8*f^4*h^8 + 211456*a^8*b^9*c^5*d^{11} \\
& e^4*f^4*h^8 - 204800*a^4*b^6*c^{10}*d^{13}*e^2*f^4*h^8 - 172032*a^8*b^8*c^6*d^{12} \\
& e^3*f^4*h^8 - 157696*a^8*b^{10}*c^4*d^{10}*e^5*f^4*h^8 - 131072*a^3*b^2*c^{10} \\
& d^{14}*e^4*f^4*h^8 + 98304*a^8*b^5*c^2*d^6*e^{14}*f^4*h^8 + 92160*a^2*b^4*c^9 \\
& d^{14}*e^4*f^4*h^8 + 84992*a^8*b^7*c^7*d^{13}*e^2*f^4*h^8 + 64512*a^8*b^{11} \\
& c^3*d^9*e^6*f^4*h^8 + 23552*a^6*b^8*c^3*d^2*e^{13}*f^4*h^8 + 18944*a^3*b^{11} \\
& c^3*d^5*e^{10}*f^4*h^8 - 13312*a^4*b^{10}*c^4*d^4*e^{11}*f^4*h^8 - 9472 \\
& a^5*b^9*c^3*d^3*e^{12}*f^4*h^8 - 8192*a^8*b^{12}*c^2*d^8*e^7*f^4*h^8 - 6144 \\
& a^2*b^{12}*c^3*d^6*e^9*f^4*h^8 - 17920*b^{11}*c^4*d^{11}*e^4*f^4*h^8 + 14336 \\
& b^{12}*c^3*d^{10}*e^5*f^4*h^8 + 14336*b^{10}*c^5*d^{12}*e^3*f^4*h^8 - 7168 \\
& b^{13}*c^2*d^9*e^6*f^4*h^8 - 7168*b^9*c^6*d^{13}*e^2*f^4*h^8 - 425984 \\
& a^9*c^6*d^4*e^{11}*f^4*h^8 - 360448*a^8*c^7*d^6*e^9*f^4*h^8 - 262144*a^{10} \\
& c^5*d^2*e^{13}*f^4*h^8 - 131072*a^7*c^8*d^8*e^7*f^4*h^8 + 98304*a^5*c^{10} \\
& d^{12}*e^3*f^4*h^8 + 65536*a^6*c^9*d^{10}*e^5*f^4*h^8 - 1536*a^5*b^{10} \\
& d^2*e^{13}*f^4*h^8 - 1536*a^2*b^{13}*d^5*e^{10}*f^4*h^8 + 768*a^4*b^{11} \\
& d^3*e^{12}*f^4*h^8 + 768*a^3*b^{12}*d^4*e^{11}*f^4*h^8 + 65536*a^{10} \\
& b^2*c^3*e^{15}*f^4*h^8 - 24576*a^9*b^4*c^2*e^{15}*f^4*h^8 - 10240 \\
& a^2*b^3*c^{10}*d^{15}*f^4*h^8 + 2048*b^{14}*c^3*d^8*e^7*f^4*h^8 + 2048 \\
& b^8*c^7*d^{14}*e^4*f^4*h^8 + 32768*a^4*c^{11}*d^{14}*e^4*f^4*h^8 + 1024 \\
& a^6*b^9*d^6*e^{14}*f^4*h^8 + 1024*a^8*b^{14}*d^6*e^9*f^4*h^8 + 4096 \\
& a^8*b^6*c^5*e^{15}*f^4*h^8 + 12288*a^3*b^6*c^{11}*d^{15}*f^4*h^8 + 2816 \\
& a^5*b^5*c^9*d^{15}*f^4*h^8 - 256*b^{15}*d^7*e^8*f^4*h^8 - 65536 \\
& a^{11}*c^4*e^{15}*f^4*h^8 - 256*b^7*c^8*d^{15}*f^4*h^8 - 256*a^7 \\
& b^8*e^{15}*f^4*h^8 - 896*a^8*b^8*c^2*d^6*e^{10}*f^2*h^4 + 192 \\
& a^8*b^6*c^9*d^8*e^3*f^2*h^4 + 11520*a^3*b^3*c^5*d^2*e^9*f^2*h^4 - 5856 \\
& a^2*b^5*c^4*d^2*e^9*f^2*h^4 - 5120*a^3*b^2*c^6*d^3*e^8*f^2*h^4 + 3200 \\
& a^2*b^4*c^5*d^3*e^8*f^2*h^4 - 640*a^2*b^3*c^6*d^4*e^7*f^2*h^4 - 96 \\
& a^2*b^2*c^7*d^5*e^6*f^2*h^4 - 10880*a^3*b^4*c^4*d^4*e^{10}*f^2*h^4 + 10240 \\
& a^4*b^2*c^5*d^4*e^{10}*f^2*h^4 - 7680*a^4*b^6*c^6*d^2*e^9*f^2*h^4 + 4672 \\
& a^2*b^6*c^3*d^4*e^{10}*f^2*h^4 + 1248*a^8*b^7*c^3*d^2*e^9*f^2*h^4 + 832 \\
& a^3*b^6*c^7*d^4*e^7*f^2*h^4 - 768*a^8*b^6*c^4*d^3*e^8*f^2*h^4 + 192 \\
& a^2*b^8*c^8*d^6*e^5*f^2*h^4 - 192*a^8*b^2*c^8*d^7*e^4*f^2*h^4 + 176 \\
& a^8*b^5*c^5*d^4*e^7*f^2*h^4 + 64*a^8*b^3*c^7*d^6*e^5*f^2*h^4 - 96 \\
& b^9*c^2*d^2*e^9*f^2*h^4 - 96*b^2*c^9*d^9*e^2*f^2*h^4 + 64*b^8*c^3 \\
& d^3*e^8*f^2*h^4
\end{aligned}$$

+ 64*b^3*c^8*d^8*e^3*f^2*h^4 - 16*b^7*c^4*d^4*e^7*f^2*h^4 - 16*b^4*c^7*d^7*e^4*f^2*h^4 + 2032*a^4*c^7*d^3*e^8*f^2*h^4 - 96*a^2*c^9*d^7*e^4*f^2*h^4 - 64*a^3*c^8*d^5*e^6*f^2*h^4 - 4480*a^4*b^3*c^4*e^11*f^2*h^4 + 3696*a^3*b^5*c^3*e^11*f^2*h^4 - 1376*a^2*b^7*c^2*e^11*f^2*h^4 - 2048*a^5*c^6*d*e^10*f^2*h^4 - 64*a*c^10*d^9*e^2*f^2*h^4 + 1792*a^5*b*c^5*e^11*f^2*h^4 + 64*b^10*c*d*e^10*f^2*h^4 + 64*b*c^10*d^10*e*f^2*h^4 + 240*a*b^9*c*e^11*f^2*h^4 - 16*c^11*d^11*f^2*h^4 - 16*b^11*e^11*f^2*h^4 - c^7*e^7, h, k), k, 1, 12)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{fx} (d + ex^2)(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)

[Out] Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)

$$3.233 \quad \int \frac{x^5 \sqrt{a+bx^2+cx^4}}{d+ex^2} dx$$

Optimal. Leaf size=272

$$\frac{(-8c^2de(bd - ae) - 2bce^2(bd - 2ae) - b^3e^3 + 16c^3d^3) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4} + \frac{\sqrt{a+bx^2+cx^4}((2cd - be)(be + 16c^2e^3))}{16c^2e^3}$$

Rubi [A] time = 0.57, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$\frac{(-8c^2de(bd - ae) - 2bce^2(bd - 2ae) - b^3e^3 + 16c^3d^3) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4} + \frac{\sqrt{a+bx^2+cx^4}((2cd - be)(be + 4cd) - 2cex^2(be + 2cd))}{16c^2e^3} + \frac{d^2\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ax^2(2cd - be) + bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2 - bde + cd^2}}\right)}{2e^4} + \frac{(a + bx^2 + cx^4)^{3/2}}{6ce}$$

Antiderivative was successfully verified.

[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt{a + bx^2 + cx^4}}{d + ex^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right) \\
&= \frac{(a + bx^2 + cx^4)^{3/2}}{6ce} + \frac{\text{Subst} \left(\int \frac{\left(-\frac{3}{2}bde - \frac{3}{2}e(2cd + be)x\right) \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right)}{6ce^2} \\
&= \frac{\left((2cd - be)(4cd + be) - 2ce(2cd + be)x^2\right) \sqrt{a + bx^2 + cx^4}}{16c^2e^3} + \frac{(a + bx^2 + cx^4)^{3/2}}{6ce} - \frac{\text{Subst} \left(\int \frac{\left(-\frac{3}{2}bde - \frac{3}{2}e(2cd + be)x\right) \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right)}{6ce^2} \\
&= \frac{\left((2cd - be)(4cd + be) - 2ce(2cd + be)x^2\right) \sqrt{a + bx^2 + cx^4}}{16c^2e^3} + \frac{(a + bx^2 + cx^4)^{3/2}}{6ce} + \frac{\left(d^2 \sqrt{a + bx^2 + cx^4}\right)}{6ce} - \frac{\text{Subst} \left(\int \frac{\left(-\frac{3}{2}bde - \frac{3}{2}e(2cd + be)x\right) \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right)}{6ce^2} \\
&= \frac{\left((2cd - be)(4cd + be) - 2ce(2cd + be)x^2\right) \sqrt{a + bx^2 + cx^4}}{16c^2e^3} + \frac{(a + bx^2 + cx^4)^{3/2}}{6ce} - \frac{\left(d^2 \sqrt{a + bx^2 + cx^4}\right)}{6ce} - \frac{\text{Subst} \left(\int \frac{\left(-\frac{3}{2}bde - \frac{3}{2}e(2cd + be)x\right) \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right)}{6ce^2} \\
&= \frac{\left((2cd - be)(4cd + be) - 2ce(2cd + be)x^2\right) \sqrt{a + bx^2 + cx^4}}{16c^2e^3} + \frac{(a + bx^2 + cx^4)^{3/2}}{6ce} - \frac{\left(d^2 \sqrt{a + bx^2 + cx^4}\right)}{6ce} - \frac{\left(16c^2 \sqrt{a + bx^2 + cx^4}\right)}{6ce}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 267, normalized size = 0.98

$$\frac{2\sqrt{c} \left(e\sqrt{a + bx^2 + cx^4} (2ce(4ae - 3bd + bex^2) - 3b^2e^2 + 4c^2(6d^2 - 3dex^2 + 2e^2x^4)) + 24c^2d^2\sqrt{e(ae - bd) + cd^2} \tanh^{-1} \left(\frac{-2ae + (d - cx^2) + 2cdx^2}{2\sqrt{a + bx^2 + cx^4}\sqrt{e(ae - bd) + cd^2}} \right) - 3(8c^2d(ea - bd) - 2bce^2(bd - 2ae) - b^3e^3 + 16c^3d^3) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{a + bx^2 + cx^4}} \right) \right)}{96c^{5/2}e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] (-3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e + b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 24*c^2*d^2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(96*c^(5/2)*e^4)

IntegrateAlgebraic [A] time = 1.33, size = 316, normalized size = 1.16

$$\frac{\sqrt{a + bx^2 + cx^4} (8ace^2 - 3b^2e^2 - 6bcde + 2bc^2x^2 + 24c^2d^2 - 12c^2dex^2 + 8c^2e^2x^4)}{48e^2e^3} + \frac{(4abc^3 + 8ac^2d^2 - b^3e^3 - 2b^2cde^2 - 8bc^2d^2e + 16c^3d^3) \log\left(-2\sqrt{c}\sqrt{a + bx^2 + cx^4} + b + 2cx^2\right)}{32c^{5/2}e^4} + \frac{d^2\sqrt{-ae^2 + bde - cd^2} \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{c\sqrt{a + bx^2 + cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2 + bde - cd^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(24*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 + 8*a*c*e^2 -
12*c^2*d*e*x^2 + 2*b*c*e^2*x^2 + 8*c^2*e^2*x^4))/(48*c^2*e^3) + (d^2*Sqrt[-
(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2]
+ (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^
4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^4 + ((16*c^3*d^3 - 8*b*c^2*d^2*e - 2
*b^2*c*d*e^2 + 8*a*c^2*d*e^2 - b^3*e^3 + 4*a*b*c*e^3)*Log[b + 2*c*x^2 - 2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^(5/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.04, size = 1049, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/8/e*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16/e*b^
2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8/e*b/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))*a+1/32/e*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))-1/4/e^2*d*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8/e^2*d/c*(c*x^4+b*x^2+a)^(
1/2)*b-1/4/e^2*d/c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a
+1/16/e^2*d/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2+1/2
*d^2/e^3*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)+1/4*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(
b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2*d^3/e^
4*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(
```

$$x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*c^{(1/2)}-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*a+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*b-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*c$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for more details)Is b*e-2*c*d zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \sqrt{c x^4 + b x^2 + a}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)

[Out] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{a + b x^2 + c x^4}}{d + e x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)

[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)

$$3.234 \quad \int \frac{x^3 \sqrt{a+bx^2+cx^4}}{d+ex^2} dx$$

Optimal. Leaf size=208

$$\frac{(-4ce(bd - ae) - b^2e^2 + 8c^2d^2) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) d\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{16c^{3/2}e^3} - \frac{d\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}$$

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{(-4ce(bd - ae) - b^2e^2 + 8c^2d^2) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) d\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right) - \frac{\sqrt{a+bx^2+cx^4}(-be+4cd-2cex^2)}{8ce^2}}{16c^{3/2}e^3} - \frac{d\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) + ((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sqrt{a + bx^2 + cx^4}}{d + ex^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x \sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right) \\
&= -\frac{(4cd - be - 2cex^2) \sqrt{a + bx^2 + cx^4}}{8ce^2} - \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}d(4bcd - b^2e - 4ace) - \frac{1}{2}(8c^2d^2 - b^2e^2 - 4ce(bd - ae))}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{8ce^2} \\
&= -\frac{(4cd - be - 2cex^2) \sqrt{a + bx^2 + cx^4}}{8ce^2} - \frac{(d(cd^2 - bde + ae^2)) \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2e^3} \\
&= -\frac{(4cd - be - 2cex^2) \sqrt{a + bx^2 + cx^4}}{8ce^2} + \frac{(d(cd^2 - bde + ae^2)) \text{Subst} \left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - ex^2} dx, x, x^2 \right)}{e^3} \\
&= -\frac{(4cd - be - 2cex^2) \sqrt{a + bx^2 + cx^4}}{8ce^2} + \frac{(8c^2d^2 - b^2e^2 - 4ce(bd - ae)) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c}\sqrt{a + bx^2 + cx^4}} \right)}{16c^{3/2}e^3}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 205, normalized size = 0.99

$$\frac{(4ce(ae - bd) - b^2e^2 + 8c^2d^2) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c}\sqrt{a + bx^2 + cx^4}} \right) + 2\sqrt{c} (4cd\sqrt{ae^2 - bde + cd^2} \tanh^{-1} \left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a + bx^2 + cx^4} \sqrt{ae^2 - bde + cd^2}} \right) + e\sqrt{a + bx^2 + cx^4} (be - 4cd + 2cex^2))}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] (((8*c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]) + 2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2))*Sqrt[a + b*x^2 + c*x^4] + 4*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]))/(16*c^(3/2)*e^3)

IntegrateAlgebraic [A] time = 0.84, size = 247, normalized size = 1.19

$$\frac{(-4ace^2 + b^2e^2 + 4bcde - 8c^2d^2) \log \left(-2c^{3/2} \sqrt{a + bx^2 + cx^4} + bc + 2c^2x^2 \right) - d\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e\sqrt{a + bx^2 + cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2 + bde - cd^2}} \right) + \frac{\sqrt{a + bx^2 + cx^4} (be - 4cd + 2cex^2)}{8ce^2}}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] ((-4*c*d + b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) - (d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(16*c^(3/2)*e^3)

)]/Sqrt[-(c*d^2) + b*d*e - a*e^2]]/e^3 + ((-8*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*a*c*e^2)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(3/2)*e^3)

fricas [A] time = 56.15, size = 1231, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")

[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e))*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e))*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/16*(8*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.01, size = 887, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d), x)

[Out] $\frac{1}{4}e(c^2x^4+b^2x^2+a)^{1/2}x^2 + \frac{1}{8}e/c(c^2x^4+b^2x^2+a)^{1/2}b + \frac{1}{4}e/c^{1/2} \ln\left(\frac{c^2x^2+1/2*b}{c^{1/2}} + (c^2x^4+b^2x^2+a)^{1/2}\right) * a - \frac{1}{16}e/c^{3/2} \ln\left(\frac{c^2x^2+1/2*b}{c^{1/2}} + (c^2x^4+b^2x^2+a)^{1/2}\right) * b^2 - \frac{1}{2}d/e^2 * ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} - \frac{1}{4}d/e^2 * \ln\left(\frac{(x^2+d/e) * c + 1/2 * (b^2e - 2*c*d) / e}{c^{1/2}} + ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2}\right) / c^{1/2} * b + \frac{1}{2}d^2/e^3 * \ln\left(\frac{(x^2+d/e) * c + 1/2 * (b^2e - 2*c*d) / e}{c^{1/2}} + ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2}\right) * c^{1/2} + \frac{1}{2}d/e^2 / ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * \ln\left(\frac{(b^2e - 2*c*d) * (x^2+d/e) / e + 2 * (a^2e^2 - b*d*e + c*d^2) / e^2 + 2 * ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2}}{(x^2+d/e)} * a - \frac{1}{2}d^2/e^3 / ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * \ln\left(\frac{(b^2e - 2*c*d) * (x^2+d/e) / e + 2 * (a^2e^2 - b*d*e + c*d^2) / e^2 + 2 * ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2}}{(x^2+d/e)} * b + \frac{1}{2}d^3/e^4 / ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * \ln\left(\frac{(b^2e - 2*c*d) * (x^2+d/e) / e + 2 * (a^2e^2 - b*d*e + c*d^2) / e^2 + 2 * ((a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2} * ((x^2+d/e)^2 * c + (b^2e - 2*c*d) * (x^2+d/e) / e + (a^2e^2 - b*d*e + c*d^2) / e^2)^{1/2}}{(x^2+d/e)} * c\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2e-2*c*d>0)', see `assume?` for more details)Is b^2e-2*c*d zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{cx^4 + bx^2 + a}}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{a + bx^2 + cx^4}}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d), x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

$$3.235 \quad \int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2} dx$$

Optimal. Leaf size=168

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} + \frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Rubi [A] time = 0.22, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1247, 734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} + \frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x]
- Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]
*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex} dx, x, x^2 \right) \\ &= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\text{Subst} \left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4e} \\ &= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4e^2} + \frac{(cd^2-bde+ae^2) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2e^2} \\ &= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be) \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}} \right)}{2e^2} - \frac{(cd^2-bde+ae^2) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2e^2} \\ &= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{4\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2} \tanh^{-1} \left(\frac{2\sqrt{c}\sqrt{a+bx+cx^2}}{2\sqrt{c}} \right)}{2e^2} \end{aligned}$$

Mathematica [A] time = 0.13, size = 167, normalized size = 0.99

$$\frac{2\sqrt{c} \left(e\sqrt{a+bx^2+cx^4} - \sqrt{ae^2 - bde + cd^2} \tanh^{-1} \left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2 - bde + cd^2}} \right) \right) + (be - 2cd) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{4\sqrt{c} e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(-b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/(4*Sqrt[c]*e^2)

IntegrateAlgebraic [A] time = 0.58, size = 203, normalized size = 1.21

$$\frac{\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c} ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c} d}{\sqrt{-ae^2 + bde - cd^2}} \right) + \frac{(2cd - be) \log(-2\sqrt{c} \sqrt{a+bx^2+cx^4} + b + 2cx^2)}{4\sqrt{c} e^2} + \frac{\sqrt{a+bx^2+cx^4}}{2e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2), x]

[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^2 + ((2*c*d - b*e)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[c]*e^2)

fricas [A] time = 4.09, size = 1050, normalized size = 6.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d), x, algorithm="fricas")

[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e +

$$\begin{aligned} & (b^2 + 4ac)e^2 x^4 - 8abd e + 8a^2 e^2 + (b^2 + 4ac)d^2 + 2(4b^2 c d^2 + 4ab^2 e^2 - (3b^2 + 4ac)d e) x^2 + 4\sqrt{c x^4 + b x^2 + a} \sqrt{c d^2 - b d e + a e^2} ((2cd - b e) x^2 + b d - 2a e) / (e^2 x^4 + 2d e x^2 + d^2) / (c e^2), \\ & 1/8(4\sqrt{c x^4 + b x^2 + a} c e + 4\sqrt{-c d^2 + b d e - a e^2} c \arctan(-1/2\sqrt{c x^4 + b x^2 + a} \sqrt{-c d^2 + b d e - a e^2}) ((2cd - b e) x^2 + b d - 2a e) / ((c^2 d^2 - b c d e + a c e^2) x^4 + a c d^2 - a b d e + a^2 e^2 + (b c d^2 - b^2 d e + a b e^2) x^2)) - \\ & (2cd - b e) \sqrt{c} \log(-8c^2 x^4 - 8b c x^2 - b^2 - 4\sqrt{c x^4 + b x^2 + a} (2c x^2 + b) \sqrt{c} - 4ac) / (c e^2), \\ & 1/4(2\sqrt{c x^4 + b x^2 + a} c e + 2\sqrt{-c d^2 + b d e - a e^2} c \arctan(-1/2\sqrt{c x^4 + b x^2 + a} \sqrt{-c d^2 + b d e - a e^2}) ((2cd - b e) x^2 + b d - 2a e) / ((c^2 d^2 - b c d e + a c e^2) x^4 + a c d^2 - a b d e + a^2 e^2 + (b c d^2 - b^2 d e + a b e^2) x^2)) + \\ & (2cd - b e) \sqrt{-c} \arctan(1/2\sqrt{c x^4 + b x^2 + a} (2c x^2 + b) \sqrt{-c} / (c^2 x^4 + b c x^2 + a c)) / (c e^2) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.00, size = 757, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)

[Out] $\frac{1}{2} e \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} + \frac{1}{4} e \ln \left(\left(\frac{x^2+d}{e} \right) c + \frac{1}{2} (b e - 2 c d) / e \right) / c^{1/2} + \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} / c^{1/2} - \frac{1}{2} e^2 \ln \left(\left(\frac{x^2+d}{e} \right) c + \frac{1}{2} (b e - 2 c d) / e \right) / c^{1/2} + \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} * c^{1/2} d - \frac{1}{2} e / \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} * \ln \left(\frac{(b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + 2 (a e^2 - b d e + c d^2) / e^2 + 2 \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}}{\left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}} \right) / \left(\frac{x^2+d}{e} \right) * a + \frac{1}{2} e^2 / \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} * \ln \left(\frac{(b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + 2 (a e^2 - b d e + c d^2) / e^2 + 2 \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}}{\left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}} \right) / \left(\frac{x^2+d}{e} \right) * d - \frac{1}{2} e^3 / \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} * \ln \left(\frac{(b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + 2 (a e^2 - b d e + c d^2) / e^2 + 2 \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}}{\left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2}} \right) / \left(\frac{x^2+d}{e} \right) + 2 (a e^2 - b d e + c d^2) / e^2 + 2 \left(\frac{x^2+d}{e} \right)^2 c + (b e - 2 c d) \left(\frac{x^2+d}{e} \right) / e + (a e^2 - b d e + c d^2) / e^2 \Big)^{1/2} * \left(\frac{x^2+d}{e} \right)$

$$\frac{(c^2 + (b^2 e - 2 c^2 d) (x^2 + d/e) / e + (a e^2 - b^2 d e + c^2 d^2) / e^2)^{1/2}}{(x^2 + d/e) c d^2}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for more details)Is a*e^2-b*d*e +c*d^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sqrt{c x^4 + b x^2 + a}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)

[Out] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{a + b x^2 + c x^4}}{d + e x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)

[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)

$$3.236 \quad \int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)} dx$$

Optimal. Leaf size=186

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}}\right)}{2de} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 895, 724, 206, 843, 621}

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}}\right)}{2de} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)), x]

[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843


```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))), x_Symbol] := Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p] && GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + bx^2 + cx^4}}{x(d + ex^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x(d + ex)} dx, x, x^2 \right) \\ &= -\frac{\text{Subst} \left(\int \frac{-bd + ae - cdx}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2d} + \frac{a \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2d} \\ &= -\frac{a \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^2}{\sqrt{a + bx^2 + cx^4}} \right)}{d} + \frac{c \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2e} - \frac{1}{2} \left(-b + \frac{cd}{e} + \frac{\sqrt{a} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2d} + \frac{c \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^2}{\sqrt{a + bx^2 + cx^4}} \right)}{e} - \left(b - \frac{cd}{e} - \frac{ae}{d} \right) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right) \right) \\ &= -\frac{\sqrt{a} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2d} + \frac{\sqrt{c} \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{2e} - \frac{\sqrt{cd^2 - bde + ae^2} \tanh^{-1} \left(\frac{\sqrt{cd^2 - bde + ae^2}}{\sqrt{a + bx^2 + cx^4}} \right)}{\sqrt{cd^2 - bde + ae^2}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.96

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right) - \sqrt{c}d \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) + \sqrt{a}e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)), x]

[Out] -1/2*(Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] - Sqrt[c]*d*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)

IntegrateAlgebraic [A] time = 0.70, size = 227, normalized size = 1.22

$$\frac{\sqrt{-ae^2 + bde - cd^2} \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right) + \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) - \sqrt{c} \log\left(-2\sqrt{c}e\sqrt{a+bx^2+cx^4} + be + 2cex^2\right)}{de} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{c} \log\left(-2\sqrt{c}e\sqrt{a+bx^2+cx^4} + be + 2cex^2\right)}{2e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)), x]

[Out] -((Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2])/(d*e)) + (Sqrt[a]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/d - (Sqrt[c]*Log[b*e + 2*c*e*x^2 - 2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]])/(2*e)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d), x, algorithm="giac")

[Out] `int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d), x)`

[Out] `Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)`

$$3.237 \quad \int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)} dx$$

Optimal. Leaf size=361

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2} + \frac{\sqrt{a} e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} - \frac{be \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}$$

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 29, number of rules / integrand size = 0.276, Rules used = {1251, 960, 732, 843, 621, 206, 724, 734}

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2} + \frac{\sqrt{a} e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} - \frac{be \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)), x]

[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 960

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)} \right) dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^2} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x} dx, x, x^2 \right)}{2d^2} + \frac{e^2 \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{\text{Subst} \left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4d} + \frac{e \text{Subst} \left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4d^2} \\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} + \frac{b \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4d} + \frac{c \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2d} \\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}} \right)}{2d} + \frac{c \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}} \right)}{d} \\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{4\sqrt{a}d} + \frac{\sqrt{a}e \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2d^2} + \frac{\sqrt{c} \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 165, normalized size = 0.46

$$\frac{2\sqrt{ae^2 - bde + cd^2} \tanh^{-1} \left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}} \right) + \frac{(2ae-bd) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{\sqrt{a}} - \frac{2d\sqrt{a+bx^2+cx^4}}{x^2}}{4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)), x]

[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/x^2 + ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/(4*d^2)

IntegrateAlgebraic [A] time = 0.78, size = 212, normalized size = 0.59

$$\frac{\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{cd}}{\sqrt{-ae^2+bde-cd^2}} \right) + \frac{(bd-2ae) \tanh^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}} \right)}{2\sqrt{a}d^2} - \frac{\sqrt{a+bx^2+cx^4}}{2dx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(d*x^2) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]))/d^2 + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(2*Sqrt[a]*d^2)
```

fricas [A] time = 2.36, size = 1094, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e))*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a))*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a))*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e))*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2)]
```


giac [A] time = 0.52, size = 216, normalized size = 0.60

$$\frac{(cd^2 - bde + ae^2) \arctan\left(\frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)}{\sqrt{-cd^2 + bde - ae^2}d^2} + \frac{(bd - 2ae) \arctan\left(\frac{-\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{2\sqrt{-a}d^2} + \frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})b + 2a\sqrt{c}}{2\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")

[Out] (c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)*d^2 + 1/2*(b*d - 2*a*e)*arctan(-sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a)/sqrt(-a)*d^2 + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)

maple [B] time = 0.02, size = 1009, normalized size = 2.80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x)

[Out] $-1/2/d^2*e*(c*x^4+b*x^2+a)^{(1/2)} - 1/4/d^2*e*b*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})/c^{(1/2)} + 1/2/d^2*e*a^{(1/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2) + 1/2*e/d^2*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} + 1/4*e/d^2*\ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^{(1/2)}+(x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/c^{(1/2)}*b - 1/2/d*\ln(((x^2+d/e)*c+1/2*(b*e-2*c*d)/e)/c^{(1/2)}+(x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})*c^{(1/2)} - 1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*a + 1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*c - 1/2/d/a/x^2*(c*x^4+b*x^2+a)^{(3/2)} + 1/2/d*b/a*(c*x^4+b*x^2+a)^{(1/2)} - 1/4/d*b/a^{(1/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2) + 1/2/d/a*c*(c*x^4+b*x^2+a)^{(1/2)}*x^2 + 1/2/d*c^{(1/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^3 (ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)

[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3 (d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)

[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)

$$3.238 \quad \int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$$

Optimal. Leaf size=482

$$\frac{(16bc^2e^3(3a^2e^2 - 3abde + b^2d^2) + 6b^3ce^4(bd - 4ae) - 384c^4d^3e(bd - ae) + 96c^3de^2(bd - ae)^2 + 3b^5e^5 + 256c^5e^5)}{512c^{7/2}e^6}$$

Rubi [A] time = 1.10, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$\frac{(16bc^2(3a^2e^2 - 3abde + b^2d^2) + 6b^3ce^4(bd - 4ae) - 384c^4d^3e(bd - ae) + 96c^3de^2(bd - ae)^2 + 3b^5e^5 + 256c^5e^5) \sqrt{a+bx^2+cx^4} \operatorname{arctanh}\left(\frac{bx^2+cx^4}{\sqrt{a+bx^2+cx^4}}\right) + (a+bx^2+cx^4)^{3/2} \sqrt{d+ex^2} \operatorname{arctanh}\left(\frac{bx^2+cx^4}{\sqrt{a+bx^2+cx^4}}\right) + (a+bx^2+cx^4)^{5/2} \sqrt{d+ex^2} \operatorname{arctanh}\left(\frac{bx^2+cx^4}{\sqrt{a+bx^2+cx^4}}\right) + (a+bx^2+cx^4)^{7/2} \sqrt{d+ex^2} \operatorname{arctanh}\left(\frac{bx^2+cx^4}{\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}e^6}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 - 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3*e^5) + (((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*d + b*e))*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^6)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:= Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p) / (c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:= Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:= With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rubi steps

$$\int \frac{x^5 (a + bx^2 + cx^4)^{3/2}}{d + ex^2} dx = \frac{1}{2} \text{Subst} \left(\int \frac{x^2 (a + bx + cx^2)^{3/2}}{d + ex} dx, x, x^2 \right)$$

$$= \frac{(a + bx^2 + cx^4)^{5/2}}{10ce} + \frac{\text{Subst} \left(\int \frac{\left(-\frac{5}{2}bde - \frac{5}{2}e(2cd + be)x\right)(a + bx + cx^2)^{3/2}}{d + ex} dx, x, x^2 \right)}{10ce^2}$$

$$= \frac{(16c^2d^2 - 6bcde - 3b^2e^2 - 6ce(2cd + be)x^2) (a + bx^2 + cx^4)^{3/2}}{96c^2e^3} + \frac{(a + bx^2 + cx^4)^5}{10ce}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 256c^2d^2e^2)}{256c^2e^3}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 256c^2d^2e^2)}{256c^2e^3}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 256c^2d^2e^2)}{256c^2e^3}$$

$$= \frac{(128c^4d^4 + 3b^4e^4 - 32c^3d^2e(5bd - 4ae) + 8bc^2de^2(2bd - 3ae) + 6b^2ce^3(bd - 2ae) - 256c^2d^2e^2)}{256c^2e^3}$$

Mathematica [A] time = 0.98, size = 545, normalized size = 1.13

$$\frac{\text{Root}\left[\left(\frac{1}{2}d + \frac{1}{2}ex^2\right)^2 - \frac{1}{4}e^2x^4, 1\right] \sqrt{\frac{1}{2}d + \frac{1}{2}ex^2} \sqrt{a + bx^2 + cx^4} - \frac{1}{2}d \sqrt{\frac{1}{2}d + \frac{1}{2}ex^2} \sqrt{a + bx^2 + cx^4} + \frac{1}{2}ex^2 \sqrt{\frac{1}{2}d + \frac{1}{2}ex^2} \sqrt{a + bx^2 + cx^4} - \frac{1}{2}e^2x^4 \sqrt{\frac{1}{2}d + \frac{1}{2}ex^2} \sqrt{a + bx^2 + cx^4}}{256c^2e^3} + \frac{1}{10ce} (a + bx^2 + cx^4)^5$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]

[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*d*e*(-2*sqrt[c]*(b + 2*c*x^2)*sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2

$$\begin{aligned} & *(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^{(3/2)} + 3*(b^2 - 4*a*c)*((2*(b + 2* \\ & c*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*\text{ArcTanh}[(b + 2*c*x^2)/(\\ & 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4]]))/c^{(3/2)})))/c^2 - (240*d^2*((2*c*d - b* \\ & e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*\text{ArcTanh}[(b + 2*c*x^2)/(2* \\ & \text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4]]) + 2*\text{Sqrt}[c]*(e*\text{Sqrt}[a + b*x^2 + c*x^4]*(- \\ & (b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8* \\ & c*(c*d^2 + e*(-(b*d) + a*e))^{(3/2)}*\text{ArcTanh}[(-(b*d) + 2*a*e - 2*c*d*x^2 + b* \\ & e*x^2)/(2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*\text{Sqrt}[a + b*x^2 + c*x^4]])))/c^{(3 \\ & /2)*e^3))/(7680*e^3) \end{aligned}$$

IntegrateAlgebraic [B] time = 30.30, size = 13888, normalized size = 28.81

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [B] time = 0.06, size = 2068, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)

```
[Out] 11/80/e*b*x^6*(c*x^4+b*x^2+a)^(1/2)-5/8*d^3/e^4*b*(c*x^4+b*x^2+a)^(1/2)+1/2
*d^4/e^5*c*(c*x^4+b*x^2+a)^(1/2)+2/3*d^2/e^3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*d^
5/e^6*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10/e*a^2/c*
(c*x^4+b*x^2+a)^(1/2)-3/512/e*b^5/c^(7/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))+1/5/e*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^4/c^3*(c*x^4+b*x
^2+a)^(1/2)+1/10/e*c*x^8*(c*x^4+b*x^2+a)^(1/2)-3/32/e*a^2*b/c^(3/2)*ln((c*x
^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/160/e*b^2*x^4/c*(c*x^4+b*x^2+a)^(
1/2)-1/128/e*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-5/64/e*a*b^2/c^2*(c*x^4+b*x
^2+a)^(1/2)+3/64/e*a*b^3/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-3/16/e^2*d*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)-5/16/e^2*d*a*x^2*(c*x^4+b*x^2+a)^(1/2)+3/128/e^2*d*b^3/c^2*(c*x^4+b*x^2+a
)^(1/2)-3/256/e^2*d*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))-3/16/e^2*d*b*x^4*(c*x^4+b*x^2+a)^(1/2)-1/8/e^2*d*c*x^6*(c*x^4+b*x^2+a)
^(1/2)+1/6*d^2/e^3*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*d^2/e^3*b*x^2*(c*x^4+b*
x^2+a)^(1/2)+1/16*d^2/e^3/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/32*d^2/e^3*b^3/c^(3
/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/4*d^3/e^4*a*c^(1/2)*l
n((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/16*d^3/e^4*b^2*ln((c*x^2+1
/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*d^4/e^5*b*c^(1/2)*ln((c*x^
2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*d^3/e^4*x^2*c*(c*x^4+b*x^2+a)^(
1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/
e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d/e)^2*
c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2-1/
2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*
e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d/e)^2*c+(b*e-
2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2*d^6/e
^7/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d
*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d/e)^2*c+(b*e-2*c*d)*
(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+7/160/e*a*b*x^2/
c*(c*x^4+b*x^2+a)^(1/2)-1/64/e^2*d*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)-5/32/e^2
*d*a*b/c*(c*x^4+b*x^2+a)^(1/2)+3/32/e^2*d*a*b^2/c^(3/2)*ln((c*x^2+1/2*b)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*d^2/e^3*a*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*b-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2
+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d
/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*
a*c+d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(
a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d/e)^2*c+(b*
e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for more details)Is b*e-2*c*d zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 (c x^4 + b x^2 + a)^{3/2}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)

[Out] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b x^2 + c x^4)^{3/2}}{d + e x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)

[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)

$$3.239 \quad \int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$$

Optimal. Leaf size=360

$$\frac{(8b^2ce^3(bd - 3ae) - 192c^3d^2e(bd - ae) + 48c^2e^2(bd - ae)^2 + 3b^4e^4 + 128c^4d^4) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) \sqrt{a+bx^2+cx^4}}{256c^{5/2}e^5}$$

Rubi [A] time = 0.70, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^2+cx^4}(-2c^2x^2(-4c(2bd-3ae)-3b^2d^2+16c^2d^2)-16c^2d(5bd-4ae)+48c^2(2bd-3ae)+3b^3d^3+64c^3d^3)}{128c^4} \cdot \frac{(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5} \cdot \frac{d(a^2-bde+ce^2) \tanh^{-1}\left(\frac{-2c^2bd-bd+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{a+bx^2+cx^4}}\right)}{2e^5} \cdot \frac{(a+bx^2+cx^4)^{3/2}(-3be+8d-6c^2x^2)}{48c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4]/(128*c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(256*c^(5/2)*e^5) - (d*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*e^5)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :=> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + bx^2 + cx^4)^{3/2}}{d + ex^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x (a + bx + cx^2)^{3/2}}{d + ex} dx, x, x^2 \right) \\
&= -\frac{(8cd - 3be - 6cex^2) (a + bx^2 + cx^4)^{3/2}}{48ce^2} - \frac{\text{Subst} \left(\int \frac{\left(\frac{1}{2}d(4ace - 2b(4cd - \frac{3be}{2}))\right) - \frac{1}{2}(16c^2d^2 - 3b^2e^2 - 3c^2d^2)}{d+ex} dx, x, x^2 \right)}{16ce^2} \\
&= -\frac{(64c^3d^3 + 3b^3e^3 - 16c^2de(5bd - 4ae) + 4bce^2(2bd - 3ae) - 2ce(16c^2d^2 - 3b^2e^2 - 3c^2d^2))}{128c^2e^4} \\
&= -\frac{(64c^3d^3 + 3b^3e^3 - 16c^2de(5bd - 4ae) + 4bce^2(2bd - 3ae) - 2ce(16c^2d^2 - 3b^2e^2 - 3c^2d^2))}{128c^2e^4} \\
&= -\frac{(64c^3d^3 + 3b^3e^3 - 16c^2de(5bd - 4ae) + 4bce^2(2bd - 3ae) - 2ce(16c^2d^2 - 3b^2e^2 - 3c^2d^2))}{128c^2e^4} \\
&= -\frac{(64c^3d^3 + 3b^3e^3 - 16c^2de(5bd - 4ae) + 4bce^2(2bd - 3ae) - 2ce(16c^2d^2 - 3b^2e^2 - 3c^2d^2))}{128c^2e^4}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 344, normalized size = 0.96

$$\frac{2\sqrt{c}\left(\sqrt{a+bx^2+cx^4}\left(8c^2e\left(a\left(15c^2-32d\right)+b\left(30d^2-14de^2+9e^2d^4\right)\right)+6bc^2\left(10ae-4bd+bcx^2\right)-9b^3e^3-16c^3\left(12d^3-6d^2ex^2+4d^2x^4-3c^3x^6\right)\right)+192c^2d\left(e\left(ae-bd\right)+cd^2\right)^{3/2}\operatorname{tanh}^{-1}\left(\frac{2ae-bd+bx^2-2cd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e\left(-bd+ae\right)}}\right)\right)+3\left(8d^2c^2\left(bd-3ae\right)-192c^3d^2e\left(bd-ae\right)+48c^2e^2\left(bd-ae\right)^2+3b^4e^4+128c^4d^4\right)\operatorname{tanh}^{-1}\left(\frac{bx^2d^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e\left(-bd+ae\right)}}\right)}{768c^5e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

[Out] (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e^2*x^4 - 3*e^3*x^6) + 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*x^2 + 9*e^2*x^4))) + 192*c^2*d*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(768*c^(5/2)*e^5)

IntegrateAlgebraic [A] time = 2.78, size = 410, normalized size = 1.14

$$\frac{(-48d^2c^4 + 24d^2e^4 + 96bd^2d^2 - 192ac^2d^2 - 3b^4e^4 - 8b^3cd^2 - 48b^2c^2d^2 + 192bc^2d^2 - 128c^4d^4)\operatorname{log}\left(-2\sqrt{a+bx^2+cx^4}\sqrt{e\left(-bd+ae\right)}, \sqrt{a+bx^2+cx^4}\left(6abbc^2-256ac^2d^2+120bc^2d^2-9b^3e^3-24d^2cd^2+6d^2c^2d^2+240c^2d^2e-112b^2d^2e^2+72b^2c^2d^2e-192c^2d^2e^2+96c^2d^2e^2-64c^2d^2e^2+48c^2d^2e^2\right)\right)}{384c^5e^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]

[Out] (Sqrt[a + b*x^2 + c*x^4]*(-192*c^3*d^3 + 240*b*c^2*d^2*e - 24*b^2*c*d*e^2 - 256*a*c^2*d*e^2 - 9*b^3*e^3 + 60*a*b*c*e^3 + 96*c^3*d^2*e*x^2 - 112*b*c^2*d*e^2*x^2 + 6*b^2*c*e^3*x^2 + 120*a*c^2*e^3*x^2 - 64*c^3*d*e^2*x^4 + 72*b*c^2*e^3*x^4 + 48*c^3*e^3*x^6))/(384*c^2*e^4) - (Sqrt[-(c*d^2) + b*d*e - a*e^2]*((c*d^3 - b*d^2*e + a*d*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e*x^2 - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^5 + ((-128*c^4*d^4 + 192*b*c^3*d^3*e - 48*b^2*c^2*d^2*e^2 - 192*a*c^3*d^2*e^2 - 8*b^3*c*d*e^3 + 96*a*b*c^2*d*e^3 - 3*b^4*e^4 + 24*a*b^2*c*e^4 - 48*a^2*c^2*e^4)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(5/2)*e^5)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [B] time = 0.01, size = 1696, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)

[Out] 1/64/e*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32/e*a*b/c*(c*x^4+b*x^2+a)^(1/2)+1/2*d^4/e^5*c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+5/8*d^2/e^3*b*(c*x^4+b*x^2+a)^(1/2)-1/2*d^3/e^4*c*(c*x^4+b*x^2+a)^(1/2)-2/3*d/e^2*a*(c*x^4+b*x^2+a)^(1/2)+3/16/e*a^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+5/16/e*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128/e*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^4/c^(5/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16/e*b*x^4*(c*x^4+b*x^2+a)^(1/2)+1/8/e*c*x^6*(c*x^4+b*x^2+a)^(1/2)+1/2*d^5/e^6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(

$$\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2 + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{c - 1/6} \frac{d}{e} \\
- \frac{2^2 c^2 x^4 (c^2 x^4 + b^2 x^2 + a)^{1/2} - 7/24 d/e^2 b^2 x^2 (c^2 x^4 + b^2 x^2 + a)^{1/2} - 3/4 d^3/e^4 b^2 c^{1/2} \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) + 1/4 d^2/e^3 x^2 c^2 (c^2 x^4 + b^2 x^2 + a)^{1/2} + 1/2 d/e^2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \ln \left(\frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \right)}{e^2 + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{a^2 + 1/2} \frac{d^3}{e^4} \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \ln \left(\frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \right) + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{b^2 - 1/16} \frac{d}{e^2} \frac{c^2 b^2 (c^2 x^4 + b^2 x^2 + a)^{1/2} + 1/32 d/e^2 b^3/c^{3/2} \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) + 3/4 d^2/e^3 a^2 c^{1/2} \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) + 3/16 d^2/e^3 b^2 \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) / c^{3/2} \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) - 3/8 d/e^2 a^2 b \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) / c^{3/2} \ln \left(\frac{c^2 x^2 + 1/2 b}{c^{1/2}} + (c^2 x^4 + b^2 x^2 + a)^{1/2} \right) - d^2/e^3 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \ln \left(\frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \right)}{e^2 + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{a^2 + b^2} \frac{d^3}{e^4} \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \ln \left(\frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \right) + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{a^2 c - d^4/e^5} \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \ln \left(\frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \right) + 2 \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2} \left(\frac{x^2 + d}{e} \right)^{2c} + \frac{(b^2 e - 2^2 c^2 d) \left(\frac{x^2 + d}{e} \right)}{e + \left(\frac{a^2 e^{-2} - b^2 d e + c^2 d^2}{e^2} \right)^{1/2}} \left(\frac{x^2 + d}{e} \right)^{b^2 c}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for more details)Is b*e-2*c*d zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 (c x^4 + b x^2 + a)^{3/2}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3 (a + bx^2 + cx^4)^{\frac{3}{2}}}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d), x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

$$3.240 \quad \int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$$

Optimal. Leaf size=269

$$\frac{\sqrt{a+bx^2+cx^4} \left(-2ce(5bd-4ae) + b^2e^2 - 2cex^2(2cd-be) + 8c^2d^2 \right) (2cd-be) \left(-4ce(2bd-3ae) - b^2e^2 + 8c^2d^2 \right)}{16ce^3} - \frac{32c^{3/2}e^4}{32c^{3/2}e^4}$$

Rubi [A] time = 0.46, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1247, 734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^2+cx^4} \left(-2ce(5bd-4ae) + b^2e^2 - 2cex^2(2cd-be) + 8c^2d^2 \right) (2cd-be) \left(-4ce(2bd-3ae) - b^2e^2 + 8c^2d^2 \right) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right) + \frac{(ae^2-bde+cd^2)^{3/2} \tanh^{-1} \left(\frac{-2ac+2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}} \right) + \frac{(a+bx^2+cx^4)^{3/2}}{6e}}{32c^{3/2}e^4}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x]
- Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]
*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x]
- Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(a+bx+cx^2)^{3/2}}{d+ex} dx, x, x^2 \right) \\
&= \frac{(a+bx^2+cx^4)^{3/2}}{6e} - \frac{\text{Subst} \left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex} dx, x, x^2 \right)}{4e} \\
&= \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x^2) \sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e} \\
&= \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x^2) \sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e} \\
&= \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x^2) \sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e} \\
&= \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x^2) \sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 255, normalized size = 0.95

$$\frac{2\sqrt{c} \left(e\sqrt{a+bx^2+cx^4} (2ce(16ae-15bd+7bex^2)+3b^2e^2+4c^2(6d^2-3dex^2+2c^2x^4)) - 24c(e(ae-bd)+cd^2)^{3/2} \tanh^{-1} \left(\frac{2ae-bd+bx^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}} \right) \right) - 3(2cd-be)(4ce(3ae-2bd)-b^2e^2+8c^2d^2) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{96c^{3/2}e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])] + 2*sqrt[c]*(e*sqrt[a + b*x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) - 24*c*(c*d^2 + e*(-b*d) + a*e)^(3/2)*ArcTanh[(-b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*sqrt[c*d^2 + e*(-b*d) + a*e])*sqrt[a + b*x^2 + c*x^4])))/(96*c^(3/2)*e^4)

IntegrateAlgebraic [A] time = 1.42, size = 291, normalized size = 1.08

$$\frac{\sqrt{a+bx^2+cx^4} (32ace^2+3b^2e^2-30bcde+14bce^2x^2+24c^2d^2-12c^2dex^2+8c^2e^2x^4)}{48ce^3} + \frac{(-12abce^3+24ac^2d^2+b^3e^3+6b^2cde-24b^2d^2e+16c^3d^3) \log(-2c^{3/2}\sqrt{a+bx^2+cx^4}+bc+2c^2x^2)}{32c^{3/2}e^4} + \frac{\sqrt{-ae^2+bde-cd^2}(ae^2-bde+cd^2) \tan^{-1} \left(\frac{-c\sqrt{a+bx^2+cx^4}+\sqrt{c}d+\sqrt{c}cx^2}{\sqrt{-ae^2+bde-cd^2}} \right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x]

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(24*c^2*d^2 - 30*b*c*d*e + 3*b^2*e^2 + 32*a*c*e^2
- 12*c^2*d*e*x^2 + 14*b*c*e^2*x^2 + 8*c^2*e^2*x^4))/(48*c*e^3) + (Sqrt[-(c*
d^2) + b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*ArcTan[(Sqrt[c]*d + Sqrt[c]*e
*x^2 - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/e^4 + ((
16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + 24*a*c^2*d*e^2 + b^3*e^3 - 12
*a*b*c*e^3)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(32*c
^(3/2)*e^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [B] time = 0.01, size = 1411, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e
/c*b^2*(c*x^4+b*x^2+a)^(1/2)-5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c
*x^4+b*x^2+a)^(1/2)*d^2-1/32/e*b^3/c^(3/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2))-3/4/e^2*a*d*c^(1/2)*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/16/e^2*b^2*d*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(
1/2)+3/4/e^3*b*c^(1/2)*d^2*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+
2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d+3/8/e*a
*b*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2/e^4*c^(3/2)*
d^3*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a
e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b
```

```

*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x^2+d/e))*a*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-
2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*c*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*
(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/
e))*b^2*d^2+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)
/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2
*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c*d
^3-1/2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a
*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e
-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2*d^4

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` for
more details)Is a*e^2-b*d*e                                +c*d^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(c x^4 + b x^2 + a)^{3/2}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b x^2 + c x^4)^{3/2}}{d + e x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

$$3.241 \quad \int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)} dx$$

Optimal. Leaf size=350

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{(-12cde(bd-ae) + be^2(3bd-4ae) + 8c^2d^3) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3} - \frac{\sqrt{a+bx^2}}{16\sqrt{c}de^3}$$

Rubi [A] time = 0.57, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, number of rules / integrand size = 0.276, Rules used = {1251, 895, 734, 843, 621, 206, 724, 814}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{(-12cde(bd-ae) + be^2(3bd-4ae) + 8c^2d^3) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3} - \frac{\sqrt{a+bx^2+cx^4} \left(-c(5bd-4ae) + 4cd^2 - 2cdex^2\right)}{8d^2} - \frac{(ae^2 - bde + cd^2)^{3/2} \tanh^{-1}\left(\frac{-2ae+x^2(2d-b)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{a^2-bd+cd^2}}\right)}{2d^2} + \frac{a\sqrt{a+bx^2+cx^4}}{2d} + \frac{ab \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]

[Out] (a*sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e*x^2)*sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])])/(4*sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])])/(16*sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*sqrt[c*d^2 - b*d*e + a*e^2]*sqrt[a + b*x^2 + c*x^4])])/(2*d*e^3)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 814

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 895

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))), x_Symbol] := Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p] && GtQ[p, 0]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2 + cx^4)^{3/2}}{x(d + ex^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x(d + ex)} dx, x, x^2 \right) \\
 &= -\frac{\text{Subst} \left(\int \frac{(-bd + ae - cdx)\sqrt{a + bx + cx^2}}{d + ex} dx, x, x^2 \right)}{2d} + \frac{a \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x} dx, x, x^2 \right)}{2d} \\
 &= \frac{a\sqrt{a + bx^2 + cx^4}}{2d} - \frac{(4cd^2 - e(5bd - 4ae) - 2cdex^2)\sqrt{a + bx^2 + cx^4}}{8de^2} - \frac{a \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2d} \\
 &= \frac{a\sqrt{a + bx^2 + cx^4}}{2d} - \frac{(4cd^2 - e(5bd - 4ae) - 2cdex^2)\sqrt{a + bx^2 + cx^4}}{8de^2} + \frac{a^2 \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2d} \\
 &= \frac{a\sqrt{a + bx^2 + cx^4}}{2d} - \frac{(4cd^2 - e(5bd - 4ae) - 2cdex^2)\sqrt{a + bx^2 + cx^4}}{8de^2} - \frac{a^2 \text{Subst} \left(\int \frac{1}{4x} dx, x, x^2 \right)}{2d} \\
 &= \frac{a\sqrt{a + bx^2 + cx^4}}{2d} - \frac{(4cd^2 - e(5bd - 4ae) - 2cdex^2)\sqrt{a + bx^2 + cx^4}}{8de^2} - \frac{a^{3/2} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a + bx^2 + cx^4}} \right)}{2d}
 \end{aligned}$$

Mathematica [A] time = 0.53, size = 251, normalized size = 0.72

$$\frac{1}{16} \left(\frac{8a^{3/2} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a + bx^2 + cx^4}} \right)}{d} + \frac{(12ce(ae - bd) + 3b^2e^2 + 8c^2d^2) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{a + bx^2 + cx^4}} \right)}{\sqrt{c}e^3} + \frac{2 \left(4(e(ae - bd) + cd^2) \tanh^{-1} \left(\frac{2ae - bd + bx^2 - 2cdx^2}{2\sqrt{a + bx^2 + cx^4}} \sqrt{e(ae - bd) + cd^2} \right) + de\sqrt{a + bx^2 + cx^4} (5be - 4cd + 2cex^2) \right)}{de^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x]

[Out] ((-8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d + ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e^3) + (2*(d*e*(-4*c*d + 5*b*e + 2*c*e*x^2))*Sqrt[a + b*x^2 + c*x^4] + 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*

$\text{ArcTanh}\left[\frac{-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2}{(2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*\text{Sqrt}[a + b*x^2 + c*x^4])}\right]/(d*e^3)/16$

IntegrateAlgebraic [A] time = 1.28, size = 267, normalized size = 0.76

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} + \frac{(-12ace^2 - 3b^2e^2 + 12bcde - 8c^2d^2) \log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4} + b + 2cx^2\right)}{16\sqrt{c}e^3} - \frac{\sqrt{-ae^2 + bde - cd^2} (ae^2 - bde + cd^2) \tan^{-1}\left(\frac{-c\sqrt{a+bx^2+cx^4} + \sqrt{c}d + \sqrt{c}cx^2}{\sqrt{-ae^2 + bde - cd^2}}\right)}{de^3} + \frac{\sqrt{a+bx^2+cx^4} (5be - 4cd + 2cex^2)}{8e^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]

[Out] $\left(\frac{-4*c*d + 5*b*e + 2*c*e*x^2}{8*e^2}\right)*\text{Sqrt}[a + b*x^2 + c*x^4] - \left(\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]\right)*\left(\frac{(c*d^2 - b*d*e + a*e^2)*\text{ArcTan}\left[\frac{\text{Sqrt}[c]*d + \text{Sqrt}[c]*e*x^2 - e*\text{Sqrt}[a + b*x^2 + c*x^4]}{\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]}\right]}{d} + \frac{(a^{3/2})*\text{ArcTanh}\left[\frac{\text{Sqrt}[c]*x^2}{\text{Sqrt}[a]} - \frac{\text{Sqrt}[a + b*x^2 + c*x^4]}{\text{Sqrt}[a]}\right]}{d} + \frac{(-8*c^2*d^2 + 12*b*c*d*e - 3*b^2*e^2 - 12*a*c*e^2)*\text{Log}[b + 2*c*x^2 - 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4]]}{16*\text{Sqrt}[c]*e^3}\right)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type

maple [B] time = 0.04, size = 1270, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x)

[Out] $\frac{5}{8} \frac{e*b*(c*x^4+b*x^2+a)^{(1/2)} - 1/2/d*a^{(3/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2})*a^{(1/2)))/x^2} - 1/2/e^2*d*c*(c*x^4+b*x^2+a)^{(1/2)} + 3/4/e*a*c^{(1/2)}*\ln(($

$c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}+3/16/e*b^2*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})+1/4/e*x^2*c*(c*x^4+b*x^2+a)^{(1/2)}+1/2/e^{3*d^2}*c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*a^2+1/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*a*c-1/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*b*c-3/4/e^2*d*b*c^{(1/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*a*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*b^2+1/2/e^4*d^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))*c^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{(ex^2 + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^2 + a)^{3/2}}{x(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)

[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d), x)

[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)

$$3.242 \quad \int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)} dx$$

Optimal. Leaf size=562

$$\frac{a^{3/2}e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{be(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} + \frac{\sqrt{a+bx^2+cx^4}(-2ce(5bd-4ae)+b^2e^2)}{16cd^2e}$$

Rubi [A] time = 0.92, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 29, number of rules / integrand size = 0.310, Rules used = {1251, 960, 732, 814, 843, 621, 206, 724, 734}

$$\frac{e^{3/2} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{a+bx^2+cx^4}(-2ce(5bd-4ae)+b^2e^2)}{16cd^2e} + \frac{(2d-b)(-4d(2d-3e)-b^2+6c^2) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} + \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} + \frac{(4ac+b^2+2c^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{3(4c+b^2) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16cd^2} + \frac{(a^2-bd+cd^2) \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{(a+b^2+cd^2) \sqrt{a+bx^2+cx^4}}{2cd} + \frac{3(3b+2c^2)\sqrt{a+bx^2+cx^4}}{8c} + \frac{3\sqrt{d} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x]

[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) + (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*e^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```

_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 960

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

```

Rule 1251

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^3(d + ex^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^2(d + ex)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{(a + bx + cx^2)^{3/2}}{dx^2} - \frac{e(a + bx + cx^2)^{3/2}}{d^2x} + \frac{e^2(a + bx + cx^2)^{3/2}}{d^2(d + ex)} \right) dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{(a+bx+cx^2)^{3/2}}{x} dx, x, x^2 \right)}{2d^2} + \frac{e^2 \text{Subst} \left(\int \frac{(a+bx+cx^2)^{3/2}}{d+ex} dx, x, x^2 \right)}{2d^2} \\
&= -\frac{(a + bx^2 + cx^4)^{3/2}}{2dx^2} + \frac{3 \text{Subst} \left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x} dx, x, x^2 \right)}{4d} + \frac{e \text{Subst} \left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{d+ex} dx, x, x^2 \right)}{4d^2} \\
&= \frac{3(3b + 2cx^2)\sqrt{a + bx^2 + cx^4}}{8d} - \frac{e(b^2 + 8ac + 2bcx^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} + \frac{(8c^2d^2 + b^2e^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} \\
&= \frac{3(3b + 2cx^2)\sqrt{a + bx^2 + cx^4}}{8d} - \frac{e(b^2 + 8ac + 2bcx^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} + \frac{(8c^2d^2 + b^2e^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} \\
&= \frac{3(3b + 2cx^2)\sqrt{a + bx^2 + cx^4}}{8d} - \frac{e(b^2 + 8ac + 2bcx^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} + \frac{(8c^2d^2 + b^2e^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} \\
&= \frac{3(3b + 2cx^2)\sqrt{a + bx^2 + cx^4}}{8d} - \frac{e(b^2 + 8ac + 2bcx^2)\sqrt{a + bx^2 + cx^4}}{16cd^2} + \frac{(8c^2d^2 + b^2e^2)\sqrt{a + bx^2 + cx^4}}{16cd^2}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 240, normalized size = 0.43

$$\frac{1}{4} \left(-\frac{2 \left(x^2 (e(ae - bd) + cd^2) \right)^{3/2} \tanh^{-1} \left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a+bx^2+cx^4} \sqrt{e(ae-bd)+cd^2}} \right) + de\sqrt{a+bx^2+cx^4} (ae - cd^2)}{d^2 e^2 x^2} + \frac{\sqrt{a} (2ae - 3bd) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right) - \sqrt{c} (2cd - 3be) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x]

[Out] ((Sqrt[a]*(-3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d^2 - (Sqrt[c]*(2*c*d - 3*b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/e^2 - (2*(d*e*(a*e - c*d*x^2)*Sqrt[a + b*x^2 + c*x^4] + (c*d^2 + e*(-(b*d) + a*e))^(3/2)*x^2*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(d^2*e^2*x^2))/4

IntegrateAlgebraic [A] time = 1.29, size = 261, normalized size = 0.46

$$\frac{(2a^{3/2}e - 3\sqrt{a}bd) \tanh^{-1}\left(\frac{\sqrt{a+bx^2+cx^4}-\sqrt{c}x^2}{\sqrt{a}}\right)}{2d^2} + \frac{(2c^{3/2}d - 3b\sqrt{c}e) \log\left(-2\sqrt{c}\sqrt{a+bx^2+cx^4} + b + 2cx^2\right)}{4e^2} + \frac{\sqrt{-ae^2 + bde - cd^2} (ae^2 - bde + cd^2) \tan^{-1}\left(\frac{-x\sqrt{a+bx^2+cx^4} + \sqrt{c}d + \sqrt{c}cx^2}{\sqrt{-ae^2 + bde - cd^2}}\right)}{d^2e^2} + \frac{\sqrt{a+bx^2+cx^4} (cdx^2 - ae)}{2dex^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]

[Out] $((-(a*e) + c*d*x^2)*\text{Sqrt}[a + b*x^2 + c*x^4])/(2*d*e*x^2) + (\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 - b*d*e + a*e^2)*\text{ArcTan}[(\text{Sqrt}[c]*d + \text{Sqrt}[c]*e*x^2 - e*\text{Sqrt}[a + b*x^2 + c*x^4])/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]])/(d^2*e^2) + ((-3*\text{Sqrt}[a]*b*d + 2*a^{(3/2)}*e)*\text{ArcTanh}[(-(\text{Sqrt}[c]*x^2) + \text{Sqrt}[a + b*x^2 + c*x^4])/\text{Sqrt}[a]])/(2*d^2) + ((2*c^{(3/2)}*d - 3*b*\text{Sqrt}[c]*e)*\text{Log}[b + 2*c*x^2 - 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4]])/(4*e^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.03, size = 1207, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x)

[Out] $1/2/e*c*(c*x^4+b*x^2+a)^{(1/2)} - 3/4/d*a^{(1/2)}*b*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2) - 1/2/d*a/x^2*(c*x^4+b*x^2+a)^{(1/2)} + 3/4/e*b*c^{(1/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}) - 1/2/e^2*d*c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)}) - 1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*$

$\ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}}{(x^2+d/e)}\right) b^2+1/2/d^2 e a^{(3/2)} \ln\left(\frac{(b x^2+2 a+2(c x^4+b x^2+a)^{(1/2)} a^{(1/2)})/x^2-1/2 e/d^2}{((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)} \ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}\right)}{(x^2+d/e)}\right) a^2+1/d/((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)} \ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}\right)}{(x^2+d/e)}\right) a b-1/e/((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)} \ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}\right)}{(x^2+d/e)}\right) a c-1/2/e^3 d^2/((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)} \ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}\right)}{(x^2+d/e)}\right) c^2+1/e^2 d/((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)} \ln\left(\frac{(b e^{-2} c d)(x^2+d/e)/e+2(a e^{-2}-b d e+c d^2)/e^2+2((a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}((x^2+d/e)^2 c+(b e^{-2} c d)(x^2+d/e)/e+(a e^{-2}-b d e+c d^2)/e^2)^{(1/2)}\right)}{(x^2+d/e)}\right) b^2 c$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c x^4 + b x^2 + a)^{\frac{3}{2}}}{(e x^2 + d) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c x^4 + b x^2 + a)^{\frac{3}{2}}}{x^3 (e x^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)

[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b x^2 + c x^4)^{\frac{3}{2}}}{x^3 (d + e x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

$$3.243 \quad \int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=173

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}} + \frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 1653, 843, 621, 206, 724}

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}} + \frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right) \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}bde - \frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2ce^2} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2 \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2e^2} - \frac{(2cd+be) \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4e^2} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2 \text{Subst} \left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2} dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}} \right)}{e^2} - \frac{(2cd+be) \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{4e^2} \\
&= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{4c^{3/2}e^2} + \frac{d^2 \tanh^{-1} \left(\frac{bd-2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}} \right)}{2e^2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 171, normalized size = 0.99

$$\frac{2\sqrt{c} \left(\frac{cd^2 \tanh^{-1} \left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}} \right) + e\sqrt{a+bx^2+cx^4}}{\sqrt{ae^2-bde+cd^2}} \right) - (be+2cd) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{4c^{3/2}e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] (-((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]) + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] + (c*d^2*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2])*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*c^(3/2)*e^2)

IntegrateAlgebraic [A] time = 0.70, size = 230, normalized size = 1.33

$$\frac{(be+2cd) \log \left(-2c^{3/2}\sqrt{a+bx^2+cx^4} + bc + 2c^2x^2 \right)}{4c^{3/2}e^2} + \frac{d^2\sqrt{-ae^2+bde-cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}} \right)}{e^2(ae^2-bde+cd^2)} + \frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) + (d^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2]] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2])/(4*c^(3/2)*e^2)

) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]))/(e^2*(c*d^2 - b*d*e + a*e^2)) + ((2*c*d + b*e)*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(3/2)*e^2)

fricas [B] time = 58.18, size = 1364, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.02, size = 267, normalized size = 1.54

$$\frac{d^2 \ln \left(\frac{(be-2cd)\left(x^2+\frac{d}{c}\right) + 2ae^2-2de+2cd^2 + 2\sqrt{\frac{a^2-deb+cd^2}{e^2}} \sqrt{\left(x^2+\frac{d}{c}\right)^2 + \frac{(be-2cd)\left(x^2+\frac{d}{c}\right) + a^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{c}} \right)}{2\sqrt{\frac{a^2-deb+cd^2}{e^2}} e^3} - \frac{b \ln \left(\frac{cx^2+\frac{b}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}}{\sqrt{c}} \right)}{4c^{\frac{3}{2}}e} - \frac{d \ln \left(\frac{cx^2+\frac{b}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}}{\sqrt{c}} \right)}{2\sqrt{c}e^2} + \frac{\sqrt{cx^4+bx^2+a}}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] $\frac{1}{2}*(c*x^4+b*x^2+a)^{(1/2)}/c/e-1/4/e*b/c^{(3/2)}*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})-1/2/e^2*d*\ln((c*x^2+1/2*b)/c^{(1/2)}+(c*x^4+b*x^2+a)^{(1/2)})/c^{(1/2)}-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2))*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(ex^2 + d)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

$$3.244 \quad \int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) - (d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843


```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2\right) \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2\right)}{2e} - \frac{d \text{Subst}\left(\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2\right)}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^2}{\sqrt{a + bx^2 + cx^4}}\right)}{e} + \frac{d \text{Subst}\left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x^2} dx, x, \frac{-bd + 2ae}{\sqrt{a + bx^2 + cx^4}}\right)}{e} \\ &= \frac{\tanh^{-1}\left(\frac{b + 2cx^2}{2\sqrt{c}\sqrt{a + bx^2 + cx^4}}\right)}{2\sqrt{c}e} - \frac{d \tanh^{-1}\left(\frac{bd - 2ae + (2cd - be)x^2}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx^2 + cx^4}}\right)}{2e\sqrt{cd^2 - bde + ae^2}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.97

$$\frac{d \tanh^{-1}\left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a + bx^2 + cx^4}\sqrt{e(ae - bd) + cd^2}}\right)}{\sqrt{e(ae - bd) + cd^2}} + \frac{\tanh^{-1}\left(\frac{b + 2cx^2}{2\sqrt{c}\sqrt{a + bx^2 + cx^4}}\right)}{\sqrt{c}}$$

2e

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] (ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c] + (d*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)])*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e)

IntegrateAlgebraic [A] time = 0.54, size = 195, normalized size = 1.42

$$\frac{d\sqrt{-ae^2 + bde - cd^2} \tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2 + bde - cd^2}}\right)}{e(ae^2 - bde + cd^2)} - \frac{\log\left(-2\sqrt{c}e\sqrt{a+bx^2+cx^4} + be + 2cex^2\right)}{2\sqrt{c}e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] $-(d*\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]*\text{ArcTan}[(\text{Sqrt}[c]*d)/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2] + (b*d*e - a*e^2) + (\text{Sqrt}[c]*e*x^2)/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2] - (e*\text{Sqrt}[a + b*x^2 + c*x^4])/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2])]/(e*(c*d^2 - b*d*e + a*e^2)) - \text{Log}[b*e + 2*c*e*x^2 - 2*\text{Sqrt}[c]*e*\text{Sqrt}[a + b*x^2 + c*x^4]]/(2*\text{Sqrt}[c]*e)$

fricas [B] time = 4.11, size = 1084, normalized size = 7.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] $[1/4*(\text{sqrt}(c*d^2 - b*d*e + a*e^2)*c*d*\log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*\text{sqrt}(c)*\log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/4*(2*\text{sqrt}(-c*d^2 + b*d*e - a*e^2)*c*d*\arctan(-1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2) - (c*d^2 - b*d*e + a*e^2)*\text{sqrt}(c)*\log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(\text{sqrt}(c*d^2 - b*d*e + a*e^2)*c*d*\log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(c*d^2 - b*d*e + a*e^2)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*\text{sqrt}(-c))/(c^2*x^4 + b*c*x^2 + a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(\text{sqrt}(-c*d^2 + b*d*e - a*e^2)*c*d*\arctan(-1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2) + (c*d^2 - b*d*e + a*e^2)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^2$

+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT>Error: Bad Argument Type

maple [A] time = 0.01, size = 204, normalized size = 1.49

$$d \ln \left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e} + \frac{2ae^2-2deb+2cd^2}{e^2} + 2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} \sqrt{\left(x^2+\frac{d}{e}\right)^2 c + \frac{(be-2cd)\left(x^2+\frac{d}{e}\right) + ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}} \right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} e^2} + \frac{\ln\left(\frac{cx^2+\frac{b}{\sqrt{c}}}{\sqrt{c}} + \sqrt{cx^4+bx^2+a}\right)}{2\sqrt{c} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/2/e*ln((c*x^2+1/2*b)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(ex^2 + d)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)`

[Out] `int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d + ex^2) \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2), x)`

[Out] `Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)`

$$3.245 \quad \int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x^2} dx, x, \frac{-bd + 2ae - (2cd - be)x^2}{\sqrt{a+bx^2+cx^4}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.01

$$-\frac{\tanh^{-1} \left(\frac{2ae-bd+bx^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}} \right)}{2\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] -1/2*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)])*Sqrt[a + b*x^2 + c*x^4]]/Sqrt[c*d^2 + e*(-(b*d) + a*e)]

IntegrateAlgebraic [A] time = 0.38, size = 141, normalized size = 1.64

$$\frac{\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c} ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c} d}{\sqrt{-ae^2+bde-cd^2}} \right)}{ae^2 - bde + cd^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] (Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2]] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2])/(c*d^2 - b*d*e + a*e^2)

fricas [B] time = 1.19, size = 357, normalized size = 4.15

$$\left[\frac{\log \left(-\frac{(8c^2d^2-8bcde+(b^2+4ac)x^4-8abde+8a^2e^2+(b^2+4ac)d^2+2(4bcd^2+4abce^2-(3b^2+4ac)de)x^2+4\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}((2cd-be)x^2+bd-2ae))}{e^2x^4+2dex^2+d^2}}{4\sqrt{cd^2-bde+ae^2}} \right)}{2(cd^2-bde+ae^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]

giac [A] time = 0.49, size = 75, normalized size = 0.87

$$\frac{\arctan\left(-\frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)}{\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)

maple [B] time = 0.01, size = 165, normalized size = 1.92

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e} + \frac{2ae^2-2deb+2cd^2}{e^2} + 2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} \sqrt{\left(x^2+\frac{d}{e}\right)^2 c + \frac{(be-2cd)\left(x^2+\frac{d}{e}\right) + ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(ex^2 + d) \sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d + ex^2) \sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)

$$3.246 \quad \int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=138

$$-\frac{e \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}} - \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1251, 960, 724, 206}

$$-\frac{e \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}} - \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a]*d) - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2]))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 960

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+bx+cx^2}} \right) dx, x, x^2 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx, x, x^2 \right)}{2d} \\
 &= -\frac{\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}} \right)}{d} + \frac{e \text{Subst} \left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2} dx, x, \frac{-bd+2a+cx^2}{\sqrt{a+bx^2+cx^4}} \right)}{d} \\
 &= -\frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}d} - \frac{e \tanh^{-1} \left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}} \right)}{2d\sqrt{cd^2-bde+ae^2}}
 \end{aligned}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.97

$$\frac{e \tanh^{-1} \left(\frac{-2ae+b(d-ex^2)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}} \right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] -1/2*(ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a] + (e*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/d

IntegrateAlgebraic [A] time = 0.52, size = 195, normalized size = 1.41

$$\frac{\tanh^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}} \right)}{\sqrt{a}d} - \frac{e\sqrt{-ae^2+bde-cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}} \right)}{d(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*
e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]))/(d*(c*d^2 - b*d*e + a*e^2))
) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/(Sqrt[
a]*d)
```

fricas [B] time = 1.74, size = 1097, normalized size = 7.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b
*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*
d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arct
an(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
+ a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/4*(sq
rt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*
d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^
2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*
c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [A] time = 0.01, size = 207, normalized size = 1.50

$$\frac{\ln\left(\frac{\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e} + \frac{2ae^2-2deb+2cd^2}{e^2} + 2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} \sqrt{\left(x^2+\frac{d}{e}\right)^2 c + \frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e} + \frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2\sqrt{\frac{ae^2-deb+cd^2}{e^2}} d} - \frac{\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{2\sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] -1/2/d/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(ex^2 + d)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(d + ex^2)\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)

$$3.247 \quad \int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=218

$$\frac{b \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e^2 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}} + \frac{e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} - \frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Rubi [A] time = 0.27, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 960, 730, 724, 206}

$$\frac{b \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e^2 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}} + \frac{e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} - \frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,

$m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2$
 $*c*d - b*e, 0] \&\& \text{EqQ}[m + 2*p + 3, 0]$

Rule 960

$\text{Int}[\{(d_.) + (e_.)*(x_.)\}^{(m_.)}*\{(f_.) + (g_.)*(x_.)\}^{(n_.)}*\{(a_.) + (b_.)*(x_.)$
 $+ (c_.)*(x_.)^2\}^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g$
 $*x)^n*(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}$
 $[e*f - d*g, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& (\text{IntegerQ}[p] \parallel (\text{ILtQ}[m, 0] \&\& \text{ILtQ}[n, 0])) \&\& !(\text{IGtQ}[m, 0] \parallel \text{IGtQ}[n, 0])$

Rule 1251

$\text{Int}[(x_.)^{(m_.)}*\{(d_.) + (e_.)*(x_.)^2\}^{(q_.)}*\{(a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)$
 $\}^4\}^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}*(d + e*x)^q*(a +$
 $b*x + c*x^2)^p, x], x, x^2], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (d + ex^2) \sqrt{a + bx^2 + cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (d + ex) \sqrt{a + bx + cx^2}} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{dx^2 \sqrt{a + bx + cx^2}} - \frac{e}{d^2 x \sqrt{a + bx + cx^2}} + \frac{e^2}{d^2 (d + ex) \sqrt{a + bx + cx^2}} \right) dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2d^2} + \frac{e^2 \text{Subst} \left(\int \frac{1}{(d + ex) \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2d^2} \\ &= -\frac{\sqrt{a + bx^2 + cx^4}}{2adx^2} - \frac{b \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{4ad} + \frac{e \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, x^2 \right)}{d^2} \\ &= -\frac{\sqrt{a + bx^2 + cx^4}}{2adx^2} + \frac{e \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{a} d^2} + \frac{e^2 \tanh^{-1} \left(\frac{bd - 2ae + (2cd^2 - bde + ae^2) \sqrt{a + bx^2 + cx^4}}{2\sqrt{cd^2 - bde + ae^2} \sqrt{a + bx^2 + cx^4}} \right)}{2d^2 \sqrt{cd^2 - bde + ae^2}} \\ &= -\frac{\sqrt{a + bx^2 + cx^4}}{2adx^2} + \frac{b \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{4a^{3/2} d} + \frac{e \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{a} d^2} \end{aligned}$$

Mathematica [A] time = 0.37, size = 175, normalized size = 0.80

$$\frac{2\sqrt{a} \left(\frac{ae^2 \tanh^{-1} \left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}} \right) - \frac{d\sqrt{a+bx^2+cx^4}}{x^2}}{\sqrt{ae^2-bde+cd^2}} \right) + (2ae+bd) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{4a^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[a]*(-(d*Sqrt[a + b*x^2 + c*x^4])/x^2) + (a*e^2*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/Sqrt[c*d^2 - b*d*e + a*e^2]))/(4*a^(3/2)*d^2)

IntegrateAlgebraic [A] time = 0.82, size = 237, normalized size = 1.09

$$\frac{(-2ae - bd) \tanh^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}} \right)}{2a^{3/2}d^2} + \frac{e^2\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}} \right)}{d^2(ae^2 - bde + cd^2)} - \frac{\sqrt{a + bx^2 + cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*d*x^2) + (e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d^2*(c*d^2 - b*d*e + a*e^2)) + ((-(b*d) - 2*a*e)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]])/(2*a^(3/2)*d^2)

fricas [A] time = 2.78, size = 1414, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2),

$$\begin{aligned} & 1/8*(4*\sqrt{-c*d^2 + b*d*e - a*e^2})*a^2*e^2*x^2*\arctan(-1/2*\sqrt{c*x^4 + b*x^2 + a}*\sqrt{-c*d^2 + b*d*e - a*e^2})*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*\sqrt{a}*x^2*\log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*\sqrt{c*x^4 + b*x^2 + a})*x^2 + a)*(b*x^2 + 2*a)*\sqrt{a} + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*\sqrt{c*x^4 + b*x^2 + a})/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(\sqrt{c*d^2 - b*d*e + a*e^2})*a^2*e^2*x^2*\log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*\sqrt{c*x^4 + b*x^2 + a})*\sqrt{c*d^2 - b*d*e + a*e^2})*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*\sqrt{-a}*x^2*\arctan(1/2*\sqrt{c*x^4 + b*x^2 + a}*(b*x^2 + 2*a)*\sqrt{-a})/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*\sqrt{c*x^4 + b*x^2 + a})/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(2*\sqrt{-c*d^2 + b*d*e - a*e^2})*a^2*e^2*x^2*\arctan(-1/2*\sqrt{c*x^4 + b*x^2 + a}*\sqrt{-c*d^2 + b*d*e - a*e^2})*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*\sqrt{-a}*x^2*\arctan(1/2*\sqrt{c*x^4 + b*x^2 + a}*(b*x^2 + 2*a)*\sqrt{-a})/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*\sqrt{c*x^4 + b*x^2 + a})/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2)] \end{aligned}$$

giac [A] time = 0.49, size = 208, normalized size = 0.95

$$\frac{\arctan\left(-\frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)e^2}{\sqrt{-cd^2 + bde - ae^2}d^2} - \frac{(bd + 2ae)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{2\sqrt{-a}ad^2} + \frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})b + 2a\sqrt{c}}{2\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*a*e)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)*a*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)

maple [A] time = 0.01, size = 276, normalized size = 1.27

$$\frac{e \ln\left(\frac{(bc-2cd)\left(x^2+\frac{d}{c}\right)+\frac{2a^2-2deb+2c d^2}{c^2}+2\sqrt{\frac{a^2-dcb+cd^2}{c^2}}\sqrt{\left(x^2+\frac{d}{c}\right)^2 c+\frac{(bc-2cd)\left(x^2+\frac{d}{c}\right)+\frac{a^2-dcb+cd^2}{c^2}}}{x^2+\frac{d}{c}}\right)}{2\sqrt{\frac{a^2-dcb+cd^2}{c^2}}d^2}}{2\sqrt{a}d^2} + \frac{e \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{2\sqrt{a}d^2} + \frac{b \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right)}{4a^2d} - \frac{\sqrt{cx^4+bx^2+a}}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)`

[Out] $\frac{1}{2} \frac{d^2 e}{a^{1/2}} \ln\left(\frac{(b x^2 + 2 a + 2 (c x^4 + b x^2 + a)^{1/2} a^{1/2})}{x^2} - \frac{1}{2} \frac{e}{d^2} \frac{(a e^2 - b d e + c d^2)}{e^2}\right)^{1/2} \ln\left(\frac{(b e - 2 c d) (x^2 + d/e)}{e + 2 (a e^2 - b d e + c d^2)} \frac{(x^2 + d/e)}{e + (a e^2 - b d e + c d^2)/e^2}\right)^{1/2} \frac{((x^2 + d/e)^2 c + (b e - 2 c d) (x^2 + d/e) + (a e^2 - b d e + c d^2)/e^2)}{(x^2 + d/e)} - \frac{1}{2} \frac{(c x^4 + b x^2 + a)^{1/2}}{a d x^2 + 1/4 d b/a^{3/2}} \ln\left(\frac{(b x^2 + 2 a + 2 (c x^4 + b x^2 + a)^{1/2} a^{1/2})}{x^2}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c x^4 + b x^2 + a} (e x^2 + d) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (e x^2 + d) \sqrt{c x^4 + b x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)`

[Out] `int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + e x^2) \sqrt{a + b x^2 + c x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)`

[Out] `Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)`

$$3.248 \quad \int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=236

$$\frac{x^2(2a^2ce - ab^2e - 3abcd + b^3d) + a(-abe - 2acd + b^2d)}{c(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}(ae^2 - bde + cd^2)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} - \frac{d^3 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e(ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 1646, 843, 621, 206, 724}

$$\frac{x^2(2a^2ce - ab^2e - 3abcd + b^3d) + a(-abe - 2acd + b^2d)}{c(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}(ae^2 - bde + cd^2)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} - \frac{d^3 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e(ae^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3 * ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}} dx, x, x^2 \right) \\
&= \frac{a(b^2d - 2acd - abe) + (b^3d - 3abcd - ab^2e + 2a^2ce)x^2}{c(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst} \left(\int \frac{(b^2-4ac)a}{2c(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} dx, x, x^2 \right)}{2c} \\
&= \frac{a(b^2d - 2acd - abe) + (b^3d - 3abcd - ab^2e + 2a^2ce)x^2}{c(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2+cx^4}} dx, x, x^2 \right)}{2c} \\
&= \frac{a(b^2d - 2acd - abe) + (b^3d - 3abcd - ab^2e + 2a^2ce)x^2}{c(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, x^2 \right)}{c} \\
&= \frac{a(b^2d - 2acd - abe) + (b^3d - 3abcd - ab^2e + 2a^2ce)x^2}{c(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right)}{2c^{3/2}e}
\end{aligned}$$

Mathematica [A] time = 0.82, size = 271, normalized size = 1.15

$$\frac{1}{2} \left(\frac{2(a^2(be + 2c(d - ex^2)) + ab(-bd + bex^2 + 3cdx^2) + b^3(-d)x^2)}{c(4ac - b^2)\sqrt{a+bx^2+cx^4}(e(ae - bd) + cd^2)} + \frac{\log(2\sqrt{c}\sqrt{a+bx^2+cx^4} + b + 2cx^2)}{c^{3/2}e} + \frac{d^3 \log(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2 - bde + cd^2} + 2ae - bd + bex^2 - 2cdx^2)}{e(e(ae - bd) + cd^2)^{3/2}} - \frac{d^3 \log(d + ex^2)}{e(e(ae - bd) + cd^2)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] ((2*(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d - e*x^2))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]) - (d^3*Log[d + e*x^2])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) + Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])/(c^(3/2)*e) + (d^3*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/2

IntegrateAlgebraic [A] time = 1.21, size = 308, normalized size = 1.31

$$\frac{a^2be + 2a^2cd - 2a^2cex^2 - ab^2d + ab^2ex^2 + 3abcdx^2 + b^3(-d)x^2}{c(4ac - b^2)\sqrt{a+bx^2+cx^4}(ae^2 - bde + cd^2)} - \frac{\log(-2c^{3/2}e\sqrt{a+bx^2+cx^4} + bce + 2c^2ex^2)}{2c^{3/2}e} - \frac{d^3\sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2 + bde - cd^2}} \right)}{e(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out]
$$\begin{aligned} &(-a*b^2*d) + 2*a^2*c*d + a^2*b*e - b^3*d*x^2 + 3*a*b*c*d*x^2 + a*b^2*e*x^2 \\ &- 2*a^2*c*e*x^2)/(c*(-b^2 + 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x^2 \\ &+ c*x^4]) - (d^3*\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]*\text{ArcTan}[(\text{Sqrt}[c]*d)/\text{Sqrt}[-(c \\ &*d^2) + b*d*e - a*e^2] + (\text{Sqrt}[c]*e*x^2)/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2] - (\\ &e*\text{Sqrt}[a + b*x^2 + c*x^4])/ \text{Sqrt}[-(c*d^2) + b*d*e - a*e^2])]/(e*(c*d^2 - b*d \\ &*e + a*e^2)^2) - \text{Log}[b*c*e + 2*c^2*e*x^2 - 2*c^(3/2)*e*\text{Sqrt}[a + b*x^2 + c*x \\ &^4]]/(2*c^(3/2)*e) \end{aligned}$$

fricas [B] time = 172.62, size = 4901, normalized size = 20.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} &[1/4*((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b \\ &^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^ \\ &3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d \\ &^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2 \\ &)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2 \\ &*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2* \\ &(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*\text{sqrt}(c)*\text{log}(- \\ &8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a))*(2*c*x^2 + b)*\text{sqrt}(\\ &c) - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 \\ &+ (a*b^2*c^2 - 4*a^2*c^3)*d^3)*\text{sqrt}(c*d^2 - b*d*e + a*e^2)*\text{log}(-((8*c^2*d^2 \\ &- 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a* \\ &c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*\text{sqrt}(c*x^4 \\ &+ b*x^2 + a)*\text{sqrt}(c*d^2 - b*d*e + a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e) \\ &)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d \\ &^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^ \\ &3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a \\ &*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*\text{sqrt}(c*x^4 \\ &+ b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)* \\ &d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 \\ &- 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6) \\ &*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c \\ &^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4) \\ &*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^ \\ &2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^ \\ &2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((b^2*c^3 - 4 \\ &*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d \\ &^3)*\text{sqrt}(-c*d^2 + b*d*e - a*e^2)*\text{arctan}(-1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(- \\ &c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d* \\ &e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e \end{aligned}$$

$$\begin{aligned}
&^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e \\
&+ (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 \\
&+ (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b* \\
&c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2 \\
&*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d \\
&^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^ \\
&2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c) \\
&*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b) \\
&*sqrt(c) - 4*a*c) + 4*(a^3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3 \\
&*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b \\
&*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^ \\
&2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a)) \\
&/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a* \\
&b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3) \\
&)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b \\
&^3*c^4 - 4*a*b*c^5)*d^3*e^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - \\
&2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + (\\
&(b^3*c^4 - 4*a*b*c^5)*d^4*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 \\
&- 2*a*b^3*c^3 - 8*a^2*b*c^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 \\
&+ (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^5)*x^2), -1/4*(2*((a*b^2*c^2 - 4*a^2*c^3)*d \\
&^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^ \\
&2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 \\
&- 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8* \\
&a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2) \\
&)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (\\
&b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a \\
&^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(\\
&2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - ((b^2*c^3 - 4*a*c^4)*d^3 \\
&*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c* \\
&d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 \\
&- 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3 \\
&*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e \\
&^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(a^ \\
&3*b*c*e^4 - (a*b^2*c^2 - 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - \\
&2*(a^2*b^2*c - a^3*c^2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2* \\
&a*b^2*c^2 - 2*a^2*c^3)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2 \\
&*c - 2*a^3*c^2)*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5) \\
&*d^4*e - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - \\
&8*a^3*c^4)*d^2*e^3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - \\
&4*a^4*c^3)*e^5 + ((b^2*c^5 - 4*a*c^6)*d^4*e - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e \\
&^2 + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^3 - 2*(a*b^3*c^3 - 4*a^2*b*c \\
&^4)*d*e^4 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^5)*x^4 + ((b^3*c^4 - 4*a*b*c^5)*d^4 \\
&*e - 2*(b^4*c^3 - 4*a*b^2*c^4)*d^3*e^2 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c \\
&^4)*d^2*e^3 - 2*(a*b^4*c^2 - 4*a^2*b^2*c^3)*d*e^4 + (a^2*b^3*c^2 - 4*a^3*b* \\
&c^3)*e^5)*x^2), -1/2*((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*
\end{aligned}$$

$$2)) + d/e^2/(c*x^4+b*x^2+a)^{(1/2)}*(b*x^2+2*a)/(4*a*c-b^2)+d^2/e^3*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}-2*d^3/e^3*c/(e*(-4*a*c+b^2)^{(1/2)}+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)}/c))^2-(-4*a*c+b^2)^{(1/2)}*(x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)}/c)))^{(1/2)}-2*d^3/e^2*c/(e*(-4*a*c+b^2)^{(1/2)}-b*e+2*c*d)/(e*(-4*a*c+b^2)^{(1/2)}+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))+2*d^3/e^3*c/(e*(-4*a*c+b^2)^{(1/2)}-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)}/c))^2+(-4*a*c+b^2)^{(1/2)}*(x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)}/c)))^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(ex^2 + d)(cx^4 + bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)

[Out] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d + ex^2)(a + bx^2 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)

$$3.249 \quad \int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=167

$$\frac{d^2 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}} - \frac{x^2(-abe-2acd+b^2d)+a(bd-2ae)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

Rubi [A] time = 0.29, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 1646, 12, 724, 206}

$$\frac{d^2 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}} - \frac{x^2(-abe-2acd+b^2d)+a(bd-2ae)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1646

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(d + ex^2)(a + bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(d + ex)(a + bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
 &= -\frac{a(bd - 2ae) + (b^2d - 2acd - abe)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{\text{Subst} \left(\int -\frac{(b^2 - 4ac)}{2(cd^2 - bde + ae^2)(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{b^2 - 4ac} \\
 &= -\frac{a(bd - 2ae) + (b^2d - 2acd - abe)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} + \frac{d^2 \text{Subst} \left(\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)} \\
 &= -\frac{a(bd - 2ae) + (b^2d - 2acd - abe)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{d^2 \text{Subst} \left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - (d + ex)^2} dx, x, x^2 \right)}{cd^2 - bde + ae^2} \\
 &= -\frac{a(bd - 2ae) + (b^2d - 2acd - abe)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} + \frac{d^2 \tanh^{-1} \left(\frac{bd - 2ae + (2cd - bde + ae^2)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx + cx^2}} \right)}{2(cd^2 - bde + ae^2)}
 \end{aligned}$$

Mathematica [A] time = 0.62, size = 204, normalized size = 1.22

$$\frac{1}{2} \left(\frac{2(-2a^2e + ab(d - ex^2) - 2acd x^2 + b^2 dx^2)}{(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} (e(bd - ae) - cd^2)} - \frac{d^2 \log(2\sqrt{a + bx^2 + cx^4} \sqrt{ae^2 - bde + cd^2} + 2ae - bd + bex^2 - 2cdx^2)}{(e(ae - bd) + cd^2)^{3/2}} + \frac{d^2 \log(d + ex^2)}{(e(ae - bd) + cd^2)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] ((2*(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2)))/((b^2 - 4*a*c)*(-c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Log[d + e*x^2])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) - (d^2*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/2

IntegrateAlgebraic [A] time = 0.78, size = 226, normalized size = 1.35

$$\frac{-2a^2e + abd - abex^2 - 2acd x^2 + b^2 dx^2}{(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} (-ae^2 + bde - cd^2)} + \frac{d^2 \sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c} ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e \sqrt{a + bx^2 + cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c} d}{\sqrt{-ae^2 + bde - cd^2}} \right)}{(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] (a*b*d - 2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 - a*b*e*x^2)/((b^2 - 4*a*c)*(-(c*d^2) + b*d*e - a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2

fricas [B] time = 2.20, size = 1381, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2

$$\begin{aligned}
& - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4* \\
& a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c \\
& ^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4 \\
&)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - \\
& 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^ \\
& 3 - 4*a^3*b*c)*e^4)*x^2), 1/2*((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c) \\
& *d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2* \\
& sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b \\
& *d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^ \\
& 2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2* \\
& a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b \\
& ^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/ \\
& ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2 \\
& *a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 \\
& - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + \\
& (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^ \\
& 3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c \\
& - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 \\
& - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
\end{aligned}$$

giac [B] time = 0.72, size = 458, normalized size = 2.74

$$\frac{d^2 \arctan\left(\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-cd^2 + bde - ae^2}}\right) + \sqrt{cd}}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}} - \frac{((b^2c^3 - 2ac^2b^3 - b^3b^2e + abcd^2e + 2ab^2d^2 - 2a^2cd^2 - a^2b^3)^2)}{(b^2c^3 - 2ac^2b^3 - b^3b^2e + abcd^2e + 2ab^2d^2 - 2a^2cd^2 - a^2b^3)^2} + \frac{abcd^3 - ab^2d^2e - 2a^2cd^2e + 3a^2bd^2 - 2a^3e^3}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

$$\begin{aligned}
& [Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c \\
& *d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2 \\
&)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2 - \\
& 2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + \\
& 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2* \\
& a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3 - a \\
& *b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 - 4*a* \\
& c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 \\
& - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^ \\
& 3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
\end{aligned}$$

maple [B] time = 0.02, size = 613, normalized size = 3.67

$$\frac{2c d^2 \ln\left(\frac{(b^2c^3 - 2ac^2b^3 - b^3b^2e + abcd^2e + 2ab^2d^2 - 2a^2cd^2 - a^2b^3)^2}{(b^2c^3 - 2ac^2b^3 - b^3b^2e + abcd^2e + 2ab^2d^2 - 2a^2cd^2 - a^2b^3)^2}\right) + \sqrt{cd}}{(b^2c^3 - 2ac^2b^3 - b^3b^2e + abcd^2e + 2ab^2d^2 - 2a^2cd^2 - a^2b^3)^2} - \frac{abcd^3 - ab^2d^2e - 2a^2cd^2e + 3a^2bd^2 - 2a^3e^3}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)`

[Out]
$$-1/e/(c*x^4+b*x^2+a)^{(1/2)}/(4*a*c-b^2)*x^2*b-2/e/(c*x^4+b*x^2+a)^{(1/2)}/(4*a*c-b^2)*a-2/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*c*x^2-1/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*b+2*d^2/e^2*c/(b*e-2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*((x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c))^2*c-(-4*a*c+b^2)^{(1/2)}*(x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c))^2+2*d^2/e*c/(-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(b*e-2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))-2*d^2/e^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c))^2*c+(-4*a*c+b^2)^{(1/2)}*(x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c))^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(ex^2 + d)(cx^4 + bx^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)`

[Out] `int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d + ex^2)(a + bx^2 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)`

[Out] `Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)`

$$3.250 \quad \int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=159

$$\frac{cx^2(bd - 2ae) + a(2cd - be)}{(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} (ae^2 - bde + cd^2)} - \frac{de \tanh^{-1} \left(\frac{-2ae + x^2(2cd - be) + bd}{2\sqrt{a + bx^2 + cx^4} \sqrt{ae^2 - bde + cd^2}} \right)}{2(ae^2 - bde + cd^2)^{3/2}}$$

Rubi [A] time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 822, 12, 724, 206}

$$\frac{cx^2(bd - 2ae) + a(2cd - be)}{(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} (ae^2 - bde + cd^2)} - \frac{de \tanh^{-1} \left(\frac{-2ae + x^2(2cd - be) + bd}{2\sqrt{a + bx^2 + cx^4} \sqrt{ae^2 - bde + cd^2}} \right)}{2(ae^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d + ex^2)(a + bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(d + ex)(a + bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
&= \frac{a(2cd - be) + c(bd - 2ae)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{\text{Subst} \left(\int \frac{(b^2 - 4ac)de}{2(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} \\
&= \frac{a(2cd - be) + c(bd - 2ae)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{(de) \text{Subst} \left(\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} \\
&= \frac{a(2cd - be) + c(bd - 2ae)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} + \frac{(de) \text{Subst} \left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - x} dx, x, x^2 \right)}{cd^2 - bde + ae^2\sqrt{a + bx^2 + cx^4}} \\
&= \frac{a(2cd - be) + c(bd - 2ae)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{de \tanh^{-1} \left(\frac{bd - 2ae + (2cd - be)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx^2 + cx^4}} \right)}{2(cd^2 - bde + ae^2)^{3/2}\sqrt{a + bx^2 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 162, normalized size = 1.02

$$\frac{a (be - 2cd + 2cex^2) - bcdx^2}{(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} (e(bd - ae) - cd^2)} + \frac{de \tanh^{-1} \left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a + bx^2 + cx^4} \sqrt{e(ae - bd) + cd^2}} \right)}{2(e(ae - bd) + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] $(-(b*c*d*x^2) + a*(-2*c*d + b*e + 2*c*e*x^2))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*\text{Sqrt}[a + b*x^2 + c*x^4]) + (d*e*\text{ArcTanh}[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]]*\text{Sqrt}[a + b*x^2 + c*x^4])]/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))$

IntegrateAlgebraic [A] time = 0.75, size = 218, normalized size = 1.37

$$\frac{abe - 2acd + 2acex^2 - bcdx^2}{(4ac - b^2) \sqrt{a + bx^2 + cx^4} (ae^2 - bde + cd^2)} - \frac{de \sqrt{-ae^2 + bde - cd^2} \tan^{-1} \left(\frac{\sqrt{c} ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e \sqrt{a + bx^2 + cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c} d}{\sqrt{-ae^2 + bde - cd^2}} \right)}{(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] $(-2*a*c*d + a*b*e - b*c*d*x^2 + 2*a*c*e*x^2)/((-b^2 + 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x^2 + c*x^4]) - (d*e*\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]*\text{ArcTan}[(\text{Sqrt}[c]*d)/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]] + (\text{Sqrt}[c]*e*x^2)/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2]) - (e*\text{Sqrt}[a + b*x^2 + c*x^4])/\text{Sqrt}[-(c*d^2) + b*d*e - a*e^2])/(c*d^2 - b*d*e + a*e^2)^2$

fricas [B] time = 2.25, size = 1349, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] $[1/4*((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2*c)*d*e)*\text{sqrt}(c*d^2 - b*d*e + a*e^2)*\log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(c*d^2 - b*d*e + a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d^2*e - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*x^2)*\text{sqrt}(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4$

$$\begin{aligned}
 & *a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 \\
 & - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2 \\
 & *(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - \\
 & 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c \\
 & c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8 \\
 & *a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c) \\
 & *e^4)*x^2), -1/2*((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a \\
 & *b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + \\
 & b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(\\
 & (c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 \\
 & - b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (\\
 & a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d^2*e - 2*a^2*c*e^3 - (b^2*c \\
 & + 2*a*c^2)*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 \\
 & - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2 \\
 & *e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - \\
 & 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a \\
 & ^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2) \\
 & *e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b \\
 & ^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^ \\
 & 2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
 \end{aligned}$$

giac [B] time = 0.64, size = 441, normalized size = 2.77

$$\frac{d \arctan\left(\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-cd^2 + bde - ae^2}}\right)e + \frac{(b^2d^3 - b^2cd^2e - 2a^2d^2e + 3abcd^2 - 2a^2ce^2)^2}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^2e + b^4d^2e^2 - 2ab^2cd^2e - 8a^2d^2e^2 - 2ab^3d^3e + 8a^2bcd^3e + a^2b^2d^4 - 4a^3cd^4 + \frac{2ac^2d^3 - 3abcd^2e + ab^2d^2e + 2a^2cd^2 - a^2b^3}{\sqrt{cx^4 + bx^2 + a}}}}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

[Out]
$$\begin{aligned}
 & -d*\arctan(-(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2 + a})*e + \sqrt{c}*d)/\sqrt{-c*d^2 + b*d*e - a*e^2} \\
 & + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d^2*e - 2*a^2*c*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d^2*e^3 + 8*a^2*b*c*d^2*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d^2*e^3 + 8*a^2*b*c*d^2*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/\sqrt{c*x^4 + b*x^2 + a}
 \end{aligned}$$

maple [B] time = 0.01, size = 506, normalized size = 3.18

$$\frac{2cd \ln\left(\frac{(b-2cd)\sqrt{c^2d^2} + 2a^2-2ab+2c^2d^2 + \sqrt{c^2d^2+cd}}{c^2d^2} \sqrt{\frac{(c^2d^2+cd)}{c^2d^2}} + \frac{(b-2cd)\sqrt{c^2d^2} + 2a^2-2ab+2c^2d^2}{c^2d^2}\right)}{(b-2cd + \sqrt{-4ac + b^2})e} - \frac{2\sqrt{\left(x^2 + \frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2 c - \sqrt{-4ac + b^2}} \left(x^2 + \frac{b+\sqrt{-4ac+b^2}}{2c}\right) cd}{(b-2cd + \sqrt{-4ac + b^2})e} + \frac{2\sqrt{\left(x^2 - \frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2 c + \sqrt{-4ac + b^2}} \left(x^2 - \frac{-b+\sqrt{-4ac+b^2}}{2c}\right) cd}{(b-2cd + \sqrt{-4ac + b^2})e} + \frac{2cx^2 + b}{(4ac - b^2)\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)`

[Out]
$$\frac{1}{e} \frac{(2cx^2+b)(4ac-b^2)}{(cx^4+bx^2+a)^{1/2}} - \frac{2d}{e} \frac{c}{(b^2e-2cd+(-4ac+b^2)^{1/2})} \frac{e}{(-4ac+b^2)} \frac{1}{(x^2+1/2b/c+1/2(-4ac+b^2)^{1/2}/c)} \left(\frac{x^2+1/2(b+(-4ac+b^2)^{1/2})}{c} \right)^{1/2} - \frac{2d}{e} \frac{c}{(-b^2e+2cd+(-4ac+b^2)^{1/2})} \frac{e}{(b^2e-2cd+(-4ac+b^2)^{1/2})} \frac{1}{((a^2e-b^2d+cd^2)/e^2)^{1/2}} \ln \left(\frac{(b^2e-2cd)(x^2+d/e)}{e+2(a^2e-b^2d+cd^2)/e^2+2((a^2e-b^2d+cd^2)/e^2)^{1/2}((x^2+d/e)^2c+(b^2e-2cd)(x^2+d/e)/e+(a^2e-b^2d+cd^2)/e^2)^{1/2}} \right) \frac{1}{(x^2+d/e)+2d/e} \frac{c}{(-b^2e+2cd+(-4ac+b^2)^{1/2})} \frac{e}{(-4ac+b^2)} \frac{1}{(x^2+1/2b/c-1/2(-4ac+b^2)^{1/2}/c)} \left(\frac{x^2-1/2(-b+(-4ac+b^2)^{1/2})}{c} \right)^{1/2} - \frac{2d}{e} \frac{c}{(-b^2e+2cd+(-4ac+b^2)^{1/2})} \frac{e}{(-4ac+b^2)} \frac{1}{(x^2+1/2b/c-1/2(-4ac+b^2)^{1/2}/c)} \left(\frac{x^2-1/2(-b+(-4ac+b^2)^{1/2})}{c} \right)^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(cx^4 + bx^2 + a)^{3/2} (ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(ex^2 + d) (cx^4 + bx^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)`

[Out] `int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d + ex^2) (a + bx^2 + cx^4)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)`

[Out] `Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)`

$$3.251 \quad \int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=166

$$\frac{e^2 \tanh^{-1} \left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}} \right)}{2(ae^2-bde+cd^2)^{3/2}} - \frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1247, 740, 12, 724, 206}

$$\frac{e^2 \tanh^{-1} \left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4} \sqrt{ae^2-bde+cd^2}} \right)}{2(ae^2-bde+cd^2)^{3/2}} - \frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]))/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```
Int[((d._) + (e._)*(x._))^(m._)*((a._) + (b._)*(x._) + (c._)*(x._)^2)^(p._), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1247

```
Int[(x._)*((d._) + (e._)*(x._)^2)^(q._)*((a._) + (b._)*(x._)^2 + (c._)*(x._)^4)^(p._), x_Symbol]
:> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{(d + ex^2)(a + bx^2 + cx^4)^{3/2}} dx = \frac{1}{2} \text{Subst} \left(\int \frac{1}{(d + ex)(a + bx + cx^2)^{3/2}} dx, x, x^2 \right)$$

$$= -\frac{bcd - b^2e + 2ace + c(2cd - be)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{\text{Subst} \left(\int \frac{(b^2 - 4ac)e^2}{2(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{(b^2 - 4ac)(cd^2 - bde + ae^2)}$$

$$= -\frac{bcd - b^2e + 2ace + c(2cd - be)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} + \frac{e^2 \text{Subst} \left(\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{2(cd^2 - bde + ae^2)}$$

$$= -\frac{bcd - b^2e + 2ace + c(2cd - be)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} - \frac{e^2 \text{Subst} \left(\int \frac{1}{4cd^2 - 4bde + 4ae^2 - (d + ex)^2} dx, x, x^2 \right)}{cd^2 - bde + ae^2}$$

$$= -\frac{bcd - b^2e + 2ace + c(2cd - be)x^2}{(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}} + \frac{e^2 \tanh^{-1} \left(\frac{bd - 2ae + (2cd - bde + ae^2)x}{2\sqrt{cd^2 - bde + ae^2}\sqrt{a + bx^2 + cx^4}} \right)}{2(cd^2 - bde + ae^2)}$$

Mathematica [A] time = 0.15, size = 167, normalized size = 1.01

$$\frac{2ace + b^2(-e) + cx^2(2cd - be) + bcd}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}(e(ae - bd) + cd^2)} - \frac{e^2 \tanh^{-1} \left(\frac{2ae - bd + bex^2 - 2cdx^2}{2\sqrt{a + bx^2 + cx^4}\sqrt{e(ae - bd) + cd^2}} \right)}{2(e(ae - bd) + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4])) - (e^2*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))

IntegrateAlgebraic [A] time = 0.75, size = 224, normalized size = 1.35

$$\frac{2ace + b^2(-e) + bcd - bcex^2 + 2c^2dx^2}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}(-ae^2 + bde - cd^2)} + \frac{e^2\sqrt{-ae^2 + bde - cd^2} \tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2 + bde - cd^2}} - \frac{e\sqrt{a + bx^2 + cx^4}}{\sqrt{-ae^2 + bde - cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2 + bde - cd^2}}\right)}{(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] (b*c*d - b^2*e + 2*a*c*e + 2*c^2*d*x^2 - b*c*e*x^2)/((b^2 - 4*a*c)*(-(c*d^2) + b*d*e - a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(c*d^2 - b*d*e + a*e^2)^2

fricas [B] time = 2.49, size = 1379, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)

$$\begin{aligned} & *e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*\sqrt{-c*d^2 + b*d*e - a*e^2}*\arctan(-1/2* \\ & \sqrt{c*x^4 + b*x^2 + a}*\sqrt{-c*d^2 + b*d*e - a*e^2})*((2*c*d - b*e)*x^2 + b \\ & *d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 \\ & + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)* \\ & d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2* \\ & d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*\sqrt{c*x^4 + b*x^2 + a})/ \\ & ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2 \\ & *a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 \\ & - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + \\ & (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 \\ & + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c \\ & - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 \\ & - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)] \end{aligned}$$

giac [B] time = 0.62, size = 454, normalized size = 2.73

$$\frac{\frac{(2c^3b^3 - 3bc^2d^2e + b^2cd^2 + 2a^2d^2 - abce^2)^2}{b^2c^2d^4 - 4ac^3d^3 - 2b^3cd^3e + 8ab^2d^3 + b^4d^3e^2 - 2ab^2cd^2e - 8a^2d^2e^2 - 2ab^2d^3e^2 + 8a^2bcd^3 + a^2b^2d^3e - 4a^3cd^4} + \frac{bc^2d^3 - 2b^2cd^2e + 2ac^2d^2e + b^3de^2 - abcd^2 - ab^2e^3 + 2a^2ce^3}{\sqrt{cx^4 + bx^2 + a}}}{\sqrt{cx^4 + bx^2 + a}} + \frac{\arctan\left(-\frac{(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)e^2}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^2 \\ & / (b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 \\ & - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 \\ & + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e + \\ & b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3) / (b^2*c^2*d^4 - 4*a*c^3* \\ & d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8 \\ & *a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c* \\ & e^4) / \sqrt{c*x^4 + b*x^2 + a} + \arctan(-((\sqrt{c})*x^2 - \sqrt{c*x^4 + b*x^2 \\ & + a})*e + \sqrt{c}*d) / \sqrt{-c*d^2 + b*d*e - a*e^2}) * e^2 / ((c*d^2 - b*d*e + a* \\ & e^2)*\sqrt{-c*d^2 + b*d*e - a*e^2}) \end{aligned}$$

maple [B] time = 0.01, size = 454, normalized size = 2.73

$$\frac{2ce \ln\left(\frac{(be-2cd)\left(x^2+\frac{d}{c}\right) + 2a^2-2db+2cd^2}{x^2+\frac{d}{c}} + 2\sqrt{\frac{a^2-db+cd^2}{c^2}}\sqrt{\left(x^2+\frac{d}{c}\right)^2 + \frac{a^2-db+cd^2}{c^2}}\right)}{(be+2cd+\sqrt{-4ac+b^2}e)(be-2cd+\sqrt{-4ac+b^2}e)\sqrt{\frac{a^2-db+cd^2}{c^2}}} + \frac{2\sqrt{\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2 c - \sqrt{-4ac+b^2}}\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{(be-2cd+\sqrt{-4ac+b^2}e)(-4ac+b^2)\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)} - \frac{2\sqrt{\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2 c + \sqrt{-4ac+b^2}}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)c}{(be+2cd+\sqrt{-4ac+b^2}e)(-4ac+b^2)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)

$$\text{[Out]} \quad 2*c/(b*e-2*c*d+(-4*a*c+b^2)^(1/2)*e)/(-4*a*c+b^2)/(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/(1/2))/c*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1$$

$$\begin{aligned} & /2*(b+(-4*a*c+b^2)^{(1/2)})/c)^{(1/2)}+2*c*e/(-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e) \\ & / (b*e-2*c*d+(-4*a*c+b^2)^{(1/2)}*e) / ((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * \ln(((b*e- \\ & 2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * \\ & ((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}) / \\ & (x^2+d/e)) - 2*c / (-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e) / (-4*a*c+b^2) / (x^2-1/2*(-b+ \\ & (-4*a*c+b^2)^{(1/2)})/c) * ((x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c)^2*c+(-4*a*c+b^2) \\ &)^{(1/2)} * (x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c))^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(ex^2 + d)(cx^4 + bx^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)

[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d + ex^2)(a + bx^2 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)

$$3.252 \quad \int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=266

$$\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} + \frac{e(2ace + b^2(-e) + cx^2(2cd - be) + bcd)}{d(b^2 - 4ac)\sqrt{a+bx^2+cx^4}(ae^2 - bde + cd^2)} + \frac{-2ac + b^2 + bcx^2}{ad(b^2 - 4ac)\sqrt{a+bx^2+cx^4}} - \frac{e^3 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d(ae^2 - bde + cd^2)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 960, 740, 12, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} + \frac{e(2ace + b^2(-e) + cx^2(2cd - be) + bcd)}{d(b^2 - 4ac)\sqrt{a+bx^2+cx^4}(ae^2 - bde + cd^2)} + \frac{-2ac + b^2 + bcx^2}{ad(b^2 - 4ac)\sqrt{a+bx^2+cx^4}} - \frac{e^3 \tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d(ae^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(d+ex)(a+bx+cx^2)^{3/2}} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{dx(a+bx+cx^2)^{3/2}} - \frac{e}{d(d+ex)(a+bx+cx^2)^{3/2}} \right) dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}} dx, x, x^2 \right)}{2d} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd - b^2e + 2ace + c(2cd - be)x^2)}{(b^2 - 4ac)d(cd^2 - bde + ae^2)\sqrt{a+bx^2}} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd - b^2e + 2ace + c(2cd - be)x^2)}{(b^2 - 4ac)d(cd^2 - bde + ae^2)\sqrt{a+bx^2}} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd - b^2e + 2ace + c(2cd - be)x^2)}{(b^2 - 4ac)d(cd^2 - bde + ae^2)\sqrt{a+bx^2}} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd - b^2e + 2ace + c(2cd - be)x^2)}{(b^2 - 4ac)d(cd^2 - bde + ae^2)\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.73, size = 236, normalized size = 0.89

$$\frac{\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} + \frac{2d(bc(3ae+cdx^2)-2ac^2(d-ex^2)+b^3(-e)+b^2c(d-ex^2))}{a(4ac-b^2)\sqrt{a+bx^2+cx^4}(e(ae-bd)+cd^2)} + \frac{e^3 \tanh^{-1}\left(\frac{-2ae+b(d-ex^2)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{(e(ae-bd)+cd^2)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] -1/2*((2*d*(-(b^3*e) + b*c*(3*a*e + c*d*x^2) + b^2*c*(d - e*x^2) - 2*a*c^2*(d - e*x^2)))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]) + ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/a^(3/2) + (e^3*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-

$(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/d$

IntegrateAlgebraic [A] time = 1.49, size = 324, normalized size = 1.22

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2 - \sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{-3abce + 2ac^2d - 2ac^2ex^2 + b^3e - b^2cd + b^2cex^2 - bc^2dx^2}{a\sqrt{a+bx^2+cx^4}(4a^2ce^2 - ab^2e^2 - 4abcde + 4a^2d^2 + b^3de - b^2cd^2)} - \frac{e^3\sqrt{-ae^2+bde-cd^2} \tan^{-1}\left(\frac{\sqrt{c}ex^2}{\sqrt{-ae^2+bde-cd^2}} - \frac{e\sqrt{a+bx^2+cx^4}}{\sqrt{-ae^2+bde-cd^2}} + \frac{\sqrt{c}d}{\sqrt{-ae^2+bde-cd^2}}\right)}{d(ae^2 - bde + cd^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] $(-(b^2*c*d) + 2*a*c^2*d + b^3*e - 3*a*b*c*e - b*c^2*d*x^2 + b^2*c*e*x^2 - 2*a*c^2*e*x^2)/(a*(-(b^2*c*d^2) + 4*a*c^2*d^2 + b^3*d*e - 4*a*b*c*d*e - a*b^2*e^2 + 4*a^2*c*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*d)/Sqrt[-(c*d^2) + b*d*e - a*e^2] + (Sqrt[c]*e*x^2)/Sqrt[-(c*d^2) + b*d*e - a*e^2] - (e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/(d*(c*d^2 - b*d*e + a*e^2)^2) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/(a^(3/2)*d)$

fricas [B] time = 10.49, size = 4909, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] $[1/4*((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 -$

$$\begin{aligned}
& 4a^5b^3c^2*d^2e^3 + (a^5b^2 - 4a^6c)*d^2e^4 + ((a^2b^2c^3 - 4a^3c^4) \\
& *d^5 - 2*(a^2b^3c^2 - 4a^3b^3c^3)*d^4e + (a^2b^4c - 2a^3b^2c^2 - 8 \\
& *a^4c^3)*d^3e^2 - 2*(a^3b^3c - 4a^4b^3c^2)*d^2e^3 + (a^4b^2c - 4a^5 \\
& *c^2)*d^2e^4)*x^4 + ((a^2b^3c^2 - 4a^3b^3c^3)*d^5 - 2*(a^2b^4c - 4a^3 \\
& *b^2c^2)*d^4e + (a^2b^5 - 2a^3b^3c - 8a^4b^3c^2)*d^3e^2 - 2*(a^3b^4 \\
& - 4a^4b^2c)*d^2e^3 + (a^4b^3 - 4a^5b^3c)*d^2e^4)*x^2, -1/4*(2*((a^2 \\
& *b^2c - 4a^3c^2)*e^3*x^4 + (a^2b^3 - 4a^3b^3c)*e^3*x^2 + (a^3b^2 - 4a \\
& *a^4c)*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a \\
&)*sqrt(-c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 \\
& - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e \\
& + a*b*e^2)*x^2)) - ((a*b^2c^2 - 4a^2c^3)*d^4 - 2*(a*b^3c - 4a^2b^3c^2) \\
&)*d^3e + (a*b^4 - 2a^2b^2c - 8a^3c^2)*d^2e^2 - 2*(a^2b^3 - 4a^3b^3c \\
&)*d^2e^3 + (a^3b^2 - 4a^4c)*e^4 + ((b^2c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 \\
& - 4a*b^3c^3)*d^3e + (b^4c - 2a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c \\
& - 4a^2b^3c^2)*d^2e^3 + (a^2b^2c - 4a^3c^2)*e^4)*x^4 + ((b^3c^2 - 4a*a \\
& *b^3c^3)*d^4 - 2*(b^4c - 4a*b^2c^2)*d^3e + (b^5 - 2a*b^3c - 8a^2b^3c^2) \\
&)*d^2e^2 - 2*(a*b^4 - 4a^2b^2c)*d^2e^3 + (a^2b^3 - 4a^3b^3c)*e^4)*x^2) \\
& *sqrt(a)*log(-((b^2 + 4a*c)*x^4 + 8a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b \\
& *x^2 + 2a)*sqrt(a) + 8a^2)/x^4) - 4*((a*b^2c^2 - 2a^2c^3)*d^4 - (2a*b \\
& ^3c - 5a^2b^3c^2)*d^3e + (a*b^4 - 2a^2b^2c - 2a^3c^2)*d^2e^2 - (a^ \\
& 2b^3 - 3a^3b^3c)*d^2e^3 + (a*b^3c^3*d^4 - 2*(a*b^2c^2 - a^2c^3)*d^3e + (\\
& a*b^3c - a^2b^3c^2)*d^2e^2 - (a^2b^2c - 2a^3c^2)*d^2e^3)*x^2)*sqrt(c*x \\
& ^4 + b*x^2 + a))/((a^3b^2c^2 - 4a^4c^3)*d^5 - 2*(a^3b^3c - 4a^4b^3c^ \\
& 2)*d^4e + (a^3b^4 - 2a^4b^2c - 8a^5c^2)*d^3e^2 - 2*(a^4b^3 - 4a^5 \\
& *b^3c)*d^2e^3 + (a^5b^2 - 4a^6c)*d^2e^4 + ((a^2b^2c^3 - 4a^3c^4)*d^5 \\
& - 2*(a^2b^3c^2 - 4a^3b^3c^3)*d^4e + (a^2b^4c - 2a^3b^2c^2 - 8a^4c^ \\
& 3)*d^3e^2 - 2*(a^3b^3c - 4a^4b^3c^2)*d^2e^3 + (a^4b^2c - 4a^5c^2) \\
&)*d^2e^4)*x^4 + ((a^2b^3c^2 - 4a^3b^3c^3)*d^5 - 2*(a^2b^4c - 4a^3b^2c^ \\
& 2)*d^4e + (a^2b^5 - 2a^3b^3c - 8a^4b^3c^2)*d^3e^2 - 2*(a^3b^4 - 4 \\
& *a^4b^2c)*d^2e^3 + (a^4b^3 - 4a^5b^3c)*d^2e^4)*x^2, 1/4*(2*((a*b^2c^2 \\
& - 4a^2c^3)*d^4 - 2*(a*b^3c - 4a^2b^3c^2)*d^3e + (a*b^4 - 2a^2b^2c \\
& - 8a^3c^2)*d^2e^2 - 2*(a^2b^3 - 4a^3b^3c)*d^2e^3 + (a^3b^2 - 4a^4c)* \\
& e^4 + ((b^2c^3 - 4a*c^4)*d^4 - 2*(b^3c^2 - 4a*b^3c^3)*d^3e + (b^4c - 2 \\
& *a*b^2c^2 - 8a^2c^3)*d^2e^2 - 2*(a*b^3c - 4a^2b^3c^2)*d^2e^3 + (a^2b^ \\
& 2c - 4a^3c^2)*e^4)*x^4 + ((b^3c^2 - 4a*b^3c^3)*d^4 - 2*(b^4c - 4a*b^2 \\
& *c^2)*d^3e + (b^5 - 2a*b^3c - 8a^2b^3c^2)*d^2e^2 - 2*(a*b^4 - 4a^2b^ \\
& 2c)*d^2e^3 + (a^2b^3 - 4a^3b^3c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 \\
& + b*x^2 + a)*(b*x^2 + 2a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + ((a^2b^2 \\
& *c - 4a^3c^2)*e^3*x^4 + (a^2b^3 - 4a^3b^3c)*e^3*x^2 + (a^3b^2 - 4a^4c \\
&)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4 \\
& *a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + \\
& 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^ \\
& 2 - b*d*e + a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 \\
& + d^2)) + 4*((a*b^2c^2 - 2a^2c^3)*d^4 - (2a*b^3c - 5a^2b^3c^2)*d^3e \\
& + (a*b^4 - 2a^2b^2c - 2a^3c^2)*d^2e^2 - (a^2b^3 - 3a^3b^3c)*d^2e^3 +
\end{aligned}$$

$$\begin{aligned}
& (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e \\
& ^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*\text{sqrt}(c*x^4 + b*x^2 + a)/((a^3*b^2 \\
& *c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^ \\
& 4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - \\
& 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b \\
& *c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c \\
& - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c \\
& ^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2* \\
& a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4 \\
& *b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/2*((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^ \\
& 2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*\text{sqrt}(-c*d^2 + b*d*e - \\
& a*e^2)*\arctan(-1/2*\text{sqrt}(c*x^4 + b*x^2 + a)*\text{sqrt}(-c*d^2 + b*d*e - a*e^2))*((\\
& 2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^ \\
& 2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 - \\
& 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - \\
& 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^ \\
& 4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a \\
& *b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2* \\
& c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c \\
& ^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2* \\
& c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*\text{sqrt}(-a)*\arctan(1/2*\text{sqrt}(c*x^4 + \\
& b*x^2 + a)*(b*x^2 + 2*a)*\text{sqrt}(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a*b^2*c \\
& ^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2* \\
& c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a* \\
& b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a \\
& ^3*c^2)*d*e^3)*x^2)*\text{sqrt}(c*x^4 + b*x^2 + a)/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 \\
& - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)* \\
& d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a \\
& ^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^ \\
& 4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2* \\
& e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 \\
& - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c \\
& ^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e \\
& ^4)*x^2)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error: Bad Argument Type

maple [B] time = 0.02, size = 612, normalized size = 2.30

$$\frac{2cx^2 \ln \left(\frac{(bx-2cd)(x^2-d)}{(bx-2cd)(x^2-d)} \sqrt{\frac{(bx-2cd)(x^2-d)}{(bx-2cd)(x^2-d)}} \right)}{(-bx+2cd+\sqrt{-4ac+b^2})(bx-2cd+\sqrt{-4ac+b^2})\sqrt{\frac{(bx-2cd)(x^2-d)}{(bx-2cd)(x^2-d)}}} - \frac{bcx^2}{(4ac-b^2)\sqrt{cx^4+bx^2+a}} - \frac{2\sqrt{\left(x^2+\frac{bx+\sqrt{-4ac+b^2}}{2}\right)^2c-\sqrt{-4ac+b^2}\left(x^2+\frac{bx+\sqrt{-4ac+b^2}}{2}\right)}}{(bx-2cd+\sqrt{-4ac+b^2})(-4ac+b^2)\left(x^2+\frac{bx+\sqrt{-4ac+b^2}}{2}\right)d} + \frac{2\sqrt{\left(x^2-\frac{bx+\sqrt{-4ac+b^2}}{2}\right)^2c+\sqrt{-4ac+b^2}\left(x^2-\frac{bx+\sqrt{-4ac+b^2}}{2}\right)}}{(bx-2cd+\sqrt{-4ac+b^2})(-4ac+b^2)\left(x^2+\frac{bx+\sqrt{-4ac+b^2}}{2}\right)d} - \frac{b^2}{2(4ac-b^2)\sqrt{cx^4+bx^2+a}} - \frac{\ln\left(\frac{(bx^2+2cd+\sqrt{-4ac+b^2})\sqrt{cx^4+bx^2+a}}{x^2}\right)}{2bx^2d} + \frac{1}{2\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2), x)

[Out] $\frac{1}{2} \frac{d}{a} \frac{1}{(cx^4+bx^2+a)^{1/2}} - \frac{1}{d} \frac{b}{a} \frac{1}{(4ac-b^2)} \frac{1}{(cx^4+bx^2+a)^{1/2}} * c * x^2 - \frac{1}{2} \frac{d}{b^2} \frac{1}{a} \frac{1}{(4ac-b^2)} \frac{1}{(cx^4+bx^2+a)^{1/2}} - \frac{1}{2} \frac{d}{a} \ln\left(\frac{(bx^2+2a+2*(cx^4+bx^2+a)^{1/2}*a^{1/2})}{x^2} - 2*e/d*c/(b*e-2*c*d+(-4*a*c+b^2)^{1/2}) * e\right) / (-4*a*c+b^2) / (x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{1/2}/c) * ((x^2+1/2*(b+(-4*a*c+b^2)^{1/2})/c)^2 * c - (-4*a*c+b^2)^{1/2} * (x^2+1/2*(b+(-4*a*c+b^2)^{1/2})/c))^{1/2} - 2*e^2/d*c / (-b*e+2*c*d+(-4*a*c+b^2)^{1/2}) * e / (b*e-2*c*d+(-4*a*c+b^2)^{1/2}) * e / ((a*e^2-b*d*e+c*d^2)/e^2)^{1/2} * \ln\left(\frac{(b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}}{(x^2+d/e)+2*e/d*c/(-b*e+2*c*d+(-4*a*c+b^2)^{1/2}) * e} / (-4*a*c+b^2) / (x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{1/2}/c) * ((x^2-1/2*(-b+(-4*a*c+b^2)^{1/2})/c)^2 * c + (-4*a*c+b^2)^{1/2} * (x^2-1/2*(-b+(-4*a*c+b^2)^{1/2})/c))^{1/2}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)^{3/2} (ex^2 + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x (ex^2 + d) (cx^4 + bx^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(d + ex^2)(a + bx^2 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2), x)

[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)

$$3.253 \quad \int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}} dx$$

Optimal. Leaf size=419

$$\frac{e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2} + \frac{3b \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d} - \frac{(3b^2 - 8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2 - 4ac)} - \frac{e^2(2ace + b^2(-e) + cx^2)}{d^2(b^2 - 4ac)\sqrt{a+bx^2+cx^4}}$$

Rubi [A] time = 0.56, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1251, 960, 740, 806, 724, 206, 12}

$$-\frac{(3b^2 - 8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2 - 4ac)} + \frac{e \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2} + \frac{3b \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d} - \frac{e^2(2ace + b^2(-e) + cx^2)}{d^2(b^2 - 4ac)\sqrt{a+bx^2+cx^4}} - \frac{e(-2ac + b^2 + bcx^2)}{ad^2(b^2 - 4ac)\sqrt{a+bx^2+cx^4}} + \frac{-2ac + b^2 + bcx^2}{ad^2(b^2 - 4ac)\sqrt{a+bx^2+cx^4}} + \frac{e^4 \tanh^{-1}\left(\frac{-2ae + b^2(2d-b) + cx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{a^2 - bde + cd^2}}\right)}{2d^2(a^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*sqrt[a + b*x^2 + c*x^4]) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*sqrt[a + b*x^2 + c*x^4]) - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2*(c*d^2 - b*d*e + a*e^2)*sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*sqrt[c*d^2 - b*d*e + a*e^2]*sqrt[a + b*x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 960

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (d + ex) (a + bx + cx^2)^{3/2}} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{dx^2 (a + bx + cx^2)^{3/2}} - \frac{e}{d^2 x (a + bx + cx^2)^{3/2}} + \frac{1}{d^2 (d + ex)} \right) dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx + cx^2)^{3/2}} dx, x, x^2 \right)}{2d} - \frac{e \text{Subst} \left(\int \frac{1}{x (a + bx + cx^2)^{3/2}} dx, x, x^2 \right)}{2d^2} + \frac{1}{2d^2} \int \frac{1}{d + ex} dx \\
&= -\frac{e (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) d^2 \sqrt{a + bx^2 + cx^4}} + \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) dx^2 \sqrt{a + bx^2 + cx^4}} - \frac{1}{2d^2} \ln |d + ex| \\
&= -\frac{e (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) d^2 \sqrt{a + bx^2 + cx^4}} + \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) dx^2 \sqrt{a + bx^2 + cx^4}} - \frac{1}{2d^2} \ln |d + ex| \\
&= -\frac{e (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) d^2 \sqrt{a + bx^2 + cx^4}} + \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) dx^2 \sqrt{a + bx^2 + cx^4}} - \frac{1}{2d^2} \ln |d + ex| \\
&= -\frac{e (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) d^2 \sqrt{a + bx^2 + cx^4}} + \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) dx^2 \sqrt{a + bx^2 + cx^4}} - \frac{1}{2d^2} \ln |d + ex|
\end{aligned}$$

Mathematica [A] time = 1.48, size = 350, normalized size = 0.84

$$\frac{(2ae+3bd) \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{a^{5/2}} + \frac{2d(-4a^3cx^2+a^2(b^2c^2+4bce(d-cx^2)-4c^2(d^2+dex^2+c^2x^4))+d(b^3c(cx^2-d)+b^2c(d^2+12dex^2+c^2x^4))-10b^2dx^2(d-cx^2)-8c^3d^2x^4)+3b^2d^2(b+cx^2)(cd-bc)}{a^2x^2(b^2-4ac)\sqrt{a+bx^2+cx^4}(e(bd-ae)-cd^2)} + \frac{2e^4 \tanh^{-1} \left(\frac{-2ae+b(d-cx^2)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ac-bd)+cd^2}} \right)}{(e(ac-bd)+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x]

[Out] ((2*d*(-4*a^3*c*e^2 + 3*b^2*d*(c*d - b*e))*x^2*(b + c*x^2) + a^2*(b^2*e^2 + 4*b*c*e*(d - e*x^2) - 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(-8*c^3*d^2*x^4 - 10*b*c^2*d*x^2*(d - e*x^2) + b^3*e*(-d + e*x^2) + b^2*c*(d^2 + 12*d*e*x^2 + e^2*x^4)))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*sqrt[a + b*x^2 + c*x^4] + ((3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(a^(5/2)) + (2*e^4*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e

$x^2)) / (2 * \text{Sqrt}[c * d^2 + e * (- (b * d) + a * e)] * \text{Sqrt}[a + b * x^2 + c * x^4]) / (c * d^2 + e * (- (b * d) + a * e))^{(3/2)} / (4 * d^2)$

IntegrateAlgebraic [A] time = 2.07, size = 479, normalized size = 1.14

$$\frac{(-2ae - 3bd) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c^2 d^2 - \sqrt{c^2 d^2 + e^2}}}{\sqrt{e}}\right) - 4a^2 b^2 + b^3 d^2 + 4a^2 b c d - 4a^2 b^2 c^2 - 4a^2 c^2 d^2 - 4a^2 c^2 d e^2 - 4a^2 c^2 d^2 e - a b^3 d e + a b^3 c^2 + a b^2 c d^2 + 12 a b^2 c d e^2 + a b^2 c d^2 e - 10 a b c^2 d^2 x^2 + 10 a b c^2 d e x^4 - 8 a c^3 d x^4 - 3 b^4 d e^2 + 3 b^3 c d e x^2 - 3 b^3 c d e^2 x^4}{2 a^2 b^2} + \frac{e^2 \sqrt{-a^2 + b d e - c d^2} \operatorname{tanh}^{-1}\left(\frac{\sqrt{c^2 d^2 - \sqrt{c^2 d^2 + e^2}}}{\sqrt{-a^2 + b d e - c d^2}}\right) + \frac{\sqrt{c d}}{\sqrt{-a^2 + b d e - c d^2}}}{d^2 (a^2 - b d e + c d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] $(a * b^2 * c * d^2 - 4 * a^2 * c^2 * d^2 - a * b^3 * d * e + 4 * a^2 * b * c * d * e + a^2 * b^2 * e^2 - 4 * a^3 * c * e^2 + 3 * b^3 * c * d^2 * x^2 - 10 * a * b * c^2 * d^2 * x^2 - 3 * b^4 * d * e * x^2 + 12 * a * b^2 * c * d * e * x^2 - 4 * a^2 * c^2 * d * e * x^2 + a * b^3 * e^2 * x^2 - 4 * a^2 * b * c * e^2 * x^2 + 3 * b^2 * c^2 * d^2 * x^4 - 8 * a * c^3 * d^2 * x^4 - 3 * b^3 * c * d * e * x^4 + 10 * a * b * c^2 * d * e * x^4 + a * b^2 * c * e^2 * x^4 - 4 * a^2 * c^2 * e^2 * x^4) / (2 * a^2 * (-b^2 + 4 * a * c) * d * (c * d^2 - b * d * e + a * e^2) * x^2 * \text{Sqrt}[a + b * x^2 + c * x^4]) + (e^4 * \text{Sqrt}[-(c * d^2) + b * d * e - a * e^2] * \text{ArcTan}[(\text{Sqrt}[c] * d) / \text{Sqrt}[-(c * d^2) + b * d * e - a * e^2]] + (\text{Sqrt}[c] * e * x^2) / \text{Sqrt}[-(c * d^2) + b * d * e - a * e^2]) - (e * \text{Sqrt}[a + b * x^2 + c * x^4]) / \text{Sqrt}[-(c * d^2) + b * d * e - a * e^2]) / (d^2 * (c * d^2 - b * d * e + a * e^2)^2) + ((-3 * b * d - 2 * a * e) * \text{ArcTanh}[(\text{Sqrt}[c] * x^2 - \text{Sqrt}[a + b * x^2 + c * x^4]) / \text{Sqrt}[a]]) / (2 * a^{(5/2)} * d^2)$

fricas [B] time = 22.94, size = 6486, normalized size = 15.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] $[1/8 * (2 * ((a^3 * b^2 * c - 4 * a^4 * c^2) * e^4 * x^6 + (a^3 * b^3 - 4 * a^4 * b * c) * e^4 * x^4 + (a^4 * b^2 - 4 * a^5 * c) * e^4 * x^2) * \text{sqrt}(c * d^2 - b * d * e + a * e^2) * \log(-((8 * c^2 * d^2 - 8 * b * c * d * e + (b^2 + 4 * a * c) * e^2) * x^4 - 8 * a * b * d * e + 8 * a^2 * e^2 + (b^2 + 4 * a * c) * d^2 + 2 * (4 * b * c * d^2 + 4 * a * b * e^2 - (3 * b^2 + 4 * a * c) * d * e) * x^2 + 4 * \text{sqrt}(c * x^4 + b * x^2 + a) * \text{sqrt}(c * d^2 - b * d * e + a * e^2) * ((2 * c * d - b * e) * x^2 + b * d - 2 * a * e)) / (e^2 * x^4 + 2 * d * e * x^2 + d^2)) + ((3 * (b^3 * c^3 - 4 * a * b * c^4) * d^5 - 2 * (3 * b^4 * c^2 - 13 * a * b^2 * c^3 + 4 * a^2 * c^4) * d^4 * e + (3 * b^5 * c - 10 * a * b^3 * c^2 - 8 * a^2 * b * c^3) * d^3 * e^2 - 4 * (a * b^4 * c - 5 * a^2 * b^2 * c^2 + 4 * a^3 * c^3) * d^2 * e^3 - (a^2 * b^3 * c - 4 * a^3 * b * c^2) * d * e^4 + 2 * (a^3 * b^2 * c - 4 * a^4 * c^2) * e^5) * x^6 + (3 * (b^4 * c^2 - 4 * a * b^2 * c^3) * d^5 - 2 * (3 * b^5 * c - 13 * a * b^3 * c^2 + 4 * a^2 * b * c^3) * d^4 * e + (3 * b^6 - 10 * a * b^4 * c - 8 * a^2 * b^2 * c^2) * d^3 * e^2 - 4 * (a * b^5 - 5 * a^2 * b^3 * c + 4 * a^3 * b * c^2) * d^2 * e^3 - (a^2 * b^4 - 4 * a^3 * b^2 * c) * d * e^4 + 2 * (a^3 * b^3 - 4 * a^4 * b * c) * e^5) * x^4 + (3 * (a * b^3 * c^2 - 4 * a^2 * b * c^3) * d^5 - 2 * (3 * a * b^4 * c - 13 * a^2 * b^2 * c^2 + 4 * a^3 * c^3) * d^4 * e + (3 * a * b^5 - 10 * a^2 * b^3 * c - 8 * a^3 * b * c^2) * d^3 * e^2 - 4 * (a^2 * b^4 - 5 * a^3 * b^2 * c + 4 * a^4 * c^2) * d^2 * e^3 - (a^3 * b^3 - 4 * a^4 * b * c) * d * e^4 + 2 * (a^4 * b^2 - 4 * a^5 * c) * e^5) * x^2) * \text{sqrt}(a) * \log(-((b^2 + 4 * a * c) * x^4 + 8 * a * b * x^2 + 4 * \text{sqrt}(c * x^4 + b * x^2 + a) * (b * x^2 + 2 * a) * \text{sqrt}(a) + 8 * a^2) / x^4) - 4 * ((a^2 * b^2 * c^2 - 4 * a^3 * c^3) * d^5 - 2 * (a^2 * b^3 * c - 4 * a^3 * b * c^2) * d^4 * e + (a^2 * b^4 - 2 * a^3 * b^2 * c$

$$\begin{aligned}
& - 8a^4c^2)d^3e^2 - 2(a^3b^3 - 4a^4b^2c)d^2e^3 + (a^4b^2 - 4a^5c) \\
&)d^4e^4 + ((3a^2b^2c^3 - 8a^2c^4)d^5 - 6(a^2b^3c^2 - 3a^2b^2c^3)d^4e \\
& e + 3(a^2b^4c - 2a^2b^2c^2 - 4a^3c^3)d^3e^2 - 2(2a^2b^3c - 7a^3 \\
& b^2c^2)d^2e^3 + (a^3b^2c - 4a^4c^2)d^2e^4)x^4 + ((3a^2b^3c^2 - 10a^2 \\
& b^2c^3)d^5 - 2(3a^2b^4c - 11a^2b^2c^2 + 2a^3c^3)d^4e + (3a^2b^5 \\
& - 8a^2b^3c - 10a^3b^2c^2)d^3e^2 - 4(a^2b^4 - 4a^3b^2c + a^4c^2) \\
&)d^2e^3 + (a^3b^3 - 4a^4b^2c)d^2e^4)x^2) \sqrt{cx^4 + bx^2 + a} / (((\\
& a^3b^2c^3 - 4a^4c^4)d^6 - 2(a^3b^3c^2 - 4a^4b^2c^3)d^5e + (a^3b^4 \\
& c - 2a^4b^2c^2 - 8a^5c^3)d^4e^2 - 2(a^4b^3c - 4a^5b^2c^2)d^3 \\
& e^3 + (a^5b^2c - 4a^6c^2)d^2e^4)x^6 + ((a^3b^3c^2 - 4a^4b^2c^3) \\
&)d^6 - 2(a^3b^4c - 4a^4b^2c^2)d^5e + (a^3b^5 - 2a^4b^3c - 8a^5b \\
& b^2c^2)d^4e^2 - 2(a^4b^4 - 4a^5b^2c)d^3e^3 + (a^5b^3 - 4a^6b^2c) \\
&)d^2e^4)x^4 + ((a^4b^2c^2 - 4a^5c^3)d^6 - 2(a^4b^3c - 4a^5b^2c^2) \\
&)d^5e + (a^4b^4 - 2a^5b^2c - 8a^6c^2)d^4e^2 - 2(a^5b^3 - 4a^6b^2 \\
& c)d^3e^3 + (a^6b^2 - 4a^7c)d^2e^4)x^2), 1/8(4((a^3b^2c - 4a^4 \\
& c^2)e^4x^6 + (a^3b^3 - 4a^4b^2c)e^4x^4 + (a^4b^2 - 4a^5c)e^4x^2) \\
&) \sqrt{-cd^2 + bde - ae^2} \arctan(-1/2 \sqrt{cx^4 + bx^2 + a} \sqrt{-cd^2 \\
& + bde - ae^2}) * ((2cd - b^2e)x^2 + bd - 2ae) / ((c^2d^2 - b^2cde \\
& + a^2e^2)x^4 + acd^2 - abde + a^2e^2 + (bcd^2 - b^2de + abe^2) \\
&)x^2)) + ((3(b^3c^3 - 4ab^2c^4)d^5 - 2(3b^4c^2 - 13ab^2c^3 + 4a^2 \\
& c^4)d^4e + (3b^5c - 10ab^3c^2 - 8a^2b^2c^3)d^3e^2 - 4(ab^4c \\
& - 5a^2b^2c^2 + 4a^3c^3)d^2e^3 - (a^2b^3c - 4a^3b^2c^2)d^2e^4 + 2 \\
&) * (a^3b^2c - 4a^4c^2)e^5)x^6 + (3(b^4c^2 - 4ab^2c^3)d^5 - 2(3b^5 \\
& c - 13ab^3c^2 + 4a^2b^2c^3)d^4e + (3b^6 - 10ab^4c - 8a^2b^2c^2) \\
&)d^3e^2 - 4(ab^5 - 5a^2b^3c + 4a^3b^2c^2)d^2e^3 - (a^2b^4 - 4 \\
& a^3b^2c)d^2e^4 + 2(a^3b^3 - 4a^4b^2c)e^5)x^4 + (3(ab^3c^2 - 4a^2 \\
& b^2c^3)d^5 - 2(3ab^4c - 13a^2b^2c^2 + 4a^3c^3)d^4e + (3ab^5 - \\
& 10a^2b^3c - 8a^3b^2c^2)d^3e^2 - 4(a^2b^4 - 5a^3b^2c + 4a^4c^2) \\
&)d^2e^3 - (a^3b^3 - 4a^4b^2c)d^2e^4 + 2(a^4b^2 - 4a^5c)e^5)x^2) * \\
& \sqrt{a} \log(-((b^2 + 4ac)x^4 + 8abx^2 + 4 \sqrt{cx^4 + bx^2 + a})(bx^2 \\
& + 2a) \sqrt{a} + 8a^2)/x^4) - 4((a^2b^2c^2 - 4a^3c^3)d^5 - 2(a^2 \\
& b^3c - 4a^3b^2c^2)d^4e + (a^2b^4 - 2a^3b^2c - 8a^4c^2)d^3e^2 \\
& - 2(a^3b^3 - 4a^4b^2c)d^2e^3 + (a^4b^2 - 4a^5c)d^2e^4 + ((3a^2b^2c \\
& ^3 - 8a^2c^4)d^5 - 6(a^2b^3c^2 - 3a^2b^2c^3)d^4e + 3(a^2b^4c - 2a^2 \\
& b^2c^2 - 4a^3c^3)d^3e^2 - 2(2a^2b^3c - 7a^3b^2c^2)d^2e^3 + (a^3 \\
& b^2c - 4a^4c^2)d^2e^4)x^4 + ((3a^2b^3c^2 - 10a^2b^2c^3)d^5 - 2(3 \\
& a^2b^4c - 11a^2b^2c^2 + 2a^3c^3)d^4e + (3a^2b^5 - 8a^2b^3c - 10a^3 \\
& b^2c^2)d^3e^2 - 4(a^2b^4 - 4a^3b^2c + a^4c^2)d^2e^3 + (a^3b^3 \\
& - 4a^4b^2c)d^2e^4)x^2) \sqrt{cx^4 + bx^2 + a} / (((a^3b^2c^3 - 4a^4c^4) \\
&)d^6 - 2(a^3b^3c^2 - 4a^4b^2c^3)d^5e + (a^3b^4c - 2a^4b^2c^2 - \\
& 8a^5c^3)d^4e^2 - 2(a^4b^3c - 4a^5b^2c^2)d^3e^3 + (a^5b^2c - 4 \\
& a^6c^2)d^2e^4)x^6 + ((a^3b^3c^2 - 4a^4b^2c^3)d^6 - 2(a^3b^4c - \\
& 4a^4b^2c^2)d^5e + (a^3b^5 - 2a^4b^3c - 8a^5b^2c^2)d^4e^2 - 2(a^4 \\
& b^4 - 4a^5b^2c)d^3e^3 + (a^5b^3 - 4a^6b^2c)d^2e^4)x^4 + ((a^4b^2 \\
& c^2 - 4a^5c^3)d^6 - 2(a^4b^3c - 4a^5b^2c^2)d^5e + (a^4b^4 - 2
\end{aligned}$$

$$\begin{aligned}
& *a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), -1/4*((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - ((a^3*b^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 2*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/4*(2*((a^3*b^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^
\end{aligned}$$

$$\begin{aligned}
& 3c^2 - 4a^2bc^3)d^5 - 2(3a^2b^4c - 13a^2b^2c^2 + 4a^3c^3)d^4e \\
& + (3ab^5 - 10a^2b^3c - 8a^3b^2c^2)d^3e^2 - 4(a^2b^4 - 5a^3b^2c \\
& + 4a^4c^2)d^2e^3 - (a^3b^3 - 4a^4bc)d^2e^4 + 2(a^4b^2 - 4a^5c) \\
&)e^5)x^2)\sqrt{-a}\arctan(1/2\sqrt{cx^4 + bx^2 + a})(bx^2 + 2a)\sqrt{(-a)} \\
& / (a^2cx^4 + abx^2 + a^2) - 2((a^2b^2c^2 - 4a^3c^3)d^5 - 2(a^2b^3c \\
& - 4a^3b^2c^2)d^4e + (a^2b^4 - 2a^3b^2c - 8a^4c^2)d^3e^2 - \\
& 2(a^3b^3 - 4a^4bc)d^2e^3 + (a^4b^2 - 4a^5c)d^2e^4 + ((3a^2b^2c^3 \\
& - 8a^2c^4)d^5 - 6(ab^3c^2 - 3a^2b^2c^3)d^4e + 3(ab^4c - 2a^2b^2c^2 \\
& - 4a^3c^3)d^3e^2 - 2(2a^2b^3c - 7a^3b^2c^2)d^2e^3 + (a^3b^2c \\
& - 4a^4c^2)d^2e^4)x^4 + ((3a^2b^3c^2 - 10a^2b^2c^3)d^5 - 2(3a^2b^4c \\
& - 11a^2b^2c^2 + 2a^3c^3)d^4e + (3ab^5 - 8a^2b^3c - 10a^3b^2c^2)d^3e^2 \\
& - 4(a^2b^4 - 4a^3b^2c + a^4c^2)d^2e^3 + (a^3b^3 - 4a^4bc)d^2e^4)x^2) \\
&)\sqrt{cx^4 + bx^2 + a}) / (((a^3b^2c^3 - 4a^4c^4)d^6 - 2(a^3b^3c^2 - 4a^4bc^3)d^5e \\
& + (a^3b^4c - 2a^4b^2c^2 - 8a^5c^3)d^4e^2 - 2(a^4b^3c - 4a^5b^2c^2)d^3e^3 + (a^5b^2c \\
& - 4a^6c^2)d^2e^4)x^6 + ((a^3b^3c^2 - 4a^4bc^3)d^6 - 2(a^3b^4c - 4a^4b^2c^2)d^5e \\
& + (a^3b^5 - 2a^4b^3c - 8a^5b^2c^2)d^4e^2 - 2(a^4b^4 - 4a^5b^2c)d^3e^3 + (a^5b^3 \\
& - 4a^6bc)d^2e^4)x^4 + ((a^4b^2c^2 - 4a^5c^3)d^6 - 2(a^4b^3c - 4a^5b^2c^2)d^5e \\
& + (a^4b^4 - 2a^5b^2c - 8a^6c^2)d^4e^2 - 2(a^5b^3 - 4a^6bc)d^3e^3 + (a^6b^2 \\
& - 4a^7c)d^2e^4)x^2)]
\end{aligned}$$

giac [B] time = 2.74, size = 762, normalized size = 1.82

$$\frac{\arctan\left(\frac{\sqrt{c^2 - \sqrt{c^4 + bx^2 + a}} + \sqrt{c^2}}{\sqrt{-a}\sqrt{bx^2 + a}}\right)x^4}{\sqrt{c^2 + bx^2 + a}} + \frac{(3bd + 2a)\arctan\left(\frac{\sqrt{c^2 - \sqrt{c^4 + bx^2 + a}}}{\sqrt{-a}}\right)}{2\sqrt{-a}d^2} + \frac{(\sqrt{c^2 - \sqrt{c^4 + bx^2 + a}})b + 2a\sqrt{c}}{2\left(\sqrt{c^2 - \sqrt{c^4 + bx^2 + a}}\right)^2 - a}x^2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -((a^2b^2c^3d^3 - 2a^3c^4d^3 - 2a^2b^3c^2d^2e + 5a^3b^2c^3d^2e \\
& + a^2b^4c^2d^2e^2 - 2a^3b^2c^2d^2e^2 - 2a^4c^3d^2e^2 - a^3b^3c^2e^3 \\
& + 3a^4b^2c^2e^3)x^2/(a^4b^2c^2d^4 - 4a^5c^3d^4 - 2a^4b^3c^2d^3e \\
& + 8a^5b^2c^2d^3e + a^4b^4d^2e^2 - 2a^5b^2c^2d^2e^2 - 8a^6c^2d^2e^2 \\
& - 2a^5b^3d^2e^3 + 8a^6b^2c^2d^2e^3 + a^6b^2e^4 - 4a^7c^2e^4) + (\\
& a^2b^3c^2d^3 - 3a^3b^2c^3d^3 - 2a^2b^4c^2d^2e + 7a^3b^2c^2d^2e \\
& - 2a^4c^3d^2e + a^2b^5d^2e^2 - 3a^3b^3c^2d^2e^2 - a^4b^2c^2d^2e^2 - \\
& a^3b^4e^3 + 4a^4b^2c^2e^3 - 2a^5c^2e^3)/(a^4b^2c^2d^4 - 4a^5c^3 \\
& d^4 - 2a^4b^3c^2d^3e + 8a^5b^2c^2d^3e + a^4b^4d^2e^2 - 2a^5b^2c^2 \\
& d^2e^2 - 8a^6c^2d^2e^2 - 2a^5b^3d^2e^3 + 8a^6b^2c^2d^2e^3 + a^6b^2 \\
& e^4 - 4a^7c^2e^4))/\sqrt{cx^4 + bx^2 + a} + \arctan(-(\sqrt{c}x^2 - \sqrt{c} \\
& (cx^4 + bx^2 + a))e + \sqrt{c}d)/\sqrt{-cd^2 + bde - ae^2})e^4/((cd \\
& ^4 - bd^3e + ad^2e^2)\sqrt{-cd^2 + bde - ae^2}) - 1/2(3bd + 2a \\
& e)\arctan(-(\sqrt{c}x^2 - \sqrt{c}x^4 + bx^2 + a))/\sqrt{-a})/(\sqrt{-a}a^2 \\
& d^2) + 1/2((\sqrt{c}x^2 - \sqrt{c}x^4 + bx^2 + a))b + 2a\sqrt{c})/(((\sqrt{c} \\
& x^2 - \sqrt{c}x^4 + bx^2 + a))^2 - a)a^2d)
\end{aligned}$$

maple [B] time = 0.03, size = 863, normalized size = 2.06

$$\frac{\frac{1}{(cx^4+bx^2+a)^{3/2}} \int \frac{1}{x^3(e^x+dx)} dx}{\frac{1}{(cx^4+bx^2+a)^{3/2}} \int \frac{1}{x^3(e^x+dx)} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2), x)

[Out]
$$-1/2/d^2*e/a/(c*x^4+b*x^2+a)^{(1/2)}+1/d^2*e*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*c*x^2+1/2/d^2*e*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}+1/2/d^2*e/a^{(3/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2)+2*e^2/d^2*c/(b*e-2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(-4*a*c+b^2)/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^{(1/2)}/c)*((x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c)^2*c-(-4*a*c+b^2)^{(1/2)}*(x^2+1/2*(b+(-4*a*c+b^2)^{(1/2)})/c))^{(1/2)}+2*e^3/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(b*e-2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(((b*e-2*c*d)*(x^2+d/e)/e+2*(a*e^2-b*d*e+c*d^2)/e^2+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x^2+d/e)^2*c+(b*e-2*c*d)*(x^2+d/e)/e+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x^2+d/e))-2*e^2/d^2*c/(-b*e+2*c*d+(-4*a*c+b^2)^{(1/2)}*e)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^{(1/2)}/c)*((x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c)^2*c+(-4*a*c+b^2)^{(1/2)}*(x^2-1/2*(-b+(-4*a*c+b^2)^{(1/2)})/c))^{(1/2)}-1/2/d/a/x^2/(c*x^4+b*x^2+a)^{(1/2)}-3/4/d*b/a^2/(c*x^4+b*x^2+a)^{(1/2)}+3/2/d*b^2/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*c*x^2+3/4/d*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}+3/4/d*b/a^{(5/2)}*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2)-4/d*c^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*x^2-2/d*c/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^{(1/2)}*b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (e x^2 + d) (c x^4 + b x^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)

[Out] `int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2), x)`

[Out] `Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)`

$$3.254 \quad \int \frac{x^7 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=406

$$\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} + 2abce - ac^2d + b^3(-e) + b^2cd \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) \left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} \right)}{\sqrt{2} c^{7/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}$$

Rubi [A] time = 8.59, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} + 2abce - ac^2d + b^2cd + b^3(-e) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) \left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} + 2abce - ac^2d + b^2cd + b^3(-e) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{\sqrt{2} c^{7/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} \sqrt{2} c^{7/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} + \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(d+ex^2)^{3/2}(be+cd)}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5c^2e^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^7*sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] ((b^2 - a*c)*sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/sqrt[b^2 - 4*a*c])*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(7/2)*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/sqrt[b^2 - 4*a*c])*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(7/2)*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra

ctionQ[m]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
 Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
 gerQ[(m - 1)/2]

Rule 1287

Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
 *a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^7 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^3 \sqrt{d+ex}}{a+bx+cx^2} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^2 \left(-\frac{d}{e} + \frac{x^2}{e} \right)^3}{\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(\frac{(b^2-ac)e}{c^3} - \frac{(cd+be)x^2}{c^2e} + \frac{x^4}{ce} - \frac{(b^2-ac)(cd^2-bde+ae^2) - (b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2} \right)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2} - \frac{\text{Subst} \left(\int \frac{(b^2-ac)(cd^2-bde+ae^2) + (b^2cd-ac^2d-b^3e+2abce)x^2}{\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2} + \frac{(b^2cd-ac^2d-b^3e+2abce - (b^2-ac)(cd^2-bde+ae^2))\sqrt{d+ex^2}}{e^3} \\
&= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3} - \frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2} + \frac{(d+ex^2)^{5/2}}{5ce^2} - \frac{(b^2cd-ac^2d-b^3e+2abce - (b^2-ac)(cd^2-bde+ae^2))\sqrt{d+ex^2}}{e^3}
\end{aligned}$$

Mathematica [B] time = 10.84, size = 943, normalized size = 2.32

Warning: Unable to verify antiderivative.

[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] ((c*(d + e*x^2)^(9/2)*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]*(105*b^3*e^3 - 35*b^2*e^2*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^2)) + 7*b*c*e*(-45*a*e^2 + (d + e*x^2)*(-5*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 3*c*(d + e*x^2))) + c*(35*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e + 2*c*(d + e*x^2)) + c*(d + e*x^2)*(7*Sqrt[b^2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(-70*d^2 + 8*4*d*(d + e*x^2) - 30*(d + e*x^2)^2))) + 105*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c]*e)

$$\begin{aligned} & - 4*a*c] - a*c*\text{Sqrt}[b^2 - 4*a*c)]*e^3*\text{ArcTanh}[\text{Sqrt}[2]*\text{Sqrt}[(c*(d + e*x^2)) \\ & /((2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e))]])/(210*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*e^4 \\ & *(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e)^3*((-\text{Sqrt}[b^2 - 4*a*c]/e) - (2*c*d - \\ & b*e)/e^2)*((c*(d + e*x^2))/(2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e))^(9/2)) + (2 \\ & *c*d^3*(d + e*x^2)^(3/2)*(((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)*(-105*b^3*e^ \\ & 3 + 35*b^2*e^2*(-3*\text{Sqrt}[b^2 - 4*a*c])*e + c*(d + e*x^2)) - 7*b*c*e*(-45*a*e^ \\ & 2 + (d + e*x^2)*(-5*c*d - 5*\text{Sqrt}[b^2 - 4*a*c]*e + 3*c*(d + e*x^2))) + c*(35 \\ & *a*e^2*(3*\text{Sqrt}[b^2 - 4*a*c]*e - 2*c*(d + e*x^2)) + c*(d + e*x^2)*(7*\text{Sqrt}[b^ \\ & 2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(70*d^2 - 84*d*(d + e*x^2) + 30*(d + \\ & e*x^2)^2)))))/(140*c^4*d^3*(d + e*x^2)) + (3*(b^3 - 3*a*b*c + b^2*\text{Sqrt}[b^2 \\ & - 4*a*c] - a*c*\text{Sqrt}[b^2 - 4*a*c])*e^3*(d + e*x^2)^3*\text{ArcTanh}[\text{Sqrt}[2]*\text{Sqrt}[(\\ & c*(d + e*x^2))/(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e))]]/(4*\text{Sqrt}[2]*d^3*(2*c*d \\ & - (b + \text{Sqrt}[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + \text{Sqrt}[b^2 - 4 \\ & *a*c])*e))^(9/2)))/(3*\text{Sqrt}[b^2 - 4*a*c]*e^4*(\text{Sqrt}[b^2 - 4*a*c]/e - (2*c*d \\ & - b*e)/e^2)))/e \end{aligned}$$

IntegrateAlgebraic [A] time = 1.65, size = 502, normalized size = 1.24

$$\frac{(-2a^2c^2 + a^2d\sqrt{b^2 - 4ac} - b^2cd\sqrt{b^2 - 4ac} + 4ab^2ce - 2abce\sqrt{b^2 - 4ac} + b^3e\sqrt{b^2 - 4ac} - 3ab^2d + b^3(-c) + b^3cd)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex^2}}{\sqrt{c^2d-4ace-2a^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{c^2d-4ace-2a^2}} + \frac{(2b^2c^2e + a^2d\sqrt{b^2-4ac} - b^2cd\sqrt{b^2-4ac} - 4ab^2ce - 2abce\sqrt{b^2-4ac} + b^3e\sqrt{b^2-4ac} + 3ab^2d + b^3(-c) + b^3cd)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex^2}}{\sqrt{c^2d-4ace-2a^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{c^2d-4ace-2a^2}} + \frac{\sqrt{d+ex^2}(-15ac^2 + 15b^2d^2 - 5b^2cd - 5b^2c^2d^2 - 2c^2d^2 + c^2d^2d^2)}{15c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]

[Out] (Sqrt[d + e*x^2]*(-2*c^2*d^2 - 5*b*c*d*e + 15*b^2*e^2 - 15*a*c*e^2 + c^2*d*e*x^2 - 5*b*c*e^2*x^2 + 3*c^2*e^2*x^4))/(15*c^3*e^2) - ((b^3*c*d - 3*a*b*c^2*d - b^2*c*Sqrt[b^2 - 4*a*c]*d + a*c^2*Sqrt[b^2 - 4*a*c]*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e + b^3*Sqrt[b^2 - 4*a*c]*e - 2*a*b*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]]/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]) - (((-b^3*c*d) + 3*a*b*c^2*d - b^2*c*Sqrt[b^2 - 4*a*c]*d + a*c^2*Sqrt[b^2 - 4*a*c]*d + b^4*e - 4*a*b^2*c*e + 2*a^2*c^2*e + b^3*Sqrt[b^2 - 4*a*c]*e - 2*a*b*c*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]]/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.65, size = 928, normalized size = 2.29

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-\left(\left(b^4c - 5ab^2c^2 + 4a^2c^3\right)de - \left(b^5 - 6ab^3c + 8a^2b^2c^2\right)e^2\right)c^2 + 2\left(b^3c^4 - 3ab^2c^5\right)d^2 - \left(3b^4c^3 - 11ab^2c^4 + 4a^2c^5\right)d^2e - 2\left(\left(b^2c^3 - ac^4\right)\sqrt{b^2 - 4ac}d^2 - \left(b^3c^2 - abc^3\right)\sqrt{b^2 - 4ac}de + \left(ab^2c^2 - a^2c^3\right)\sqrt{b^2 - 4ac}e^2\right)\text{abs}(c) + \left(b^5c^2 - 4ab^3c^3 + 2a^2b^2c^4\right)e^2\text{arctan}\left(2\sqrt{\frac{1}{2}}\sqrt{x^2e + d}\right)/\sqrt{-\left(2c^6de^{12} - bc^5e^{13} + \sqrt{-4\left(c^6d^2e^{12} - bc^5de^{13} + ac^5e^{14}\right)}c^6e^{12} + \left(2c^6de^{12} - bc^5e^{13}\right)^2\right)e^{-12}/c^6} \Big/ \left(\left(2\sqrt{b^2 - 4ac}\right)c^4d + \left(b^2c^3 - 4ac^4 - \sqrt{b^2 - 4ac}\right)bc^3\right)e\sqrt{-4c^2d + 2(bc + \sqrt{b^2 - 4ac})c}e^2 + \left(\left(b^4c - 5ab^2c^2 + 4a^2c^3\right)de - \left(b^5 - 6ab^3c + 8a^2b^2c^2\right)e^2\right)c^2 + 2\left(b^3c^4 - 3ab^2c^5\right)d^2 - \left(3b^4c^3 - 11ab^2c^4 + 4a^2c^5\right)d^2e + 2\left(\left(b^2c^3 - ac^4\right)\sqrt{b^2 - 4ac}d^2 - \left(b^3c^2 - abc^3\right)\sqrt{b^2 - 4ac}de + \left(ab^2c^2 - a^2c^3\right)\sqrt{b^2 - 4ac}e^2\right)\text{abs}(c) + \left(b^5c^2 - 4ab^3c^3 + 2a^2b^2c^4\right)e^2\text{arctan}\left(2\sqrt{\frac{1}{2}}\sqrt{x^2e + d}\right)/\sqrt{-\left(2c^6de^{12} - bc^5e^{13} - \sqrt{-4\left(c^6d^2e^{12} - bc^5de^{13} + ac^5e^{14}\right)}c^6e^{12} + \left(2c^6de^{12} - bc^5e^{13}\right)^2\right)e^{-12}/c^6} \Big/ \left(\left(2\sqrt{b^2 - 4ac}\right)c^4d - \left(b^2c^3 - 4ac^4 + \sqrt{b^2 - 4ac}\right)bc^3\right)e\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac})c}e^2 + 1/15\left(3\left(x^2e + d\right)^{5/2}c^4e^8 - 5\left(x^2e + d\right)^{3/2}c^4de^8 - 5\left(x^2e + d\right)^{3/2}bc^3e^9 + 15\sqrt{x^2e + d}b^2c^2e^{10} - 15\sqrt{x^2e + d}ac^3e^{10}\right)e^{-10}/c^5$$

maple [C] time = 0.06, size = 496, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)

[Out]
$$\frac{1}{5}c^2x^2(e^2x^2+d)^{3/2}/e - \frac{2}{15}c^2d/e^2(e^2x^2+d)^{3/2} - \frac{1}{3}c^2b(e^2x^2+d)^{3/2}/e + \frac{1}{2}c^2e^{1/2}x^2a - \frac{1}{2}c^3e^{1/2}x^2b^2 - \frac{1}{2}c^2(e^2x^2+d)^{1/2}a + \frac{1}{2}c^3(e^2x^2+d)^{1/2}b^2 - \frac{1}{4}c^3\text{sum}\left(\left(-2abc^2e + ac^2d + b^3e - b^2cd\right)_R^6 + \left(-4a^2c^2e^2 + 4ab^2e^2 + 2abc^2d - 3ac^2d^2 - 3b^3d^2e + 3b^2cd^2\right)_R^4 + d\left(4a^2c^2e^2 - 4ab^2e^2 - 2abc^2d + 3ac^2d^2 + 3b^3d^2e - 3b^2cd^2\right)_R^2 + 2abc^2d^3e - ac^2d^4 - b^3d^3e + b^2cd^4\right)/\left(\sqrt{c^2x^2 + b^2}^7 + 3\sqrt{c^2x^2 + b^2}^5 + b^2e - 3\sqrt{c^2x^2 + b^2}^3 + cd + 8\sqrt{c^2x^2 + b^2}^3 + a^2e^2 - 4\sqrt{c^2x^2 + b^2}^3 + b^2d^2 + 3\sqrt{c^2x^2 + b^2}^3 + cd^2 + \sqrt{c^2x^2 + b^2}^3 + cd^2 + 4\sqrt{c^2x^2 + b^2}^3 + cd^2 + 4\sqrt{c^2x^2 + b^2}^3 + cd^2\right) - \frac{1}{2}c^2d/\left(\left(e^2x^2+d\right)^{1/2} - e^{1/2}x\right)a + \frac{1}{2}c^3d/\left(\left(e^2x^2+d\right)^{1/2} - e^{1/2}x\right)b^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} x^7}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 2.48, size = 11195, normalized size = 27.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] $(d + e*x^2)^{1/2} * ((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^4) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4)) * (b*e^3 - 2*c*d*e^2))/(c*e^2)) - (d + e*x^2)^{3/2} * (d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)) + \operatorname{atan}(\frac{(16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^{1/2} * (-b^9*e - 8*a^4*c^5*d - b^6*e * (-4*a*c - b^2)^3)^{1/2} - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e * (-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d * (-4*a*c - b^2)^3)^{1/2} + 5*a*b^4*c*e * (-4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c^2*d * (-4*a*c - b^2)^3)^{1/2} + 3*a^2*b*c^3*d * (-4*a*c - b^2)^3)^{1/2} - 6*a^2*b^2*c^2*e * (-4*a*c - b^2)^3)^{1/2}}{(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{1/2} * (4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2)/c^5 * (-b^9*e - 8*a^4*c^5*d - b^6*e * (-4*a*c - b^2)^3)^{1/2} - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e * (-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d * (-4*a*c - b^2)^3)^{1/2} + 5*a*b^4*c*e * (-4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c^2*d * (-4*a*c - b^2)^3)^{1/2} + 3*a^2*b*c^3*d * (-4*a*c - b^2)^3)^{1/2} - 6*a^2*b^2*c^2*e * (-4*a*c - b^2)^3)^{1/2}}{(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{1/2} - (2*(d + e*x^2)^{1/2} * (b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5 * (-b^9*e - 8*a^4*c^5*d - b^6*e * (-4*a*c - b^2)^3)^{1/2} - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e * (-4*a*c - b^2)^3)^{1/2} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d * (-4*a*c - b^2)^3)^{1/2} + 5*a*b^4*c*e * (-4*a*c - b^2)^3)^{1/2} - 4*a*b^3*c^2*d * (-4*a*c - b^2)^3)^{1/2} + 3*a^2*b*c^3*d * (-4*a*c - b^2)^3)^{1/2}}$

$$\begin{aligned}
& 3*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8 \\
& *(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)}*1i - (((16*a^3*c^6*e^4 + 4*a* \\
& b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4 \\
& *b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d \\
& ^2*e^2)/c^5 + (2*(d + e*x^2)^{(1/2)}*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^ \\
& 5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7* \\
& c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)}))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)}*(4*b^3*c^7*e^3 \\
& - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2)/c^5)*(-(b^9*e - 8*a^ \\
& 4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c*d - 33*a^2*b^4*c^3*d + 38* \\
& a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b^3*c^2 \\
& *d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^ \\
& 2*b^2*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^ \\
& 8))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2 \\
& *e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c \\
& *e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3* \\
& b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - \\
& 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c*d - 33*a^2*b^4*c^3*d + \\
& 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b^3 \\
& *c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^ \\
& 2*c^8))^{(1/2)}*1i)/(((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - \\
& 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5 \\
& *d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^{(1 \\
& /2)}*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^8*c*d - 33* \\
& a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^ \\
& 3*c^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b \\
& *c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^ \\
& (1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^3*d*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(16*a^2*c^9 + b \\
& ^4*c^7 - 8*a*b^2*c^8))^{(1/2)}*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8 \\
& *e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2* \\
& e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^7*c*e + \\
& 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 5*a*b^ \\
& 4*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3 \\
& *a^2*b*c^3*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^ \\
& (1/2)))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2) * (b^8 e^4 + 2 a^4 c^4 e^4 + 20 a^2 b^4 c^2 e^4 - 16 a^3 b^2 c^3 e^4 - 2 a^3 c^5 d^2 e^2 + b^6 c^2 d^2 e^2 - 8 a b^6 c e^4 - 2 b^7 c d e^3 + 9 a^2 b^2 c^4 d^2 e^2 + 14 a b^5 c^2 d e^3 + 14 a^3 b c^4 d e^3 - 6 a b^4 c^3 d^2 e^2 - 28 a^2 b^3 c^3 d e^3) / c^5 * (- (b^9 e - 8 a^4 c^5 d - b^6 e * (- (4 a c - b^2)^3)^{1/2} - b^8 c d - 33 a^2 b^4 c^3 d + 38 a^3 b^2 c^4 d + 42 a^2 b^5 c^2 e - 63 a^3 b^3 c^3 e + a^3 c^3 e * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c e + 10 a b^6 c^2 d + 28 a^4 b c^4 e + b^5 c d * (- (4 a c - b^2)^3)^{1/2} + 5 a b^4 c e * (- (4 a c - b^2)^3)^{1/2} - 4 a b^3 c^2 d * (- (4 a c - b^2)^3)^{1/2} + 3 a^2 b c^3 d * (- (4 a c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a b^2 c^8))^{1/2} + (((16 a^3 c^6 e^4 + 4 a b^4 c^4 e^4 - 4 b^5 c^4 d e^3 - 20 a^2 b^2 c^5 e^4 + 16 a^2 c^7 d^2 e^2 + 4 b^4 c^5 d^2 e^2 + 20 a b^3 c^5 d e^3 - 16 a^2 b c^6 d e^3 - 20 a b^2 c^6 d^2 e^2) / c^5 + (2 * (d + e x^2)^{1/2} * (- (b^9 e - 8 a^4 c^5 d - b^6 e * (- (4 a c - b^2)^3)^{1/2} - b^8 c d - 33 a^2 b^4 c^3 d + 38 a^3 b^2 c^4 d + 42 a^2 b^5 c^2 e - 63 a^3 b^3 c^3 e + a^3 c^3 e * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c e + 10 a b^6 c^2 d + 28 a^4 b c^4 e + b^5 c d * (- (4 a c - b^2)^3)^{1/2} + 5 a b^4 c e * (- (4 a c - b^2)^3)^{1/2} - 4 a b^3 c^2 d * (- (4 a c - b^2)^3)^{1/2} + 3 a^2 b c^3 d * (- (4 a c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a b^2 c^8))^{1/2} * (4 b^3 c^7 e^3 - 8 b^2 c^8 d e^2 - 16 a b c^8 e^3 + 32 a c^9 d e^2) / c^5 * (- (b^9 e - 8 a^4 c^5 d - b^6 e * (- (4 a c - b^2)^3)^{1/2} - b^8 c d - 33 a^2 b^4 c^3 d + 38 a^3 b^2 c^4 d + 42 a^2 b^5 c^2 e - 63 a^3 b^3 c^3 e + a^3 c^3 e * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c e + 10 a b^6 c^2 d + 28 a^4 b c^4 e + b^5 c d * (- (4 a c - b^2)^3)^{1/2} + 5 a b^4 c e * (- (4 a c - b^2)^3)^{1/2} - 4 a b^3 c^2 d * (- (4 a c - b^2)^3)^{1/2} + 3 a^2 b c^3 d * (- (4 a c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a b^2 c^8))^{1/2} + (2 * (d + e x^2)^{1/2} * (b^8 e^4 + 2 a^4 c^4 e^4 + 20 a^2 b^4 c^2 e^4 - 16 a^3 b^2 c^3 e^4 - 2 a^3 c^5 d^2 e^2 + b^6 c^2 d^2 e^2 - 8 a b^6 c e^4 - 2 b^7 c d e^3 + 9 a^2 b^2 c^4 d^2 e^2 + 14 a b^5 c^2 d e^3 + 14 a^3 b c^4 d e^3 - 6 a b^4 c^3 d^2 e^2 - 28 a^2 b^3 c^3 d e^3) / c^5 * (- (b^9 e - 8 a^4 c^5 d - b^6 e * (- (4 a c - b^2)^3)^{1/2} - b^8 c d - 33 a^2 b^4 c^3 d + 38 a^3 b^2 c^4 d + 42 a^2 b^5 c^2 e - 63 a^3 b^3 c^3 e + a^3 c^3 e * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c e + 10 a b^6 c^2 d + 28 a^4 b c^4 e + b^5 c d * (- (4 a c - b^2)^3)^{1/2} + 5 a b^4 c e * (- (4 a c - b^2)^3)^{1/2} - 4 a b^3 c^2 d * (- (4 a c - b^2)^3)^{1/2} + 3 a^2 b c^3 d * (- (4 a c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a b^2 c^8))^{1/2} - (2 * (a^4 b^3 e^5 - a^3 b^4 d e^4 + a^5 c^2 d e^4 + a^4 c^3 d^3 e^2 - 2 a^5 b c e^5 - a^3 b^2 c^2 d^3 e^2 + a^4 b^2 c d e^4 + 2 a^3 b^3 c d^2 e^3 - 3 a^4 b c^2 d^2 e^3) / c^5) * (- (b^9 e - 8 a^4 c^5 d - b^6 e * (- (4 a c - b^2)^3)^{1/2} - b^8 c d - 33 a^2 b^4 c^3 d + 38 a^3 b^2 c^4 d + 42 a^2 b^5 c^2 e - 63 a^3 b^3 c^3 e + a^3 c^3 e * (- (4 a c - b^2)^3)^{1/2} - 11 a b^7 c e + 10 a b^6 c^2 d + 28 a^4 b c^4 e + b^5 c d * (- (4 a c - b^2)^3)^{1/2} + 5 a b^4 c e * (- (4 a c - b^2)^3)^{1/2} - 4 a b^3 c^2 d * (- (4 a c - b^2)^3)^{1/2} + 3 a^2 b c^3 d * (- (4 a c - b^2)^3)^{1/2} - 6 a^2 b^2 c^2 e * (- (4 a c - b^2)^3)^{1/2}) / (8 * (16 a^2 c^9 + b^4 c^7 - 8 a b^2 c^8))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& ^5d - b^9e - b^6e*(-(4ac - b^2)^3)^{(1/2)} + b^8cd + 33a^2b^4c^3d \\
& - 38a^3b^2c^4d - 42a^2b^5c^2e + 63a^3b^3c^3e + a^3c^3e*(-(4a \\
& *c - b^2)^3)^{(1/2)} + 11ab^7c^2e - 10ab^6c^2d - 28a^4b^4c^4e + b^5c \\
& *d*(-(4ac - b^2)^3)^{(1/2)} + 5ab^4c^2e*(-(4ac - b^2)^3)^{(1/2)} - 4ab^ \\
& 3c^2d*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^3d*(-(4ac - b^2)^3)^{(1/2)} - \\
& 6a^2b^2c^2e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^9 + b^4c^7 - 8ab \\
& ^2c^8))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^8e^4 + 2a^4c^4e^4 + 20a^2b^ \\
& 4c^2e^4 - 16a^3b^2c^3e^4 - 2a^3c^5d^2e^2 + b^6c^2d^2e^2 - 8ab \\
& b^6c^2e^4 - 2b^7c^2d^2e^3 + 9a^2b^2c^4d^2e^2 + 14ab^5c^2d^2e^3 + 14 \\
& a^3b^4c^4d^2e^3 - 6ab^4c^3d^2e^2 - 28a^2b^3c^3d^2e^3))/c^5)*((8a^ \\
& 4c^5d - b^9e - b^6e*(-(4ac - b^2)^3)^{(1/2)} + b^8cd + 33a^2b^4c^3 \\
& *d - 38a^3b^2c^4d - 42a^2b^5c^2e + 63a^3b^3c^3e + a^3c^3e*(-(4 \\
& ac - b^2)^3)^{(1/2)} + 11ab^7c^2e - 10ab^6c^2d - 28a^4b^4c^4e + b^ \\
& 5c^2d*(-(4ac - b^2)^3)^{(1/2)} + 5ab^4c^2e*(-(4ac - b^2)^3)^{(1/2)} - 4a \\
& *b^3c^2d*(-(4ac - b^2)^3)^{(1/2)} + 3a^2b^3c^3d*(-(4ac - b^2)^3)^{(1/2)} \\
&) - 6a^2b^2c^2e*(-(4ac - b^2)^3)^{(1/2)})/(8*(16a^2c^9 + b^4c^7 - 8 \\
& ab^2c^8))^{(1/2)} - (2*(a^4b^3e^5 - a^3b^4d^2e^4 + a^5c^2d^2e^4 + a^4c \\
& ^3d^3e^2 - 2a^5b^2c^2d^3e^2 + a^4b^2c^2d^2e^4 + 2a^3 \\
& *b^3c^2d^2e^3 - 3a^4b^2c^2d^2e^3))/c^5)*((8a^4c^5d - b^9e - b^6e \\
& *(- (4ac - b^2)^3)^{(1/2)} + b^8cd + 33a^2b^4c^3d - 38a^3b^2c^4d - \\
& 42a^2b^5c^2e + 63a^3b^3c^3e + a^3c^3e*(-(4ac - b^2)^3)^{(1/2)} + \\
& 11ab^7c^2e - 10ab^6c^2d - 28a^4b^4c^4e + b^5c^2d*(-(4ac - b^2)^3) \\
& ^{(1/2)} + 5ab^4c^2e*(-(4ac - b^2)^3)^{(1/2)} - 4ab^3c^2d*(-(4ac - b^ \\
& 2)^3)^{(1/2)} + 3a^2b^3c^3d*(-(4ac - b^2)^3)^{(1/2)} - 6a^2b^2c^2e*(-(4 \\
& ac - b^2)^3)^{(1/2)})/(8*(16a^2c^9 + b^4c^7 - 8ab^2c^8))^{(1/2)}*2i + \\
& (d + e*x^2)^{(5/2)}/(5c^2e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.255 \quad \int \frac{x^5 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=324

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e)\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e}}$$

Rubi [A] time = 3.53, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e(b+\sqrt{b^2-4ac})}} - \frac{b\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

[In] Int[(x^5*sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] -((b*sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/sqrt[b^2 - 4*a*c])*ArcTan h[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(5/2)*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/sqrt[b^2 - 4*a*c])*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(5/2)*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :=> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :=> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 \sqrt{d+ex}}{a+bx+cx^2} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^2 \left(-\frac{d}{e} + \frac{x^2}{e} \right)^2}{\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(-\frac{be}{c^2} + \frac{x^2}{c} + \frac{b(cd^2-bde+ae^2) - (bcd-b^2e+ace)x^2}{c^2 e \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2} \right)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3ce} + \frac{\text{Subst} \left(\int \frac{b(cd^2-bde+ae^2) + (-bcd+b^2e-ace)x^2}{\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}} dx, x, \sqrt{d+ex^2} \right)}{c^2 e^2} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3ce} - \frac{\left(bcd - b^2e + ace - \frac{b^2cd - 2ac^2d - b^3e + 3abce}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \left(\int \frac{1}{-\frac{\sqrt{b^2 - 4ac}}{2e} - \frac{x}{2c}} \right)}{2c^2 e^2} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3ce} + \frac{\left(bcd - b^2e + ace - \frac{b^2cd - 2ac^2d - b^3e + 3abce}{\sqrt{b^2 - 4ac}} \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} \right)}{\sqrt{2} c^{5/2} \sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}}
\end{aligned}$$

Mathematica [A] time = 7.25, size = 591, normalized size = 1.82

$$\frac{(d+ex^2)^{7/2} \left(\frac{c^2 \sqrt{2} \sqrt{\frac{d+ex^2}{\sqrt{b^2-4ac}}}}{\sqrt{2cd-(b-\sqrt{b^2-4ac})e}} \left(5b(3c\sqrt{b^2-4ac}+c(d+ex^2)) + (-5c\sqrt{b^2-4ac}+4cd-6cx^2)+30bc^2-15b^2d \right) - 15c^2 \left(b\sqrt{b^2-4ac}+2ac \right) \tanh^{-1} \left(\sqrt{\frac{d+ex^2}{c\sqrt{b^2-4ac}}} \right) \right) - c^2 \sqrt{2} \sqrt{\frac{d+ex^2}{\sqrt{b^2-4ac}}} \left(-5b(3c\sqrt{b^2-4ac}-(d+ex^2)) + (4cd-6cx^2)+30bc^2-15b^2d \right) + 15c^2 \left(b\sqrt{b^2-4ac}-2ac \right) \tanh^{-1} \left(\sqrt{\frac{d+ex^2}{2cd-(b-\sqrt{b^2-4ac})e}} \right) \right)}{\left((b-\sqrt{b^2-4ac})-2cd \right) \left(\sqrt{b^2-4ac} \right) \left(\frac{d+ex^2}{\sqrt{b^2-4ac}} \right)^{3/2}} \frac{30\sqrt{2}c^4\sqrt{b^2-4ac}}{\left(\sqrt{b^2-4ac} \right) \left(\sqrt{2cd-(b-\sqrt{b^2-4ac})e} \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] (c*(d + e*x^2)^(7/2)*((e^2*(sqrt[2]*sqrt[(c*(d + e*x^2))/(2*c*d + (-b + sqrt[b^2 - 4*a*c])*e]))*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d + e*x^2)*(4*c*d - 5*sqrt[b^2 - 4*a*c]*e - 6*c*e*x^2) + 5*b*e*(3*sqrt[b^2 - 4*a*c]*e + c*(d + e*x^2))) - 15*(-b^2 + 2*a*c + b*sqrt[b^2 - 4*a*c])*e^2*ArcTanh[sqrt[2]*sqrt[(c*(d + e*x^2))/(2*c*d - b*e + sqrt[b^2 - 4*a*c]*e)]])/((-2*c*d + (b - sqrt[b^2 - 4*a*c])*e)*(2*c*d + (-b + sqrt[b^2 - 4*a*c])*e)^2*((c*(d + e*x^2))/(2*

$$\begin{aligned} &^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c^4)*d*e - 2*(\text{sqrt}(b^2 - 4*a*c)*b*c^3*d^2 - \text{sqrt}(b^2 - 4*a*c)*b^2*c^2*d*e + \text{sqrt}(b^2 - 4*a*c)*a*b*c^2*e^2)*\text{abs}(c) \\ &+ (b^4*c^2 - 3*a*b^2*c^3)*e^2)*\text{arctan}(2*\text{sqrt}(1/2)*\text{sqrt}(x^2*e + d)/\text{sqrt}(- \\ &(2*c^4*d*e^4 - b*c^3*e^5 + \text{sqrt}(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)* \\ &c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^{(-4)/c^4})/((2*\text{sqrt}(b^2 - 4*a*c)* \\ &c^3*d + (b^2*c^2 - 4*a*c^3 - \text{sqrt}(b^2 - 4*a*c)*b*c^2)*e)*\text{sqrt}(-4*c^2*d + 2* \\ &(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*e)*c^2) - (((b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5* \\ &a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8* \\ &a*b*c^4)*d*e + 2*(\text{sqrt}(b^2 - 4*a*c)*b*c^3*d^2 - \text{sqrt}(b^2 - 4*a*c)*b^2*c^2*d \\ &*e + \text{sqrt}(b^2 - 4*a*c)*a*b*c^2*e^2)*\text{abs}(c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*a \\ &\text{rctan}(2*\text{sqrt}(1/2)*\text{sqrt}(x^2*e + d)/\text{sqrt}(-(2*c^4*d*e^4 - b*c^3*e^5 - \text{sqrt}(-4* \\ &(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5) \\ &^2))*e^{(-4)/c^4})/((2*\text{sqrt}(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + \text{sqrt}(b \\ &^2 - 4*a*c)*b*c^2)*e)*\text{sqrt}(-4*c^2*d + 2*(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*e)*c^2) \end{aligned}$$

maple [C] time = 0.03, size = 332, normalized size = 1.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*(e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $\frac{1}{3}*(e*x^2+d)^{(3/2)}/c/e+1/2/c^2*b*e^{(1/2)}*x-1/2*b*(e*x^2+d)^{(1/2)}/c^2-1/4/c^2*\text{sum}(((a*c*e-b^2*e+b*c*d)*_R^6+(-4*a*b*e^2+a*c*d*e+3*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2-a*c*d*e-3*b^2*d*e+3*b*c*d^2)*_R^2-a*c*d^3*e+b^2*d^3*e-c*d^4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*\ln(-e^{(1/2)}*x-_R+(e*x^2+d)^{(1/2)}), _R=\text{RootOf}(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/c^2*b*d/(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} x^5}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\text{sqrt}(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)$

mupad [B] time = 1.99, size = 8222, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^5*(d + e*x^2)^{(1/2)})/(a + b*x^2 + c*x^4), x)$

[Out] $(d + ex^2)^{3/2}/(3ce) - \operatorname{atan}\left(\frac{(4ab^3c^3e^4 - 16a^2b^4c^4e^4 - 4b^4c^3d^2e^3 + 4b^3c^4d^2e^2 - 16ab^5c^5d^2e^2 + 16ab^2c^4d^2e^3)/c^3 - (2(d + ex^2)^{1/2}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}(4b^3c^5e^3 - 8b^2c^6d^2e^2 - 16ab^5c^6e^3 + 32a^7c^7d^2e^2)/c^3}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} - (2(d + ex^2)^{1/2})(b^6e^4 - 2a^3c^3e^4 + 9a^2b^2c^2e^4 + 2a^2c^4d^2e^2 + b^4c^2d^2e^2 - 6ab^4c^4e^4 - 2b^5cd^2e^3 + 10ab^3c^2d^2e^3 - 10a^2b^3c^3d^2e^3 - 4ab^2c^3d^2e^2)/c^3}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} * i - \left(\frac{(4ab^3c^3e^4 - 16a^2b^4c^4e^4 - 4b^4c^3d^2e^3 + 4b^3c^4d^2e^2 - 16ab^5c^5d^2e^2 + 16ab^2c^4d^2e^3)/c^3 + (2(d + ex^2)^{1/2}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} * (4b^3c^5e^3 - 8b^2c^6d^2e^2 - 16ab^5c^6e^3 + 32a^7c^7d^2e^2)/c^3}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} + (2(d + ex^2)^{1/2})(b^6e^4 - 2a^3c^3e^4 + 9a^2b^2c^2e^4 + 2a^2c^4d^2e^2 + b^4c^2d^2e^2 - 6ab^4c^4e^4 - 2b^5cd^2e^3 + 10ab^3c^2d^2e^3 - 10a^2b^3c^3d^2e^3 - 4ab^2c^3d^2e^2)/c^3}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} * i) / ((2(a^4c^5e^5 - a^3b^2e^5 + a^2b^3d^2e^4 + a^3c^2d^2e^3 + a^2b^3c^2d^3e^2 - 2a^2b^2c^3d^2e^3))/c^3 + ((4ab^3c^3e^4 - 16a^2b^4c^4e^4 - 4b^4c^3d^2e^3 + 4b^3c^4d^2e^2 - 16ab^5c^5d^2e^2 + 16ab^2c^4d^2e^3)/c^3 - (2(d + ex^2)^{1/2}(-b^7e + 8a^3c^4d + b^4e(-4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e(-4ac - b^2)^3)^{1/2} - 9ab^5c^5e + 8ab^4c^2d - 20a^3b^3c^3e - b^3cd(-4ac - b^2)^3)^{1/2} + 2abc^2d(-4ac - b^2)^3)^{1/2} - 3ab^2c^2e(-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}} * i)$

$$\begin{aligned}
& *(- (4ac - b^2)^3)^{1/2} - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - \\
& b^3c^2d * (- (4ac - b^2)^3)^{1/2} + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3 \\
& *ab^2c^2e * (- (4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6 \\
&))^{1/2} * (4b^3c^5e^3 - 8b^2c^6d^2e^2 - 16ab^2c^6e^3 + 32a^2c^7d^2e^2) / c^3 * \\
& (- (b^7e + 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} - b^6cd - \\
& 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} \\
& + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} - (2(d + ex^2) \\
&)^{1/2} * (b^6e^4 - 2a^3c^3e^4 + 9a^2b^2c^2e^4 + 2a^2c^4d^2e^2 + b \\
& ^4c^2d^2e^2 - 6ab^4c^2e^4 - 2b^5c^2d^2e^3 + 10ab^3c^2d^2e^3 - 10a^2 \\
& b^2c^3d^2e^3 - 4ab^2c^3d^2e^2) / c^3 * (- (b^7e + 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} \\
& - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} \\
& + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (((4ab^3c^3e^4 - 16a^2b^2c^4e^4 - 4b^4c^3d^2e^3 \\
& + 4b^3c^4d^2e^2 - 16ab^2c^5d^2e^2 + 16ab^2c^4d^2e^3) / c^3 + (2(d \\
& + ex^2)^{1/2} * (- (b^7e + 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} - b \\
& ^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} \\
& + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - b^2c^3e^3 - 8b^2c^6d^2e^2 - 16ab^2c^6e^3 + 32a^2c^7d^2e^2) / c^3 * (- (b^7e + \\
& 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} - b^6cd - 18a^2b^2c^3d \\
& + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} - 9ab^5c^2e + 8a \\
& *b^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} + 2ab^2c^2 \\
& d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2}) / (8(16a \\
& ^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (2(d + ex^2)^{1/2} * (b^6e^4 - 2 \\
& *a^3c^3e^4 + 9a^2b^2c^2e^4 + 2a^2c^4d^2e^2 + b^4c^2d^2e^2 - 6 \\
& *ab^4c^2e^4 - 2b^5c^2d^2e^3 + 10ab^3c^2d^2e^3 - 10a^2b^2c^3d^2e^3 - 4 \\
& *ab^2c^3d^2e^2) / c^3 * (- (b^7e + 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} \\
& - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} \\
& + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * (- (b^7e + 8a^3c^4d + b^4e * (- (4ac - b^2)^3)^{1/2} \\
& - b^6cd - 18a^2b^2c^3d + 25a^2b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
& - 9ab^5c^2e + 8ab^4c^2d - 20a^3b^2c^3e - b^3c^2d * (- (4ac - b^2)^3)^{1/2} \\
& + 2ab^2c^2d * (- (4ac - b^2)^3)^{1/2} - 3ab^2c^2e * (- (4ac - b^2)^3)^{1/2} \\
&) / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * 2i - \operatorname{atan}((((4ab^3c^3 \\
& e^4 - 16a^2b^2c^4e^4 - 4b^4c^3d^2e^3 + 4b^3c^4d^2e^2 - 16ab^2c^5 \\
& d^2e^2 + 16ab^2c^4d^2e^3) / c^3 - (2(d + ex^2)^{1/2} * (b^4e * (- (4ac - b^2)^3)^{1/2} \\
& - 8a^3c^4d - b^7e + b^6cd + 18a^2b^2c^3d - 25a^2 \\
& *b^3c^2e + a^2c^2e * (- (4ac - b^2)^3)^{1/2} + 9ab^5c^2e - 8ab^4c^2
\end{aligned}$$

$$\begin{aligned}
& d - b^7e + b^6c*d + 18a^2b^2c^3*d - 25a^2b^3c^2*e + a^2c^2*e*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + 9a*b^5c*e - 8a*b^4c^2*d + 20a^3b*c^3*e - b^3c* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)} + 2a*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3a*b^2 \\
& *c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8a*b^2c^6))^{(1 \\
& /2)} - (2*(d + e*x^2)^{(1/2)}*(b^6e^4 - 2a^3c^3e^4 + 9a^2b^2c^2e^4 + 2 \\
& *a^2c^4d^2e^2 + b^4c^2d^2e^2 - 6a*b^4c*e^4 - 2b^5c*d*e^3 + 10a*b \\
& ^3c^2d*e^3 - 10a^2b*c^3d*e^3 - 4a*b^2c^3d^2e^2))/c^3)*((b^4e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 8a^3c^4d - b^7e + b^6c*d + 18a^2b^2c^3*d - 2 \\
& 5a^2b^3c^2*e + a^2c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9a*b^5c*e - 8a*b^ \\
& 4c^2*d + 20a^3b*c^3*e - b^3c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 2a*b*c^2*d*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 3a*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16a^2* \\
& c^7 + b^4c^5 - 8a*b^2c^6))^{(1/2)} + (((4a*b^3c^3e^4 - 16a^2b*c^4e^ \\
& 4 - 4b^4c^3d*e^3 + 4b^3c^4d^2e^2 - 16a*b*c^5d^2e^2 + 16a*b^2c^4 \\
& *d*e^3)/c^3 + (2*(d + e*x^2)^{(1/2)}*((b^4e*(-(4*a*c - b^2)^3)^{(1/2)} - 8a^3 \\
& *c^4d - b^7e + b^6c*d + 18a^2b^2c^3*d - 25a^2b^3c^2*e + a^2c^2*e* \\
& (-(4*a*c - b^2)^3)^{(1/2)} + 9a*b^5c*e - 8a*b^4c^2*d + 20a^3b*c^3*e - b \\
& ^3c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 2a*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3* \\
& a*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)}))/(8*(16a^2c^7 + b^4c^5 - 8a*b^2c^6) \\
&))^{(1/2)}*(4b^3c^5e^3 - 8b^2c^6d*e^2 - 16a*b*c^6e^3 + 32a*c^7d*e^2 \\
&))/c^3)*((b^4e*(-(4*a*c - b^2)^3)^{(1/2)} - 8a^3c^4d - b^7e + b^6c*d + \\
& 18a^2b^2c^3*d - 25a^2b^3c^2*e + a^2c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 9a*b^5c*e - 8a*b^4c^2*d + 20a^3b*c^3*e - b^3c*d*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} + 2a*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3a*b^2*c*e*(-(4*a*c - b^2)^3 \\
&)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8a*b^2c^6))^{(1/2)} + (2*(d + e*x^2)^{(\\
& 1/2)}*(b^6e^4 - 2a^3c^3e^4 + 9a^2b^2c^2e^4 + 2a^2c^4d^2e^2 + b^4 \\
& *c^2d^2e^2 - 6a*b^4c*e^4 - 2b^5c*d*e^3 + 10a*b^3c^2d*e^3 - 10a^2* \\
& b*c^3d*e^3 - 4a*b^2c^3d^2e^2))/c^3)*((b^4e*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 8a^3c^4d - b^7e + b^6c*d + 18a^2b^2c^3*d - 25a^2b^3c^2*e + a^2* \\
& c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9a*b^5c*e - 8a*b^4c^2*d + 20a^3b*c^3 \\
& *e - b^3c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 2a*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2) \\
&) - 3a*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8a*b^ \\
& 2c^6))^{(1/2)}))*((b^4e*(-(4*a*c - b^2)^3)^{(1/2)} - 8a^3c^4d - b^7e + b \\
& ^6c*d + 18a^2b^2c^3*d - 25a^2b^3c^2*e + a^2c^2*e*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 9a*b^5c*e - 8a*b^4c^2*d + 20a^3b*c^3*e - b^3c*d*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 2a*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3a*b^2*c*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(8*(16a^2c^7 + b^4c^5 - 8a*b^2c^6))^{(1/2)}*2i - ((2* \\
& d)/(c*e) + (b*e^2 - 2c*d*e)/(c^2e^2))*(d + e*x^2)^{(1/2)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.256 \quad \int \frac{x^3 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=292

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be) + 2ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right) \left(\sqrt{b^2-4ac}(cd-be) + 2ace + b^2(-e)\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} \sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}$$

Rubi [A] time = 3.60, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be) + 2ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} - \frac{\left(\sqrt{b^2-4ac}(cd-be) + 2ace + b^2(-e) + bcd\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b+\sqrt{b^2-4ac})}} + \frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 824

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m-1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x \sqrt{d+ex}}{a+bx+cx^2} dx, x, x^2 \right) \\
 &= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst} \left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)} dx, x, x^2 \right)}{2c} \\
 &= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst} \left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{c} \\
 &= \frac{\sqrt{d+ex^2}}{c} - \frac{(bcd - b^2e + 2ace - \sqrt{b^2 - 4ac}(cd - be)) \text{Subst} \left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e + \frac{1}{2}(-2cd+be)+cx^2} dx \right)}{2c\sqrt{b^2 - 4ac}} \\
 &= \frac{\sqrt{d+ex^2}}{c} + \frac{(bcd - b^2e + 2ace - \sqrt{b^2 - 4ac}(cd - be)) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} \right)}{\sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} - \frac{(bcd - b^2e + 2ace - \sqrt{b^2 - 4ac}(cd - be))}{2c\sqrt{b^2 - 4ac}}
 \end{aligned}$$

Mathematica [A] time = 0.55, size = 308, normalized size = 1.05

$$\frac{(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}} + \frac{(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \sqrt{d+ex^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[d + e*x^2] + ((b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c]*e)]) + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e - 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]*e)]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]*e)]])/c

IntegrateAlgebraic [C] time = 1.11, size = 352, normalized size = 1.21

$$\frac{(cd\sqrt{4ac-b^2}-be\sqrt{4ac-b^2}+2iace-ib^2e+ibcd)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}} + \frac{(cd\sqrt{4ac-b^2}-be\sqrt{4ac-b^2}-2iace+ib^2e-ibcd)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}} + \frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] Sqrt[d + e*x^2]/c + ((I*b*c*d + c*Sqrt[-b^2 + 4*a*c]*d - I*b^2*e + (2*I)*a*c*e - b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*b*c*d + c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*e - (2*I)*a*c*e - b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])

fricas [B] time = 138.37, size = 2435, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +

$$\begin{aligned}
& a^2c^2)e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4))*\log((2ab^2cd^2 \\
& - 2ab^3d^2e + 2(a^2b^2 - a^3c)e^2 + (ab^2cd^2e - (ab^3 - a^2b^2c) \\
& *e^2)*x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4c - 4ab^2c^2)*d - (b^5 - 5 \\
& *ab^3c + 4a^2b^2c^2)*e - (b^4c^3 - 6ab^2c^4 + 8a^2c^5)*\sqrt{(b^2c \\
& ^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 \\
& - 4a^2c^7)))*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e + (b^2c^3 - 4 \\
& *a^2c^4)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2 \\
& c^2)*e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4)) - ((ab^2c^3 - 4a^2 \\
& c^4)*e*x^2 + 2*(ab^2c^3 - 4a^2c^4)*d)*\sqrt{(b^2c^2d^2 - 2*(b^3c - \\
& ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - 4a^2c^7)))/x^2) \\
& - \sqrt{1/2}*c*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e + (b^2c^3 - 4 \\
& a^2c^4)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2 \\
& c^2)*e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4))*\log((2ab^2cd^2 - \\
& 2ab^3d^2e + 2(a^2b^2 - a^3c)e^2 + (ab^2cd^2e - (ab^3 - a^2b^2c)*e^2) \\
& *x^2 - 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4c - 4ab^2c^2)*d - (b^5 - 5ab^3 \\
& c + 4a^2b^2c^2)*e - (b^4c^3 - 6ab^2c^4 + 8a^2c^5)*\sqrt{(b^2c^2d^2 \\
& d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - \\
& 4a^2c^7)))*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e + (b^2c^3 - 4a^2 \\
& c^4)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2) \\
& ^2)*e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4)) - ((ab^2c^3 - 4a^2c^4) \\
& *e*x^2 + 2*(ab^2c^3 - 4a^2c^4)*d)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2 \\
& c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - 4a^2c^7)))/x^2) + \sqrt{ \\
& 1/2}*c*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e - (b^2c^3 - 4a^2c^4) \\
& *\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2) \\
& ^2)*e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4))*\log((2ab^2cd^2 - 2a \\
& *b^3d^2e + 2(a^2b^2 - a^3c)e^2 + (ab^2cd^2e - (ab^3 - a^2b^2c)*e^2)* \\
& x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4c - 4ab^2c^2)*d - (b^5 - 5ab^3 \\
& c + 4a^2b^2c^2)*e + (b^4c^3 - 6ab^2c^4 + 8a^2c^5)*\sqrt{(b^2c^2d^2 \\
& - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - 4 \\
& a^2c^7)))*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e - (b^2c^3 - 4a^2c^4) \\
&)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2) \\
& *e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4)) + ((ab^2c^3 - 4a^2c^4) \\
& *e*x^2 + 2*(ab^2c^3 - 4a^2c^4)*d)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2) \\
& ^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - 4a^2c^7)))/x^2) - \sqrt{ \\
& 1/2}*c*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e - (b^2c^3 - 4a^2c^4) \\
& *\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)* \\
& e^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4))*\log((2ab^2cd^2 - 2ab^3 \\
& d^2e + 2(a^2b^2 - a^3c)e^2 + (ab^2cd^2e - (ab^3 - a^2b^2c)*e^2)*x^2 \\
& - 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4c - 4ab^2c^2)*d - (b^5 - 5ab^3c \\
& + 4a^2b^2c^2)*e + (b^4c^3 - 6ab^2c^4 + 8a^2c^5)*\sqrt{(b^2c^2d^2 - \\
& 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2)/(b^2c^6 - 4a^2c \\
& ^7)))*\sqrt{((b^2c - 2a^2c^2)*d - (b^3 - 3ab^2c)*e - (b^2c^3 - 4a^2c^4)* \\
& \sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*d^2 + (b^4 - 2ab^2c + a^2c^2)*e^2) \\
& ^2)/(b^2c^6 - 4a^2c^7)))/(b^2c^3 - 4a^2c^4)) + ((ab^2c^3 - 4a^2c^4)*e \\
& *x^2 + 2*(ab^2c^3 - 4a^2c^4)*d)*\sqrt{(b^2c^2d^2 - 2*(b^3c - ab^2c^2)*
\end{aligned}$$

$d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + 4*\text{sqrt}(e*x^2 + d)/c$

giac [B] time = 0.78, size = 619, normalized size = 2.12

$$\frac{\sqrt{e+d}}{c} \frac{(2bc^2e + (b^2 - 4ac^2)e - (b^2 - 4ac^2)^2 - (3b^2 - 4ac^2)e - 2(\sqrt{b^2 - 4ac^2}e - \sqrt{b^2 - 4ac^2}e + \sqrt{b^2 - 4ac^2}e)(b^2 - 2ac^2)) \arctan\left(\frac{2\sqrt{e+d}}{2bc + \sqrt{2e+d} + \sqrt{2e+d}}\right)}{(2\sqrt{b^2 - 4ac^2}e - (b^2 - 4ac^2)e + \sqrt{b^2 - 4ac^2}e + \sqrt{b^2 - 4ac^2}e + \sqrt{b^2 - 4ac^2}e)(b^2 - 2ac^2)) \arctan\left(\frac{2\sqrt{e+d}}{2bc + \sqrt{2e+d} + \sqrt{2e+d}}\right)}{(2\sqrt{b^2 - 4ac^2}e - (b^2 - 4ac^2)e + \sqrt{b^2 - 4ac^2}e + \sqrt{b^2 - 4ac^2}e + \sqrt{b^2 - 4ac^2}e)(b^2 - 2ac^2)) \arctan\left(\frac{2\sqrt{e+d}}{2bc + \sqrt{2e+d} + \sqrt{2e+d}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $\text{sqrt}(x^2*e + d)/c - (2*b*c^4*d^2 + ((b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e - 2*(\text{sqrt}(b^2 - 4*a*c)*c^3*d^2 - \text{sqrt}(b^2 - 4*a*c)*b*c^2*d*e + \text{sqrt}(b^2 - 4*a*c)*a*c^2*e^2)*\text{abs}(c) + (b^3*c^2 - 2*a*b*c^3)*e^2)*\text{arctan}(2*\text{sqrt}(1/2)*\text{sqrt}(x^2*e + d)/\text{sqrt}(-(2*c^2*d - b*c*e + \text{sqrt}(-4*(c^2*d^2 - b*c*d*e + a*c*e^2))*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*\text{sqrt}(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + \text{sqrt}(b^2 - 4*a*c)*b*c)*e)*\text{sqrt}(-4*c^2*d + 2*(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*e)*c^2 + (2*b*c^4*d^2 + ((b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e + 2*(\text{sqrt}(b^2 - 4*a*c)*c^3*d^2 - \text{sqrt}(b^2 - 4*a*c)*b*c^2*d*e + \text{sqrt}(b^2 - 4*a*c)*a*c^2*e^2)*\text{abs}(c) + (b^3*c^2 - 2*a*b*c^3)*e^2)*\text{arctan}(2*\text{sqrt}(1/2)*\text{sqrt}(x^2*e + d)/\text{sqrt}(-(2*c^2*d - b*c*e - \text{sqrt}(-4*(c^2*d^2 - b*c*d*e + a*c*e^2))*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*\text{sqrt}(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - \text{sqrt}(b^2 - 4*a*c)*b*c)*e)*\text{sqrt}(-4*c^2*d + 2*(b*c + \text{sqrt}(b^2 - 4*a*c)*c)*e)*c^2)$

maple [C] time = 0.03, size = 275, normalized size = 0.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)

[Out] $-1/2/c*e^{(1/2)*x} + 1/2*(e*x^2+d)^{(1/2)}/c + 1/4/c*\text{sum}(((b*e+c*d)*_R^6 + (-4*a*e^2 + 3*b*d*e - 3*c*d^2)*_R^4 + d*(4*a*e^2 - 3*b*d*e + 3*c*d^2)*_R^2 + b*d^3*e - c*d^4)/(_R^7*c + 3*_R^5*b*e - 3*_R^5*c*d + 8*_R^3*a*e^2 - 4*_R^3*b*d*e + 3*_R^3*c*d^2 + _R*b*d^2*e - _R*c*d^3)*\ln(-e^{(1/2)*x}*_R + (e*x^2+d)^{(1/2)}), _R = \text{RootOf}(_Z^8*c + (4*b*e - 4*c*d)*_Z^6 + c*d^4 + (16*a*e^2 - 8*b*d*e + 6*c*d^2)*_Z^4 + (4*b*d^2*e - 4*c*d^3)*_Z^2) + 1/2/c*d/(-e^{(1/2)*x} + (e*x^2+d)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} x^3}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 2.34, size = 5705, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] $(d + e*x^2)^{1/2}/c - \operatorname{atan}\left(\frac{((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^{1/2}*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * (4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} - (2*(d + e*x^2)^{1/2}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * i - \left(\frac{((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^{1/2}*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} + (2*(d + e*x^2)^{1/2}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * i\right) / \left(\frac{((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^{1/2}*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * (4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} + (2*(d + e*x^2)^{1/2}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c * ((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^{1/2} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{1/2} + b*c*d*(-(4*a*c - b^2)^3)^{1/2} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{1/2} * i\right)$

$$\begin{aligned}
& 4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12 \\
& *a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} - (2*(d + e*x \\
& ^2)^{(1/2)}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4* \\
& a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c*((8*a^2*c^3*d - b^5*e - \\
& b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12*a^2*b*c^ \\
& 2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} - (2*(a*c^2*d^3*e^2 - \\
& a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3 \\
& *e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2 \\
& *e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^{(1/2)}*((8*a^2*c^3*d - b^5*e - b \\
& ^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12*a^2*b*c^2 \\
& *e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2* \\
& c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c*((8*a^2*c^3*d - b^5*e - b^ \\
& 2*e*(-(4*a*c - b^2)^3)^{(1/2)} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12*a^2*b*c^2* \\
& e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(\\
& b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 \\
& - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16 \\
& *a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}))*((8*a^2*c^3*d - b^5*e - b^2*e*(- \\
& -(4*a*c - b^2)^3)^{(1/2)} + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^ \\
& (1/2) + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8 \\
& *(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*2i - \operatorname{atan}((((16*a^2*c^3*e^4 \\
& - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 \\
& - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^{(1/2)}*(-(b^5*e - 8*a^2*c^3*d - b^2*e \\
& *(-4*a*c - b^2)^3)^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/ \\
& (8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4* \\
& d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c*(-(b^5*e - 8*a^2*c^3*d - b^2*e \\
& *(-4*a*c - b^2)^3)^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/ \\
& (8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}*(b^4 \\
& *e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - \\
& 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a \\
& ^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^ \\
& 2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3 \\
& *d*e^3)/c + (2*(d + e*x^2)^{(1/2)}*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b* \\
& c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c \\
& ^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a \\
& *b*c^4*e^3 + 32*a*c^5*d*e^2))/c*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c -
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b* \\
& c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c \\
& ^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^4*e^4 + 2*a^2 \\
& *c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^ \\
& 3 + 6*a*b*c^2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& ^{(1/2)} - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4 \\
& *c^3 - 8*a*b^2*c^4))^{(1/2)}*1i)/((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a \\
& *c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - \\
& (2*(d + e*x^2)^{(1/2)}*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 \\
& - 8*a*b^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + \\
& 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 \\
& - 8*a*b^2*c^4))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2 \\
& *a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^ \\
& 2*d*e^3))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^4* \\
& c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b \\
& ^2*c^4))^{(1/2)} - (2*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 \\
& - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^ \\
& 2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + \\
& e*x^2)^{(1/2)}*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^4* \\
& c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b \\
& ^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^ \\
& 5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^4* \\
& c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b \\
& ^2*c^4))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d \\
& ^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3) \\
&)/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c*d - 7* \\
& a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) \\
&)^{(1/2)}))*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - b^4*c*d \\
& - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*c*d*(-(4*a*c - b^2)^3)^{(1/2)} \\
& ^{(1/2)} + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2* \\
& c^4))^{(1/2)}*2i
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

$$3.257 \quad \int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=202

$$\frac{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2cd-e(b-\sqrt{b^2-4ac})} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2cd-e(b-\sqrt{b^2-4ac})} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 699

Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1130

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2

+ q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx, x, x^2 \right) \\ &= e \text{Subst} \left(\int \frac{x^2}{cd^2 - bde + ae^2 + (-2cd + be)x^2 + cx^4} dx, x, \sqrt{d+ex^2} \right) \\ &= - \left(\frac{1}{2} \left(-e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2 - 4ac}e + \frac{1}{2}(-2cd + be) + cx^2} dx, x, \sqrt{d+ex^2} \right) \right) \\ &\quad - \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})} e \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})} e} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} + \frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})} e \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})} e} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.33, size = 179, normalized size = 0.89

$$\frac{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}} \right) - \sqrt{e\sqrt{b^2 - 4ac} - be + 2cd} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2 - 4ac} - be + 2cd}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] (-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [C] time = 0.73, size = 279, normalized size = 1.38

$$\frac{\left(e\sqrt{4ac-b^2} + ibe - 2icd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}} + \frac{\left(e\sqrt{4ac-b^2} - ibe + 2icd\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]

[Out] (((-2*I)*c*d + I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((2*I)*c*d - I*b*e + Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])

fricas [B] time = 21.45, size = 1085, normalized size = 5.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e

$$-\frac{(b^2c - 4a^2c^2)\sqrt{e^2/(b^2c^2 - 4a^2c^3)}}{(b^2c - 4a^2c^2)} - \left((b^2c - 4a^2c^2)ex^2 + 2(b^2c - 4a^2c^2)d \right) \sqrt{e^2/(b^2c^2 - 4a^2c^3)} \Big/ x^2$$

giac [A] time = 0.55, size = 228, normalized size = 1.13

$$-\frac{\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac}c)}e \arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{\frac{2cd-be + \sqrt{(2cd-be)^2 - 4(cd^2-bde+ae^2)c}}{c}}}\right)}{2\sqrt{b^2 - 4ac}|c|} + \frac{\sqrt{-4c^2d + 2(bc + \sqrt{b^2 - 4ac}c)}e \arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{\frac{2cd-be - \sqrt{(2cd-be)^2 - 4(cd^2-bde+ae^2)c}}{c}}}\right)}{2\sqrt{b^2 - 4ac}|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-\frac{1}{2}\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac}c)*e}*\arctan\left(\frac{2*\sqrt{1/2}*\sqrt{x^2*e + d}}{\sqrt{-(2*c*d - b*e + \sqrt{(2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c})/c}}\right)/(\sqrt{b^2 - 4*a*c}*abs(c)) + \frac{1}{2}\sqrt{-4c^2d + 2(bc + \sqrt{b^2 - 4ac}c)*e}*\arctan\left(\frac{2*\sqrt{1/2}*\sqrt{x^2*e + d}}{\sqrt{-(2*c*d - b*e - \sqrt{(2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c})/c}}\right)/(\sqrt{b^2 - 4*a*c}*abs(c))$

maple [C] time = 0.02, size = 177, normalized size = 0.88

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)

[Out] $\frac{1}{4}*e*\sum\left(\frac{R^6 + R^4*d - R^2*d^2 - d^3}{R^7*c + 3*R^5*b*e - 3*R^5*c*d + 8*R^3*a*e^2 - 4*R^3*b*d*e + 3*R^3*c*d^2 + R*b*d^2*e - R*c*d^3}\right)*\ln\left(-e^{1/2}*x - R + (e*x^2 + d)^{1/2}\right), R = \text{RootOf}\left(\frac{Z^8*c + (4*b*e - 4*c*d)*Z^6 + c*d^4 + (16*a*e^2 - 8*b*d*e + 6*c*d^2)*Z^4 + (4*b*d^2*e - 4*c*d^3)*Z^2}{1}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}x}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.72, size = 717, normalized size = 3.55

$$\frac{\left(\frac{\sqrt{e x^2 + d} \operatorname{arctan}\left(\frac{\sqrt{e x^2 + d}}{\sqrt{c x^2 + a}}\right)}{2 \sqrt{c x^2 + a}} + \frac{\sqrt{e x^2 + d} \operatorname{arctan}\left(\frac{\sqrt{e x^2 + d}}{\sqrt{c x^2 + a}}\right)}{2 \sqrt{c x^2 + a}} \right)}{8 (b x^2 + a)^2} + \frac{\left(\frac{\sqrt{e x^2 + d} \operatorname{arctan}\left(\frac{\sqrt{e x^2 + d}}{\sqrt{c x^2 + a}}\right)}{2 \sqrt{c x^2 + a}} + \frac{\sqrt{e x^2 + d} \operatorname{arctan}\left(\frac{\sqrt{e x^2 + d}}{\sqrt{c x^2 + a}}\right)}{2 \sqrt{c x^2 + a}} \right)}{8 (b x^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)`

[Out]
$$- 2*\operatorname{atanh}\left(\frac{2*((d + e*x^2)^{1/2}*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) + ((d + e*x^2)^{1/2}*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 3*2*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^{1/2} + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e))}{8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)}\right)*(- (b^3*e + e*(-(4*a*c - b^2)^3)^{1/2} + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^{1/2})/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4)*(- (b^3*e + e*(-(4*a*c - b^2)^3)^{1/2} + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^{1/2} - 2*\operatorname{atanh}\left(\frac{2*((d + e*x^2)^{1/2}*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) - ((d + e*x^2)^{1/2}*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^{1/2} - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e))}{8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)}\right)*((e*(-(4*a*c - b^2)^3)^{1/2} - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^{1/2})/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4)*((e*(-(4*a*c - b^2)^3)^{1/2} - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^{1/2}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)`

[Out] `Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)`

$$3.258 \quad \int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=281

$$\frac{\sqrt{c} \left(d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) - \sqrt{c} \left(-d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(\sqrt{b^2 - 4ac} + b \right)}}$$

Rubi [A] time = 1.35, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$\frac{\sqrt{c} \left(d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) - \sqrt{c} \left(-d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right) - \sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(\sqrt{b^2 - 4ac} + b \right)} - a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)), x]

[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_)^n)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^p, x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, S

```

ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]

```

Rule 1166

```

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

```

Rule 1251

```

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)
^4)^(p_), x_Symbol] :=> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]

```

Rule 1287

```

Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 +
(c_)*(x_)^4), x_Symbol] :=> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^2}{\left(-\frac{d}{e} + \frac{x^2}{e}\right) \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}\right)} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(-\frac{de}{a(d-x^2)} + \frac{e(cd^2-bde+ae^2-cdx^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{a} - \frac{d \text{Subst} \left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a} + \frac{\left(c \left(bd - \sqrt{b^2 - 4ac} d - 2ae \right) \right) \text{Subst} \left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}e + \frac{1}{2}(-2cd+be) + \dots} dx \right)}{2a\sqrt{b^2-4ac}} \\
&= -\frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a} + \frac{\sqrt{c} \left(bd + \sqrt{b^2 - 4ac} d - 2ae \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 241, normalized size = 0.86

$$\frac{\sqrt{2} \left(\left(\sqrt{b^2-4ac} + b \right) \sqrt{e \sqrt{b^2-4ac} - be + 2cd} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{e \sqrt{b^2-4ac} - be + 2cd}} \right) + \left(\sqrt{b^2-4ac} - b \right) \sqrt{2cd - e \left(\sqrt{b^2-4ac} + b \right)} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - e \left(\sqrt{b^2-4ac} + b \right)}} \right) \right)}{\sqrt{c} \sqrt{b^2-4ac}} - 4\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)$$

4a

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)), x]

[Out] (-4*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (Sqrt[2]*((b + Sqrt[b^2 - 4*a*c])*Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]))/(4*a)

IntegrateAlgebraic [A] time = 0.73, size = 280, normalized size = 1.00

$$\frac{\sqrt{c} \left(d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}} \right) - \sqrt{c} \left(d\sqrt{b^2 - 4ac} + 2ae - bd \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}} \right) - \sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{-e\sqrt{b^2 - 4ac} + be - 2cd} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{e\sqrt{b^2 - 4ac} + be - 2cd} - a}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]

[Out] -((Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e])) - (Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]) - (Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.68, size = 717, normalized size = 2.55

$$\frac{\sqrt{a^2+2b^2-4ac} \sqrt{b^2-4ac} \sqrt{d+ex^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{-e\sqrt{b^2-4ac}+be-2cd}}\right) - \sqrt{a^2+2b^2-4ac} \sqrt{b^2-4ac} \sqrt{d+ex^2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}+be-2cd}}\right) - \sqrt{d} \operatorname{tanh}^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{-e\sqrt{b^2 - 4ac} + be - 2cd} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{e\sqrt{b^2 - 4ac} + be - 2cd} - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] d*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2)))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d


```
*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e
+ d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2))*a*c +
(2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*
a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

maple [C] time = 0.03, size = 294, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/2/a*(e*x^2+d)^(1/2)+1/
2/a*e^(1/2)*x-1/4/a*sum((_R^6*c*d+(-4*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^
2-4*b*d*e+3*c*d^2)*_R^2-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4
*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R+(e*x^2+d)^(1
/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_
Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 6.87, size = 10964, normalized size = 39.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9
+ 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
+ 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(((b^4*d +
8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a
*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c
```

$$\begin{aligned}
& *e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32* \\
& a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4* \\
& c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^ \\
& 2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^{10} - 192*a^3*c^5*d^3*e^8 \\
& + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 \\
& + 48*a^3*b^2*c^3*d*e^{10}) - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^ \\
& 4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - \\
& 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3* \\
& c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a \\
& ^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^ \\
& (1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) \\
&)^{(1/2)} + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^ \\
& 4*c^2*d^2*e^{10} + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^ \\
& 11 - 24*a*b^2*c^3*d^2*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e \\
&)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*1i + ((d + e*x^2)^{(1/2)}*(\\
& 2*a^2*c^3*e^{12} + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4*a \\
& *b*c^3*d*e^{11}) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^ \\
& 4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2* \\
& c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(\\
& 1/2)}*((d + e*x^2)^{(1/2)}*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8* \\
& (a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4 \\
& *c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2 \\
& *e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 \\
& + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^{10} + 192*a^3*c^5*d^3*e^8 - 48*a \\
& ^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^ \\
& 3*b^2*c^3*d*e^{10}) - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^1 \\
& 0 - 8*a^2*b^3*c^2*e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5* \\
& c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2 \\
& *e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2* \\
& d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} \\
& + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4 \\
& *d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{10} \\
&))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))^{(1/2)}*1i)/(((d + e*x^2)^{(1/2)}*(2*a^2*c^3*e^{12} + 6*c^5*d^4 \\
& *e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4*a*b*c^3*d*e^{11}) - ((b^4*d + \\
& 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2 \\
& *c)))^{(1/2)}*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b
\end{aligned}$$

$$\begin{aligned}
& *c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*((d + e*x^2)^{(1/2)}*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) \\
& + 192*a^4*c^4*d*e^{10} + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^{10}) - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} - ((d + e*x^2)^{(1/2)}*(2*a^2*c^3*e^{12} + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4*a*b*c^3*d*e^{11}) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*((d + e*x^2)^{(1/2)}*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^{10} - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^{10}) - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} + 2*c^4*d^3*e^{10} - 2*b*c^3*d^2*e^{11} + 2*a*c^3*d*e^{12}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^{(1/2)} - b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*2i + \operatorname{atan}(((d + e*x^2)^{(1/2)}*
\end{aligned}$$

$$\begin{aligned}
& (2*a^2*c^3*e^{12} + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4* \\
& a*b*c^3*d*e^{11}) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{ \\
& (1/2) + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 \\
& ^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*(((b^4*d + 8*a^2*c^2*d - a*b^3*e - \\
& a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + \\
& 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*((d + e*x^2)^{ \\
& (1/2)}*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d* \\
& (- (4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c \\
& ^2 - 8*a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2 \\
& *c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3 \\
& *d*e^9) - 192*a^4*c^4*d*e^{10} - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 \\
& - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^{10}) \\
& - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2* \\
& e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a \\
& *b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c \\
& ^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a \\
& ^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} + 12*a*c^5*d^4*e^8 \\
& + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4 \\
& *d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{1 \\
& 0}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))^{(1/2)}*1i + ((d + e*x^2)^{(1/2)}*(2*a^2*c^3*e^{12} + 6*c^5*d^4 \\
& *e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4*a*b*c^3*d*e^{11}) - ((b^4*d \\
& + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2 \\
& ^2*c)))^{(1/2)}*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2* \\
& b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)}*((d + e*x^2)^{(1/2)}*(\\
& (b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8 \\
& *a^3*b^2*c)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3 \\
& *e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2 \\
& *e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) \\
& + 192*a^4*c^4*d*e^{10} + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a \\
& ^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^{10}) - (d + \\
& e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + \\
& 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2 \\
& *d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2* \\
& e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c* \\
& e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^{(1/2)} + 12*a^2*c^4*d^2*e^{10} - \\
& 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d* \\
& e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{10}))*((b^4*d + 8*a^2*c^2*d
\end{aligned}$$

$$\begin{aligned}
& - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6 \\
& *a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)}*1 \\
& i)/(((d + e*x^2)^{(1/2)}*(2*a^2*c^3*e^{12} + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + \\
& 4*b^2*c^3*d^2*e^{10} - 4*a*b*c^3*d*e^{11}) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e - \\
& a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + \\
& 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)}*(12*a*c^5*d^4 \\
& *e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^ \\
& 4*c^2 - 8*a^3*b^2*c))^{(1/2)}*((d + e*x^2)^{(1/2)}*((b^4*d + 8*a^2*c^2*d - a*b \\
& ^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^ \\
& 2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)}*(512*a \\
& ^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2* \\
& e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^ \\
& 9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^{10} + 19 \\
& 2*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a \\
& ^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^{10}) - (d + e*x^2)^{(1/2)}*(32*a^3*b*c^3 \\
& *e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + 144*a^2*c^5*d^3*e^8 + 16*b \\
& ^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^{10} - 96*a*b^2*c^4*d^ \\
& 3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^1 \\
& 0))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^ \\
& 2 - 8*a^3*b^2*c))^{(1/2)} + 12*a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c \\
& ^2*d^2*e^{10} + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} \\
& - 24*a*b^2*c^3*d^2*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/ \\
& (8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)} - ((d + e*x^2)^{(1/2)}*(2*a^2 \\
& *c^3*e^{12} + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^{10} - 4*a*b*c^ \\
& 3*d*e^{11}) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 1 \\
& 6*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)}*((((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(- \\
& (4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2 \\
& *b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)}*((d + e*x^2)^{(1/2)}* \\
& ((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - \\
& 8*a^3*b^2*c))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2* \\
& c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d \\
& ^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9 \\
&) - 192*a^4*c^4*d*e^{10} - 192*a^3*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48* \\
& a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^{10}) - (d + \\
& e*x^2)^{(1/2)}*(32*a^3*b*c^3*e^{11} + 48*a^3*c^4*d*e^{10} - 8*a^2*b^3*c^2*e^{11} + \\
& 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c \\
& ^2*d*e^{10} - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2 \\
& *e^9 - 72*a^2*b^2*c^3*d*e^{10}))*((b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c \\
& e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))^{(1/2)} + 12*a*c^5*d^4*e^8 + 12
\end{aligned}$$

$$\begin{aligned}
& *a^2*c^4*d^2*e^{10} - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^{10} + 8*a*b*c^4*d^3* \\
& e^9 - 4*a*b^3*c^2*d*e^{11} + 20*a^2*b*c^3*d*e^{11} - 24*a*b^2*c^3*d^2*e^{10}))*((\\
& b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8* \\
& a^3*b^2*c)))^{(1/2)} + 2*c^4*d^3*e^{10} - 2*b*c^3*d^2*e^{11} + 2*a*c^3*d*e^{12}))* (\\
& (b^4*d + 8*a^2*c^2*d - a*b^3*e - a*e*(-(4*a*c - b^2)^3)^{(1/2)} + b*d*(-(4*a* \\
& c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8 \\
& *a^3*b^2*c)))^{(1/2)} *2i - (d^{(1/2)}*atanh((20*c^4*d^{(5/2)}*e^{10}*(d + e*x^2)^{(1 \\
& /2)))/(20*c^4*d^3*e^{10} - 12*b*c^3*d^2*e^{11} + (18*c^5*d^5*e^8)/a + 2*a*c^3*d* \\
& e^{12} + (6*b^2*c^3*d^3*e^{10})/a + (2*b^3*c^2*d^2*e^{11})/a - (4*b^2*c^4*d^5*e^8 \\
&)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^{10})/a^2 - (28*b*c^4*d^4* \\
& e^9)/a) + (18*c^5*d^{(9/2)}*e^8*(d + e*x^2)^{(1/2)))/(18*c^5*d^5*e^8 + 20*a*c^4 \\
& *d^3*e^{10} + 2*a^2*c^3*d*e^{12} - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^{10} + 2*b^ \\
& 3*c^2*d^2*e^{11} - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2 \\
& *d^3*e^{10})/a - 12*a*b*c^3*d^2*e^{11} - (28*b*c^4*d^{(7/2)}*e^9*(d + e*x^2)^{(1/ \\
& 2)))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^{10} + 2*a^2*c^3*d*e^{12} - 28*b*c^4*d^4*e \\
& ^9 + 6*b^2*c^3*d^3*e^{10} + 2*b^3*c^2*d^2*e^{11} - (4*b^2*c^4*d^5*e^8)/a + (6*b \\
& ^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^{10})/a - 12*a*b*c^3*d^2*e^{11}) + (2*b^3* \\
& c^2*d^{(3/2)}*e^{11}*(d + e*x^2)^{(1/2)))/(18*c^5*d^5*e^8 + 20*a*c^4*d^3*e^{10} + 2 \\
& *a^2*c^3*d*e^{12} - 28*b*c^4*d^4*e^9 + 6*b^2*c^3*d^3*e^{10} + 2*b^3*c^2*d^2*e^{1 \\
& 1} - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4*e^9)/a - (2*b^4*c^2*d^3*e^{10})/a \\
& - 12*a*b*c^3*d^2*e^{11}) + (6*b^2*c^3*d^{(5/2)}*e^{10}*(d + e*x^2)^{(1/2)))/(18*c^5 \\
& *d^5*e^8 + 20*a*c^4*d^3*e^{10} + 2*a^2*c^3*d*e^{12} - 28*b*c^4*d^4*e^9 + 6*b^2* \\
& c^3*d^3*e^{10} + 2*b^3*c^2*d^2*e^{11} - (4*b^2*c^4*d^5*e^8)/a + (6*b^3*c^3*d^4* \\
& e^9)/a - (2*b^4*c^2*d^3*e^{10})/a - 12*a*b*c^3*d^2*e^{11}) - (2*b^4*c^2*d^{(5/2)} \\
& *e^{10}*(d + e*x^2)^{(1/2)))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^{12} + 20*a^2*c^4* \\
& d^3*e^{10} - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^{10} - 28* \\
& a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^{10} + 2*a*b^3*c^2*d^2*e^{11} - 12*a^2*b*c^ \\
& 3*d^2*e^{11}) + (6*b^3*c^3*d^{(7/2)}*e^9*(d + e*x^2)^{(1/2)))/(18*a*c^5*d^5*e^8 + \\
& 2*a^3*c^3*d*e^{12} + 20*a^2*c^4*d^3*e^{10} - 4*b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4 \\
& *e^9 - 2*b^4*c^2*d^3*e^{10} - 28*a*b*c^4*d^4*e^9 + 6*a*b^2*c^3*d^3*e^{10} + 2*a \\
& *b^3*c^2*d^2*e^{11} - 12*a^2*b*c^3*d^2*e^{11}) - (4*b^2*c^4*d^{(9/2)}*e^8*(d + e \\
& x^2)^{(1/2)))/(18*a*c^5*d^5*e^8 + 2*a^3*c^3*d*e^{12} + 20*a^2*c^4*d^3*e^{10} - 4* \\
& b^2*c^4*d^5*e^8 + 6*b^3*c^3*d^4*e^9 - 2*b^4*c^2*d^3*e^{10} - 28*a*b*c^4*d^4*e \\
& ^9 + 6*a*b^2*c^3*d^3*e^{10} + 2*a*b^3*c^2*d^2*e^{11} - 12*a^2*b*c^3*d^2*e^{11}) + \\
& (2*a*c^3*d^{(1/2)}*e^{12}*(d + e*x^2)^{(1/2)))/(20*c^4*d^3*e^{10} - 12*b*c^3*d^2*e \\
& ^{11} + (18*c^5*d^5*e^8)/a + 2*a*c^3*d*e^{12} + (6*b^2*c^3*d^3*e^{10})/a + (2*b^3 \\
& *c^2*d^2*e^{11})/a - (4*b^2*c^4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b \\
& ^4*c^2*d^3*e^{10})/a^2 - (28*b*c^4*d^4*e^9)/a) - (12*b*c^3*d^{(3/2)}*e^{11}*(d + \\
& e*x^2)^{(1/2)))/(20*c^4*d^3*e^{10} - 12*b*c^3*d^2*e^{11} + (18*c^5*d^5*e^8)/a + 2 \\
& *a*c^3*d*e^{12} + (6*b^2*c^3*d^3*e^{10})/a + (2*b^3*c^2*d^2*e^{11})/a - (4*b^2*c^ \\
& 4*d^5*e^8)/a^2 + (6*b^3*c^3*d^4*e^9)/a^2 - (2*b^4*c^2*d^3*e^{10})/a^2 - (28*b \\
& *c^4*d^4*e^9)/a))/a
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a), x)

[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)

$$3.259 \quad \int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=382

$$\frac{\sqrt{c} \left(\sqrt{b^2 - 4ac} (bd - ae) - abe - 2acd + b^2d \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) \sqrt{c} \left(-b \left(d\sqrt{b^2 - 4ac} + ae \right) - a \left(2cd \right) \right)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)}} + \frac{\sqrt{2} a^2 \sqrt{b^2 - 4ac}}{\sqrt{2} a^2 \sqrt{b^2 - 4ac}}$$

Rubi [A] time = 4.13, antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$\frac{\sqrt{c} \left(\sqrt{b^2 - 4ac} (bd - ae) - abe - 2acd + b^2d \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) + \sqrt{c} \left(-\sqrt{b^2 - 4ac} (bd - ae) - abe - 2acd + b^2d \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)}} + \frac{(bd - ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2 \sqrt{d}} - \frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{2a \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] $-\text{Sqrt}[d + e*x^2]/(2*a*x^2) + (e*\text{ArcTanh}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[d]])/(2*a*\text{Sqrt}[d]) + ((b*d - a*e)*\text{ArcTanh}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[d]])/(a^2*\text{Sqrt}[d]) - (\text{Sqrt}[c]*(b^2*d - 2*a*c*d - a*b*e + \text{Sqrt}[b^2 - 4*a*c]*(b*d - a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[c]*(b^2*d - 2*a*c*d - a*b*e - \text{Sqrt}[b^2 - 4*a*c]*(b*d - a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1287

Int[(((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^2}{\left(-\frac{d}{e} + \frac{x^2}{e}\right)^2 \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}\right)} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(\frac{de^2}{a(d-x^2)^2} - \frac{e(-bd+ae)}{a^2(d-x^2)} + \frac{e(-b(cd^2-bde+ae^2)+c(bd-ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \frac{-b(cd^2-bde+ae^2)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{a^2} + \frac{(de) \text{Subst} \left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2 \sqrt{d}} + \frac{e \text{Subst} \left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2} \right)}{2a} - \frac{c(b^2d-2acd-ab^2)}{\sqrt{2} a^2} \\
&= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{2a \sqrt{d}} + \frac{(bd-ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2 \sqrt{d}} - \frac{\sqrt{c} (b^2d-2acd-ab^2)}{\sqrt{2} a^2}
\end{aligned}$$

Mathematica [A] time = 1.39, size = 349, normalized size = 0.91

$$\frac{\sqrt{2} \sqrt{c} \left(\frac{(-bd \sqrt{b^2-4ac} + ae \sqrt{b^2-4ac} + abe + 2acd + b^2(-d)) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{e \sqrt{b^2-4ac} - be + 2cd}} \right) - (bd \sqrt{b^2-4ac} - ae \sqrt{b^2-4ac} + abe + 2acd + b^2(-d)) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - e \sqrt{b^2-4ac} + b}} \right)}{\sqrt{e \sqrt{b^2-4ac} - be + 2cd}} - \frac{(bd \sqrt{b^2-4ac} - ae \sqrt{b^2-4ac} + abe + 2acd + b^2(-d)) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - e \sqrt{b^2-4ac} + b}} \right)}{\sqrt{2cd - e \sqrt{b^2-4ac} + b}} \right)}{\sqrt{b^2-4ac}} + \frac{(2bd-ae) \log(\sqrt{d+ex^2} + d)}{\sqrt{d}} + \frac{\log(x)(ae-2bd)}{\sqrt{d}} - \frac{a \sqrt{d+ex^2}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d) + 2*a*c*d - b*Sqrt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - (((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]*e)]))

)*e]))/Sqrt[b^2 - 4*a*c] + ((-2*b*d + a*e)*Log[x])/Sqrt[d] + ((2*b*d - a*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/(2*a^2)

IntegrateAlgebraic [A] time = 1.50, size = 398, normalized size = 1.04

$$\frac{(-b\sqrt{c}d\sqrt{b^2-4ac} + a\sqrt{c}e\sqrt{b^2-4ac} + ab\sqrt{c}e + 2ac^{3/2}d + b^2(-\sqrt{c})d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-c\sqrt{b^2-4ac}+be-2cd}}\right) + (b\sqrt{c}d\sqrt{b^2-4ac} - a\sqrt{c}e\sqrt{b^2-4ac} + ab\sqrt{c}e + 2ac^{3/2}d + b^2(-\sqrt{c})d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-c\sqrt{b^2-4ac}+be-2cd}}\right) + \frac{(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2\sqrt{d}} - \frac{\sqrt{d+ex^2}}{2ax^2}}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{-c\sqrt{b^2-4ac}+be-2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]

[Out]
$$-1/2*\text{Sqrt}[d + e*x^2]/(a*x^2) - ((-(b^2*\text{Sqrt}[c]*d) + 2*a*c^{(3/2)*d} - b*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c]*d + a*b*\text{Sqrt}[c]*e + a*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c]*e)*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/(\text{Sqrt}[-2*c*d + b*e - \text{Sqrt}[b^2 - 4*a*c]*e])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[-2*c*d + b*e - \text{Sqrt}[b^2 - 4*a*c]*e]) + ((-(b^2*\text{Sqrt}[c]*d) + 2*a*c^{(3/2)*d} + b*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c]*d + a*b*\text{Sqrt}[c]*e - a*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c]*e)*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/(\text{Sqrt}[-2*c*d + b*e + \text{Sqrt}[b^2 - 4*a*c]*e])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[-2*c*d + b*e + \text{Sqrt}[b^2 - 4*a*c]*e]) + ((2*b*d - a*e)*\text{ArcTanh}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[d]])/(2*a^2*\text{Sqrt}[d])$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e ,const index_m & i,const vecteur & l) Error: Bad Argument ValueDone

maple [C] time = 0.03, size = 401, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x)`

[Out] $\frac{1}{a^2 b d^{1/2}} \ln\left(\frac{(2d+2(e^2x^2+d)^{1/2})d^{1/2}}{x}\right) - \frac{1}{2a^2 b} (e^2x^2+d)^{1/2} - \frac{1}{2a^2 e^{1/2}} x b + \frac{1}{4a^2} \sum\left(\frac{(c(-ae+bd))_R^6 + (-4ab^2e^2 - ac^2d^2e + 4b^2d^2e - 3b^2c^2d^2)_R^4 + d(4ab^2e^2 + ac^2d^2e - 4b^2d^2e + 3b^2c^2d^2)_R^2 + ac^2d^3e - b^2c^2d^4}{(R^7c + 3R^5b^2e - 3R^5c^2d + 8R^3a^2e^2 - 4R^3b^2d^2e + 3R^3c^2d^2 + R^2bd^2e - R^2c^2d^3)} \ln(-e^{1/2}x - R + (e^2x^2+d)^{1/2})\right), R = \text{RootOf}(_Z^8c + (4b^2e - 4c^2d)_Z^6 + c^2d^4 + (16a^2e^2 - 8b^2d^2e + 6c^2d^2)_Z^4 + (4b^2d^2e - 4c^2d^3)_Z^2) + \frac{1}{2a^2 b d} (-e^{1/2}x + (e^2x^2+d)^{1/2}) - \frac{1}{2a} \frac{d}{x^2} (e^2x^2+d)^{3/2} - \frac{1}{2a} \frac{e}{d^{1/2}} \ln\left(\frac{(2d+2(e^2x^2+d)^{1/2})d^{1/2}}{x}\right) + \frac{1}{2a} \frac{e}{d} (e^2x^2+d)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)`

mupad [B] time = 5.46, size = 19959, normalized size = 52.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)`

[Out] $\left(\text{atan}\left(\frac{((ae - 2bd) * ((d + e^2x^2)^{1/2}) * (6a^4c^5e^{12} + 4a^2c^7d^4e^8 + 6a^3c^6d^2e^{10} + 4b^4c^5d^4e^8 + 21a^2b^2c^5d^2e^{10} - 18a^3b^2c^5d^2e^{11} - 8ab^2c^6d^4e^8 - 12ab^3c^5d^3e^9))}{(2a^4) - ((16a^5b^2c^4e^{12} + 20a^5c^5d^2e^{11} + a^3b^5c^2e^{12} - 8a^4b^3c^3e^{12} + 20a^4c^6d^3e^9 + 40a^2b^3c^5d^4e^8 - 20a^2b^4c^4d^3e^9 - 27a^2b^5c^3d^2e^{10} - 20a^3b^2c^5d^3e^9 + 84a^3b^3c^4d^2e^{10} - 8ab^5c^4d^4e^8 + 6ab^6c^3d^3e^9 + 2ab^7c^2d^2e^{10} - 3a^2b^6c^2d^2e^{11} - 32a^3b^2c^6d^4e^8 + 28a^3b^4c^3d^2e^{11} - 36a^4b^2c^5d^2e^{10} - 68a^4b^2c^4d^2e^{11})}{a^4} - ((ae - 2bd) * ((d + e^2x^2)^{1/2}) * (240a^6b^2c^4e^{11} + 64a^6c^5d^2e^{10} + 20a^4b^5c^2e^{11} - 140a^5b^3c^3e^{11} + 160a^5c^6d^3e^8 - 32a^2b^6c^3d^3e^8 + 32a^2b^7c^2d^2e^9 + 224a^3b^4c^4d^3e^8 - 208a^3b^5c^3d^2e^9 - 432a^4b^2c^5d^3e^8 + 272a^4b^3c^4d^2e^9 - 48a^3b^6c^2d^2e^{10} + 348a^4b^4c^3d^2e^{10} + 224a^5b^2c^5d^2e^9 - 648a^5b^2c^4d^2e^{10}))}{(2a^4) -$

$$\begin{aligned}
& ((a^2e - 2*bd)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2))*(a^2e - 2*bd)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2))*(a^2e - 2*bd))/(4*a^2*d^(1/2))*1i)/(4*a^2*d^(1/2)) + ((a^2e - 2*bd)*(((d + e*x^2)^(1/2))*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a^2e - 2*bd)*(((d + e*x^2)^(1/2))*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) + ((a^2e - 2*bd)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2))*(a^2e - 2*bd)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2))*(a^2e - 2*bd))/(4*a^2*d^(1/2))*1i)/(4*a^2*d^(1/2)))/(((a^3*c^5*e^13)/2 + a*c^7*d^4*e^9 - 2*b*c^7*d^5*e^8 + (3*a^2*c^6*d^2*e^11)/2 + 2*b^2*c^6*d^4*e^9 - 4*a*b*c^6*d^3*e^10 - (3*a^2*b*c^5*d*e^12)/2 + a*b^2*c^5*d^2*e^11)/a^4 - ((a^2e - 2*bd)*(((d + e*x^2)^(1/2))*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a^2e - 2*bd)*(((d + e*x^2)^(1/2))*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e
\end{aligned}$$

$$\begin{aligned}
&^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10)/(2*a^4) - ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2))*(a*e - 2*b*d))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2)))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2))*(a*e - 2*b*d))/(4*a^2*d^(1/2)))/(4*a^2*d^(1/2))*(a*e - 2*b*d)*i)/(2*a^2*d^(1/2)) - atan((((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) - ((d + e*x^2)^(1/2)*(-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^(1/2) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2} \right) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} + ((d + e*x^2)^{(1/2)} * (240*a^6*b*c^4*e^{11} + 64*a^6*c^5*d*e^{10} + \\
& 20*a^4*b^5*c^2*e^{11} - 140*a^5*b^3*c^3*e^{11} + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - \\
& 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^{10} + 348*a^4*b^4*c^3*d*e^{10} + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^{10})) / (2*a^4) * (-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} * (-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} + ((d + e*x^2)^{(1/2)} * (6*a^4*c^5*e^{12} + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^{10} + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^{10} - 18*a^3*b*c^5*d*e^{11} - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9)) / (2*a^4) * (-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} * i) / ((a^3*c^5*e^{13} + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^{11} + 4*b^2*c^6*d^4*e^9 - 8*a*b*c^6*d^3*e^{10} - 3*a^2*b*c^5*d*e^{12} + 2*a*b^2*c^5*d^2*e^{11}) / (2*a^4) + (((64*a^5*b*c^4*e^{12} + 80*a^5*c^5*d*e^{11} + 4*a^3*b^5*c^2*e^{12} - 32*a^4*b^3*c^3*e^{12} + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^{10} - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^{10} - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^{10} - 12*a^2*b^6*c^2*d*e^{11} - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^{11} - 144*a^4*b*c^5*d^2*e^{10} - 272*a^4*b^2*c^4*d*e^{11}) / (4*a^4) + (((512*a^8*c^4*e^{11} + 32*a^6*b^4*c^2*e^{11} - 256*a^7*b^2*c^3*e^{11} + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^{10} - 32*a^5*b^5*c^2*d*e^{10} - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^{10}) / (4*a^4) - ((d + e*x^2)^{(1/2)} * (-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} * (1024*a^9*c^4*e^{10} + 64*a^7*b^4*c^2*e^{10} - 512*a^8*b^2*c^3*e^{10} + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9)) / (2*a^4) * (-(8*a^3*c^3*d - b^6*d - b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*
\end{aligned}$$

$$\begin{aligned}
& a^5 b^2 c))^{(1/2)} - ((d + e x^2)^{(1/2)} * (240 a^6 b^3 c^4 e^{11} + 64 a^6 c^5 d^5 e^{10} + 20 a^4 b^5 c^2 e^{11} - 140 a^5 b^3 c^3 e^{11} + 160 a^5 c^6 d^3 e^8 - 3 \\
& 2 a^2 b^6 c^3 d^3 e^8 + 32 a^2 b^7 c^2 d^2 e^9 + 224 a^3 b^4 c^4 d^3 e^8 - 208 a^3 b^5 c^3 d^2 e^9 - 432 a^4 b^2 c^5 d^3 e^8 + 272 a^4 b^3 c^4 d^2 e^9 \\
& - 48 a^3 b^6 c^2 d e^{10} + 348 a^4 b^4 c^3 d e^{10} + 224 a^5 b^3 c^5 d^2 e^9 - 648 a^5 b^2 c^4 d e^{10})) / (2 a^4)) * (- (8 a^3 c^3 d - b^6 d - b^3 d * (- (4 a^3 c \\
& - b^2)^3)^{(1/2)} + a b^5 e - 18 a^2 b^2 c^2 d + 8 a b^4 c d + a b^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} - 7 a^2 b^3 c e + 12 a^3 b^3 c^2 e - a^2 c e * (- (4 a^3 c - b^2)^3)^{(1/2)} \\
& + 2 a b^3 c d * (- (4 a^3 c - b^2)^3)^{(1/2)}) / (8 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c))^{(1/2)} * (- (8 a^3 c^3 d - b^6 d - b^3 d * (- (4 a^3 c - b^2)^3)^{(1/2)} \\
& + a b^5 e - 18 a^2 b^2 c^2 d + 8 a b^4 c d + a b^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} - 7 a^2 b^3 c e + 12 a^3 b^3 c^2 e - a^2 c e * (- (4 a^3 c - b^2)^3)^{(1/2)} \\
& + 2 a b^3 c d * (- (4 a^3 c - b^2)^3)^{(1/2)}) / (8 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c))^{(1/2)} - ((d + e x^2)^{(1/2)} * (6 a^4 c^5 e^{12} + 4 a^2 c^7 d^4 e^8 + 6 a^3 c^6 d^2 e^{10} + 4 b^4 c^5 d^4 e^8 + 21 a^2 b^2 c^5 d^2 e^{10} - 18 a^3 b^3 c^5 \\
& d e^{11} - 8 a b^2 c^6 d^4 e^8 - 12 a b^3 c^5 d^3 e^9)) / (2 a^4)) * (- (8 a^3 c^3 d - b^6 d - b^3 d * (- (4 a^3 c - b^2)^3)^{(1/2)} + a b^5 e - 18 a^2 b^2 c^2 d + \\
& 8 a b^4 c d + a b^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} - 7 a^2 b^3 c e + 12 a^3 b^3 c^2 e - a^2 c e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 2 a b^3 c d * (- (4 a^3 c - b^2)^3)^{(1/2)}) / (8 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c))^{(1/2)} + (((64 a^5 b^3 c^4 e^{12} \\
& + 80 a^5 c^5 d e^{11} + 4 a^3 b^5 c^2 e^{12} - 32 a^4 b^3 c^3 e^{12} + 80 a^4 c^6 d^3 e^9 + 160 a^2 b^3 c^5 d^4 e^8 - 80 a^2 b^4 c^4 d^3 e^9 - 108 a^2 b^5 c^3 d^2 e^{10} - 80 a^3 b^2 c^5 d^3 e^9 + 336 a^3 b^3 c^4 d^2 e^{10} - 32 a b^5 c^4 d^4 e^8 + 24 a b^6 c^3 d^3 e^9 + 8 a b^7 c^2 d^2 e^{10} - 12 a^2 b^6 c^2 d e^{11} - 128 a^3 b^3 c^6 d^4 e^8 + 112 a^3 b^4 c^3 d e^{11} - 144 a^4 b^3 c^5 d^2 e^{10} - 272 a^4 b^2 c^4 d e^{11}) / (4 a^4) + (((512 a^8 c^4 e^{11} + 32 a^6 b^4 c^2 e^{11} - 256 a^7 b^2 c^3 e^{11} + 512 a^7 c^5 d^2 e^9 + 128 a^5 b^3 c^4 d^3 e^8 - 96 a^5 b^4 c^3 d^2 e^9 + 256 a^6 b^2 c^4 d^2 e^9 - 1024 a^7 b^3 c^4 d e^{10} - 32 a^5 b^5 c^2 d e^{10} - 512 a^6 b^3 c^5 d^3 e^8 + 384 a^6 b^3 c^3 d e^{10}) / (4 a^4) + ((d + e x^2)^{(1/2)} * (- (8 a^3 c^3 d - b^6 d - b^3 d * (- (4 a^3 c - b^2)^3)^{(1/2)} + a b^5 e - 18 a^2 b^2 c^2 d + 8 a b^4 c d + a b^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} - 7 a^2 b^3 c e + 12 a^3 b^3 c^2 e - a^2 c e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 2 a b^3 c d * (- (4 a^3 c - b^2)^3)^{(1/2)}) / (8 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c))^{(1/2)} * (1024 a^9 c^4 e^{10} + 64 a^7 b^4 c^2 e^{10} - 512 a^8 b^2 c^3 e^{10} + 1536 a^8 c^5 d^2 e^8 + 128 a^6 b^4 c^3 d^2 e^8 - 896 a^7 b^2 c^4 d^2 e^8 - 1792 a^8 b^3 c^4 d e^9 - 128 a^6 b^5 c^2 d e^9 + 960 a^7 b^3 c^3 d e^9)) / (2 a^4)) * (- (8 a^3 c^3 d - b^6 d - b^3 d * (- (4 a^3 c - b^2)^3)^{(1/2)} + a b^5 e - 18 a^2 b^2 c^2 d + 8 a b^4 c d + a b^2 e * (- (4 a^3 c - b^2)^3)^{(1/2)} - 7 a^2 b^3 c e + 12 a^3 b^3 c^2 e - a^2 c e * (- (4 a^3 c - b^2)^3)^{(1/2)} + 2 a b^3 c d * (- (4 a^3 c - b^2)^3)^{(1/2)}) / (8 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c))^{(1/2)} + ((d + e x^2)^{(1/2)} * (240 a^6 b^3 c^4 e^{11} + 64 a^6 c^5 d^5 e^{10} + 20 a^4 b^5 c^2 e^{11} - 140 a^5 b^3 c^3 e^{11} + 160 a^5 c^6 d^3 e^8 - 32 a^2 b^6 c^3 d^3 e^8 + 32 a^2 b^7 c^2 d^2 e^9 + 224 a^3 b^4 c^4 d^3 e^8 - 208 a^3 b^5 c^3 d^2 e^9 - 432 a^4 b^2 c^5 d^3 e^8 + 272 a^4 b^3 c^4 d^2 e^9 - 48 a^3 b^6 c^2 d e^{10} + 348 a^4 b^4 c^3 d e^{10} + 224 a^5 b^3 c^5 d^2 e^9 - 648 a^5 b^2 c^4 d e^{10}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{4*d*e^{10}}{(2*a^4)} * (- (8*a^3*c^3*d - b^6*d - b^3*d * (- (4*a*c - b^2)^3)^{1/2}) \right. \\
& + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} \\
& \left. - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} + 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2} \right) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)) \\
& \left. \right)^{1/2} * (- (8*a^3*c^3*d - b^6*d - b^3*d * (- (4*a*c - b^2)^3)^{1/2}) + a*b^5*e \\
& - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2 \\
& *b^3*c*e + 12*a^3*b*c^2*e - a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} + 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2} \\
& \left. \right) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2} + ((d + e*x^2)^{1/2} * (6*a^4*c^5*e^{12} + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^{10} \\
& + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^{10} - 18*a^3*b*c^5*d*e^{11} - 8*a*b^2*c^6*d^4*e^8 \\
& - 12*a*b^3*c^5*d^3*e^9)) / (2*a^4) * (- (8*a^3*c^3*d - b^6*d - b^3*d * (- (4*a*c - b^2)^3)^{1/2}) \\
& + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2*b^3*c*e \\
& + 12*a^3*b*c^2*e - a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} + 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2} \\
& \left. \right) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2} * (- (8*a^3*c^3*d - b^6*d - b^3*d * (- (4*a*c - b^2)^3)^{1/2}) \\
& + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d + a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2*b^3*c*e \\
& + 12*a^3*b*c^2*e - a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} + 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2} \\
& \left. \right) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2} * 2i - (d + e*x^2)^{1/2} / (2*a*x^2) - \operatorname{atan}\left(\frac{(64*a^5*b*c^4*e^{12} + 80*a^5*c^5*d*e^{11} + 4*a^3*b^5*c^2*e^{12} - 32*a^4*b^3*c^3*e^{12} + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^{10} - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^{10} - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^{10} - 12*a^2*b^6*c^2*d*e^{11} - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^{11} - 144*a^4*b*c^5*d^2*e^{10} - 272*a^4*b^2*c^4*d*e^{11})}{(4*a^4)} + \frac{(512*a^8*c^4*e^{11} + 32*a^6*b^4*c^2*e^{11} - 256*a^7*b^2*c^3*e^{11} + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^{10} - 32*a^5*b^5*c^2*d*e^{10} - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^{10})}{(4*a^4)} - ((d + e*x^2)^{1/2} * (- (8*a^3*c^3*d - b^6*d + b^3*d * (- (4*a*c - b^2)^3)^{1/2}) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} - 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2})}{(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2}} * (1024*a^9*c^4*e^{10} + 64*a^7*b^4*c^2*e^{10} - 512*a^8*b^2*c^3*e^{10} + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9) / (2*a^4) * (- (8*a^3*c^3*d - b^6*d + b^3*d * (- (4*a*c - b^2)^3)^{1/2}) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} - 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2} - ((d + e*x^2)^{1/2} * (240*a^6*b*c^4*e^{11} + 64*a^6*c^5*d^2*e^{10} + 20*a^4*b^5*c^2*e^{11} - 140*a^5*b^3*c^3*e^{11} + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^{10} + 348*a^4*b^4*c^3*d*e^{10} + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^{10})) / (2*a^4) * (- (8*a^3*c^3*d - b^6*d + b^3*d * (- (4*a*c - b^2)^3)^{1/2}) + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e * (- (4*a*c - b^2)^3)^{1/2} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e * (- (4*a*c - b^2)^3)^{1/2} - 2*a*b*c*d * (- (4*a*c - b^2)^3)^{1/2}) / (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} - ((d + e*x^2)^{(1/2)}*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*1i - (((64*a^5*b*c^4*e^12 + 80*a^5*c^5*d*e^11 + 4*a^3*b^5*c^2*e^12 - 32*a^4*b^3*c^3*e^12 + 80*a^4*c^6*d^3*e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d^2*e^10 - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^10 - 32*a*b^5*c^4*d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^10 - 12*a^2*b^6*c^2*d*e^11 - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^11 - 144*a^4*b*c^5*d^2*e^10 - 272*a^4*b^2*c^4*d*e^11)/(4*a^4) + (((512*a^8*c^4*e^11 + 32*a^6*b^4*c^2*e^11 - 256*a^7*b^2*c^3*e^11 + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^10 - 32*a^5*b^5*c^2*d*e^10 - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^10)/(4*a^4) + ((d + e*x^2)^{(1/2)}*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(2*a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4)*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} -
\end{aligned}$$

$$\begin{aligned}
& 6*d^2*e^{10} + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^{10} - 18*a^3*b*c^5*d*e \\
& ^{11} - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9)/(2*a^4))*(-(8*a^3*c^3*d \\
& - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a \\
& *b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2* \\
& e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/ \\
& (8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^{(1/2)} + (((64*a^5*b*c^4*e^{12} + 80 \\
& *a^5*c^5*d*e^{11} + 4*a^3*b^5*c^2*e^{12} - 32*a^4*b^3*c^3*e^{12} + 80*a^4*c^6*d^3 \\
& *e^9 + 160*a^2*b^3*c^5*d^4*e^8 - 80*a^2*b^4*c^4*d^3*e^9 - 108*a^2*b^5*c^3*d \\
& ^2*e^{10} - 80*a^3*b^2*c^5*d^3*e^9 + 336*a^3*b^3*c^4*d^2*e^{10} - 32*a*b^5*c^4* \\
& d^4*e^8 + 24*a*b^6*c^3*d^3*e^9 + 8*a*b^7*c^2*d^2*e^{10} - 12*a^2*b^6*c^2*d*e^ \\
& ^{11} - 128*a^3*b*c^6*d^4*e^8 + 112*a^3*b^4*c^3*d*e^{11} - 144*a^4*b*c^5*d^2*e^ \\
& ^{10} - 272*a^4*b^2*c^4*d*e^{11})/(4*a^4) + (((512*a^8*c^4*e^{11} + 32*a^6*b^4*c^2* \\
& e^{11} - 256*a^7*b^2*c^3*e^{11} + 512*a^7*c^5*d^2*e^9 + 128*a^5*b^3*c^4*d^3*e^8 \\
& - 96*a^5*b^4*c^3*d^2*e^9 + 256*a^6*b^2*c^4*d^2*e^9 - 1024*a^7*b*c^4*d*e^{10} \\
& - 32*a^5*b^5*c^2*d*e^{10} - 512*a^6*b*c^5*d^3*e^8 + 384*a^6*b^3*c^3*d*e^{10})/ \\
& (4*a^4) + ((d + e*x^2)^{(1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^ \\
& ^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^ \\
& 5*b^2*c)))^{(1/2)}*(1024*a^9*c^4*e^{10} + 64*a^7*b^4*c^2*e^{10} - 512*a^8*b^2*c^3 \\
& *e^{10} + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^ \\
& ^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^ \\
& ^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^ \\
& ^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7 \\
& *a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^{(1/2)} \\
& + ((d + e*x^2)^{(1/2)}*(240*a^6*b*c^4*e^{11} + 64*a^6*c^5*d*e^{10} + 20*a^4*b^5* \\
& c^2*e^{11} - 140*a^5*b^3*c^3*e^{11} + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3* \\
& e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^ \\
& ^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2* \\
& d*e^{10} + 348*a^4*b^4*c^3*d*e^{10} + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d \\
& *e^{10}))/ (2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a* \\
& b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^{(\\
& 1/2))*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 1 \\
& 8*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^ \\
& ^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4* \\
& a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^{(1/2)} + ((d \\
& + e*x^2)^{(1/2)}*(6*a^4*c^5*e^{12} + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^{10} + 4 \\
& *b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^{10} - 18*a^3*b*c^5*d*e^{11} - 8*a*b^2* \\
& c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4))*(-(8*a^3*c^3*d - b^6*d + b^3* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b \\
& ^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(- \\
& (4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^4*b^4 +
\end{aligned}$$

$$16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2))}*(-(8*a^3*c^3*d - b^6*d + b^3*d*(-(4*a*c - b^2)^3)^{(1/2)} + a*b^5*e - 18*a^2*b^2*c^2*d + 8*a*b^4*c*d - a*b^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 7*a^2*b^3*c*e + 12*a^3*b*c^2*e + a^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a*b*c*d*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*2i$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^3(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a), x)

[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)

$$3.260 \quad \int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=552

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(-abe - acd + b^2d) \sqrt{c} \left(b^2(d\sqrt{b^2 - 4ac} - ae) - ab(e\sqrt{b^2 - 4ac} + 3cd) - ac(d\sqrt{b^2 - 4ac} - \dots)\right)}{a^3\sqrt{d}} + \frac{\sqrt{2}a^3\sqrt{b^2 - 4ac} \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\dots}$$

Rubi [A] time = 4.24, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(-abe - acd + b^2d)}{a^3\sqrt{d}} + \frac{\sqrt{c} \left(b^2(d\sqrt{b^2 - 4ac} - ae) - ab(e\sqrt{b^2 - 4ac} + 3cd) - ac(d\sqrt{b^2 - 4ac} - \dots)\right)}{\sqrt{2}a^3\sqrt{b^2 - 4ac} \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}} + \frac{\sqrt{2}a^3\sqrt{b^2 - 4ac} \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d - a*e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[n, p] && FractionQ[m]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1287

Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^2}{\left(-\frac{d}{e} + \frac{x^2}{e}\right)^3 \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}\right)} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(-\frac{de^3}{a(d-x^2)^3} + \frac{e^2(-bd+ae)}{a^2(d-x^2)^2} + \frac{e(-b^2d+acd+abe)}{a^3(d-x^2)} + \frac{e((b^2-ac)(cd^2-bde+ae^2)-c(b^2d-acd-abe)x^2}{a^3(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)-c(b^2d-acd-abe)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{a^3} - \frac{(de^2) \text{Subst} \left(\int \frac{1}{(d-x^2)^3} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{\sqrt{d+ex^2}}{4ax^4} + \frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2} - \frac{(b^2d-acd-abe) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}} - \frac{(3e^2) \text{Subst} \left(\int \frac{1}{(d-x^2)^3} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{\sqrt{d+ex^2}}{4ax^4} + \frac{3e\sqrt{d+ex^2}}{8adx^2} + \frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2} - \frac{e(bd-ae) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}} - \frac{(b^2d-acd-abe) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}} \\
&= -\frac{\sqrt{d+ex^2}}{4ax^4} + \frac{3e\sqrt{d+ex^2}}{8adx^2} + \frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}} - \frac{e(bd-ae) \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 1.97, size = 466, normalized size = 0.84

$$\frac{\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\sqrt{4abd+e(a^2+8cd^2)-8d^2d^2} - \log(x)\sqrt{4abd+e(a^2+8cd^2)-8d^2d^2}}{d^{5/2}} - \frac{\log(x)\sqrt{4abd+e(a^2+8cd^2)-8d^2d^2}}{d^{5/2}} - \frac{4\sqrt{e}\sqrt{c}\left(\frac{(\sqrt{2}(a-d\sqrt{d-4ac})-a)(-\sqrt{d-4ac}+3cd)+c(\sqrt{d-4ac}-2ae)+b^2(-a)}{\sqrt{(d-4ac)-2cd}}\right)\sqrt{e}\sqrt{d+ex^2}}{\sqrt{(d-4ac)-2cd}} - \frac{(\sqrt{2}(a-d\sqrt{d-4ac})+a)(-\sqrt{d-4ac}-3cd)+c(\sqrt{d-4ac}+2ae)+b^2(-a)}{\sqrt{(d-4ac)+2cd}}\sqrt{e}\sqrt{d+ex^2}}{\sqrt{(d-4ac)+2cd}} + \frac{a\sqrt{d+ex^2}(4bd^2-e(2d+ex^2))}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) - (4*Sqrt[2]*Sqrt[c]*(((b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d - 2*a*e))))/d^3 - (3*e*Sqrt[d+e*x^2])/d^3 - (e*(b*d - a*e)*ArcTanh[Sqrt[d+e*x^2]/Sqrt[d]])/d^3 - (e*(b*d - a*e)*ArcTanh[Sqrt[d+e*x^2]/Sqrt[d]])/d^3)

$$a*c]*d) + a*e) + a*b*(3*c*d + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]]/\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e] + ((b^3*d - b^2*(\text{Sqrt}[b^2 - 4*a*c]*d + a*e) + a*c*(\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x^2])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]))/\text{Sqrt}[b^2 - 4*a*c] - ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*\text{Log}[x])/d^{(3/2)} + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*\text{Log}[d + \text{Sqrt}[d]*\text{Sqrt}[d + e*x^2]])/d^{(3/2)})/(8*a^3)$$

IntegrateAlgebraic [A] time = 2.39, size = 579, normalized size = 1.05

$$\frac{\sqrt{d+e^2}(-2ad-ae^2+dbd^2)}{8a^2c^2} + \frac{\text{atanh}\left(\frac{\sqrt{2cd}}{d}\right)(b^2+4bd+bae^2-8d^2)}{8a^2b^2} + \frac{(-2\sqrt{2}d^{3/2}e + \sqrt{2}ae^{3/2}\sqrt{b^2-4ac} - \sqrt{2}b^2\sqrt{c}\sqrt{b^2-4ac} + \sqrt{2}ab^2\sqrt{c}e + \sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac} + 3\sqrt{2}ab^2d - \sqrt{2}b^2\sqrt{c}d)\text{atan}\left(\frac{\sqrt{2}d\sqrt{2cd}}{\sqrt{d+e^2}-2a}\right)}{2a^2\sqrt{b^2-4ac}\sqrt{d+e^2}-4ac+be-2ad} + \frac{(2\sqrt{2}d^{3/2}e + \sqrt{2}ae^{3/2}\sqrt{b^2-4ac} - \sqrt{2}b^2\sqrt{c}\sqrt{b^2-4ac} - \sqrt{2}ab^2\sqrt{c}e + \sqrt{2}ab\sqrt{c}e\sqrt{b^2-4ac} - 3\sqrt{2}ab^2d + \sqrt{2}b^2\sqrt{c}d)\text{atan}\left(\frac{\sqrt{2}d\sqrt{2cd}}{\sqrt{d+e^2}+2a}\right)}{2a^2\sqrt{b^2-4ac}\sqrt{d+e^2}-4ac+be-2ad}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)), x]

[Out] (Sqrt[d + e*x^2]*(-2*a*d + 4*b*d*x^2 - a*e*x^2))/(8*a^2*d*x^4) + ((-(Sqrt[2]*b^3*Sqrt[c]*d) + 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[2]*b^2*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*b^2*Sqrt[c]*e - 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(2*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]) + ((Sqrt[2]*b^3*Sqrt[c]*d - 3*Sqrt[2]*a*b*c^(3/2)*d - Sqrt[2]*b^2*Sqrt[c]*Sqrt[b^2 - 4*a*c]*d + Sqrt[2]*a*c^(3/2)*Sqrt[b^2 - 4*a*c]*d - Sqrt[2]*a*b^2*Sqrt[c]*e + 2*Sqrt[2]*a^2*c^(3/2)*e + Sqrt[2]*a*b*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]])/(2*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e]) + ((-8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e + a^2*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*a^3*d^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.90, size = 1055, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8*(\sqrt{-4*c^2*d + 2*(b*c - \sqrt{b^2 - 4*a*c})*c}*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*\sqrt{b^2 - 4*a*c}*d^2 - (a*b^3 - a^2*b*c)*\sqrt{b^2 - 4*a*c}*d*e + (a^2*b^2 - a^3*c)*\sqrt{b^2 - 4*a*c}*e^2)*\sqrt{-4*c^2*d + 2*(b*c - \sqrt{b^2 - 4*a*c})*c}*e \\ &)*abs(a) - \sqrt{-4*c^2*d + 2*(b*c - \sqrt{b^2 - 4*a*c})*c}*e*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*\arctan(2*\sqrt{1/2}*\sqrt{x^2*e + d}/\sqrt{-(2*a^3*c*d - a^3*b*e + \sqrt{-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)}*a^3*c + (2*a^3*c*d - a^3*b*e)^2}))/((\sqrt{b^2 - 4*a*c})*a^4*c*d^2 - \sqrt{b^2 - 4*a*c})*a^4*b*d*e + \sqrt{b^2 - 4*a*c})*a^5*e^2)*abs(a)*abs(c)) + 1/8*(\sqrt{-4*c^2*d + 2*(b*c + \sqrt{b^2 - 4*a*c})*c}*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*\sqrt{b^2 - 4*a*c}*d^2 - (a*b^3 - a^2*b*c)*\sqrt{b^2 - 4*a*c}*d*e + (a^2*b^2 - a^3*c)*\sqrt{b^2 - 4*a*c}*e^2)*\sqrt{-4*c^2*d + 2*(b*c + \sqrt{b^2 - 4*a*c})*c}*e)*abs(a) - \sqrt{-4*c^2*d + 2*(b*c + \sqrt{b^2 - 4*a*c})*c}*e*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*\arctan(2*\sqrt{1/2}*\sqrt{x^2*e + d}/\sqrt{-(2*a^3*c*d - a^3*b*e - \sqrt{-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)}*a^3*c + (2*a^3*c*d - a^3*b*e)^2}))/((\sqrt{b^2 - 4*a*c})*a^4*c*d^2 - \sqrt{b^2 - 4*a*c})*a^4*b*d*e + \sqrt{b^2 - 4*a*c})*a^5*e^2)*abs(a)*abs(c)) + 1/8*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*\arctan(\sqrt{x^2*e + d}/\sqrt{-d}))/((\sqrt{b^2 - 4*a*c})*a^4*c*d^2 - \sqrt{b^2 - 4*a*c})*a^4*b*d*e + \sqrt{b^2 - 4*a*c})*a^5*e^2)*abs(a)*abs(c)) + 1/8*(4*(x^2*e + d)^(3/2)*b*d*e - 4*\sqrt{x^2*e + d}*b*d^2*e - (x^2*e + d)^(3/2)*a*e^2 - \sqrt{x^2*e + d}*a*d*e^2)*e^(-2))/(a^2*d*x^4) \end{aligned}$$

maple [C] time = 0.04, size = 655, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x)

[Out]
$$\begin{aligned} & 1/a^2*d^(1/2)*\ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*c-1/a^3*d^(1/2)*\ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)*b^2-1/2/a^2*(e*x^2+d)^(1/2)*c+1/2/a^3*(e*x^2+d)^(1/2)*b^2-1/2/a^2*e^(1/2)*x*c+1/2/a^3*e^(1/2)*x*b^2+1/4/a^3*\sum((c*(a*b*e+a*c*d-b^2*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^3*d*e+3*b^2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^2+4*b^3*d*e-3*b^2*c*d^2)*_R^2-a*b*c*d^3*e-a*c^2*d^4+b^2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*\ln(-e^(1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))+1/2/a^2*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*c-1/2/a^3*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2-1/4/a/d/x^4*(e*x^2+d)^(3/2)+1/8/a*e/d^2/x^2*(e*x^2+d)^(3/2)+1/8/a*e^2/d^(3/2)*\ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)-1/8/a*e^2/d^2*(e*x^2+d)^(1/2)+1/2/a^2*b/d/x^2*(e*x^2+d)^(3/2)+1/2/a^2*b*e/d^(1/2)*\ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2) \end{aligned}$$

))/x)-1/2/a^2*b*e/d*(e*x^2+d)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)

mupad [B] time = 7.30, size = 33925, normalized size = 61.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)

[Out] atan(((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^2 - ((d + e*x^2)^(1/2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2)))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9)/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2)))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*

$$\begin{aligned}
& d^2e^{11} - 16384a^7b^2c^6d^5e^8 + 7168a^7b^3c^5d^4e^9 + 19776a^7 \\
& *b^4c^4d^3e^{10} + 2272a^7b^5c^3d^2e^{11} - 18048a^8b^2c^5d^3e^{10} \\
& - 6144a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^3c^6d^4e^9 \\
& + 228a^8b^4c^3d^5e^{12} + 4608a^9b^3c^5d^2e^{11} - 408a^9b^2c^4d^5e^{12} \\
& 12)) / (32a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} - \\
& a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2 \\
& *e * (-4ac - b^2)^3)^{1/2} - 10ab^6cd + a^4b^4e * (-4ac - b^2)^3)^{1/2} \\
& + 9a^2b^5c^3e + 20a^4b^3c^3e + 4ab^3cd * (-4ac - b^2)^3)^{1/2} \\
& - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2} \\
& ^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} + (16a^9c^5e^{14} \\
& - 4a^6b^6c^2e^{14} + 28a^7b^4c^3e^{14} - 52a^8b^2c^4e^{14} - 768a^6c^8d^6e^8 \\
& - 768a^7c^7d^4e^{10} + 16a^8c^6d^2e^{12} - 512a^2b^8c^4d^6e^8 + 384a^2b^9c^3d^5e^9 \\
& + 128a^2b^10c^2d^4e^{10} + 3840a^3b^6c^5d^6e^8 - 2048a^3b^7c^4d^5e^9 \\
& - 2208a^3b^8c^3d^4e^{10} - 224a^3b^9c^2d^3e^{11} - 8704a^4b^4c^6d^6e^8 + 896a^4b^5c^5d^5e^9 \\
& + 10752a^4b^6c^4d^4e^{10} + 2688a^4b^7c^3d^3e^{11} + 96a^4b^8c^2d^2 \\
& e^{12} + 6400a^5b^2c^7d^6e^8 + 5632a^5b^3c^6d^5e^9 - 18144a^5b^4 \\
& c^5d^4e^{10} - 10464a^5b^5c^4d^3e^{11} - 836a^5b^6c^3d^2e^{12} + 93 \\
& 44a^6b^2c^6d^4e^{10} + 14592a^6b^3c^5d^3e^{11} + 2236a^6b^4c^4d^2 \\
& e^{12} - 1716a^7b^2c^5d^2e^{12} - 528a^8b^3c^5d^2e^{13} + 4a^5b^7c^2d^2 \\
& e^{13} - 4352a^6b^3c^7d^5e^9 - 92a^6b^5c^3d^2e^{13} - 5632a^7b^3c^6d^3 \\
& e^{11} + 436a^7b^3c^4d^5e^{13}) / (64a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * \\
& (-4ac - b^2)^3)^{1/2} - a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - \\
& 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10ab^6cd + a^4b^4 \\
& e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^3e + 20a^4b^3c^3e + 4ab^3cd * \\
& (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2 \\
& c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} \\
& - ((d + ex^2)^{1/2} * (a^6b^2c^5e^{14} - 2a^7c^6e^{14} + 192a^4c^9d^6e^8 \\
& + 32a^5c^8d^4e^{10} + 34a^6c^7d^2e^{12} + 64b^8c^5d^6e^8 + 704a^2b^4c^7d^6e^8 \\
& + 960a^2b^5c^6d^5e^9 + 192a^2b^6c^5d^4e^{10} - 512a^3b^2c^8d^6e^8 - 1280a^3b^3c^7d^5e^9 \\
& - 752a^3b^4c^6d^4e^{10} - 56a^3b^5c^5d^3e^{11} + 704a^4b^2c^7d^4e^{10} + 128a^4b^3 \\
& c^6d^3e^{11} - 15a^4b^4c^5d^2e^{12} + 60a^5b^2c^6d^2e^{12} - 10a^6b^3c^6d^2e^{13} \\
& - 384a^6b^6c^6d^6e^8 - 192a^6b^7c^5d^5e^9 + 384a^4b^3c^8d^5e^9 - 144a^5b^3c^7d^3e^{11} \\
& + 6a^5b^3c^5d^5e^{13})) / (32a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} \\
& - a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{1/2} \\
& - 10ab^6cd + a^4b^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^3e + 20a^4b^3c^3e \\
& + 4ab^3cd * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2 \\
& e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} * i - (((((2048a^{12}c^4d^5e^{12} \\
& + 12288a^{10}c^6d^5e^8 + 14336a^{11}c^5d^3e^{10} + 2048a^8b^4c^4d^5e^8 - 153 \\
& 6a^8b^5c^3d^4e^9 - 512a^8b^6c^2d^3e^{10} - 11264a^9b^2c^5d^5e^8 + 7168a^9b^3c^4d^4e^9 \\
& + 6272a^9b^4c^3d^3e^{10} + 384a^9b^5c^2d^2e^{11} - 20480a^{10}b^2c^4d^3e^{10} - 3072a^{10}b^3c^3d^2e^{11} - 4096*
\end{aligned}$$

$$\begin{aligned}
& a^{10}b^5c^5d^4e^9 + 128a^{10}b^4c^2d^2e^{12} + 6144a^{11}b^5c^4d^2e^{11} - 1024a^{11}b^2c^3d^2e^{12}) / (64a^8d^2) + ((d + ex^2)^{1/2}) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10a^4b^6cd + ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} * (24576a^{12}c^5d^4e^8 + 16384a^{13}c^4d^2e^{10} + 2048a^{10}b^4c^3d^4e^8 - 2048a^{10}b^5c^2d^3e^9 - 14336a^{11}b^2c^4d^4e^8 + 15360a^{11}b^3c^3d^3e^9 + 1024a^{11}b^4c^2d^2e^{10} - 8192a^{12}b^2c^3d^2e^{10} - 28672a^{12}b^2c^4d^3e^9) / (32a^8d^2)) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10a^4b^6cd + ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} - ((d + ex^2)^{1/2}) * (32a^{10}c^5d^2e^{12} - 48a^{10}b^2c^4e^{13} - 4a^8b^5c^2e^{13} + 28a^9b^3c^3e^{13} + 4608a^8c^7d^5e^8 + 2048a^9c^6d^3e^{10} + 512a^4b^8c^3d^5e^8 - 512a^4b^9c^2d^4e^9 - 4608a^5b^6c^4d^5e^8 + 4352a^5b^7c^3d^4e^9 + 768a^5b^8c^2d^3e^{10} + 14080a^6b^4c^5d^5e^8 - 11264a^6b^5c^4d^4e^9 - 6912a^6b^6c^3d^3e^{10} - 256a^6b^7c^2d^2e^{11} - 16384a^7b^2c^6d^5e^8 + 7168a^7b^3c^5d^4e^9 + 19776a^7b^4c^4d^3e^{10} + 2272a^7b^5c^3d^2e^{11} - 18048a^8b^2c^5d^3e^{10} - 6144a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^2c^6d^4e^9 + 228a^8b^4c^3d^2e^{12} + 4608a^9b^2c^5d^2e^{11} - 408a^9b^2c^4d^2e^{12})) / (32a^8d^2)) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10a^4b^6cd + ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} + (16a^9c^5e^{14} - 4a^6b^6c^2e^{14} + 28a^7b^4c^3e^{14} - 52a^8b^2c^4e^{14} - 768a^6c^8d^6e^8 - 768a^7c^7d^4e^{10} + 16a^8c^6d^2e^{12} - 512a^2b^8c^4d^6e^8 + 384a^2b^9c^3d^5e^9 + 128a^2b^10c^2d^4e^{10} + 3840a^3b^6c^5d^6e^8 - 2048a^3b^7c^4d^5e^9 - 2208a^3b^8c^3d^4e^{10} - 224a^3b^9c^2d^3e^{11} - 8704a^4b^4c^6d^6e^8 + 896a^4b^5c^5d^5e^9 + 10752a^4b^6c^4d^4e^{10} + 2688a^4b^7c^3d^3e^{11} + 96a^4b^8c^2d^2e^{12} + 6400a^5b^2c^7d^6e^8 + 5632a^5b^3c^6d^5e^9 - 18144a^5b^4c^5d^4e^{10} - 10464a^5b^5c^4d^3e^{11} - 836a^5b^6c^3d^2e^{12} + 9344a^6b^2c^6d^4e^{10} + 14592a^6b^3c^5d^3e^{11} + 2236a^6b^4c^4d^2e^{12} - 1716a^7b^2c^5d^2e^{12} - 528a^8b^2c^5d^2e^{13} + 4a^5b^7c^2d^2e^{13} - 4352a^6b^2c^7d^5e^9 - 92a^6b^5c^3d^2e^{13} - 5632a^7b^2c^6d^3e^{11} + 436a^7b^3c^4d^2e^{13}) / (64a^8d^2)) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac -
\end{aligned}$$

$$\begin{aligned}
& b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5 \\
& *c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b*c^2* \\
& d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^ \\
& 6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(a^6*b^2*c^5 \\
& *e^{14} - 2*a^7*c^6*e^{14} + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6 \\
& *c^7*d^2*e^{12} + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5* \\
& c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^{10} - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3 \\
& *b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704 \\
& *a^4*b^2*c^7*d^4*e^{10} + 128*a^4*b^3*c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} \\
& + 60*a^5*b^2*c^6*d^2*e^{12} - 10*a^6*b*c^6*d*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 1 \\
& 92*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a \\
& ^5*b^3*c^5*d*e^{13}))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3* \\
& c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)}*i)/ \\
& ((((((2048*a^12*c^4*d^5*e^{12} + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^3*e^ \\
& 10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6*c^2* \\
& d^3*e^{10} - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272*a^9* \\
& b^4*c^3*d^3*e^{10} + 384*a^9*b^5*c^2*d^2*e^{11} - 20480*a^10*b^2*c^4*d^3*e^{10} - \\
& 3072*a^10*b^3*c^3*d^2*e^{11} - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^2*d* \\
& e^{12} + 6144*a^11*b*c^4*d^2*e^{11} - 1024*a^11*b^2*c^3*d*e^{12}))/((64*a^8*d^2) - \\
& ((d + e*x^2)^{(1/2)}*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2 \\
& *e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^{(1/ \\
& 2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3) \\
& ^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)}*(24576*a^12*c^5*d^4 \\
& *e^8 + 16384*a^13*c^4*d^2*e^{10} + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5* \\
& c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 102 \\
& 4*a^11*b^4*c^2*d^2*e^{10} - 8192*a^12*b^2*c^3*d^2*e^{10} - 28672*a^12*b*c^4*d^3 \\
& *e^9))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^ \\
& ^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^{(\\
& 1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2) \\
&) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + ((d + e*x^2)^{(1 \\
& /2)}*(32*a^10*c^5*d^5*e^{12} - 48*a^10*b*c^4*e^{13} - 4*a^8*b^5*c^2*e^{13} + 28*a^9* \\
& b^3*c^3*e^{13} + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^{10} + 512*a^4*b^8*c \\
& ^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5* \\
& b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^{10} + 14080*a^6*b^4*c^5*d^5*e^8 - 11 \\
& 264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^{10} - 256*a^6*b^7*c^2*d^2*e \\
& ^{11} - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4* \\
& c^4*d^3*e^{10} + 2272*a^7*b^5*c^3*d^2*e^{11} - 18048*a^8*b^2*c^5*d^3*e^{10} - 614
\end{aligned}$$

$$\begin{aligned}
& 4a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^3c^6d^4e^9 + 2 \\
& 28a^8b^4c^3d^2e^{12} + 4608a^9b^3c^5d^2e^{11} - 408a^9b^2c^4d^2e^{12}) / \\
& (32a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{(1/2)} - ab^7e \\
& + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (- \\
& (4ac - b^2)^3)^{(1/2)} - 10ab^6cd + ab^4e * (-4ac - b^2)^3)^{(1/2)} + \\
& 9a^2b^5c^2e + 20a^4b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2 \\
& b^2c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)} \\
&)) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} + (16a^9c^5e^{14} - 4a^6 \\
& b^6c^2e^{14} + 28a^7b^4c^3e^{14} - 52a^8b^2c^4e^{14} - 768a^6c^8d^6e^8 - 768a^7 \\
& c^7d^4e^{10} + 16a^8c^6d^2e^{12} - 512a^2b^8c^4d^6e^8 + 384a^2b^9c^3d^5e^9 + 128a^2 \\
& b^{10}c^2d^4e^{10} + 3840a^3b^6c^5d^6e^8 - 2048a^3b^7c^4d^5e^9 - 2208a^3b^8c^3d^4e^{10} - 224a^3 \\
& b^9c^2d^3e^{11} - 8704a^4b^4c^6d^6e^8 + 896a^4b^5c^5d^5e^9 + 10752a^4b^6c^4d^4e^{10} + 2688a^4 \\
& b^7c^3d^3e^{11} + 96a^4b^8c^2d^2e^{12} + 6400a^5b^2c^7d^6e^8 + 5632a^5b^3c^6d^5e^9 - 18144a^5 \\
& b^4c^5d^4e^{10} - 10464a^5b^5c^4d^3e^{11} - 836a^5b^6c^3d^2e^{12} + 9344a^6b^2c^6d^4e^{10} + 14592a^6 \\
& b^3c^5d^3e^{11} + 2236a^6b^4c^4d^2e^{12} - 1716a^7b^2c^5d^2e^{12} - 528a^8b^3c^5d^2e^{13} + 4a^5 \\
& b^7c^2d^2e^{13} - 4352a^6b^3c^7d^5e^9 - 92a^6b^5c^3d^2e^{13} - 5632a^7b^3c^6d^3e^{11} + 436a^7 \\
& b^3c^4d^2e^{13}) / (64a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{(1/2)} - ab^7e \\
& + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{(1/2)} - 10ab^6cd + ab^4e * (- \\
& (4ac - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2 \\
& b^2c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)} / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} \\
&) - ((d + ex^2)^{(1/2)} * (a^6b^2c^5e^{14} - 2a^7c^6e^{14} + 192a^4c^9d^6e^8 + 32a^5c^8d^4e^{10} + 34a^6c^7d^2e^{12} + 64b^8c^5d^6e^8 + 704 \\
& a^2b^4c^7d^6e^8 + 960a^2b^5c^6d^5e^9 + 192a^2b^6c^5d^4e^{10} - 512a^3b^2c^8d^6e^8 - 1280a^3b^3c^7d^5e^9 - 752a^3b^4c^6d^4e^{10} - 56a^3 \\
& b^5c^5d^3e^{11} + 704a^4b^2c^7d^4e^{10} + 128a^4b^3c^6d^3e^{11} - 15a^4b^4c^5d^2e^{12} + 60a^5b^2c^6d^2e^{12} - 10a^6b^3c^6d^2e^{13} - 384a^6 \\
& b^4c^6d^6e^8 - 192a^6b^7c^5d^5e^9 + 384a^4b^3c^8d^5e^9 - 144a^5b^3c^7d^3e^{11} + 6a^5b^3c^5d^2e^{13}) / (32a^8d^2) * ((b^8d + 8a^4c^4d - b^5d * (-4ac - b^2)^3)^{(1/2)} - ab^7e \\
& + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e + a^3c^2e * (-4ac - b^2)^3)^{(1/2)} - 10ab^6cd + ab^4e * (-4ac - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4 \\
& b^3c^3e + 4ab^3c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2d * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)} / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} + (((((2048a^12c^4d^2e^{12} + 12288a^{10}c^6 \\
& d^5e^8 + 14336a^{11}c^5d^3e^{10} + 2048a^8b^4c^4d^5e^8 - 1536a^8b^5c^3d^4e^9 - 512a^8b^6c^2d^3e^{10} - 11264a^9b^2c^5d^5e^8 + 7168 \\
& a^9b^3c^4d^4e^9 + 6272a^9b^4c^3d^3e^{10} + 384a^9b^5c^2d^2e^{11} - 20480a^{10}b^2c^4d^3e^{10} - 3072a^{10}b^3c^3d^2e^{11} - 4096a^{10}b^4c^5d^4e^9 + 128a^{10}b^4c^2d^2e^{12} + 6144a^{11}b^3c^4d^2e^{11} - 1024a^{11} \\
& b^2c^3d^2e^{12}) / (64a^8d^2) + ((d + ex^2)^{(1/2)} * ((b^8d + 8a^4c^4d -
\end{aligned}$$

$$\begin{aligned}
& b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d \\
& + a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))^{(1/2)}*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10* \\
& b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3 \\
& *d^2*e^10 - 28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - \\
& 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a \\
& *b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7* \\
& b^2*c))^{(1/2)} - ((d + e*x^2)^{(1/2)}*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048* \\
& a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 \\
& + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7* \\
& b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e \\
& ^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2)*((b^8*d + 8*a^4*c^4*d - b^5*d \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b \\
& ^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2 \\
& *b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))^{(1/2)} + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52* \\
& a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10 \\
& *c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 \\
& + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3 \\
& *c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3* \\
& e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d^2*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d \\
& *e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - \\
& 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^6*b^4 + \\
& 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(a^6*b^2*c^5*e^{14} - \\
& 2*a^7*c^6*e^{14} + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6*c^7*d^2 \\
& *e^{12} + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5* \\
& e^9 + 192*a^2*b^6*c^5*d^4*e^{10} - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7 \\
& *d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704*a^4*b^2 \\
& *c^7*d^4*e^{10} + 128*a^4*b^3*c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} + 60*a^5 \\
& *b^2*c^6*d^2*e^{12} - 10*a^6*b*c^6*d*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7 \\
& *c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a^5*b^3*c \\
& ^5*d*e^{13}))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^ \\
& (1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + \\
& a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + (7*a^5*c^7 \\
& *d*e^{14} + 56*a^3*c^9*d^5*e^{10} + 63*a^4*c^8*d^3*e^{12} - 64*b^4*c^8*d^7*e^8 + \\
& 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^{10} + 224*a^2*b^3*c^7*d^4*e^{11} - 1 \\
& 12*a^3*b^2*c^7*d^3*e^{12} + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192 \\
& *a*b^4*c^7*d^5*e^{10} - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^{11} + 9*a^4 \\
& *b*c^7*d^2*e^{13}))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^ \\
& 2)^3)^{(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^ \\
& 2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*c*e*(-(4* \\
& a*c - b^2)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)}*2i + a \\
& \tan(((((((2048*a^12*c^4*d*e^{12} + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d^ \\
& 3*e^{10} + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6* \\
& c^2*d^3*e^{10} - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272* \\
& a^9*b^4*c^3*d^3*e^{10} + 384*a^9*b^5*c^2*d^2*e^{11} - 20480*a^10*b^2*c^4*d^3*e^ \\
& 10 - 3072*a^10*b^3*c^3*d^2*e^{11} - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c^ \\
& 2*d*e^{12} + 6144*a^11*b*c^4*d^2*e^{11} - 1024*a^11*b^2*c^3*d*e^{12}))/((64*a^8*d^2 \\
&) - ((d + e*x^2)^{(1/2)}*(b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^{(1/ \\
& 2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3 \\
& *c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1 \\
& /2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e*(-(4*a*c - b^2 \\
&)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)}*(24576*a^12*c^5 \\
& *d^4*e^8 + 16384*a^13*c^4*d^2*e^{10} + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10* \\
& b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + \\
& 1024*a^11*b^4*c^2*d^2*e^{10} - 8192*a^12*b^2*c^3*d^2*e^{10} - 28672*a^12*b*c^4 \\
& *d^3*e^9))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^{(\\
& 1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a \\
& ^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^ \\
& (1/2) + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e*(-(4*a*c - b
\end{aligned}$$

$$\begin{aligned}
& \left. \right)^2)^3)^{(1/2)) / (8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))^{(1/2)} + ((d + e*x^2) \\
&)^{(1/2)} * (32*a^{10}*c^5*d*e^{12} - 48*a^{10}*b*c^4*e^{13} - 4*a^8*b^5*c^2*e^{13} + 28* \\
& a^9*b^3*c^3*e^{13} + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^{10} + 512*a^4*b \\
& ^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352* \\
& a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^{10} + 14080*a^6*b^4*c^5*d^5*e^8 \\
& - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^{10} - 256*a^6*b^7*c^2*d \\
& ^2*e^{11} - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7* \\
& b^4*c^4*d^3*e^{10} + 2272*a^7*b^5*c^3*d^2*e^{11} - 18048*a^8*b^2*c^5*d^3*e^{10} - \\
& 6144*a^8*b^3*c^4*d^2*e^{11} - 32*a^7*b^6*c^2*d*e^{12} + 3584*a^8*b*c^6*d^4*e^9 \\
& + 228*a^8*b^4*c^3*d*e^{12} + 4608*a^9*b*c^5*d^2*e^{11} - 408*a^9*b^2*c^4*d*e^{11} \\
& 2)) / (32*a^8*d^2)) * ((b^8*d + 8*a^4*c^4*d + b^5*d * (- (4*a*c - b^2)^3)^{(1/2)} - \\
& a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2* \\
& e * (- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e * (- (4*a*c - b^2)^3)^{(1/2)} \\
&) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d * (- (4*a*c - b^2)^3)^{(1/2)} + \\
& 3*a^2*b*c^2*d * (- (4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e * (- (4*a*c - b^2)^3)^{(1/2)} \\
& (1/2)) / (8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))^{(1/2)} + (16*a^9*c^5*e^{14} - \\
& 4*a^6*b^6*c^2*e^{14} + 28*a^7*b^4*c^3*e^{14} - 52*a^8*b^2*c^4*e^{14} - 768*a^6*c \\
& ^8*d^6*e^8 - 768*a^7*c^7*d^4*e^{10} + 16*a^8*c^6*d^2*e^{12} - 512*a^2*b^8*c^4*d \\
& ^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^{10} + 3840*a^3*b^6 \\
& *c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^{10} - 224*a \\
& ^3*b^9*c^2*d^3*e^{11} - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + \\
& 10752*a^4*b^6*c^4*d^4*e^{10} + 2688*a^4*b^7*c^3*d^3*e^{11} + 96*a^4*b^8*c^2*d^2 \\
& *e^{12} + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4 \\
& *c^5*d^4*e^{10} - 10464*a^5*b^5*c^4*d^3*e^{11} - 836*a^5*b^6*c^3*d^2*e^{12} + 934 \\
& 4*a^6*b^2*c^6*d^4*e^{10} + 14592*a^6*b^3*c^5*d^3*e^{11} + 2236*a^6*b^4*c^4*d^2* \\
& e^{12} - 1716*a^7*b^2*c^5*d^2*e^{12} - 528*a^8*b*c^5*d*e^{13} + 4*a^5*b^7*c^2*d*e \\
& ^{13} - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^{13} - 5632*a^7*b*c^6*d^3*e \\
& ^{11} + 436*a^7*b^3*c^4*d*e^{13}) / (64*a^8*d^2)) * ((b^8*d + 8*a^4*c^4*d + b^5*d * (\\
& - (4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 2 \\
& 5*a^3*b^3*c^2*e - a^3*c^2*e * (- (4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4 \\
& *e * (- (4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d * \\
& (- (4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^2*d * (- (4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b \\
& ^2*c*e * (- (4*a*c - b^2)^3)^{(1/2)) / (8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c))^{(1/2)} - \\
& ((d + e*x^2)^{(1/2)} * (a^6*b^2*c^5*e^{14} - 2*a^7*c^6*e^{14} + 192*a^4*c^9 \\
& *d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6*c^7*d^2*e^{12} + 64*b^8*c^5*d^6*e^8 + \\
& 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^ \\
& 10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d \\
& ^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704*a^4*b^2*c^7*d^4*e^{10} + 128*a^4*b^3* \\
& c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} + 60*a^5*b^2*c^6*d^2*e^{12} - 10*a^6*b \\
& *c^6*d*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8 \\
& *d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a^5*b^3*c^5*d*e^{13})) / (32*a^8*d^2)) * ((\\
& b^8*d + 8*a^4*c^4*d + b^5*d * (- (4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4 \\
& *c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e * (- (4*a*c - b^2)^3)^{(1/2)} - \\
& 10*a*b^6*c*d - a*b^4*e * (- (4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + \\
& 20*a^4*b*c^3*e - 4*a*b^3*c*d * (- (4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^2*d * (- (4*
\end{aligned}$$

$$\begin{aligned}
& a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2e^2(-4a^2c - b^2)^3)^{(1/2)))/(8(a^6b^4 + \\
& 16a^8c^2 - 8a^7b^2c))^{(1/2)}*i - (((((2048a^{12}c^4d^2e^{12} + 12288a^{10}c^6d^5e^8 + 14336a^{11}c^5d^3e^{10} + 2048a^8b^4c^4d^5e^8 - 1536 \\
& a^8b^5c^3d^4e^9 - 512a^8b^6c^2d^3e^{10} - 11264a^9b^2c^5d^5e^8 \\
& + 7168a^9b^3c^4d^4e^9 + 6272a^9b^4c^3d^3e^{10} + 384a^9b^5c^2d^2e^{11} - 20480a^{10}b^2c^4d^3e^{10} - 3072a^{10}b^3c^3d^2e^{11} - 4096a^{10}b^4c^2d^2e^{11} \\
& + 128a^{10}b^4c^2d^2e^{12} + 6144a^{11}b^3c^4d^2e^{11} - 1024a^{11}b^2c^3d^2e^{12}))/((64a^8d^2) + ((d + e^2x^2)^{(1/2)}*((b^8d + 8a^4c^4d + b^5d*(-4a^2c - b^2)^3)^{(1/2)} - a^2b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e*(-4a^2c - b^2)^3)^{(1/2)} - 10a^2b^6c^2d - a^2b^4e*(-4a^2c - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^2b^3c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2e*(-4a^2c - b^2)^3)^{(1/2)))/(8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)}*(24576a^{12}c^5d^4e^8 + 16384a^{13}c^4d^2e^{10} + 2048a^{10}b^4c^3d^4e^8 - 2048a^{10}b^5c^2d^3e^9 - 14336a^{11}b^2c^4d^4e^8 + 15360a^{11}b^3c^3d^3e^9 + 1024a^{11}b^4c^2d^2e^{10} - 8192a^{12}b^2c^3d^2e^{10} - 28672a^{12}b^3c^4d^3e^9))/((32a^8d^2))*((b^8d + 8a^4c^4d + b^5d*(-4a^2c - b^2)^3)^{(1/2)} - a^2b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e*(-4a^2c - b^2)^3)^{(1/2)} - 10a^2b^6c^2d - a^2b^4e*(-4a^2c - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^2b^3c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2e*(-4a^2c - b^2)^3)^{(1/2)))/(8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} - ((d + e^2x^2)^{(1/2)}*(32a^{10}c^5d^2e^{12} - 48a^{10}b^2c^4e^{13} - 4a^8b^5c^2e^{13} + 28a^9b^3c^3e^{13} + 4608a^8c^7d^5e^8 + 2048a^9c^6d^3e^{10} + 512a^4b^8c^3d^5e^8 - 512a^4b^9c^2d^4e^9 - 4608a^5b^6c^4d^5e^8 + 4352a^5b^7c^3d^4e^9 + 768a^5b^8c^2d^3e^{10} + 14080a^6b^4c^5d^5e^8 - 11264a^6b^5c^4d^4e^9 - 6912a^6b^6c^3d^3e^{10} - 256a^6b^7c^2d^2e^{11} - 16384a^7b^2c^6d^5e^8 + 7168a^7b^3c^5d^4e^9 + 19776a^7b^4c^4d^3e^{10} + 2272a^7b^5c^3d^2e^{11} - 18048a^8b^2c^5d^3e^{10} - 6144a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^3c^6d^4e^9 + 228a^8b^4c^3d^2e^{12} + 4608a^9b^3c^5d^2e^{11} - 408a^9b^2c^4d^2e^{12}))/((32a^8d^2))*((b^8d + 8a^4c^4d + b^5d*(-4a^2c - b^2)^3)^{(1/2)} - a^2b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e*(-4a^2c - b^2)^3)^{(1/2)} - 10a^2b^6c^2d - a^2b^4e*(-4a^2c - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^2b^3c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2d*(-4a^2c - b^2)^3)^{(1/2)} + 3a^2b^2c^2e*(-4a^2c - b^2)^3)^{(1/2)))/(8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} + (16a^9c^5e^{14} - 4a^6b^6c^2e^{14} + 28a^7b^4c^3e^{14} - 52a^8b^2c^4e^{14} - 768a^6c^8d^6e^8 - 768a^7c^7d^4e^{10} + 16a^8c^6d^2e^{12} - 512a^2b^8c^4d^6e^8 + 384a^2b^9c^3d^5e^9 + 128a^2b^10c^2d^4e^{10} + 3840a^3b^6c^5d^6e^8 - 2048a^3b^7c^4d^5e^9 - 2208a^3b^8c^3d^4e^{10} - 224a^3b^9c^2d^3e^{11} - 8704a^4b^4c^6d^6e^8 + 896a^4b^5c^5d^5e^9 + 10752a^4b^6c^4d^4e^{10} + 2688a^4b^7c^3d^3e^{11} + 96a^4b^8c^2d^2e^{12} + 6400a^5b^2c^7d^6e^8 + 5632a^5b^3c^6d^5e^9 - 18144a^5b^4c^5d^4e^{10} - 10464a^5b^5c^4d^3e^
\end{aligned}$$

$$\begin{aligned}
& 11 - 836a^5b^6c^3d^2e^{12} + 9344a^6b^2c^6d^4e^{10} + 14592a^6b^3c^5d^3e^{11} + 2236a^6b^4c^4d^2e^{12} - 1716a^7b^2c^5d^2e^{12} - 528a^8b^3c^5d^2e^{13} \\
& + 4a^5b^7c^2d^2e^{13} - 4352a^6b^3c^7d^5e^9 - 92a^6b^5c^3d^2e^{13} - 5632a^7b^3c^6d^3e^{11} + 436a^7b^3c^4d^2e^{13}) / (64a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d \\
& - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10ab^6cd - ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4ab^3c^3d * (-4ac - b^2)^3)^{1/2} \\
& + 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} + ((d + ex^2)^{1/2} * (a^6b^2c^5e^{14} - 2a^7c^6e^{14} \\
& + 192a^4c^9d^6e^8 + 32a^5c^8d^4e^{10} + 34a^6c^7d^2e^{12} + 64b^8c^5d^6e^8 + 704a^2b^4c^7d^6e^8 + 960a^2b^5c^6d^5e^9 + 192a^2b^6c^5d^4e^{10} - 512a^3b^2c^8d^6e^8 - 1280a^3b^3c^7d^5e^9 \\
& - 752a^3b^4c^6d^4e^{10} - 56a^3b^5c^5d^3e^{11} + 704a^4b^2c^7d^4e^{10} + 128a^4b^3c^6d^3e^{11} - 15a^4b^4c^5d^2e^{12} + 60a^5b^2c^6d^2e^{12} - 10a^6b^3c^6d^2e^{13} - 384a^6b^6c^6d^6e^8 - 192a^6b^7c^5d^5e^9 \\
& + 384a^4b^3c^8d^5e^9 - 144a^5b^3c^7d^3e^{11} + 6a^5b^3c^5d^2e^{13})) / (32a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{1/2} \\
& - 10ab^6cd - ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4ab^3c^3d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} * ii) / ((((((2048a^{12}c^4d^2e^{12} + 12288a^{10}c^6d^5e^8 + 14336a^{11}c^5d^3e^{11} + 2048a^8b^4c^4d^5e^8 - 1536a^8b^5c^3d^4e^9 - 512a^8b^6c^2d^3e^{10} - 11264a^9b^2c^5d^5e^8 + 7168a^9b^3c^4d^4e^9 + 6272a^9b^4c^3d^3e^{10} + 384a^9b^5c^2d^2e^{11} - 20480a^{10}b^2c^4d^3e^{10} - 3072a^{10}b^3c^3d^2e^{11} - 4096a^{10}b^4c^2d^2e^{12} + 6144a^{11}b^2c^3d^2e^{12}) / (64a^8d^2) - (d + ex^2)^{1/2} * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10ab^6cd - ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4ab^3c^3d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} * (24576a^{12}c^5d^4e^8 + 16384a^{13}c^4d^2e^{10} + 2048a^{10}b^4c^3d^4e^8 - 2048a^{10}b^5c^2d^3e^9 - 14336a^{11}b^2c^4d^4e^8 + 15360a^{11}b^3c^3d^3e^9 + 1024a^{11}b^4c^2d^2e^{10} - 8192a^{12}b^2c^3d^2e^{10} - 28672a^{12}b^3c^4d^3e^9)) / (32a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{1/2} - ab^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{1/2} - 10ab^6cd - ab^4e * (-4ac - b^2)^3)^{1/2} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4ab^3c^3d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{1/2} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{1/2}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{1/2} + ((d + ex^2)^{1/2} * (32a^{10}c^5d^2e^{12} - 48a^{10}b^3c^4e^{13} - 4a^8b^5c^2e^{13} + 28a^9b
\end{aligned}$$

$$\begin{aligned}
& ^3c^3e^{13} + 4608a^8c^7d^5e^8 + 2048a^9c^6d^3e^{10} + 512a^4b^8c^3d^5e^8 - 512a^4b^9c^2d^4e^9 - 4608a^5b^6c^4d^5e^8 + 4352a^5b^7c^3d^4e^9 + 768a^5b^8c^2d^3e^{10} + 14080a^6b^4c^5d^5e^8 - 11264a^6b^5c^4d^4e^9 - 6912a^6b^6c^3d^3e^{10} - 256a^6b^7c^2d^2e^{11} - 16384a^7b^2c^6d^5e^8 + 7168a^7b^3c^5d^4e^9 + 19776a^7b^4c^4d^3e^{10} + 2272a^7b^5c^3d^2e^{11} - 18048a^8b^2c^5d^3e^{10} - 6144a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^5c^6d^4e^9 + 228a^8b^4c^3d^2e^{12} + 4608a^9b^3c^5d^2e^{11} - 408a^9b^2c^4d^2e^{12}) / (32a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{(1/2)} - a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{(1/2)} - 10a^6b^6cd - a^7b^4e * (-4ac - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^5b^3c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} + (16a^9c^5e^{14} - 4a^6b^6c^2e^{14} + 28a^7b^4c^3e^{14} - 52a^8b^2c^4e^{14} - 768a^6c^8d^6e^8 - 768a^7c^7d^4e^{10} + 16a^8c^6d^2e^{12} - 512a^2b^8c^4d^6e^8 + 384a^2b^9c^3d^5e^9 + 128a^2b^10c^2d^4e^{10} + 3840a^3b^6c^5d^6e^8 - 2048a^3b^7c^4d^5e^9 - 2208a^3b^8c^3d^4e^{10} - 224a^3b^9c^2d^3e^{11} - 8704a^4b^4c^6d^6e^8 + 896a^4b^5c^5d^5e^9 + 10752a^4b^6c^4d^4e^{10} + 2688a^4b^7c^3d^3e^{11} + 96a^4b^8c^2d^2e^{12} + 6400a^5b^2c^7d^6e^8 + 5632a^5b^3c^6d^5e^9 - 18144a^5b^4c^5d^4e^{10} - 10464a^5b^5c^4d^3e^{11} - 836a^5b^6c^3d^2e^{12} + 9344a^6b^2c^6d^4e^{10} + 14592a^6b^3c^5d^3e^{11} + 2236a^6b^4c^4d^2e^{12} - 1716a^7b^2c^5d^2e^{12} - 528a^8b^3c^5d^2e^{13} + 4a^5b^7c^2d^2e^{13} - 4352a^6b^3c^7d^5e^9 - 92a^6b^5c^3d^2e^{13} - 5632a^7b^3c^6d^3e^{11} + 436a^7b^3c^4d^2e^{13}) / (64a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{(1/2)} - a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{(1/2)} - 10a^6b^6cd - a^7b^4e * (-4ac - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^5b^3c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} - ((d + e*x^2)^{(1/2)} * (a^6b^2c^5e^{14} - 2a^7c^6e^{14} + 192a^4c^9d^6e^8 + 32a^5c^8d^4e^{10} + 34a^6c^7d^2e^{12} + 64b^8c^5d^6e^8 + 704a^2b^4c^7d^6e^8 + 960a^2b^5c^6d^5e^9 + 192a^2b^6c^5d^4e^{10} - 512a^3b^2c^8d^6e^8 - 1280a^3b^3c^7d^5e^9 - 752a^3b^4c^6d^4e^{10} - 56a^3b^5c^5d^3e^{11} + 704a^4b^2c^7d^4e^{10} + 128a^4b^3c^6d^3e^{11} - 15a^4b^4c^5d^2e^{12} + 60a^5b^2c^6d^2e^{12} - 10a^6b^3c^6d^2e^{13} - 384a^6b^6c^6d^6e^8 - 192a^6b^7c^5d^5e^9 + 384a^4b^3c^8d^5e^9 - 144a^5b^3c^7d^3e^{11} + 6a^5b^3c^5d^2e^{13})) / (32a^8d^2) * ((b^8d + 8a^4c^4d + b^5d * (-4ac - b^2)^3)^{(1/2)} - a^7b^7e + 33a^2b^4c^2d - 38a^3b^2c^3d - 25a^3b^3c^2e - a^3c^2e * (-4ac - b^2)^3)^{(1/2)} - 10a^6b^6cd - a^7b^4e * (-4ac - b^2)^3)^{(1/2)} + 9a^2b^5c^2e + 20a^4b^3c^3e - 4a^5b^3c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2d * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^2c^2e * (-4ac - b^2)^3)^{(1/2)}) / (8(a^6b^4 + 16a^8c^2 - 8a^7b^2c))^{(1/2)} + (((((2048a^12c^4d^2e^{12} + 12288a^10c^6
\end{aligned}$$

$$\begin{aligned}
& d^5 e^8 + 14336 a^{11} c^5 d^3 e^{10} + 2048 a^8 b^4 c^4 d^5 e^8 - 1536 a^8 b^5 c^3 d^4 e^9 - 512 a^8 b^6 c^2 d^3 e^{10} - 11264 a^9 b^2 c^5 d^5 e^8 + 7168 a^9 b^3 c^4 d^4 e^9 + 6272 a^9 b^4 c^3 d^3 e^{10} + 384 a^9 b^5 c^2 d^2 e^{11} \\
& - 20480 a^{10} b^2 c^4 d^3 e^{10} - 3072 a^{10} b^3 c^3 d^2 e^{11} - 4096 a^{10} b^4 c^2 d e^9 + 128 a^{10} b^4 c^2 d e^{12} + 6144 a^{11} b^2 c^4 d^2 e^{11} - 1024 a^{11} b^2 c^3 d e^{12} \\
& / (64 a^8 d^2) + ((d + e x^2)^{1/2}) * ((b^8 d + 8 a^4 c^4 d + b^5 d * (-4 a c - b^2)^3)^{1/2} - a b^7 e + 33 a^2 b^4 c^2 d - 38 a^3 b^2 c^3 d - 25 a^3 b^3 c^2 e - a^3 c^2 e * (-4 a c - b^2)^3)^{1/2} - 10 a b^6 c d - a b^4 e * (-4 a c - b^2)^3)^{1/2} + 9 a^2 b^5 c e + 20 a^4 b^3 c^3 e - 4 a a b^3 c d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c^2 d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c e * (-4 a c - b^2)^3)^{1/2} / (8 (a^6 b^4 + 16 a^8 c^2 - 8 a^7 b^2 c))^{1/2} * (24576 a^{12} c^5 d^4 e^8 + 16384 a^{13} c^4 d^2 e^{10} + 2048 a^{10} b^4 c^3 d^4 e^8 - 2048 a^{10} b^5 c^2 d^3 e^9 - 14336 a^{11} b^2 c^4 d^4 e^8 + 15360 a^{11} b^3 c^3 d^3 e^9 + 1024 a^{11} b^4 c^2 d^2 e^{10} - 8192 a^{12} b^2 c^3 d^2 e^{10} - 28672 a^{12} b^3 c^4 d^3 e^9) / (32 a^8 d^2) * ((b^8 d + 8 a^4 c^4 d + b^5 d * (-4 a c - b^2)^3)^{1/2} - a b^7 e + 33 a^2 b^4 c^2 d - 38 a^3 b^2 c^3 d - 25 a^3 b^3 c^2 e - a^3 c^2 e * (-4 a c - b^2)^3)^{1/2} - 10 a b^6 c d - a b^4 e * (-4 a c - b^2)^3)^{1/2} + 9 a^2 b^5 c e + 20 a^4 b^3 c^3 e - 4 a a b^3 c d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c^2 d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c e * (-4 a c - b^2)^3)^{1/2} / (8 (a^6 b^4 + 16 a^8 c^2 - 8 a^7 b^2 c))^{1/2} - ((d + e x^2)^{1/2}) * (32 a^{10} c^5 d e^{12} - 48 a^{10} b^4 c^4 e^{13} - 4 a^8 b^5 c^2 e^{13} + 28 a^9 b^3 c^3 e^{13} + 4608 a^8 c^7 d^5 e^8 + 2048 a^9 c^6 d^3 e^{10} + 512 a^4 b^8 c^3 d^5 e^8 - 512 a^4 b^9 c^2 d^4 e^9 - 4608 a^5 b^6 c^4 d^5 e^8 + 4352 a^5 b^7 c^3 d^4 e^9 + 768 a^5 b^8 c^2 d^3 e^{10} + 14080 a^6 b^4 c^5 d^5 e^8 - 11264 a^6 b^5 c^4 d^4 e^9 - 6912 a^6 b^6 c^3 d^3 e^{10} - 256 a^6 b^7 c^2 d^2 e^{11} - 16384 a^7 b^2 c^6 d^5 e^8 + 7168 a^7 b^3 c^5 d^4 e^9 + 19776 a^7 b^4 c^4 d^3 e^{10} + 2272 a^7 b^5 c^3 d^2 e^{11} - 18048 a^8 b^2 c^5 d^3 e^{10} - 6144 a^8 b^3 c^4 d^2 e^{11} - 32 a^7 b^6 c^2 d e^{12} + 3584 a^8 b^3 c^6 d^4 e^9 + 228 a^8 b^4 c^3 d e^{12} + 4608 a^9 b^3 c^5 d^2 e^{11} - 408 a^9 b^2 c^4 d e^{12}) / (32 a^8 d^2) * ((b^8 d + 8 a^4 c^4 d + b^5 d * (-4 a c - b^2)^3)^{1/2} - a b^7 e + 33 a^2 b^4 c^2 d - 38 a^3 b^2 c^3 d - 25 a^3 b^3 c^2 e - a^3 c^2 e * (-4 a c - b^2)^3)^{1/2} - 10 a b^6 c d - a b^4 e * (-4 a c - b^2)^3)^{1/2} + 9 a^2 b^5 c e + 20 a^4 b^3 c^3 e - 4 a a b^3 c d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c^2 d * (-4 a c - b^2)^3)^{1/2} + 3 a^2 b^2 c e * (-4 a c - b^2)^3)^{1/2} / (8 (a^6 b^4 + 16 a^8 c^2 - 8 a^7 b^2 c))^{1/2} + (16 a^9 c^5 e^{14} - 4 a^6 b^6 c^2 e^{14} + 28 a^7 b^4 c^3 e^{14} - 52 a^8 b^2 c^4 e^{14} - 768 a^6 c^8 d^6 e^8 - 768 a^7 c^7 d^4 e^{10} + 16 a^8 c^6 d^2 e^{12} - 512 a^2 b^8 c^4 d^6 e^8 + 384 a^2 b^9 c^3 d^5 e^9 + 128 a^2 b^{10} c^2 d^4 e^{10} + 3840 a^3 b^6 c^5 d^6 e^8 - 2048 a^3 b^7 c^4 d^5 e^9 - 2208 a^3 b^8 c^3 d^4 e^{10} - 224 a^3 b^9 c^2 d^3 e^{11} - 8704 a^4 b^4 c^6 d^6 e^8 + 896 a^4 b^5 c^5 d^5 e^9 + 10752 a^4 b^6 c^4 d^4 e^{10} + 2688 a^4 b^7 c^3 d^3 e^{11} + 96 a^4 b^8 c^2 d^2 e^{12} + 6400 a^5 b^2 c^7 d^6 e^8 + 5632 a^5 b^3 c^6 d^5 e^9 - 18144 a^5 b^4 c^5 d^4 e^{10} - 10464 a^5 b^5 c^4 d^3 e^{11} - 836 a^5 b^6 c^3 d^2 e^{12} + 9344 a^6 b^2 c^6 d^4 e^{10} + 14592 a^6 b^3 c^5 d^3 e^{11} + 2236 a^6 b^4 c^4 d^2 e^{12} - 1716 a^7 b^2 c^5 d^2 e^{12} - 528 a^8 b^3 c^5
\end{aligned}$$

$$\begin{aligned}
& *d^*e^{13} + 4*a^5*b^7*c^2*d^*e^{13} - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d^*e^{13} - 5632*a^7*b*c^6*d^3*e^{11} + 436*a^7*b^3*c^4*d^*e^{13})/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(a^6*b^2*c^5*e^{14} - 2*a^7*c^6*e^{14} + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6*c^7*d^2*e^{12} + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^{10} - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704*a^4*b^2*c^7*d^4*e^{10} + 128*a^4*b^3*c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} + 60*a^5*b^2*c^6*d^2*e^{12} - 10*a^6*b*c^6*d^*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a^5*b^3*c^5*d^*e^{13}))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)} + (7*a^5*c^7*d^*e^{14} + 56*a^3*c^9*d^5*e^{10} + 63*a^4*c^8*d^3*e^{12} - 64*b^4*c^8*d^7*e^8 + 64*b^5*c^7*d^6*e^9 + 64*a^2*b^2*c^8*d^5*e^{10} + 224*a^2*b^3*c^7*d^4*e^{11} - 112*a^3*b^2*c^7*d^3*e^{12} + 64*a*b^2*c^9*d^7*e^8 + 64*a*b^3*c^8*d^6*e^9 - 192*a*b^4*c^7*d^5*e^{10} - 96*a^2*b*c^9*d^6*e^9 - 136*a^3*b*c^8*d^4*e^{11} + 9*a^4*b*c^7*d^2*e^{13}))/((32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d + b^5*d*(-(4*a*c - b^2)^3)^{(1/2)} - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e - a^3*c^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^6*c*d - a*b^4*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e - 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^{(1/2)}*2i - (((d + e*x^2)^{(1/2)}*(a*e^2 + 4*b*d*e))/(8*a^2) + ((d + e*x^2)^{(3/2)}*(a*e^2 - 4*b*d*e))/(8*a^2*d))/((d + e*x^2)^2 - 2*d*(d + e*x^2) + d^2) + (atan(-((((d + e*x^2)^{(1/2)}*(a^6*b^2*c^5*e^{14} - 2*a^7*c^6*e^{14} + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6*c^7*d^2*e^{12} + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^{10} - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704*a^4*b^2*c^7*d^4*e^{10} + 128*a^4*b^3*c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} + 60*a^5*b^2*c^6*d^2*e^{12} - 10*a^6*b*c^6*d^*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a^5*b^3*c^5*d^*e^{13}))/((32*a^8*d^2) - (((a^9*c^5*e^{14})/4 - (a^6*b^6*c^2*e^{14})/16 + (7*a^7*b^4*c^3*e^{14})/16 - (13*a^8*b^2*c^4*e^{14})/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^{10} + (a^8*c^6*d^2*e^{12})/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^{10} + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^
\end{aligned}$$

$$\begin{aligned}
& 10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7*c^3*d^3*e^11 + (3*a^4*b^8*c^2*d^2*e^12)/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^10)/2 - (327*a^5*b^5*c^4*d^3*e^11)/2 - (209*a^5*b^6*c^3*d^2*e^12)/16 + 146*a^6*b^2*c^6*d^4*e^10 + 228*a^6*b^3*c^5*d^3*e^11 + (559*a^6*b^4*c^4*d^2*e^12)/16 - (429*a^7*b^2*c^5*d^2*e^12)/16 - (33*a^8*b^3*c^5*d^2*e^13)/4 + (a^5*b^7*c^2*d^2*e^13)/16 - 68*a^6*b^3*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d^2*e^13)/16 - 88*a^7*b^3*c^6*d^3*e^11 + (109*a^7*b^3*c^4*d^2*e^13)/16)/(a^8*d^2) + ((((((32*a^12*c^4*d^2*e^12 + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^10 + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^10 - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^10 + 6*a^9*b^5*c^2*d^2*e^11 - 320*a^10*b^2*c^4*d^3*e^10 - 48*a^10*b^3*c^3*d^2*e^11 - 64*a^10*b^4*c^2*d^2*e^12 + 96*a^11*b^3*c^4*d^2*e^11 - 16*a^11*b^2*c^3*d^2*e^12)/(a^8*d^2) - ((d + e*x^2)^(1/2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b^3*c^4*d^3*e^9)))/(512*a^11*d^2*(d^3)^(1/2)))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d^2*e^12 - 48*a^10*b^4*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d^2*e^12 + 3584*a^8*b^3*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d^2*e^12 + 4608*a^9*b^3*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d^2*e^12))/(32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^(1/2)) + (((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b^3*c^6*d^2*e^13 - 384*a^6*b^4*c^6*d^6*e^8 - 192*a^6*b^7*c^5*d^5*e^9 + 384*a^4*b^3*c^8*d^5*e^9 - 144*a^5*b^3*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d^2*e^13))/(32*a^8*d^2) + (((((a^9*c^5*e^14)/4 - (a^6*b^6*c^2*e^14)/16 + (7*a^7*b^4*c^3*e^14)/16 - (13*a^8*b^2*c^4*e^14)/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^10 + (a^8*c^6*d^2*e^12)/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^9 + 2*a^2*b^10*c^2*d^4*e^10 + 60*a^3*b^6*c^5*d^6*e^8 - 32*a^3*b^7*c^4*d^5*e^9 - (69*a^3*b^8*c^3*d^4*e^10)/2 - (7*a^3*b^9*c^2*d^3*e^11)/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^10 + 42*a^4*b^7
\end{aligned}$$

$$\begin{aligned}
& *c^3*d^3*e^{11} + (3*a^4*b^8*c^2*d^2*e^{12})/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^{10})/2 - (327*a^5*b^5*c^4*d^3*e^{11})/2 - (209*a^5*b^6*c^3*d^2*e^{12})/16 + 146*a^6*b^2*c^6*d^4*e^{10} + 228*a^6*b^3*c^5*d^3*e^{11} + (559*a^6*b^4*c^4*d^2*e^{12})/16 - (429*a^7*b^2*c^5*d^2*e^{12})/16 - (33*a^8*b*c^5*d*e^{13})/4 + (a^5*b^7*c^2*d*e^{13})/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^{13})/16 - 88*a^7*b*c^6*d^3*e^{11} + (109*a^7*b^3*c^4*d*e^{13})/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^{12} + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^{10} + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^{10} - 176*a^9*b^2*c^5*d^5*e^8 + 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^{10} + 6*a^9*b^5*c^2*d^2*e^{11} - 320*a^10*b^2*c^4*d^3*e^{10} - 48*a^10*b^3*c^3*d^2*e^{11} - 64*a^10*b*c^5*d^4*e^9 + 2*a^10*b^4*c^2*d*e^{12} + 96*a^11*b*c^4*d^2*e^{11} - 16*a^11*b^2*c^3*d*e^{12})/(a^8*d^2) + ((d + e*x^2)^{(1/2)}*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^{10} + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^{10} - 8192*a^12*b^2*c^3*d^2*e^{10} - 28672*a^12*b*c^4*d^3*e^9))/(512*a^11*d^2*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}) - ((d + e*x^2)^{(1/2)}*(32*a^10*c^5*d*e^{12} - 48*a^10*b*c^4*e^{13} - 4*a^8*b^5*c^2*e^{13} + 28*a^9*b^3*c^3*e^{13} + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^{10} + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^{10} + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^{10} - 256*a^6*b^7*c^2*d^2*e^{11} - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^{10} + 2272*a^7*b^5*c^3*d^2*e^{11} - 18048*a^8*b^2*c^5*d^3*e^{10} - 6144*a^8*b^3*c^4*d^2*e^{11} - 32*a^7*b^6*c^2*d*e^{12} + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^{12} + 4608*a^9*b*c^5*d^2*e^{11} - 408*a^9*b^2*c^4*d*e^{12}))/((32*a^8*d^2)*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/((7*a^5*c^7*d*e^{14})/32 + (7*a^3*c^9*d^5*e^{10})/4 + (63*a^4*c^8*d^3*e^{12})/32 - 2*b^4*c^8*d^7*e^8 + 2*b^5*c^7*d^6*e^9 + 2*a^2*b^2*c^8*d^5*e^{10} + 7*a^2*b^3*c^7*d^4*e^{11} - (7*a^3*b^2*c^7*d^3*e^{12})/2 + 2*a*b^2*c^9*d^7*e^8 + 2*a*b^3*c^8*d^6*e^9 - 6*a*b^4*c^7*d^5*e^{10} - 3*a^2*b*c^9*d^6*e^9 - (17*a^3*b*c^8*d^4*e^{11})/4 + (9*a^4*b*c^7*d^2*e^{13})/32)/(a^8*d^2) - (((((d + e*x^2)^{(1/2)}*(a^6*b^2*c^5*e^{14} - 2*a^7*c^6*e^{14} + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^{10} + 34*a^6*c^7*d^2*e^{12} + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^{10} - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^{10} - 56*a^3*b^5*c^5*d^3*e^{11} + 704*a^4*b^2*c^7*d^4*e^{10} + 128*a^4*b^3*c^6*d^3*e^{11} - 15*a^4*b^4*c^5*d^2*e^{12} + 60*a^5*b^2*c^6*d^2*e^{12} - 10*a^6*b*c^6*d*e^{13} - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^{11} + 6*a^5*b^3*c^5*d*e^{13}))/((32*a^8*d^2) - (((a^9*c^5*e^{14})/4 - (a^6*b^6*c^2*e^{14})/16 + (7*a^7*b^4*c^3*e^{14})/16 - (13*a^8*b^2*c^4*e^{14})/16 - 12*a^6*c^8*d^6*e^8 - 12*a^7*c^7*d^4*e^{10} + (a^8*c^6*d^2*e^{12})/4 - 8*a^2*b^8*c^4*d^6*e^8 + 6*a^2*b^9*c^3*d^5*e^
\end{aligned}$$

$$\begin{aligned}
& 9 + 2a^2b^{10}c^2d^4e^{10} + 60a^3b^6c^5d^6e^8 - 32a^3b^7c^4d^5e^9 - (69a^3b^8c^3d^4e^{10})/2 - (7a^3b^9c^2d^3e^{11})/2 - 136a^4b^4c^6d^6e^8 + 14a^4b^5c^5d^5e^9 + 168a^4b^6c^4d^4e^{10} + 42a^4b^7c^3d^3e^{11} + (3a^4b^8c^2d^2e^{12})/2 + 100a^5b^2c^7d^6e^8 + 88a^5b^3c^6d^5e^9 - (567a^5b^4c^5d^4e^{10})/2 - (327a^5b^5c^4d^3e^{11})/2 - (209a^5b^6c^3d^2e^{12})/16 + 146a^6b^2c^6d^4e^{10} + 228a^6b^3c^5d^3e^{11} + (559a^6b^4c^4d^2e^{12})/16 - (429a^7b^2c^5d^2e^{12})/16 - (33a^8b^3c^5d^2e^{13})/4 + (a^5b^7c^2d^2e^{13})/16 - 68a^6b^3c^7d^5e^9 - (23a^6b^5c^3d^2e^{13})/16 - 88a^7b^3c^6d^3e^{11} + (109a^7b^3c^4d^2e^{13})/16)/(a^8d^2) + (((((32a^{12}c^4d^2e^{12} + 192a^{10}c^6d^5e^8 + 224a^{11}c^5d^3e^{10} + 32a^8b^4c^4d^5e^8 - 24a^8b^5c^3d^4e^9 - 8a^8b^6c^2d^3e^{10} - 176a^9b^2c^5d^5e^8 + 112a^9b^3c^4d^4e^9 + 98a^9b^4c^3d^3e^{10} + 6a^9b^5c^2d^2e^{11} - 320a^{10}b^2c^4d^3e^{10} - 48a^{10}b^3c^3d^2e^{11} - 64a^{10}b^4c^2d^2e^{12} + 96a^{11}b^2c^4d^2e^{11} - 16a^{11}b^2c^3d^2e^{12})/(a^8d^2) - ((d + e*x^2)^{(1/2)}*(a^2e^2 - 8b^2d^2 + 8a*c*d^2 + 4a*b*d*e))*(24576a^{12}c^5d^4e^8 + 16384a^{13}c^4d^2e^{10} + 2048a^{10}b^4c^3d^4e^8 - 2048a^{10}b^5c^2d^3e^9 - 14336a^{11}b^2c^4d^4e^8 + 15360a^{11}b^3c^3d^3e^9 + 1024a^{11}b^4c^2d^2e^{10} - 8192a^{12}b^2c^3d^2e^{10} - 28672a^{12}b^3c^4d^3e^9))/(512a^{11}d^2*(d^3)^{(1/2)}))*(a^2e^2 - 8b^2d^2 + 8a*c*d^2 + 4a*b*d*e))/(16a^3*(d^3)^{(1/2)}) + ((d + e*x^2)^{(1/2)}*(32a^{10}c^5d^2e^{12} - 48a^{10}b^3c^4e^{13} - 4a^8b^5c^2e^{13} + 28a^9b^3c^3e^{13} + 4608a^8c^7d^5e^8 + 2048a^9c^6d^3e^{10} + 512a^4b^8c^3d^5e^8 - 512a^4b^9c^2d^4e^9 - 4608a^5b^6c^4d^5e^8 + 4352a^5b^7c^3d^4e^9 + 768a^5b^8c^2d^3e^{10} + 14080a^6b^4c^5d^5e^8 - 11264a^6b^5c^4d^4e^9 - 6912a^6b^6c^3d^3e^{10} - 256a^6b^7c^2d^2e^{11} - 16384a^7b^2c^6d^5e^8 + 7168a^7b^3c^5d^4e^9 + 19776a^7b^4c^4d^3e^{10} + 2272a^7b^5c^3d^2e^{11} - 18048a^8b^2c^5d^3e^{10} - 6144a^8b^3c^4d^2e^{11} - 32a^7b^6c^2d^2e^{12} + 3584a^8b^3c^6d^4e^9 + 228a^8b^4c^3d^2e^{12} + 4608a^9b^3c^5d^2e^{11} - 408a^9b^2c^4d^2e^{12}))/((32a^8d^2))*(a^2e^2 - 8b^2d^2 + 8a*c*d^2 + 4a*b*d*e))/(16a^3*(d^3)^{(1/2)}))*(a^2e^2 - 8b^2d^2 + 8a*c*d^2 + 4a*b*d*e))/(16a^3*(d^3)^{(1/2)})) + (((((d + e*x^2)^{(1/2)}*(a^6b^2c^5e^{14} - 2a^7c^6e^{14} + 192a^4c^9d^6e^8 + 32a^5c^8d^4e^{10} + 34a^6c^7d^2e^{12} + 64b^8c^5d^6e^8 + 704a^2b^4c^7d^6e^8 + 960a^2b^5c^6d^5e^9 + 192a^2b^6c^5d^4e^{10} - 512a^3b^2c^8d^6e^8 - 1280a^3b^3c^7d^5e^9 - 752a^3b^4c^6d^4e^{10} - 56a^3b^5c^5d^3e^{11} + 704a^4b^2c^7d^4e^{10} + 128a^4b^3c^6d^3e^{11} - 15a^4b^4c^5d^2e^{12} + 60a^5b^2c^6d^2e^{12} - 10a^6b^3c^6d^2e^{13} - 384a^6b^6c^6d^6e^8 - 192a^6b^7c^5d^5e^9 + 384a^4b^3c^8d^5e^9 - 144a^5b^3c^7d^3e^{11} + 6a^5b^3c^5d^2e^{13}))/((32a^8d^2) + (((a^9c^5e^{14})/4 - (a^6b^6c^2e^{14})/16 + (7a^7b^4c^3e^{14})/16 - (13a^8b^2c^4e^{14})/16 - 12a^6c^8d^6e^8 - 12a^7c^7d^4e^{10} + (a^8c^6d^2e^{12})/4 - 8a^2b^8c^4d^6e^8 + 6a^2b^9c^3d^5e^9 + 2a^2b^10c^2d^4e^{10} + 60a^3b^6c^5d^6e^8 - 32a^3b^7c^4d^5e^9 - (69a^3b^8c^3d^4e^{10})/2 - (7a^3b^9c^2d^3
\end{aligned}$$

$$\begin{aligned}
& *e^{11})/2 - 136*a^4*b^4*c^6*d^6*e^8 + 14*a^4*b^5*c^5*d^5*e^9 + 168*a^4*b^6*c^4*d^4*e^{10} + 42*a^4*b^7*c^3*d^3*e^{11} + (3*a^4*b^8*c^2*d^2*e^{12})/2 + 100*a^5*b^2*c^7*d^6*e^8 + 88*a^5*b^3*c^6*d^5*e^9 - (567*a^5*b^4*c^5*d^4*e^{10})/2 - \\
& (327*a^5*b^5*c^4*d^3*e^{11})/2 - (209*a^5*b^6*c^3*d^2*e^{12})/16 + 146*a^6*b^2*c^6*d^4*e^{10} + 228*a^6*b^3*c^5*d^3*e^{11} + (559*a^6*b^4*c^4*d^2*e^{12})/16 - \\
& (429*a^7*b^2*c^5*d^2*e^{12})/16 - (33*a^8*b*c^5*d*e^{13})/4 + (a^5*b^7*c^2*d*e^{13})/16 - 68*a^6*b*c^7*d^5*e^9 - (23*a^6*b^5*c^3*d*e^{13})/16 - 88*a^7*b*c^6*d^3*e^{11} + (109*a^7*b^3*c^4*d*e^{13})/16)/(a^8*d^2) + (((((32*a^12*c^4*d*e^{12} \\
& + 192*a^10*c^6*d^5*e^8 + 224*a^11*c^5*d^3*e^{10} + 32*a^8*b^4*c^4*d^5*e^8 - 24*a^8*b^5*c^3*d^4*e^9 - 8*a^8*b^6*c^2*d^3*e^{10} - 176*a^9*b^2*c^5*d^5*e^8 + \\
& 112*a^9*b^3*c^4*d^4*e^9 + 98*a^9*b^4*c^3*d^3*e^{10} + 6*a^9*b^5*c^2*d^2*e^{11} - 320*a^{10}*b^2*c^4*d^3*e^{10} - 48*a^{10}*b^3*c^3*d^2*e^{11} - 64*a^{10}*b*c^5*d^4*e^9 + 2*a^{10}*b^4*c^2*d*e^{12} + 96*a^{11}*b*c^4*d^2*e^{11} - 16*a^{11}*b^2*c^3*d*e^{12})/(a^8*d^2) + ((d + e*x^2)^{(1/2)}*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*(24576*a^{12}*c^5*d^4*e^8 + 16384*a^{13}*c^4*d^2*e^{10} + 2048*a^{10}*b^4*c^3*d^4*e^8 - 2048*a^{10}*b^5*c^2*d^3*e^9 - 14336*a^{11}*b^2*c^4*d^4*e^8 + 15360*a^{11}*b^3*c^3*d^3*e^9 + 1024*a^{11}*b^4*c^2*d^2*e^{10} - 8192*a^{12}*b^2*c^3*d^2*e^{10} - 28672*a^{12}*b*c^4*d^3*e^9))/(512*a^{11}*d^2*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}) - ((d + e*x^2)^{(1/2)}*(32*a^{10}*c^5*d*e^{12} - 48*a^{10}*b*c^4*e^{13} - 4*a^8*b^5*c^2*e^{13} + 28*a^9*b^3*c^3*e^{13} + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^{10} + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^{10} + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^{10} - 256*a^6*b^7*c^2*d^2*e^{11} - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^{10} + 2272*a^7*b^5*c^3*d^2*e^{11} - 18048*a^8*b^2*c^5*d^3*e^{10} - 6144*a^8*b^3*c^4*d^2*e^{11} - 32*a^7*b^6*c^2*d*e^{12} + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^{12} + 4608*a^9*b*c^5*d^2*e^{11} - 408*a^9*b^2*c^4*d*e^{12}))/((32*a^8*d^2))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)})*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e))/(16*a^3*(d^3)^{(1/2)}))*(a^2*e^2 - 8*b^2*d^2 + 8*a*c*d^2 + 4*a*b*d*e)*1i)/(8*a^3*(d^3)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.261 \quad \int \frac{x^4 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=390

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}$$

Rubi [A] time = 2.92, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1291, 388, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \frac{(cd-2be) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) + \frac{x\sqrt{d+ex^2}}{2c}}{2c^2\sqrt{e}}}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-e(b+\sqrt{b^2-4ac})}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] (x*sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])]/(c^2*sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])]/(c^2*sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/(2*c^2*sqrt[e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^4 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{\int \frac{cd-be+cx^2}{\sqrt{d+ex^2}} dx}{c^2} - \frac{\int \frac{a(cd-be)+(bcd-b^2e+ace)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{c^2} \\
 &= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{bcd-b^2e+ace + \frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace - \frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{c^2} + \frac{(cd - \dots)}{c^2} \\
 &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be) \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace - \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)}{2c^2} \\
 &= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be) \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace - \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right) \operatorname{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} \\
 &= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace - \frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})e}} \left(bcd - \dots\right)
 \end{aligned}$$

Mathematica [B] time = 6.40, size = 10915, normalized size = 27.99

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 91.94, size = 566, normalized size = 1.45

$\sqrt{2}\operatorname{RootSum}\left[4e^2x^2 + 4b^2bx - 4b^2cd + 16b^2ac^2 - 8b^2bd + 6b^2cd^2 + 4b^2bde - 4b^2cd^2 + cd^2e, \dots\right], \frac{d(b-c)\log\left(\sqrt{d+ex^2} - \sqrt{e}\right)}{2c^2\sqrt{e}}, \frac{e\sqrt{d+ex^2}}{2c}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] $(x\sqrt{d+ex^2})/(2c) + ((-(c*d) + 2*b*e)*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d+e*x^2]])/(2*c^2*\operatorname{Sqrt}[e]) - (\operatorname{Sqrt}[e]*\operatorname{RootSum}[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1$

$$\begin{aligned} &^6 + c\#1^8 \& , (- (b*c*d^3*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]) + b^2*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - a*c*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] + 2*b*c*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 2*b^2*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 2*a*c*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*a*b*e^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - b*c*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 + b^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 - a*c*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(- (c*d^3) + b*d^2*e + 3*c*d^2*\#1^2 - 4*b*d*e*\#1^2 + 8*a*e^2*\#1^2 - 3*c*d*\#1^4 + 3*b*e*\#1^4 + c*\#1^6) \&) / (2*c^2) \end{aligned}$$

fricas [B] time = 104.32, size = 6534, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} &[1/4*(\text{sqrt}(1/2)*c^2*e*\text{sqrt}(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*\text{log}(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 + 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*\text{sqrt}(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*\text{log}(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*\text{sqrt}(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2))/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8* \end{aligned}$$

$$\begin{aligned}
& - 2*a^3*b*c)*e^2)*x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4*c^4 - 6*a*b^2*c^5 \\
& + 8*a^2*c^6)*x*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3* \\
& a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2* \\
& c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c \\
& + 8*a^2*b^2*c^2)*e)*x)*\sqrt{-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2 \\
& *a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d \\
& ^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b \\
& ^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))/x^2) - \sqrt{1/2}*c \\
& ^2*e*\sqrt{-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2* \\
& c^4 - 4*a*c^5)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a \\
& *b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c \\
& ^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*\log(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*\sqrt{ \\
& t(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b \\
& *c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 2 \\
& *(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2* \\
& b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c \\
&)*e^2)*x^2 - 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6) \\
& *x*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + \\
& 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c \\
& ^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^ \\
& 2*c^2)*e)*x)*\sqrt{-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e \\
& + (b^2*c^4 - 4*a*c^5)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5 \\
& *c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2 \\
&)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))/x^2) + \sqrt{1/2}*c^2*e*\sqrt{- \\
& ((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c \\
& ^5)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + \\
& 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^ \\
& 9)))/(b^2*c^4 - 4*a*c^5))*\log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*\sqrt{((b^4*c^ \\
& 2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e \\
& + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2* \\
& c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 \\
& - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 \\
& + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*\sqrt{ \\
& ((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c \\
& ^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b \\
& ^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)* \\
& x)*\sqrt{-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^ \\
& 4 - 4*a*c^5)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b \\
& ^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 \\
& - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))/x^2) - \sqrt{1/2}*c^2*e*\sqrt{-((b^3*c - \\
& 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*\sqrt{((\\
& b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^ \\
& 3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2* \\
& c^4 - 4*a*c^5))*\log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*\sqrt{((b^4*c^2 - 2*a*b^ \\
& 2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
\end{aligned}$$

$$\begin{aligned} & *a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c - a^3*c^2) \\ & *d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 \\ & + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*\sqrt{c} \\ & (1/2)*\sqrt{e*x^2 + d}*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*\sqrt{((b^4*c^2 \\ & - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + \\ & (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a \\ & *b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*\sqrt{-((\\ & (b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5) \\ &)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2 \\ & *a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9 \\ &)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*\sqrt{e*x^2 + d}*c*e*x - 2*(c*d - 2*b*e)* \\ & \sqrt{-e}*\arctan(\sqrt{-e}*x/\sqrt{e*x^2 + d}))/c^2*e] \end{aligned}$$

giac [A] time = 2.04, size = 53, normalized size = 0.14

$$-\frac{(cd - 2be)e^{(-\frac{1}{2})} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4c^2} + \frac{\sqrt{x^2e + d}x}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/4*(c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/2*sqrt(x^2*e + d)*x/c

maple [C] time = 0.04, size = 290, normalized size = 0.74

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)

[Out] 1/2*x*(e*x^2+d)^(1/2)/c+1/2/c*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/c^2*e^(1/2)*sum(((a*c*e-b^2*e+b*c*d)*_R^2+2*(-2*a*b*e^2+a*c*d*e+b^2*d*e-b*c*d^2)*_R+e*c*d^2*a-b^2*d^2*e+b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln((-e^(1/2)*x+(e*x^2+d)^(1/2))^2-_R), _R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/c^2*e^(1/2)*b*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}x^4}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \sqrt{ex^2 + d}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)

$$3.262 \quad \int \frac{x^2 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=324

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Rubi [A] time = 1.52, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1293, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c}}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(c*sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(c*sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]) + (sqrt[e]*ArcTanh[(sqrt[e]*x)/sqrt[d + e*x^2]])/c

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1293

```
Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(
q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{c} + \frac{e \int \frac{1}{\sqrt{d+ex^2}} dx}{c} \\
&= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{c} + \frac{e \operatorname{Subst} \left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{c} \\
&= \frac{\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right)}{c} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{c} - \frac{(-cd+be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{c} \\
&= \frac{\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right)}{c} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \operatorname{Subst} \left(\int \frac{1}{b+\sqrt{b^2-4ac} - (-2cd+(b+\sqrt{b^2-4ac})x^2)} dx \right)}{c} - \frac{\left(-cd+be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \operatorname{Subst} \left(\int \frac{1}{b-\sqrt{b^2-4ac} - (-2cd+(b-\sqrt{b^2-4ac})x^2)} dx \right)}{c} \\
&= \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{c\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})e}} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{c\sqrt{b+\sqrt{b^2-4ac}} \sqrt{2cd-(b+\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [B] time = 6.17, size = 7768, normalized size = 23.98

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 57.34, size = 425, normalized size = 1.31

$$\frac{\sqrt{e} \operatorname{RootSum} \left[\#1^3 c + 4 \#1^2 b e - 4 \#1^2 c d + 16 \#1^2 a c^2 - 8 \#1^2 b d e + 6 \#1^2 c d^2 + 4 \#1^2 b d^2 e - 4 \#1^2 c d^3 + c d^4 e, \frac{-\#1^2 b e \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) + \#1^2 b e \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) - 4 \#1^2 a c^2 \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) + 2 \#1^2 b d e \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) + 2 \#1^2 c d^2 \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) + 2 \#1^2 c d^3 \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right) + c d^4 \operatorname{Log} \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{\sqrt{d+ex^2} + \sqrt{c}} \right)}{\sqrt{d+ex^2} + \sqrt{c}} \right]}{c} \sqrt{e} \log \left(\frac{\sqrt{d+ex^2} - \sqrt{c}}{c} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]

[Out] -((Sqrt[e]*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/c) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d

$$+ e*x^2] - \#1] - b*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - 2*c*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 2*b*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 4*a*e^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + c*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 - b*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*\#1^2 + 4*b*d*e*\#1^2 - 8*a*e^2*\#1^2 + 3*c*d*\#1^4 - 3*b*e*\#1^4 - c*\#1^6) \&])/(2*c)$$

fricas [B] time = 13.84, size = 3260, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(\text{sqrt}(1/2)*c*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*\text{sqrt} \\ & ((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3) \\ &)*\text{log}(-((b^2*c^2 - 4*a*c^3)*d*x^2*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2 \\ & *c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4* \\ & a*c)*d*e)*x^2 + 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*\text{sqrt}((\\ & c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d \\ & - (b^3 - 4*a*b*c)*e)*x)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3) \\ &)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a \\ & *c^3)))/x^2) - \text{sqrt}(1/2)*c*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a* \\ & c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - \\ & 4*a*c^3))*\text{log}(-((b^2*c^2 - 4*a*c^3)*d*x^2*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e \\ & ^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (\\ & b^2 + 4*a*c)*d*e)*x^2 - 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^3*c^2 - 4*a* \\ & b*c^3)*x*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c \\ & - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - \\ & 4*a*c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c \\ & ^2 - 4*a*c^3)))/x^2) + \text{sqrt}(1/2)*c*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 \\ & - 4*a*c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 \\ & - 4*a*c^3))*\text{log}(((b^2*c^2 - 4*a*c^3)*d*x^2*\text{sqrt}((c^2*d^2 - 2*b*c*d*e \\ & + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b* \\ & e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^3*c^2 - 4*a* \\ & b*c^3)*x*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*c \\ & - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 \\ & - 4*a*c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/ \\ & (b^2*c^2 - 4*a*c^3)))/x^2) - \text{sqrt}(1/2)*c*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - \\ & (b^2*c^2 - 4*a*c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5) \\ &)))/(b^2*c^2 - 4*a*c^3))*\text{log}(((b^2*c^2 - 4*a*c^3)*d*x^2*\text{sqrt}((c^2*d^2 - 2*b \\ & *c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + \\ & 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*((b^3*c^2 \\ & - 4*a*b*c^3)*x*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + \\ & ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)* \\ & e - (b^2*c^2 - 4*a*c^3)*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a \end{aligned}$$


```

*c^5)))/(b^2*c^2 - 4*a*c^3))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e +
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*
b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2
+ 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^
2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a
*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (
b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2
- 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*
c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*
d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c
^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*
x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 +
2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)
/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + 4*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d))/c]

```

giac [A] time = 1.83, size = 27, normalized size = 0.08

$$\frac{e^{\frac{1}{2}} \log \left(\left(x e^{\frac{1}{2}} - \sqrt{x^2 e + d} \right)^2 \right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-1/2*e^{(1/2)}*\log((x*e^{(1/2)} - \sqrt{x^2*e + d})^2)/c$

maple [C] time = 0.03, size = 224, normalized size = 0.69

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $1/2*e^{(1/2)}/c*\text{sum}(((b*e-c*d)*_R^2+2*(2*a*e^2-b*d*e+c*d^2)*_R+b*d^2*e-c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-e^{(1/2)}/c*\ln(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\sqrt{e*x^2 + d}*x^2/(c*x^4 + b*x^2 + a), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{ex^2 + d}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2*(d + e*x^2)^{(1/2)})/(a + b*x^2 + c*x^4), x)$

[Out] $\text{int}((x^2*(d + e*x^2)^{(1/2)})/(a + b*x^2 + c*x^4), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)$

[Out] $\text{Integral}(x**2*\sqrt{d + e*x**2}/(a + b*x**2 + c*x**4), x)$

$$3.263 \quad \int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tan^{-1} \left(\frac{x \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}{\sqrt{\sqrt{b^2 - 4ac} + b} \sqrt{d + ex^2}} \right)}{\sqrt{b^2 - 4ac} \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Rubi [A] time = 0.32, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1174, 402, 217, 206, 377, 205}

$$\frac{\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tan^{-1} \left(\frac{x \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}{\sqrt{\sqrt{b^2 - 4ac} + b} \sqrt{d + ex^2}} \right)}{\sqrt{b^2 - 4ac} \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 1174

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b - r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx &= \frac{(2c) \int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}} \\
 &= \frac{(2cd - (b - \sqrt{b^2-4ac})e) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{\sqrt{b^2-4ac}} + \frac{(-2cd + (b + \sqrt{b^2-4ac})e) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{\sqrt{b^2-4ac}} \\
 &= \frac{(2cd - (b - \sqrt{b^2-4ac})e) \text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac} - (-2cd + (b - \sqrt{b^2-4ac})e)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} + \frac{(-2cd + (b + \sqrt{b^2-4ac})e) \text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac} - (-2cd + (b + \sqrt{b^2-4ac})e)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} \\
 &= \frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})e} \tan^{-1}\left(\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})e} x}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac} \sqrt{b - \sqrt{b^2-4ac}}} - \frac{\sqrt{2cd - (b + \sqrt{b^2-4ac})e} \tan^{-1}\left(\frac{\sqrt{2cd - (b + \sqrt{b^2-4ac})e} x}{\sqrt{b + \sqrt{b^2-4ac}} \sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac} \sqrt{b + \sqrt{b^2-4ac}}}
 \end{aligned}$$

Mathematica [B] time = 5.12, size = 2585, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]

[Out] (Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2]) + x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2] + x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2]) + x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2]) + x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2]) + x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2] + x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2] + x] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])/Sqrt[2] + x] + 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c])

```

rt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt
[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2
- 4*a*c])/c)]]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^
2 - 4*a*c]*e)/c]*Log[2*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]]*e*x +
Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[(-b
+ Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2
*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]]*e*x + Sqrt[4*d - (2*(b + Sq
rt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt
[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d + S
qrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2
- 4*a*c])*e)/c]*Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]]*Sqrt[(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)/c]*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c])

```

IntegrateAlgebraic [C] time = 17.05, size = 250, normalized size = 1.04

$$\frac{1}{2}e^{3/2}\text{RootSum}\left[\#1^5c + 4\#1^6be - 4\#1^6cd + 16\#1^4ae^2 - 8\#1^4bde + 6\#1^4cd^2 + 4\#1^2bd^2e - 4\#1^2cd^3 + cd^4\&, \frac{\#1^4\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex}) + 2\#1^2d\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex}) + d^2\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex})}{\#1^6(-c) - 3\#1^6be + 3\#1^4cd - 8\#1^2ae^2 + 4\#1^2bde - 3\#1^2cd^2 - bd^2e + cd^3}\right]\&$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]

```

[Out] (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b
*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^
2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
- c*#1^6) & ])/2

```

fricas [B] time = 3.34, size = 985, normalized size = 4.10



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

```

[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c))))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c))))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) -
1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -

```

$$\begin{aligned}
& 4*a^3*c)) * x^2 - 4*\sqrt{1/2}*(a^2*b^2 - 4*a^3*c)*\sqrt{e*x^2 + d}*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))} * x*\sqrt{-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))})/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2 \\
& + 1/4*\sqrt{1/2}*\sqrt{-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))})/(a*b^2 - 4*a^2*c))} * \log(((a*b^2 - 4*a^2*c)*d*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))} * x^2 + 4*\sqrt{1/2}*(a^2*b^2 - 4*a^3*c)*\sqrt{e*x^2 + d}*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))} * x*\sqrt{-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))})/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2 \\
& - 1/4*\sqrt{1/2}*\sqrt{-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))})/(a*b^2 - 4*a^2*c))} * \log(((a*b^2 - 4*a^2*c)*d*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))} * x^2 - 4*\sqrt{1/2}*(a^2*b^2 - 4*a^3*c)*\sqrt{e*x^2 + d}*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))} * x*\sqrt{-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*\sqrt{d^2/(a^2*b^2 - 4*a^3*c))})/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due to rounding error%%{18446744069414584320, [4,7,8,2,3,14,2]%%}+%%{-2147483648, [3,8,10,8,3,10,1]%%}+%%{-12884901888, [3,8,10,7,2,12,2]%%}+%%{463856467968, [3,8,10,6,1,14,3]%%}+%%{1924145348608, [3,8,10,5,0,16,4]%%}+%%{536870912, [3,8,9,8,5,10,0]%%}+%%{20401094656, [3,8,9,7,4,12,1]%%}+%%{-150323855360, [3,8,9,6,3,14,2]%%}+%%{-3135326126080, [3,8,9,5,2,16,3]%%}+%%{-4672924418048, [3,8,9,4,1,18,4]%%}+%%{6047313952768, [3,8,9,3,0,20,5]%%}+%%{-4294967296, [3,8,8,7,6,12,0]%%}+%%{-42412802048, [3,8,8,6,5,14,1]%%}+%%{1046898278400, [3,8,8,5,4,16,2]%%}+%%{6210522710016, [3,8,8,4,3,18,3]%%}+%%{-1786706395136, [3,8,8,3,2,20,4]%%}+%%{-11544872091648, [3,8,8,2,1,22,5]%%}+%%{4398046511104, [3,8,8,1,0,24,6]%%}+%%{12750684160, [3,8,7,6,7,14,0]%%}+%%{-23890755584, [3,8,7,5,6,16,1]%%}+%%{-2103460233216, [3,8,7,4,5,18,2]%%}+%%{-3324304687104, [3,8,7,3,4,20,3]%%}+%%{9758165696512, [3,8,7,2,3,22,4]%%}+%%{1649267441664, [3,8,7,1,2,24,5]%%}+%%{-4398046511104, [3,8,7,0,1,26,6]%%}+%%{-17985175552, [3,8,6,5,8,16,0]%%}+%%{161866579968, [3,8,6,4,7,18,1]%%}+%%{1586990415872, [3,8,6,3,6,20,2]%%}+%%{-1795296329728, [3,8,6,2,5,22,3]%%}+%%{-4123168604160, [3,8,6,1,4,24,4]%%}+%%{3848290697216, [3,8,6,0,3,26,5]%%}+%%{12213813248, [3,8,5,4,9,18,0]%%}+%%{-171798691840, [3,8,5,3,8,20,1]%%}+%%{-212600881152, [3,8,5,2,7,22,2]%%}+%%{14

77468749824, [3,8,5,1,6,24,3]%%}+%%{-1099511627776, [3,8,5,0,5,26,4]%%}+%%
%{-3221225472, [3,8,4,3,10,20,0]%%}+%%{57982058496, [3,8,4,2,9,22,1]%%}+%%
%{-154618822656, [3,8,4,1,8,24,2]%%}+%%{103079215104, [3,8,4,0,7,26,3]%%}+
%%{1048576, [3,6,10,4,2,4,0]%%}+%%{8388608, [3,6,10,3,1,6,1]%%}+%%{16777
216, [3,6,10,2,0,8,2]%%}+%%{-5242880, [3,6,9,3,3,6,0]%%}+%%{-29360128, [3,
6,9,2,2,8,1]%%}+%%{-33554432, [3,6,9,1,1,10,2]%%}+%%{9699328, [3,6,8,2,4,
8,0]%%}+%%{33554432, [3,6,8,1,3,10,1]%%}+%%{16777216, [3,6,8,0,2,12,2]%%
}+%%{-7864320, [3,6,7,1,5,10,0]%%}+%%{-12582912, [3,6,7,0,4,12,1]%%}+%%{
2359296, [3,6,6,0,6,12,0]%%}+%%{536870912, [2,7,10,6,2,8,1]%%}+%%{6710886
400, [2,7,10,5,1,10,2]%%}+%%{18253611008, [2,7,10,4,0,12,3]%%}+%%{-134217
728, [2,7,9,6,4,8,0]%%}+%%{-5502926848, [2,7,9,5,3,10,1]%%}+%%{-369098752
00, [2,7,9,4,2,12,2]%%}+%%{-42949672960, [2,7,9,3,1,14,3]%%}+%%{429496729
60, [2,7,9,2,0,16,4]%%}+%%{956301312, [2,7,8,5,5,10,0]%%}+%%{18656264192,
[2,7,8,4,4,12,1]%%}+%%{64961380352, [2,7,8,3,3,14,2]%%}+%%{-8589934592, [2,
7,8,2,2,16,3]%%}+%%{-85899345920, [2,7,8,1,1,18,4]%%}+%%{-2642411520, [2,
7,7,4,6,12,0]%%}+%%{-27783069696, [2,7,7,3,5,14,1]%%}+%%{-33957085184,
[2,7,7,2,4,16,2]%%}+%%{73014444032, [2,7,7,1,3,18,3]%%}+%%{42949672960, [2,
7,7,0,2,20,4]%%}+%%{3556769792, [2,7,6,3,7,14,0]%%}+%%{17716740096, [2,
7,6,2,6,16,1]%%}+%%{-12884901888, [2,7,6,1,5,18,2]%%}+%%{-39728447488, [2,
7,6,0,4,20,3]%%}+%%{-2340421632, [2,7,5,2,8,16,0]%%}+%%{-2415919104, [2,
7,5,1,7,18,1]%%}+%%{12079595520, [2,7,5,0,6,20,2]%%}+%%{603979776, [2,7,4
,1,9,18,0]%%}+%%{-1207959552, [2,7,4,0,8,20,1]%%}+%%{2147483648, [1,8,10,
9,3,10,1]%%}+%%{38654705664, [1,8,10,8,2,12,2]%%}+%%{51539607552, [1,8,10
,7,1,14,3]%%}+%%{-274877906944, [1,8,10,6,0,16,4]%%}+%%{-536870912, [1,8,
9,9,5,10,0]%%}+%%{-26843545600, [1,8,9,8,4,12,1]%%}+%%{-188978561024, [1,
8,9,7,3,14,2]%%}+%%{146028888064, [1,8,9,6,2,16,3]%%}+%%{962072674304, [1
,8,9,5,1,18,4]%%}+%%{-549755813888, [1,8,9,4,0,20,5]%%}+%%{4294967296, [1
,8,8,8,6,12,0]%%}+%%{95026151424, [1,8,8,7,5,14,1]%%}+%%{239444426752, [1
,8,8,6,4,16,2]%%}+%%{-858993459200, [1,8,8,5,3,18,3]%%}+%%{-618475290624
, [1,8,8,4,2,20,4]%%}+%%{1099511627776, [1,8,8,3,1,22,5]%%}+%%{-127506841
60, [1,8,7,7,7,14,0]%%}+%%{-136633647104, [1,8,7,6,6,16,1]%%}+%%{62277025
792, [1,8,7,5,5,18,2]%%}+%%{936302870528, [1,8,7,4,4,20,3]%%}+%%{-5497558
13888, [1,8,7,3,3,22,4]%%}+%%{-549755813888, [1,8,7,2,2,24,5]%%}+%%{17985
175552, [1,8,6,6,8,16,0]%%}+%%{71940702208, [1,8,6,5,7,18,1]%%}+%%{-26736
1714176, [1,8,6,4,6,20,2]%%}+%%{-137438953472, [1,8,6,3,5,22,3]%%}+%%{481
036337152, [1,8,6,2,4,24,4]%%}+%%{-12213813248, [1,8,5,5,9,18,0]%%}+%%{72
47757312, [1,8,5,4,8,20,1]%%}+%%{103079215104, [1,8,5,3,7,22,2]%%}+%%{-13
7438953472, [1,8,5,2,6,24,3]%%}+%%{3221225472, [1,8,4,4,10,20,0]%%}+%%{-1
2884901888, [1,8,4,3,9,22,1]%%}+%%{12884901888, [1,8,4,2,8,24,2]%%}+%%{-1
048576, [1,6,10,5,2,4,0]%%}+%%{-8388608, [1,6,10,4,1,6,1]%%}+%%{-16777216
, [1,6,10,3,0,8,2]%%}+%%{8388608, [1,6,9,4,3,6,0]%%}+%%{62914560, [1,6,9,3
,2,8,1]%%}+%%{150994944, [1,6,9,2,1,10,2]%%}+%%{134217728, [1,6,9,1,0,12,
3]%%}+%%{-26476544, [1,6,8,3,4,8,0]%%}+%%{-163577856, [1,6,8,2,3,10,1]%%
}+%%{-301989888, [1,6,8,1,2,12,2]%%}+%%{-134217728, [1,6,8,0,1,14,3]%%}+
%%{41156608, [1,6,7,2,5,10,0]%%}+%%{178257920, [1,6,7,1,4,12,1]%%}+%%{167


```

772160, [1, 6, 7, 0, 3, 14, 2]%%}+%%{-31457280, [1, 6, 6, 1, 6, 12, 0]%%}+%%{-6920601
6, [1, 6, 6, 0, 5, 14, 1]%%}+%%{9437184, [1, 6, 5, 0, 7, 14, 0]%%}+%%{-402653184, [0, 7
, 10, 7, 2, 8, 1]%%}+%%{-5637144576, [0, 7, 10, 6, 1, 10, 2]%%}+%%{-16106127360, [0,
7, 10, 5, 0, 12, 3]%%}+%%{100663296, [0, 7, 9, 7, 4, 8, 0]%%}+%%{4160749568, [0, 7, 9,
6, 3, 10, 1]%%}+%%{30198988800, [0, 7, 9, 5, 2, 12, 2]%%}+%%{28991029248, [0, 7, 9, 4
, 1, 14, 3]%%}+%%{-68719476736, [0, 7, 9, 3, 0, 16, 4]%%}+%%{-687865856, [0, 7, 8, 6,
5, 10, 0]%%}+%%{-13925089280, [0, 7, 8, 5, 4, 12, 1]%%}+%%{-48184164352, [0, 7, 8, 4
, 3, 14, 2]%%}+%%{49392123904, [0, 7, 8, 3, 2, 16, 3]%%}+%%{120259084288, [0, 7, 8, 2
, 1, 18, 4]%%}+%%{-68719476736, [0, 7, 8, 1, 0, 20, 5]%%}+%%{1845493760, [0, 7, 7, 5,
6, 12, 0]%%}+%%{19964887040, [0, 7, 7, 4, 5, 14, 1]%%}+%%{11542724608, [0, 7, 7, 3, 4
, 16, 2]%%}+%%{-113816633344, [0, 7, 7, 2, 3, 18, 3]%%}+%%{8589934592, [0, 7, 7, 1, 2
, 20, 4]%%}+%%{68719476736, [0, 7, 7, 0, 1, 22, 5]%%}+%%{-2432696320, [0, 7, 6, 4, 7,
14, 0]%%}+%%{-11207180288, [0, 7, 6, 3, 6, 16, 1]%%}+%%{28185722880, [0, 7, 6, 2, 5,
18, 2]%%}+%%{34359738368, [0, 7, 6, 1, 4, 20, 3]%%}+%%{-60129542144, [0, 7, 6, 0, 3,
22, 4]%%}+%%{1577058304, [0, 7, 5, 3, 8, 16, 0]%%}+%%{-201326592, [0, 7, 5, 2, 7, 18,
1]%%}+%%{-14495514624, [0, 7, 5, 1, 6, 20, 2]%%}+%%{17179869184, [0, 7, 5, 0, 5, 22,
3]%%}+%%{-402653184, [0, 7, 4, 2, 9, 18, 0]%%}+%%{1610612736, [0, 7, 4, 1, 8, 20, 1]%%
%%}+%%{-1610612736, [0, 7, 4, 0, 7, 22, 2]%%} / %%{-1024, [0, 3, 4, 2, 1, 2, 0]%%}+%%
%{-4096, [0, 3, 4, 1, 0, 4, 1]%%}+%%{2560, [0, 3, 3, 1, 2, 4, 0]%%}+%%{4096, [0, 3, 3, 0,
1, 6, 1]%%}+%%{-1536, [0, 3, 2, 0, 3, 6, 0]%%} Error: Bad Argument Value

```

maple [C] time = 0.02, size = 161, normalized size = 0.67

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $-1/2*e^{(3/2)}*\text{sum}((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^2+d)^{(1/2)}/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(\text{sqrt}(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)`

[Out] `int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a), x)`

[Out] `Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)`

$$3.264 \quad \int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=291

$$\frac{c \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) - c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) \frac{\sqrt{d+ex^2}}{ax}}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} - a \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}}$$

Rubi [A] time = 0.67, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1295, 264, 1692, 377, 205}

$$\frac{c \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) - c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) \frac{\sqrt{d+ex^2}}{ax}}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} - a \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_)^(m_))*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x], x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e + c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)} dx &= -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{a} + \frac{d \int \frac{1}{x^2\sqrt{d+ex^2}} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{cd + \frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{cd - \frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{a} - \frac{\left(c \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b+\sqrt{b^2-4ac} - (-2cd+(b+\sqrt{b^2-4ac})e)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{a} \\
&= -\frac{\sqrt{d+ex^2}}{ax} - \frac{c \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})e} x}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a\sqrt{b - \sqrt{b^2-4ac}}} - \frac{c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd - (b + \sqrt{b^2-4ac})e} x}{\sqrt{b + \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a\sqrt{b + \sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [B] time = 6.32, size = 4644, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] $-\frac{\sqrt{d+ex^2}}{ax} - \frac{(-1/2*(b*d*(\text{Log}[\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] + x)/\text{Sqrt}[d + ((-b/c) - \text{Sqrt}[b^2 - 4*a*c]/c)*e]/2) - \text{Log}[2*d - \text{Sqrt}[2]*\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e*x + 2*\text{Sqrt}[d + ((-b/c) - \text{Sqrt}[b^2 - 4*a*c]/c)*e]/2]}{\text{Sqrt}[2]*c*\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*(-(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2]) - \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2])} + \frac{(\text{Sqrt}[b^2 - 4*a*c]*d*(\text{Log}[\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2] + x)/\text{Sqrt}[d + ((-b/c) - \text{Sqrt}[b^2 - 4*a*c]/c)*e]/2) - \text{Log}[2*d - \text{Sqrt}[2]*\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*e*x + 2*\text{Sqrt}[d + ((-b/c) - \text{Sqrt}[b^2 - 4*a*c]/c)*e]/2]}{2*\text{Sqrt}[2]*c*\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]*(-(\text{Sqrt}[-(b/c) - \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2]) - \text{Sqrt}[-(b/c) + \text{Sqrt}[b^2 - 4*a*c]/c]/\text{Sqrt}[2])}$

$$\begin{aligned}
& c/\sqrt{2}) * (-(\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c)/\sqrt{2}) + \sqrt{-(b/c)} + \\
& \sqrt{b^2 - 4ac}/c)/\sqrt{2})) + (a * (\log[\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] / \\
& \sqrt{2} + x)/\sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d - \\
& \sqrt{2} * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2} * \sqrt{d + e * x^2}] / \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2)) / (\sqrt{2} * c * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c) * (-\sqrt{-(b/c)} \\
& - \sqrt{b^2 - 4ac}/c)/\sqrt{2}) - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2} \\
&] * (-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c)/\sqrt{2}) + \sqrt{-(b/c)} + \sqrt{b^2 - \\
& - 4ac}/c)/\sqrt{2})) + (b * d * (\log[-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] / \sqrt{2} \\
& [2]) + x)/\sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d + \sqrt{2} \\
&] * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2} * \sqrt{d + e * x^2}] / \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * \\
& e}/2)) / (2 * \sqrt{2} * c * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c) * (\sqrt{-(b/c)} - \sqrt{ \\
& b^2 - 4ac}/c)/\sqrt{2} - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2}) * (\sqrt{ \\
& -(b/c)} - \sqrt{b^2 - 4ac}/c)/\sqrt{2} + \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c \\
&]/\sqrt{2})) - (\sqrt{b^2 - 4ac} * d * (\log[-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] / \sqrt{2} \\
& [2]) + x)/\sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d + \\
& \sqrt{2} * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2} * \sqrt{d + e * x^2}] / \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2)) / (2 * \sqrt{2} * c * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c) * (\sqrt{-(b/c)} \\
& - \sqrt{b^2 - 4ac}/c)/\sqrt{2} - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2} \\
&) * (\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c)/\sqrt{2} + \sqrt{-(b/c)} + \sqrt{b^2 - 4ac} \\
& /c)/\sqrt{2})) - (a * (\log[-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c] / \sqrt{2} [2]) \\
& + x)/\sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d + \sqrt{2} * \sqrt{ \\
& -(b/c)} - \sqrt{b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac} \\
& /c) * e}/2} * \sqrt{d + e * x^2}] / \sqrt{d + ((-b/c) - \sqrt{b^2 - 4ac}/c) * e}/2 \\
&])) / (\sqrt{2} * c * \sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c) * (\sqrt{-(b/c)} - \sqrt{b^2 - \\
& 4ac}/c)/\sqrt{2} - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2}) * (\sqrt{-(b/c)} \\
& - \sqrt{b^2 - 4ac}/c)/\sqrt{2} + \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2} \\
& [2])) - (b * d * (\log[\sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c] / \sqrt{2} [2] + x)/\sqrt{d + \\
& ((-b/c) + \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d - \sqrt{2} * \sqrt{-(b/c)} + \sqrt{ \\
& b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) + \sqrt{b^2 - 4ac}/c) * e}/2} * \sqrt{ \\
& d + e * x^2}] / \sqrt{d + ((-b/c) + \sqrt{b^2 - 4ac}/c) * e}/2)) / (2 * \sqrt{2} * \\
& c * \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c) * (-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac}/c) / \\
& \sqrt{2}) - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2}) * (\sqrt{-(b/c)} - \sqrt{b^2 - \\
& 4ac}/c)/\sqrt{2} - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2})) - (\sqrt{ \\
& b^2 - 4ac} * d * (\log[\sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c] / \sqrt{2} [2] + x)/\sqrt{ \\
& d + ((-b/c) + \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d - \sqrt{2} * \sqrt{-(b/c)} \\
& + \sqrt{b^2 - 4ac}/c] * e * x + 2 * \sqrt{d + ((-b/c) + \sqrt{b^2 - 4ac}/c) * e} \\
& /2} * \sqrt{d + e * x^2}] / \sqrt{d + ((-b/c) + \sqrt{b^2 - 4ac}/c) * e}/2)) / (2 * \sqrt{ \\
& 2} * c * \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c) * (-\sqrt{-(b/c)} - \sqrt{b^2 - 4ac} \\
& /c)/\sqrt{2}) - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2}) * (\sqrt{-(b/c)} - \\
& \sqrt{b^2 - 4ac}/c)/\sqrt{2} - \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c)/\sqrt{2})) + (a * (\log[\sqrt{-(b/c)} + \sqrt{b^2 - 4ac}/c] / \sqrt{2} [2] + x)/\sqrt{d + ((- \\
& b/c) + \sqrt{b^2 - 4ac}/c) * e}/2} - \log[2 * d - \sqrt{2} * \sqrt{-(b/c)} + \sqrt{b^2 - 4ac}
\end{aligned}$$

$$\begin{aligned}
& 2 - 4*a*c]/c]*e*x + 2*sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]*sqrt[d \\
& + e*x^2]]/sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]))/((sqrt[2]*c*sqrt \\
& [-(b/c) + sqrt[b^2 - 4*a*c]/c]*(-(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/c]/sqrt[2] \\
&]) - sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]))*(sqrt[-(b/c) - sqrt[b^2 - \\
& 4*a*c]/c]/sqrt[2] - sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2])) + (b*d*(Lo \\
& g[-(sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]) + x]/sqrt[d + ((-(b/c) + sq \\
& rt[b^2 - 4*a*c]/c)*e)/2] - log[2*d + sqrt[2]*sqrt[-(b/c) + sqrt[b^2 - 4*a*c \\
&]/c]*e*x + 2*sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]*sqrt[d + e*x^2 \\
&]/sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]))/((2*sqrt[2]*c*sqrt[-(b/c) \\
& + sqrt[b^2 - 4*a*c]/c]*(-(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/c]/sqrt[2])) + sq \\
& rt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]))*(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/ \\
& c]/sqrt[2] + sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2])) + (sqrt[b^2 - 4*a \\
& *c]*d*(log[-(sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]) + x]/sqrt[d + ((- \\
& b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2] - log[2*d + sqrt[2]*sqrt[-(b/c) + sqrt[b^ \\
& 2 - 4*a*c]/c]*e*x + 2*sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]*sqrt[d \\
& + e*x^2]]/sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]))/((2*sqrt[2]*c*sq \\
& rt[-(b/c) + sqrt[b^2 - 4*a*c]/c]*(-(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/c]/sqrt \\
& [2])) + sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]))*(sqrt[-(b/c) - sqrt[b^2 \\
& - 4*a*c]/c]/sqrt[2] + sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2])) - (a*e*(\\
& log[-(sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]) + x]/sqrt[d + ((-(b/c) + \\
& sqrt[b^2 - 4*a*c]/c)*e)/2] - log[2*d + sqrt[2]*sqrt[-(b/c) + sqrt[b^2 - 4*a \\
& *c]/c]*e*x + 2*sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]*sqrt[d + e*x^ \\
& 2]]/sqrt[d + ((-(b/c) + sqrt[b^2 - 4*a*c]/c)*e)/2]))/((sqrt[2]*c*sqrt[-(b/c) \\
& + sqrt[b^2 - 4*a*c]/c]*(-(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/c]/sqrt[2])) + sq \\
& rt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2]))*(sqrt[-(b/c) - sqrt[b^2 - 4*a*c]/ \\
& c]/sqrt[2] + sqrt[-(b/c) + sqrt[b^2 - 4*a*c]/c]/sqrt[2])))/a
\end{aligned}$$

IntegrateAlgebraic [C] time = 70.51, size = 348, normalized size = 1.20

$$\frac{\sqrt{c} \operatorname{RootSum} \left[\#1^8 c + 4\#1^6 b c - 4\#1^4 c d + 16\#1^4 a c^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{\#1^5 d \log(-\#1 + \sqrt{d+e x^2} - \sqrt{c x}) - 4\#1^2 a^2 \log(-\#1 + \sqrt{d+e x^2} - \sqrt{c x}) + 4\#1^2 b d e \log(-\#1 + \sqrt{d+e x^2} - \sqrt{c x}) - 2\#1^2 c d^2 \log(-\#1 + \sqrt{d+e x^2} - \sqrt{c x}) + c d^3 \log(-\#1 + \sqrt{d+e x^2} - \sqrt{c x})}{\#1^7 (-c) - 3\#1^5 b c + 3\#1^3 c d - 8\#1^2 a c^2 + 4\#1^2 b d e - 3\#1^2 c d^2 - 4\beta c e d} \right]}{2a} \frac{\sqrt{d+e x^2}}{a x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]

[Out] -(sqrt[d + e*x^2]/(a*x)) - (sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*
e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#
1^6 + c*#1^8 & , (c*d^3*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1] - 2*c*d^2*
Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1]*#1^2 + 4*b*d*e*Log[-(sqrt[e]*x) +
sqrt[d + e*x^2] - #1]*#1^2 - 4*a*e^2*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #
1]*#1^2 + c*d*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2
*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 -
c*#1^6) &])/(2*a)

fricas [B] time = 7.22, size = 2402, normalized size = 8.25

$$4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.03, size = 272, normalized size = 0.93

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x)

[Out]
$$-1/a/d/x*(e*x^2+d)^{(3/2)}+1/a*e/d*x*(e*x^2+d)^{(1/2)}+1/a*e^{(1/2)}*\ln(e^{(1/2)}*x+(e*x^2+d)^{(1/2)})+1/2/a*e^{(1/2)}*\sum((_R^2*c*d+2*(-2*a*e^2+2*b*d*e-c*d^2)*_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/a*e^{(1/2)}*\ln(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d}}{x^2 (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)`

[Out] `int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^2 (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a), x)`

[Out] `Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)`

$$3.265 \quad \int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=373

$$\frac{c \left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{c \left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Rubi [A] time = 2.53, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$\frac{c \left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{c \left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{\sqrt{d+ex^2}(bd-ae)}{a^2 dx} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\sqrt{d+ex^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)), x]

[Out] $-\text{Sqrt}[d + e*x^2]/(3*a*x^3) + (2*e*\text{Sqrt}[d + e*x^2])/(3*a*d*x) + ((b*d - a*e) * \text{Sqrt}[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])]*e)*x]/(\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[d + e*x^2]))/(a^2*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])]*e) + (c*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])]*e)*x]/(\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[d + e*x^2]))/(a^2*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])]*e)$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_)^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)
^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)} dx &= -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{a} + \frac{d \int \frac{1}{x^4\sqrt{d+ex^2}} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} - \frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}(a+bx^2+cx^4)} \right) dx}{3a} - \frac{(2e) \int \frac{1}{x^2\sqrt{d+ex^2}} dx}{3a} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{a^2} - \frac{(bd-ae) \int \frac{1}{x^2\sqrt{d+ex^2}} dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2 dx} - \frac{\int \left(\frac{-c(bd-ae) - \frac{c(b^2d-2acd-abe)}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{-c(bd-ae)}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2 dx} + \frac{\left(c \left(bd-ae - \frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2 dx} + \frac{\left(c \left(bd-ae - \frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \right) \text{Subst}\left(\frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}}, \frac{b+\sqrt{b^2-4ac}+2cx^2}{2c}, \frac{d+ex^2}{4c}\right)}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2 dx} + \frac{c \left(bd-ae + \frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{\sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})x^2}}
\end{aligned}$$

Mathematica [B] time = 6.39, size = 7777, normalized size = 20.85

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 111.74, size = 479, normalized size = 1.28

$\frac{\sqrt{d+ex^2}(-cd-ae^2+3bd^2)}{3e^2d^3} - \frac{\sqrt{c}\text{RootSum}\left[0t^3c+40t^2bc-40t^2cd+160t^2ac^2-80t^2bd+60t^2cd^2+40t^2bc^2-40t^2cd^2+c^2d^2, \frac{-e^2bx\sqrt{d+ex^2}-\sqrt{c}}{e^2}\right] + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}-\sqrt{c}}{e^2}\right) + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}+\sqrt{c}}{e^2}\right) + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}-\sqrt{c}}{e^2}\right) + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}+\sqrt{c}}{e^2}\right) + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}-\sqrt{c}}{e^2}\right) + e^2\sqrt{c}\log\left(\frac{e^2bx\sqrt{d+ex^2}+\sqrt{c}}{e^2}\right)}{e^2\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})x^2}} + \frac{-c(bd-ae) - \frac{c(b^2d-2acd-abe)}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{-c(bd-ae)}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - a*e*x^2))/(3*a^2*d*x^3) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^2*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 2*a*c*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*b*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) &])/(2*a^2)

fricas [B] time = 31.16, size = 4095, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/x^2)


```

- 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2
- 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c
^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x
^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt
t(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4
*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 1
3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)
*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*
b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1
1*c))))/(a^5*b^2 - 4*a^6*c))/x^2) + 4*((3*b*d - a*e)*x^2 - a*d)*sqrt(e*x^2
+ d))/(a^2*d*x^3)

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.04, size = 322, normalized size = 0.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a^2*b/d/x*(e*x^2+d)^(3/2)-1/a^2*b*e/d*x*(e*x^2+d)^(1/2)-1/a^2*b*e^(1/2)*1
n(e^(1/2)*x+(e*x^2+d)^(1/2))+1/2/a^2*e^(1/2)*sum((c*(a*e-b*d)*_R^2+2*(2*a*b
*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+a*c*d^2*e-b*c*d^3)/(_R^3*c+3*_R^2*b*e-3*
_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x
+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*
b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/a^2*e^(1/2)*b*ln(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/3/a/d/x^3*(e*x^2+d)^(3/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d}}{x^4 (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)

[Out] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^4 (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)

[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)

$$3.266 \quad \int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=512

$$\frac{\sqrt{d+ex^2}(-abe-acd+b^2d)}{a^3dx} - \frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x} + \frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3} - \frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd}}$$

Rubi [A] time = 4.94, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$\frac{\sqrt{d+ex^2}(-abe-acd+b^2d)}{a^3dx} - \frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} - \frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}} - abe - acd + b^2d\right) \tan^{-1}\left(\frac{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x} + \frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\sqrt{d+ex^2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)), x]

[Out] $-\sqrt{d+ex^2}/(5ax^5) + (4e\sqrt{d+ex^2})/(15ad^2x^3) + ((bd-ae)\sqrt{d+ex^2})/(3a^2d^2x^3) - (8e^2\sqrt{d+ex^2})/(15ad^2x) - (2e(bd-ae)\sqrt{d+ex^2})/(3a^2d^2x) - ((b^2d-abc-d-ab^2e)\sqrt{d+ex^2})/(a^3dx) - (c(b^2d-abc-d-ab^2e+(b^3d-3abc*d-d-ab^2e+2a^2c*e)/\sqrt{b^2-4ac})\text{ArcTan}[(\sqrt{2cd-(b-\sqrt{b^2-4ac})}e)x]/(\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2})))/(a^3\sqrt{b-\sqrt{b^2-4ac}}) - (c(b^2d-abc-d-ab^2e+(b^3d-3abc*d-d-ab^2e+2a^2c*e)/\sqrt{b^2-4ac})\text{ArcTan}[(\sqrt{2cd-(b+\sqrt{b^2-4ac})}e)x]/(\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2})))/(a^3\sqrt{b+\sqrt{b^2-4ac}}) - (8e^2\sqrt{d+ex^2})/(15ad^2x) + (4e\sqrt{d+ex^2})/(15adx^3) - \sqrt{d+ex^2}/(5ax^5)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)} dx &= -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{a} + \frac{d \int \frac{1}{x^6\sqrt{d+ex^2}} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} - \frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}} + \frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-abe)x^2}{a^2\sqrt{d+ex^2}(a+bx^2+cx^4)} \right) dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-abe)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{a^3} + \frac{(8e^2) \int \frac{1}{x^2\sqrt{d+ex^2}} dx}{15ad} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{(b^2d-acd-abe)}{a^3dx} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x}
\end{aligned}$$

Mathematica [B] time = 6.59, size = 10933, normalized size = 21.35

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 160.94, size = 687, normalized size = 1.34

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[d + e*x^2]*(-3*a^2*d^2 + 5*a*b*d^2*x^2 - a^2*d*e*x^2 - 15*b^2*d^2*x^4 + 15*a*c*d^2*x^4 + 5*a*b*d*e*x^4 + 2*a^2*e^2*x^4))/(15*a^3*d^2*x^5) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^2*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*b*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^2*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*c^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^3*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 6*a*b*c*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a*b^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a^2*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^2*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*b*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) &])/(2*a^3)

fricas [B] time = 77.19, size = 5773, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c))

$$\begin{aligned}
& 2*b^{10} - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2 \\
&)/(a^{14}*b^2 - 4*a^{15}*c)) + ((a*b^{10} - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4 \\
& 4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4 \\
& *b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*\sqrt{-((b^7 - 7*a*b^5*c + 1 \\
& 4*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a \\
& ^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*\sqrt{((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 \\
& - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} \\
& - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6 \\
& *b*c^5)*d*e + (a^2*b^{10} - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9 \\
& *a^6*b^2*c^4)*e^2)/(a^{14}*b^2 - 4*a^{15}*c)))/(a^7*b^2 - 4*a^8*c))/x^2) - 15* \\
& \sqrt{1/2)*a^3*d^2*x^5*\sqrt{-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3) \\
& *d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a \\
& ^8*c)*\sqrt{((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4* \\
& b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} - 9*a^2*b^9*c + 29*a^3* \\
& b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^{10} - \\
& 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^{14}*b \\
& ^2 - 4*a^{15}*c)))/(a^7*b^2 - 4*a^8*c))*\log(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2 \\
& *\sqrt{((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c \\
& ^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} - 9*a^2*b^9*c + 29*a^3*b^7*c \\
& ^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^{10} - 8*a^3 \\
& *b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^{14}*b^2 - \\
& 4*a^{15}*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 - \\
& 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4 \\
& + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3* \\
& b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^ \\
& 2)*x^2 - 2*\sqrt{1/2)*\sqrt{e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^{10}*b*c^ \\
& 2)*x*\sqrt{((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b \\
& ^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} - 9*a^2*b^9*c + 29*a^3*b \\
& ^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^{10} - 8 \\
& *a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^{14}*b^ \\
& 2 - 4*a^{15}*c)) + ((a*b^{10} - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 \\
& + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - \\
& 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*\sqrt{-((b^7 - 7*a*b^5*c + 14*a^2*b^3* \\
& c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e \\
& - (a^7*b^2 - 4*a^8*c)*\sqrt{((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b \\
& ^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} - 9*a^2 \\
& *b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d* \\
& e + (a^2*b^{10} - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c \\
& ^4)*e^2)/(a^{14}*b^2 - 4*a^{15}*c)))/(a^7*b^2 - 4*a^8*c))/x^2) + 15*\sqrt{1/2)* \\
& a^3*d^2*x^5*\sqrt{-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a \\
& b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*\sqrt{ \\
& ((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - \\
& 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^{11} - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - \\
& 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^{10} - 8*a^3*b^8* \\
& c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^{14}*b^2 - 4*a^{15}
\end{aligned}$$

$$\begin{aligned} & \sqrt{6b^6c^4}e) * x) * \sqrt{-(b^7 - 7ab^5c + 14a^2b^3c^2 - 7a^3b^2c^3)d - \\ & (ab^6 - 6a^2b^4c + 9a^3b^2c^2 - 2a^4c^3)e + (a^7b^2 - 4a^8c) * \\ & \sqrt{((b^{12} - 10a^2b^{10}c + 37a^4b^8c^2 - 62a^6b^6c^3 + 46a^8b^4c^4 - 12a^{10}b^2c^5 + a^{12}c^6)d^2 - 2(a^{11}b - 9a^2b^9c + 29a^3b^7c^2 - 40a^4b^5c^3 + 22a^5b^3c^4 - 3a^6b^2c^5)d * e + (a^2b^{10} - 8a^3b^8c + 22a^4b^6c^2 - 24a^5b^4c^3 + 9a^6b^2c^4)e^2) / (a^{14}b^2 - 4a^{15}c))} / (a^7b^2 - 4a^8c)) / x^2 - 4((5ab^2d * e + 2a^2e^2 - 15(b^2 - ac)d^2) * x^4 - 3a^2d^2 + (5ab^2d^2 - a^2d * e) * x^2) * \sqrt{e * x^2 + d} / (a^3d^2 * x^5) \end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 503, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x)

[Out] $\frac{1}{a^2 d x} (e x^2 + d)^{3/2} c - \frac{1}{a^3 d x} (e x^2 + d)^{3/2} b^2 - \frac{1}{a^2 e d x} (e x^2 + d)^{1/2} c + \frac{1}{a^3 e d x} (e x^2 + d)^{1/2} b^2 - \frac{1}{a^2 e^{1/2}} \ln(e^{1/2} x + (e x^2 + d)^{1/2}) c + \frac{1}{a^3 e^{1/2}} \ln(e^{1/2} x + (e x^2 + d)^{1/2}) b^2 - \frac{1}{2 a^3 e^{1/2}} \sum((c(a b e + a c d - b^2 d) * _R^2 + 2(-2 a^2 c e^2 + 2 a b^2 e^2 + 3 a b c d e - a c^2 d^2 - 2 b^3 d e + b^2 c d^2) * _R + a b c d^2 e + a c^2 d^3 - b^2 c d^3) / (_R^3 c + 3 _R^2 b e - 3 _R^2 c d + 8 _R a e^2 - 4 _R b d e + 3 _R c d^2 + b d^2 e - c d^3) * \ln(-_R + (-e^{1/2} x + (e x^2 + d)^{1/2})^2), _R = \text{RootOf}(_Z^4 c + c d^4 + (4 b e - 4 c d) _Z^3 + (16 a e^2 - 8 b d e + 6 c d^2) _Z^2 + (4 b d^2 e - 4 c d^3) _Z)) - \frac{1}{a^2 e^{1/2}} \ln(-e^{1/2} x + (e x^2 + d)^{1/2}) c + \frac{1}{a^3 e^{1/2}} \ln(-e^{1/2} x + (e x^2 + d)^{1/2}) b^2 + \frac{1}{3 a^2 b d x^3} (e x^2 + d)^{3/2} - \frac{1}{5 a d x^5} (e x^2 + d)^{3/2} + \frac{2}{15 a e d^2 x^3} (e x^2 + d)^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d}}{x^6 (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)

[Out] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^6 (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a), x)

[Out] Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)

$$3.267 \quad \int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=460

$$\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)$$

$$\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}$$

Rubi [A] time = 5.08, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{bc\left(\frac{2d\sqrt{b^2-4ac}-3ae}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}+cd^2\right)+c\left(\frac{ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}\right)-b^2e\left(\frac{e\sqrt{b^2-4ac}+2cd}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}\right)+b^3e^2}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} \frac{bc\left(\frac{2d\sqrt{b^2-4ac}-3ae}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}\right)+c\left(\frac{ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}\right)-b^2e\left(\frac{e\sqrt{b^2-4ac}+2cd}{\sqrt{2d-e(b-\sqrt{b^2-4ac})}}\right)+b^3e^2}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} \frac{\frac{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 824

Int[(((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[(d + e*x)^(m-1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]]/(a + b*x + c*x^2), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (d + ex^2)^{3/2}}{a + bx^2 + cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x(d + ex)^{3/2}}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{(d + ex^2)^{3/2}}{3c} + \frac{\text{Subst} \left(\int \frac{\sqrt{d+ex} (-ae+(cd-be)x)}{a+bx+cx^2} dx, x, x^2 \right)}{2c} \\
&= \frac{(cd - be)\sqrt{d + ex^2}}{c^2} + \frac{(d + ex^2)^{3/2}}{3c} + \frac{\text{Subst} \left(\int \frac{-ae(2cd-be) + (c^2d^2 + b^2e^2 - ce(2bd+ae))x}{\sqrt{d+ex}(a+bx+cx^2)} dx, x, x^2 \right)}{2c^2} \\
&= \frac{(cd - be)\sqrt{d + ex^2}}{c^2} + \frac{(d + ex^2)^{3/2}}{3c} + \frac{\text{Subst} \left(\int \frac{-ae^2(2cd-be) - d(c^2d^2 + b^2e^2 - ce(2bd+ae)) + (c^2d^2 + b^2e^2 - ce(2cd-be)x^2 + cx^4)}{cd^2 - bde + ae^2 + (-2cd+be)x^2 + cx^4} dx, x, x^2 \right)}{c^2} \\
&= \frac{(cd - be)\sqrt{d + ex^2}}{c^2} + \frac{(d + ex^2)^{3/2}}{3c} - \frac{(b^3e^2 - b^2e(2cd + \sqrt{b^2 - 4ac}e) + c(a\sqrt{b^2 - 4ac}e^2 - \dots))}{c^2} \\
&= \frac{(cd - be)\sqrt{d + ex^2}}{c^2} + \frac{(d + ex^2)^{3/2}}{3c} + \frac{(b^3e^2 - b^2e(2cd + \sqrt{b^2 - 4ac}e) + c(a\sqrt{b^2 - 4ac}e^2 - \dots))}{c^2}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 457, normalized size = 0.99

$$\frac{(-bc(e(2d\sqrt{b^2-4ac}-3ae)+ae^2)+c(cd(d\sqrt{b^2-4ac}-4ae)-ae^2\sqrt{b^2-4ac})+b^2e(e\sqrt{b^2-4ac}+2cd)+b^3(-e^2))\text{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex^2}}{\sqrt{b^2-4ac}-bx+2cd}\right)+bc(ae^2-e(2d\sqrt{b^2-4ac}+3ae))+c(cd(d\sqrt{b^2-4ac}+4ae)-ae^2\sqrt{b^2-4ac})+b^2e(e\sqrt{b^2-4ac}-2cd)+b^3e^2)\text{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex^2}}{\sqrt{b^2-4ac}+bx}\right)+\sqrt{d+ex^2}\frac{(cd-be)}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}}{\sqrt{2}e^{3/2}\sqrt{b^2-4ac}\sqrt{e}\sqrt{b^2-4ac-b}+2cd}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) - ((- (b^3*e^2) + b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))) * ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - ((b^3*e^2 + b^2*e*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e))) * ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

IntegrateAlgebraic [C] time = 2.12, size = 524, normalized size = 1.14

$$\frac{(c^2d^2\sqrt{4ac-b^2} - 2bdce\sqrt{4ac-b^2} + b^2d^2\sqrt{4ac-b^2} - ac^2\sqrt{4ac-b^2} - 3abce^2 + 4aa^2de + b^3d^2 - 2b^2cde + bc^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{4ac-b^2}}{\sqrt{c^2d^2+4ac-b^2}}\right) + (c^2d^2\sqrt{4ac-b^2} - 2bdce\sqrt{4ac-b^2} + b^2d^2\sqrt{4ac-b^2} - ac^2\sqrt{4ac-b^2} + 3abce^2 - 4aa^2de - b^3d^2 + 2b^2cde - bc^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{4ac-b^2}}{\sqrt{c^2d^2+4ac-b^2}}\right) + \sqrt{d+e^2}\frac{(-3be+4cd+ce^2)}{3c^2}}{\sqrt{2}c^2\sqrt{4ac-b^2}\sqrt{-be-2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]

[Out] (Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2))/(3*c^2) + ((I*b*c^2*d^2 + c^2*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*b^2*c*d*e + (4*I)*a*c^2*d*e - 2*b*c*Sqrt[-b^2 + 4*a*c]*d*e + I*b^3*e^2 - (3*I)*a*b*c*e^2 + b^2*Sqrt[-b^2 + 4*a*c]*e^2 - a*c*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]]/(Sqrt[2]*c^(5/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((-I)*b*c^2*d^2 + c^2*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*b^2*c*d*e - (4*I)*a*c^2*d*e - 2*b*c*Sqrt[-b^2 + 4*a*c]*d*e - I*b^3*e^2 + (3*I)*a*b*c*e^2 + b^2*Sqrt[-b^2 + 4*a*c]*e^2 - a*c*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]]/(Sqrt[2]*c^(5/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.25, size = 857, normalized size = 1.86

$$\frac{(b^2c^2d^3 - 5b^2c^2d^2e + (b^2c^2 - 4a^2c^3)d^2e - 2(b^3c - 4a^2b^2c^2)d^2e^2 + (b^4 - 5a^2b^2c + 4a^2c^2)e^3)c^2 + 2(2b^3c^3 - 5a^2b^2c^4)d^2e^2 - 2(\sqrt{b^2 - 4a^2c})c^4d^3 - 2\sqrt{b^2 - 4a^2c}b^2c^3d^2e - \sqrt{b^2 - 4a^2c}a^2b^2c^2e^3 + (b^2c^2 + a^2c^3)\sqrt{b^2 - 4a^2c}d^2e^2)\text{abs}(c) - (b^4c^2 - 3a^2b^2c^3)e^3)\text{arctan}(2\sqrt{1/2})\sqrt{x^2e + d}/\sqrt{-(2c^4d - b^2c^3e + \sqrt{-4(c^4d^2 - b^2c^3d^2e + a^2c^3e^2)}c^4 + (2c^4d - b^2c^3e)^2)/c^4)}}{(2\sqrt{b^2 - 4a^2c})c^3d - (b^2c^2 - 4a^2c^3 + \sqrt{b^2 - 4a^2c})b^2c^2e)\sqrt{-4c^2d + 2(b^2c^2 - 4a^2c^3 + \sqrt{b^2 - 4a^2c})b^2c^2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -(2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e + ((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b^2*c^2)*d^2*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2*b^3*c^3 - 5*a*b^2*c^4)*d^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b^2*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a^2*b^2*c^2*e^3 + (b^2*c^2 + a^2*c^3)*sqrt(b^2 - 4*a*c)*d^2*e^2)*abs(c) - (b^4*c^2 - 3*a*b^2*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b^2*c^3*e + sqrt(-4*(c^4*d^2 - b^2*c^3*d^2*e + a^2*c^3*e^2))*c^4 + (2*c^4*d - b^2*c^3*e)^2)/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b^2*c^2)*e)*sqrt(-4*c^2*d + 2*(b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b^2*c^2)*e))

$$\begin{aligned}
& - \sqrt{(b^2 - 4ac)c}e^{c^2} + (2b^2c^5d^3 - (5b^2c^4 - 8a^2c^5)d^2e \\
& + ((b^2c^2 - 4a^2c^3)d^2e - 2(b^3c - 4ab^2c^2)d^2e + (b^4 - 5a^2b^2c + 4a^2c^2)e^3)c^2 + 2(2b^3c^3 - 5a^2b^2c^4)d^2e + 2(\sqrt{b^2 - 4ac}c^4d^3 - 2\sqrt{b^2 - 4ac}b^2c^3d^2e - \sqrt{b^2 - 4ac}a^2b^2c^2e^3 + (b^2c^2 + a^2c^3)\sqrt{b^2 - 4ac}d^2e^2)\text{abs}(c) - (b^4c^2 - 3a^2b^2c^3)e^3)\arctan(2\sqrt{1/2}\sqrt{x^2e + d}/\sqrt{-(2c^4d - b^2c^3e - \sqrt{-4(c^4d^2 - b^2c^3d^2e + a^2c^3e^2)}c^4 + (2c^4d - b^2c^3e)^2)/c^4})/(2\sqrt{b^2 - 4ac}c^3d + (b^2c^2 - 4a^2c^3 - \sqrt{b^2 - 4ac}b^2c^2)e)\sqrt{-4c^2d + 2(b^2c + \sqrt{b^2 - 4ac}c)e}c^2) + 1/3((x^2e + d)^{3/2}c^2 + 3\sqrt{x^2e + d}c^2d - 3\sqrt{x^2e + d}b^2c^2e)/c^3
\end{aligned}$$

maple [C] time = 0.03, size = 490, normalized size = 1.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a), x)`

[Out]
$$\begin{aligned}
& -1/6/c^2e^{3/2}x^3 + 1/8/c^2e^{3/2}x^2 + 1/24*(e*x^2+d)^{3/2}/c + 1/2/c^2e^{3/2}x^2b - 1/2/c^2*(e*x^2+d)^{1/2}b^2e + 5/8/c^2*(e*x^2+d)^{1/2}d + 1/4/c^2*\text{sum}(((-a^2c^2e^2 + b^2e^2 - 2b^2c^2d^2e + c^2d^2)*_R^6 + (4a^2b^2e^3 - 5a^2c^2d^2e - 3b^2d^2e + 6b^2c^2d^2e - 3c^2d^3)*_R^4 + d*(-4a^2b^2e^3 + 5a^2c^2d^2e^2 + 3b^2d^2e^2 - 6b^2c^2d^2e + 3c^2d^3)*_R^2 + a^2c^2d^3e^2 - b^2d^3e^2 + 2b^2c^2d^4e - c^2d^5)/(_R^7*c + 3*_R^5*b^2e - 3*_R^5*c^2d + 8*_R^3*a^2e^2 - 4*_R^3*b^2d^2e + 3*_R^3*c^2d^2 + *_R*b^2d^2e - *_R*c^2d^3)*\ln(-e^{1/2}x - *_R + (e*x^2+d)^{1/2}), *_R = \text{RootOf}(_Z^8*c + (4*b^2e - 4*c*d)*_Z^6 + c*d^4 + (16*a^2e^2 - 8*b^2d^2e + 6*c*d^2)*_Z^4 + (4*b^2d^2e - 4*c*d^3)*_Z^2) - 1/2/c^2*d/(-e^{1/2}x + (e*x^2+d)^{1/2})*b^2e + 5/8/c^2*d^2/(-e^{1/2}x + (e*x^2+d)^{1/2}) + 1/24/c^2*d^3/(-e^{1/2}x + (e*x^2+d)^{1/2})^3
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}x^3}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a), x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)`

mupad [B] time = 3.41, size = 16951, normalized size = 36.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)`

$$\begin{aligned}
& + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2* \\
& e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^{2/4} - \\
& (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4* \\
& c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4* \\
& *b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1 \\
& /2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^ \\
& 3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18* \\
& a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72* \\
& a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^ \\
& 2*c^6)))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7 \\
& *d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^ \\
& 4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^ \\
& 3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4 \\
& *c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^{2/4} - (256*a^2*c^ \\
& 7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - \\
& a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3 \\
& *a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/2)} + 2*b^7* \\
& e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 \\
& + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 \\
& - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^ \\
& 2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/ \\
& 2)} + (2*(d + e*x^2)^{(1/2)}*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^ \\
& 2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + \\
& 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20 \\
& *a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4* \\
& b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3* \\
& e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5* \\
& c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2* \\
& b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^{2/4} - (256*a^2*c^7 + 16*b^4*c^5 - 128* \\
& a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a \\
& ^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6* \\
& a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 \\
& - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + \\
& 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42* \\
& a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3 \\
& *d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)}*ii)/((((4*a*b^3*c^ \\
& 3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3* \\
& d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b \\
& ^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^{(1/2)}*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - \\
& 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12 \\
& *b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84 \\
& *a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^ \\
& 3*d*e^2)^{2/4} - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^ \\
& 3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2* \\
& d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2
\end{aligned}$$

$$\begin{aligned}
& *c*d^4*e^2))^{(1/2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2)/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2)))^{(1/2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2)))^{(1/2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)} + ((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^{(1/2)}*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2)))^{(1/2)} + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2
\end{aligned}$$

$$\begin{aligned}
& - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2)/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - \\
& 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12 \\
& *b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84 \\
& *a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c \\
& ^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c \\
& ^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2* \\
& d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2 \\
& *c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2* \\
& c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^ \\
& 3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4* \\
& c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b \\
& ^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e \\
& ^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4 \\
& *e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^ \\
& 5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2 \\
& *c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a* \\
& b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100* \\
& a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96 \\
& *a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256* \\
& a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2* \\
& e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e \\
& ^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + \\
& 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^ \\
& 3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5* \\
& c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b* \\
& c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6) \\
&))^(1/2) + (2*(a^4*c*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*d*e^7 - \\
& a*c^4*d^6*e^2 - a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6 + 4*a*b*c^3*d^5*e^3 + 4*a \\
& *b^3*c*d^3*e^5 - 6*a*b^2*c^2*d^4*e^4 + 4*a^2*b*c^2*d^3*e^5 - 5*a^2*b^2*c*d^ \\
& 2*e^6))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^ \\
& 4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^ \\
& 3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4 \\
& *c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^ \\
& 7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - \\
& a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3 \\
& *a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7* \\
& e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 \\
& + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 \\
& - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^ \\
& 2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/ \\
& 2)*2i - (d + e*x^2)^(1/2)*(d/c + (b*e - 2*c*d)/c^2) - atan((((4*a*b^3*c^3* \\
& e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d* \\
& e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2 \\
& *c^4*d*e^4)/c^3 - (2*(d + e*x^2)^(1/2)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b \\
& ^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^
\end{aligned}$$

$$\begin{aligned}
& 5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a* \\
& b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3* \\
& d*e^2)^{2/4} - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3* \\
& d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4* \\
& *e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c* \\
& d^4*e^2))^{(1/2)} - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4 \\
& *d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c \\
& ^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2 \\
& *d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c \\
& ^5 - 8*a*b^2*c^6)))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^ \\
& 3 + 32*a*c^7*d*e^2))/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + \\
& 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + \\
& 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e \\
& + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^{2/4} - \\
& (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c \\
& *d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b* \\
& b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/ \\
& 2)} - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3 \\
& *b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a \\
& *b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a \\
& ^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2 \\
& *c^6)))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4 \\
& *e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3 \\
& *d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d \\
& ^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c \\
& ^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80* \\
& a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - \\
& 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 \\
& + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^{2/4} - (256*a^2*c^7 + 16*b^4*c \\
& ^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3 \\
& *e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2* \\
& d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/2)} - 2*b^7*e^3 + 16*a^ \\
& 2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^ \\
& 4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d \\
& *e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a \\
& ^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^{(1/2)}*i - (((\\
& 4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - \\
& 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^ \\
& 3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^{(1/2)}*(((4*b^7*e^3 - 32*a^2*c \\
& ^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d \\
& *e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d \\
& *e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216* \\
& a^2*b^2*c^3*d*e^2)^{2/4} - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^ \\
& 6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3 \\
& *a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 +
\end{aligned}$$

$$\begin{aligned}
& 3*a^2*b^2*c*d^4*e^2)^{(1/2)} - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - \\
& 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - \\
& 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - \\
& 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^ \\
& 2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 1 \\
& 6*a*b*c^6*e^3 + 32*a*c^7*d*e^2)/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^ \\
& 4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5 \\
& *c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b \\
& ^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d \\
& *e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d \\
& ^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4* \\
& e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d \\
& ^4*e^2))^{(1/2)} - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4* \\
& d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^ \\
& 2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2* \\
& d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c \\
& ^5 - 8*a*b^2*c^6))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^6*e^6 - 2*a^3*c^3*e^6 - \\
& 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 \\
& - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + \\
& 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3 \\
& *d^2*e^4))/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c \\
& ^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b \\
& ^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^ \\
& 4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c \\
& ^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - \\
& a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - \\
& 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/2)} - 2*b^7 \\
& *e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 \\
& - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 \\
& + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d \\
& ^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1 \\
& /2)}*i)/(((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2* \\
& c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a* \\
& b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*x^2)^{(1/2)}*((((4*b^7*e^ \\
& 3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + \\
& 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 \\
& - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4* \\
& d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2* \\
& c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2 \\
& *d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b* \\
& c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^{(1/2)} - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^ \\
& 4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5* \\
& c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3* \\
& c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2 \\
&))/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))^{(1/2)}*(4*b^3*c^5*e^3 - 8*b^2*c
\end{aligned}$$

$$\begin{aligned}
&^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2)/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 + (2*(d + e*x^2)^(1/2)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) - 2*b^7*e^3 + 16*a^2*c^5*d^3 + 2*b^4*c^3*d^3 - 12*a*b^2*c^4*d^3 + 40*a^3*b*c^3*e^3 - 48*a^3*c^4*d*e^2 - 6*b^5*c^2*d^2*e - 50*a^2*b^3*c^2*e^3 + 18*a*b^5*c*e^3 + 6*b^6*c*d*e^2 + 42*a*b^3*c^3*d^2*e - 48*a*b^4*c^2*d*e^2 - 72*a^2*b*c^4*d^2*e + 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)
\end{aligned}$$

$$\begin{aligned}
& * (a^5 e^6 + a^2 c^3 d^6 + 3 a^4 c d^2 e^4 - a^2 b^3 d^3 e^3 + 3 a^3 b^2 d^2 e^4 + 3 a^3 c^2 d^4 e^2 - 3 a^4 b d e^5 - 3 a^2 b c^2 d^5 e - 6 a^3 b c d^3 e^3 + 3 a^2 b^2 c d^4 e^2))^{(1/2)} - 2 b^7 e^3 + 16 a^2 c^5 d^3 + 2 b^4 c^3 d^3 - 12 a b^2 c^4 d^3 + 40 a^3 b c^3 e^3 - 48 a^3 c^4 d e^2 - 6 b^5 c^2 d^2 e - 50 a^2 b^3 c^2 e^3 + 18 a b^5 c e^3 + 6 b^6 c d e^2 + 42 a b^3 c^3 d^2 e - 48 a b^4 c^2 d e^2 - 72 a^2 b c^4 d^2 e + 108 a^2 b^2 c^3 d e^2) / (16 (16 a^2 c^7 + b^4 c^5 - 8 a b^2 c^6))^{(1/2)} + (2 (d + e x^2)^{(1/2)} (b^6 e^6 - 2 a^3 c^3 e^6 - 2 a c^5 d^4 e^2 + 9 a^2 b^2 c^2 e^6 + 12 a^2 c^4 d^2 e^4 + b^2 c^4 d^4 e^2 - 4 b^3 c^3 d^3 e^3 + 6 b^4 c^2 d^2 e^4 - 6 a b^4 c e^6 - 4 b^5 c d e^5 + 12 a b c^4 d^3 e^3 + 20 a b^3 c^2 d e^5 - 20 a^2 b c^3 d e^5 - 24 a b^2 c^3 d^2 e^4)) / c^3) * (((4 b^7 e^3 - 32 a^2 c^5 d^3 - 4 b^4 c^3 d^3 + 24 a b^2 c^4 d^3 - 80 a^3 b c^3 e^3 + 96 a^3 c^4 d e^2 + 12 b^5 c^2 d^2 e + 100 a^2 b^3 c^2 e^3 - 36 a b^5 c e^3 - 12 b^6 c d e^2 - 84 a b^3 c^3 d^2 e + 96 a b^4 c^2 d e^2 + 144 a^2 b c^4 d^2 e - 216 a^2 b^2 c^3 d e^2))^2 / 4 - (256 a^2 c^7 + 16 b^4 c^5 - 128 a b^2 c^6) * (a^5 e^6 + a^2 c^3 d^6 + 3 a^4 c d^2 e^4 - a^2 b^3 d^3 e^3 + 3 a^3 b^2 d^2 e^4 + 3 a^3 c^2 d^4 e^2 - 3 a^4 b d e^5 - 3 a^2 b c^2 d^5 e - 6 a^3 b c d^3 e^3 + 3 a^2 b^2 c d^4 e^2))^{(1/2)} - 2 b^7 e^3 + 16 a^2 c^5 d^3 + 2 b^4 c^3 d^3 - 12 a b^2 c^4 d^3 + 40 a^3 b c^3 e^3 - 48 a^3 c^4 d e^2 - 6 b^5 c^2 d^2 e - 50 a^2 b^3 c^2 e^3 + 18 a b^5 c e^3 + 6 b^6 c d e^2 + 42 a b^3 c^3 d^2 e - 48 a b^4 c^2 d e^2 - 72 a^2 b c^4 d^2 e + 108 a^2 b^2 c^3 d e^2) / (16 (16 a^2 c^7 + b^4 c^5 - 8 a b^2 c^6))^{(1/2)} + (2 (a^4 c e^8 - a^3 b^2 e^8 - a b^4 d^2 e^6 + 2 a^2 b^3 d e^7 - a c^4 d^6 e^2 - a^2 c^3 d^4 e^4 + a^3 c^2 d^2 e^6 + 4 a a b c^3 d^5 e^3 + 4 a a b^3 c d^3 e^5 - 6 a a b^2 c^2 d^4 e^4 + 4 a^2 b c^2 d^3 e^5 - 5 a^2 b^2 c d^2 e^6)) / c^3) * (((4 b^7 e^3 - 32 a^2 c^5 d^3 - 4 b^4 c^3 d^3 + 24 a b^2 c^4 d^3 - 80 a^3 b c^3 e^3 + 96 a^3 c^4 d e^2 + 12 b^5 c^2 d^2 e + 100 a^2 b^3 c^2 e^3 - 36 a b^5 c e^3 - 12 b^6 c d e^2 - 84 a b^3 c^3 d^2 e + 96 a b^4 c^2 d e^2 + 144 a^2 b c^4 d^2 e - 216 a^2 b^2 c^3 d e^2))^2 / 4 - (256 a^2 c^7 + 16 b^4 c^5 - 128 a b^2 c^6) * (a^5 e^6 + a^2 c^3 d^6 + 3 a^4 c d^2 e^4 - a^2 b^3 d^3 e^3 + 3 a^3 b^2 d^2 e^4 + 3 a^3 c^2 d^4 e^2 - 3 a^4 b d e^5 - 3 a^2 b c^2 d^5 e - 6 a^3 b c d^3 e^3 + 3 a^2 b^2 c d^4 e^2))^{(1/2)} - 2 b^7 e^3 + 16 a^2 c^5 d^3 + 2 b^4 c^3 d^3 - 12 a b^2 c^4 d^3 + 40 a^3 b c^3 e^3 - 48 a^3 c^4 d e^2 - 6 b^5 c^2 d^2 e - 50 a^2 b^3 c^2 e^3 + 18 a b^5 c e^3 + 6 b^6 c d e^2 + 42 a b^3 c^3 d^2 e - 48 a b^4 c^2 d e^2 - 72 a^2 b c^4 d^2 e + 108 a^2 b^2 c^3 d e^2) / (16 (16 a^2 c^7 + b^4 c^5 - 8 a b^2 c^6))^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.268 \quad \int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=327

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b+\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}\right)+\frac{e\sqrt{d+ex^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}+\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 1.46, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1247, 703, 826, 1166, 208}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)+\frac{e\sqrt{d+ex^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b+\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}\right)+\frac{e\sqrt{d+ex^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] (e*sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d - sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(3/2)*sqrt[b^2 - 4*a*c]*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + sqrt[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[d + e*x^2])/sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]]/(sqrt[2]*c^(3/2)*sqrt[b^2 - 4*a*c]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 703

Int[((d_) + (e_)*(x_)^m)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 826

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

Rule 1166

Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2} dx, x, x^2 \right) \\
 &= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst} \left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}(a+bx+cx^2)} dx, x, x^2 \right)}{2c} \\
 &= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst} \left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{c} \\
 &= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2 + b(b - \sqrt{b^2 - 4ac})e^2 - 2ce(bd - \sqrt{b^2 - 4ac}d + ae)\right) \text{Subst} \left(\int \frac{-\frac{1}{2}\sqrt{b}}{\sqrt{2cd-}} \right)}{2c\sqrt{b^2 - 4ac}} \\
 &= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2 + b(b - \sqrt{b^2 - 4ac})e^2 - 2ce(bd - \sqrt{b^2 - 4ac}d + ae)\right) \tanh^{-1} \left(\frac{\sqrt{2}}{\sqrt{2cd-}} \right)}{\sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}}
 \end{aligned}$$

Mathematica [A] time = 0.57, size = 324, normalized size = 0.99

$$\frac{(2ce(-d\sqrt{b^2-4ac}+ae+bd)+be^2(\sqrt{b^2-4ac}-b)-2c^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)+(-2ce(d\sqrt{b^2-4ac}+ae+bd)+be^2(\sqrt{b^2-4ac}+b)+2c^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)+\frac{e\sqrt{d+ex^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}+\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]

[Out] (e*Sqrt[d + e*x^2])/c + ((-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c]))*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

IntegrateAlgebraic [C] time = 1.55, size = 447, normalized size = 1.37

$$\frac{(2\sqrt{2}cde\sqrt{4ac-b^2}-\sqrt{2}be^2\sqrt{4ac-b^2}+2i\sqrt{2}ace^2-i\sqrt{2}b^2e^2+2i\sqrt{2}bcde-2i\sqrt{2}c^2d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right)+\frac{e\sqrt{d+ex^2}}{c}}{2c^{3/2}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}+2c^{3/2}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]

[Out] (e*Sqrt[d + e*x^2])/c + (((-2*I)*Sqrt[2]*c^2*d^2 + (2*I)*Sqrt[2]*b*c*d*e + 2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e - I*Sqrt[2]*b^2*e^2 + (2*I)*Sqrt[2]*a*c*e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]]/(2*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + (((2*I)*Sqrt[2]*c^2*d^2 - (2*I)*Sqrt[2]*b*c*d*e + 2*Sqrt[2]*c*Sqrt[-b^2 + 4*a*c]*d*e + I*Sqrt[2]*b^2*e^2 - (2*I)*Sqrt[2]*a*c*e^2 - Sqrt[2]*b*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]]/(2*c^(3/2)*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.09, size = 649, normalized size = 1.98

$$\frac{\left(4c^2d^2 - 4b^2cd - (2(b^2 - 4ac)^2 - (b^2 - 4ac)^2)^2 + 4(b^2 - ac)^2d^2 + 2\left(\sqrt{b^2 - 4ac}cd - \sqrt{b^2 - 4ac}bd + \sqrt{b^2 - 4ac}ac^2\right)\left(-b^2 - 2ab^2\right)\right)\arctan\left(\frac{2\sqrt{d}\sqrt{cx^2 + d}}{\sqrt{2bx^2 + 2ax + a}}\right)}{\left(2\sqrt{b^2 - 4ac}cd - (b^2 - 4ac + \sqrt{b^2 - 4ac}b)\sqrt{-4cd + 2(b^2 - 4ac)c}\right)^2} \frac{\left(4c^2d^2 - 4b^2cd - (2(b^2 - 4ac)^2 - (b^2 - 4ac)^2)^2 + 4(b^2 - ac)^2d^2 + 2\left(\sqrt{b^2 - 4ac}cd - \sqrt{b^2 - 4ac}bd + \sqrt{b^2 - 4ac}ac^2\right)\left(-b^2 - 2ab^2\right)\right)\arctan\left(\frac{2\sqrt{d}\sqrt{cx^2 + d}}{\sqrt{2bx^2 + 2ax + a}}\right)}{\left(2\sqrt{b^2 - 4ac}cd + (b^2 - 4ac - \sqrt{b^2 - 4ac}b)\sqrt{-4cd + 2(b^2 - 4ac)c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] sqrt(x^2*e + d)*e/c + (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*abs(c) - (b^3*c^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2))*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) - (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*abs(c) - (b^3*c^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2))*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2)

maple [C] time = 0.02, size = 279, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)

[Out] -1/2*e^(3/2)/c*x+1/2*e*(e*x^2+d)^(1/2)/c+1/4*e/c*sum(((b*e+2*c*d)*_R^6+(-4*a*e^2+3*b*d*e-2*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+2*c*d^2)*_R^2+b*d^3*e-2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(1/2)*x-_R*(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2)+1/2*e/c*d/(-e^(1/2)*x+(e*x^2+d)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}x}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

$$\begin{aligned}
& 2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3 \\
& *e^3))^{(1/2)} + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 \\
& - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24* \\
& a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4 \\
&)))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^ \\
& 2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 9 \\
& 6*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a \\
& *b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a* \\
& b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e \\
& ^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6* \\
& a*b*c*d^3*e^3))^{(1/2)} + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b \\
& *c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d* \\
& e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8* \\
& a*b^2*c^4)))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4* \\
& d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2* \\
& c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 \\
& - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - \\
& 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2 \\
& /4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^ \\
& 3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4* \\
& e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^5*e^3 + \\
& 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c \\
& ^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^ \\
& 2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*1i)/((((16*a^2*c^ \\
& 3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^ \\
& 2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^{(1/2)}*(-(((4*b^5*e^3 + 32*a*c^ \\
& 4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^ \\
& 2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e \\
& ^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b \\
& ^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c* \\
& d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^5*e \\
& ^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6 \\
& *b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b \\
& ^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*(4*b^3*c^3*e \\
& ^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 \\
& + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b \\
& ^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^ \\
& 2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^ \\
& 3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + \\
& 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} \\
& + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3* \\
& d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e \\
& + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} - (\\
& 2*(d + e*x^2)^{(1/2)}*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2 \\
& *e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5
\end{aligned}$$

$$\begin{aligned}
& + 12*a*b*c^2*d*e^5)/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48* \\
& a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b \\
& ^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16 \\
& *b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3 \\
& *b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c \\
& ^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c \\
& *e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c \\
& ^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + \\
& 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4) \\
& /c + (2*(d + e*x^2)^(1/2))*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 4 \\
& 8*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12 \\
& *b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + \\
& 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2* \\
& e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - \\
& 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2 \\
& *c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3 \\
& *c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2 \\
& *c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16 \\
& *a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^ \\
& 3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c \\
& *e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a \\
& ^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a \\
& *b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2* \\
& b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^ \\
& 3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - \\
& 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(1 \\
& 6*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2))*(b^4* \\
& e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + \\
& 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(- \\
& (((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^ \\
& 3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d \\
& ^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4) \\
& *(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a \\
& ^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^ \\
& 3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 \\
& - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24 \\
& *a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^ \\
& 4)))^(1/2) - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 \\
& - 5*b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b \\
& *c*d^2*e^6))/c))*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c \\
& ^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d* \\
& e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^ \\
& 3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a \\
& *c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
\end{aligned}$$

$$\begin{aligned}
& d^5e - 6*ab*cd^3e^3))^{(1/2)} + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 \\
& + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - \\
& 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4 \\
& 4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*2i - \operatorname{atan}(\frac{((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^{(1/2)}*((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c*((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} - (2*(d + e*x^2)^{(1/2)}*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c*((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^{(1/2)}*((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c*((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3
\end{aligned}$$

$$\begin{aligned}
& - 96a^2c^3d^2e^2 + 12b^3c^2d^2e - 28ab^3c^2e^3 - 12b^4c^2d^2e^2 - 4 \\
& 8ab^2c^3d^2e + 72ab^2c^2d^2e^2)^2/4 - (256a^2c^5 + 16b^4c^3 - 128 \\
& ab^2c^4)(a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4 \\
& 4e^2 + 3a^2cd^2e^4 + 3b^2cd^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - \\
& 6ab^2cd^3e^3))^{(1/2)} - 2b^5e^3 - 16ac^4d^3 + 4b^2c^3d^3 - 24a^2 \\
& 2b^2c^2e^3 + 48a^2c^3d^2e^2 - 6b^3c^2d^2e + 14ab^3c^2e^3 + 6b^4c^2 \\
& d^2e^2 + 24ab^2c^3d^2e - 36ab^2c^2d^2e^2)/(16(16a^2c^5 + b^4c^3 - \\
& 8ab^2c^4))^{(1/2)} + (2(d + ex^2)^{(1/2)}(b^4e^6 + 2a^2c^2e^6 + 2c^4 \\
& d^4e^2 - 12ac^3d^2e^4 - 4b^2c^3d^3e^3 + 6b^2c^2d^2e^4 - 4ab^2 \\
& c^2e^6 - 4b^3cd^2e^5 + 12ab^2c^2d^2e^5))/c * (((4b^5e^3 + 32ac^4d^3 \\
& - 8b^2c^3d^3 + 48a^2b^2c^2e^3 - 96a^2c^3d^2e^2 + 12b^3c^2d^2e \\
& - 28ab^3c^2e^3 - 12b^4c^2d^2e + 72ab^2c^2d^2e^2)^2/4 - (256a^2c^5 + 16b^4c^3 - 128ab^2c^4) \\
& (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + 3b^2cd^4 \\
& e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6ab^2cd^3e^3))^{(1/2)} - 2b^5e^3 \\
& - 16ac^4d^3 + 4b^2c^3d^3 - 24a^2b^2c^2e^3 + 48a^2c^3d^2e^2 - 6b^3 \\
& c^2d^2e + 14ab^3c^2e^3 + 6b^4c^2d^2e^2 + 24ab^2c^3d^2e - 36ab^2c^2 \\
& c^2d^2e^2)/(16(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{(1/2)} * i) / (((16a^2c^3 \\
& e^5 - 4ab^2c^2e^5 + 16ac^4d^2e^3 + 4b^3c^2d^2e^4 - 4b^2c^3d^2 \\
& d^2e^3 - 16ab^2c^3d^2e^4)/c - (2(d + ex^2)^{(1/2)} * (((4b^5e^3 + 32ac^4 \\
& d^3 - 8b^2c^3d^3 + 48a^2b^2c^2e^3 - 96a^2c^3d^2e^2 + 12b^3c^2d^2 \\
& e^2 - 28ab^3c^2e^3 - 12b^4c^2d^2e + 72ab^2c^2d^2e^2)^2/4 - (256a^2c^5 + 16b^4c^3 - 128ab^2c^4) \\
& (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + 3b^2cd^4 \\
& e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6ab^2cd^3e^3))^{(1/2)} - 2b^5e^3 \\
& e^3 - 16ac^4d^3 + 4b^2c^3d^3 - 24a^2b^2c^2e^3 + 48a^2c^3d^2e^2 - 6b^3 \\
& c^2d^2e + 14ab^3c^2e^3 + 6b^4c^2d^2e^2 + 24ab^2c^3d^2e - 36ab^2c^2 \\
& b^2c^2d^2e^2)/(16(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{(1/2)} * (4b^3c^3e^3 - 8b^2c^4 \\
& d^2e^2 - 16ab^2c^4e^3 + 32ac^5d^2e^2))/c * (((4b^5e^3 + 32ac^4d^3 - 8b^2c^3 \\
& d^3 + 48a^2b^2c^2e^3 - 96a^2c^3d^2e^2 + 12b^3c^2d^2e - 28ab^3c^2e^3 - 12b^4 \\
& c^2d^2e + 72ab^2c^2d^2e^2)^2/4 - (256a^2c^5 + 16b^4c^3 - 128ab^2c^4) \\
& (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + 3b^2cd^4 \\
& e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6ab^2cd^3e^3))^{(1/2)} - 2b^5e^3 \\
& - 16ac^4d^3 + 4b^2c^3d^3 - 24a^2b^2c^2e^3 + 48a^2c^3d^2e^2 - 6b^3c^2d^2e \\
& + 14ab^3c^2e^3 + 6b^4c^2d^2e^2 + 24ab^2c^3d^2e - 36ab^2c^2d^2e^2)/(16(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{(1/2)} - (\\
& 2(d + ex^2)^{(1/2)}(b^4e^6 + 2a^2c^2e^6 + 2c^4d^4e^2 - 12ac^3d^2 \\
& e^4 - 4b^2c^3d^3e^3 + 6b^2c^2d^2e^4 - 4ab^2c^2e^6 - 4b^3cd^2e^5 \\
& + 12ab^2c^2d^2e^5))/c * (((4b^5e^3 + 32ac^4d^3 - 8b^2c^3d^3 + 48a^2 \\
& b^2c^2e^3 - 96a^2c^3d^2e^2 + 12b^3c^2d^2e - 28ab^3c^2e^3 - 12b^4 \\
& c^2d^2e - 48ab^2c^3d^2e + 72ab^2c^2d^2e^2)^2/4 - (256a^2c^5 + 16b^4c^3 - 128ab^2c^4) \\
& (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + 3b^2cd^4 \\
& e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6ab^2cd^3e^3))^{(1/2)} - 2b^5e^3 - 16ac^4d^3 + 4b^2c^2c^
\end{aligned}$$

$$\begin{aligned}
& 3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c* \\
& e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^ \\
& 5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + \\
& 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/ \\
& c + (2*(d + e*x^2)^{(1/2)}*(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48* \\
& a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b \\
& ^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16 \\
& *b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^ \\
& 4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3 \\
& *b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c \\
& ^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c* \\
& e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c \\
& ^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a \\
& *b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d \\
& ^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^ \\
& 3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2* \\
& c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^ \\
& 2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d \\
& *e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + \\
& 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14 \\
& *a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(\\
& 16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (2*(d + e*x^2)^{(1/2)}*(b^4*e^6 \\
& + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b \\
& ^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(((\\
& 4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d* \\
& e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e \\
& + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^ \\
& 3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c \\
& *d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^ \\
& 3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24*a^2*b*c^2*e^3 + 4 \\
& 8*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4*c*d*e^2 + 24*a*b \\
& *c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) \\
& ^{(1/2)} - (2*(2*c^3*d^5*e^3 - b^3*d^2*e^6 - a^2*b*e^8 + 4*a*c^2*d^3*e^5 - 5* \\
& b*c^2*d^4*e^4 + 4*b^2*c*d^3*e^5 + 2*a*b^2*d*e^7 + 2*a^2*c*d*e^7 - 6*a*b*c*d \\
& ^2*e^6))/c)*(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^ \\
& 3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - \\
& 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^{2/4} - (256*a^2*c^5 + 16*b^4*c^3 - 1 \\
& 28*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2* \\
& d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e \\
& - 6*a*b*c*d^3*e^3))^{(1/2)} - 2*b^5*e^3 - 16*a*c^4*d^3 + 4*b^2*c^3*d^3 - 24* \\
& a^2*b*c^2*e^3 + 48*a^2*c^3*d*e^2 - 6*b^3*c^2*d^2*e + 14*a*b^3*c*e^3 + 6*b^4 \\
& *c*d*e^2 + 24*a*b*c^3*d^2*e - 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 \\
& - 8*a*b^2*c^4))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.269 \quad \int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=346

$$\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}\right)\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Rubi [A] time = 1.74, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}} - \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])]/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)
^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^4}{\left(-\frac{d}{e} + \frac{x^2}{e}\right) \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}\right)} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(-\frac{d^2e}{a(d-x^2)} + \frac{e(d(cd^2-bde+ae^2)-(cd^2-ae^2)x^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \frac{d(cd^2-bde+ae^2)+(-cd^2+ae^2)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{e} - \frac{d^2 \text{Subst} \left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{d^{3/2} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a} + \frac{\left(a\sqrt{b^2-4ac}e^2 - cd \left(\sqrt{b^2-4ac}d - 4ae \right) - b(cd^2+ae^2) \right) \text{Subst} \left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2} \right)}{2a\sqrt{b^2-4ac}} \\
&= -\frac{d^{3/2} \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a} - \frac{\left(a\sqrt{b^2-4ac}e^2 - cd \left(\sqrt{b^2-4ac}d - 4ae \right) - b(cd^2+ae^2) \right) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd - (b - \sqrt{b^2-4ac})}}
\end{aligned}$$

Mathematica [A] time = 1.38, size = 333, normalized size = 0.96

$$\frac{\left(-cd(d\sqrt{b^2-4ac}+4ae)+ae^2\sqrt{b^2-4ac}+b(ae^2+cd^2) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right) - \left(cd(d\sqrt{b^2-4ac}-4ae)-ae^2\sqrt{b^2-4ac}+b(ae^2+cd^2) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-b+2cd}} \right)}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}} - \frac{d^{3/2} \log(\sqrt{d}\sqrt{d+ex^2}+d)}{a} + \frac{d^{3/2} \log(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)), x]

[Out] -(((---(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (d^(3/2)*Log[x])/a - (d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/a

IntegrateAlgebraic [C] time = 1.53, size = 389, normalized size = 1.12

$$\frac{\left(cd^2\sqrt{4ac-b^2} - ae^2\sqrt{4ac-b^2} - iabe^2 + 4iacde - ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-ie\sqrt{4ac-b^2}+be-2cd}}\right) - \left(cd^2\sqrt{4ac-b^2} - ae^2\sqrt{4ac-b^2} + iabe^2 - 4iacde + ibcd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}\right) - d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{4ac-b^2}\sqrt{-ie\sqrt{4ac-b^2}+be-2cd} - \sqrt{2}a\sqrt{c}\sqrt{4ac-b^2}\sqrt{ie\sqrt{4ac-b^2}+be-2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]

[Out] -((((-I)*b*c*d^2 + c*Sqrt[-b^2 + 4*a*c]*d^2 + (4*I)*a*c*d*e - I*a*b*e^2 - a*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e])) - ((I*b*c*d^2 + c*Sqrt[-b^2 + 4*a*c]*d^2 - (4*I)*a*c*d*e + I*a*b*e^2 - a*Sqrt[-b^2 + 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e])) - (d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.02, size = 827, normalized size = 2.39

$$\frac{\left(d^2\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)\right)}{a\sqrt{-d}} - \frac{1}{8}\left(\left(b^2c - 4a^2c^2\right)d^2e - (ab^2 - 4a^2c)e^3\right)\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac})c}e) a^2 - 2\left(\sqrt{b^2 - 4ac}a^2c^2d^3 - \sqrt{b^2 - 4ac}a^2b^2c^2d^2e + \sqrt{b^2 - 4ac}a^2c^2d^2e^2\right)\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac})c}e) \operatorname{abs}(a) - \left(2a^2b^2c^2d^3 + 6a^3b^2c^2d^2e - a^3b^2c^2e^3 - (a^2b^2c + 8a^3c^2)d^2e\right)\sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac})c}e) \operatorname{arctan}\left(\frac{2\sqrt{1/2}\sqrt{x^2e+d}}{\sqrt{-(2ac^2d - ab^2e + \sqrt{-4(a^2c^2d^2 - ab^2d^2e + a^2e^2)ac + (2ac^2d - ab^2e)^2})/ac}}\right) / \left(\sqrt{b^2 - 4ac}a^2c^2d^2 - \sqrt{b^2 - 4ac}a^2b^2c^2d^2e + \sqrt{b^2 - 4ac}a^3c^2e^2\right) \operatorname{abs}(a) \operatorname{abs}(c) + \frac{1}{8}\left(\left(b^2c - 4a^2c^2\right)d^2e - (ab^2 - 4a^2c)e^3\right) \operatorname{abs}(a) \operatorname{abs}(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] d^2*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c))*c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a^2*c^2*d^3 - sqrt(b^2 - 4*a*c)*a^2*b^2*c^2*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c^2*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c))*c)*e)*abs(a) - (2*a^2*b^2*c^2*d^3 + 6*a^3*b^2*c^2*d^2*e - a^3*b^2*c^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c))*c)*e)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c^2*d - a*b^2*e + sqrt(-4*(a^2*c^2*d^2 - a*b^2*d^2*e + a^2*e^2)*a*c + (2*a*c^2*d - a*b^2*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b^2*c^2*d^2*e + sqrt(b^2 - 4*a*c)*a^3*c^2*e^2)*abs(a)*abs(c)) + 1/8*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*abs(a)*abs(c))

```

qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*
a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3
+ 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2))*a*c + (2*a*c
*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))

```

maple [C] time = 0.03, size = 388, normalized size = 1.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 7/24/a*(e*x^2+d)^(3/2)-1/a*d^(3/2)*ln((2*d+2*(e*x^2+d)^(1/2)*d^(1/2))/x)+3/
8/a*(e*x^2+d)^(1/2)*d+1/6/a*e^(3/2)*x^3-1/8/a*e*(e*x^2+d)^(1/2)*x^2+3/4/a*e
^(1/2)*x*d-1/4/a*sum(((a*e^2+c*d^2)*_R^6+d*(-5*a*e^2+4*b*d*e-3*c*d^2)*_R^4
+d^2*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^2+a*d^3*e^2-c*d^5)/(_R^7*c+3*_R^5*b*e-3*_
R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(-e^(
1/2)*x-_R+(e*x^2+d)^(1/2)),_R=RootOf(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*
e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-5/8/a*d^2/(-e^(1/2)*x+
(e*x^2+d)^(1/2))-1/24/a*d^3/(-e^(1/2)*x+(e*x^2+d)^(1/2))^3

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

mupad [B] time = 7.67, size = 28434, normalized size = 82.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*

```

$$\begin{aligned}
& b^3*c*d^3*e^{13} + 16*a*b^2*c^2*d^4*e^{12} - 24*a^2*b*c^2*d^3*e^{13} + 12*a^2*b^2*c*d^2*e^{14}) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)))^{(1/2)} - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^{(1/2)}*((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)))^{(1/2)} - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^{(1/2)}*(d + e*x^2)^{(1/2)}*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)))^{(1/2)} - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^{(1/2)}*(512*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^{10} + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^{10} - 16*a^3*b^2*c^3*d^2*e^{10} + 64*a^4*b*c^3*d*e^{11} + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^{11}) + (d + e*x^2)^{(1/2)}*(8*a^3*b^3*c*e^{13} - 32*a^4*b*c^2*e^{13} + 176*a^4*c^3*d*e^{12} - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^{10} - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^{10} + 112*a^2*b^3*c^2*d^2*e^{11} - 16*a^2*b^4*c*d*e^{12} + 96*a*b^2*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^{11} + 16*a^3*b^2*c^2*d*e^{12}))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)))^{(1/2)} - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^{(1/2)} + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^{14} - 84*a^2*c^4*d^5*e^{10} - 9
\end{aligned}$$

$$\begin{aligned}
& 2*a^3*c^3*d^3*e^{12} - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5* \\
& e^{10} - 12*a^2*b^2*c^2*d^3*e^{12} + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^{14} - \\
& 36*a*b^2*c^3*d^5*e^{10} - 20*a*b^3*c^2*d^4*e^{11} + 160*a^2*b*c^3*d^4*e^{11} + 4* \\
& a^2*b^3*c*d^2*e^{13} + 16*a^3*b*c^2*d^2*e^{13}) * (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 \\
& ^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 \\
& - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4 \\
& *c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + \\
& 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2 \\
& ^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2* \\
& b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c* \\
& e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4 \\
& *c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c \\
& *e^{16} + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^{10} - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4* \\
& e^{12} + 16*a^2*c^3*d^4*e^{12} + 8*a^3*c^2*d^2*e^{14} + 24*b^2*c^3*d^6*e^{10} - 16* \\
& b^3*c^2*d^5*e^{11} - 8*a^3*b*c*d*e^{15} - 8*a*b^3*c*d^3*e^{13} + 16*a*b^2*c^2*d^4 \\
& *e^{12} - 24*a^2*b*c^2*d^3*e^{13} + 12*a^2*b^2*c*d^2*e^{14}) + (-(((4*b^4*c*d^3 - \\
& 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16* \\
& a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2 \\
& /4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3 \\
& *d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4 \\
& ^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c* \\
& d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 \\
& - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2 \\
&)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^(1/2)*(((4*b^4*c*d^3 - \\
& 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16* \\
& a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2 \\
& /4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3 \\
& *d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4 \\
& ^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c* \\
& d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 \\
& - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2 \\
&)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^(1/2)*((d + e*x^2)^(1/2)* \\
& -(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3 \\
& *c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2 \\
& ^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 \\
& + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2 \\
& ^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)) \\
& ^{(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + \\
& 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 1 \\
& 2*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^(1/2)*(51 \\
& 2*a^5*c^4*e^{10} + 32*a^3*b^4*c^2*e^{10} - 256*a^4*b^2*c^3*e^{10} + 768*a^4*c^5*d \\
& ^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d \\
& *e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 \\
& + 192*a^4*c^4*d^2*e^{10} - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - \\
& 16*a^2*b^4*c^2*d^2*e^{10} + 16*a^3*b^2*c^3*d^2*e^{10} - 64*a^4*b*c^3*d*e^{11} - 2
\end{aligned}$$

$$\begin{aligned}
& 56a^3b^3c^4d^3e^9 + 16a^3b^3c^2d^2e^{11}) + (d + ex^2)^{(1/2)} * (8a^3b^3c^3e^{13} - 32a^4b^3c^2e^{13} + 176a^4c^3d^3e^{12} - 144a^2c^5d^5e^8 + 2 \\
& 24a^3c^4d^3e^{10} - 16b^4c^3d^5e^8 + 16b^5c^2d^4e^9 + 48a^2b^2c^3d^3e^{10} + 112a^2b^3c^2d^2e^{11} - 16a^2b^4c^3d^3e^{12} + 96a^2b^2c^4d^4 \\
& * e^9 - 416a^3b^3c^3d^2e^{11} + 16a^3b^2c^2d^2e^{12})) * (-(((4b^4c^3d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24a^2b^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3 \\
& b^3c^2e^3 - 12a^2b^3c^2d^2e + 48a^2b^2c^2d^2e + 24a^2b^2c^2d^2e)^2 / 4 - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2) * (a^3e^6 + c^3d^6 - b^3 \\
& d^3e^3 + 3a^2b^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6a^2b^2c^2d^3e^3))^{(1/2)} - 2b^4c^3d^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12a^2b^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6a^2b^3c^2d^2e - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e) \\
& / (16 * (16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} - 12a^3c^5d^7e^8 - 4a^4c^2d^2e^{14} + 84a^2c^4d^5e^{10} + 92a^3c^3d^3e^{12} + 4b^2c^4d^7e^8 + 4b^3c^3d^6e^9 - 8b^4c^2d^5e^{10} + 12a^2b^2c^2d^3e^{12} - \\
& 32a^2b^3c^4d^6e^9 + 4a^3b^2c^2d^2e^{14} + 36a^2b^2c^3d^5e^{10} + 20a^2b^3c^2d^4e^{11} - 160a^2b^3c^3d^4e^{11} - 4a^2b^3c^3d^2e^{13} - 16a^3b^3c^2 \\
& d^2e^{13})) * (-(((4b^4c^3d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24a^2b^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3b^3c^2e^3 - 12a^2b^3c^2d^2e + 48a^2b^2c^2 \\
& d^2e + 24a^2b^2c^2d^2e)^2 / 4 - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2) * (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3a^2b^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6a^2 \\
& b^2c^2d^3e^3))^{(1/2)} - 2b^4c^3d^3 + 2a^2b^3e^3 - 16a^2c^3d^3 + 12a^2b^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6a^2b^3c^2d^2e - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e) / (16 * (16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} * i) / (((d + ex^2)^{(1/2)} * (2a^4c^3e^{16} + 6c^5d^8e^8 - 16a^3c^4d^6e^{10} - 16b^3c^4d^7e^9 + 4b^4c^3d^4e^{12} + 16a^2c^3d^4e^{12} + 8a^3c^2d^2e^{14} + 24b^2c^3d^6e^{10} - 16b^3c^2d^5e^{11} - 8a^3b^3c^2d^5e^{11} - 8a^3b^3c^2d^3e^{13} + 16a^2b^2c^2d^4e^{12} - 24a^2b^2c^2d^3e^{13} + 12a^2b^2c^2d^2e^{14}) + (-(((4b^4c^3d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24a^2b^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3b^3c^2e^3 - 12a^2b^3c^2d^2e + 48a^2b^2c^2d^2e + 24a^2b^2c^2d^2e)^2 / 4 - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2) * (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3a^2b^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6a^2b^2c^2d^3e^3))^{(1/2)} - 2b^4c^3d^3 + 2a^2b^3e^3 - 16a^2c^3d^3 + 12a^2b^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6a^2b^3c^2d^2e - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e) / (16 * (16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} * (((-(((4b^4c^3d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24a^2b^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3b^3c^2e^3 - 12a^2b^3c^2d^2e + 48a^2b^2c^2d^2e + 24a^2b^2c^2d^2e)^2 / 4 - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2) * (a^3e^6 + c^3d^6 - b^3d^3e^3 + 3a^2b^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6a^2b^2c^2d^3e^3))^{(1/2)} - 2b^4c^3d^3 + 2a^2b^3e^3 - 16a^2c^3d^3 + 12a^2b^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6a^2b^3c^2d^2e
\end{aligned}$$

$$\begin{aligned}
& - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e^2)/(16*(16a^4c^3 + a^2b^4c - 8 \\
& a^3b^2c^2))^{(1/2)}*((d + ex^2)^{(1/2)}*(-(((4b^4cd^3 - 4a^2b^3e^3 + \\
& 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3b^2c^2e^3 - 12 \\
& ab^3c^2d^2e + 48a^2b^2c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 \\
& + 16a^2b^4c - 128a^3b^2c^2)*(a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b \\
& d^2e^5 - 3b^2c^2d^5e - 6ab^2c^2d^3e^3))^{(1/2)} - 2b^4cd^3 + 2a^2b^3e^3 \\
& e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^2c^2e^3 \\
& + 6ab^3c^2d^2e - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e^2)/(16*(16a^4c^3 \\
& + a^2b^4c - 8a^3b^2c^2))^{(1/2)}*(512a^5c^4e^10 + 32a^3b^4c^2e \\
& ^10 - 256a^4b^2c^3e^10 + 768a^4c^5d^2e^8 + 64a^2b^4c^3d^2e^8 - \\
& 448a^3b^2c^4d^2e^8 - 896a^4b^2c^4d^2e^9 - 64a^2b^5c^2d^2e^9 + 480 \\
& a^3b^3c^3d^2e^9) + 192a^3c^5d^4e^8 + 192a^4c^4d^2e^10 - 48a^2b \\
& ^2c^4d^4e^8 + 64a^2b^3c^3d^3e^9 - 16a^2b^4c^2d^2e^10 + 16a^3b \\
& ^2c^3d^2e^10 - 64a^4b^2c^3d^2e^11 - 256a^3b^2c^4d^3e^9 + 16a^3b^3 \\
& c^2d^2e^11) + (d + ex^2)^{(1/2)}*(8a^3b^3c^2e^13 - 32a^4b^2c^2e^13 + 17 \\
& 6a^4c^3d^2e^12 - 144a^2c^5d^5e^8 + 224a^3c^4d^3e^10 - 16b^4c^3d \\
& ^5e^8 + 16b^5c^2d^4e^9 + 48a^2b^2c^3d^3e^10 + 112a^2b^3c^2d^2 \\
& e^11 - 16a^2b^4c^2d^2e^12 + 96ab^2c^4d^5e^8 - 80ab^3c^3d^4e^9 \\
& - 32ab^4c^2d^3e^10 + 96a^2b^2c^4d^4e^9 - 416a^3b^2c^3d^2e^11 + 1 \\
& 6a^3b^2c^2d^2e^12)*(-(((4b^4cd^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - \\
& 24ab^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3b^2c^2e^3 - 12ab^3c^2d^2e + 4 \\
& 8a^2b^2c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 + 16a^2b^4c - \\
& 128a^3b^2c^2)*(a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d \\
& ^5e - 6ab^2c^2d^3e^3))^{(1/2)} - 2b^4cd^3 + 2a^2b^3e^3 - 16a^2c^3d^3 \\
& ^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^2c^2e^3 + 6ab^3c^2d^2e \\
& - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e^2)/(16*(16a^4c^3 + a^2b^4c - 8 \\
& a^3b^2c^2))^{(1/2)} - 12a^2c^5d^7e^8 - 4a^4c^2d^2e^14 + 84a^2c^4d^5 \\
& e^10 + 92a^3c^3d^3e^12 + 4b^2c^4d^7e^8 + 4b^3c^3d^6e^9 - 8b^4 \\
& c^2d^5e^10 + 12a^2b^2c^2d^3e^12 - 32ab^2c^4d^6e^9 + 4a^3b^2c^2 \\
& d^2e^14 + 36ab^2c^3d^5e^10 + 20ab^3c^2d^4e^11 - 160a^2b^2c^3d^4e \\
& ^11 - 4a^2b^3c^2d^2e^13 - 16a^3b^2c^2d^2e^13))*(-(((4b^4cd^3 - 4 \\
& a^2b^3e^3 + 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 + 16a^3 \\
& b^2c^2e^3 - 12ab^3c^2d^2e + 48a^2b^2c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} \\
& - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2)*(a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2d^2e^5 - 3b^2c^2d^5e - 6ab^2c^2d^3e^3))^{(1/2)} - 2b^4cd^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8 \\
& a^3b^2c^2e^3 + 6ab^3c^2d^2e - 24a^2b^2c^2d^2e - 12a^2b^2c^2d^2e^2)/(\\
& 16*(16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} - ((d + ex^2)^{(1/2)}*(2 \\
& a^4c^2e^16 + 6c^5d^8e^8 - 16a^2c^4d^6e^10 - 16b^2c^4d^7e^9 + 4b^4c \\
& ^2d^4e^12 + 16a^2c^3d^4e^12 + 8a^3c^2d^2e^14 + 24b^2c^3d^6e^10 \\
& - 16b^3c^2d^5e^11 - 8a^3b^2c^2d^2e^15 - 8ab^3c^2d^3e^13 + 16ab^2c^2 \\
& ^2d^4e^12 - 24a^2b^2c^2d^3e^13 + 12a^2b^2c^2d^2e^14) + (-(((4b^4c
\end{aligned}$$

$$\begin{aligned}
& d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 \\
& + 16a^3b^3c^2e^3 - 12ab^3c^2d^2e + 48a^2b^3c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2)(a^3e^6 + c^3d^6 \\
& - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^3d^2e^5 - 3b^3c^2d^5e - 6ab^3c^2d^3e^3)^{(1/2)} - 2b^4c^2d^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6ab^3c^2d^2e - 24a^2b^3c^2d^2e - 12a^2b^2c^2d^2e^2 \\
& *d^2e^2)/(16(16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} * (((-(((4b^4c^2d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 \\
& + 16a^3b^3c^2e^3 - 12ab^3c^2d^2e + 48a^2b^3c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2)(a^3e^6 + c^3d^6 \\
& - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^3d^2e^5 - 3b^3c^2d^5e - 6ab^3c^2d^3e^3)^{(1/2)} - 2b^4c^2d^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6ab^3c^2d^2e - 24a^2b^3c^2d^2e - 12a^2b^2c^2d^2e^2 \\
& *d^2e^2)/(16(16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} * ((d + ex^2)^{(1/2)} * (-(((4b^4c^2d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 \\
& + 16a^3b^3c^2e^3 - 12ab^3c^2d^2e + 48a^2b^3c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2)(a^3e^6 + c^3d^6 \\
& - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^3d^2e^5 - 3b^3c^2d^5e - 6ab^3c^2d^3e^3)^{(1/2)} - 2b^4c^2d^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6ab^3c^2d^2e - 24a^2b^3c^2d^2e - 12a^2b^2c^2d^2e^2 \\
& *d^2e^2)/(16(16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} * (512a^5c^4e^10 + 32a^3b^4c^2e^10 - 256a^4b^2c^3e^10 + 768a^4c^5d^2e^8 + 64a^2b^4c^3d^2e^8 - 448a^3b^2c^4d^2e^8 - 896a^4b^3c^4d^2e^9 \\
& - 64a^2b^5c^2d^2e^9 + 480a^3b^3c^3d^2e^9) - 192a^3c^5d^4e^8 - 192a^4c^4d^2e^10 + 48a^2b^2c^4d^4e^8 - 64a^2b^3c^3d^3e^9 + 16a^2b^4c^2d^2e^10 - 16a^3b^2c^3d^2e^10 + 64a^4b^3c^3d^2e^11 \\
& + 256a^3b^3c^4d^3e^9 - 16a^3b^3c^2d^2e^11) + (d + ex^2)^{(1/2)} * (8a^3b^3c^2e^13 - 32a^4b^3c^2e^13 + 176a^4c^3d^2e^12 - 144a^2c^5d^5e^8 + 224a^3c^4d^3e^10 - 16b^4c^3d^5e^8 + 16b^5c^2d^4e^9 + 48a^2b^2c^3d^3e^10 \\
& + 112a^2b^3c^2d^2e^11 - 16a^2b^4c^2d^2e^12 + 96a^2b^2c^4d^5e^8 - 80ab^3c^3d^4e^9 - 32ab^4c^2d^3e^10 + 96a^2b^3c^4d^4e^9 - 416a^3b^3c^3d^2e^11 + 16a^3b^2c^2d^2e^12)) * (-(((4b^4c^2d^3 - 4a^2b^3e^3 + 32a^2c^3d^3 - 24ab^2c^2d^3 - 96a^3c^2d^2e^2 \\
& + 16a^3b^3c^2e^3 - 12ab^3c^2d^2e + 48a^2b^3c^2d^2e + 24a^2b^2c^2d^2e^2)^{2/4} - (256a^4c^3 + 16a^2b^4c - 128a^3b^2c^2)(a^3e^6 + c^3d^6 - b^3d^3e^3 + 3ab^2d^2e^4 + 3ac^2d^4e^2 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^3d^2e^5 - 3b^3c^2d^5e - 6ab^3c^2d^3e^3)^{(1/2)} - 2b^4c^2d^3 \\
& + 2a^2b^3e^3 - 16a^2c^3d^3 + 12ab^2c^2d^3 + 48a^3c^2d^2e^2 - 8a^3b^3c^2e^3 + 6ab^3c^2d^2e - 24a^2b^3c^2d^2e - 12a^2b^2c^2d^2e^2 *d^2e^2)/(16(16a^4c^3 + a^2b^4c - 8a^3b^2c^2))^{(1/2)} + 12a^3c^5d^7e^8 + 4a^4c^2d^2e^14 - 84a^2c^4d^5e^10 - 92a^3c^3d^3e^12 - 4b^2c^4d^7e^8 - 4b^3c^3d^6e^9 + 8b^4c^2d^5e^10 - 12a^2b^2c^2d^3e
\end{aligned}$$

$$\begin{aligned}
& ^{12} + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^{14} - 36*a*b^2*c^3*d^5*e^{10} - 20* \\
& a*b^3*c^2*d^4*e^{11} + 160*a^2*b^2*c^3*d^4*e^{11} + 4*a^2*b^3*c*d^2*e^{13} + 16*a^3 \\
& *b*c^2*d^2*e^{13}) * (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a* \\
& b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2 \\
& *b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128* \\
& a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d \\
& ^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e \\
& - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + \\
& 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24* \\
& a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c \\
& ^2*c^2)))^(1/2) + 6*c^4*d^8*e^{10} + 14*a*c^3*d^6*e^{12} + 2*a^3*c*d^2*e^{16} - 1 \\
& 6*b*c^3*d^7*e^{11} - 4*b^3*c*d^5*e^{13} + 10*a^2*c^2*d^4*e^{14} + 14*b^2*c^2*d^6* \\
& e^{12} - 24*a*b*c^2*d^5*e^{13} + 10*a*b^2*c*d^4*e^{14} - 8*a^2*b*c*d^3*e^{15})) * (- \\
& ((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3* \\
& c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2 \\
& *b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 \\
& + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2* \\
& e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(\\
& 1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48 \\
& *a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12* \\
& a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*2i + \\
& \operatorname{atan}(((d + e*x^2)^(1/2)*(2*a^4*c*e^{16} + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^{10} \\
& - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^{12} + 16*a^2*c^3*d^4*e^{12} + 8*a^3*c^2*d^2 \\
& *e^{14} + 24*b^2*c^3*d^6*e^{10} - 16*b^3*c^2*d^5*e^{11} - 8*a^3*b*c*d*e^{15} - 8*a* \\
& b^3*c*d^3*e^{13} + 16*a*b^2*c^2*d^4*e^{12} - 24*a^2*b*c^2*d^3*e^{13} + 12*a^2*b^2 \\
& *c*d^2*e^{14}) + (((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2* \\
& c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c \\
& ^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3* \\
& b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e \\
& ^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6* \\
& a*b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a \\
& *b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2* \\
& b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2 \\
& ^2)))^(1/2)*(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c \\
& ^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2 \\
& *d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b \\
& ^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 \\
& + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a \\
& *b*c*d^3*e^3))^(1/2) + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a* \\
& b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b \\
& *c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2 \\
& ^2)))^(1/2)*((d + e*x^2)^(1/2)*(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3* \\
& d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2 \\
& *e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b \\
& ^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4
\end{aligned}$$

$$\begin{aligned}
& + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b \\
& *c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2 \\
& *c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c* \\
& d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c \\
& - 8*a^3*b^2*c^2)))^{(1/2)}*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^ \\
& 4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2 \\
& *c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3 \\
& *d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e \\
& ^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2* \\
& e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) \\
& + (d + e*x^2)^{(1/2)}*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d* \\
& e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16 \\
& *b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16* \\
& a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c \\
& ^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^ \\
& 2*d*e^12))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2* \\
& d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d \\
& ^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2* \\
& c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + \\
& 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b* \\
& c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2 \\
& *c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^ \\
& 2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)) \\
&)^{(1/2)} + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 92*a^ \\
& 3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 \\
& - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a \\
& *b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*a^2* \\
& b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + \\
& 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12* \\
& a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 \\
& + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b \\
& ^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b* \\
& d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e \\
& ^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - \\
& 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 \\
& + a^2*b^4*c - 8*a^3*b^2*c^2)))^{(1/2)}*1i + ((d + e*x^2)^{(1/2)}*(2*a^4*c*e^16 \\
& + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 \\
& + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c \\
& ^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 \\
& - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (((4*b^4*c*d^3 - 4*a^2 \\
& *b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b* \\
& c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (\\
& 256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3* \\
& e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 \\
& - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 -
\end{aligned}$$

$$\begin{aligned}
& 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} * ((((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} * ((d + e*x^2)^{(1/2)} * (((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} * (512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^3*c^5*d^4*e^8 + 192*a^4*c^4*d^2*e^10 - 48*a^2*b^2*c^4*d^4*e^8 + 64*a^2*b^3*c^3*d^3*e^9 - 16*a^2*b^4*c^2*d^2*e^10 + 16*a^3*b^2*c^3*d^2*e^10 - 64*a^4*b*c^3*d*e^11 - 256*a^3*b*c^4*d^3*e^9 + 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^{(1/2)} * (8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12)) * (((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} - 12*a*c^5*d^7*e^8 - 4*a^4*c^2*d*e^14 + 84*a^2*c^4*d^5*e^10 + 92*a^3*c^3*d^3*e^12 + 4*b^2*c^4*d^7*e^8 + 4*b^3*c^3*d^6*e^9 - 8*b^4*c^2*d^5*e^10 + 12*a^2*b^2*c^2*d^3*e^12 - 32*a*b*c^4*d^6*e^9 + 4*a^3*b^2*c*d*e^14 + 36*a*b^2*c^3*d^5*e^10 + 20*a*b^3*c^2*d^4*e^11 - 160*a^2*b*c^3*d^4*e^11 - 4*a^2*b^3*c*d^2*e^13 - 16*a^3*b*c^2*d^2*e^13)) * (((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3
\end{aligned}$$

$$\begin{aligned}
&^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d \\
&*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c* \\
&d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)}*(512*a^5*c^4*e^ \\
&10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64* \\
&a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^ \\
&2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^ \\
&4*d^2*e^10 + 48*a^2*b^2*c^4*d^4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c \\
&^2*d^2*e^10 - 16*a^3*b^2*c^3*d^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4 \\
&*d^3*e^9 - 16*a^3*b^3*c^2*d*e^11) + (d + e*x^2)^{(1/2)}*(8*a^3*b^3*c*e^13 - 3 \\
&2*a^4*b*c^2*e^13 + 176*a^4*c^3*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d \\
&^3*e^10 - 16*b^4*c^3*d^5*e^8 + 16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 \\
&+ 112*a^2*b^3*c^2*d^2*e^11 - 16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - \\
&80*a*b^3*c^3*d^4*e^9 - 32*a*b^4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a \\
&^3*b*c^3*d^2*e^11 + 16*a^3*b^2*c^2*d*e^12))*(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 \\
&+ 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - \\
&12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^{2/4} - (256*a^4* \\
&c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3* \\
&a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2 \\
&*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^ \\
&3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^3*b*c*e^ \\
&3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16*(16*a^4* \\
&c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e \\
&^14 - 84*a^2*c^4*d^5*e^10 - 92*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3 \\
&*c^3*d^6*e^9 + 8*b^4*c^2*d^5*e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^ \\
&6*e^9 - 4*a^3*b^2*c*d*e^14 - 36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 \\
&+ 160*a^2*b*c^3*d^4*e^11 + 4*a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*((\\
&((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3 \\
&*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^ \\
&2*b^2*c*d*e^2)^{2/4} - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^ \\
&6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2 \\
&*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{ \\
&(1/2)} + 2*b^4*c*d^3 - 2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 4 \\
&8*a^3*c^2*d*e^2 + 8*a^3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12 \\
&*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)} + 6* \\
&c^4*d^8*e^10 + 14*a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 16*b*c^3*d^7*e^11 - 4 \\
&*b^3*c*d^5*e^13 + 10*a^2*c^2*d^4*e^14 + 14*b^2*c^2*d^6*e^12 - 24*a*b*c^2*d^ \\
&5*e^13 + 10*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15))*(((4*b^4*c*d^3 - 4*a^2 \\
&*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b* \\
&c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^{2/4} - (\\
&256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3* \\
&e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 \\
&- 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^{(1/2)} + 2*b^4*c*d^3 - \\
&2*a^2*b^3*e^3 + 16*a^2*c^3*d^3 - 12*a*b^2*c^2*d^3 - 48*a^3*c^2*d*e^2 + 8*a^ \\
&3*b*c*e^3 - 6*a*b^3*c*d^2*e + 24*a^2*b*c^2*d^2*e + 12*a^2*b^2*c*d*e^2)/(16* \\
&(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))^{(1/2)}*2i - (\operatorname{atanh}((72*c^4*d^6*e^
\end{aligned}$$

$$\begin{aligned}
& 10*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(72*c^4*d^8*e^{10} + 60*a*c^3*d^6*e^{12} + 2* \\
& a^3*c*d^2*e^{16} - 104*b*c^3*d^7*e^{11} - 6*b^3*c*d^5*e^{13} + 8*a^2*c^2*d^4*e^{14} \\
& + (18*c^5*d^{10}*e^8)/a + 20*b^2*c^2*d^6*e^{12} + (20*b^2*c^3*d^8*e^{10})/a + (1 \\
& 2*b^3*c^2*d^7*e^{11})/a - (4*b^2*c^4*d^{10}*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 \\
& - (6*b^4*c^2*d^8*e^{10})/a^2 - 32*a*b*c^2*d^5*e^{13} + 12*a*b^2*c*d^4*e^{14} - 8 \\
& *a^2*b*c*d^3*e^{15} - (48*b*c^4*d^9*e^9)/a + (2*a^3*c*e^{16}*(d + e*x^2)^{(1/2)} \\
& *(d^3)^{(1/2)})/(72*c^4*d^8*e^{10} + 60*a*c^3*d^6*e^{12} + 2*a^3*c*d^2*e^{16} - 104 \\
& *b*c^3*d^7*e^{11} - 6*b^3*c*d^5*e^{13} + 8*a^2*c^2*d^4*e^{14} + (18*c^5*d^{10}*e^8) \\
& /a + 20*b^2*c^2*d^6*e^{12} + (20*b^2*c^3*d^8*e^{10})/a + (12*b^3*c^2*d^7*e^{11})/ \\
& a - (4*b^2*c^4*d^{10}*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^ \\
& 10)/a^2 - 32*a*b*c^2*d^5*e^{13} + 12*a*b^2*c*d^4*e^{14} - 8*a^2*b*c*d^3*e^{15} - \\
& (48*b*c^4*d^9*e^9)/a + (18*c^5*d^8*e^8*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(18* \\
& c^5*d^{10}*e^8 + 72*a*c^4*d^8*e^{10} + 2*a^4*c*d^2*e^{16} - 48*b*c^4*d^9*e^9 + 60 \\
& *a^2*c^3*d^6*e^{12} + 8*a^3*c^2*d^4*e^{14} + 20*b^2*c^3*d^8*e^{10} + 12*b^3*c^2*d \\
& ^7*e^{11} - (4*b^2*c^4*d^{10}*e^8)/a + (10*b^3*c^3*d^9*e^9)/a - (6*b^4*c^2*d^8* \\
& e^{10})/a - 104*a*b*c^3*d^7*e^{11} - 6*a*b^3*c*d^5*e^{13} - 8*a^3*b*c*d^3*e^{15} + \\
& 20*a*b^2*c^2*d^6*e^{12} - 32*a^2*b*c^2*d^5*e^{13} + 12*a^2*b^2*c*d^4*e^{14}) + (8 \\
& *a^2*c^2*d^2*e^{14}*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(72*c^4*d^8*e^{10} + 60*a*c^ \\
& 3*d^6*e^{12} + 2*a^3*c*d^2*e^{16} - 104*b*c^3*d^7*e^{11} - 6*b^3*c*d^5*e^{13} + 8*a \\
& ^2*c^2*d^4*e^{14} + (18*c^5*d^{10}*e^8)/a + 20*b^2*c^2*d^6*e^{12} + (20*b^2*c^3*d \\
& ^8*e^{10})/a + (12*b^3*c^2*d^7*e^{11})/a - (4*b^2*c^4*d^{10}*e^8)/a^2 + (10*b^3*c \\
& ^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^{10})/a^2 - 32*a*b*c^2*d^5*e^{13} + 12*a*b^2 \\
& *c*d^4*e^{14} - 8*a^2*b*c*d^3*e^{15} - (48*b*c^4*d^9*e^9)/a + (20*b^2*c^2*d^4* \\
& e^{12}*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(72*c^4*d^8*e^{10} + 60*a*c^3*d^6*e^{12} + \\
& 2*a^3*c*d^2*e^{16} - 104*b*c^3*d^7*e^{11} - 6*b^3*c*d^5*e^{13} + 8*a^2*c^2*d^4*e^ \\
& 14 + (18*c^5*d^{10}*e^8)/a + 20*b^2*c^2*d^6*e^{12} + (20*b^2*c^3*d^8*e^{10})/a + \\
& (12*b^3*c^2*d^7*e^{11})/a - (4*b^2*c^4*d^{10}*e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a \\
& ^2 - (6*b^4*c^2*d^8*e^{10})/a^2 - 32*a*b*c^2*d^5*e^{13} + 12*a*b^2*c*d^4*e^{14} - \\
& 8*a^2*b*c*d^3*e^{15} - (48*b*c^4*d^9*e^9)/a - (48*b*c^4*d^7*e^9*(d + e*x^2) \\
& ^{(1/2)}*(d^3)^{(1/2)})/(18*c^5*d^{10}*e^8 + 72*a*c^4*d^8*e^{10} + 2*a^4*c*d^2*e^{16} \\
& - 48*b*c^4*d^9*e^9 + 60*a^2*c^3*d^6*e^{12} + 8*a^3*c^2*d^4*e^{14} + 20*b^2*c^3 \\
& *d^8*e^{10} + 12*b^3*c^2*d^7*e^{11} - (4*b^2*c^4*d^{10}*e^8)/a + (10*b^3*c^3*d^9* \\
& e^9)/a - (6*b^4*c^2*d^8*e^{10})/a - 104*a*b*c^3*d^7*e^{11} - 6*a*b^3*c*d^5*e^{13} \\
& - 8*a^3*b*c*d^3*e^{15} + 20*a*b^2*c^2*d^6*e^{12} - 32*a^2*b*c^2*d^5*e^{13} + 12* \\
& a^2*b^2*c*d^4*e^{14}) - (4*b^2*c^4*d^8*e^8*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(18 \\
& *a*c^5*d^{10}*e^8 + 2*a^5*c*d^2*e^{16} + 72*a^2*c^4*d^8*e^{10} + 60*a^3*c^3*d^6*e \\
& ^{12} + 8*a^4*c^2*d^4*e^{14} - 4*b^2*c^4*d^{10}*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4* \\
& c^2*d^8*e^{10} + 20*a^2*b^2*c^2*d^6*e^{12} - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3 \\
& *e^{15} + 20*a*b^2*c^3*d^8*e^{10} + 12*a*b^3*c^2*d^7*e^{11} - 104*a^2*b*c^3*d^7*e \\
& ^{11} - 6*a^2*b^3*c*d^5*e^{13} - 32*a^3*b*c^2*d^5*e^{13} + 12*a^3*b^2*c*d^4*e^{14}) \\
& + (10*b^3*c^3*d^7*e^9*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(18*a*c^5*d^{10}*e^8 + \\
& 2*a^5*c*d^2*e^{16} + 72*a^2*c^4*d^8*e^{10} + 60*a^3*c^3*d^6*e^{12} + 8*a^4*c^2*d^ \\
& 4*e^{14} - 4*b^2*c^4*d^{10}*e^8 + 10*b^3*c^3*d^9*e^9 - 6*b^4*c^2*d^8*e^{10} + 20* \\
& a^2*b^2*c^2*d^6*e^{12} - 48*a*b*c^4*d^9*e^9 - 8*a^4*b*c*d^3*e^{15} + 20*a*b^2*c \\
& ^3*d^8*e^{10} + 12*a*b^3*c^2*d^7*e^{11} - 104*a^2*b*c^3*d^7*e^{11} - 6*a^2*b^3*c*
\end{aligned}$$

$$\begin{aligned} & e^8)/a^2 + (10*b^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2 \\ & *d^5*e^13 + 12*a*b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a \\ &) - (8*a^2*b*c*d*e^15*(d + e*x^2)^{(1/2)}*(d^3)^{(1/2)})/(72*c^4*d^8*e^10 + 60* \\ & a*c^3*d^6*e^12 + 2*a^3*c*d^2*e^16 - 104*b*c^3*d^7*e^11 - 6*b^3*c*d^5*e^13 + \\ & 8*a^2*c^2*d^4*e^14 + (18*c^5*d^10*e^8)/a + 20*b^2*c^2*d^6*e^12 + (20*b^2*c \\ & ^3*d^8*e^10)/a + (12*b^3*c^2*d^7*e^11)/a - (4*b^2*c^4*d^10*e^8)/a^2 + (10*b \\ & ^3*c^3*d^9*e^9)/a^2 - (6*b^4*c^2*d^8*e^10)/a^2 - 32*a*b*c^2*d^5*e^13 + 12*a \\ & *b^2*c*d^4*e^14 - 8*a^2*b*c*d^3*e^15 - (48*b*c^4*d^9*e^9)/a))*(d^3)^{(1/2)})/ \\ & a \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.270 \quad \int \frac{(d+ex^2)^{3/2}}{x^3(ax^2+cx^4)} dx$$

Optimal. Leaf size=417

$$\frac{\sqrt{c} \left(-2a \left(e \left(d\sqrt{b^2 - 4ac} - ae \right) + cd^2 \right) + bd \left(d\sqrt{b^2 - 4ac} - 2ae \right) + b^2 d^2 \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) + \sqrt{c} \left(-2a \right)}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)}}$$

Rubi [A] time = 3.24, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$\frac{\sqrt{c} \left(bd \left(d\sqrt{b^2 - 4ac} - 2ae \right) - 2ae \left(d\sqrt{b^2 - 4ac} - ae \right) - 2acd^2 + b^2 d^2 \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right) + \sqrt{c} \left(-bd \left(d\sqrt{b^2 - 4ac} + 2ae \right) + 2ae \left(d\sqrt{b^2 - 4ac} + ae \right) - 2acd^2 + b^2 d^2 \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd-e(b+\sqrt{b^2-4ac})}} \right) + \frac{\sqrt{d} (bd - 2ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2} - \frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d} e \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{2a}}{\sqrt{2} a^2 \sqrt{b^2 - 4ac} \sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac} \right)}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] $-(d\sqrt{d+ex^2})/(2ax^2) + (\sqrt{d}e\text{ArcTanh}[\sqrt{d+ex^2}/\sqrt{d}])/(2a) + (\sqrt{d}(bd-2ae)\text{ArcTanh}[\sqrt{d+ex^2}/\sqrt{d}])/a^2 - (\sqrt{c}(b^2d^2-2acd^2+bd(\sqrt{b^2-4ac}d-2ae)-2ae(\sqrt{b^2-4ac}d+ae))\text{ArcTanh}[(\sqrt{2}\sqrt{c}\sqrt{d+ex^2})/\sqrt{2cd-e(b-\sqrt{b^2-4ac})}]) + (\sqrt{c}(bd-2ae)\text{ArcTanh}[(\sqrt{2}\sqrt{c}\sqrt{d+ex^2})/\sqrt{2cd-e(b+\sqrt{b^2-4ac})}]) + (\sqrt{d}(bd-2ae)\text{ArcTanh}[\sqrt{d+ex^2}/\sqrt{d}]) - d\sqrt{d+ex^2}/(2ax^2) + \sqrt{d}e\text{ArcTanh}[\sqrt{d+ex^2}/\sqrt{d}]/(2a)$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rule 208

$\text{Int}[\frac{(a_.) + (b_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.)^2 + (c_.) \cdot (x_.)^4}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]}{a}, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

$\text{Int}[\frac{(d_.) + (e_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.)^2 + (c_.) \cdot (x_.)^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q/e, \text{Subst}[\text{Int}[x^{q(m+1)-1} \cdot ((e \cdot f - d \cdot g)/e + (g \cdot x^q)/e)^n \cdot ((c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2)/e^2 - ((2 \cdot c \cdot d - b \cdot e) \cdot x^q)/e^2 + (c \cdot x^{(2 \cdot q)})/e^2)^p], x], (d + e \cdot x)^{(1/q)}, x]] /;$ FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e \cdot f - d \cdot g, 0] && NeQ[b^2 - 4 \cdot a \cdot c, 0] && NeQ[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1166

$\text{Int}[\frac{(d_.) + (e_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.)^2 + (c_.) \cdot (x_.)^4}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4 \cdot a \cdot c, 2]\}, \text{Dist}[e/2 + (2 \cdot c \cdot d - b \cdot e)/(2 \cdot q), \text{Int}[1/(b/2 - q/2 + c \cdot x^2), x], x] + \text{Dist}[e/2 - (2 \cdot c \cdot d - b \cdot e)/(2 \cdot q), \text{Int}[1/(b/2 + q/2 + c \cdot x^2), x], x]] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4 \cdot a \cdot c, 0] && NeQ[c \cdot d^2 - a \cdot e^2, 0] && PosQ[b^2 - 4 \cdot a \cdot c]

Rule 1251

$\text{Int}[(x_.)^{(m_.)} \cdot ((d_.) + (e_.) \cdot (x_.)^2)^{(q_.)} \cdot ((a_.) + (b_.) \cdot (x_.)^2 + (c_.) \cdot (x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2} \cdot (d + e \cdot x)^q \cdot (a + b \cdot x + c \cdot x^2)^p], x], x^2], x] /;$ FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1287

$\text{Int}[\frac{((f_.) \cdot (x_.)^m) \cdot ((d_.) + (e_.) \cdot (x_.)^2)^{(q_.)}}{(a_.) + (b_.) \cdot (x_.)^2 + (c_.) \cdot (x_.)^4}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(f \cdot x)^m \cdot (d + e \cdot x^2)^q}{(a + b \cdot x^2 + c \cdot x^4)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4 \cdot a \cdot c, 0] && IntegerQ[q] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{x^4}{\left(-\frac{d}{e} + \frac{x^2}{e}\right)^2 \left(\frac{cd^2-bde+ae^2}{e^2} - \frac{(2cd-be)x^2}{e^2} + \frac{cx^4}{e^2}\right)} dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \left(\frac{d^2e^2}{a(d-x^2)^2} - \frac{de(-bd+2ae)}{a^2(d-x^2)} + \frac{e(-((bd-ae)(cd^2-bde+ae^2))+cd(bd-2ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)} \right) dx, x, \sqrt{d+ex^2} \right)}{e} \\
&= \frac{\text{Subst} \left(\int \frac{-((bd-ae)(cd^2-bde+ae^2))+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4} dx, x, \sqrt{d+ex^2} \right)}{a^2} + \frac{(d^2e) \text{Subst} \left(\int \frac{1}{(d-x^2)^2} dx, x, \sqrt{d+ex^2} \right)}{a} \\
&= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}(bd-2ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2} + \frac{(de) \text{Subst} \left(\int \frac{1}{d-x^2} dx, x, \sqrt{d+ex^2} \right)}{2a} \\
&= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}e \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{2a} + \frac{\sqrt{d}(bd-2ae) \tanh^{-1} \left(\frac{\sqrt{d+ex^2}}{\sqrt{d}} \right)}{a^2} - \frac{\sqrt{c}(b^2d^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 1.60, size = 380, normalized size = 0.91

$$\frac{\sqrt{2} \sqrt{c} \left(\frac{2a \left((d\sqrt{b^2-4ac}-a) + c d^2 \right) + b \left(2ac-d\sqrt{b^2-4ac} \right) - d^2 a^2}{\sqrt{c} \sqrt{b^2-4ac-bx+2cd}} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{c} \sqrt{b^2-4ac-bx+2cd}} \right) - \frac{b \left(d\sqrt{b^2-4ac}+2ae \right) - 2ae \left(d\sqrt{b^2-4ac}+ae \right) + 2ac d^2 - d^2 a^2}{\sqrt{2cd - (b^2-4ac+b)}} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex^2}}{\sqrt{2cd - (b^2-4ac+b)}} \right) \right)}{\sqrt{b^2-4ac}} + \frac{\sqrt{d}(2bd-3ae) \log(\sqrt{d}\sqrt{d+ex^2}+d) - \sqrt{d} \log(x)(2bd-3ae) - \frac{ad\sqrt{d+ex^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] $\left(-\left(\frac{a d \sqrt{d+e x^2}}{x^2} \right) + \left(\sqrt{2} \sqrt{c} \sqrt{d+e x^2} \right) \left(\frac{-\left(b^2 d^2 \right) + b d \left(-\left(\sqrt{b^2-4 a c} \right) d\right) + 2 a e\right) + 2 a \left(c d^2 + e \left(\sqrt{b^2-4 a c} \right) d - a e\right) \right) \text{ArcTanh} \left[\frac{\left(\sqrt{2} \sqrt{c} \sqrt{d+e x^2} \right) / \sqrt{2 c d - b e} + \sqrt{b^2-4 a c} e}{\sqrt{2 c d + (-b + \sqrt{b^2-4 a c}) e}} \right] - \left(\left(-\left(b^2 d^2 \right) + 2 a c d^2 - 2 a e \left(\sqrt{b^2-4 a c} \right) d + a e \right) + b d \left(\sqrt{b^2-4 a c} \right) d + 2 a e \right) \text{ArcTanh} \left[\frac{\left(\sqrt{2} \sqrt{c} \sqrt{d+e x^2} \right) / \sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e}}{\sqrt{2 c d - (b + \sqrt{b^2-4 a c}) e}} \right] \right)$

)]*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[b^2 - 4*a*c] - Sqrt[d]*(2*b*d - 3*a*e)*Log[x] + Sqrt[d]*(2*b*d - 3*a*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(2*a^2)

IntegrateAlgebraic [C] time = 2.12, size = 548, normalized size = 1.31

$$\frac{(-2\sqrt{2}e^2\sqrt{c^2d^2 + \sqrt{2}b\sqrt{c^2d^2 + \sqrt{4ac-d^2}} - 2\sqrt{2}a\sqrt{c^2d^2 + \sqrt{4ac-d^2}} + 2\sqrt{2}ab\sqrt{c^2d^2 + \sqrt{4ac-d^2}} + 2\sqrt{2}ac^2d^2 - 1\sqrt{2}b^2\sqrt{c^2d^2}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c^2d^2 + \sqrt{4ac-d^2}}}{\sqrt{a\sqrt{4ac-d^2} + b - 2cd}}\right) + (2\sqrt{2}e^2\sqrt{c^2d^2 + \sqrt{2}b\sqrt{c^2d^2 + \sqrt{4ac-d^2}} - 2\sqrt{2}a\sqrt{c^2d^2 + \sqrt{4ac-d^2}} - 2\sqrt{2}ab\sqrt{c^2d^2 + \sqrt{4ac-d^2}} - 2\sqrt{2}ac^2d^2 + 1\sqrt{2}b^2\sqrt{c^2d^2}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c^2d^2 + \sqrt{4ac-d^2}}}{\sqrt{a\sqrt{4ac-d^2} + b - 2cd}}\right) + (2b^2d^2 - 3a\sqrt{d^2}) \tanh^{-1}\left(\frac{\sqrt{d^2 + e^2}}{\sqrt{d}}\right) + \frac{d\sqrt{d + e^2}}{2a^2}}{2a^2\sqrt{4ac-d^2}\sqrt{a\sqrt{4ac-d^2} + b - 2cd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] -1/2*(d*Sqrt[d + e*x^2])/(a*x^2) + (((-I)*Sqrt[2]*b^2*Sqrt[c]*d^2 + (2*I)*Sqrt[2]*a*c^(3/2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 + (2*I)*Sqrt[2]*a*b*Sqrt[c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e - (2*I)*Sqrt[2]*a^2*Sqrt[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]]/(2*a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]) + ((I*Sqrt[2]*b^2*Sqrt[c]*d^2 - (2*I)*Sqrt[2]*a*c^(3/2)*d^2 + Sqrt[2]*b*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d^2 - (2*I)*Sqrt[2]*a*b*Sqrt[c]*d*e - 2*Sqrt[2]*a*Sqrt[c]*Sqrt[-b^2 + 4*a*c]*d*e + (2*I)*Sqrt[2]*a^2*Sqrt[c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]]/(2*a^2*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]) + ((2*b*d^(3/2) - 3*a*Sqrt[d]*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.72, size = 433, normalized size = 1.04

$$\frac{(2b^2 - 3abd) \arctan\left(\frac{\sqrt{2c^2d}}{\sqrt{d}}\right) + \sqrt{-4c^2d + 2(bc - \sqrt{b^2 - 4ac})} \left((b^2 - 2ac + \sqrt{b^2 - 4ac})d - (ab + \sqrt{b^2 - 4ac})e \right) \arctan\left(\frac{\sqrt{2c^2d}}{\sqrt{2c^2d + \sqrt{4ac-d^2}}}\right) + \sqrt{-4c^2d + 2(bc + \sqrt{b^2 - 4ac})} \left((b^2 - 2ac - \sqrt{b^2 - 4ac})d - (ab - \sqrt{b^2 - 4ac})e \right) \arctan\left(\frac{\sqrt{2c^2d}}{\sqrt{2c^2d + \sqrt{4ac-d^2}}}\right) + \frac{\sqrt{d^2 + e^2}}{2a^2}}{4\sqrt{b^2 - 4ac}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] -1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a^2*sqrt(-d)) - 1/4*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2)*sqrt(x

$$\sqrt{x^2e + d} / \sqrt{-(2a^2cd - a^2b^2e + \sqrt{-4(a^2cd^2 - a^2b^2de + a^3e^2)}a^2c + (2a^2cd - a^2b^2e)^2) / (a^2c))} / (\sqrt{b^2 - 4ac} a^2 \operatorname{abs}(c)) + 1/4 \sqrt{-4c^2d + 2(bc + \sqrt{b^2 - 4ac})c} e) * ((b^2 - 2ac - \sqrt{b^2 - 4ac})b) * d - (ab - \sqrt{b^2 - 4ac})a) * e) * \arctan(2\sqrt{1/2} \sqrt{x^2e + d} / \sqrt{-(2a^2cd - a^2b^2e - \sqrt{-4(a^2cd^2 - a^2b^2de + a^3e^2)}a^2c + (2a^2cd - a^2b^2e)^2) / (a^2c))} / (\sqrt{b^2 - 4ac}) * a^2 \operatorname{abs}(c)) - 1/2 \sqrt{x^2e + d} * d / (a * x^2)$$

maple [C] time = 0.04, size = 555, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x)`

[Out]
$$-7/24/a^2b*(e*x^2+d)^{(3/2)}+1/a^2b*d^{(3/2)}*\ln((2*d+2*(e*x^2+d)^{(1/2)}*d^{(1/2)})/x)-3/8/a^2b*(e*x^2+d)^{(1/2)}*d-1/6/a^2e^{(3/2)}*x^3b+1/8/a^2e*(e*x^2+d)^{(1/2)}*x^2b-3/4/a^2e^{(1/2)}*x*b*d+1/2/a^{(3/2)}*x+1/a*(e*x^2+d)^{(1/2)}*e+1/4/a^2*\sum((c*d*(-2*a*e+b*d)*_R^6+(4*a^2*e^3-8*a*b*d*e^2+2*a*c*d^2*e+4*b^2*d^2*e-3*b*c*d^3)*_R^4+d*(-4*a^2*e^3+8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b*c*d^3)*_R^2+2*a*c*d^4*e-b*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*\ln(-e^{(1/2)}*x-_R+(e*x^2+d)^{(1/2)}),_R=\operatorname{RootOf}(_Z^8*c+(4*b*e-4*c*d)*_Z^6+c*d^4+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2))-1/2/a*d/(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})*e+5/8/a^2*d^2/(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})*b+1/24/a^2*b*d^3/(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^3-1/2/a/d/x^2*(e*x^2+d)^{(5/2)}+1/2/a*e/d*(e*x^2+d)^{(3/2)}-3/2/a*e*d^{(1/2)}*\ln((2*d+2*(e*x^2+d)^{(1/2)}*d^{(1/2)})/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)`

mupad [B] time = 6.10, size = 35855, normalized size = 85.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)`

[Out] $(d^{(1/2)} \operatorname{atan}(((d^{(1/2)}(3ae - 2bd)) * (((d + ex^2)^{(1/2)}(4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^2c^3d^2e^{14} - 8a^5b^2c^6d^8e^8 - 28a^5b^3c^5d^7e^9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5d^5e^{11} - 60a^4b^3c^4d^3e^{13}))/ (2a^4) - (d^{(1/2)}((56a^4c^6d^6e^9 - 44a^5c^5d^4e^{11} - 100a^6c^4d^2e^{13} + 40a^2b^3c^5d^7e^8 - 39a^2b^5c^3d^5e^{10} - 11a^2b^6c^2d^4e^{11} - 108a^3b^2c^5d^6e^9 + 96a^3b^3c^4d^5e^{10} + 111a^3b^4c^3d^4e^{11} + 22a^3b^5c^2d^3e^{12} - 237a^4b^2c^4d^4e^{11} - 161a^4b^3c^3d^3e^{12} - 19a^4b^4c^2d^2e^{13} + 111a^5b^2c^3d^2e^{13} - 28a^6b^3c^3d^2e^{14} - 8a^5b^5c^4d^7e^8 + 6a^5b^6c^3d^6e^9 + 2a^5b^7c^2d^5e^{10} - 32a^3b^3c^6d^7e^8 + 92a^4b^3c^5d^5e^{10} + 252a^5b^3c^4d^3e^{12} + 6a^5b^3c^2d^2e^{14}))/ a^4 + (d^{(1/2)}(3ae - 2bd)) * (((d + ex^2)^{(1/2)}(64a^7b^3c^3e^{13} + 352a^7c^4d^5e^{12} - 16a^6b^3c^2e^{13} - 160a^5c^6d^5e^8 + 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 224a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4e^9 + 112a^3b^6c^2d^3e^{10} + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3d^2e^{11} - 896a^5b^3c^5d^4e^9 + 64a^5b^4c^2d^2e^{12} - 1392a^6b^3c^4d^2e^{11} - 336a^6b^2c^3d^2e^{12}))/ (2a^4) + (d^{(1/2)}((320a^8c^4d^5e^{11} + 320a^7c^5d^3e^9 + 32a^5b^3c^4d^4e^8 - 24a^5b^4c^3d^3e^9 - 8a^5b^5c^2d^2e^{10} + 16a^6b^2c^4d^3e^9 + 144a^6b^3c^3d^2e^{10} - 128a^6b^3c^5d^4e^8 + 8a^6b^4c^2d^2e^{11} - 448a^7b^3c^4d^2e^{10} - 112a^7b^2c^3d^2e^{11}))/ a^4 - (d^{(1/2)}(d + ex^2)^{(1/2)}(3ae - 2bd)) * (1024a^9c^4e^{10} + 64a^7b^4c^2e^{10} - 512a^8b^2c^3e^{10} + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^3c^4d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9))/ (8a^6)) * (3ae - 2bd)) / (4a^2)) / (4a^2) * (3ae - 2bd)) / (4a^2) * i) / (4a^2) + (d^{(1/2)}(3ae - 2bd)) * (((d + ex^2)^{(1/2)}(4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^2c^3d^2e^{14} - 8a^5b^2c^6d^8e^8 - 28a^5b^3c^5d^7e^9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5d^5e^{11} - 60a^4b^3c^4d^3e^{13}))/ (2a^4) + (d^{(1/2)}((56a^4c^6d^6e^9 - 44a^5c^5d^4e^{11} - 100a^6c^4d^2e^{13} + 40a^2b^3c^5d^7e^8 - 39a^2b^5c^3d^5e^{10} - 11a^2b^6c^2d^4e^{11} - 108a^3b^2c^5d^6e^9 + 96a^3b^3c^4d^5e^{10} + 111a^3b^4c^3d^4e^{11} + 22a^3b^5c^2d^3e^{12} - 237a^4b^2c^4d^4e^{11} - 161a^4b^3c^3d^3e^{12} - 19a^4b^4c^2d^2e^{13} + 111a^5b^2c^3d^2e^{13} - 28a^6b^3c^3d^2e^{14} - 8a^5b^5c^4d^7e^8 + 6a^5b^6c^3d^6e^9 + 2a^5b^7c^2d^5e^{10} - 32a^3b^3c^6d^7e^8 + 92a^4b^3c^5d^5e^{10} + 252a^5b^3c^4d^3e^{12} + 6a^5b^3c^2d^2e^{14}))/ a^4 - (d^{(1/2)}(3ae - 2bd)) * (((d + ex^2)^{(1/2)}(64a^7b^3c^3e^{13} + 352a^7c^4d^5e^{12} - 16a^6b^3c^2e^{13} - 160a^5c^6d^5e^8 +$

$$\begin{aligned}
& 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 22 \\
& 4a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4e^9 + 112a^3b^6c^2d^3e^{10} \\
& + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} \\
& - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3 \\
& *d^2e^{11} - 896a^5b^4c^2d^2e^{12} - 1392a^6b^3c^4 \\
& d^2e^{11} - 336a^6b^2c^3d^2e^{12}))/ (2a^4) - (d^{(1/2)} * ((320a^8c^4d^4e^{11} \\
& + 320a^7c^5d^3e^9 + 32a^5b^3c^4d^4e^8 - 24a^5b^4c^3d^3e^9 - \\
& 8a^5b^5c^2d^2e^{10} + 16a^6b^2c^4d^3e^9 + 144a^6b^3c^3d^2e^{10} \\
& - 128a^6b^4c^5d^4e^8 + 8a^6b^4c^2d^2e^{11} - 448a^7b^3c^4d^2e^{10} - 1 \\
& 12a^7b^2c^3d^2e^{11})/a^4 + (d^{(1/2)} * (d + e*x^2)^{(1/2)} * (3a*e - 2*b*d) * (10 \\
& 24a^9c^4e^{10} + 64a^7b^4c^2e^{10} - 512a^8b^2c^3e^{10} + 1536a^8c^5 \\
& *d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^3c^4 \\
& *d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9)) / (8a^6)) * (3a*e \\
& - 2*b*d) / (4a^2)) / (4a^2)) * (3a*e - 2*b*d) / (4a^2)) * i / (4a^2)) / ((3a*c \\
& ^7d^9e^9 + 3a^5c^3d^2e^{17} - 2*b*c^7d^10e^8 + 3a^2c^6d^7e^{11} + 3a \\
& ^4c^4d^3e^{15} + 4b^2c^6d^9e^9 - 2b^3c^5d^8e^{10} + 2a^2b^2c^4d^ \\
& 5e^{13} - (11a^2b^3c^3d^4e^{14})/2 + 11a^3b^2c^3d^3e^{15} - 8a*b*c^6 \\
& d^8e^{10} + 4a*b^2c^5d^7e^{11} + a*b^4c^3d^5e^{13} - (3a^2b^3c^5d^6e^{1} \\
& 2)/2 - 5a^3b^3c^4d^4e^{14} - (19a^4b^3c^3d^2e^{16})/2) / a^4 - (d^{(1/2)} * (3a \\
& *e - 2*b*d) * (((d + e*x^2)^{(1/2)} * (4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^ \\
& 3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8 * \\
& e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^ \\
& 4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3 \\
& *d^2e^{14} - 16a^5b^3c^3d^2e^{15} - 8a*b^2c^6d^8e^8 - 28a*b^3c^5d^7e^ \\
& 9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5d^5e^{11} - 60a^4b^3c^4d^3e^{13})) / \\
& (2a^4) - (d^{(1/2)} * ((56a^4c^6d^6e^9 - 44a^5c^5d^4e^{11} - 100a^6c^4 \\
& *d^2e^{13} + 40a^2b^3c^5d^7e^8 - 39a^2b^5c^3d^5e^{10} - 11a^2b^6c^ \\
& ^2d^4e^{11} - 108a^3b^2c^5d^6e^9 + 96a^3b^3c^4d^5e^{10} + 111a^3b \\
& ^4c^3d^4e^{11} + 22a^3b^5c^2d^3e^{12} - 237a^4b^2c^4d^4e^{11} - 161a \\
& ^4b^3c^3d^3e^{12} - 19a^4b^4c^2d^2e^{13} + 111a^5b^2c^3d^2e^{13} - \\
& 28a^6b^3c^3d^2e^{14} - 8a*b^5c^4d^7e^8 + 6a*b^6c^3d^6e^9 + 2a*b^7 * \\
& c^2d^5e^{10} - 32a^3b^3c^6d^7e^8 + 92a^4b^3c^5d^5e^{10} + 252a^5b^3c^4 \\
& *d^3e^{12} + 6a^5b^3c^2d^2e^{14}) / a^4 + (d^{(1/2)} * (3a*e - 2*b*d) * (((d + e*x \\
& ^2)^{(1/2)} * (64a^7b^3c^3e^{13} + 352a^7c^4d^2e^{12} - 16a^6b^3c^2e^{13} - 1 \\
& 60a^5c^6d^5e^8 + 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2 \\
& *b^7c^2d^4e^9 - 224a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4e^9 + 112 * \\
& a^3b^6c^2d^3e^{10} + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - \\
& 716a^4b^4c^3d^3e^{10} - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e \\
& ^{10} + 860a^5b^3c^3d^2e^{11} - 896a^5b^4c^2d^2e^{12} - 1392a^6b^3c^4 \\
& *d^2e^{11} - 336a^6b^2c^3d^2e^{12})) / (2a^4) + (d^{(1/2)} \\
& * ((320a^8c^4d^4e^{11} + 320a^7c^5d^3e^9 + 32a^5b^3c^4d^4e^8 - 24a \\
& ^5b^4c^3d^3e^9 - 8a^5b^5c^2d^2e^{10} + 16a^6b^2c^4d^3e^9 + 144 * \\
& a^6b^3c^3d^2e^{10} - 128a^6b^4c^5d^4e^8 + 8a^6b^4c^2d^2e^{11} - 448a \\
& ^7b^3c^4d^2e^{10} - 112a^7b^2c^3d^2e^{11}) / a^4 - (d^{(1/2)} * (d + e*x^2)^{(1/2)} \\
&) * (3a*e - 2*b*d) * (1024a^9c^4e^{10} + 64a^7b^4c^2e^{10} - 512a^8b^2c^
\end{aligned}$$

$$\begin{aligned}
& 3e^{10} + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^2c^4d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9) / (8a^6) * (3ae - 2bd) / (4a^2) / (4a^2) * (3ae - 2bd) / (4a^2) / (4a^2) + (d^{1/2}) * (3ae - 2bd) * ((d + e^2)^{1/2}) * (4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^2c^3d^2e^{15} - 8a^2b^2c^6d^8e^8 - 28a^2b^3c^5d^7e^9 + 8a^2b^2c^6d^7e^9 - 228a^3b^2c^5d^5e^{11} - 60a^4b^2c^4d^3e^{13}) / (2a^4) + (d^{1/2}) * ((56a^4c^6d^6e^9 - 44a^5c^5d^4e^{11} - 100a^6c^4d^2e^{13} + 40a^2b^3c^5d^7e^8 - 39a^2b^5c^3d^5e^{10} - 11a^2b^6c^2d^4e^{11} - 108a^3b^2c^5d^6e^9 + 96a^3b^3c^4d^5e^{10} + 111a^3b^4c^3d^4e^{11} + 22a^3b^5c^2d^3e^{12} - 237a^4b^2c^4d^4e^{11} - 161a^4b^3c^3d^3e^{12} - 19a^4b^4c^2d^2e^{13} + 111a^5b^2c^3d^2e^{13} - 28a^6b^2c^3d^2e^{14} - 8a^2b^5c^4d^7e^8 + 6a^2b^6c^3d^6e^9 + 2a^2b^7c^2d^5e^{10} - 32a^3b^2c^6d^7e^8 + 92a^4b^2c^5d^5e^{10} + 252a^5b^2c^4d^3e^{12} + 6a^5b^3c^2d^2e^{14}) / a^4 - (d^{1/2}) * (3ae - 2bd) * ((d + e^2)^{1/2}) * (64a^7b^2c^3e^{13} + 352a^7c^4d^2e^{12} - 16a^6b^3c^2e^{13} - 160a^5c^6d^5e^8 + 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 224a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4e^9 + 112a^3b^6c^2d^3e^{10} + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3d^2e^{11} - 896a^5b^2c^5d^4e^9 + 64a^5b^4c^2d^2e^{12} - 1392a^6b^2c^4d^2e^{11} - 336a^6b^2c^3d^2e^{12}) / (2a^4) - (d^{1/2}) * ((320a^8c^4d^2e^{11} + 320a^7c^5d^3e^9 + 32a^5b^3c^4d^4e^8 - 24a^5b^4c^3d^3e^9 - 8a^5b^5c^2d^2e^{10} + 16a^6b^2c^4d^3e^9 + 144a^6b^3c^3d^2e^{10} - 128a^6b^2c^5d^4e^8 + 8a^6b^4c^2d^2e^{11} - 448a^7b^2c^4d^2e^{10} - 112a^7b^2c^3d^2e^{11}) / a^4 + (d^{1/2}) * (d + e^2)^{1/2} * (3ae - 2bd) * (1024a^9c^4e^{10} + 64a^7b^4c^2e^{10} - 512a^8b^2c^3e^{10} + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^2c^4d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9) / (8a^6) * (3ae - 2bd) / (4a^2) / (4a^2) * (3ae - 2bd) / (4a^2) / (4a^2) * (3ae - 2bd) * i) / (2a^2) - \operatorname{atan}((((224a^4c^6d^6e^9 - 176a^5c^5d^4e^{11} - 400a^6c^4d^2e^{13} + 160a^2b^3c^5d^7e^8 - 156a^2b^5c^3d^5e^{10} - 44a^2b^6c^2d^4e^{11} - 432a^3b^2c^5d^6e^9 + 384a^3b^3c^4d^5e^{10} + 444a^3b^4c^3d^4e^{11} + 88a^3b^5c^2d^3e^{12} - 948a^4b^2c^4d^4e^{11} - 644a^4b^3c^3d^3e^{12} - 76a^4b^4c^2d^2e^{13} + 444a^5b^2c^3d^2e^{13} - 112a^6b^2c^3d^2e^{14} - 32a^2b^5c^4d^7e^8 + 24a^2b^6c^3d^6e^9 + 8a^2b^7c^2d^5e^{10} - 128a^3b^2c^6d^7e^8 + 368a^4b^2c^5d^5e^{10} + 1008a^5b^2c^4d^3e^{12} + 24a^5b^3c^2d^2e^{14}) / (4a^4) + (((1280a^8c^4d^2e^{11} + 1280a^7c^5d^3e^9 + 128a^5b^3c^4d^4e^8 - 96a^5b^4c^3d^3e^9 - 32a^5b^5c^2d^2e^{10} + 64a^6b^2c^4d^3e^9 + 576a^6b^3c^3d^2e^{10} - 512a^6b^2c^5d^4e^8 + 32a^6b^4c^2d^2e^{11} - 1792a^7b^2c^4d^2e^{10} - 448a^7b^2c^3d^2e^{11}) / (4a^4) - ((d + e^2)^{1/2}) * (((4b^6d^3 - 4a^3b^3e^3 - 32a
\end{aligned}$$

$$\begin{aligned}
& *d^2e + 36a^2b^2c^2d^3 - 16ab^4cd^3 + 8a^4b^2c^2e^3 - 6ab^5d^2e \\
& + 42a^2b^3cd^2e - 72a^3b^2c^2d^2e - 36a^3b^2cd^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} - ((d + ex^2)^{(1/2)}(4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^2c^3d^2e^{15} - 8ab^2c^6d^8e^8 - 28ab^3c^5d^7e^9 + 8a^2b^2c^6d^7e^9 - 228a^3b^2c^5d^5e^{11} - 60a^4b^2c^4d^3e^{13}))/((2a^4)*(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^2c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^2/4 - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)*c^4d^6 + a^3c^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2cd^2e^5 - 6ab^2c^2d^3e^3 + 3ab^2cd^2e^4))^{(1/2)} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^2e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16ab^4cd^3 + 8a^4b^2c^2e^3 - 6ab^5d^2e + 42a^2b^3cd^2e - 72a^3b^2cd^2e^2 - 36a^3b^2cd^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} * \\
& 1i - (((224a^4c^6d^6e^9 - 176a^5c^5d^4e^{11} - 400a^6c^4d^2e^{13} + 160a^2b^3c^5d^7e^8 - 156a^2b^5c^3d^5e^{10} - 44a^2b^6c^2d^4e^{11} - 432a^3b^2c^5d^6e^9 + 384a^3b^3c^4d^5e^{10} + 444a^3b^4c^3d^4e^{11} + 88a^3b^5c^2d^3e^{12} - 948a^4b^2c^4d^4e^{11} - 644a^4b^3c^3d^3e^{12} - 76a^4b^4c^2d^2e^{13} + 444a^5b^2c^3d^2e^{13} - 112a^6b^2c^3d^2e^{14} - 32ab^5c^4d^7e^8 + 24ab^6c^3d^6e^9 + 8ab^7c^2d^5e^{10} - 128a^3b^2c^6d^7e^8 + 368a^4b^2c^5d^5e^{10} + 1008a^5b^2c^4d^3e^{12} + 24a^5b^3c^2d^2e^{14}))/((4a^4) + (((1280a^8c^4d^2e^{11} + 1280a^7c^5d^3e^9 + 128a^5b^3c^4d^4e^8 - 96a^5b^4c^3d^3e^9 - 32a^5b^5c^2d^2e^{10} + 64a^6b^2c^4d^3e^9 + 576a^6b^3c^3d^2e^{10} - 512a^6b^2c^5d^4e^8 + 32a^6b^4c^2d^2e^{11} - 1792a^7b^2c^4d^2e^{10} - 448a^7b^2c^3d^2e^{11}))/((4a^4) + ((d + ex^2)^{(1/2)}*(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^2c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^2/4 - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)*c^4d^6 + a^3c^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2cd^2e^5 - 6ab^2c^2d^3e^3 + 3ab^2cd^2e^4))^{(1/2)} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^2e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16ab^4cd^3 + 8a^4b^2c^2e^3 - 6ab^5d^2e + 42a^2b^3cd^2e - 72a^3b^2cd^2e^2 - 36a^3b^2cd^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} * (1024a^9c^4e^{10} + 64a^7b^4c^2e^{10} - 512a^8b^2c^3e^{10} + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^2c^4d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9))/(2a^4)*(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^2c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)
\end{aligned}$$

$$\begin{aligned}
& 2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)(c^4d^6 + a^3c^6 + \\
& 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3 \\
& *b^3c^3d^5e - 3a^2b^3cd^5e^5 - 6a^2b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1 \\
& /2)} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^4e^2 + 48a^4 \\
& *c^2d^2e^2 + 36a^2b^2c^2d^3 - 16a^2b^4cd^3 + 8a^4b^3c^3e^3 - 6a^2b^5d^2 \\
& *e + 42a^2b^3cd^2e - 72a^3b^3c^2d^2e - 36a^3b^2c^2d^2e^2)/(16*(\\
& a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} - ((d + ex^2)^{(1/2)}*(64a^7b^3 \\
& c^3e^{13} + 352a^7c^4d^4e^{12} - 16a^6b^3c^2e^{13} - 160a^5c^6d^5e^8 + \\
& 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 2 \\
& 24a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4e^9 + 112a^3b^6c^2d^3e^{10} \\
& + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} - \\
& 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3 \\
& d^2e^{11} - 896a^5b^3c^5d^4e^9 + 64a^5b^4c^2d^2e^{12} - 1392a^6b^3c^4 \\
& *d^2e^{11} - 336a^6b^2c^3d^2e^{12}))/((2a^4))*(((4b^6d^3 - 4a^3b^3e^3 \\
& - 32a^3c^3d^3 + 12a^2b^4d^4e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 \\
& - 32a^2b^4cd^3 + 16a^4b^3c^3e^3 - 12a^2b^5d^2e + 84a^2b^3cd^2e - \\
& 144a^3b^3c^2d^2e - 72a^3b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 \\
& - 128a^5b^2c)(c^4d^6 + a^3c^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3 \\
& *a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3c^3d^5e - 3a^2b^3cd^5e^5 - 6 \\
& *a^2b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} + 2b^6d^3 - 2a^3b^3e^3 - 1 \\
& 6a^3c^3d^3 + 6a^2b^4d^4e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 1 \\
& 6a^2b^4cd^3 + 8a^4b^3c^3e^3 - 6a^2b^5d^2e + 42a^2b^3cd^2e - 72a^3 \\
& *b^3c^2d^2e - 36a^3b^2c^2d^2e^2)/(16*(a^4b^4 + 16a^6c^2 - 8a^5b^2c) \\
&))^{(1/2))*(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^4e^2 \\
& + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32a^2b^4cd^3 + 16a^4b^3c^3e^3 \\
& - 12a^2b^5d^2e + 84a^2b^3cd^2e - 144a^3b^3c^2d^2e - 72a^3b^2c^2 \\
& *d^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)(c^4d^6 + a^3c^6 \\
& + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3c^3 \\
& d^5e - 3a^2b^3cd^5e^5 - 6a^2b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} + 2b^6d^3 - 2a^3b^3e^3 - 16 \\
& *a^3c^3d^3 + 6a^2b^4d^4e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16 \\
& *a^2b^4cd^3 + 8a^4b^3c^3e^3 - 6a^2b^5d^2e + 42a^2b^3cd^2e - 72a^3 \\
& *b^3c^2d^2e - 36a^3b^2c^2d^2e^2)/(16*(a^4b^4 + 16a^6c^2 - 8a^5b^2c) \\
&))^{(1/2)} + ((d + ex^2)^{(1/2)}*(4a^6 \\
& *c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - \\
& 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2 \\
& *b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^ \\
& ^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^3c^3d^2e^{15} - 8a^2b^ \\
& ^2c^6d^8e^8 - 28a^2b^3c^5d^7e^9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5 \\
& *d^5e^{11} - 60a^4b^3c^4d^3e^{13}))/((2a^4))*(((4b^6d^3 - 4a^3b^3e^3 \\
& - 32a^3c^3d^3 + 12a^2b^4d^4e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 \\
& - 32a^2b^4cd^3 + 16a^4b^3c^3e^3 - 12a^2b^5d^2e + 84a^2b^3cd^2e - \\
& 144a^3b^3c^2d^2e - 72a^3b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - \\
& 128a^5b^2c)(c^4d^6 + a^3c^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3 \\
& *a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3c^3d^5e - 3a^2b^3cd^5e^5 - 6a^2 \\
& *b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} + 2b^6d^3 - 2a^3b^3e^3 - 16
\end{aligned}$$

$$\begin{aligned}
& *a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16 \\
& *a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3* \\
& b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)) \\
&)^{(1/2)}*i)/((6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2 \\
& *c^6*d^7*e^11 + 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 \\
& + 4*a^2*b^2*c^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^ \\
& 15 - 16*a*b*c^6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3* \\
& a^2*b*c^5*d^6*e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16)/(2*a^4) \\
& + (((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^6*c^4*d^2*e^13 + 1 \\
& 60*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^2*b^6*c^2*d^4*e^11 \\
& - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 444*a^3*b^4*c^3*d^4 \\
& *e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^11 - 644*a^4*b^3*c^ \\
& 3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^2*e^13 - 112*a^6*b \\
& *c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 + 8*a*b^7*c^2*d^5 \\
& *e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + 1008*a^5*b*c^4*d^3 \\
& *e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4*d*e^11 + 1280*a^7* \\
& c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^5*b^5 \\
& *c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^10 - 512*a^6 \\
& *b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d^2*e^10 - 448*a^7* \\
& b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^{(1/2)}*(((4*b^6*d^3 - 4*a^3*b^3*e^3 \\
& - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 \\
& - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - \\
& 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - \\
& 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3* \\
& a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a \\
& *b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16 \\
& *a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16 \\
& *a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3* \\
& b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)) \\
&)^{(1/2)}*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1 \\
& 536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1 \\
& 792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9)/(2*a^ \\
& 4))*(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96* \\
& a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a \\
& *b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2) \\
& ^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3* \\
& a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b \\
& *c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^{(1/2)} \\
&) + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c \\
& ^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^ \\
& 2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^ \\
& 4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(64*a^7*b*c^ \\
& 3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 7 \\
& 36*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224 \\
& *a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 +
\end{aligned}$$

$$\begin{aligned}
& 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3d^2e^{11} - 896a^5b^4c^2d^2e^{12} - 1392a^6b^3c^4d^2e^{11} - 336a^6b^2c^3d^2e^{12}) / (2a^4) * (((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^3c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) * (c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3cd^5e - 3a^2b^3cd^2e^5 - 6a^2b^3cd^3e^3 + 3a^2b^2cd^2e^4))^{1/2} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^2e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16a^2b^4cd^3 + 8a^4b^3c^2e^3 - 6a^2b^5d^2e + 42a^2b^3cd^2e - 72a^3b^2cd^2e - 36a^3b^2cd^2e^2) / (16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{1/2}) * (((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^3c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) * (c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3cd^5e - 3a^2b^3cd^2e^5 - 6a^2b^3cd^3e^3 + 3a^2b^2cd^2e^4))^{1/2} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^2e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16a^2b^4cd^3 + 8a^4b^3c^2e^3 - 6a^2b^5d^2e + 42a^2b^3cd^2e - 72a^3b^2cd^2e - 36a^3b^2cd^2e^2) / (16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{1/2} - ((d + ex^2)^{1/2} * (4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^3c^3d^2e^{15} - 8a^2b^2c^6d^8e^8 - 28a^2b^3c^5d^7e^9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5d^5e^{11} - 60a^4b^3c^4d^3e^{13})) / (2a^4)) * (((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^3c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) * (c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3cd^5e - 3a^2b^3cd^2e^5 - 6a^2b^3cd^3e^3 + 3a^2b^2cd^2e^4))^{1/2} + 2b^6d^3 - 2a^3b^3e^3 - 16a^3c^3d^3 + 6a^2b^4d^2e^2 + 48a^4c^2d^2e^2 + 36a^2b^2c^2d^3 - 16a^2b^4cd^3 + 8a^4b^3c^2e^3 - 6a^2b^5d^2e + 42a^2b^3cd^2e - 72a^3b^2cd^2e - 36a^3b^2cd^2e^2) / (16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{1/2} + (((224a^4c^6d^6e^9 - 176a^5c^5d^4e^{11} - 400a^6c^4d^2e^{13} + 160a^2b^3c^5d^7e^8 - 156a^2b^5c^3d^5e^{10} - 44a^2b^6c^2d^4e^{11} - 432a^3b^2c^5d^6e^9 + 384a^3b^3c^4d^5e^{10} + 444a^3b^4c^3d^4e^{11} + 88a^3b^5c^2d^3e^{12} - 948a^4b^2c^4d^4e^{11} - 644a^4b^3c^3d^3e^{12} - 76a^4b^4c^2d^2e^{13} + 444a^5b^2c^3d^2e^{13} - 112a^6b^3c^3d^2e^{14} - 32a^2b^5c^4d^7e^8 + 24a^2b^6c^3d^6e^9 + 8a^2b^7c^2d^5e^{10} - 128a^3b^3c^6d^7e^8 + 368a^4b^3c^5d^5e^{10} + 1008a^5b^3c^
\end{aligned}$$

$$\begin{aligned}
& 4*d^3*e^{12} + 24*a^5*b^3*c^2*d*e^{14})/(4*a^4) + (((1280*a^8*c^4*d*e^{11} + 1280 \\
& *a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^3*e^9 - 32*a^ \\
& 5*b^5*c^2*d^2*e^{10} + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3*d^2*e^{10} - 51 \\
& 2*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^{11} - 1792*a^7*b*c^4*d^2*e^{10} - 448 \\
& *a^7*b^2*c^3*d*e^{11})/(4*a^4) + ((d + e*x^2)^{(1/2)}*(((4*b^6*d^3 - 4*a^3*b^3 \\
& *e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^ \\
& 2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2 \\
& *e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c \\
& ^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 \\
& + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 \\
& - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 \\
& - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 \\
& - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72 \\
& *a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^ \\
& 2*c)))^{(1/2)}*(1024*a^9*c^4*e^{10} + 64*a^7*b^4*c^2*e^{10} - 512*a^8*b^2*c^3*e^1 \\
& 0 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^ \\
& 8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/ \\
& (2*a^4))*(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 \\
& + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - \\
& 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d \\
& *e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 \\
& + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 \\
& - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4)) \\
& ^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48* \\
& a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b \\
& ^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(1 \\
& 6*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c)))^{(1/2)} - ((d + e*x^2)^{(1/2)}*(64*a^7 \\
& *b*c^3*e^{13} + 352*a^7*c^4*d*e^{12} - 16*a^6*b^3*c^2*e^{13} - 160*a^5*c^6*d^5*e^ \\
& 8 + 736*a^6*c^5*d^3*e^{10} + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 \\
& - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e \\
& ^{10} + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d \\
& ^3*e^{10} - 132*a^4*b^5*c^2*d^2*e^{11} + 936*a^5*b^2*c^4*d^3*e^{10} + 860*a^5*b^3 \\
& *c^3*d^2*e^{11} - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^{12} - 1392*a^6*b* \\
& c^4*d^2*e^{11} - 336*a^6*b^2*c^3*d*e^{12}))/((2*a^4))*(((4*b^6*d^3 - 4*a^3*b^3* \\
& e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2 \\
& *d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2* \\
& e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^2/4 - (16*a^4*b^4 + 256*a^6*c \\
& ^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 \\
& + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - \\
& 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 \\
& - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 \\
& - 16*a*b^4*c*d^3 + 8*a^4*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72* \\
& a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2 \\
& *c)))^{(1/2))*(((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d* \\
& e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*c*e
\end{aligned}$$

$$\begin{aligned}
&^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2 \\
&*c*d*e^2)^{2/4} - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c \\
&*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4 \\
&*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e \\
&^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + \\
&48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*b*c*e^3 - 6 \\
&*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2 \\
&)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)} + ((d + e*x^2)^{(1/2)}*(4* \\
&a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^1 \\
&2 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32* \\
&a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 2 \\
&8*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8* \\
&a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b* \\
&c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4))*((((4*b^6*d^3 - 4*a^3*b^3*e \\
&^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d*e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2* \\
&d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*b*c*e^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e \\
&- 144*a^3*b*c^2*d^2*e - 72*a^3*b^2*c*d*e^2)^{2/4} - (16*a^4*b^4 + 256*a^6*c^ \\
&2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + \\
&3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - \\
&6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 - \\
&16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + 48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - \\
&16*a*b^4*c*d^3 + 8*a^4*b*b*c*e^3 - 6*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a \\
&^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2* \\
&c))^{(1/2)}))*((((4*b^6*d^3 - 4*a^3*b^3*e^3 - 32*a^3*c^3*d^3 + 12*a^2*b^4*d* \\
&e^2 + 96*a^4*c^2*d*e^2 + 72*a^2*b^2*c^2*d^3 - 32*a*b^4*c*d^3 + 16*a^4*b*b*c* \\
&^3 - 12*a*b^5*d^2*e + 84*a^2*b^3*c*d^2*e - 144*a^3*b*c^2*d^2*e - 72*a^3*b^2 \\
&*c*d*e^2)^{2/4} - (16*a^4*b^4 + 256*a^6*c^2 - 128*a^5*b^2*c)*(c^4*d^6 + a^3*c \\
&*e^6 + 3*a*c^3*d^4*e^2 - b^3*c*d^3*e^3 + 3*a^2*c^2*d^2*e^4 + 3*b^2*c^2*d^4* \\
&*e^2 - 3*b*c^3*d^5*e - 3*a^2*b*c*d*e^5 - 6*a*b*c^2*d^3*e^3 + 3*a*b^2*c*d^2*e \\
&^4))^{(1/2)} + 2*b^6*d^3 - 2*a^3*b^3*e^3 - 16*a^3*c^3*d^3 + 6*a^2*b^4*d*e^2 + \\
&48*a^4*c^2*d*e^2 + 36*a^2*b^2*c^2*d^3 - 16*a*b^4*c*d^3 + 8*a^4*b*b*c*e^3 - 6 \\
&*a*b^5*d^2*e + 42*a^2*b^3*c*d^2*e - 72*a^3*b*c^2*d^2*e - 36*a^3*b^2*c*d*e^2 \\
&)/(16*(a^4*b^4 + 16*a^6*c^2 - 8*a^5*b^2*c))^{(1/2)}*2i - (d*(d + e*x^2)^{(1/2)} \\
&))/(2*a*x^2) - atan((((224*a^4*c^6*d^6*e^9 - 176*a^5*c^5*d^4*e^11 - 400*a^ \\
&6*c^4*d^2*e^13 + 160*a^2*b^3*c^5*d^7*e^8 - 156*a^2*b^5*c^3*d^5*e^10 - 44*a^ \\
&2*b^6*c^2*d^4*e^11 - 432*a^3*b^2*c^5*d^6*e^9 + 384*a^3*b^3*c^4*d^5*e^10 + 4 \\
&44*a^3*b^4*c^3*d^4*e^11 + 88*a^3*b^5*c^2*d^3*e^12 - 948*a^4*b^2*c^4*d^4*e^1 \\
&1 - 644*a^4*b^3*c^3*d^3*e^12 - 76*a^4*b^4*c^2*d^2*e^13 + 444*a^5*b^2*c^3*d^ \\
&2*e^13 - 112*a^6*b*c^3*d*e^14 - 32*a*b^5*c^4*d^7*e^8 + 24*a*b^6*c^3*d^6*e^9 \\
&+ 8*a*b^7*c^2*d^5*e^10 - 128*a^3*b*c^6*d^7*e^8 + 368*a^4*b*c^5*d^5*e^10 + \\
&1008*a^5*b*c^4*d^3*e^12 + 24*a^5*b^3*c^2*d*e^14)/(4*a^4) + (((1280*a^8*c^4* \\
&d*e^11 + 1280*a^7*c^5*d^3*e^9 + 128*a^5*b^3*c^4*d^4*e^8 - 96*a^5*b^4*c^3*d^ \\
&3*e^9 - 32*a^5*b^5*c^2*d^2*e^10 + 64*a^6*b^2*c^4*d^3*e^9 + 576*a^6*b^3*c^3* \\
&d^2*e^10 - 512*a^6*b*c^5*d^4*e^8 + 32*a^6*b^4*c^2*d*e^11 - 1792*a^7*b*c^4*d \\
&^2*e^10 - 448*a^7*b^2*c^3*d*e^11)/(4*a^4) - ((d + e*x^2)^{(1/2)}*(-((4*b^6*d
\end{aligned}$$

$$\begin{aligned}
& 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16ab^4cd^3 - \\
& 8a^4b^2c^2e^3 + 6ab^5d^2e - 42a^2b^3cd^2e + 72a^3b^2c^2d^2e + 3 \\
& 6a^3b^2cd^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} - ((d + \\
& ex^2)^{(1/2)}(4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} + 13 \\
& 2a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c \\
& ^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} + 88a^3b^2 \\
& *c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b \\
& *c^3d^2e^{15} - 8ab^2c^6d^8e^8 - 28ab^3c^5d^7e^9 + 8a^2b^2c^6d^7 \\
& *e^9 - 228a^3b^2c^5d^5e^{11} - 60a^4b^2c^4d^3e^{13}))/((2a^4)*(-(((4b^6 \\
& *d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 \\
& + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^2c^2e^3 - 12ab^5d^2e + \\
& 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^2/4 - (16a^4 \\
& 4b^4 + 256a^6c^2 - 128a^5b^2c)*(c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 \\
& - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - \\
& 3a^2b^2cd^2e^5 - 6ab^2c^2d^3e^3 + 3ab^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 \\
& + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36 \\
& a^2b^2c^2d^3 + 16ab^4cd^3 - 8a^4b^2c^2e^3 + 6ab^5d^2e - 42a^2b \\
& ^3cd^2e + 72a^3b^2c^2d^2e + 36a^3b^2cd^2e^2)/(16(a^4b^4 + 16a^6 \\
& c^2 - 8a^5b^2c))^{(1/2)}*i - (((224a^4c^6d^6e^9 - 176a^5c^5d^4e \\
& ^{11} - 400a^6c^4d^2e^{13} + 160a^2b^3c^5d^7e^8 - 156a^2b^5c^3d^5 \\
& *e^{10} - 44a^2b^6c^2d^4e^{11} - 432a^3b^2c^5d^6e^9 + 384a^3b^3c^4 \\
& *d^5e^{10} + 444a^3b^4c^3d^4e^{11} + 88a^3b^5c^2d^3e^{12} - 948a^4b^ \\
& 2c^4d^4e^{11} - 644a^4b^3c^3d^3e^{12} - 76a^4b^4c^2d^2e^{13} + 444a \\
& ^5b^2c^3d^2e^{13} - 112a^6b^2c^3d^2e^{14} - 32ab^5c^4d^7e^8 + 24ab^ \\
& 6c^3d^6e^9 + 8ab^7c^2d^5e^{10} - 128a^3b^2c^6d^7e^8 + 368a^4b^2c^ \\
& 5d^5e^{10} + 1008a^5b^2c^4d^3e^{12} + 24a^5b^3c^2d^2e^{14}))/((4a^4) + (((\\
& 1280a^8c^4d^2e^{11} + 1280a^7c^5d^3e^9 + 128a^5b^3c^4d^4e^8 - 96a \\
& ^5b^4c^3d^3e^9 - 32a^5b^5c^2d^2e^{10} + 64a^6b^2c^4d^3e^9 + 576 \\
& *a^6b^3c^3d^2e^{10} - 512a^6b^2c^5d^4e^8 + 32a^6b^4c^2d^2e^{11} - 179 \\
& 2a^7b^2c^4d^2e^{10} - 448a^7b^2c^3d^2e^{11}))/((4a^4) + ((d + ex^2)^{(1/2)} \\
& *(-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4 \\
& c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^2c^2e^3 - 12ab^ \\
& ^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2cd^2e^2)^2 \\
& /4 - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)*(c^4d^6 + a^3c^2e^6 + 3a^2 \\
& c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^ \\
& ^3d^5e - 3a^2b^2cd^2e^5 - 6ab^2c^2d^3e^3 + 3ab^2c^2d^2e^4))^{(1/2)} \\
& - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2 \\
& *d^2e^2 - 36a^2b^2c^2d^3 + 16ab^4cd^3 - 8a^4b^2c^2e^3 + 6ab^5d^2e \\
& - 42a^2b^3cd^2e + 72a^3b^2c^2d^2e + 36a^3b^2cd^2e^2)/(16(a^4b \\
& ^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)}*(1024a^9c^4e^{10} + 64a^7b^4c^2 \\
& *e^{10} - 512a^8b^2c^3e^{10} + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e \\
& ^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^2c^4d^2e^9 - 128a^6b^5c^2d^2e^9 \\
& + 960a^7b^3c^3d^2e^9))/((2a^4)*(-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3 \\
& c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab \\
& ^4cd^3 + 16a^4b^2c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3
\end{aligned}$$

$$\begin{aligned}
& b^2c^2d^2e - 72a^3b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5 \\
& *b^2c^2)(c^4d^6 + a^3c^2e^6 + 3a^3c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 \\
& + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2c^2d^5e^5 - 6a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4) \\
& ^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 \\
& + 16a^2b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^2b^5d^2e - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e \\
& + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} \\
& - ((d + ex^2)^{(1/2)}(64a^7b^3c^3e^{13} + 352a^7c^4d^2e^{12} - 16a^6b^3c^2e^{13} - 160a^5c^6d^5e^8 \\
& + 736a^6c^5d^3e^{10} + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 224a^3b^4c^4d^5e^8 + 144a^3b^5c^3d^4 \\
& e^9 + 112a^3b^6c^2d^3e^{10} + 432a^4b^2c^5d^5e^8 + 144a^4b^3c^4d^4e^9 - 716a^4b^4c^3d^3e^{10} \\
& - 132a^4b^5c^2d^2e^{11} + 936a^5b^2c^4d^3e^{10} + 860a^5b^3c^3d^2e^{11} - 896a^5b^2c^5d^4e^9 + 64a^5b^4 \\
& c^2d^2e^{12} - 1392a^6b^2c^4d^2e^{11} - 336a^6b^2c^3d^2e^{12}))/((2a^4) * (-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 \\
& + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32a^2b^4c^2d^3 + 16a^4b^2c^2e^3 - 12a^2b^5 \\
& d^2e + 84a^2b^3c^2d^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) \\
& *(c^4d^6 + a^3c^2e^6 + 3a^3c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e \\
& - 3a^2b^2c^2d^5e^5 - 6a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 \\
& - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16a^2b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^2b^5d^2e \\
& - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} \\
& * (-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32a^2b^4 \\
& c^2d^3 + 16a^4b^2c^2e^3 - 12a^2b^5d^2e + 84a^2b^3c^2d^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) \\
& *(c^4d^6 + a^3c^2e^6 + 3a^3c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3a^2b^2c^2d^5e \\
& - 3a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 \\
& - 36a^2b^2c^2d^3 + 16a^2b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^2b^5d^2e - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e \\
& + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} + ((d + ex^2)^{(1/2)}(4a^6c^3e^{16} + 4a^2c^7d^8e^8 - 2a^3c^6d^6e^{10} \\
& + 132a^4c^5d^4e^{12} - 2a^5c^4d^2e^{14} + 4b^4c^5d^8e^8 + 129a^2b^2c^5d^6e^{10} - 32a^2b^3c^4d^5e^{11} + 8a^2b^4c^3d^4e^{12} \\
& + 88a^3b^2c^4d^4e^{12} - 28a^3b^3c^3d^3e^{13} + 33a^4b^2c^3d^2e^{14} - 16a^5b^2c^3d^2e^{15} - 8a^2b^2c^6d^8e^8 - 28a^2b^3c^5d^7e^9 + 8a^2 \\
& b^2c^6d^7e^9 - 228a^3b^2c^5d^5e^{11} - 60a^4b^2c^4d^3e^{13}))/((2a^4) * (-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 \\
& + 72a^2b^2c^2d^3 - 32a^2b^4c^2d^3 + 16a^4b^2c^2e^3 - 12a^2b^5d^2e + 84a^2b^3c^2d^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) \\
& *(c^4d^6 + a^3c^2e^6 + 3a^3c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2c^2d^5e^5 - 6a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
&) - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16a^4b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^4b^5d^2e - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} \cdot i) / ((6a^7c^9d^9e^9 + 6a^5c^3d^9e^17 - 4b^7c^7d^10e^8 + 6a^2c^6d^7e^11 + 6a^4c^4d^3e^15 + 8b^2c^6d^9e^9 - 4b^3c^5d^8e^10 + 4a^2b^2c^4d^5e^13 - 11a^2b^3c^3d^4e^14 + 22a^3b^2c^3d^3e^15 - 16a^2b^2c^6d^8e^10 + 8a^2b^2c^5d^7e^11 + 2a^2b^4c^3d^5e^13 - 3a^2b^2c^5d^6e^12 - 10a^3b^2c^4d^4e^14 - 19a^4b^2c^3d^2e^16)/(2a^4) + (((224a^4c^6d^6e^9 - 176a^5c^5d^4e^11 - 400a^6c^4d^2e^13 + 160a^2b^3c^5d^7e^8 - 156a^2b^5c^3d^5e^10 - 44a^2b^6c^2d^4e^11 - 432a^3b^2c^5d^6e^9 + 384a^3b^3c^4d^5e^10 + 444a^3b^4c^3d^4e^11 + 88a^3b^5c^2d^3e^12 - 948a^4b^2c^4d^4e^11 - 644a^4b^3c^3d^3e^12 - 76a^4b^4c^2d^2e^13 + 444a^5b^2c^3d^2e^13 - 112a^6b^2c^3d^2e^14 - 32a^2b^5c^4d^7e^8 + 24a^2b^6c^3d^6e^9 + 8a^2b^7c^2d^5e^10 - 128a^3b^2c^6d^7e^8 + 368a^4b^2c^5d^5e^10 + 1008a^5b^2c^4d^3e^12 + 24a^5b^3c^2d^2e^14)/(4a^4) + (((1280a^8c^4d^4e^11 + 1280a^7c^5d^3e^9 + 128a^5b^3c^4d^4e^8 - 96a^5b^4c^3d^3e^9 - 32a^5b^5c^2d^2e^10 + 64a^6b^2c^4d^3e^9 + 576a^6b^3c^3d^2e^10 - 512a^6b^2c^5d^4e^8 + 32a^6b^4c^2d^2e^11 - 1792a^7b^2c^4d^2e^10 - 448a^7b^2c^3d^2e^11)/(4a^4) - ((d + e^2)^{(1/2)} * (-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32a^2b^4c^2d^3 + 16a^4b^2c^2e^3 - 12a^2b^5d^2e + 84a^2b^3c^2d^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e^2)^2/4 - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) * (c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2c^2d^2e^5 - 6a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16a^4b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^4b^5d^2e - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} * (1024a^9c^4e^10 + 64a^7b^4c^2e^10 - 512a^8b^2c^3e^10 + 1536a^8c^5d^2e^8 + 128a^6b^4c^3d^2e^8 - 896a^7b^2c^4d^2e^8 - 1792a^8b^2c^4d^2e^9 - 128a^6b^5c^2d^2e^9 + 960a^7b^3c^3d^2e^9))/(2a^4)) * (-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32a^2b^4c^2d^3 + 16a^4b^2c^2e^3 - 12a^2b^5d^2e + 84a^2b^3c^2d^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e^2)^2/4 - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c) * (c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3c^2d^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^2c^3d^5e - 3a^2b^2c^2d^2e^5 - 6a^2b^2c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16a^4b^4c^2d^3 - 8a^4b^2c^2e^3 + 6a^4b^5d^2e - 42a^2b^3c^2d^2e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16(a^4b^4 + 16a^6c^2 - 8a^5b^2c))^{(1/2)} + ((d + e^2)^{(1/2)} * (64a^7b^3c^3e^13 + 352a^7c^4d^4e^12 - 16a^6b^3c^2e^13 - 160a^5c^6d^5e^8 + 736a^6c^5d^3e^10 + 32a^2b^6c^3d^5e^8 - 32a^2b^7c^2d^4e^9 - 224a^3b^4c^4d^5e^8 + 144a^3b^5c^3
\end{aligned}$$

$$\begin{aligned}
& d^4 e^9 + 112 a^3 b^6 c^2 d^3 e^{10} + 432 a^4 b^2 c^5 d^5 e^8 + 144 a^4 b^3 c^4 d^4 e^9 - 716 a^4 b^4 c^3 d^3 e^{10} - 132 a^4 b^5 c^2 d^2 e^{11} + 936 a^5 b^2 c^4 d^3 e^{10} + 860 a^5 b^3 c^3 d^2 e^{11} - 896 a^5 b^4 c^2 d^2 e^{12} - 1392 a^6 b^3 c^4 d^2 e^{11} - 336 a^6 b^2 c^3 d^2 e^{12} \\
& \left. \right) / (2 a^4) \left. \right) \left(- \left(\left((4 b^6 d^3 - 4 a^3 b^3 e^3 - 32 a^3 c^3 d^3 + 12 a^2 b^4 d^2 e^2 + 96 a^4 c^2 d^2 e^2 + 72 a^2 b^2 c^2 d^3 - 32 a^2 b^4 c^2 d^3 + 16 a^4 b^3 c^2 e^3 - 12 a^2 b^5 d^2 e + 84 a^2 b^3 c^2 d^2 e - 144 a^3 b^3 c^2 d^2 e - 72 a^3 b^2 c^2 d^2 e \right)^2 / 4 - (16 a^4 b^4 + 256 a^6 c^2 - 128 a^5 b^2 c) (c^4 d^6 + a^3 c^2 e^6 + 3 a^2 c^3 d^4 e^2 - b^3 c^2 d^3 e^3 + 3 a^2 c^2 d^2 e^4 + 3 b^2 c^2 d^4 e^2 - 3 b^2 c^3 d^5 e - 3 a^2 b^3 c^2 d^3 e^3 + 3 a^2 b^2 c^2 d^2 e^4) \right)^{1/2} - 2 b^6 d^3 + 2 a^3 b^3 e^3 + 16 a^3 c^3 d^3 - 6 a^2 b^4 d^2 e^2 - 48 a^4 c^2 d^2 e^2 - 36 a^2 b^2 c^2 d^3 + 16 a^2 b^4 c^2 d^3 - 8 a^4 b^3 c^2 e^3 + 6 a^2 b^5 d^2 e - 42 a^2 b^3 c^2 d^2 e + 72 a^3 b^3 c^2 d^2 e + 36 a^3 b^2 c^2 d^2 e^2) / (16 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c)) \right)^{1/2} \left. \right) \left(- \left(\left((4 b^6 d^3 - 4 a^3 b^3 e^3 - 32 a^3 c^3 d^3 + 12 a^2 b^4 d^2 e^2 + 96 a^4 c^2 d^2 e^2 + 72 a^2 b^2 c^2 d^3 - 32 a^2 b^4 c^2 d^3 + 16 a^4 b^3 c^2 e^3 - 12 a^2 b^5 d^2 e + 84 a^2 b^3 c^2 d^2 e - 144 a^3 b^3 c^2 d^2 e - 72 a^3 b^2 c^2 d^2 e \right)^2 / 4 - (16 a^4 b^4 + 256 a^6 c^2 - 128 a^5 b^2 c) (c^4 d^6 + a^3 c^2 e^6 + 3 a^2 c^3 d^4 e^2 - b^3 c^2 d^3 e^3 + 3 a^2 c^2 d^2 e^4 + 3 b^2 c^2 d^4 e^2 - 3 b^2 c^3 d^5 e - 3 a^2 b^3 c^2 d^3 e^3 + 3 a^2 b^2 c^2 d^2 e^4) \right)^{1/2} - 2 b^6 d^3 + 2 a^3 b^3 e^3 + 16 a^3 c^3 d^3 - 6 a^2 b^4 d^2 e^2 - 48 a^4 c^2 d^2 e^2 - 36 a^2 b^2 c^2 d^3 + 16 a^2 b^4 c^2 d^3 - 8 a^4 b^3 c^2 e^3 + 6 a^2 b^5 d^2 e - 42 a^2 b^3 c^2 d^2 e + 72 a^3 b^3 c^2 d^2 e + 36 a^3 b^2 c^2 d^2 e^2) / (16 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c)) \right)^{1/2} - ((d + e x^2)^{1/2} (4 a^6 c^3 e^{16} + 4 a^2 c^7 d^8 e^8 - 2 a^3 c^6 d^6 e^{10} + 132 a^4 c^5 d^4 e^{12} - 2 a^5 c^4 d^2 e^{14} + 4 b^4 c^5 d^8 e^8 + 129 a^2 b^2 c^5 d^6 e^{10} - 32 a^2 b^3 c^4 d^5 e^{11} + 8 a^2 b^4 c^3 d^4 e^{12} + 88 a^3 b^2 c^4 d^4 e^{12} - 28 a^3 b^3 c^3 d^3 e^{13} + 33 a^4 b^2 c^3 d^2 e^{14} - 16 a^5 b^3 c^3 d^2 e^{15} - 8 a^2 b^2 c^6 d^8 e^8 - 28 a^2 b^3 c^5 d^7 e^9 + 8 a^2 b^3 c^6 d^7 e^9 - 228 a^3 b^3 c^5 d^5 e^{11} - 60 a^4 b^3 c^4 d^3 e^{13})) / (2 a^4) \left. \right) \left(- \left(\left((4 b^6 d^3 - 4 a^3 b^3 e^3 - 32 a^3 c^3 d^3 + 12 a^2 b^4 d^2 e^2 + 96 a^4 c^2 d^2 e^2 + 72 a^2 b^2 c^2 d^3 - 32 a^2 b^4 c^2 d^3 + 16 a^4 b^3 c^2 e^3 - 12 a^2 b^5 d^2 e + 84 a^2 b^3 c^2 d^2 e - 144 a^3 b^3 c^2 d^2 e - 72 a^3 b^2 c^2 d^2 e \right)^2 / 4 - (16 a^4 b^4 + 256 a^6 c^2 - 128 a^5 b^2 c) (c^4 d^6 + a^3 c^2 e^6 + 3 a^2 c^3 d^4 e^2 - b^3 c^2 d^3 e^3 + 3 a^2 c^2 d^2 e^4 + 3 b^2 c^2 d^4 e^2 - 3 b^2 c^3 d^5 e - 3 a^2 b^3 c^2 d^3 e^3 + 3 a^2 b^2 c^2 d^2 e^4) \right)^{1/2} - 2 b^6 d^3 + 2 a^3 b^3 e^3 + 16 a^3 c^3 d^3 - 6 a^2 b^4 d^2 e^2 - 48 a^4 c^2 d^2 e^2 - 36 a^2 b^2 c^2 d^3 + 16 a^2 b^4 c^2 d^3 - 8 a^4 b^3 c^2 e^3 + 6 a^2 b^5 d^2 e - 42 a^2 b^3 c^2 d^2 e + 72 a^3 b^3 c^2 d^2 e + 36 a^3 b^2 c^2 d^2 e^2) / (16 (a^4 b^4 + 16 a^6 c^2 - 8 a^5 b^2 c)) \right)^{1/2} + \left(\left((224 a^4 c^6 d^6 e^9 - 176 a^5 c^5 d^4 e^{11} - 400 a^6 c^4 d^2 e^{13} + 160 a^2 b^3 c^5 d^7 e^8 - 156 a^2 b^5 c^3 d^5 e^{10} - 44 a^2 b^6 c^2 d^4 e^{11} - 432 a^3 b^2 c^5 d^6 e^9 + 384 a^3 b^3 c^4 d^5 e^{10} + 444 a^3 b^4 c^3 d^4 e^{11} + 88 a^3 b^5 c^2 d^3 e^{12} - 948 a^4 b^2 c^4 d^4 e^{11} - 644 a^4 b^3 c^3 d^3 e^{12} - 76 a^4 b^4 c^2 d^2 e^{13} + 444 a^5 b^2 c^3 d^2 e^{13} - 112 a^6 b^3 c^3 d^2 e^{14} - 32 a^2 b^5 c^4 d^7 e^8 + 24 a^2 b^6 c^3 d^6 e^9 + 8 a^2 b^7 c^2 d^5 e^{10} - 128 a^3 b^3 c^6 d^7 e^8 + \right.
\end{aligned}$$

$$\begin{aligned}
& b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^3c^2e^3 - 12ab^5d^2e + 84a^2b^3 \\
& *cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + 25 \\
& 6a^6c^2 - 128a^5b^2c)*(c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd \\
& ^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3c^3d^5e - 3a^2b^3c \\
& *d^5e - 6a^2b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b \\
& ^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2c \\
& ^2d^3 + 16ab^4cd^3 - 8a^4b^3c^2e^3 + 6ab^5d^2e - 42a^2b^3cd^2e \\
& + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16*(a^4b^4 + 16a^6c^2 - 8 \\
& a^5b^2c)))^{(1/2)} + ((d + ex^2)^{(1/2)}*(4a^6c^3e^16 + 4a^2c^7d^8e^8 \\
& - 2a^3c^6d^6e^10 + 132a^4c^5d^4e^12 - 2a^5c^4d^2e^14 + 4b^4c \\
& ^5d^8e^8 + 129a^2b^2c^5d^6e^10 - 32a^2b^3c^4d^5e^11 + 8a^2b^4 \\
& *c^3d^4e^12 + 88a^3b^2c^4d^4e^12 - 28a^3b^3c^3d^3e^13 + 33a^4b \\
& ^2c^3d^2e^14 - 16a^5b^3c^3d^2e^15 - 8a^2b^2c^6d^8e^8 - 28a^2b^3c^5 \\
& *d^7e^9 + 8a^2b^3c^6d^7e^9 - 228a^3b^3c^5d^5e^11 - 60a^4b^3c^4d^3 \\
& e^13))/(2a^4))*(-(((4b^6d^3 - 4a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^ \\
& 4d^2e^2 + 96a^4c^2d^2e^2 + 72a^2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b \\
& *c^2e^3 - 12ab^5d^2e + 84a^2b^3cd^2e - 144a^3b^2c^2d^2e - 72a^3 \\
& *b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + 256a^6c^2 - 128a^5b^2c)*(c^4d^6 + a \\
& ^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d \\
& ^4e^2 - 3b^3c^3d^5e - 3a^2b^3cd^5e - 6a^2b^3cd^5e - 6a^2b^3cd^5e \\
& - 6a^2b^3cd^5e + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e \\
& ^2 - 48a^4c^2d^2e^2 - 36a^2b^2c^2d^3 + 16ab^4cd^3 - 8a^4b^3c^2e^3 \\
& + 6ab^5d^2e - 42a^2b^3cd^2e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d \\
& *e^2)/(16*(a^4b^4 + 16a^6c^2 - 8a^5b^2c)))^{(1/2)}))*(-(((4b^6d^3 - 4 \\
& a^3b^3e^3 - 32a^3c^3d^3 + 12a^2b^4d^2e^2 + 96a^4c^2d^2e^2 + 72a^ \\
& 2b^2c^2d^3 - 32ab^4cd^3 + 16a^4b^3c^2e^3 - 12ab^5d^2e + 84a^2b \\
& ^3cd^2e - 144a^3b^2c^2d^2e - 72a^3b^2c^2d^2e^2)^{2/4} - (16a^4b^4 + \\
& 256a^6c^2 - 128a^5b^2c)*(c^4d^6 + a^3c^2e^6 + 3a^2c^3d^4e^2 - b^3cd \\
& ^3e^3 + 3a^2c^2d^2e^4 + 3b^2c^2d^4e^2 - 3b^3c^3d^5e - 3a^2b^3c \\
& *d^5e - 6a^2b^3c^2d^3e^3 + 3a^2b^2c^2d^2e^4))^{(1/2)} - 2b^6d^3 + 2a^3 \\
& *b^3e^3 + 16a^3c^3d^3 - 6a^2b^4d^2e^2 - 48a^4c^2d^2e^2 - 36a^2b^2 \\
& *c^2d^3 + 16ab^4cd^3 - 8a^4b^3c^2e^3 + 6ab^5d^2e - 42a^2b^3cd^2 \\
& e + 72a^3b^2c^2d^2e + 36a^3b^2c^2d^2e^2)/(16*(a^4b^4 + 16a^6c^2 - \\
& 8a^5b^2c)))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.271 \quad \int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=595

$$\frac{\sqrt{2cd - e} \left(b - \sqrt{b^2 - 4ac} \right) \left(-\frac{3abce - 2ac^2d + b^3(-e) + b^2cd}{\sqrt{b^2 - 4ac}} + ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e} \left(b - \sqrt{b^2 - 4ac} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right) + \sqrt{2cd - e}}{2c^3 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 3.28, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1291, 388, 195, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd - e} \left(b - \sqrt{b^2 - 4ac} \right) \left(-\frac{3abce - 2ac^2d + b^3(-e) + b^2cd}{\sqrt{b^2 - 4ac}} + ace + b^2(-e) + bcd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e} \left(b - \sqrt{b^2 - 4ac} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right) + \sqrt{2cd - e}}{2c^3 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 1291

Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d + e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]

Rule 1692

```
Int[(Px_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (d + ex^2)^{3/2}}{a + bx^2 + cx^4} dx &= \frac{\int \sqrt{d + ex^2} (cd - be + cex^2) dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2} (a(cd-be)+(bcd-b^2e+ace)x^2)}{a+bx^2+cx^4} dx}{c^2} \\
&= \frac{x(d + ex^2)^{3/2}}{4c} - \frac{\int \left(\frac{(bcd-b^2e+ace + \frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}})\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{(bcd-b^2e+ace - \frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}})}{b+\sqrt{b^2-4ac}+2cx^2} \right) dx}{c^2} \\
&= \frac{(3cd - 4be)x\sqrt{d + ex^2}}{8c^2} + \frac{x(d + ex^2)^{3/2}}{4c} + \frac{(d(3cd - 4be)) \int \frac{1}{\sqrt{d+ex^2}} dx}{8c^2} - \frac{(bcd - b^2e + ace)}{8c^2} \\
&= \frac{(3cd - 4be)x\sqrt{d + ex^2}}{8c^2} + \frac{x(d + ex^2)^{3/2}}{4c} + \frac{(d(3cd - 4be)) \text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{8c^2} \\
&= \frac{(3cd - 4be)x\sqrt{d + ex^2}}{8c^2} + \frac{x(d + ex^2)^{3/2}}{4c} + \frac{d(3cd - 4be) \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{e(bcd - b^2e)}{8c^2} \\
&= \frac{(3cd - 4be)x\sqrt{d + ex^2}}{8c^2} + \frac{x(d + ex^2)^{3/2}}{4c} - \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} (bcd - b^2e + ace)}{2c^3\sqrt{b - \dots}}
\end{aligned}$$

Mathematica [B] time = 6.49, size = 18689, normalized size = 31.41

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [F] time = 180.17, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]

[Out] \$Aborted

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.87, size = 104, normalized size = 0.17

$$\frac{1}{8} \sqrt{x^2 e + d} \left(\frac{2x^2 e}{c} + \frac{(5c^5 d e^2 - 4bc^4 e^3) e^{(-2)}}{c^6} \right) x - \frac{(3c^2 d^2 - 12bcde + 8b^2 e^2 - 8ace^2) e^{(-\frac{1}{2})} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2 e + d}\right)^2\right)}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e/c + (5*c^5*d*e^2 - 4*b*c^4*e^3)*e^(-2)/c^6)*x - 1/16*(3*c^2*d^2 - 12*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^3

maple [C] time = 0.04, size = 516, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)

[Out] 1/4*x*(e*x^2+d)^(3/2)/c+3/8/c*d*x*(e*x^2+d)^(1/2)+3/8/c*d^2/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/4/c^2*e^(3/2)*b*x^2-1/4/c^2*e*b*(e*x^2+d)^(1/2)*x+1/8/c^2*e^(1/2)*b*d-1/2/c^3*e^(1/2)*sum(((2*a*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2*b^2*c*d*e-b*c^2*d^2)*_R^2+2*(2*a^2*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)*_R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^2+2*b^2*c*d^3*e-b*c^2*d^4)/(_R^3+c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/c^2*e^(3/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*a-1/c^3*e^(3/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b^2+3/2/c^2*e^(1/2)*ln(-e^(1/2)*x+(e*x^2+d)^(1/2))*b*d-1/8/c^2*e^(1/2)*b*d^2/(-e^(1/2)*x+(e*x^2+d)^(1/2))^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} x^4}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (e x^2 + d)^{3/2}}{c x^4 + b x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)

[Out] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (d + ex^2)^{\frac{3}{2}}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)

$$3.272 \quad \int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=491

$$\frac{\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \left(-\frac{2ace + b^2(-e) + bcd}{\sqrt{b^2 - 4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right) + \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \left(\frac{2cd - e(b + \sqrt{b^2 - 4ac})}{\sqrt{b^2 - 4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{2c^2 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.80, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1293, 195, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \left(\frac{2ace + b^2(-e) + bcd}{\sqrt{b^2 - 4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right) + \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \left(\frac{2ace + b^2(-e) + bcd}{\sqrt{b^2 - 4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right) + \frac{\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) \left(-\frac{2ace + b^2(-e) + bcd}{\sqrt{b^2 - 4ac}} - be + cd \right) + \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) \left(\frac{2ace + b^2(-e) + bcd}{\sqrt{b^2 - 4ac}} - be + cd \right) + \frac{ex \sqrt{d + ex^2}}{2c} + \frac{d \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right)}{2c}}{2c^2 \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2])]/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2])]/(2*c^2) + (Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2])]/(2*c^2)

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 1293

Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1] && LeQ[m, 3]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (d + ex^2)^{3/2}}{a + bx^2 + cx^4} dx &= -\frac{\int \frac{\sqrt{d+ex^2} (ae-(cd-be)x^2)}{a+bx^2+cx^4} dx}{c} + \frac{e \int \sqrt{d+ex^2} dx}{c} \\
&= \frac{ex\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}})\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}})\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2} \right) dx}{c} + \frac{(de) \int \frac{1}{\sqrt{d+ex^2}} dx}{2c} \\
&= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{(de) \operatorname{Subst} \left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{2c} + \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2} dx}{c} \\
&= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{2c} + \frac{\left(e \left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{\sqrt{d+ex^2}} dx}{2c^2} + \frac{\left(2cd - \left(b - \sqrt{b^2-4ac} \right) e \left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \right) \int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2} dx}{2c^2} \\
&= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{2c} + \frac{\left(e \left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right) \right) \operatorname{Subst} \left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{2c^2} \\
&= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{\sqrt{2cd - \left(b - \sqrt{b^2-4ac} \right) e \left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right)} \tan^{-1} \left(\frac{\sqrt{2cd - \left(b - \sqrt{b^2-4ac} \right) e \left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}} \right)}}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{2c^2 \sqrt{b - \sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [B] time = 6.26, size = 14032, normalized size = 28.58

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 154.73, size = 692, normalized size = 1.41

RootSum[...]

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]

[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + ((-3*c*d*Sqrt[e] + 2*b*e^(3/2))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*c^2) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4

```
*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 +
4*b*e*#1^6 + c*#1^8 & , (c^2*d^4*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] -
2*b*c*d^3*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + b^2*d^2*e^2*Log[-(S
qrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c*d^2*e^2*Log[-(Sqrt[e]*x) + Sqrt[d +
e*x^2] - #1] - 2*c^2*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4
*b*c*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 2*b^2*d*e^2*Log[
-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 6*a*c*d*e^2*Log[-(Sqrt[e]*x) +
Sqrt[d + e*x^2] - #1]*#1^2 + 4*a*b*e^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] -
#1]*#1^2 + c^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - 2*b*c*d
*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 + b^2*e^2*Log[-(Sqrt[e]*x)
+ Sqrt[d + e*x^2] - #1]*#1^4 - a*c*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]
- #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 +
3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) & ])/(2*c^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.97, size = 58, normalized size = 0.12

$$\frac{\sqrt{x^2e + d}xe}{2c} - \frac{(3cde - 2be^2)e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/2*sqrt(x^2*e + d)*x*e/c - 1/4*(3*c*d*e - 2*b*e^2)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2

maple [C] time = 0.03, size = 382, normalized size = 0.78

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)

[Out] -1/4*e^(3/2)/c*x^2+1/4*e*x*(e*x^2+d)^(1/2)/c-1/8*e^(1/2)/c*d+1/2*e^(1/2)/c^2*sum(((a*c*e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^2+2*(-2*a*b*e^3+3*a*c*d*e^2+b

$$\frac{d^2 e^{-2} - 2 b c d^2 e + c^2 d^3}{R + a c d^2 e^{-2} - b^2 d^2 e^{-2} + 2 b c d^3 e - c^2 d^4} / (R^3 c + 3 R^2 b e - 3 R^2 c d + 8 R a e^{-2} - 4 R b d e + 3 R c d^2 + b d^2 e - c d^3) \ln(-R + (-e^{1/2} x + (e x^2 + d)^{1/2})^2), R = \text{RootOf}(Z^4 c + c d^4 + (4 b e - 4 c d) Z^3 + (16 a e^{-2} - 8 b d e + 6 c d^2) Z^2 + (4 b d^2 e - 4 c d^3) Z) + 1/8 e^{1/2} / c d^2 / (-e^{1/2} x + (e x^2 + d)^{1/2})^2 + e^{3/2} / c^2 \ln(-e^{1/2} x + (e x^2 + d)^{1/2}) * b - 3/2 e^{1/2} / c \ln(-e^{1/2} x + (e x^2 + d)^{1/2}) * d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e x^2 + d)^{\frac{3}{2}} x^2}{c x^4 + b x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (e x^2 + d)^{3/2}}{c x^4 + b x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)

[Out] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + e x^2)^{\frac{3}{2}}}{a + b x^2 + c x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)

$$3.273 \quad \int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=487

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)\left(-2ce\left(d\sqrt{b^2-4ac}\right)+be^2\left(b+\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}+c\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}$$

Rubi [A] time = 1.57, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 26, number of rules / integrand size = 0.269, Rules used = {1174, 416, 523, 217, 206, 377, 205}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b+\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b+\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b+\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x]

[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)(x_)^2], x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; } \text{FreeQ}\{a, b\}, x\} \&\& \text{!GtQ}[a, 0]$

Rule 377

$\text{Int}[(a_) + (b_.)(x_)^{(n_)}]^{(p_)} / ((c_) + (d_.)(x_)^{(n_)}), x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$

Rule 416

$\text{Int}[(a_) + (b_.)(x_)^{(n_)}]^{(p_)} * ((c_) + (d_.)(x_)^{(n_)})^{(q_)}, x_Symbol] \text{ :> } \text{Simp}[(d*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}) / (b*(n*(p+q) + 1)), x] + \text{Dist}[1/(b*(n*(p+q) + 1)), \text{Int}[(a + b*x^n)^p * (c + d*x^n)^{(q-2)} * \text{Simp}[c*(b*c*(n*(p+q) + 1) - a*d) + d*(b*c*(n*(p+2*q-1) + 1) - a*d*(n*(q-1) + 1)) * x^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, n, p\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[q, 1] \&\& \text{NeQ}[n*(p+q) + 1, 0] \&\& \text{!IGtQ}[p, 1] \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 523

$\text{Int}[(e_) + (f_.)(x_)^{(n_)}] / (((a_) + (b_.)(x_)^{(n_)})*\text{Sqrt}[(c_) + (d_.)(x_)^{(n_)}]), x_Symbol] \text{ :> } \text{Dist}[f/b, \text{Int}[1/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/((a + b*x^n)*\text{Sqrt}[c + d*x^n]), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 1174

$\text{Int}[(d_) + (e_.)(x_)^2]^{(q_)} / ((a_) + (b_.)(x_)^2 + (c_.)(x_)^4), x_Symbol] \text{ :> } \text{With}\{r = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/r, \text{Int}[(d + e*x^2)^q / (b - r + 2*c*x^2), x], x] - \text{Dist}[(2*c)/r, \text{Int}[(d + e*x^2)^q / (b + r + 2*c*x^2), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{!IntegerQ}[q]$

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4} dx &= \frac{(2c) \int \frac{(d+ex^2)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{(d+ex^2)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}} \\
&= \frac{\int \frac{d(4cd-(b-\sqrt{b^2-4ac})e)+2e(3cd-(b-\sqrt{b^2-4ac})e)x^2}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d(4cd-(b+\sqrt{b^2-4ac})e)+2e(3cd-(b+\sqrt{b^2-4ac})e)x^2}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{2\sqrt{b^2-4ac}} \\
&= \frac{\left(e(3cd-(b-\sqrt{b^2-4ac})e)\right) \int \frac{1}{\sqrt{d+ex^2}} dx}{2c\sqrt{b^2-4ac}} - \frac{\left(e(3cd-(b+\sqrt{b^2-4ac})e)\right) \int \frac{1}{\sqrt{d+ex^2}} dx}{2c\sqrt{b^2-4ac}} \\
&= \frac{\left(e(3cd-(b-\sqrt{b^2-4ac})e)\right) \text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} - \frac{\left(e(3cd-(b+\sqrt{b^2-4ac})e)\right) \text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} \\
&= \frac{\left(2c^2d^2 + b(b-\sqrt{b^2-4ac})e^2 - 2ce(bd - \sqrt{b^2-4ac}d + ae)\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [B] time = 6.16, size = 9290, normalized size = 19.08

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 121.93, size = 392, normalized size = 0.80

$$\frac{e^{3/2} \text{RootSum}\left[\#1^3 c + 4\#1^2 b e - 4\#1^2 c d + 16\#1^4 a^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 e, \frac{-\#1^3 e \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c}) + 2\#1^2 c d \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c}) - 4\#1^2 a e^2 \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c}) + 2\#1^2 b d e \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c}) - b d^2 e \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c}) + 2a d^3 \log(-\#1 + \sqrt{d+ex^2} - \sqrt{c})}{\#1^3 (-c) - 3\#1^2 b e + 3\#1^2 c d - 3\#1^2 a^2 + 4\#1^2 b d e - 3\#1^2 c d^2 - b d^2 e + c d^3}\right] e}{c} - \frac{e^{3/2} \log(\sqrt{d+ex^2} - \sqrt{c} x)}{c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x]

[Out] -((e^(3/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/c) + (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4

$$\begin{aligned}
&^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2* \\
&b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a \\
&^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*\sqrt{-(b*c^2*d^3 \\
&- 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4* \\
&a^2*c^3)*\sqrt{(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2 \\
&e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c \\
&^2 - 4*a^2*c^3))/x^2) - \sqrt{1/2}*c*\sqrt{-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a \\
&*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(c^4*d^6 \\
&- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 \\
&+ a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*\log((2 \\
&*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^ \\
&3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - \\
&(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*\sqrt{ \\
&(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2 \\
&*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c \\
&^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c \\
&^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)* \\
&x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^ \\
&3*c^2 - 4*a^3*b*c^3)*e)*x*\sqrt{(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e \\
&^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^ \\
&3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 \\
&+ (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*\sqrt{-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c* \\
&d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(c^4*d^6 - 6*a \\
&*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^ \\
&2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))/x^2) + \sqrt{ \\
&t(1/2)*c*\sqrt{-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2* \\
&c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^ \\
&2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 \\
&- 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*\log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e \\
&- 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a \\
&^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^ \\
&2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*\sqrt{(c^4*d^6 - 6*a*c^3*d^4*e^2 \\
&+ 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a \\
&^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (\\
&b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^ \\
&2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*\sqrt{1/2}*\sqrt{e*x^2 \\
&+ d}*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*\sqrt{ \\
&(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a \\
&^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^ \\
&2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4) \\
&)*x)*\sqrt{-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^ \\
&3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3 \\
&*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4* \\
&a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))/x^2) + 2*e^(3/2)*\log(-2*e*x^2 - 2*\sqrt{ \\
&(e*x^2 + d)*\sqrt{e}*x - d))/c, 1/4*(\sqrt{1/2})*c*\sqrt{-(b*c^2*d^3 - 6*a*c^2*}
\end{aligned}$$

$$\begin{aligned}
& d^2e + 3abcde^2 - (ab^2 - 2a^2c)e^3 - (ab^2c^2 - 4a^2c^3)\sqrt{t} \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) / (ab^2c^2 - 4a^2c^3) \\
& \log\left(\frac{2ac^3d^6 - 2abc^2d^5e - 4a^2c^2d^4e^2 + 8a^2b^2c^2d^3e^3 + 2a^3b^2d^2e^5 - 2(a^2b^2 + 3a^3c)d^2e^4 + ((ab^2c^3 - 4a^2c^4)d^3 - (ab^3c^2 - 4a^2b^2c^3)d^2e + (a^2b^2c^2 - 4a^3c^3)d^2e^2)}{x^2\sqrt{t}} \right) \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) - (bc^3d^6 + 2abc^2d^4e^2 - 4a^3b^2e^6 - (b^2c^2 + 4ac^3)d^5e + 4(ab^2c + 2a^2c^2)d^3e^3 - (ab^3 + 19a^2b^2c)d^2e^4 + (5a^2b^2 + 12a^3c)d^2e^5) \\
& x^2 + 2\sqrt{1/2}\sqrt{ex^2 + d} \left((2(a^2b^2c^3 - 4a^3c^4)d - (a^2b^3c^2 - 4a^3b^2c^3)e) x \sqrt{t} \right) \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) + ((ab^2c^2 - 4a^2c^3)d^3e - 3(a^2b^2c - 4a^3c^2)d^2e^3 + (a^2b^3 - 4a^3b^2c)e^4) x \sqrt{t} \\
& \sqrt{-(bc^2d^3 - 6ac^2d^2e + 3abc^2d^2e^2 - (ab^2 - 2a^2c)e^3 - (ab^2c^2 - 4a^2c^3)\sqrt{t}} \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) / (ab^2c^2 - 4a^2c^3) \\
& \sqrt{1/2} c \sqrt{-(bc^2d^3 - 6ac^2d^2e + 3abc^2d^2e^2 - (ab^2 - 2a^2c)e^3 - (ab^2c^2 - 4a^2c^3)\sqrt{t}} \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) / (ab^2c^2 - 4a^2c^3) \\
& \log\left(\frac{2ac^3d^6 - 2abc^2d^5e - 4a^2c^2d^4e^2 + 8a^2b^2c^2d^3e^3 + 2a^3b^2d^2e^5 - 2(a^2b^2 + 3a^3c)d^2e^4 + ((ab^2c^3 - 4a^2c^4)d^3 - (ab^3c^2 - 4a^2b^2c^3)d^2e + (a^2b^2c^2 - 4a^3c^3)d^2e^2)}{x^2\sqrt{t}} \right) \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) - (bc^3d^6 + 2abc^2d^4e^2 - 4a^3b^2e^6 - (b^2c^2 + 4ac^3)d^5e + 4(ab^2c + 2a^2c^2)d^3e^3 - (ab^3 + 19a^2b^2c)d^2e^4 + (5a^2b^2 + 12a^3c)d^2e^5) \\
& x^2 - 2\sqrt{1/2}\sqrt{ex^2 + d} \left((2(a^2b^2c^3 - 4a^3c^4)d - (a^2b^3c^2 - 4a^3b^2c^3)e) x \sqrt{t} \right) \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) + ((ab^2c^2 - 4a^2c^3)d^3e - 3(a^2b^2c - 4a^3c^2)d^2e^3 + (a^2b^3 - 4a^3b^2c)e^4) x \sqrt{t} \\
& \sqrt{-(bc^2d^3 - 6ac^2d^2e + 3abc^2d^2e^2 - (ab^2 - 2a^2c)e^3 - (ab^2c^2 - 4a^2c^3)\sqrt{t}} \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) / (ab^2c^2 - 4a^2c^3) \\
& \sqrt{1/2} c \sqrt{-(bc^2d^3 - 6ac^2d^2e + 3abc^2d^2e^2 - (ab^2 - 2a^2c)e^3 + (ab^2c^2 - 4a^2c^3)\sqrt{t}} \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right) / (ab^2c^2 - 4a^2c^3) \\
& \log\left(\frac{2ac^3d^6 - 2abc^2d^5e - 4a^2c^2d^4e^2 + 8a^2b^2c^2d^3e^3 + 2a^3b^2d^2e^5 - 2(a^2b^2 + 3a^3c)d^2e^4 - ((ab^2c^3 - 4a^2c^4)d^3 - (ab^3c^2 - 4a^2b^2c^3)d^2e + (a^2b^2c^2 - 4a^3c^3)d^2e^2)}{x^2\sqrt{t}} \right) \\
& \left(\frac{c^4d^6 - 6ac^3d^4e^2 + 2abc^2d^3e^3 + 9a^2c^2d^2e^4 - 6a^2b^2c^2d^2e^5 + a^2b^2e^6}{a^2b^2c^4 - 4a^3c^5} \right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)`

[Out] $\frac{1}{2}e^{3/2}/c \cdot \text{sum}(((b \cdot e^{-2} \cdot c \cdot d) \cdot R^2 + 2 \cdot e \cdot (2 \cdot a \cdot e^{-b} \cdot d) \cdot R + b \cdot d^2 \cdot e^{-2} \cdot c \cdot d^3) / (R^3 \cdot c + 3 \cdot R^2 \cdot b \cdot e^{-3} \cdot R^2 \cdot c \cdot d + 8 \cdot R \cdot a \cdot e^2 - 4 \cdot R \cdot b \cdot d \cdot e + 3 \cdot R \cdot c \cdot d^2 + b \cdot d^2 \cdot e - c \cdot d^3) \cdot \ln(-R + (-e^{1/2} \cdot x + (e \cdot x^2 + d)^{1/2}))^2), R = \text{RootOf}(_Z^4 \cdot c + c \cdot d^4 + (4 \cdot b \cdot e - 4 \cdot c \cdot d) \cdot _Z^3 + (16 \cdot a \cdot e^2 - 8 \cdot b \cdot d \cdot e + 6 \cdot c \cdot d^2) \cdot _Z^2 + (4 \cdot b \cdot d^2 \cdot e - 4 \cdot c \cdot d^3) \cdot _Z)) - e^{3/2} / c \cdot \ln(-e^{1/2} \cdot x + (e \cdot x^2 + d)^{1/2}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)`

[Out] `int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^{\frac{3}{2}}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)`

[Out] `Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)`

$$3.274 \quad \int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=260

$$\frac{\left(2cd - e\left(b - \sqrt{b^2 - 4ac}\right)\right)^{3/2} \tan^{-1}\left(\frac{x\sqrt{2cd - e\left(b - \sqrt{b^2 - 4ac}\right)}}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2 - 4ac}\left(b - \sqrt{b^2 - 4ac}\right)^{3/2}} + \frac{\left(2cd - e\left(\sqrt{b^2 - 4ac} + b\right)\right)^{3/2} \tan^{-1}\left(\frac{x\sqrt{2cd - e\left(\sqrt{b^2 - 4ac}\right)}}{\sqrt{\sqrt{b^2 - 4ac} + b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2 - 4ac}\left(\sqrt{b^2 - 4ac} + b\right)^{3/2}}$$

Rubi [A] time = 0.85, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, number of rules / integrand size = 0.276, Rules used = {1295, 277, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd - e\left(b - \sqrt{b^2 - 4ac}\right)}\left(\frac{bd - 2ae}{\sqrt{b^2 - 4ac}} + d\right) \tan^{-1}\left(\frac{x\sqrt{2cd - e\left(b - \sqrt{b^2 - 4ac}\right)}}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2cd - e\left(\sqrt{b^2 - 4ac} + b\right)}\left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{x\sqrt{2cd - e\left(\sqrt{b^2 - 4ac}\right)}}{\sqrt{\sqrt{b^2 - 4ac} + b}\sqrt{d+ex^2}}\right)}{2a\sqrt{\sqrt{b^2 - 4ac} + b}} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)\left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)}{2a} - \frac{\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)\left(\frac{bd - 2ae}{\sqrt{b^2 - 4ac}} + d\right)}{2a} - \frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e} \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 277

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^p / (c*(m+1)), x] - \text{Dist}[(b*n*p) / (c^n*(m+1)), \text{Int}[(c*x)^{(m+n)}*(a + b*x^n)^{(p-1)}, x], x] \text{ /; FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1] \&\& \text{!ILtQ}[(m + n*p + n + 1)/n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 377

$\text{Int}[((a_) + (b_)*(x_)^{(n_)})^{(p_)}/((c_) + (d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] \text{ /; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$

Rule 402

$\text{Int}[((a_) + (b_)*(x_)^2)^{(p_)} / ((c_) + (d_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[b/d, \text{Int}[(a + b*x^2)^{(p-1)}, x], x] - \text{Dist}[(b*c - a*d)/d, \text{Int}[(a + b*x^2)^{(p-1)} / (c + d*x^2), x], x] \text{ /; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{EqQ}[p, 1/2] \text{ || } \text{EqQ}[\text{Denominator}[p], 4])$

Rule 1295

$\text{Int}[(((f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^2)^{(q_)} / ((a_)+(b_)*(x_)^2+(c_)*(x_)^4), x_Symbol] \rightarrow \text{Dist}[d/a, \text{Int}[(f*x)^m*(d + e*x^2)^{(q-1)}, x], x] - \text{Dist}[1/(a*f^2), \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^{(q-1)}*\text{Simp}[b*d - a*e + c*d*x^2, x]] / (a + b*x^2 + c*x^4), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!IntegerQ}[q] \&\& \text{GtQ}[q, 0] \&\& \text{LtQ}[m, 0]$

Rule 1692

$\text{Int}[(P_x)*((d_)+(e_)*(x_)^2)^{(q_)}*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[P_x*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{PolyQ}[P_x, x^2] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)} dx &= -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{a+bx^2+cx^4} dx}{a} + \frac{d \int \frac{\sqrt{d+ex^2}}{x^2} dx}{a} \\
&= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{\left(cd + \frac{c(bd-2ae)}{\sqrt{b^2-4ac}} \right) \sqrt{d+ex^2}}{b - \sqrt{b^2-4ac} + 2cx^2} + \frac{\left(cd - \frac{c(bd-2ae)}{\sqrt{b^2-4ac}} \right) \sqrt{d+ex^2}}{b + \sqrt{b^2-4ac} + 2cx^2} \right) dx}{a} + \frac{(de) \int \frac{1}{\sqrt{d+ex^2}} dx}{a} \\
&= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{(de) \operatorname{Subst} \left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{a} - \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \int \frac{\sqrt{d+ex^2}}{b + \sqrt{b^2-4ac} + 2cx^2} dx}{a} \\
&= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{a} - \frac{\left(e \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{\sqrt{d+ex^2}} dx}{2a} - \frac{\left((2cd - (b - \sqrt{b^2-4ac})) \sqrt{d+ex^2} \right)}{2a} \\
&= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{a} - \frac{\left(e \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \operatorname{Subst} \left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}} \right)}{2a} \\
&= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})} e \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})} ex}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{2a\sqrt{b - \sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [B] time = 6.30, size = 7789, normalized size = 29.96

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 121.56, size = 423, normalized size = 1.63

$$\frac{\sqrt{c} \operatorname{RootSum} \left[\#1^3 c + 4\#1^2 b e - 4\#1^4 a c^2 + 16\#1^4 a c^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 c - 4\#1^2 c d^3 + c d^4 \&, \frac{-\#1^2 a^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) + \#1^2 a^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) - 4\#1^2 a b^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) + 4\#1^2 b d^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) - 2\#1^2 c d^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) + a b^2 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) + c d^4 \log(\#1 + \sqrt{d+ex^2} - \sqrt{c}) \right]}{2a} - \frac{d\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] $-\left(\frac{d \sqrt{d+ex^2}}{ax} \right) - \left(\frac{\sqrt{e} \operatorname{RootSum} [c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b$

$$\frac{e^{6x} + c^{8x} \left((c^4 d \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] - a d^2 e^2 \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] - 2 c d^3 \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] \#1^2 + 4 b d^2 e \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] \#1^2 - 6 a d e^2 \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] \#1^2 + c d^2 \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] \#1^4 - a e^2 \log[-(\sqrt{e}x) + \sqrt{d + e x^2}] - \#1] \#1^4 \right) / (c d^3 - b d^2 e - 3 c d^2 \#1^2 + 4 b d e \#1^2 - 8 a e^2 \#1^2 + 3 c d \#1^4 - 3 b e \#1^4 - c \#1^6) \&] / (2 a)$$

fricas [B] time = 37.77, size = 4059, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4 * (\text{sqrt}(1/2) * a * x * \text{sqrt}(- (3 * a^2 * b * d * e^2 - 2 * a^3 * e^3 + (b^3 - 3 * a * b * c) * d^3 \\ & - 3 * (a * b^2 - 2 * a^2 * c) * d^2 * e + (a^3 * b^2 - 4 * a^4 * c) * \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - \\ & 9 * a^4 * d^2 * e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5 * \\ & e - 3 * (5 * a^2 * b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - 4 * a^7 * c))) / (a^3 * b^2 - 4 * a^4 * \\ & c)) * \log(- (12 * a^3 * b * d^3 * e^3 - 6 * a^4 * d^2 * e^4 - 2 * (a * b^2 * c - a^2 * c^2) * d^6 + 2 * \\ & (a * b^3 + 2 * a^2 * b * c) * d^5 * e - 4 * (2 * a^2 * b^2 + a^3 * c) * d^4 * e^2 + ((a^3 * b^2 * c - \\ & 4 * a^4 * c^2) * d^3 - (a^3 * b^3 - 4 * a^4 * b * c) * d^2 * e + (a^4 * b^2 - 4 * a^5 * c) * d * e^2) * x \\ & ^2 * \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - 9 * a^4 * d^2 * e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 \\ & + 6 * (a * b^3 - a^2 * b * c) * d^5 * e - 3 * (5 * a^2 * b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - \\ & 4 * a^7 * c)) + (27 * a^3 * b * d^2 * e^4 - 12 * a^4 * d * e^5 + (b^3 * c - a * b * c^2) * d^6 - (b^4 \\ & + 6 * a * b^2 * c - 4 * a^2 * c^2) * d^5 * e + 2 * (4 * a * b^3 + 5 * a^2 * b * c) * d^4 * e^2 - 2 * (11 * \\ & a^2 * b^2 + 4 * a^3 * c) * d^3 * e^3) * x^2 + 2 * \text{sqrt}(1/2) * \text{sqrt}(e * x^2 + d) * ((a^4 * b^3 - \\ & 4 * a^5 * b * c) * d - 2 * (a^5 * b^2 - 4 * a^6 * c) * e) * x * \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - 9 * a^4 * d^2 * \\ & e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5 * e - 3 * (5 * \\ & a^2 * b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - 4 * a^7 * c)) - ((a * b^4 - 5 * a^2 * b^2 * c + \\ & 4 * a^3 * c^2) * d^4 - 3 * (a^2 * b^3 - 4 * a^3 * b * c) * d^3 * e + 3 * (a^3 * b^2 - 4 * a^4 * c) * d^2 \\ & * e^2) * x) * \text{sqrt}(- (3 * a^2 * b * d * e^2 - 2 * a^3 * e^3 + (b^3 - 3 * a * b * c) * d^3 - 3 * (a * b^2 \\ & - 2 * a^2 * c) * d^2 * e + (a^3 * b^2 - 4 * a^4 * c) * \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - 9 * a^4 * d^2 * \\ & e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5 * e - 3 * (5 * a^2 * \\ & b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - 4 * a^7 * c))) / (a^3 * b^2 - 4 * a^4 * c)) / x^2 \\ & - \text{sqrt}(1/2) * a * x * \text{sqrt}(- (3 * a^2 * b * d * e^2 - 2 * a^3 * e^3 + (b^3 - 3 * a * b * c) * d^3 - 3 * \\ & (a * b^2 - 2 * a^2 * c) * d^2 * e + (a^3 * b^2 - 4 * a^4 * c) * \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - 9 * a^4 * d^2 * \\ & e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5 * e - 3 * (5 * a^2 * \\ & b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - 4 * a^7 * c))) / (a^3 * b^2 - 4 * a^4 * c)) \\ & * \log(- (12 * a^3 * b * d^3 * e^3 - 6 * a^4 * d^2 * e^4 - 2 * (a * b^2 * c - a^2 * c^2) * d^6 + 2 * (a * \\ & b^3 + 2 * a^2 * b * c) * d^5 * e - 4 * (2 * a^2 * b^2 + a^3 * c) * d^4 * e^2 + ((a^3 * b^2 * c - 4 * a^4 * \\ & c^2) * d^3 - (a^3 * b^3 - 4 * a^4 * b * c) * d^2 * e + (a^4 * b^2 - 4 * a^5 * c) * d * e^2) * x^2 * \\ & \text{sqrt}(- (18 * a^3 * b * d^3 * e^3 - 9 * a^4 * d^2 * e^4 - (b^4 - 2 * a * b^2 * c + a^2 * c^2) * d^6 + \\ & 6 * (a * b^3 - a^2 * b * c) * d^5 * e - 3 * (5 * a^2 * b^2 - 2 * a^3 * c) * d^4 * e^2) / (a^6 * b^2 - 4 * a^7 * \\ & c)) + (27 * a^3 * b * d^2 * e^4 - 12 * a^4 * d * e^5 + (b^3 * c - a * b * c^2) * d^6 - (b^4 + \\ & 6 * a * b^2 * c - 4 * a^2 * c^2) * d^5 * e + 2 * (4 * a * b^3 + 5 * a^2 * b * c) * d^4 * e^2 - 2 * (11 * a^2 * \end{aligned}$$

$$\begin{aligned}
& b^2 + 4a^3c) * d^3e^3) * x^2 - 2 * \text{sqrt}(1/2) * \text{sqrt}(e * x^2 + d) * (((a^4b^3 - 4a^5b^3c) * d - 2 * (a^5b^2 - 4a^6c) * e) * x * \text{sqrt}(-(18a^3b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c))) - ((a * b^4 - 5a^2 * b^2c + 4a^3c^2) * d^4 - 3 * (a^2 * b^3 - 4a^3 * b * c) * d^3e + 3 * (a^3 * b^2 - 4a^4c) * d^2e^2) * x) * \text{sqrt}(-(3a^2 * b * d * e^2 - 2a^3e^3 + (b^3 - 3a * b * c) * d^3 - 3 * (a * b^2 - 2a^2c) * d^2e + (a^3 * b^2 - 4a^4c) * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c)))) / (a^3 * b^2 - 4a^4c))) / x^2) - \text{sqrt}(1/2) * a * x * \text{sqrt}(-(3a^2 * b * d * e^2 - 2a^3e^3 + (b^3 - 3a * b * c) * d^3 - 3 * (a * b^2 - 2a^2c) * d^2e - (a^3 * b^2 - 4a^4c) * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c)))) / (a^3 * b^2 - 4a^4c))) * \log(-(12a^3 * b * d^3e^3 - 6a^4d^2e^4 - 2 * (a * b^2c - a^2c^2) * d^6 + 2 * (a * b^3 + 2a^2 * b * c) * d^5e - 4 * (2a^2 * b^2 + a^3c) * d^4e^2 - ((a^3 * b^2c - 4a^4c^2) * d^3 - (a^3 * b^3 - 4a^4 * b * c) * d^2e + (a^4 * b^2 - 4a^5 * c) * d * e^2) * x^2 * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c))) + (27a^3 * b * d^2e^4 - 12a^4d * e^5 + (b^3c - a * b * c^2) * d^6 - (b^4 + 6a * b^2c - 4a^2c^2) * d^5e + 2 * (4a * b^3 + 5a^2 * b * c) * d^4e^2 - 2 * (11a^2 * b^2 + 4a^3c) * d^3e^3) * x^2 + 2 * \text{sqrt}(1/2) * \text{sqrt}(e * x^2 + d) * (((a^4b^3 - 4a^5b^3c) * d - 2 * (a^5b^2 - 4a^6c) * e) * x * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c))) + ((a * b^4 - 5a^2 * b^2c + 4a^3c^2) * d^4 - 3 * (a^2 * b^3 - 4a^3 * b * c) * d^3e + 3 * (a^3 * b^2 - 4a^4c) * d^2e^2) * x) * \text{sqrt}(-(3a^2 * b * d * e^2 - 2a^3e^3 + (b^3 - 3a * b * c) * d^3 - 3 * (a * b^2 - 2a^2c) * d^2e - (a^3 * b^2 - 4a^4c) * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c)))) / (a^3 * b^2 - 4a^4c))) / x^2) + \text{sqrt}(1/2) * a * x * \text{sqrt}(-(3a^2 * b * d * e^2 - 2a^3e^3 + (b^3 - 3a * b * c) * d^3 - 3 * (a * b^2 - 2a^2c) * d^2e - (a^3 * b^2 - 4a^4c) * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c)))) / (a^3 * b^2 - 4a^4c))) * \log(-(12a^3 * b * d^3e^3 - 6a^4d^2e^4 - 2 * (a * b^2c - a^2c^2) * d^6 + 2 * (a * b^3 + 2a^2 * b * c) * d^5e - 4 * (2a^2 * b^2 + a^3c) * d^4e^2 - ((a^3 * b^2c - 4a^4c^2) * d^3 - (a^3 * b^3 - 4a^4 * b * c) * d^2e + (a^4 * b^2 - 4a^5 * c) * d * e^2) * x^2 * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c))) + (27a^3 * b * d^2e^4 - 12a^4d * e^5 + (b^3c - a * b * c^2) * d^6 - (b^4 + 6a * b^2c - 4a^2c^2) * d^5e + 2 * (4a * b^3 + 5a^2 * b * c) * d^4e^2 - 2 * (11a^2 * b^2 + 4a^3c) * d^3e^3) * x^2 - 2 * \text{sqrt}(1/2) * \text{sqrt}(e * x^2 + d) * (((a^4b^3 - 4a^5b^3c) * d - 2 * (a^5b^2 - 4a^6c) * e) * x * \text{sqrt}(-(18a^3 * b * d^3e^3 - 9a^4d^2e^4 - (b^4 - 2a * b^2c + a^2c^2) * d^6 + 6 * (a * b^3 - a^2 * b * c) * d^5e - 3 * (5a^2 * b^2 - 2a^3c) * d^4e^2) / (a^6b^2 - 4a^7c))) + ((a * b^4 - 5a^2 * b^2c + 4a^3c^2) * d^4 - 3 * (a^2 * b^3 - 4a^3 * b * c) * d^3e + 3 * (a^3 * b^2 - 4a^4c) * d^2e^2) * x) * \text{sqrt}
\end{aligned}$$

$$t(-3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))/x^2) + 4*sqrt(e*x^2 + d)*d)/(a*x)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 360, normalized size = 1.38

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x)

[Out]
$$-1/a/d/x*(e*x^2+d)^{5/2}+1/a*e/d*x*(e*x^2+d)^{3/2}+5/4/a*e*x*(e*x^2+d)^{1/2}+3/2/a*e^{1/2}*d*\ln(e^{1/2}*x+(e*x^2+d)^{1/2})+1/4/a*e^{3/2}*x^2+1/8/a*e^{1/2}*d-1/2/a*e^{1/2}*sum(((a*e^2-c*d^2)*_R^2+2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R+a*d^2*e^2-c*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{1/2}*x+(e*x^2+d)^{1/2})^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/8/a*e^{1/2}*d^2/(-e^{1/2}*x+(e*x^2+d)^{1/2})^2+3/2/a*e^{1/2}*d*\ln(-e^{1/2}*x+(e*x^2+d)^{1/2})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2}}{x^2 (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)`

[Out] `int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^{\frac{3}{2}}}{x^2(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a), x)`

[Out] `Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)`

$$3.275 \quad \int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=523

$$\frac{\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)+\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)\left(-\frac{1}{2a^2}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Rubi [A] time = 2.62, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1295, 264, 6728, 277, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)+\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)\left(-\frac{1}{2a^2}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x]

[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(2*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 277

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_)*(x_)^2)^(p_)/((c_) + (d_)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 1295

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x], x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e + c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]

Rule 1692

Int[(Px_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -

$4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[p]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] \text{:> With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^(2*n)), x]\}, \text{Int}[v, x] \text{/; SumQ}[v]] \text{/; FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)} dx &= -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)} dx}{a} + \frac{d \int \frac{\sqrt{d+ex^2}}{x^4} dx}{a} \\
 &= -\frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2} + \frac{\sqrt{d+ex^2}(-b^2d+acd+abe-c(bd-ae)x^2)}{a(a+bx^2+cx^4)} \right) dx}{a} \\
 &= -\frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \frac{\sqrt{d+ex^2}(-b^2d+acd+abe-c(bd-ae)x^2)}{a+bx^2+cx^4} dx}{a^2} - \frac{(bd-ae) \int \frac{\sqrt{d+ex^2}}{x^2} dx}{a^2} \\
 &= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \left(\frac{\left(-c(bd-ae) - \frac{c(b^2d-2acd-abe)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-c(bd-ae) + \frac{c(b^2d-2acd-abe)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2} \right) dx}{a^2} \\
 &= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{(e(bd-ae)) \text{Subst}\left(\int \frac{1}{1-ex^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{c(b^2d-2acd-abe)\sqrt{d+ex^2}}{a^2\sqrt{b^2-4ac}} \\
 &= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e(bd-ae) - \frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{a^2} \\
 &= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e(bd-ae) - \frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{a^2} \\
 &= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})} e \left(bd - ae + \frac{b^2d - 2acd - abe}{\sqrt{b^2 - 4ac}}\right)\sqrt{d+ex^2}}{2a^2\sqrt{b - \sqrt{b^2 - 4ac}}}
 \end{aligned}$$

Mathematica [B] time = 6.41, size = 9321, normalized size = 17.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 179.82, size = 484, normalized size = 0.93

$$\frac{\sqrt{d+ex^2} \operatorname{RootSum}\left[4t^4+4bt^3e-4bt^2d+16b^2a^2-8bt^3de+6bt^4d^2+4bt^3be^2-4bt^2de^2+ce^2d\right]}{3a^2x^3} \cdot \frac{\sqrt{d+ex^2} \operatorname{RootSum}\left[4t^4+4bt^3e-4bt^2d+16b^2a^2-8bt^3de+6bt^4d^2+4bt^3be^2-4bt^2de^2+ce^2d\right]}{2a^2} \cdot \frac{\sqrt{d+ex^2} \operatorname{RootSum}\left[4t^4+4bt^3e-4bt^2d+16b^2a^2-8bt^3de+6bt^4d^2+4bt^3be^2-4bt^2de^2+ce^2d\right]}{4a^2 \sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - 4*a*e*x^2))/(3*a^2*x^3) - (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b*c*d^4*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*a*c*d^3*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b*c*d^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^2*d^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 8*a*b*d*e^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a^2*e^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - 2*a*c*d*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) & ])/(2*a^2)
```

fricas [B] time = 149.20, size = 7830, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*
```

$$\begin{aligned}
& a^3 b^3 c - 4 a^4 b^2 c^2) d^3 e^3 - 2(4 a^4 b^2 c - 3 a^5 c^2) d^2 e^4 + ((\\
& a^5 b^2 c^2 - 4 a^6 c^3) d^3 - (a^5 b^3 c - 4 a^6 b^2 c^2) d^2 e + (a^6 b^2 c \\
& - 4 a^7 c^2) d e^2) x^2 \sqrt{(a^6 b^2 e^6 + (b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 \\
& - 6 a^3 b^2 c^3 + a^4 c^4) d^6 - 6(a b^7 - 5 a^2 b^5 c + 7 a^3 b^3 c^2 \\
& - 2 a^4 b^2 c^3) d^5 e + 3(5 a^2 b^6 - 20 a^3 b^4 c + 20 a^4 b^2 c^2 - 2 a^5 \\
& 5 c^3) d^4 e^2 - 2(10 a^3 b^5 - 30 a^4 b^3 c + 19 a^5 b^2 c^2) d^3 e^3 + 3(\\
& 5 a^4 b^4 - 10 a^5 b^2 c + 3 a^6 c^2) d^2 e^4 - 6(a^5 b^3 - a^6 b^2 c) d e^5 \\
&) / (a^{10} b^2 - 4 a^{11} c) + (4 a^5 b^2 c e^6 + (b^5 c^2 - 3 a b^3 c^3 + a^2 b^2 c^4) \\
& d^6 - (b^6 c + 4 a b^4 c^2 - 17 a^2 b^2 c^3 + 4 a^3 c^4) d^5 e + 2(4 a \\
& a b^5 c - 3 a^2 b^3 c^2 - 11 a^3 b^2 c^3) d^4 e^2 - 2(11 a^2 b^4 c - 16 a^3 b^2 c^2 \\
& - 4 a^4 c^3) d^3 e^3 + 7(4 a^3 b^3 c - 5 a^4 b^2 c^2) d^2 e^4 - (17 a^4 b^2 c \\
& - 12 a^5 c^2) d e^5) x^2 + 2 \sqrt{1/2} \sqrt{e x^2 + d} ((a^6 b^4 \\
& - 6 a^7 b^2 c + 8 a^8 c^2) d - (a^7 b^3 - 4 a^8 b^2 c) e) x \sqrt{(a^6 b^2 e^6 \\
& + (b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) d^6 - 6(a \\
& a b^7 - 5 a^2 b^5 c + 7 a^3 b^3 c^2 - 2 a^4 b^2 c^3) d^5 e + 3(5 a^2 b^6 - 20 \\
& a^3 b^4 c + 20 a^4 b^2 c^2 - 2 a^5 c^3) d^4 e^2 - 2(10 a^3 b^5 - 30 a^4 b^3 c \\
& + 19 a^5 b^2 c^2) d^3 e^3 + 3(5 a^4 b^4 - 10 a^5 b^2 c + 3 a^6 c^2) d^2 \\
& e^4 - 6(a^5 b^3 - a^6 b^2 c) d e^5) / (a^{10} b^2 - 4 a^{11} c) - ((a b^7 - 7 a^2 \\
& b^5 c + 13 a^3 b^3 c^2 - 4 a^4 b^2 c^3) d^4 - (4 a^2 b^6 - 25 a^3 b^4 c + 3 \\
& 7 a^4 b^2 c^2 - 4 a^5 c^3) d^3 e + 3(2 a^3 b^5 - 11 a^4 b^3 c + 12 a^5 b^2 c^2) \\
& d^2 e^2 - (4 a^4 b^4 - 19 a^5 b^2 c + 12 a^6 c^2) d e^3 + (a^5 b^3 - 4 a^6 b^2 c) \\
& e^4) x) \sqrt{-(b^5 - 5 a b^3 c + 5 a^2 b^2 c^2) d^3 - 3(a b^4 - 4 a^2 b^2 c \\
& + 2 a^3 c^2) d^2 e + 3(a^2 b^3 - 3 a^3 b^2 c) d e^2 - (a^3 b^2 - 2 a^4 c) e^3 \\
& + (a^5 b^2 - 4 a^6 c) \sqrt{(a^6 b^2 e^6 + (b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 \\
& + a^4 c^4) d^6 - 6(a b^7 - 5 a^2 b^5 c + 7 a^3 b^3 c^2 - 2 a^4 b^2 c^3) d^5 e \\
& + 3(5 a^2 b^6 - 20 a^3 b^4 c + 20 a^4 b^2 c^2 - 2 a^5 c^3) d^4 e^2 - 2(10 a^3 b^5 - 30 a^4 b^3 c \\
& + 19 a^5 b^2 c^2) d^3 e^3 + 3(5 a^4 b^4 - 10 a^5 b^2 c + 3 a^6 c^2) d^2 e^4 - 6(a^5 b^3 - a^6 b^2 c) \\
& d e^5) / (a^{10} b^2 - 4 a^{11} c)) / (a^5 b^2 - 4 a^6 c)) / x^2) - 3 \sqrt{1/2} \\
& a^2 x^3 \sqrt{-(b^5 - 5 a b^3 c + 5 a^2 b^2 c^2) d^3 - 3(a b^4 - 4 a^2 b^2 c \\
& + 2 a^3 c^2) d^2 e + 3(a^2 b^3 - 3 a^3 b^2 c) d e^2 - (a^3 b^2 - 2 a^4 c) e^3 \\
& + (a^5 b^2 - 4 a^6 c) \sqrt{(a^6 b^2 e^6 + (b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 \\
& + a^4 c^4) d^6 - 6(a b^7 - 5 a^2 b^5 c + 7 a^3 b^3 c^2 - 2 a^4 b^2 c^3) d^5 e \\
& + 3(5 a^2 b^6 - 20 a^3 b^4 c + 20 a^4 b^2 c^2 - 2 a^5 c^3) d^4 e^2 - 2(10 a^3 b^5 - 30 a^4 b^3 c \\
& + 19 a^5 b^2 c^2) d^3 e^3 + 3(5 a^4 b^4 - 10 a^5 b^2 c + 3 a^6 c^2) d^2 e^4 - 6(a^5 b^3 - a^6 b^2 c) \\
& d e^5) / (a^{10} b^2 - 4 a^{11} c)) / (a^5 b^2 - 4 a^6 c)) * \log((2 a^5 b^2 c^2 d e^5 - 2(a \\
& b^4 c^2 - 3 a^2 b^2 c^3 + a^3 c^4) d^6 + 2(a b^5 c - 5 a^3 b^2 c^3) d^5 e - \\
& 4(2 a^2 b^4 c - 3 a^3 b^2 c^2 - a^4 c^3) d^4 e^2 + 4(3 a^3 b^3 c - 4 a^4 b^2 c^2) d^3 e^3 - \\
& 2(4 a^4 b^2 c - 3 a^5 c^2) d^2 e^4 + ((a^5 b^2 c^2 - 4 a^6 c^3) d^3 - (a^5 b^3 c - 4 a^6 b^2 c) d^2 e \\
& + (a^6 b^2 c - 4 a^7 c^2) d e^2) x^2 \sqrt{(a^6 b^2 e^6 + (b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 \\
& + a^4 c^4) d^6 - 6(a b^7 - 5 a^2 b^5 c + 7 a^3 b^3 c^2 - 2 a^4 b^2 c^3) d^5 e \\
& + 3(5 a^2 b^6 - 20 a^3 b^4 c + 20 a^4 b^2 c^2 - 2 a^5 c^3) d^4 e^2 - 2(10 a^3 b^5 - 30 a^4 b^3 c \\
& + 19 a^5 b^2 c^2) d^3 e^3 + 3(5 a^4 b^4 - 10 a^5 b^2 c + 3 a^6 c^2) d^2 e^4 - 6(a^5 b^3 - a^6 b^2 c) d e^5) / (a^{10} b^2 - 4 a^{11} c)
\end{aligned}$$

$$\begin{aligned}
& 5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c)))/((a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c)))/((a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a
\end{aligned}$$

$$\begin{aligned}
& ^7b^3 - 4a^8bc) * e) * x * \text{sqrt}((a^6b^2e^6 + (b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) * d^6 - 6(a^2b^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3) * d^5e + 3(5a^2b^6 - 20a^3b^4c + 20a^4b^2c^2 - 2a^5c^3) * d^4e^2 - 2(10a^3b^5 - 30a^4b^3c + 19a^5b^2c^2) * d^3e^3 + 3(5a^4b^4 - 10a^5b^2c + 3a^6c^2) * d^2e^4 - 6(a^5b^3 - a^6bc) * de^5) / (a^10b^2 - 4a^11c)) + ((a^2b^7 - 7a^2b^5c + 13a^3b^3c^2 - 4a^4b^2c^3) * d^4 - (4a^2b^6 - 25a^3b^4c + 37a^4b^2c^2 - 4a^5c^3) * d^3e + 3(2a^3b^5 - 11a^4b^3c + 12a^5b^2c^2) * d^2e^2 - (4a^4b^4 - 19a^5b^2c + 12a^6c^2) * de^3 + (a^5b^3 - 4a^6bc) * e^4) * x) * \text{sqrt}(-((b^5 - 5a^2b^3c + 5a^2b^2c^2) * d^3 - 3(a^2b^4 - 4a^2b^2c + 2a^3c^2) * d^2e + 3(a^2b^3 - 3a^3bc) * de^2 - (a^3b^2 - 2a^4c) * e^3 - (a^5b^2 - 4a^6c) * \text{sqrt}((a^6b^2e^6 + (b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) * d^6 - 6(a^2b^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3) * d^5e + 3(5a^2b^6 - 20a^3b^4c + 20a^4b^2c^2 - 2a^5c^3) * d^4e^2 - 2(10a^3b^5 - 30a^4b^3c + 19a^5b^2c^2) * d^3e^3 + 3(5a^4b^4 - 10a^5b^2c + 3a^6c^2) * d^2e^4 - 6(a^5b^3 - a^6bc) * de^5) / (a^10b^2 - 4a^11c))) / (a^5b^2 - 4a^6c))) / x^2) + 3 * \text{sqrt}(1/2) * a^2 * x^3 * \text{sqrt}(-((b^5 - 5a^2b^3c + 5a^2b^2c^2) * d^3 - 3(a^2b^4 - 4a^2b^2c + 2a^3c^2) * d^2e + 3(a^2b^3 - 3a^3bc) * de^2 - (a^3b^2 - 2a^4c) * e^3 - (a^5b^2 - 4a^6c) * \text{sqrt}((a^6b^2e^6 + (b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) * d^6 - 6(a^2b^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3) * d^5e + 3(5a^2b^6 - 20a^3b^4c + 20a^4b^2c^2 - 2a^5c^3) * d^4e^2 - 2(10a^3b^5 - 30a^4b^3c + 19a^5b^2c^2) * d^3e^3 + 3(5a^4b^4 - 10a^5b^2c + 3a^6c^2) * d^2e^4 - 6(a^5b^3 - a^6bc) * de^5) / (a^10b^2 - 4a^11c))) / (a^5b^2 - 4a^6c))) * \log((2a^5bc * de^5 - 2(a^2b^4c^2 - 3a^2b^2c^3 + a^3c^4) * d^6 + 2(a^2b^5c - 5a^3bc^3) * d^5e - 4(2a^2b^4c - 3a^3b^2c^2 - a^4c^3) * d^4e^2 + 4(3a^3b^3c - 4a^4bc^2) * d^3e^3 - 2(4a^4b^2c - 3a^5c^2) * d^2e^4 - ((a^5b^2c^2 - 4a^6c^3) * d^3 - (a^5b^3c - 4a^6bc^2) * d^2e + (a^6b^2c - 4a^7c^2) * de^2) * x^2 * \text{sqrt}((a^6b^2e^6 + (b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) * d^6 - 6(a^2b^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3) * d^5e + 3(5a^2b^6 - 20a^3b^4c + 20a^4b^2c^2 - 2a^5c^3) * d^4e^2 - 2(10a^3b^5 - 30a^4b^3c + 19a^5b^2c^2) * d^3e^3 + 3(5a^4b^4 - 10a^5b^2c + 3a^6c^2) * d^2e^4 - 6(a^5b^3 - a^6bc) * de^5) / (a^10b^2 - 4a^11c))) + (4a^5bc * e^6 + (b^5c^2 - 3a^2b^3c^3 + a^2b^2c^4) * d^6 - (b^6c + 4a^2b^4c^2 - 17a^2b^2c^3 + 4a^3c^4) * d^5e + 2(4a^2b^5c - 3a^2b^3c^2 - 11a^3bc^3) * d^4e^2 - 2(11a^2b^4c - 16a^3b^2c^2 - 4a^4c^3) * d^3e^3 + 7(4a^3b^3c - 5a^4bc^2) * d^2e^4 - (17a^4b^2c - 12a^5c^2) * de^5) * x^2 - 2 * \text{sqrt}(1/2) * \text{sqrt}(e * x^2 + d) * (((a^6b^4 - 6a^7b^2c + 8a^8c^2) * d - (a^7b^3 - 4a^8bc) * e) * x * \text{sqrt}((a^6b^2e^6 + (b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) * d^6 - 6(a^2b^7 - 5a^2b^5c + 7a^3b^3c^2 - 2a^4b^2c^3) * d^5e + 3(5a^2b^6 - 20a^3b^4c + 20a^4b^2c^2 - 2a^5c^3) * d^4e^2 - 2(10a^3b^5 - 30a^4b^3c + 19a^5b^2c^2) * d^3e^3 + 3(5a^4b^4 - 10a^5b^2c + 3a^6c^2) * d^2e^4 - 6(a^5b^3 - a^6bc) * de^5) / (a^10b^2 - 4a^11c))) + ((a^2b^7 - 7a^2b^5c + 13a^3b^3c^2 - 4a^4b^2c^3) * d^4 - (4a^
\end{aligned}$$

$2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*\sqrt{-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*\sqrt{(a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^{10}*b^2 - 4*a^{11}*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d - 4*a*e)*x^2 - a*d)*\sqrt{e*x^2 + d})/(a^2*x^3)$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 511, normalized size = 0.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x)

[Out] $1/a^2*b/d/x*(e*x^2+d)^{(5/2)} - 1/a^2*b*e/d*x*(e*x^2+d)^{(3/2)} - 5/4/a^2*b*e*x*(e*x^2+d)^{(1/2)} - 3/2/a^2*b*e^{(1/2)}*d*\ln(e^{(1/2)}*x+(e*x^2+d)^{(1/2)}) - 1/4/a^2*e^{(3/2)}*x^2*b - 1/8/a^2*e^{(1/2)}*b*d + 1/2/a^2*e^{(1/2)}*\sum((c*d*(2*a*e-b*d)*_R^2+2*(-2*a^2*e^3+4*a*b*d*e^2-2*b^2*d^2*e+b*c*d^3)*_R+2*a*c*d^3*e-b*c*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/a*e^{(3/2)}*\ln(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)}) - 3/2/a^2*e^{(1/2)}*\ln(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})*b*d + 1/8/a^2*e^{(1/2)}*b*d^2/(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2 - 1/3/a/d/x^3*(e*x^2+d)^{(5/2)} - 2/3/a*e/d^2/x*(e*x^2+d)^{(5/2)} + 2/3/a*e^2/d^2*x*(e*x^2+d)^{(3/2)} + 1/a*e^2/d*x*(e*x^2+d)^{(1/2)} + 1/a*e^{(3/2)}*\ln(e^{(1/2)}*x+(e*x^2+d)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2}}{x^4 (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)

[Out] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.276 \quad \int \frac{x^5 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=281

$$\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right) + \left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c} + \sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Rubi [A] time = 7.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right) + \left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right) - \frac{b\sqrt{1-x^2}}{c^2} - \frac{(1-x^2)^{3/2}}{3c}}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c} + \sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] -((b*sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/sqrt[b^2 - 4*a*c])*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[1 - x^2])/sqrt[b + 2*c - sqrt[b^2 - 4*a*c]])/(sqrt[2]*c^(5/2)*sqrt[b + 2*c - sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/sqrt[b^2 - 4*a*c])*ArcTanh[(sqrt[2]*sqrt[c]*sqrt[1 - x^2])/sqrt[b + 2*c + sqrt[b^2 - 4*a*c]])/(sqrt[2]*c^(5/2)*sqrt[b + 2*c + sqrt[b^2 - 4*a*c]])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :=> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :=> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \sqrt{1-x^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-x} x^2}{a+bx+cx^2} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{x^2 (1-x^2)^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right) \\
&= -\text{Subst} \left(\int \left(\frac{b}{c^2} + \frac{x^2}{c} - \frac{b(a+b+c) - (b^2-ac+bc)x^2}{c^2(a+b+c+(-b-2c)x^2+cx^4)} \right) dx, x, \sqrt{1-x^2} \right) \\
&= -\frac{b\sqrt{1-x^2}}{c^2} - \frac{(1-x^2)^{3/2}}{3c} + \frac{\text{Subst} \left(\int \frac{b(a+b+c)+(-b^2+ac-bc)x^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right)}{c^2} \\
&= -\frac{b\sqrt{1-x^2}}{c^2} - \frac{(1-x^2)^{3/2}}{3c} - \frac{\left(b^2-ac+bc - \frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx} \right)}{2c^2} \\
&= -\frac{b\sqrt{1-x^2}}{c^2} - \frac{(1-x^2)^{3/2}}{3c} + \frac{\left(b^2-ac+bc - \frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.54, size = 354, normalized size = 1.26

$$\frac{3\sqrt{2}\left(b^2(\sqrt{b^2-4ac}+c)+bc(\sqrt{b^2-4ac}-3a)-ac(\sqrt{b^2-4ac}+2c)+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)-3\sqrt{2}\left(b^2(\sqrt{b^2-4ac}-c)+bc(\sqrt{b^2-4ac}+3a)+ac(2c-\sqrt{b^2-4ac})-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)-6b\sqrt{c}\sqrt{1-x^2}-2c^{3/2}(1-x^2)^{3/2}}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}{6c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*Sqrt[1-x^2])/(a+b*x^2+c*x^4),x]

[Out] (-6*b*Sqrt[c]*Sqrt[1-x^2]-2*c^(3/2)*(1-x^2)^(3/2)-(3*Sqrt[2]*(b^3+b*c*(-3*a+Sqrt[b^2-4*a*c]))+b^2*(c+Sqrt[b^2-4*a*c])-a*c*(2*c+Sqrt[b^2-4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1-x^2])/Sqrt[-b-2*c-Sqrt[b^2-4*a*c]])/(Sqrt[b^2-4*a*c]*Sqrt[-b-2*c-Sqrt[b^2-4*a*c]])-(3*Sqrt[2]*(-b^3+a*c*(2*c-Sqrt[b^2-4*a*c]))+b*c*(3*a+Sqrt[b^2-4*a*c])+b^2*(-c+Sqrt[b^2-4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1-x^2])/Sqrt[-b-2*c+Sqrt[b^2-4*a*c]])/(Sqrt[b^2-4*a*c]*Sqrt[-b-2*c+Sqrt[b^2-4*a*c]])/(6*c^(5/2))

IntegrateAlgebraic [A] time = 0.98, size = 425, normalized size = 1.51

$$\frac{(-\sqrt{2}b^2\sqrt{b^2-4ac}-\sqrt{2}bc\sqrt{b^2-4ac}+\sqrt{2}ac\sqrt{b^2-4ac}+3\sqrt{2}abc+2\sqrt{2}ac^2-\sqrt{2}b^3-\sqrt{2}b^2c)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)+(-\sqrt{2}b^2\sqrt{b^2-4ac}-\sqrt{2}bc\sqrt{b^2-4ac}+\sqrt{2}ac\sqrt{b^2-4ac}-3\sqrt{2}abc-2\sqrt{2}ac^2+\sqrt{2}b^3+\sqrt{2}b^2c)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)+\sqrt{1-x^2}(-3b+cx^2-c)}{2c^{5/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{2c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[1 - x^2]*(-3*b - c + c*x^2))/(3*c^2) + ((-(Sqrt[2]*b^3) + 3*Sqrt[2]*a
*b*c - Sqrt[2]*b^2*c + 2*Sqrt[2]*a*c^2 - Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c] + Sq
rt[2]*a*c*Sqrt[b^2 - 4*a*c] - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]])/(2*c^(5/2)*Sq
rt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + ((Sqrt[2]*b^3 - 3*Sqr
t[2]*a*b*c + Sqrt[2]*b^2*c - 2*Sqrt[2]*a*c^2 - Sqrt[2]*b^2*Sqrt[b^2 - 4*a*c
] + Sqrt[2]*a*c*Sqrt[b^2 - 4*a*c] - Sqrt[2]*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])/(2*c^(5
/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 16.82, size = 3615, normalized size = 12.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sq
rt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
```


$$\begin{aligned}
&) * c^2 + (a^2 * b^5 + (a^4 * b - 2 * a^3 * b^2) * c^2 - (3 * a^3 * b^3 - a^2 * b^4) * c) * x^2 - \\
& 2 * (3 * a^4 * b^2 - a^3 * b^3) * c - \sqrt{1/2} * ((b^5 * c^5 - 7 * a * b^3 * c^6 + 12 * a^2 * b * c^7) * x^2 * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11})) + (b^8 + 4 * (a^4 - 2 * a^3 * b) * c^4 - (17 * a^3 * b^2 - 14 * a^2 * b^3) * c^3 + (20 * a^2 * b^4 - 7 * a * b^5) * c^2 - (8 * a * b^6 - b^7) * c) * x^2) * \sqrt{((b^5 + 2 * a^2 * c^3 + (5 * a^2 * b - 4 * a * b^2) * c^2 - (5 * a * b^3 - b^4) * c - (b^2 * c^5 - 4 * a * c^6) * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11}))} / (b^2 * c^5 - 4 * a * c^6)) - 2 * (a^3 * b^4 + (a^5 - 2 * a^4 * b) * c^2 - (3 * a^4 * b^2 - a^3 * b^3) * c) * \sqrt{-x^2 + 1} / x^2) - 3 * \sqrt{1/2} * c^2 * \sqrt{(b^5 + 2 * a^2 * c^3 + (5 * a^2 * b - 4 * a * b^2) * c^2 - (5 * a * b^3 - b^4) * c + (b^2 * c^5 - 4 * a * c^6) * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11}))} / (b^2 * c^5 - 4 * a * c^6)) * \log(-(2 * a^3 * b^4 - (a^2 * b^2 * c^5 - 4 * a^3 * c^6) * x^2 * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11})) + 2 * (a^5 - 2 * a^4 * b) * c^2 + (a^2 * b^5 + (a^4 * b - 2 * a^3 * b^2) * c^2 - (3 * a^3 * b^3 - a^2 * b^4) * c) * x^2 - 2 * (3 * a^4 * b^2 - a^3 * b^3) * c + \sqrt{1/2} * ((b^5 * c^5 - 7 * a * b^3 * c^6 + 12 * a^2 * b * c^7) * x^2 * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11})) - (b^8 + 4 * (a^4 - 2 * a^3 * b) * c^4 - (17 * a^3 * b^2 - 14 * a^2 * b^3) * c^3 + (20 * a^2 * b^4 - 7 * a * b^5) * c^2 - (8 * a * b^6 - b^7) * c) * x^2) * \sqrt{(b^5 + 2 * a^2 * c^3 + (5 * a^2 * b - 4 * a * b^2) * c^2 - (5 * a * b^3 - b^4) * c + (b^2 * c^5 - 4 * a * c^6) * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11}))} / (b^2 * c^5 - 4 * a * c^6)) - 2 * (a^3 * b^4 + (a^5 - 2 * a^4 * b) * c^2 - (3 * a^4 * b^2 - a^3 * b^3) * c) * \sqrt{-x^2 + 1} / x^2) + 3 * \sqrt{1/2} * c^2 * \sqrt{(b^5 + 2 * a^2 * c^3 + (5 * a^2 * b - 4 * a * b^2) * c^2 - (5 * a * b^3 - b^4) * c + (b^2 * c^5 - 4 * a * c^6) * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11}))} / (b^2 * c^5 - 4 * a * c^6)) * \log(-(2 * a^3 * b^4 - (a^2 * b^2 * c^5 - 4 * a^3 * c^6) * x^2 * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11})) + 2 * (a^5 - 2 * a^4 * b) * c^2 + (a^2 * b^5 + (a^4 * b - 2 * a^3 * b^2) * c^2 - (3 * a^3 * b^3 - a^2 * b^4) * c) * x^2 - 2 * (3 * a^4 * b^2 - a^3 * b^3) * c - \sqrt{1/2} * ((b^5 * c^5 - 7 * a * b^3 * c^6 + 12 * a^2 * b * c^7) * x^2 * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11})) - (b^8 + 4 * (a^4 - 2 * a^3 * b) * c^4 - (17 * a^3 * b^2 - 14 * a^2 * b^3) * c^3 + (20 * a^2 * b^4 - 7 * a * b^5) * c^2 - (8 * a * b^6 - b^7) * c) * x^2) * \sqrt{(b^5 + 2 * a^2 * c^3 + (5 * a^2 * b - 4 * a * b^2) * c^2 - (5 * a * b^3 - b^4) * c + (b^2 * c^5 - 4 * a * c^6) * \sqrt{(b^8 + (a^4 - 4 * a^3 * b + 4 * a^2 * b^2) * c^4 - 2 * (3 * a^3 * b^2 - 7 * a^2 * b^3 + 2 * a * b^4) * c^3 + (11 * a^2 * b^4 - 10 * a * b^5 + b^6) * c^2 - 2 * (3 * a * b^6 - b^7) * c}) / (b^2 * c^{10} - 4 * a * c^{11}))} / (b^2 * c^5 - 4 * a * c^6))
\end{aligned}$$

11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 2*(c*x^2 - 3*b - c)*sqrt(-x^2 + 1))/c^2

giac [B] time = 4.48, size = 4637, normalized size = 16.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 + 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c))*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c

$$\begin{aligned}
& - 2c^2 - \sqrt{b^2 - 4ac}c) * a * b^5c^2 + \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c} \\
& (b^2 - 4ac)c) * b^6c^2 - 8 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c} \\
& c) * a^2b^3c^3 - 6 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a * b^4c \\
& ^3 + 3 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * b^5c^3 + 2 * a * b^5c \\
& ^3 + 2 * b^6c^3 + 16 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a^3b \\
& c^4 + 8 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a^2b^2c^4 - 11 * s \\
& \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a * b^3c^4 - 16 * a^2b^3c^4 \\
& + 7 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * b^4c^4 - 16 * a * b^4c^4 \\
& + 2 * b^5c^4 - 4 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a^2b * c^5 \\
& + 32 * a^3b * c^5 - 28 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * a * b^2 \\
& * c^5 + 32 * a^2b^2c^5 + 5 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) * \\
& b^3c^5 - 16 * a * b^3c^5 - 20 * \sqrt{2} * \sqrt{-bc - 2c^2 - \sqrt{b^2 - 4ac}c) \\
&) * a * b * c^6 + 32 * a^2b * c^6 - 2 * (b^2 - 4ac) * a * b^3c^3 - 2 * (b^2 - 4ac) * b^4 \\
& c^3 + 8 * (b^2 - 4ac) * a^2b * c^4 + 8 * (b^2 - 4ac) * a * b^2c^4 - 2 * (b^2 - 4ac) \\
& c) * b^3c^4 + 8 * (b^2 - 4ac) * a * b * c^5) * \text{abs}(c) * \arctan(2 * \sqrt{1/2} * \sqrt{-x^2 \\
& + 1} / \sqrt{-(b^3c^3 + 2c^4 + \sqrt{-4 * (ac^3 + b^3c^3 + c^4) * c^4 + (b^3c^3 + 2 \\
& c^4)^2}) / c^4}) / ((a * b^4c^4 + b^5c^4 - 8 * a^2b^2c^5 - 6 * a * b^3c^5 + 3 * b^4 \\
& c^5 + 16 * a^3c^6 + 8 * a^2b * c^6 - 11 * a * b^2c^6 + 7 * b^3c^6 - 4 * a^2c^7 - 28 * \\
& a * b * c^7 + 5 * b^2c^7 - 20 * ac^8) * c^2) + 1/8 * (2 * b^6c^4 - 14 * a * b^4c^5 + 6 * b^ \\
& 5c^5 + 24 * a^2b^2c^6 - 40 * a * b^3c^6 + 4 * b^4c^6 + 64 * a^2b * c^7 - 24 * a * b^2 \\
& * c^7 + 32 * a^2c^8 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 \\
& - 4ac}c) * b^6c^2 + 7 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 \\
& - 4ac}c) * a * b^4c^3 - 5 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \\
& \sqrt{b^2 - 4ac}c) * b^5c^3 - 12 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2 \\
& c^2 + \sqrt{b^2 - 4ac}c) * a^2b^2c^4 + 26 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{ \\
& -bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a * b^3c^4 - 13 * \sqrt{2} * \sqrt{b^2 - 4ac} \\
&) * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * b^4c^4 - 32 * \sqrt{2} * \sqrt{b^2 - \\
& 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a^2b * c^5 + 43 * \sqrt{2} * \sqrt{ \\
& b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a * b^2c^5 - 19 * \sqrt{2} * \sqrt{ \\
& b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * b^3c^5 - 16 * \\
& \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a^2c^6 \\
& + 48 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a * b \\
& * c^6 - 10 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) \\
&) * b^2c^6 + 20 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac} \\
& c) * ac^7 - 2 * (b^2 - 4ac) * b^4c^4 + 6 * (b^2 - 4ac) * a * b^2c^5 - 6 * (b^2 \\
& - 4ac) * b^3c^5 + 16 * (b^2 - 4ac) * a * b * c^6 - 4 * (b^2 - 4ac) * b^2c^6 + 8 * \\
& (b^2 - 4ac) * ac^7 - (2 * b^6c^2 - 18 * a * b^4c^3 + 2 * b^5c^3 + 48 * a^2b^2c^ \\
& 4 - 16 * a * b^3c^4 - 32 * a^3c^5 + 32 * a^2b * c^5 - \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{ \\
& -bc - 2c^2 + \sqrt{b^2 - 4ac}c) * b^6 + 9 * \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{ \\
& -bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a * b^4c - 3 * \sqrt{2} * \sqrt{b^2 - 4ac} \\
&) * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * b^5c - 24 * \sqrt{2} * \sqrt{b^2 - 4 \\
& ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a^2b^2c^2 + 18 * \sqrt{2} * \sqrt{ \\
& b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a * b^3c^2 - 7 * \sqrt{2} \\
&) * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * b^4c^2 + 16 * s \\
& \sqrt{2} * \sqrt{b^2 - 4ac} * \sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}c) * a^3c^3 -
\end{aligned}$$

$$\begin{aligned}
& 24\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a^2* \\
& b^3c^3 + 33\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)a*b^2c^3 - 5\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)b^3c^3 - 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)a^2c^4 + 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)a*b^4c^4 - 2*(b^2 - 4ac)b^4c^2 + 10*(b^2 - 4ac)a*b^2c^3 - \\
& 2*(b^2 - 4ac)b^3c^3 - 8*(b^2 - 4ac)a^2c^4 + 8*(b^2 - 4ac) \\
&)a*b^4c^4)c^2 + 2*(\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a*b^5c^2 \\
& + \sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)b^6c^2 - 8\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)a^2b^3c^3 - 6\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a*b^4c^3 + 3\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)b^5c^3 - 2a*b^5c^3 - 2b^6c^3 + 16\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)a^3b^4c^4 + 8\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a^2b^2c^4 - \\
& 11\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a*b^3c^4 + 16a^2b^3c^4 + 7\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)b^4c^4 + 16a*b^4c^4 - 2b^5c^4 - 4\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a^2b^5c^5 - \\
& 32a^3b^5c^5 - 28\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a*b^2c^5 - 32a^2b^2c^5 + 5\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}* \\
& c)b^3c^5 + 16a*b^3c^5 - 20\sqrt{2}\sqrt{-bc - 2c^2 + \sqrt{b^2 - 4ac}}c)a*b^4c^6 - \\
& 32a^2b^4c^6 + 2*(b^2 - 4ac)a*b^3c^3 + 2*(b^2 - 4ac)b^4c^3 - 8*(b^2 - 4ac)a^2b^4c^4 - \\
& 8*(b^2 - 4ac)a*b^2c^4 + 2*(b^2 - 4ac)b^3c^4 - 8*(b^2 - 4ac)a*b^5c^5) \\
& * \arctan(2\sqrt{1/2}\sqrt{-x^2 + 1}/\sqrt{-(b^3c^3 + 2c^4 - \sqrt{-4a^3c^3 + b^3c^3 + c^4})c^4 + (b^3c^3 + 2c^4)^2})/c^4)/((a^4b^4c^4 + b^5c^4 - 8a^2b^2c^5 - 6a^3b^3c^5 + 3b^4c^5 + 16a^3c^6 + 8a^2b^4c^6 - 11a^2b^2c^6 + 7b^3c^6 - 4a^2c^7 - 28a^3b^7 + 5b^2c^7 - 20a^4c^8)c^2) - 1/3*((-x^2 + 1)^{3/2})c^2 + 3\sqrt{-x^2 + 1}b^3c)/c^3
\end{aligned}$$

maple [B] time = 0.10, size = 2134, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5(-x^2+1)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $\begin{aligned}
& -1/3*(-x^2+1)^{(3/2)}/c+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)} \\
&)a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2 \\
& a+2*(-4*a*c+b^2)^{(1/2)}-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2 \\
& *b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}-2/c^2*a/(8*a*c-2*b^2) \\
&)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*a \\
& rctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}-2*a-2*b)/(4*a \\
& *c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*b^2*(- \\
& 4*a*c+b^2)^{(1/2)}-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2 \\
& *b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2*a \\
& +2*(-4*a*c+b^2)^{(1/2)}-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4
\end{aligned}$

$$\begin{aligned}
& a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*b+8/c*a^2/(8*a*c-2*b^2)/(4* \\
& a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan \\
& (1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}-2*a-2*b)/(4*a*c-2* \\
& b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*b+8*a^2/(8* \\
& a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b \\
&)^{(1/2)}*\arctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}-2*a- \\
& 2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)} \\
&)-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^ \\
& 2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b \\
& ^2)^{(1/2)}-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b*(-4*a*c+b^2)^{(1/ \\
& 2)}-2*a*b)^{(1/2)})*b^3-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}* \\
& a+2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(-2*((-x^2+1)^{(1/2)}-1)^2/x \\
& ^2*a+2*(-4*a*c+b^2)^{(1/2)}-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*b* \\
& (-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*b^2+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(\\
& -4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^ \\
& 2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+ \\
& b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}-2/c^2*a \\
& / (8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}- \\
& 2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}+ \\
& 2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^ \\
& (1/2))*b^2*(-4*a*c+b^2)^{(1/2)}-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^ \\
& 2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^2+1)^{(1/2)} \\
&)-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2) \\
&)*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*b-8/c*a^2/(8*a* \\
& c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^ \\
& (1/2)*\arctan(1/2*(2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}+2*a+2*b \\
&)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})* \\
& b-8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^ \\
& (1/2)-2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2*(-4*a*c+b^2)^ \\
& (1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2 \\
& *a*b)^{(1/2)}+2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b* \\
& (-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^2+1)^{(1/2)}-1)^2/x^2*a+2* \\
& (-4*a*c+b^2)^{(1/2)}+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*b*(-4*a*c \\
& +b^2)^{(1/2)}-2*a*b)^{(1/2)})*b^3+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^ \\
& 2)^{(1/2)}*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)}*\arctan(1/2*(2*((-x^2+1)^{(1/2)} \\
&)-1)^2/x^2*a+2*(-4*a*c+b^2)^{(1/2)}+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2) \\
&)*a-2*b*(-4*a*c+b^2)^{(1/2)}-2*a*b)^{(1/2)})*b^2-2/c^2*b/(2/x^2-2/x^2*(-x^2+1)^ \\
& (1/2))
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1} x^5}{cx^4+bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.45, size = 917, normalized size = 3.26



Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] $(1 - x^2)^{1/2} * (2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (\log(\frac{(x * (-b - (b^2 - 4*a*c)^{1/2}))^{1/2}}{(2*c)} - 1) * i)}{((b - (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}} - (1 - x^2)^{1/2} * i)}{(x - ((b - (b^2 - 4*a*c)^{1/2})/(2*c))^{1/2})} * (b^3*c + b^4 - b^3 * (b^2 - 4*a*c)^{1/2} + 4*a^2*c^2 + 2*a*c^2 * (b^2 - 4*a*c)^{1/2} - b^2*c * (b^2 - 4*a*c)^{1/2} - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c * (b^2 - 4*a*c)^{1/2})) / (4*c^3 * ((b - (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2} * (4*a*c - b^2)) - (\log(\frac{(x * (-b + (b^2 - 4*a*c)^{1/2}))^{1/2}}{(2*c)} + 1) * i)}{((b + (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}} + (1 - x^2)^{1/2} * i)}{(x + ((b + (b^2 - 4*a*c)^{1/2})/(2*c))^{1/2})} * (b^3*c + b^4 + b^3 * (b^2 - 4*a*c)^{1/2} + 4*a^2*c^2 - 2*a*c^2 * (b^2 - 4*a*c)^{1/2} + b^2*c * (b^2 - 4*a*c)^{1/2} - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c * (b^2 - 4*a*c)^{1/2})) / (4*c^3 * (4*a*c - b^2) * ((b + (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}) - (\log(\frac{(x * (-b + (b^2 - 4*a*c)^{1/2}))^{1/2}}{(2*c)} - 1) * i)}{((b + (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}} - (1 - x^2)^{1/2} * i)}{(x - ((b + (b^2 - 4*a*c)^{1/2})/(2*c))^{1/2})} * (b^3*c + b^4 + b^3 * (b^2 - 4*a*c)^{1/2} + 4*a^2*c^2 - 2*a*c^2 * (b^2 - 4*a*c)^{1/2} + b^2*c * (b^2 - 4*a*c)^{1/2} - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c * (b^2 - 4*a*c)^{1/2})) / (4*c^3 * (4*a*c - b^2) * ((b + (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}) - (\log(\frac{(x * (-b - (b^2 - 4*a*c)^{1/2}))^{1/2}}{(2*c)} + 1) * i)}{((b - (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2}} + (1 - x^2)^{1/2} * i)}{(x + ((b - (b^2 - 4*a*c)^{1/2})/(2*c))^{1/2})} * (b^3*c + b^4 - b^3 * (b^2 - 4*a*c)^{1/2} + 4*a^2*c^2 + 2*a*c^2 * (b^2 - 4*a*c)^{1/2} - b^2*c * (b^2 - 4*a*c)^{1/2} - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c * (b^2 - 4*a*c)^{1/2})) / (4*c^3 * ((b - (b^2 - 4*a*c)^{1/2})/(2*c) + 1)^{1/2} * (4*a*c - b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)

$$3.277 \quad \int \frac{x^3 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=229

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right) - \left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right) + \frac{\sqrt{1-x^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Rubi [A] time = 1.75, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right) - \left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right) + \frac{\sqrt{1-x^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 824

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]

Rule 826

Int[((f_) + (g_)*(x_))/(Sqrt[(d_) + (e_)*(x_)]*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b

*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \sqrt{1-x^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-xx}}{a+bx+cx^2} dx, x, x^2 \right) \\
 &= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst} \left(\int \frac{a+(b+c)x}{\sqrt{1-x}(a+bx+cx^2)} dx, x, x^2 \right)}{2c} \\
 &= \frac{\sqrt{1-x^2}}{c} + \frac{\text{Subst} \left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right)}{c} \\
 &= \frac{\sqrt{1-x^2}}{c} + \frac{\left(b+c - \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, \sqrt{1-x^2} \right)}{2c} + \frac{\left(b+c + \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, \sqrt{1-x^2} \right)}{2c} \\
 &= \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c - \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c}-\sqrt{b^2-4ac}} \right)}{\sqrt{2}c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c + \frac{b^2-2ac+bc}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c}+\sqrt{b^2-4ac}} \right)}{\sqrt{2}c^{3/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}
 \end{aligned}$$

Mathematica [A] time = 0.39, size = 276, normalized size = 1.21

$$\frac{\left(b(c - \sqrt{b^2 - 4ac}) - c(\sqrt{b^2 - 4ac} + 2a) + b^2\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2 - 4ac} + b + 2c}}\right) - \left(b(\sqrt{b^2 - 4ac} + c) + c(\sqrt{b^2 - 4ac} - 2a) + b^2\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2 - 4ac} + b + 2c}}\right) + \sqrt{1-x^2}}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{-\sqrt{b^2 - 4ac} + b + 2c} - \sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{\sqrt{b^2 - 4ac} + b + 2c}} + \frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[1 - x^2] + ((b^2 + b*(c - Sqrt[b^2 - 4*a*c]) - c*(2*a + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c

IntegrateAlgebraic [A] time = 0.68, size = 285, normalized size = 1.24

$$\frac{\left(b\sqrt{b^2 - 4ac} + c\sqrt{b^2 - 4ac} - 2ac + b^2 + bc\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2 - 4ac} - b - 2c}}\right) + \left(b\sqrt{b^2 - 4ac} + c\sqrt{b^2 - 4ac} + 2ac - b^2 - bc\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2 - 4ac} - b - 2c}}\right) + \frac{\sqrt{1-x^2}}{c}}{\sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{-\sqrt{b^2 - 4ac} - b - 2c} + \sqrt{2}c^{3/2}\sqrt{b^2 - 4ac}\sqrt{\sqrt{b^2 - 4ac} - b - 2c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] Sqrt[1 - x^2]/c + ((b^2 - 2*a*c + b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])

fricas [B] time = 6.46, size = 2053, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4)*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3

$$\begin{aligned}
& - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*\sqrt{-x^2 + 1))/x^2) - \sqrt{1/2}*c*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*\log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - \sqrt{1/2}*((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*\sqrt{-x^2 + 1))/x^2) - \sqrt{1/2}*c*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*\log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + \sqrt{1/2}*((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*\sqrt{-x^2 + 1))/x^2) + \sqrt{1/2}*c*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*\log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - \sqrt{1/2}*((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*\sqrt{(b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*\sqrt{-x^2 + 1))/x^2) + 2*\sqrt{-x^2 + 1))/c
\end{aligned}$$

giac [B] time = 4.10, size = 4060, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] sqrt(-x^2 + 1)/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6

$$\begin{aligned}
& - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^3 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^4 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^4 - 13*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^3*c^4 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*c^5 + 26*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b*c^5 - 19*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^2*c^5 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*c^6 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^2 - 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^3*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b*c^3 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^2*c^3 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 - 2*(\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^4*c^2 + \sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^5*c^2 - 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*b^2*c^3 - 6*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^3*c^3 + 3*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^4*c^3 + 2*a*b^4*c^3 + 2*b^5*c^3 + 16*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^3*c^4 + 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*b*c^4 - 11*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b^2*c^4 - 16*a^2*b^2*c^4 + 7*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^3*c^4 - 16*a*b^3*c^4 + 2*b^4*c^4 - 4*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a^2*c^5 + 32*a^3*c^5 - 28*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*b*c^5 + 32*a^2*b*c^5 + 5*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*b^2*c^5 - 16*a*b^2*c^5 - 20*\sqrt{2}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}*c}*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*\text{abs}(c))*\arctan(2*\sqrt{1/2}*\sqrt{-x^2 + 1}/\sqrt{-(b*c + 2*c^2 + \sqrt{-4*(a*c + b*c + c^2)*c^2 + (b*c + 2*c^2)^2})/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 -
\end{aligned}$$

$$\begin{aligned}
& 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6 \\
& *b^4*c^5 + 16*a^2*b*c^6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 \\
& - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*b^5*c^2 \\
& + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}* \\
& a*b^3*c^3 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *c)*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a^2*b*c^4 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a*b^2*c^4 - 13*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *b^3*c^4 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a^2*c^5 + 26*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a*b*c^5 - 19*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *b^2*c^5 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a*c^6 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 \\
& + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 \\
& + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a*b^3*c - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a*b^2*c^2 - 7*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*b^3*c^2 - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a^2*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a*b*c^3 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*b^2*c^3 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 \\
& - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2 + 2*(\sqrt{2}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a*b^4*c^2 + \sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *b^5*c^2 - 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*a^2*b^2*c^3 - 6* \\
& \sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*a*b^3*c^3 + 3*\sqrt{2}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*b^4*c^3 - 2*a*b^4*c^3 - 2*b^5*c^3 + 16 \\
& *\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*a^3*c^4 + 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 \\
& + \sqrt{b^2 - 4*a*c})*c}*a^2*b*c^4 - 11*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a*b^2*c^4 + 16*a^2*b^2*c^4 + 7*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *b^3*c^4 + 16*a*b^3*c^4 - 2*b^4*c^4 - 4*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c} \\
& *a^2*c^5 - 32*a^3*c^5 - 28*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*a*b*c^5 \\
& - 32*a^2*b*c^5 + 5*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*b^2*c^5 + 16*a*b^2*c^5 \\
& - 20*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c})*c}*a*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a*c) \\
& *a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 - 8*(b^2 - 4*a*c) \\
& *a*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*\text{abs}(c))*\arctan(2*\sqrt{1/2} \\
& *\sqrt{-x^2 + 1}/\sqrt{-(b*c + 2*c^2 - \sqrt{-4*(a*c + b*c + c^2)}*c^2 + (b*c + 2*c^2)^2)}/(c^2)))/(a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^6)
\end{aligned}$$

$$c^4 - 6*a*b^3*c^4 + 3*b^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 - 28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2)$$

maple [B] time = 0.06, size = 1223, normalized size = 5.34

$$\frac{b^2 \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{b^2 \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{2b^2 \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{2b^2 \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{2\sqrt{-x^2+1} \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{2\sqrt{-x^2+1} \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{4\sqrt{-x^2+1} \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{4\sqrt{-x^2+1} \arctan\left(\frac{c \sqrt{-x^2+1}}{c^2+bx^2+a}\right)}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}} - \frac{1}{(b^2-2b\sqrt{-x^2+1}+2c^2)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(-x^2+1)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $\frac{2}{(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*a*b/c*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+4*a/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}-8/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*a^2*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*a*b^2/c*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*a*b/c*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+4*a/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}+8/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*a^2*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}-2/(8*a*c-2*b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*a*b^2/c*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)})*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+2/c/(1/x^2*(-x^2+1)-2*(-x^2+1)^{(1/2)}/x^2+1/x^2+1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1} x^3}{cx^4+bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.31, size = 776, normalized size = 3.39

$$\frac{\ln\left(\frac{\sqrt{-x^2+1} + \sqrt{c x^4 + b x^2 + a}}{\sqrt{-x^2+1} - \sqrt{c x^4 + b x^2 + a}}\right)}{4c^2 \sqrt{-x^2+1} \sqrt{c x^4 + b x^2 + a}} + \frac{\ln\left(\frac{\sqrt{-x^2+1} + \sqrt{c x^4 + b x^2 + a}}{\sqrt{-x^2+1} - \sqrt{c x^4 + b x^2 + a}}\right)}{4c^2 (4ac - b) \sqrt{-x^2+1}} + \frac{\ln\left(\frac{\sqrt{-x^2+1} + \sqrt{c x^4 + b x^2 + a}}{\sqrt{-x^2+1} - \sqrt{c x^4 + b x^2 + a}}\right)}{4c^2 \sqrt{-x^2+1} \sqrt{c x^4 + b x^2 + a}} + \frac{\ln\left(\frac{\sqrt{-x^2+1} + \sqrt{c x^4 + b x^2 + a}}{\sqrt{-x^2+1} - \sqrt{c x^4 + b x^2 + a}}\right)}{4c^2 (4ac - b) \sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] (1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - ((b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))^(1/2))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - ((b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))^(1/2))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + ((b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))^(1/2))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + ((b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))^(1/2))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)

$$3.278 \quad \int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=182

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{-\sqrt{b^2-4ac}+b+2c} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.27, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{-\sqrt{b^2-4ac}+b+2c} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 699

Int[Sqrt[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1130

Int[((d_.)*(x_)^m)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m-2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m-2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G

eQ[m, 2]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-x}}{a+bx+cx^2} dx, x, x^2 \right) \\
 &= -\text{Subst} \left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right) \\
 &= \frac{1}{2} \left(-1 - \frac{b+2c}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{1}{2}(-b-2c) - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, \sqrt{1-x^2} \right) - \frac{1}{2} \left(1 - \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}}{\sqrt{b+2c+\sqrt{b^2-4ac}}} \right) \text{tanh}^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}} \right) \\
 &= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} \text{tanh}^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+2c+\sqrt{b^2-4ac}} \text{tanh}^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 169, normalized size = 0.93

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}} \right) - \sqrt{\sqrt{b^2-4ac}-b-2c} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/ (Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/2*(2*b^2*c^2 - 8*a*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}}*c)*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}}*a*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}}*b*c - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 - \sqrt{b^2 - 4*a*c}}*c^2 - 2*(b^2 - 4*a*c)*c^2*\arctan(2*\sqrt{1/2}*\sqrt{-x^2 + 1}/\sqrt{-(b + 2*c + \sqrt{(b + 2*c)^2 - 4*(a + b + c)*c})/c})/((b^4 - 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*\text{abs}(c)) + 1/2*(2*b^2*c^2 - 8*a*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*a*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*b*c - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c^2 - 2*(b^2 - 4*a*c)*c^2*\arctan(2*\sqrt{1/2}*\sqrt{-x^2 + 1}/\sqrt{-(b + 2*c - \sqrt{(b + 2*c)^2 - 4*(a + b + c)*c})/c})/((b^4 - 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*\text{abs}(c))$$

maple [B] time = 0.05, size = 1167, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)

[Out]
$$-2*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*b-4*a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*c+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*(-4*a*c+b^2)^{(1/2)}-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\arctan(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})$$

$$\begin{aligned} & \sqrt{-x^2+1} \cdot a - 2 \cdot (-4ac + b^2)^{1/2} \cdot b \cdot \sqrt{-x^2+1} \cdot (-4ac + b^2)^{1/2} \cdot b + 4ac / (4ac - b^2) \\ & / (-2ab + 4ac - 2b^2 - 2 \cdot (-4ac + b^2)^{1/2} \cdot a - 2 \cdot (-4ac + b^2)^{1/2} \cdot b)^{1/2} \cdot \arctan(1/2 \cdot (2a + 2b + 2 \cdot (-x^2+1)^{1/2} - 1)^{1/2} \cdot a / x^2 + 2 \cdot (-4ac + b^2)^{1/2}) / \\ & (-2ab + 4ac - 2b^2 - 2 \cdot (-4ac + b^2)^{1/2} \cdot a - 2 \cdot (-4ac + b^2)^{1/2} \cdot b)^{1/2} \cdot c - 1 / (4ac - b^2) / (-2ab + 4ac - 2b^2 - 2 \cdot (-4ac + b^2)^{1/2} \cdot a - 2 \cdot (-4ac + b^2)^{1/2} \cdot b)^{1/2} \cdot \arctan(1/2 \cdot (2a + 2b + 2 \cdot (-x^2+1)^{1/2} - 1)^{1/2} \cdot a / x^2 + 2 \cdot (-4ac + b^2)^{1/2}) \\ & / (-2ab + 4ac - 2b^2 - 2 \cdot (-4ac + b^2)^{1/2} \cdot a - 2 \cdot (-4ac + b^2)^{1/2} \cdot b)^{1/2} \cdot b^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1} x}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.29, size = 649, normalized size = 3.57

$$\frac{\ln\left(\frac{\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}}}{\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}}}\right) \left(4ac + b\sqrt{b^2-4ac} + 2c\sqrt{b^2-4ac} - b^2\right)}{4c\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}} + 1(4ac - b^2)} - \frac{\ln\left(\frac{\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}}}{\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}}}\right) \left(b\sqrt{b^2-4ac} - 4ac + 2c\sqrt{b^2-4ac} + b^2\right)}{4c(4ac - b^2)\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}} + 1} + \frac{\ln\left(\frac{\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}}}{\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}}}\right) \left(4ac + b\sqrt{b^2-4ac} + 2c\sqrt{b^2-4ac} - b^2\right)}{4c\sqrt{\frac{b\sqrt{-x^2+1}+1}{2c}} + 1(4ac - b^2)} - \frac{\ln\left(\frac{\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}}}{\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}}}\right) \left(b\sqrt{b^2-4ac} - 4ac + 2c\sqrt{b^2-4ac} + b^2\right)}{4c(4ac - b^2)\sqrt{\frac{b\sqrt{-x^2+1}-1}{2c}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out]
$$\begin{aligned} & (\log(\frac{((x \cdot (-b - (b^2 - 4ac)^{1/2}))^{1/2}) / (2c))^{1/2} - 1}{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1})^{1/2} - (1 - x^2)^{1/2} \cdot 1i) / (x - (-b - (b^2 - 4ac)^{1/2}) / (2c))^{1/2}) \cdot (4ac + b \cdot (b^2 - 4ac)^{1/2} + 2c \cdot (b^2 - 4ac)^{1/2} - b^2) / (4c \cdot ((b - (b^2 - 4ac)^{1/2}) / (2c) + 1)^{1/2} \cdot (4ac - b^2)) \\ & - (\log(\frac{((x \cdot (-b + (b^2 - 4ac)^{1/2}))^{1/2}) / (2c))^{1/2} - 1}{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1})^{1/2} - (1 - x^2)^{1/2} \cdot 1i) / (x - (-b + (b^2 - 4ac)^{1/2}) / (2c))^{1/2}) \cdot (b \cdot (b^2 - 4ac)^{1/2} - 4ac + 2c \cdot (b^2 - 4ac)^{1/2} + b^2) / (4c \cdot (4ac - b^2) \cdot ((b + (b^2 - 4ac)^{1/2}) / (2c) + 1)^{1/2}) \\ & + (\log(\frac{((x \cdot (-b - (b^2 - 4ac)^{1/2}))^{1/2}) / (2c))^{1/2} + 1}{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1})^{1/2} + (1 - x^2)^{1/2} \cdot 1i) / (x + (-b - (b^2 - 4ac)^{1/2}) / (2c))^{1/2}) \cdot (4ac + b \cdot (b^2 - 4ac)^{1/2} + 2c \cdot (b^2 - 4ac)^{1/2} - b^2) / (4c \cdot ((b - (b^2 - 4ac)^{1/2}) / (2c) + 1)^{1/2} \cdot (4ac - b^2)) \\ & - (\log(\frac{((x \cdot (-b + (b^2 - 4ac)^{1/2}))^{1/2}) / (2c))^{1/2} + 1}{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1})^{1/2} + (1 - x^2)^{1/2} \cdot 1i) / (x + (-b + (b^2 - 4ac)^{1/2}) / (2c))^{1/2}) \cdot (b \cdot (b^2 - 4ac)^{1/2} - 4ac + 2c \cdot (b^2 - 4ac)^{1/2} + b^2) / (4c \cdot (4ac - b^2) \cdot ((b + (b^2 - 4ac)^{1/2}) / (2c) + 1)^{1/2}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a), x)

[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)

$$3.279 \quad \int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=241

$$\frac{\sqrt{c} \left(\sqrt{b^2 - 4ac} + 2a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right) - \sqrt{c} \left(-\sqrt{b^2 - 4ac} + 2a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{-\sqrt{b^2 - 4ac} + b + 2c} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{\sqrt{b^2 - 4ac} + b + 2c}} \tanh^{-1} \left(\sqrt{1-x^2} \right)$$

Rubi [A] time = 1.65, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$\frac{\sqrt{c} \left(\sqrt{b^2 - 4ac} + 2a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right) - \sqrt{c} \left(-\sqrt{b^2 - 4ac} + 2a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{-\sqrt{b^2 - 4ac} + b + 2c} - \sqrt{2} a \sqrt{b^2 - 4ac} \sqrt{\sqrt{b^2 - 4ac} + b + 2c}} \tanh^{-1} \left(\sqrt{1-x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)), x]

[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a + b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :=> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1287

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :=> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-x}}{x(a+bx+cx^2)} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{x^2}{(1-x^2)(a+b+c+(-b-2c)x^2+cx^4)} dx, x, \sqrt{1-x^2} \right) \\
&= -\text{Subst} \left(\int \left(-\frac{1}{a(-1+x^2)} + \frac{-a-b-c+cx^2}{a(a+b+c-(b+2c)x^2+cx^4)} \right) dx, x, \sqrt{1-x^2} \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1-x^2} \right)}{a} - \frac{\text{Subst} \left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right)}{a} \\
&= -\frac{\tanh^{-1}(\sqrt{1-x^2})}{a} + \frac{c(2a+b-\sqrt{b^2-4ac}) \text{Subst} \left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, \sqrt{1-x^2} \right)}{2a\sqrt{b^2-4ac}} \\
&= -\frac{\tanh^{-1}(\sqrt{1-x^2})}{a} + \frac{\sqrt{c}(2a+b+\sqrt{b^2-4ac}) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(2a-\sqrt{b^2-4ac}) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 212, normalized size = 0.88

$$\frac{\sqrt{2} \left(\sqrt{-\sqrt{b^2-4ac}+b+2c} (\sqrt{b^2-4ac}+b) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right) + (\sqrt{b^2-4ac}-b) \sqrt{\sqrt{b^2-4ac}+b+2c} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right) \right)}{\sqrt{c}\sqrt{b^2-4ac}} - 4 \tanh^{-1}(\sqrt{1-x^2})$$

4a

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)), x]

[Out] (-4*ArcTanh[Sqrt[1 - x^2]] + (Sqrt[2]*(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])*(b + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]))/(4*a)

IntegrateAlgebraic [A] time = 0.65, size = 277, normalized size = 1.15

$$\frac{\sqrt{c}(\sqrt{b^2-4ac}-2a-b) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}} \right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2a+b) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}} \right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}} + \frac{\log(\sqrt{1-x^2}-1)}{2a} - \frac{\log(a\sqrt{1-x^2}+a)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]) + Log[-1 + Sqrt[1 - x^2]]/(2*a) - Log[a + a*Sqrt[1 - x^2]]/(2*a)
```

fricas [B] time = 11.65, size = 1232, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)))*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

giac [B] time = 3.67, size = 3639, normalized size = 15.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*\log(\sqrt{-x^2 + 1} + 1)/a + 1/2*\log(-\sqrt{-x^2 + 1} + 1)/a + 1/8*(4*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^4*b*c - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^4*c^2 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 - 9*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 - 10*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - \sqrt{2})*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*b^4 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4 + \sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b^5 - 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c - 6*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c + 3*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 2*a^2*b^4*c - 2*a*b^5*c + 16*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^4*c^2 + 8*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^2 - 11*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 7*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 - 32*a^4*c^3 - 28*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 5*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 20*\sqrt{2}*\sqrt{-b*c - 2*c^2 + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*\arctan(2*\sqrt{1/2}*\sqrt{-x^2 + 1}/\sqrt{-(a*b + 2*a*c + \sqrt{-4*(a^2 + a*b + a*c)*a*c + (a*b + 2*a*c)^2})/(a*c)})))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b$$

$$\begin{aligned}
&^4c + 16a^5c^2 + 8a^4b^2c^2 - 11a^3b^2c^2 + 7a^2b^3c^2 - 4a^4c^3 \\
&- 28a^3b^2c^3 + 5a^2b^2c^3 - 20a^3c^4) \operatorname{abs}(a) \operatorname{abs}(c)) - 1/8(4a^3b^3c^2 + 2a^2b^4c^2 - 16a^4b^2c^3 + 4a^2b^3c^3 - 32a^4c^4 - 16a^3b^2c^4 - 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) \\
& *c) a^3b^3 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^4 + 8\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^4b^2c - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3b^2c - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^3c + 16\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^4c^2 - 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3b^2c^2 - 9\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^2 - 20\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3c^3 - 10\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^3 - 4(b^2 - 4ac) a^3b^2c^2 - 2(b^2 - 4ac) a^2b^2c^2 - 8(b^2 - 4ac) a^3c^3 - 4(b^2 - 4ac) a^2b^2c^3 + (2b^4c^2 - 16a^2b^2c^3 + 32a^2c^4 - \sqrt{2}) \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) b^4 + 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c - 2\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) b^3c - 16\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2c^2 + 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^2 - 5\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) b^2c^2 + 20\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2c^3 - 2(b^2 - 4ac) b^2c^2 + 8(b^2 - 4ac) a^2c^3) a^2 - 2(\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^4 + \sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^5 - 8\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3b^2c - 6\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^3c + 3\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^4c + 2a^2b^4c + 2a^2b^5c + 16\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^4c^2 + 8\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3b^2c^2 - 11\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^2 - 16a^3b^2c^2 + 7\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^3c^2 - 16a^2b^3c^2 + 2a^2b^4c^2 - 4\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^3c^3 + 32a^4c^3 - 28\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^3 + 32a^3b^2c^3 + 5\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2b^2c^3 - 16a^2b^2c^3 - 20\sqrt{2} \sqrt{-b^2c - 2c^2 - \sqrt{b^2 - 4ac}}) *c) a^2c^4 + 32a^3c^4 - 2(b^2 - 4ac) a^2b^2c - 2(b^2 - 4ac) a^2b^3c + 8(b^2 - 4ac) a^3c^2 + 8(b^2 - 4ac) a^2b^2c^2 - 2(b^2 - 4ac) a^2b^2c^2 + 8(b^2 - 4ac) a^2c^3) \operatorname{abs}(a) \operatorname{arctan}(2\sqrt{1/2} \sqrt{-x^2 + 1} / \sqrt{-(ab + 2ac - \sqrt{-4(a^2 + ab + ac)ac + (ab + 2ac)^2}) / (ac)}) / ((a^3b^4 + a^2b^5 - 8a^4b^2c - 6a^3b^3c + 3a^2b^4c + 16a^5c^2 + 8a^4b^2c^2 - 11a^3b^2c^2 + 7a^2b^3c^2 - 4a^4c^3 - 28a^3b^2c^3 + 5a^2b^2c^3 - 20a^3c^4) \operatorname{abs}(a) \operatorname{abs}(c)))
\end{aligned}$$

maple [B] time = 0.06, size = 2099, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((-x^2+1)^{(1/2)}/x/(c*x^4+b*x^2+a), x)$

[Out]
$$\begin{aligned} & 1/a*(-x^2+1)^{(1/2)}-1/a*\text{arctanh}(1/(-x^2+1)^{(1/2)})+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)} \\ & *b*\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ &)-2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*c*(-4*a*c+b^2)^{(1/2)}+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)} \\ & *\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ & *b^2*(-4*a*c+b^2)^{(1/2)}+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*a*c*\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*b^2*\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)} \\ & *\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ & *b*c-1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\text{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*b^3+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)} \\ & *(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*b*\text{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ &)-2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\text{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ & *c*(-4*a*c+b^2)^{(1/2)}+1/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\text{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ & *a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\text{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}) \\ &)+1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*b^2*\text{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)}) \\ &)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})- \end{aligned}$$

$$\frac{4/(4ac-b^2)/(-2ab+4ac-2b^2-2*(-4ac+b^2)^{1/2})a-2*(-4ac+b^2)^{1/2}b)^{1/2}*\arctan(1/2*(2a+2b+2*((-x^2+1)^{1/2}-1)^2a/x^2+2*(-4ac+b^2)^{1/2}b)/(-2ab+4ac-2b^2-2*(-4ac+b^2)^{1/2})a-2*(-4ac+b^2)^{1/2}b)^{1/2})*b*c+1/a/(4ac-b^2)/(-2ab+4ac-2b^2-2*(-4ac+b^2)^{1/2})a-2*(-4ac+b^2)^{1/2}b)^{1/2}*\arctan(1/2*(2a+2b+2*((-x^2+1)^{1/2}-1)^2a/x^2+2*(-4ac+b^2)^{1/2}b)/(-2ab+4ac-2b^2-2*(-4ac+b^2)^{1/2})a-2*(-4ac+b^2)^{1/2}b)^{1/2})*b^3-2/a/(2/x^2-2*(-x^2+1)^{1/2}/x^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2+1)/((c*x^4+b*x^2+a)*x),x)

mupad [B] time = 1.30, size = 669, normalized size = 2.78

$$\frac{\ln\left(\frac{\sqrt{\frac{1}{2}-1}-\sqrt{\frac{1}{2}}}{a}\right)}{4a(4ac-b^2)\sqrt{\frac{2a\sqrt{b^2-4ac}}{2c}+1}} + \frac{\ln\left(\frac{\sqrt{\frac{1}{2}-1}-\sqrt{\frac{1}{2}}}{a}\right)}{4a\sqrt{\frac{2a\sqrt{b^2-4ac}}{2c}+1}(4ac-b^2)} + \frac{\ln\left(\frac{\sqrt{\frac{1}{2}-1}-\sqrt{\frac{1}{2}}}{a}\right)}{4a(4ac-b^2)\sqrt{\frac{2a\sqrt{b^2-4ac}}{2c}+1}} + \frac{\ln\left(\frac{\sqrt{\frac{1}{2}-1}-\sqrt{\frac{1}{2}}}{a}\right)}{4a\sqrt{\frac{2a\sqrt{b^2-4ac}}{2c}+1}(4ac-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x^2)^(1/2)/(x*(a+b*x^2+c*x^4)),x)

[Out] $\log\left(\frac{(1/x^2-1)^{1/2}-(1/x^2)^{1/2}}{a} + \frac{\log\left(\frac{(x*(-(b+(b^2-4ac)^{1/2}))/2c)^{1/2}+1}{(b+(b^2-4ac)^{1/2})/2c+1}\right)^{1/2} + (1-x^2)^{1/2}}{x + \frac{(b+(b^2-4ac)^{1/2})/2c+1}{(b+(b^2-4ac)^{1/2})/2c}}\right) * (4ac+2a*(b^2-4ac)^{1/2} + b*(b^2-4ac)^{1/2} - b^2) / (4a*(4ac-b^2) * ((b+(b^2-4ac)^{1/2})/2c+1)^{1/2}) - \frac{\log\left(\frac{(x*(-(b-(b^2-4ac)^{1/2}))/2c)^{1/2}+1}{(b-(b^2-4ac)^{1/2})/2c+1}\right)^{1/2} + (1-x^2)^{1/2}}{x + \frac{(b-(b^2-4ac)^{1/2})/2c+1}{(b-(b^2-4ac)^{1/2})/2c}}\right) * (2a*(b^2-4ac)^{1/2} - 4ac + b*(b^2-4ac)^{1/2} + b^2) / (4a*((b-(b^2-4ac)^{1/2})/2c+1)^{1/2} * (4ac-b^2)) + \frac{\log\left(\frac{(x*(-(b+(b^2-4ac)^{1/2}))/2c)^{1/2}-1}{(b+(b^2-4ac)^{1/2})/2c+1}\right)^{1/2} - (1-x^2)^{1/2}}{x - \frac{(b+(b^2-4ac)^{1/2})/2c+1}{(b+(b^2-4ac)^{1/2})/2c}}\right) * (4ac+2a*(b^2-4ac)^{1/2} + b*(b^2-4ac)^{1/2} - b^2) / (4a*(4ac-b^2) * ((b+(b^2-4ac)^{1/2})/2c+1)^{1/2}) - \frac{\log\left(\frac{(x*(-(b-(b^2-4ac)^{1/2}))/2c)^{1/2}-1}{(b-(b^2-4ac)^{1/2})/2c+1}\right)^{1/2} - (1-x^2)^{1/2}}{x - \frac{(b-(b^2-4ac)^{1/2})/2c+1}{(b-(b^2-4ac)^{1/2})/2c}}\right) * (2a*(b^2-4ac)^{1/2} - 4ac + b*(b^2-4ac)^{1/2} + b^2) / (4a*((b-(b^2-4ac)^{1/2})/2c+1)^{1/2} * (4ac-b^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x(a+bx^2+cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a), x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

$$3.280 \quad \int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=290

$$\frac{\sqrt{c} \left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}} + a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac} + b + 2c}} \right) - \sqrt{c} \left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}} + a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac} + b + 2c}} \right) (a + 2b) \tanh^{-1} \left(\sqrt{1-x^2} \right)}{\sqrt{2} a^2 \sqrt{-\sqrt{b^2-4ac} + b + 2c} - \sqrt{2} a^2 \sqrt{\sqrt{b^2-4ac} + b + 2c}}$$

Rubi [A] time = 2.36, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$\frac{\sqrt{c} \left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}} + a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac} + b + 2c}} \right) - \sqrt{c} \left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}} + a + b \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac} + b + 2c}} \right) + \frac{(a + 2b) \tanh^{-1} \left(\sqrt{1-x^2} \right)}{2a^2} - \frac{1}{4a(1 - \sqrt{1-x^2})} + \frac{1}{4a(\sqrt{1-x^2} + 1)}}{\sqrt{2} a^2 \sqrt{-\sqrt{b^2-4ac} + b + 2c} - \sqrt{2} a^2 \sqrt{\sqrt{b^2-4ac} + b + 2c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]

[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +

$a*e^2/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^{(2*q)})/e^2)^p, x], x, (d + e*x)^{(1/q)], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IntegersQ}[n, p] \ \&\& \ \text{FractionQ}[m]$

Rule 1166

$\text{Int}[\frac{(d) + (e)*(x)^2}{(a) + (b)*(x)^2 + (c)*(x)^4}, x_Symbol] :> \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^2), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 1251

$\text{Int}[(x)^{(m)}*((d) + (e)*(x)^2)^{(q)}*((a) + (b)*(x)^2 + (c)*(x)^4)^{(p)}, x_Symbol] :> \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rule 1287

$\text{Int}[\frac{((f)*(x))^{(m)}*((d) + (e)*(x)^2)^{(q)}}{(a) + (b)*(x)^2 + (c)*(x)^4}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[\frac{(f*x)^m*(d + e*x^2)^q}{(a + b*x^2 + c*x^4)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-x}}{x^2(a+bx+cx^2)} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{x^2}{(1-x^2)^2(a+b+c+(-b-2c)x^2+cx^4)} dx, x, \sqrt{1-x^2} \right) \\
&= -\text{Subst} \left(\int \left(\frac{1}{4a(-1+x)^2} + \frac{1}{4a(1+x)^2} + \frac{a+2b}{2a^2(-1+x^2)} + \frac{b(a+b+c)-(a+b)c}{a^2(a+b+c-(b+2c)x^2-cx^4)} \right) dx, x, \sqrt{1-x^2} \right) \\
&= -\frac{1}{4a(1-\sqrt{1-x^2})} + \frac{1}{4a(1+\sqrt{1-x^2})} - \frac{\text{Subst} \left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4} dx, x, \sqrt{1-x^2} \right)}{a^2} \\
&= -\frac{1}{4a(1-\sqrt{1-x^2})} + \frac{1}{4a(1+\sqrt{1-x^2})} + \frac{(a+2b) \tanh^{-1}(\sqrt{1-x^2})}{2a^2} + \frac{\left(c(a+b) - \frac{b^2}{\sqrt{1-x^2}} \right)}{\sqrt{1-x^2}} \\
&= -\frac{1}{4a(1-\sqrt{1-x^2})} + \frac{1}{4a(1+\sqrt{1-x^2})} + \frac{(a+2b) \tanh^{-1}(\sqrt{1-x^2})}{2a^2} - \frac{\sqrt{c}(a+b)}{\sqrt{1-x^2}}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 292, normalized size = 1.01

$$\frac{\sqrt{2} \sqrt{c} \left(\frac{\left(b(\sqrt{b^2-4ac}+b)+a(\sqrt{b^2-4ac}+b-2c) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right) - \left(b(\sqrt{b^2-4ac}-b)+a(\sqrt{b^2-4ac}-b+2c) \right) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{b^2-4ac}} \right)}{2a^2} + (a+2b) \log(\sqrt{1-x^2}+1) - (a+2b) \log(x) - \frac{a\sqrt{1-x^2}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)), x]

[Out] $\left(-\frac{(a\sqrt{1-x^2})/x^2 + (\sqrt{2}\sqrt{c})\left(-\frac{(b(b+\sqrt{b^2-4ac}+b)+a(b-2c+\sqrt{b^2-4ac}+b-2c))\text{ArcTanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right) + \frac{(b(b+\sqrt{b^2-4ac}-b)+a(b-2c+\sqrt{b^2-4ac}-b+2c))\text{ArcTanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}} \right)}{2a^2} + (a+2b)\log(\sqrt{1-x^2}+1) - (a+2b)\log(x) - \frac{a\sqrt{1-x^2}}{x^2} \right)$

IntegrateAlgebraic [A] time = 2.09, size = 360, normalized size = 1.24

$$\frac{(b\sqrt{c}\sqrt{b^2-4ac} + a\sqrt{c}\sqrt{b^2-4ac} - ab\sqrt{c} + 2ac^{3/2} + b^2(-\sqrt{c}))\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right) + (b\sqrt{c}\sqrt{b^2-4ac} + a\sqrt{c}\sqrt{b^2-4ac} + ab\sqrt{c} - 2ac^{3/2} + b^2\sqrt{c})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right) + \frac{(a+2b)\tanh^{-1}(\sqrt{1-x^2})}{2a^2} - \frac{\sqrt{1-x^2}}{2ax^2}}{\sqrt{2a^2\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}-b-2c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]

[Out]
$$-1/2*\text{Sqrt}[1 - x^2]/(a*x^2) + ((-(a*b*\text{Sqrt}[c]) - b^2*\text{Sqrt}[c] + 2*a*c^{(3/2)} + a*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c] + b*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[1 - x^2])/\text{Sqrt}[-b - 2*c - \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[-b - 2*c - \text{Sqrt}[b^2 - 4*a*c]]) + ((a*b*\text{Sqrt}[c] + b^2*\text{Sqrt}[c] - 2*a*c^{(3/2)} + a*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c] + b*\text{Sqrt}[c]*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[1 - x^2])/\text{Sqrt}[-b - 2*c + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[-b - 2*c + \text{Sqrt}[b^2 - 4*a*c]]) + ((a + 2*b)*\text{ArcTanh}[\text{Sqrt}[1 - x^2]])/(2*a^2)$$

fricas [B] time = 32.22, size = 2799, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out]
$$-1/2*(\text{sqrt}(1/2)*a^2*x^2*\text{sqrt}((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c))*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/((a^4*b^2 - 4*a^5*c))*\log(((a^4*b^2*c - 4*a^5*c^2)*x^2*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + \text{sqrt}(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*\text{sqrt}((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c))*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/((a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*\text{sqrt}(-x^2 + 1)/x^2 - \text{sqrt}(1/2)*a^2*x^2*\text{sqrt}((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c))*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/((a^4*b^2 - 4*a^5*c))*\log(((a^4*b^2*c - 4*a^5*c^2)*x^2*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c - \text{sqrt}(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*\text{sqrt}((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/((a^4*b^2 - 4*a^5*c))$$

$$\begin{aligned}
&^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b) \\
&)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*\sqrt{(a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*\sqrt{-x^2 + 1))/x^2) + \sqrt{1/2}*a^2*x^2*\sqrt{(a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*\log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c + \sqrt{1/2}*((a^5*b^3 - 4*a^6*b*c)*x^2*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*\sqrt{(a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*\sqrt{-x^2 + 1))/x^2) - \sqrt{1/2}*a^2*x^2*\sqrt{(a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*\log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - \sqrt{1/2}*((a^5*b^3 - 4*a^6*b*c)*x^2*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*\sqrt{(a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*\sqrt{(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*\sqrt{-x^2 + 1))/x^2) + (a + 2*b)*x^2*\log((\sqrt{-x^2 + 1} - 1)/x) + \sqrt{-x^2 + 1})*a)/(a^2*x^2)
\end{aligned}$$

giac [B] time = 7.12, size = 1675, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] -1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2

```

- sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c))*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 12*a*b^2*c^3 - 16*a^2*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c^2 - 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c - sqrt(-4*(a^3 + a^2*b + a^2*c))*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) + 1/4*(a + 2*b)*log(sqrt(-x^2 + 1) + 1)/a^2 - 1/4*(a + 2*b)*log(-sqrt(-x^2 + 1) + 1)/a^2 - 1/2*sqrt(-x^2 + 1)/(a*x^2)

```

maple [B] time = 0.08, size = 2770, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a), x)

[Out]
$$\begin{aligned}
& -1/a^2*b*(-x^2+1)^{(1/2)}+1/a^2*b*\operatorname{arctanh}(1/(-x^2+1)^{(1/2)})+2/(4*a*c-b^2)/(-2 \\
& *a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a \\
& *c+b^2)^{(1/2)}*c*\operatorname{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c \\
& +b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)} \\
&)*b)^{(1/2)}-1/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4* \\
& a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}/a*b^2*\operatorname{arctan}(1/2*(-2*a-2*b-2*((- \\
& x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c \\
& +b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c- \\
& 2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(-2*a \\
& -2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2 \\
& +2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*b*c*(-4*a*c+b^2)^{(1/ \\
& 2)}-1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b \\
& ^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4* \\
& a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(\\
& 1/2)}*b)^{(1/2)})*b^3*(-4*a*c+b^2)^{(1/2)}-4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(\\
& -4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*b*c*\operatorname{arctan}(1/2*(-2*a-2*b- \\
& 2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(- \\
& 4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}+4/(4*a*c-b^2)/(-2*a*b+4*a \\
& *c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(- \\
& 2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2* \\
& b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*c^2+1/(4*a*c-b^2) \\
& /(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}/a \\
& *b^3*\operatorname{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)} \\
&)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)} \\
&)-5/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(\\
& 1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+ \\
& b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2*(-4*a*c+b^2)^{(1/2)} \\
&)*b)^{(1/2)})*b^2*c+1/a^2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)} \\
& *a+2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(-2*a-2*b-2*((-x^2+1)^{(1/2)}-1)^ \\
& 2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2+2*(-4*a*c+b^2)^{(1/2)}*a+2* \\
& (-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*b^4+2/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a* \\
& c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*c*\operatorname{arctan}(1/ \\
& 2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c \\
& -2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}-1/(4*a*c-b^2)/ \\
& (-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*(- \\
& 4*a*c+b^2)^{(1/2)}/a*b^2*\operatorname{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(\\
& -4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2 \\
&)^{(1/2)}*b)^{(1/2)}+3/a/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}* \\
& a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2* \\
& a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(- \\
& 4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*b*c*(-4*a*c+b^2)^{(1/2)}-1/a^2/(4*a*c-b^2)/(-2*a*b \\
& +4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)}*\operatorname{arctan}(1/ \\
& 2*(2*a+2*b+2*((-x^2+1)^{(1/2)}-1)^2*a/x^2+2*(-4*a*c+b^2)^{(1/2)})/(-2*a*b+4*a*c \\
& -2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c+b^2)^{(1/2)}*b)^{(1/2)})*b^3*(-4*a*c+b^ \\
& 2)^{(1/2)}+4/(4*a*c-b^2)/(-2*a*b+4*a*c-2*b^2-2*(-4*a*c+b^2)^{(1/2)}*a-2*(-4*a*c
\end{aligned}$$

$$\begin{aligned}
 & +b^2)^{(1/2)} * b)^{(1/2)} * b * c * \arctan(1/2 * (2 * a + 2 * b + 2 * ((-x^2 + 1)^{(1/2)} - 1)^2 * a / x^2 + 2 \\
 & * (-4 * a * c + b^2)^{(1/2)}) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (-4 * a * c + b \\
 & ^2)^{(1/2)} * b)^{(1/2)} - 4 / (4 * a * c - b^2) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * \\
 & a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * \arctan(1/2 * (2 * a + 2 * b + 2 * ((-x^2 + 1)^{(1/2)} - 1)^2 * \\
 & a / x^2 + 2 * (-4 * a * c + b^2)^{(1/2)}) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (- \\
 & 4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * c^2 - 1 / (4 * a * c - b^2) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + \\
 & b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} / a * b^3 * \arctan(1/2 * (2 * a + 2 * b + 2 * ((-x \\
 & ^2 + 1)^{(1/2)} - 1)^2 * a / x^2 + 2 * (-4 * a * c + b^2)^{(1/2)}) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + \\
 & b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} + 5 / a / (4 * a * c - b^2) / (-2 * a * b + 4 * a * c - 2 \\
 & * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * \arctan(1/2 * (2 * a + 2 \\
 & * b + 2 * ((-x^2 + 1)^{(1/2)} - 1)^2 * a / x^2 + 2 * (-4 * a * c + b^2)^{(1/2)}) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 \\
 & * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * b^2 * c - 1 / a^2 / (4 * a * c - b^2 \\
 &) / (-2 * a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * \\
 & \arctan(1/2 * (2 * a + 2 * b + 2 * ((-x^2 + 1)^{(1/2)} - 1)^2 * a / x^2 + 2 * (-4 * a * c + b^2)^{(1/2)}) / (-2 * \\
 & a * b + 4 * a * c - 2 * b^2 - 2 * (-4 * a * c + b^2)^{(1/2)} * a - 2 * (-4 * a * c + b^2)^{(1/2)} * b)^{(1/2)} * b^4 + 2 \\
 & / a^2 * b / (2 / x^2 - 2 * (-x^2 + 1)^{(1/2)} / x^2) - 1 / 2 / a / x^2 * (-x^2 + 1)^{(3/2)} - 1 / 2 * (-x^2 + 1)^{(\\
 & 1/2)} / a + 1 / 2 / a * \operatorname{arctanh}(1 / (-x^2 + 1)^{(1/2)})
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1}}{(cx^4 + bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)

mupad [B] time = 1.41, size = 825, normalized size = 2.84

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)

$$\begin{aligned}
 & [\text{Out}] \log((1/x^2 - 1)^{(1/2)} - (1/x^2)^{(1/2)}) / (2 * a) - (\log((1/x^2 - 1)^{(1/2)} - (1 / \\
 & x^2)^{(1/2)}) * (a + b)) / a^2 - (1 - x^2)^{(1/2)} / (2 * a * x^2) - (\log(((x * (-b + (b^2 \\
 & - 4 * a * c)^{(1/2)}) / (2 * c))^2 + 1) * i) / ((b + (b^2 - 4 * a * c)^{(1/2)}) / (2 * c) + \\
 & 1)^{(1/2)} + (1 - x^2)^{(1/2)} * i) / (x + (-b + (b^2 - 4 * a * c)^{(1/2)}) / (2 * c))^2 + 1) \\
 &)) * (4 * a^2 * c - a * b^2 - b^3 + b^2 * (b^2 - 4 * a * c)^{(1/2)} + 4 * a * b * c + a * b * (b^2 - \\
 & 4 * a * c)^{(1/2)} - 2 * a * c * (b^2 - 4 * a * c)^{(1/2)}) / (4 * a^2 * (4 * a * c - b^2) * ((b + (b^2 \\
 & - 4 * a * c)^{(1/2)}) / (2 * c) + 1)^{(1/2)}) + (\log(((x * (-b - (b^2 - 4 * a * c)^{(1/2)}) / \\
 & (2 * c))^2 + 1) * i) / ((b - (b^2 - 4 * a * c)^{(1/2)}) / (2 * c) + 1)^{(1/2)} + (1 - x^
 \end{aligned}$$

$$\begin{aligned}
& 2)^{(1/2)} * 1i) / (x + (-(b - (b^2 - 4*a*c)^{(1/2})) / (2*c))^{(1/2)}) * (a*b^2 - 4*a^2 \\
& *c + b^3 + b^2*(b^2 - 4*a*c)^{(1/2)} - 4*a*b*c + a*b*(b^2 - 4*a*c)^{(1/2)} - 2* \\
& a*c*(b^2 - 4*a*c)^{(1/2})) / (4*a^2*((b - (b^2 - 4*a*c)^{(1/2})) / (2*c) + 1)^{(1/2)} \\
&) * (4*a*c - b^2)) - (\log((((x*(-(b + (b^2 - 4*a*c)^{(1/2})) / (2*c))^{(1/2)} - 1)* \\
& 1i) / ((b + (b^2 - 4*a*c)^{(1/2})) / (2*c) + 1)^{(1/2)} - (1 - x^2)^{(1/2)} * 1i) / (x - \\
& (-(b + (b^2 - 4*a*c)^{(1/2})) / (2*c))^{(1/2)})) * (4*a^2*c - a*b^2 - b^3 + b^2*(b^ \\
& 2 - 4*a*c)^{(1/2)} + 4*a*b*c + a*b*(b^2 - 4*a*c)^{(1/2)} - 2*a*c*(b^2 - 4*a*c)^ \\
& (1/2))) / (4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^{(1/2})) / (2*c) + 1)^{(1/2)} + \\
& (\log((((x*(-(b - (b^2 - 4*a*c)^{(1/2})) / (2*c))^{(1/2)} - 1)* 1i) / ((b - (b^2 - 4 \\
& *a*c)^{(1/2})) / (2*c) + 1)^{(1/2)} - (1 - x^2)^{(1/2)} * 1i) / (x - (-(b - (b^2 - 4*a* \\
& c)^{(1/2})) / (2*c))^{(1/2)})) * (a*b^2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^{(1/2)} - \\
& 4*a*b*c + a*b*(b^2 - 4*a*c)^{(1/2)} - 2*a*c*(b^2 - 4*a*c)^{(1/2}))) / (4*a^2*((b \\
& - (b^2 - 4*a*c)^{(1/2})) / (2*c) + 1)^{(1/2)} * (4*a*c - b^2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a), x)

[Out] Timed out

$$3.281 \quad \int \frac{x^4 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=325

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c} - c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Rubi [A] time = 5.39, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1291, 388, 216, 1692, 377, 205}

$$\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}} - ac + b^2 + bc\right) \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c} - c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} + \frac{\sqrt{1-x^2}x}{2c}$$

Antiderivative was successfully verified.

[In] Int[(x^4*sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] (x*sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[b + 2*c - sqrt[b^2 - 4*a*c]]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[1 - x^2])])/(c^2*sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[b + 2*c - sqrt[b^2 - 4*a*c]]) - ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[b + 2*c + sqrt[b^2 - 4*a*c]]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[1 - x^2])])/(c^2*sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[b + 2*c + sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1291

```
Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \sqrt{1-x^2}}{a+bx^2+cx^4} dx &= \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}} dx}{c^2} - \frac{\int \frac{a(b+c)+(b^2-ac+bc)x^2}{\sqrt{1-x^2}(a+bx^2+cx^4)} dx}{c^2} \\
&= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} \right) dx}{c^2} + \frac{(2b+c) \int \frac{1}{\sqrt{1-x^2}}}{2c^2} \\
&= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c) \sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc - \frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)}}{c^2} \\
&= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c) \sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc - \frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{b-\sqrt{b^2-4ac}-(-b-\sqrt{b^2-4ac}x)} \right)}{c^2} \\
&= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c) \sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc - \frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}} \right)}{c^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [B] time = 6.31, size = 10606, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 0.97, size = 594, normalized size = 1.83

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] (x*sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcTan[x/(-1 + sqrt[1 - x^2])])/c^2 + RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-a*b*Log[x]) - a*c*Log[x] + a*b*Log[-1 + sqrt[1 - x^2] - x*#1] + a*c*Log[-1 + sqrt[1 - x^2] - x*#1] - 3*a*b*Log[x]*#1^2 - 4*b^2*Log[x]*#1^2 + a*c*Log[x]*#1^2 - 4*b*c*Log[x]*#1^2 + 3*a*b*Log[-1 + Sq

```

rt[1 - x^2] - x**1]**#1^2 + 4*b^2*Log[-1 + Sqrt[1 - x^2] - x**1]**#1^2 - a*c*
Log[-1 + Sqrt[1 - x^2] - x**1]**#1^2 + 4*b*c*Log[-1 + Sqrt[1 - x^2] - x**1]*
#1^2 - 3*a*b*Log[x]**#1^4 - 4*b^2*Log[x]**#1^4 + a*c*Log[x]**#1^4 - 4*b*c*Log[
x]**#1^4 + 3*a*b*Log[-1 + Sqrt[1 - x^2] - x**1]**#1^4 + 4*b^2*Log[-1 + Sqrt[1
- x^2] - x**1]**#1^4 - a*c*Log[-1 + Sqrt[1 - x^2] - x**1]**#1^4 + 4*b*c*Log[
-1 + Sqrt[1 - x^2] - x**1]**#1^4 - a*b*Log[x]**#1^6 - a*c*Log[x]**#1^6 + a*b*L
og[-1 + Sqrt[1 - x^2] - x**1]**#1^6 + a*c*Log[-1 + Sqrt[1 - x^2] - x**1]**#1^
6)/(a**1 + b**1 + 3*a**1^3 + 4*b**1^3 + 8*c**1^3 + 3*a**1^5 + 3*b**1^5 + a*
#1^7) & ]/(4*c^2)

```

fricas [B] time = 4.25, size = 2860, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(
-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2
- 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a
^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*
(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^
3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*
b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 +
a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (
4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
- 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*
b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c
^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2

```

$$\begin{aligned}
& *b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b \\
& ^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*\log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a \\
& ^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + s \\
& \text{qrt}(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)* \\
& c)*\text{sqrt}(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a \\
& *b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*\text{sqrt}(-x^2 + 1)*x - \\
& (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*\text{sqrt}((b^6 + a^2*c^4 + 2*(2*a^2*b - a \\
& *b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 \\
& - 4*a*c^9)))*\text{sqrt}(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c \\
& ^4 - 4*a*c^5)*\text{sqrt}((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - \\
& 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - \\
& 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*\text{sqrt}(-x^2 + 1))/x \\
& ^2) - \text{sqrt}(1/2)*c^2*\text{sqrt}(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - \\
& (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2* \\
& b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2* \\
& c^4 - 4*a*c^5))*\log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3 \\
& *b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - \text{sqrt}(1/2)*((b^6 + 4*a^2*b* \\
& c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*\text{sqrt}(-x^2 + 1)*x - (b^ \\
& 6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4* \\
& c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*\text{sqrt}(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + \\
& 8*a^2*c^6)*x)*\text{sqrt}((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - \\
& 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*\text{sqrt}(-(b^4 \\
& + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^6 + \\
& a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2 \\
& *a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - \\
& a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*\text{sqrt}(-x^2 + 1))/x^2) - \text{sqrt}(-x^2 + 1)*c*x \\
& + 2*(2*b + c)*\text{arctan}((\text{sqrt}(-x^2 + 1) - 1)/x))/c^2
\end{aligned}$$

giac [B] time = 6.69, size = 1710, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $1/4*(3*\text{sqrt}(2)*\text{sqrt}(2*a^2 + a*b + \text{sqrt}(b^2 - 4*a*c))*a)*a^2*b^3 + 2*\text{sqrt}(2)*$
 $\text{sqrt}(2*a^2 + a*b + \text{sqrt}(b^2 - 4*a*c))*a)*a*b^4 - 2*a^2*b^4 - \text{sqrt}(2)*\text{sqrt}(2*$
 $a^2 + a*b + \text{sqrt}(b^2 - 4*a*c))*a)*b^5 + 2*a*b^5 - 12*\text{sqrt}(2)*\text{sqrt}(2*a^2 + a*$
 $b + \text{sqrt}(b^2 - 4*a*c))*a)*a^3*b*c - 8*\text{sqrt}(2)*\text{sqrt}(2*a^2 + a*b + \text{sqrt}(b^2 -$
 $4*a*c))*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*\text{sqrt}(2)*\text{sqrt}(2*a^2 + a*b + \text{sqrt}(b^2$
 $- 4*a*c))*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*\text{sqrt}(2)*\text{sqrt}(2*a^2 + a$
 $*b + \text{sqrt}(b^2 - 4*a*c))*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*\text{sqrt}(2)*\text{sqrt}(2*a^2 +$
 $a*b + \text{sqrt}(b^2 - 4*a*c))*a)*\text{sqrt}(b^2 - 4*a*c)*a^2*b^2 - 2*\text{sqrt}(2)*\text{sqrt}(2*a^$
 $2 + a*b + \text{sqrt}(b^2 - 4*a*c))*a)*\text{sqrt}(b^2 - 4*a*c)*a*b^3 + \text{sqrt}(2)*\text{sqrt}(2*a^2$
 $+ a*b + \text{sqrt}(b^2 - 4*a*c))*a)*\text{sqrt}(b^2 - 4*a*c)*b^4 + 6*\text{sqrt}(2)*\text{sqrt}(2*a^2$

$$\begin{aligned}
& + a*b + \sqrt{b^2 - 4*a*c}*a*\sqrt{b^2 - 4*a*c}*a^3*c + 4*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}*a}*\sqrt{b^2 - 4*a*c}*a^2*b*c - 6*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}*a}*\sqrt{b^2 - 4*a*c}*a*b^2*c + 8*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}*a}*\sqrt{b^2 - 4*a*c}*a^2*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c + 8*(b^2 - 4*a*c)*a^2*b*c)*\text{abs}(a)*\arctan(-1/2*\sqrt{2}*(x/(\sqrt{-x^2 + 1}) - 1) - (\sqrt{-x^2 + 1}) - 1)/x)/\sqrt{((2*a*c^2 + b*c^2 + \sqrt{-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 + b*c^2)^2})/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/4*(3*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a^2*b^3 + 2*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a*b^4 + 2*a^2*b^4 - \sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*b^5 - 2*a*b^5 - 12*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a^3*b*c - 8*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a^2*b^2*c - 12*a^3*b^2*c + 8*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a*b^3*c + 16*a^2*b^3*c + 16*a^4*c^2 - 16*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*a^2*b*c^2 - 32*a^3*b*c^2 + 3*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a^2*b^2 + 2*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a*b^3 - \sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*b^4 - 6*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a^3*c - 4*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a^2*b*c + 6*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a*b^2*c - 8*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}*a})*\sqrt{b^2 - 4*a*c}*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 2*(b^2 - 4*a*c)*a*b^3 + 4*(b^2 - 4*a*c)*a^3*c - 8*(b^2 - 4*a*c)*a^2*b*c)*\text{abs}(a)*\arctan(-1/2*\sqrt{2}*(x/(\sqrt{-x^2 + 1}) - 1) - (\sqrt{-x^2 + 1}) - 1)/x)/\sqrt{((2*a*c^2 + b*c^2 - \sqrt{-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 + b*c^2)^2})/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/2*\sqrt{-x^2 + 1}*x/c + 1/4*(\pi*\text{sgn}(x) + 2*\arctan(-1/2*x*((\sqrt{-x^2 + 1}) - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1}) - 1)))*(2*b + c)/c^2
\end{aligned}$$

maple [C] time = 0.04, size = 222, normalized size = 0.68

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(-x^2+1)^{(1/2)}/(c*x^4+b*x^2+a), x)$

[Out] $1/2*x*(-x^2+1)^{(1/2)}/c+1/2*\arcsin(x)/c+1/4/c^2*\text{sum}((a*(b+c)*_R^6+(3*a*b-a*c+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^2+4*b*c)*_R^2+a*b+a*c)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*\ln(((x^2+1)^{(1/2)}-1)/x-_R), _R=\text{RootOf}(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/c^2*b*\arctan(((x^2+1)^{(1/2)}-1)/x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1} x^4}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.30, size = 1024, normalized size = 3.15

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)

[Out] asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*i)/(x - ((b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) + 1)*i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*i)/(x + ((b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) + 1)*i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*i)/(x + ((b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*i)/(x - ((b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a), x)

[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)

$$3.282 \quad \int \frac{x^2 \sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=263

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\sin^{-1}(x)}{c}$$

Rubi [A] time = 2.13, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1293, 216, 1692, 377, 205}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}} + b + c\right) \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\sin^{-1}(x)}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]

[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1293

```
Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.))/((a_.) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(
q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)
]^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sqrt{1-x^2}}{a+bx^2+cx^4} dx &= -\frac{\int \frac{1}{\sqrt{1-x^2}} dx}{c} - \frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}(a+bx^2+cx^4)} dx}{c} \\
&= -\frac{\sin^{-1}(x)}{c} - \frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} + \frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} \right) dx}{c} \\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} dx}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} dx}{c} \\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-(-b-2c+\sqrt{b^2-4ac})x^2} dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} dx}{c} \\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}} \sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [B] time = 6.14, size = 7543, normalized size = 28.68

Result too large to show

Antiderivative was successfully verified.

$$c^2 - (4ab - b^2)c \sqrt{-x^2 + 1} x - (b^3 - 4ac^2 - (4ab - b^2)c) \\ *x + ((b^3c^2 - 4abc^3) \sqrt{-x^2 + 1} x - (b^3c^2 - 4abc^3)x) \sqrt{t} \\ t((b^2 + 2bc + c^2)/(b^2c^4 - 4ac^5)) \sqrt{-(b^2 - (2a - b)c - (b^2 \\ *c^2 - 4ac^3) \sqrt{(b^2 + 2bc + c^2)/(b^2c^4 - 4ac^5)})} / (b^2c^2 - 4 \\ *ac^3) + 2(ab + ac) \sqrt{-x^2 + 1} / x^2 - \sqrt{1/2} c \sqrt{-(b^2 - (2 \\ *a - b)c - (b^2c^2 - 4ac^3) \sqrt{(b^2 + 2bc + c^2)/(b^2c^4 - 4ac^5 \\))} / (b^2c^2 - 4ac^3) * \log(-(2(ab + ac)x^2 - 2ab - 2ac - \sqrt{1/2} \\) * ((b^3 - 4ac^2 - (4ab - b^2)c) \sqrt{-x^2 + 1} x - (b^3 - 4ac^2 - (4 \\ *ab - b^2)c)x + ((b^3c^2 - 4abc^3) \sqrt{-x^2 + 1} x - (b^3c^2 - 4a \\ *bc^3)x) \sqrt{(b^2 + 2bc + c^2)/(b^2c^4 - 4ac^5)}) \sqrt{-(b^2 - (2a \\ - b)c - (b^2c^2 - 4ac^3) \sqrt{(b^2 + 2bc + c^2)/(b^2c^4 - 4ac^5)})} \\) / (b^2c^2 - 4ac^3) + 2(ab + ac) \sqrt{-x^2 + 1} / x^2 - 4 \arctan(\sqrt{ \\ -x^2 + 1} - 1/x) / c$$

giac [B] time = 4.73, size = 3580, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/2*(\pi \operatorname{sgn}(x) + 2 \arctan(-1/2*x*((\sqrt{-x^2 + 1} - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1} - 1)))/c + 1/8*((2a^2b^4 - 16a^3b^2c + 32a^4c^2 + 3\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^2 + 2\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^3 - \sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} b^4 - 12\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3c - 8\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2bc + 8\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} ab^2c - 16\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2c^2 - 2(b^2 - 4ac) a^2b^2 + 8(b^2 - 4ac) a^3c) c^2 \operatorname{abs}(a) - 2(3\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^3b^2c + 5\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^2b^3c + \sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^2b^4c + 2a^2b^4c - \sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) b^5c + 2ab^5c - 12\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^4c^2 - 20\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^3bc^2 + 3\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^2b^2c^2 - 16a^3b^2c^2 + 10\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) ab^3c^2 - 16a^2b^3c^2 - \sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) b^4c^2 + 2ab^4c^2 - 28\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^3c^3 + 32a^4c^3 - 24\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^2bc^3 + 32a^3bc^3 + 8\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) ab^2c^3 - 16a^2b^2c^3 - 16\sqrt{2} * \sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) a^2c^4 + 32a^3c^4 - 2(b^2 - 4ac) a^2b^2c - 2(b^2 - 4ac) ab^3c + 8(b^2 - 4ac) a^3c^2 + 8(b^2 - 4ac) a^2bc^2 - 2(b^2 - 4ac) ab^2c$$

$$\begin{aligned}
&^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^2 \\
&- 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sq \\
&rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(\\
&2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 - \\
&sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 + \\
&12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*c \\
&^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a \\
&^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a \\
&*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 \\
&- 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sqrt(\\
&b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c))*a)*sq \\
&rt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b \\
&^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arcta \\
&n(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1)) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a* \\
&c + b*c + sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5 \\
&*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5* \\
&b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4*c^3 - 28*a^5*c^4 - 24*a^4* \\
&b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c)) - 1/8*((2*a^2*b^4 - 16*a^3*b^2* \\
&c + 32*a^4*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 \\
&- 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(\\
&b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b \\
&^2 - 4*a*c)*b^4 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b \\
&^2 - 4*a*c)*a^3*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(\\
&b^2 - 4*a*c)*a^2*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sq \\
&rt(b^2 - 4*a*c)*a*b^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a \\
&)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3 \\
&*c)^2*abs(a) + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^3*b \\
&^2*c + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^2*b^3*c + sqrt(2 \\
&)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a*b^4*c - 2*a^2*b^4*c - sqrt(2)*s \\
&qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*b^5*c - 2*a*b^5*c - 12*sqrt(2)*sqrt(\\
&2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b - \\
&sqrt(b^2 - 4*a*c))*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4* \\
&a*c))*a)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b \\
&^2 - 4*a*c))*a)*a*b^3*c^2 + 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt \\
&(b^2 - 4*a*c))*a)*b^4*c^2 - 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt \\
&(b^2 - 4*a*c))*a)*a^3*c^3 - 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(\\
&b^2 - 4*a*c))*a)*a^2*b*c^3 - 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqr \\
&t(b^2 - 4*a*c))*a)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b \\
&- sqrt(b^2 - 4*a*c))*a)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2 \\
&*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^ \\
&2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4 \\
&*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - \\
&16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - \\
&4*a*c)*a^3*b*c^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(\\
&b^2 - 4*a*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*
\end{aligned}$$

```

sqrt(b^2 - 4*a*c)*b^4*c^2 + 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)
*a)*sqrt(b^2 - 4*a*c)*a^3*c^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*
a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*
b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*
c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*
c)*a^2*b*c^3)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-
x^2 + 1) - 1)/x)/sqrt((2*a*c + b*c - sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c
+ b*c)^2))/(a*c)))/((3*a^5*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5
*c^2 - 12*a^6*c^3 - 20*a^5*b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4
*c^3 - 28*a^5*c^4 - 24*a^4*b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c))

```

maple [C] time = 0.02, size = 175, normalized size = 0.67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a), x)

[Out]
$$-1/4/c*\text{sum}((_R^6*a+(4*c+3*a+4*b)*_R^4+(4*c+3*a+4*b)*_R^2+a)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*\ln(-_R+((-x^2+1)^{(1/2)}-1)/x), _R=\text{RootOf}(_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))+2/c*\arctan(((x^2+1)^{(1/2)}-1)/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+1} x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.27, size = 870, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4), x)

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2)))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
+ (-b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/
2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - asin(x
)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2)))/(2*c))^(1/2) - 1)*1i)/((b + (b^
2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
+ b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/((8*a*c -
2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a), x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

$$3.283 \quad \int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=220

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c} \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{\sqrt{b^2-4ac}+b+2c} \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1174, 402, 216, 377, 205}

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c} \tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{\sqrt{b^2-4ac}+b+2c} \tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]

[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
- 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4} dx = \frac{(2c) \int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(b+2c-\sqrt{b^2-4ac}) \int \frac{1}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} dx}{\sqrt{b^2-4ac}} - \frac{(b+2c+\sqrt{b^2-4ac}) \int \frac{1}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(b+2c-\sqrt{b^2-4ac}) \operatorname{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-(-b-2c+\sqrt{b^2-4ac})x^2} dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(b+2c+\sqrt{b^2-4ac}) \operatorname{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-(-b+2c+\sqrt{b^2-4ac})x^2} dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac} \sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 5.42, size = 2266, normalized size = 10.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4), x]
```

```
[Out] -1/2*(Sqrt[2]*(-b - 2*c + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b
^2 - 4*a*c])) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*Log[-(Sqrt[(-b + Sqrt[b^2 -
```


IntegrateAlgebraic [C] time = 0.17, size = 155, normalized size = 0.70

$$\frac{1}{2}i\text{RootSum}\left[\#1^4c - 4\#1^3b - 8\#1^3c + 16\#1^2a + 20\#1^2b + 24\#1^2c - 32\#1a - 32\#1b - 32\#1c + 16a + 16b + 16c\&, \frac{\#1^2\log\left(-\#1 - 2x^2 - 2i\sqrt{1-x^2}x + 2\right)}{\#1^3c - 3\#1^2b - 6\#1^2c + 8\#1a + 10\#1b + 12\#1c - 8a - 8b - 8c}\& \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4), x]

[Out] (I/2)*RootSum[16*a + 16*b + 16*c - 32*a*#1 - 32*b*#1 - 32*c*#1 + 16*a*#1^2 + 20*b*#1^2 + 24*c*#1^2 - 4*b*#1^3 - 8*c*#1^3 + c*#1^4 & , (Log[2 - 2*x^2 - (2*I)*x*Sqrt[1 - x^2] - #1]*#1^2)/(-8*a - 8*b - 8*c + 8*a*#1 + 10*b*#1 + 12*c*#1 - 3*b*#1^2 - 6*c*#1^2 + c*#1^3) &]

fricas [B] time = 0.93, size = 759, normalized size = 3.45

$$\frac{1}{2}\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2)

giac [B] time = 5.12, size = 641, normalized size = 2.91

$$\frac{1}{2}\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right) + \frac{1}{2}\sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \log\left(\frac{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}{\sqrt{\frac{2a+b+\sqrt{4ac-b^2}}{a^2}} \sqrt{\frac{2a+b-\sqrt{4ac-b^2}}{a^2}} \sqrt{1-x^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-1/2*(2*a^2*b^2 - 8*a^3*c + 3*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}}*a)*\sqrt{b^2 - 4*a*c}*a^2 + 2*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*a*b - \sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*b^2 + 4*\sqrt{2}*\sqrt{2*a^2 + a*b + \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*a*c - 2*(b^2 - 4*a*c)*a^2)*\text{abs}(a)*\arctan(-1/2*\sqrt{2}*(x/(\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)/\sqrt{((2*a + b + \sqrt{(2*a + b)^2 - 4*(a + b + c)*a})/a)})/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) + 1/2*(2*a^2*b^2 - 8*a^3*c + 3*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}}*a)*\sqrt{b^2 - 4*a*c}*a^2 + 2*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*a*b - \sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*b^2 + 4*\sqrt{2}*\sqrt{2*a^2 + a*b - \sqrt{b^2 - 4*a*c}}*a*\sqrt{b^2 - 4*a*c}*a*c - 2*(b^2 - 4*a*c)*a^2)*\text{abs}(a)*\arctan(-1/2*\sqrt{2}*(x/(\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)/\sqrt{((2*a + b - \sqrt{(2*a + b)^2 - 4*(a + b + c)*a})/a)})/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2)$$

maple [C] time = 0.01, size = 130, normalized size = 0.59

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)

[Out]
$$-1/4*\text{sum}((_R^6 - _R^4 - _R^2 + 1)/(_R^7*a + 3*_R^5*a + 3*_R^5*b + 3*_R^3*a + 4*_R^3*b + 8*_R^3*c + _R*a + _R*b)*\ln(-_R + (-x^2+1)^(1/2) - 1)/x, _R = \text{RootOf}(_Z^8*a + (4*a+4*b)*_Z^6 + (6*a+8*b+16*c)*_Z^4 + (4*a+4*b)*_Z^2 + a))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1}}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 1.27, size = 989, normalized size = 4.50

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4), x)`

[Out]
$$\begin{aligned} & \left(\log\left(\frac{(x \cdot (-b + (b^2 - 4ac)^{1/2}) / (2c))^{1/2} + 1}{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1}\right)^{1/2} + 1 \right) \cdot i / \left((b + (b^2 - 4ac)^{1/2}) / (2c) + 1 \right)^{1/2} \\ & + \left(1 - x^2 \right)^{1/2} \cdot i / \left(x + \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right) \cdot \left(b^2 \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + a \cdot b \right. \\ & \cdot \left. \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} - 2ac \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + 2ac \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right. \\ & \left. + bc \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right) / \left(2a \cdot (4ac - b^2) \cdot \left(\frac{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1}{(2c) + 1} \right)^{1/2} \right) \\ & - \left(\log\left(\frac{(x \cdot (-b - (b^2 - 4ac)^{1/2}) / (2c))^{1/2} - 1}{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1}\right)^{1/2} - 1 \right) \cdot i / \left((b - (b^2 - 4ac)^{1/2}) / (2c) + 1 \right)^{1/2} \\ & - \left(1 - x^2 \right)^{1/2} \cdot i / \left(x - \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right) \cdot \left(b^2 \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + a \cdot b \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right. \\ & \left. - 2ac \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + 2ac \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right. \\ & \left. + bc \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right) / \left(2a \cdot \left(\frac{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1}{(2c) + 1} \right)^{1/2} \cdot (4ac - b^2) \right) \\ & + \left(\log\left(\frac{(x \cdot (-b - (b^2 - 4ac)^{1/2}) / (2c))^{1/2} + 1}{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1}\right)^{1/2} + 1 \right) \cdot i / \left((b - (b^2 - 4ac)^{1/2}) / (2c) + 1 \right)^{1/2} \\ & + \left(1 - x^2 \right)^{1/2} \cdot i / \left(x + \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right) \cdot \left(b^2 \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + a \cdot b \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right. \\ & \left. - 2ac \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + 2ac \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right. \\ & \left. + bc \cdot \frac{-(b - (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right) / \left(2a \cdot \left(\frac{(b - (b^2 - 4ac)^{1/2}) / (2c) + 1}{(2c) + 1} \right)^{1/2} \cdot (4ac - b^2) \right) \\ & - \left(\log\left(\frac{(x \cdot (-b + (b^2 - 4ac)^{1/2}) / (2c))^{1/2} - 1}{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1}\right)^{1/2} - 1 \right) \cdot i / \left((b + (b^2 - 4ac)^{1/2}) / (2c) + 1 \right)^{1/2} \\ & - \left(1 - x^2 \right)^{1/2} \cdot i / \left(x - \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right) \cdot \left(b^2 \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + a \cdot b \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} \right. \\ & \left. - 2ac \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{1/2}} + 2ac \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right. \\ & \left. + bc \cdot \frac{-(b + (b^2 - 4ac)^{1/2}) / (2c)}{(2c)^{3/2}} \right) / \left(2a \cdot (4ac - b^2) \cdot \left(\frac{(b + (b^2 - 4ac)^{1/2}) / (2c) + 1}{(2c) + 1} \right)^{1/2} \right) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a), x)`

[Out] `Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)`

$$3.284 \quad \int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=265

$$\frac{c \left(\frac{2a+b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}} \right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{c \left(1 - \frac{2a+b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}} \right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{1-x^2}}{ax}$$

Rubi [A] time = 0.78, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1295, 264, 1692, 377, 205}

$$\frac{c \left(\frac{2a+b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}} \right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{c \left(1 - \frac{2a+b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}} \right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)), x]

[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1295

Int[((f_)*(x_))^(m_)*((d_)+(e_)*(x_)^2)^(q_)/((a_)+(b_)*(x_)^2+(c_)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d+e*x^2)^(q-1), x], x] - Dist[1/(a*f^2), Int[((f*x)^(m+2)*(d+e*x^2)^(q-1)*Simp[b*d-a*e+c*d*x^2, x])/(a+b*x^2+c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2-4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]

Rule 1692

Int[(Px_)*((d_)+(e_)*(x_)^2)^(q_)*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(d+e*x^2)^q*(a+b*x^2+c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2-4*a*c, 0] && NeQ[c*d^2-b*d*e+a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)} dx &= \frac{\int \frac{1}{x^2\sqrt{1-x^2}} dx}{a} - \frac{\int \frac{a+bx^2}{\sqrt{1-x^2}(a+bx^2+cx^4)} dx}{a} \\
 &= -\frac{\sqrt{1-x^2}}{ax} - \frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} + \frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} \right) dx}{a} \\
 &= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{1-x^2}(b-\sqrt{b^2-4ac}+2cx^2)} dx}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{1-x^2}(b+\sqrt{b^2-4ac}+2cx^2)} dx}{a} \\
 &= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-(-b-2c-\sqrt{b^2-4ac})x^2} dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{a} - \frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{b+\sqrt{b^2-4ac}+(-b+2c+\sqrt{b^2-4ac})x^2} dx, x, \frac{x}{\sqrt{1-x^2}}}{a} \\
 &= -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}} \sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{1-x^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}} \sqrt{b+2c+\sqrt{b^2-4ac}}}
 \end{aligned}$$

Mathematica [B] time = 4.95, size = 2661, normalized size = 10.04

Result too large to show

$$\begin{aligned}
& 5*a*b^2)*c)*\sqrt{-x^2 + 1}*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*\sqrt{-x^2 + 1}*x - (a^3*b^3 - 4*a^4*b*c)*x) \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)} - 2*(a*c^2 - (a*b + b^2)*c)*\sqrt{-x^2 + 1))/x^2 \\
& - \sqrt{1/2}*a*x*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)}*\log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c) \\
&)*x^2 - 2*(a*b + b^2)*c - \sqrt{1/2}*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*\sqrt{-x^2 + 1}*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*x \\
& - ((a^3*b^3 - 4*a^4*b*c)*\sqrt{-x^2 + 1}*x - (a^3*b^3 - 4*a^4*b*c)*x)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)} - 2*(a*c^2 - (a*b + b^2)*c)*\sqrt{-x^2 + 1))/x^2 \\
& + \sqrt{1/2}*a*x*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)}*\log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c) \\
&)*x^2 - 2*(a*b + b^2)*c + \sqrt{1/2}*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*\sqrt{-x^2 + 1}*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*x \\
& + ((a^3*b^3 - 4*a^4*b*c)*\sqrt{-x^2 + 1}*x - (a^3*b^3 - 4*a^4*b*c)*x)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)} - 2*(a*c^2 - (a*b + b^2)*c)*\sqrt{-x^2 + 1))/x^2 \\
& - \sqrt{1/2}*a*x*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)}*\log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c) \\
&)*x^2 - 2*(a*b + b^2)*c - \sqrt{1/2}*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*\sqrt{-x^2 + 1}*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*x \\
& + ((a^3*b^3 - 4*a^4*b*c)*\sqrt{-x^2 + 1}*x - (a^3*b^3 - 4*a^4*b*c)*x)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c))})} \\
&)*\sqrt{((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)} - 2*(a*c^2 - (a*b + b^2)*c)*\sqrt{-x^2 + 1))/x^2 \\
& - 2*\sqrt{-x^2 + 1))/(a*x)
\end{aligned}$$

giac [B] time = 5.04, size = 3965, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")


```
[Out] 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5
*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 +
a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*
a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(
b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*
(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16
*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^
2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*s
qrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2
- 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c
+ 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c))*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^4
*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^3*b^4 - 2*a^4*b^4
- sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^2*b^5 - 2*a^3*b^5 - 12*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^6*c - 20*sqrt(2)*sqrt(2*a
^2 + a*b - sqrt(b^2 - 4*a*c))*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c))*a)*a^4*b^2*c + 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c))*a)*a^3*b^3*c + 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c))*a)*a^2*b^4*c - 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c))*a)*a^5*c^2 - 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c))*a)*a^4*b*c^2 - 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c))*a)*a^3*b^2*c^2 + 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(2*a^2
+ a*b - sqrt(b^2 - 4*a*c))*a)*a^4*c^3 - 32*a^5*c^3 + 2*(b^2 - 4*a*c)*a^4*b^
2 + 2*(b^2 - 4*a*c)*a^3*b^3 - 8*(b^2 - 4*a*c)*a^5*c - 8*(b^2 - 4*a*c)*a^4*b
*c + 2*(b^2 - 4*a*c)*a^3*b^2*c - 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arctan(-1
/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a^2 +
a*b + sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b^2 +
5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c + 10*a^
6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a^7*c^

```

$$\begin{aligned}
& 3) - \frac{1}{8} (4a^6b^3 + 6a^5b^4 + 2a^4b^5 - 16a^7bc - 32a^6b^2c - 12a^5b^3c + 32a^7c^2 + 16a^6b^2c^2 + 6\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^6b + 13\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5b^2 + 7\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4b^3 - \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^4 - \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^5 - 12\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^6c - 6\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5bc + 12\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4b^2c + 6\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^3c - 16\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5c^2 - 8\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4bc^2 - 4(b^2 - 4ac)a^6b - 6(b^2 - 4ac)a^5b^2 - 2(b^2 - 4ac)a^4b^3 + 8(b^2 - 4ac)a^6c + 4(b^2 - 4ac)a^5bc - (2a^3b^4 + 2a^2b^5 - 16a^4b^2c - 16a^3b^3c + 32a^5c^2 + 32a^4bc^2 + 3\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^2 + 5\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^3 + \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^4 - \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} b^5 - 12\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4c - 20\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3bc + 8\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^3c - 16\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3c^2 - 16\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2bc^2 - 2(b^2 - 4ac)a^3b^2 - 2(b^2 - 4ac)a^2b^3 + 8(b^2 - 4ac)a^4c + 8(b^2 - 4ac)a^3bc)a^2 - 2(3\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5b^2 + 5\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4b^3 + \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^4 + 2a^4b^4 - \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^5 + 2a^3b^5 - 12\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^6c - 20\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5bc + 3\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4b^2c - 16a^5b^2c + 10\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^3c - 16a^4b^3c - \sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^2b^4c + 2a^3b^4c - 28\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^5c^2 + 32a^6c^2 - 24\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4bc^2 + 32a^5bc^2 + 8\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^3b^2c^2 - 16a^4b^2c^2 - 16\sqrt{2}\sqrt{2a^2 + ab + \sqrt{b^2 - 4ac}}a) \sqrt{b^2 - 4ac} a^4c^3 + 32a^5c^3 - 2(b^2 - 4ac)a^4b^2 - 2(b^2 - 4ac)a^3b^3 + 8(b^2 - 4ac)a^5c + 8(b^2 - 4ac)a^4bc - 2(b^2 - 4ac)a^3b^2c + 8(b^2 - 4ac)a^4c^2) \operatorname{abs}(a) \operatorname{arctan}(-\frac{1}{2}\sqrt{2})(\frac{x}{\sqrt{-x^2 + 1}} - 1) - (\sqrt{-x^2 + 1} - 1)/x) / \sqrt{(2a^2 + ab - \sqrt{-4(a^2 + ab + ac)a^2 + (2a^2 + ab)^2})/a^2}) / (3a^8b^2 + 5a^7b^3 + a^6b^4 - a^5b^5 - 12a^9c - 20a^8bc + 3a^7b^2c + 10a^6b^3c - a^5b^4c - 28a^8c^2 - 24a^7bc^2 + 8a^6b^2c^2 - 16a
\end{aligned}$$

$^7*c^3) + 1/2*(x/\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)/a$

maple [C] time = 0.03, size = 217, normalized size = 0.82

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x)`

[Out]
$$-1/a/x*(-x^2+1)^{3/2}-1/a*x*(-x^2+1)^{1/2}-1/a*\arcsin(x)+1/4/a*\sum(((a+b)*_R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*\ln(-_R+((-x^2+1)^{1/2}-1)/x),_R=\text{RootOf}(_Z^8*a+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/a*\arctan(((x^2+1)^{1/2}-1)/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1}}{(cx^4 + bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)`

mupad [B] time = 1.21, size = 1234, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)`

[Out]
$$\begin{aligned} & (\log(\frac{(x*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} - 1)*i}{(b + (b^2 - 4*a*c)^{1/2})/2*c + 1})^{1/2} - (1 - x^2)^{1/2}*i)/(x - (-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2}) * (b^3*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} + a*b^2*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} - 2*a^2*c*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} - 2*a*c^2*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{3/2} + b^2*c*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{3/2} - 3*a*b*c*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} + a*b*c*(-(b + (b^2 - 4*a*c)^{1/2}))/2*c)^{3/2}) / (2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^{1/2}))/2*c + 1)^{1/2} - (1 - x^2)^{1/2} / (a*x) + (\log(\frac{(x*(-(b - (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} - 1)*i}{(b - (b^2 - 4*a*c)^{1/2}))/2*c + 1})^{1/2} - (1 - x^2)^{1/2}*i)/(x - (-(b - (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2}) * (b^3*(-(b - (b^2 - 4*a*c)^{1/2}))/2*c)^{1/2} \end{aligned}$$

$$\begin{aligned} & \left. \right)^{(1/2)} + a*b^2*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} - 2*a^2*c*(-(b - \\ & (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} - 2*a*c^2*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2* \\ & c))^{(3/2)} + b^2*c*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)} - 3*a*b*c*(-(b - \\ & (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} + a*b*c*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c \\ &))^{(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^{(1/2)))/(2*c) + 1)^{(1/2)}*(4*a*c - b^2) \\ &) - (\log(((x*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} + 1)*i)/((b + (b^2 \\ & - 4*a*c)^{(1/2)))/(2*c) + 1)^{(1/2)} + (1 - x^2)^{(1/2)}*i)/(x + (-(b + (b^2 - 4 \\ & *a*c)^{(1/2)))/(2*c))^{(1/2)}))*(b^3*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} + \\ & a*b^2*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} - 2*a^2*c*(-(b + (b^2 - 4*a \\ & *c)^{(1/2)))/(2*c))^{(1/2)} - 2*a*c^2*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)} \\ & + b^2*c*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)} - 3*a*b*c*(-(b + (b^2 - 4* \\ & a*c)^{(1/2)))/(2*c))^{(1/2)} + a*b*c*(-(b + (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2))} \\ & / (2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^{(1/2)))/(2*c) + 1)^{(1/2)}) - (\log((\\ & ((x*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} + 1)*i)/((b - (b^2 - 4*a*c)^{(\\ & 1/2)))/(2*c) + 1)^{(1/2)} + (1 - x^2)^{(1/2)}*i)/(x + (-(b - (b^2 - 4*a*c)^{(1/2) \\ &))/(2*c))^{(1/2)}))*(b^3*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} + a*b^2*(-(\\ & b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(1/2)} - 2*a^2*c*(-(b - (b^2 - 4*a*c)^{(1/2)) \\ &)/(2*c))^{(1/2)} - 2*a*c^2*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)} + b^2*c*(- \\ & (b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)} - 3*a*b*c*(-(b - (b^2 - 4*a*c)^{(1/2) \\ &))/(2*c))^{(1/2)} + a*b*c*(-(b - (b^2 - 4*a*c)^{(1/2)))/(2*c))^{(3/2)))/(2*a^2*((\\ & b - (b^2 - 4*a*c)^{(1/2)))/(2*c) + 1)^{(1/2)}*(4*a*c - b^2)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2(a+bx^2+cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a), x)

[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)

$$3.285 \quad \int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} dx$$

Optimal. Leaf size=96

$$\sqrt{\frac{1}{5}}(2 + \sqrt{5}) \tan^{-1}\left(\frac{\sqrt{\frac{1}{2}(1 + \sqrt{5})}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}}(\sqrt{5} - 2) \tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}(\sqrt{5} - 1)}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Rubi [A] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1293, 216, 1692, 377, 207, 203}

$$\sqrt{\frac{1}{5}}(2 + \sqrt{5}) \tan^{-1}\left(\frac{\sqrt{\frac{1}{2}(1 + \sqrt{5})}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}}(\sqrt{5} - 2) \tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}(\sqrt{5} - 1)}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4), x]

[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1293

Int[(((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1] && LeQ[m, 3]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{1-x^2}}{-1+x^2+x^4} dx &= - \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{1-2x^2}{\sqrt{1-x^2}(-1+x^2+x^4)} dx \\
 &= -\sin^{-1}(x) - \int \left(\frac{-2 + \frac{4}{\sqrt{5}}}{\sqrt{1-x^2}(1-\sqrt{5}+2x^2)} + \frac{-2 - \frac{4}{\sqrt{5}}}{\sqrt{1-x^2}(1+\sqrt{5}+2x^2)} \right) dx \\
 &= -\sin^{-1}(x) + \frac{1}{5}(2(5-2\sqrt{5})) \int \frac{1}{\sqrt{1-x^2}(1-\sqrt{5}+2x^2)} dx + \frac{1}{5}(2(5+2\sqrt{5})) \int \frac{1}{\sqrt{1-x^2}(1+\sqrt{5}+2x^2)} dx \\
 &= -\sin^{-1}(x) + \frac{1}{5}(2(5-2\sqrt{5})) \text{Subst} \left(\int \frac{1}{1-\sqrt{5}-(-3+\sqrt{5})x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right) + \frac{1}{5}(2(5+2\sqrt{5})) \text{Subst} \left(\int \frac{1}{1+\sqrt{5}-(-3+\sqrt{5})x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right) \\
 &= -\sin^{-1}(x) + \sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1} \left(\frac{\sqrt{\frac{1}{2}(1+\sqrt{5})}x}{\sqrt{1-x^2}} \right) - \sqrt{\frac{1}{5}(-2+\sqrt{5})} \tanh^{-1} \left(\frac{\sqrt{\frac{1}{2}(-1+\sqrt{5})}x}{\sqrt{1-x^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.50, size = 743, normalized size = 7.74

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]

[Out] (-2*Sqrt[5]*ArcSin[x] + (-2 + Sqrt[5])*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sqrt[5])/2] + x] + 2*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - Sqrt[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - (2*I)*Sqrt[-2 + Sqrt[5]]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - I*Sqrt[5*(-2 + Sqrt[5])]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] + (2*I)*Sqrt[-2 + Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (2*I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (2*I)*Sqrt[-2 + Sqrt[5]]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - I*Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] + 2*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - Sqrt[5*(2 + Sqrt[5])]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 2*Sqrt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] + Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(2*Sqrt[5])

IntegrateAlgebraic [B] time = 0.59, size = 202, normalized size = 2.10

$$-2 \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right) + \sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\sqrt{5}-2}x}{\sqrt{1-x^2}-1}\right) + \sqrt{\frac{1}{5}(2+\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{5}}x}{\sqrt{1-x^2}-1}\right) + \sqrt{\frac{1}{5}(\sqrt{5}-2)} \tanh^{-1}\left(\frac{\sqrt{\sqrt{5}-2}x}{\sqrt{1-x^2}-1}\right) - \sqrt{\frac{1}{5}(\sqrt{5}-2)} \tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}x}{\sqrt{1-x^2}-1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]

[Out] -2*ArcTan[x/(-1 + Sqrt[1 - x^2])] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[-2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[-2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])]

fricas [B] time = 1.07, size = 290, normalized size = 3.02

$$\frac{2}{5} \sqrt{5} \sqrt{\sqrt{5}+2} \arctan\left(\frac{\sqrt{2}(\sqrt{\sqrt{5}+1}(\sqrt{5}-3)+\sqrt{5}-3)\sqrt{\sqrt{5}+2}\sqrt{\frac{2x^2-\sqrt{5}x-1}{4x}}}{2\sqrt{\sqrt{5}+1}\sqrt{5}+2\sqrt{\sqrt{5}+1}\sqrt{5}+2\sqrt{\sqrt{5}+1}}\right) + \frac{1}{10} \sqrt{5} \sqrt{\sqrt{5}-2} \log\left(\frac{2x^2+(\sqrt{\sqrt{5}+1}(\sqrt{5}x+1)-\sqrt{5}x-1)\sqrt{\sqrt{5}-2+2\sqrt{\sqrt{5}+1}-2}}{x^2}\right) - \frac{1}{10} \sqrt{5} \sqrt{\sqrt{5}-2} \log\left(\frac{2x^2-(\sqrt{\sqrt{5}+1}(\sqrt{5}x+1)-\sqrt{5}x-1)\sqrt{\sqrt{5}-2+2\sqrt{\sqrt{5}+1}-2}}{x^2}\right) + 2 \arctan\left(\frac{\sqrt{\sqrt{5}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")

[Out] 2/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*(sqrt(2)*(sqrt(-x^2 + 1)*(sqrt(5) - 3) + sqrt(5) - 3)*sqrt(sqrt(5) + 2)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*x^2) - 2*(sqrt(5)*x^2 - x^2 + 2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1))*sqrt(sqrt(5) + 2)*(sqrt(5) - 3))/x + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(

$$-(2*x^2 + (\sqrt{-x^2 + 1})*(\sqrt{5}*x + x) - \sqrt{5}*x - x)*\sqrt{\sqrt{5} - 2} + 2*\sqrt{-x^2 + 1} - 2)/x^2) - 1/10*\sqrt{5}*\sqrt{\sqrt{5} - 2}*\log(-(2*x^2 - (\sqrt{-x^2 + 1})*(\sqrt{5}*x + x) - \sqrt{5}*x - x)*\sqrt{\sqrt{5} - 2} + 2*\sqrt{-x^2 + 1} - 2)/x^2) + 2*\arctan((\sqrt{-x^2 + 1} - 1)/x)$$

giac [B] time = 0.72, size = 209, normalized size = 2.18

$$\frac{1}{2} \pi \operatorname{sgn}(x) - \frac{1}{5} \sqrt{5\sqrt{5} + 10} \arctan\left(\frac{x}{\sqrt{2+\sqrt{5}}}\right) - \frac{1}{10} \sqrt{5\sqrt{5} - 10} \log\left(\left|\sqrt{2\sqrt{5} - 2} - \frac{x}{\sqrt{-x^2 + 1}} + \frac{\sqrt{-x^2 + 1} - 1}{x}\right|\right) + \frac{1}{10} \sqrt{5\sqrt{5} - 10} \log\left(\left|-\sqrt{2\sqrt{5} - 2} - \frac{x}{\sqrt{-x^2 + 1}} + \frac{\sqrt{-x^2 + 1} - 1}{x}\right|\right) - \arctan\left(\frac{x\left(\frac{(\sqrt{-x^2 + 1})^2}{x^2} - 1\right)}{2(\sqrt{-x^2 + 1} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")

[Out]
$$-1/2*\pi*\operatorname{sgn}(x) - 1/5*\sqrt{5*\sqrt{5} + 10}*\arctan(-(x/(\sqrt{-x^2 + 1} - 1) - (\sqrt{-x^2 + 1} - 1)/x)/\sqrt{2*\sqrt{5} + 2})) - 1/10*\sqrt{5*\sqrt{5} - 10}*\log(\operatorname{abs}(\sqrt{2*\sqrt{5} - 2} - x/(\sqrt{-x^2 + 1} - 1) + (\sqrt{-x^2 + 1} - 1)/x)) + 1/10*\sqrt{5*\sqrt{5} - 10}*\log(\operatorname{abs}(-\sqrt{2*\sqrt{5} - 2} - x/(\sqrt{-x^2 + 1} - 1) + (\sqrt{-x^2 + 1} - 1)/x)) - \arctan(-1/2*x*((\sqrt{-x^2 + 1} - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1} - 1))$$

maple [B] time = 0.09, size = 160, normalized size = 1.67

$$-\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{2+\sqrt{5}}x}\right)}{5\sqrt{2+\sqrt{5}}} + \frac{\sqrt{\sqrt{5}-2}\sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{\sqrt{5}-2}x}\right)}{5} + 2\operatorname{arctan}\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \frac{\sqrt{2+\sqrt{5}}\sqrt{5} \operatorname{arctan}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{2+\sqrt{5}}x}\right)}{5} - \frac{\sqrt{5} \operatorname{arctan}\left(\frac{\sqrt{-x^2+1}-1}{\sqrt{\sqrt{5}-2}x}\right)}{5\sqrt{\sqrt{5}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x)

[Out]
$$-1/5*(2+5^{(1/2)})^{(1/2)}*5^{(1/2)}*\arctan(((x^2+1)^{(1/2)}-1)/x/(2+5^{(1/2)}))^{(1/2)}) + 1/5*(5^{(1/2)}-2)^{(1/2)}*5^{(1/2)}*\operatorname{arctanh}(((x^2+1)^{(1/2)}-1)/x/(5^{(1/2)}-2))^{(1/2)}) - 1/5*5^{(1/2)}/(2+5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(((x^2+1)^{(1/2)}-1)/x/(2+5^{(1/2)}))^{(1/2)}) - 1/5*5^{(1/2)}/(5^{(1/2)}-2)^{(1/2)}*\arctan(((x^2+1)^{(1/2)}-1)/x/(5^{(1/2)}-2))^{(1/2)}) + 2*\operatorname{arctan}(((x^2+1)^{(1/2)}-1)/x)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1} x^2}{x^4 + x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)

mupad [B] time = 1.50, size = 383, normalized size = 3.99

$$\begin{aligned}
 & \frac{\ln\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{11}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}\sqrt{1-x^2}\right)}{x\sqrt{\frac{\sqrt{5}-1}{2}}}\left(\sqrt{5}-2\right) + \frac{\ln\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{11}}{\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}}\sqrt{1-x^2}\right)}{x\sqrt{\frac{\sqrt{5}-1}{2}}}\left(\sqrt{5}+2\right) \\
 & + \frac{\ln\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}+1\right)^{11}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}\sqrt{1-x^2}\right)}{x+\sqrt{\frac{\sqrt{5}-1}{2}}}\left(\sqrt{5}-2\right) + \frac{\ln\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}+1\right)^{11}}{\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}}\sqrt{1-x^2}\right)}{x+\sqrt{\frac{\sqrt{5}-1}{2}}}\left(\sqrt{5}+2\right) \\
 & - \frac{\operatorname{asin}(x)}{\left(2\sqrt{\frac{\sqrt{5}-1}{2}}+4\left(\frac{\sqrt{5}-1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} + \frac{\operatorname{asin}(x)}{\left(2\sqrt{-\frac{\sqrt{5}-1}{2}}+4\left(-\frac{\sqrt{5}-1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}} + \frac{\operatorname{asin}(x)}{\left(2\sqrt{\frac{\sqrt{5}-1}{2}}+4\left(\frac{\sqrt{5}-1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} - \frac{\operatorname{asin}(x)}{\left(2\sqrt{-\frac{\sqrt{5}-1}{2}}+4\left(-\frac{\sqrt{5}-1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1), x)`

[Out]
$$\begin{aligned}
 & \left(\log\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{11}}{\left(5^{1/2}/2+3/2\right)^{1/2}}\right)-\left(1-x^2\right)^{1/2}\right) \cdot \frac{\left(5^{1/2}+2\right)}{\left(2\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{1/2}+4\sqrt{\frac{\sqrt{5}-1}{2}}\left(\frac{\sqrt{5}-1}{2}\right)^{3/2}} \\
 & - \left(\log\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}+1\right)^{11}}{\left(3/2-5^{1/2}/2\right)^{1/2}}\right)-\left(1-x^2\right)^{1/2}\right) \cdot \frac{\left(5^{1/2}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}-1}{2}}+1\right)^{1/2}+4\sqrt{\frac{\sqrt{5}-1}{2}}\left(\frac{\sqrt{5}-1}{2}\right)^{3/2}} \\
 & + \left(\log\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}+1\right)^{11}}{\left(3/2-5^{1/2}/2\right)^{1/2}}\right)+\left(1-x^2\right)^{1/2}\right) \cdot \frac{\left(5^{1/2}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{1/2}+4\sqrt{\frac{\sqrt{5}-1}{2}}\left(\frac{\sqrt{5}-1}{2}\right)^{3/2}} \\
 & - \left(\log\left(\frac{\left(x\sqrt{\frac{\sqrt{5}-1}{2}}-1\right)^{11}}{\left(5^{1/2}/2+3/2\right)^{1/2}}\right)+\left(1-x^2\right)^{1/2}\right) \cdot \frac{\left(5^{1/2}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}-1}{2}}-1\right)^{1/2}+4\sqrt{-\frac{\sqrt{5}-1}{2}}\left(-\frac{\sqrt{5}-1}{2}\right)^{3/2}}
 \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-(x-1)(x+1)}}{x^4 + x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1), x)`

[Out] `Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)`

$$3.286 \quad \int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=479

$$\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Rubi [A] time = 1.86, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}} - 2abc + b^3\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + \frac{(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) + b\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - b\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{8c^{5/2}} - \frac{3dx\sqrt{d+ex^2}}{8c^2} + \frac{x^3\sqrt{d+ex^2}}{4c}}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]

[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan[h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^3*Sqrt[e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1303

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && IntegerQ[m]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx &= \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a(b^2-ac)+b(b^2-2ac)x^2}{c^3\sqrt{d+ex^2} (a+bx^2+cx^4)} \right) dx \\
&= -\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx}{c^3} - \frac{b \int \frac{x^2}{\sqrt{d+ex^2}} dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}} dx}{c} + \frac{(b^2-ac) \int \frac{1}{\sqrt{d+ex^2}} dx}{c^3} \\
&= -\frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\int \left(\frac{b(b^2-2ac)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{b(b^2-2ac)-\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{c^3} \\
&= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{(b^2-ac) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}} \\
&= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{bd \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}} + \frac{(b^2-ac) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}} \\
&= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd}}
\end{aligned}$$

Mathematica [A] time = 1.87, size = 461, normalized size = 0.96

$$\frac{8\left(-\frac{2a^2c^2-4a^2c^2+4a^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{\frac{2\sqrt{e}\sqrt{b^2-4ac}-b+2cd}{\sqrt{b-\sqrt{b^2-4ac}}}\sqrt{d+ex^2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)-8\left(\frac{2a^2c^2-4a^2c^2+4a^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{\frac{2\sqrt{e}\sqrt{b^2-4ac}+b-2cd}{\sqrt{b+\sqrt{b^2-4ac}}}\sqrt{d+ex^2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e}\sqrt{2cd-(b^2-4ac)+b}}+\frac{8(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{4bcd\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{4bcx\sqrt{d+ex^2}}{e}+\frac{3c^2d\left(d\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)-\sqrt{ex}\sqrt{d+ex^2}\right)}{e^{3/2}}+\frac{2c^2x^3\sqrt{d+ex^2}}{e}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]

[Out] ((-4*b*c*x*Sqrt[d + e*x^2])/e + (2*c^2*x^3*Sqrt[d + e*x^2])/e - (8*(b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (8*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[

$$b^2 - 4ac) * e) + (4 * b * c * d * \text{ArcTanh}[(\text{Sqrt}[e] * x) / \text{Sqrt}[d + e * x^2]]) / e^{(3/2)} + (8 * (b^2 - a * c) * \text{ArcTanh}[(\text{Sqrt}[e] * x) / \text{Sqrt}[d + e * x^2]]) / \text{Sqrt}[e] + (3 * c^2 * d * (-\text{Sqrt}[e] * x * \text{Sqrt}[d + e * x^2]) + d * \text{ArcTanh}[(\text{Sqrt}[e] * x) / \text{Sqrt}[d + e * x^2]])) / e^{(5/2)})) / (8 * c^3)$$

IntegrateAlgebraic [C] time = 60.19, size = 538, normalized size = 1.12

$\sqrt{d+ex^2} \sqrt{e} + 4d\sqrt{e} - 4d^2\sqrt{e} + 16d^2\sqrt{e} - 8d^3\sqrt{e} + 4d^4\sqrt{e} - 4d^5\sqrt{e} + 2d^6$ $\frac{2d^5 \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right) + 4d^4 \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right) + 8d^3 \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right) + 16d^2 \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right) + 32d \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right) + 64 \text{ArcTanh}\left(\frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2} + d}\right)}{e^2 \sqrt{d+ex^2} \sqrt{e} + 4d^2 \sqrt{e} - 4d^3 \sqrt{e} + 16d^4 \sqrt{e} - 8d^5 \sqrt{e} + 4d^6}$ $\log\left(\frac{\sqrt{d+ex^2} + \sqrt{e}}{\sqrt{d+ex^2} - \sqrt{e}}\right) \left(\log\left(\frac{\sqrt{d+ex^2} + \sqrt{e}}{\sqrt{d+ex^2} - \sqrt{e}}\right) - 4d\sqrt{e} - 3d^2\sqrt{e}\right) \frac{\sqrt{d+ex^2} (-4d\sqrt{e} - 3d^2\sqrt{e})}{8d^3}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[d + e*x^2]*(-3*c*d*x - 4*b*e*x + 2*c*e*x^3))/(8*c^2*e^2) + ((-3*c^2*d^2 - 4*b*c*d*e - 8*b^2*e^2 + 8*a*c*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(8*c^3*e^(5/2)) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^3*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*a*b*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^3*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a*b*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*a*b^2*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 4*a^2*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^3*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - 2*a*b*c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) &])/(2*c^3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.03, size = 105, normalized size = 0.22

$$\frac{1}{8} \sqrt{x^2 e + d} \left(\frac{2 x^2 e^{(-1)}}{c} - \frac{(3 c^5 d e + 4 b c^4 e^2) e^{(-3)}}{c^6} \right) x - \frac{(3 c^2 d^2 + 4 b c d e + 8 b^2 e^2 - 8 a c e^2) e^{(-\frac{5}{2})} \log \left(\left(x e^{\frac{1}{2}} - \sqrt{x^2 e + d} \right)^2 \right)}{16 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{8}\sqrt{x^2e + d}(2x^2e^{-1})/c - (3c^5de + 4b^4c^4e^2)e^{-3}/c^6 * x - 1/16(3c^2d^2 + 4b^2cd^2 + 8b^2e^2 - 8ac^2e^2)e^{-5/2}\log((xe^{1/2} - \sqrt{x^2e + d})^2)/c^3$

maple [C] time = 0.03, size = 377, normalized size = 0.79

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^{(1/2)}, x)$

[Out] $\frac{1}{4}x^3(e*x^2+d)^{(1/2)}/c/e-3/8*d*x*(e*x^2+d)^{(1/2)}/c/e^2+3/8/c*d^2/e^{(5/2)} * \ln(e^{(1/2)*x+(e*x^2+d)^{(1/2)})}-1/2*b*x*(e*x^2+d)^{(1/2)}/c^2/e+1/2/c^2*b*d/e^{(3/2)} * \ln(e^{(1/2)*x+(e*x^2+d)^{(1/2)})}-1/c^2*a*\ln(e^{(1/2)*x+(e*x^2+d)^{(1/2)})/e^{(1/2)}+1/c^3*b^2*\ln(e^{(1/2)*x+(e*x^2+d)^{(1/2)})/e^{(1/2)}-1/2/c^3*e^{(1/2)} * \text{sum}((b*(2*a*c-b^2)*_R^2+2*(2*a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-b^3*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)*x+(e*x^2+d)^{(1/2)})^2}), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x^8/((c*x^4 + b*x^2 + a)*\sqrt{e*x^2 + d}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/((d + e*x^2)^{(1/2)}*(a + b*x^2 + c*x^4)), x)$

[Out] $\text{int}(x^8/((d + e*x^2)^{(1/2)}*(a + b*x^2 + c*x^4)), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

$$3.287 \quad \int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=366

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + b \tanh^{-1}\left(\frac{x\sqrt{d+ex^2}}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{b \tanh^{-1}\left(\frac{x\sqrt{d+ex^2}}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}$$

Rubi [A] time = 1.17, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) + \left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) - \frac{b \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} + \frac{x\sqrt{d+ex^2}}{2ce}}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} + c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a, 0]$

Rule 321

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] \text{ /; FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 377

$\text{Int}[((a_) + (b_)*(x_)^{(n_)})^{(p_)}/((c_) + (d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] \text{ /; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$

Rule 1303

$\text{Int}[(((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(q_)})/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^q, (f*x)^m/(a + b*x^2 + c*x^4), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!IntegerQ}[q] \&\& \text{IntegerQ}[m]$

Rule 1692

$\text{Int}[(P_x)*((d_) + (e_)*(x_)^2)^{(q_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[P_x*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{PolyQ}[P_x, x^2] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx &= \int \left(-\frac{b}{c^2\sqrt{d+ex^2}} + \frac{x^2}{c\sqrt{d+ex^2}} + \frac{ab+(b^2-ac)x^2}{c^2\sqrt{d+ex^2}(a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{ab+(b^2-ac)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{c^2} - \frac{b \int \frac{1}{\sqrt{d+ex^2}} dx}{c^2} + \frac{\int \frac{x^2}{\sqrt{d+ex^2}} dx}{c} \\
&= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\int \left(\frac{b^2-ac+\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{b^2-ac-\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{c^2} - \frac{b \operatorname{Subst} \int \frac{1}{\sqrt{d+ex^2}} dx}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{b \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{d \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \operatorname{Subst} \int \frac{1}{\sqrt{d+ex^2}} dx}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})e}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-(b+\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [A] time = 1.03, size = 355, normalized size = 0.97

$$\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right) \tan^{-1}\left(\frac{x\sqrt{e}\sqrt{b^2-4ac-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e}\sqrt{b^2-4ac-b}+2cd} + \frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e}\sqrt{b^2-4ac+b}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-e}\sqrt{b^2-4ac+b}} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{cd \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} + \frac{cx\sqrt{d+ex^2}}{e}$$

2c²

Antiderivative was successfully verified.

[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]

[Out] ((c*x*Sqrt[d + e*x^2])/e + (2*(b^2 - a*c - (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b +

$\text{Sqrt}[b^2 - 4*a*c]*e) - (c*d*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/e^{(3/2)}$
 $) - (2*b*\text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]])/\text{Sqrt}[e]/(2*c^2)$

IntegrateAlgebraic [C] time = 45.10, size = 458, normalized size = 1.25

$$\frac{\sqrt{c}\text{RootSum}\left[\#1^3c + 4\#1^2bc - 4\#1^2cd + 16\#1^2a^2 - 8\#1^2bdc + 6\#1^2cd^2 + 4\#1^2bd^2c - 4\#1^2cd^3 + cd^3e, \frac{-\#1^4e\log\left(\frac{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)+\#1^4b\log\left(\frac{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)+\#1^4c\log\left(\frac{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)+\#1^4d\log\left(\frac{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)+\#1^4e\log\left(\frac{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}{\#1+\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)}{\#1^4c+4\#1^3bc-4\#1^3cd+16\#1^2a^2-8\#1^2bdc+6\#1^2cd^2+4\#1^2bd^2c-4\#1^2cd^3+cd^3e}\right]}{2c^2}, \frac{(2b+cd)\log\left(\frac{\sqrt{d+e}\sqrt{e}\sqrt{c}}{\sqrt{d+e}\sqrt{e}\sqrt{c}}\right)}{2c^2}, \frac{\sqrt{d+e}\sqrt{e}\sqrt{c}}{2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] $(x*\text{Sqrt}[d + e*x^2])/(2*c*e) + ((c*d + 2*b*e)*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2]])/(2*c^2*e^{(3/2)}) - (\text{Sqrt}[e]*\text{RootSum}[c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b*e*\#1^6 + c*\#1^8 \& , (b^2*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - a*c*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - 2*b^2*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 2*a*c*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*a*b*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + b^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 - a*c*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*\#1^2 - 4*b*d*e*\#1^2 + 8*a*e^2*\#1^2 - 3*c*d*\#1^4 + 3*b*e*\#1^4 + c*\#1^6) \&])/(2*c^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.14, size = 55, normalized size = 0.15

$$\frac{\sqrt{x^2e + d}xe^{(-1)}}{2c} + \frac{(cd + 2be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] $1/2*\text{sqrt}(x^2*e + d)*x*e^{(-1)}/c + 1/4*(c*d + 2*b*e)*e^{(-3/2)}*\log((x*e^{(1/2)} - \text{sqrt}(x^2*e + d))^2)/c^2$

maple [C] time = 0.03, size = 269, normalized size = 0.73

$$\frac{\sqrt{x^2e + d}xe^{(-1)}}{2c} + \frac{(cd + 2be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)`

[Out] $\frac{1}{2}x(e^{\frac{1}{2}}x^2+d)^{\frac{1}{2}}/c/e^{-\frac{1}{2}}/c*d/e^{\frac{3}{2}}*\ln(e^{\frac{1}{2}}*x+(e^{\frac{1}{2}}x^2+d)^{\frac{1}{2}})-1/c^2*b*\ln(e^{\frac{1}{2}}*x+(e^{\frac{1}{2}}x^2+d)^{\frac{1}{2}})/e^{\frac{1}{2}}+1/2/c^2*e^{\frac{1}{2}}*\text{sum}(((a*c-b^2)*_R^2+2*(-2*a*b*e-a*c*d+b^2*d)*_R+a*c*d^2-b^2*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{\frac{1}{2}}*x+(e^{\frac{1}{2}}x^2+d)^{\frac{1}{2}})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)`

[Out] `int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)`

[Out] `Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)`

$$3.288 \quad \int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=298

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} - c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Rubi [A] time = 0.72, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1303, 217, 206, 1692, 377, 205}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} - c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2])/(c*Sqrt[e])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx &= \int \left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}(a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{1}{\sqrt{d+ex^2}} dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{c} \\
&= -\frac{\int \left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{c} + \frac{\text{Subst}\left(\int \frac{1}{1-ex^2} dx\right)}{c} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{c} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-(-2cd+(b-\sqrt{b^2-4ac})e)x^2} dx\right)}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}+(-2cd+(b+\sqrt{b^2-4ac})e)x^2} dx\right)}{c} \\
&= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-(b+\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 292, normalized size = 0.98

$$\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right) \tan^{-1}\left(\frac{x\sqrt{e}\sqrt{b^2-4ac-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e}\left(\sqrt{b^2-4ac}-b\right)+2cd} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{b^2-4ac}+b\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}+b\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

c

Antiderivative was successfully verified.

[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]

[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[e]

$4*a*c]]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e)) + \text{ArcTanh}[(\text{Sqrt}[e]*x)/\text{Sqrt}[d + e*x^2]]/\text{Sqrt}[e])/c$

IntegrateAlgebraic [C] time = 30.97, size = 320, normalized size = 1.07

$$\frac{\sqrt{e} \text{RootSum}\left[\#1^5 c + 4\#1^4 b e - 4\#1^3 c d + 16\#1^2 a e^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{\#1^3 b \log(-\#1 + \sqrt{d+ex^2} - \sqrt{ex}) + 4\#1^2 b e \log(-\#1 + \sqrt{d+ex^2} - \sqrt{ex}) - 2\#1^2 b d \log(-\#1 + \sqrt{d+ex^2} - \sqrt{ex}) + b d^2 \log(-\#1 + \sqrt{d+ex^2} - \sqrt{ex})}{\#1^5 c + 3\#1^4 b e - 3\#1^3 c d + 8\#1^2 a e^2 - 4\#1^2 b d e + 3\#1^2 c d^2 + b d^2 e - c d^3}\right]}{2c} - \frac{\log(\sqrt{d+ex^2} - \sqrt{ex})}{c\sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] $-(\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2]]/(c*\text{Sqrt}[e])) + (\text{Sqrt}[e]*\text{RootSum}[c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b*e*\#1^6 + c*\#1^8 \&, (b*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - 2*b*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*a*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + b*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*\#1^2 - 4*b*d*e*\#1^2 + 8*a*e^2*\#1^2 - 3*c*d*\#1^4 + 3*b*e*\#1^4 + c*\#1^6) \&])/(2*c)$

fricas [B] time = 51.77, size = 11094, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] $[1/4*(\text{sqrt}(1/2)*c*e*\text{sqrt}(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*\text{sqrt}((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e))/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\text{log}((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*\text{sqrt}((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e))/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*\text{sqrt}((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e))/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e$

$$\begin{aligned}
& + (a^2b^3 - 4a^3bc)e^2)x) \sqrt{-(b^3 - 3abc)d - (ab^2 - 2a^2c)e} \\
& - ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& / ((b^2c^6 - 4a^7c)d^4 - 2(b^3c^5 - 4abc^6)d^3e + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4) \\
& / ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) / x^2) - \sqrt{1/2}ce \sqrt{-(b^3 - 3abc)d - (ab^2 - 2a^2c)e} \\
& - ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& / ((b^2c^6 - 4a^7c)d^4 - 2(b^3c^5 - 4abc^6)d^3e + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4) \\
& / ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \log((2a^3bd^2e + ((ab^2c^3 - 4a^2c^4)d^3 - (ab^3c^2 - 4a^2bc^3)d^2e + (a^2b^2c^2 - 4a^3c^3)d^2e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& - 2(a^2b^2 - a^3c)d^2 + (4a^3b^2e^2 + (ab^3 - a^2bc)d^2 - (5a^2b^2 - 4a^3c)d^2e) \sqrt{1/2} \sqrt{ex^2 + d} \left((b^4c^3 - 6ab^2c^4 + 8a^2c^5)d^3 - (b^5c^2 - 5ab^3c^3 + 4a^2b^4c^4)d^2e + 2(ab^4c^2 - 5a^2b^2c^3 + 4a^3c^4)d^2e^2 - (a^2b^3c^2 - 4a^3b^2c^3)e^3 \right) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& + ((b^5 - 5ab^3c + 4a^2b^2c^2)d^2 - (2ab^4 - 9a^2b^2c + 4a^3c^2)d^2e + (a^2b^3 - 4a^3bc)e^2) \sqrt{-(b^3 - 3abc)d - (ab^2 - 2a^2c)e} \\
& - ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& / ((b^2c^6 - 4a^7c)d^4 - 2(b^3c^5 - 4abc^6)d^3e + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4) \\
& / ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) / x^2) - \sqrt{1/2}ce \sqrt{-(b^3 - 3abc)d - (ab^2 - 2a^2c)e} \\
& + ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& / ((b^2c^6 - 4a^7c)d^4 - 2(b^3c^5 - 4abc^6)d^3e + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4) \\
& / ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \log((2a^3bd^2e - ((ab^2c^3 - 4a^2c^4)d^3 - (ab^3c^2 - 4a^2bc^3)d^2e + (a^2b^2c^2 - 4a^3c^3)d^2e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& - 2(a^2b^2 - a^3c)d^2 + (4a^3b^2e^2 + (ab^3 - a^2bc)d^2 - (5a^2b^2 - 4a^3c)d^2e) \sqrt{1/2} \sqrt{ex^2 + d} \left((b^4c^3 - 6ab^2c^4 + 8a^2c^5)d^3 - (b^5c^2 - 5ab^3c^3 + 4a^2b^4c^4)d^2e + 2(ab^4c^2 - 5a^2b^2c^3 + 4a^3c^4)d^2e^2 - (a^2b^3c^2 - 4a^3b^2c^3)e^3 \right) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& + ((b^5 - 5ab^3c + 4a^2b^2c^2)d^2 - (2ab^4 - 9a^2b^2c + 4a^3c^2)d^2e + (a^2b^3 - 4a^3bc)e^2) \sqrt{-(b^3 - 3abc)d - (ab^2 - 2a^2c)e} \\
& - ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2) \sqrt{(a^2b^2e^2 + (b^4 - 2ab^2c + a^2c^2)d^2 - 2(ab^3 - a^2bc)d^2 + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4)} \\
& / ((b^2c^6 - 4a^7c)d^4 - 2(b^3c^5 - 4abc^6)d^3e + (b^4c^4 - 2ab^2c^5 - 8a^2c^6)d^2e^2 - 2(ab^3c^4 - 4a^2bc^5)d^2e^3 + (a^2b^2c^4 - 4a^3c^5)e^4) \\
& / ((b^2c^3 - 4a^4c)d^2 - (b^3c^2 - 4abc^3)d^2 + (ab^2c^2 - 4a^2c^3)e^2)
\end{aligned}$$

$$\begin{aligned}
& 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*\sqrt{-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))/x^2) + \sqrt{1/2}*c*e*\sqrt{-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*\sqrt{-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\sqrt{((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))/x^2) + 2*\sqrt{e}*\log(-2*e*x^2 - 2*\sqrt{e*x^2 + d}*\sqrt{e}*x - d)/(c*e), 1/4*(\sqrt{1/2}*c*e*\sqrt{-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*\sqrt{((
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 e^2 + (b^4 - 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 - a^2 b c) d e) / ((\\
& b^2 c^6 - 4 a c^7) d^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + (b^4 c^4 - 2 a b^2 \\
& c^5 - 8 a^2 c^6) d^2 e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d e^3 + (a^2 b^2 c^4 \\
& - 4 a^3 c^5) e^4)) / ((b^2 c^3 - 4 a c^4) d^2 - (b^3 c^2 - 4 a b c^3) d e \\
& + (a b^2 c^2 - 4 a^2 c^3) e^2)) * \log((2 a^3 b d e + ((a b^2 c^3 - 4 a^2 c^4) \\
& d^3 - (a b^3 c^2 - 4 a^2 b c^3) d^2 e + (a^2 b^2 c^2 - 4 a^3 c^3) d e^2) * x \\
& ^2 * \sqrt{(a^2 b^2 e^2 + (b^4 - 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 - a^2 b c) \\
&) d e) / ((b^2 c^6 - 4 a c^7) d^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + (b^4 c^4 \\
& - 2 a b^2 c^5 - 8 a^2 c^6) d^2 e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d e^3 + (a \\
& ^2 b^2 c^4 - 4 a^3 c^5) e^4)) - 2 (a^2 b^2 - a^3 c) d^2 + (4 a^3 b e^2 + (a \\
& * b^3 - a^2 b c) d^2 - (5 a^2 b^2 - 4 a^3 c) d e) * x^2 + 2 * \sqrt{1/2} * \sqrt{e * x \\
& ^2 + d} * ((b^4 c^3 - 6 a b^2 c^4 + 8 a^2 c^5) d^3 - (b^5 c^2 - 5 a b^3 c^3 \\
& + 4 a^2 b c^4) d^2 e + 2 (a b^4 c^2 - 5 a^2 b^2 c^3 + 4 a^3 c^4) d e^2 - (a \\
& ^2 b^3 c^2 - 4 a^3 b c^3) e^3) * x * \sqrt{(a^2 b^2 e^2 + (b^4 - 2 a b^2 c + a^2 \\
& c^2) d^2 - 2 (a b^3 - a^2 b c) d e) / ((b^2 c^6 - 4 a c^7) d^4 - 2 (b^3 c^5 \\
& - 4 a b c^6) d^3 e + (b^4 c^4 - 2 a b^2 c^5 - 8 a^2 c^6) d^2 e^2 - 2 (a b^3 \\
& c^4 - 4 a^2 b c^5) d e^3 + (a^2 b^2 c^4 - 4 a^3 c^5) e^4)) + ((b^5 - 5 a b \\
& ^3 c + 4 a^2 b c^2) d^2 - (2 a b^4 - 9 a^2 b^2 c + 4 a^3 c^2) d e + (a^2 b^3 \\
& - 4 a^3 b c) e^2) * x) * \sqrt{-((b^3 - 3 a b c) d - (a b^2 - 2 a^2 c) e - ((b \\
& ^2 c^3 - 4 a c^4) d^2 - (b^3 c^2 - 4 a b c^3) d e + (a b^2 c^2 - 4 a^2 c^3) \\
& e^2) * \sqrt{(a^2 b^2 e^2 + (b^4 - 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 - a^2 b \\
& c) d e) / ((b^2 c^6 - 4 a c^7) d^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + (b^4 c^4 \\
& - 2 a b^2 c^5 - 8 a^2 c^6) d^2 e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d e^3 + \\
& (a^2 b^2 c^4 - 4 a^3 c^5) e^4)) / ((b^2 c^3 - 4 a c^4) d^2 - (b^3 c^2 - 4 a \\
& b c^3) d e + (a b^2 c^2 - 4 a^2 c^3) e^2)) / x^2) - \sqrt{1/2} * c * e * \sqrt{-((b \\
& ^3 - 3 a b c) d - (a b^2 - 2 a^2 c) e - ((b^2 c^3 - 4 a c^4) d^2 - (b^3 c^2 \\
& - 4 a b c^3) d e + (a b^2 c^2 - 4 a^2 c^3) e^2) * \sqrt{(a^2 b^2 e^2 + (b^4 - \\
& 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 - a^2 b c) d e) / ((b^2 c^6 - 4 a c^7) d \\
& ^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + (b^4 c^4 - 2 a b^2 c^5 - 8 a^2 c^6) d^2 \\
& e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d e^3 + (a^2 b^2 c^4 - 4 a^3 c^5) e^4)) \\
&) / ((b^2 c^3 - 4 a c^4) d^2 - (b^3 c^2 - 4 a b c^3) d e + (a b^2 c^2 - 4 a^2 \\
& c^3) e^2)) * \log((2 a^3 b d e + ((a b^2 c^3 - 4 a^2 c^4) d^3 - (a b^3 c^2 - \\
& 4 a^2 b c^3) d^2 e + (a^2 b^2 c^2 - 4 a^3 c^3) d e^2) * x^2 * \sqrt{(a^2 b^2 e^2 \\
& + (b^4 - 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 - a^2 b c) d e) / ((b^2 c^6 - 4 \\
& a c^7) d^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + (b^4 c^4 - 2 a b^2 c^5 - 8 a^2 \\
& c^6) d^2 e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d e^3 + (a^2 b^2 c^4 - 4 a^3 c^5) \\
& e^4)) - 2 (a^2 b^2 - a^3 c) d^2 + (4 a^3 b e^2 + (a b^3 - a^2 b c) d^2 \\
& - (5 a^2 b^2 - 4 a^3 c) d e) * x^2 - 2 * \sqrt{1/2} * \sqrt{e * x^2 + d} * ((b^4 c^3 - \\
& 6 a b^2 c^4 + 8 a^2 c^5) d^3 - (b^5 c^2 - 5 a b^3 c^3 + 4 a^2 b c^4) d^2 e \\
& + 2 (a b^4 c^2 - 5 a^2 b^2 c^3 + 4 a^3 c^4) d e^2 - (a^2 b^3 c^2 - 4 a^3 b \\
& c^3) e^3) * x * \sqrt{(a^2 b^2 e^2 + (b^4 - 2 a b^2 c + a^2 c^2) d^2 - 2 (a b^3 \\
& - a^2 b c) d e) / ((b^2 c^6 - 4 a c^7) d^4 - 2 (b^3 c^5 - 4 a b c^6) d^3 e + \\
& (b^4 c^4 - 2 a b^2 c^5 - 8 a^2 c^6) d^2 e^2 - 2 (a b^3 c^4 - 4 a^2 b c^5) d \\
& e^3 + (a^2 b^2 c^4 - 4 a^3 c^5) e^4)) + ((b^5 - 5 a b^3 c + 4 a^2 b c^2) d^2 \\
& - (2 a b^4 - 9 a^2 b^2 c + 4 a^3 c^2) d e + (a^2 b^3 - 4 a^3 b c) e^2) *
\end{aligned}$$

$$\begin{aligned}
& x) \sqrt{-((b^3 - 3a*b*c)*d - (a*b^2 - 2a^2*c)*e - ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2) \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) / ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2)) / x^2) - \sqrt{1/2} * c * e * \sqrt{-((b^3 - 3a*b*c)*d - (a*b^2 - 2a^2*c)*e + ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2) \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) / ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2)) * \log((2a^3*b*d*e - ((a*b^2*c^3 - 4a^2*c^4)*d^3 - (a*b^3*c^2 - 4a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4a^3*c^3)*d*e^2) * x^2 * \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5a^2*b^2 - 4a^3*c)*d*e) * x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*((b^4*c^3 - 6a*b^2*c^4 + 8a^2*c^5)*d^3 - (b^5*c^2 - 5a*b^3*c^3 + 4a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5a^2*b^2*c^3 + 4a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4a^3*b*c^3)*e^3) * x * \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) - ((b^5 - 5a*b^3*c + 4a^2*b*c^2)*d^2 - (2a*b^4 - 9a^2*b^2*c + 4a^3*c^2)*d*e + (a^2*b^3 - 4a^3*b*c)*e^2) * x) * \sqrt{-((b^3 - 3a*b*c)*d - (a*b^2 - 2a^2*c)*e + ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2) \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) / ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2)) / x^2) + \sqrt{1/2} * c * e * \sqrt{-((b^3 - 3a*b*c)*d - (a*b^2 - 2a^2*c)*e + ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2) \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) / ((b^2*c^3 - 4a*c^4)*d^2 - (b^3*c^2 - 4a*b*c^3)*d*e + (a*b^2*c^2 - 4a^2*c^3)*e^2)) * \log((2a^3*b*d*e - ((a*b^2*c^3 - 4a^2*c^4)*d^3 - (a*b^3*c^2 - 4a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4a^3*c^3)*d*e^2) * x^2 * \sqrt{(a^2*b^2*e^2 + (b^4 - 2a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e) / ((b^2*c^6 - 4a*c^7)*d^4 - 2*(b^3*c^5 - 4a*b*c^6)*d^3*e + (b^4*c^4 - 2a*b^2*c^5 - 8a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4
\end{aligned}$$

```

*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt
t(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2
- 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*
c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 -
2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^
4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2
*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
- ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2
)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2
*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c
^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)
*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2
*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 -
(b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - 4*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/(c*e)]

```

giac [A] time = 2.05, size = 27, normalized size = 0.09

$$-\frac{e^{\left(-\frac{1}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c

maple [C] time = 0.02, size = 200, normalized size = 0.67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)

[Out] 1/c*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((_R^2*b+2*(2*a*
e-b*d)*_R+b*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(-_R+(-e^(1/2)*x+(e*x^2+d)^(1/2))^2),_R=RootOf(_Z^4*c
+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^
3)*_Z))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)

[Out] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)

[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)

$$3.289 \quad \int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}$$

Rubi [A] time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1303, 377, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])]*e)*x]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])]*e)) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])]*e)*x]/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])]*e))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1303

```
Int[(((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 +
(c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx &= \int \left(\frac{1 - \frac{b}{\sqrt{b^2-4ac}}}{\left(b - \sqrt{b^2-4ac} + 2cx^2\right) \sqrt{d+ex^2}} + \frac{1 + \frac{b}{\sqrt{b^2-4ac}}}{\left(b + \sqrt{b^2-4ac} + 2cx^2\right) \sqrt{d+ex^2}} \right) dx \\ &= \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\left(b - \sqrt{b^2-4ac} + 2cx^2\right) \sqrt{d+ex^2}} dx + \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\left(b + \sqrt{b^2-4ac} + 2cx^2\right) \sqrt{d+ex^2}} dx \\ &= \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \text{Subst} \left[\int \frac{1}{b - \sqrt{b^2-4ac} - \left(-2cd + \left(b - \sqrt{b^2-4ac}\right)e\right)x^2} dx, \right. \\ &\quad \left. \frac{\sqrt{b - \sqrt{b^2-4ac}} \tan^{-1} \left(\frac{\sqrt{2cd - \left(b - \sqrt{b^2-4ac}\right)ex}}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{2cd - \left(b - \sqrt{b^2-4ac}\right)e}} \right] \\ &\quad + \frac{\sqrt{b + \sqrt{b^2-4ac}} \tan^{-1} \left(\frac{\sqrt{2cd - \left(b + \sqrt{b^2-4ac}\right)ex}}{\sqrt{b + \sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{2cd - \left(b + \sqrt{b^2-4ac}\right)e}} \end{aligned}$$

Mathematica [A] time = 0.48, size = 227, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1} \left(\frac{x \sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}}}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]
]*e)*x]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/Sqrt[2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/Sqrt[b^2 - 4*a*c]
```


IntegrateAlgebraic [C] time = 9.82, size = 250, normalized size = 1.04

$$\frac{1}{2}\sqrt{e}\text{RootSum}\left[\#1^8c + 4\#1^6be - 4\#1^6cd + 16\#1^4ae^2 - 8\#1^4bde + 6\#1^4cd^2 + 4\#1^2bd^2e - 4\#1^2cd^3 + cd^4\&, \frac{\#1^4\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex}) - 2\#1^2d\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex}) + d^2\log(-\#1 + \sqrt{d + ex^2} - \sqrt{ex})}{\#1^6(-c) - 3\#1^4be + 3\#1^4cd - 8\#1^2ae^2 + 4\#1^2bde - 3\#1^2cd^2 - bd^2e + cd^3}\&\right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) &])/2

fricas [B] time = 12.71, size = 3395, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2))*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2)

$$*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))/x^2)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x); OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[-72,-7,6]Evaluation time: 0.44Unable to divide, perhaps due to rounding error%%{18446744069414584320, [4,7,8,2,3,14,2]%%}+%%{-2147483648, [3,8,10,8,3,10,1]%%}+%%{-12884901888, [3,8,10,7,2,12,2]%%}+%%{463856467968, [3,8,10,6,1,14,3]%%}+%%{1924145348608, [3,8,10,5,0,16,4]%%}+%%{536870912, [3,8,9,8,5,10,0]%%}+%%{20401094656, [3,8,9,7,4,12,1]%%}+%%{-150323855360, [3,8,9,6,3,14,2]%%}+%%{-3135326126080, [3,8,9,5,2,16,3]%%}+%%{-4672924418048, [3,8,9,4,1,18,4]%%}+%%{6047313952768, [3,8,9,3,0,20,5]%%}+%%{-4294967296, [3,8,8,7,6,12,0]%%}+%%{-42412802048, [3,8,8,6,5,14,1]%%}+%%{1046898278400, [3,8,8,5,4,16,2]%%}+%%{6210522710016, [3,8,8,4,3,18,3]%%}+%%{-1786706395136, [3,8,8,3,2,20,4]%%}+%%{-11544872091648, [3,8,8,2,1,22,5]%%}+%%{4398046511104, [3,8,8,1,0,24,6]%%}+%%{12750684160, [3,8,7,6,7,14,0]%%}+%%{-23890755584, [3,8,7,5,6,16,1]%%}+%%{-2103460233216, [3,8,7,4,5,18,2]%%}+%%{-3324304687104, [3,8,7,3,4,20,3]%%}+%%{9758165696512, [3,8,7,2,3,22,4]%%}+%%{1649267441664, [3,8,7,1,2,24,5]%%}+%%{-4398046511104, [3,8,7,0,1,26,6]%%}+%%{-17985175552, [3,8,6,5,8,16,0]%%}+%%{161866579968, [3,8,6,4,7,18,1]%%}+%%{1586990415872, [3,8,6,3,6,20,2]%%}+%%{-1795296329728, [3,8,6,2,5,22,3]%%}+%%{-4123168604160, [3,8,6,1,4,24,4]%%}+%%{3848290697216, [3,8,6,0,3,26,5]%%}+%%{12213813248, [3,8,5,4,9,18,0]%%}+%%{-171798691840, [3,8,5,3,8,20,1]%%}+%%{-212600881152, [3,8,5,2,7,22,2]%%}+%%{1477468749824, [3,8,5,1,6,24,3]%%}+%%{-1099511627776, [3,8,5,0,5,26,4]%%}+%%{-3221225472, [3,8,4,3,10,20,0]%%}+%%{57982058496, [3,8,4,2,9,22,1]%%}+%%{-154618822656, [3,8,4,1,8,24,2]%%}+%%{103079215104, [3,8,4,0,7,26,3]%%}+%%{1048576, [3,6,10,4,2,4,0]%%}+%%{8388608, [3,6,10,3,1,6,1]%%}+%%{16777216, [3,6,10,2,0,8,2]%%}+%%{-5242880, [3,6,9,3,3,6,0]%%}+%%{-29360128, [3,6,9,2,2,8,1]%%}+%%{-33554432, [3,6,9,1,1,10,2]%%}+%%{9699328, [3,6,8,2,4,8,0]%%}+%%{33554432, [3,6,8,1,3,10,1]%%}+%%{16777216, [3,6,8,0,2,12,2]%%}+%%{-7864320, [3,6,7,1,5,10,0]%%}+%%{-12582912, [3,6,7,0,4,12,1]%%}+%%{2359296, [3,6,6,0,6,12,0]%%}+%%{536870912, [2,7,10,6,2,8,1]%%}+%%{6710886400, [2,7,10,5,1,10,2]%%}+%%{18253611008, [2,7,10,4,0,12,3]%%}+%%{-134217728, [2,7,9,6,4,8,0]%%}+%%{-5502926848, [2,7,9,5,3,10,1]%%}+%%{-36909875200, [2,7,9,4,2,12,2]%%}+%%{-42949672960, [2,7,9,3,1,14,3]%%}+%%{429496729

60, [2,7,9,2,0,16,4]%%}+%%{956301312, [2,7,8,5,5,10,0]%%}+%%{18656264192, [2,7,8,4,4,12,1]%%}+%%{64961380352, [2,7,8,3,3,14,2]%%}+%%{-8589934592, [2,7,8,2,2,16,3]%%}+%%{-85899345920, [2,7,8,1,1,18,4]%%}+%%{-2642411520, [2,7,7,4,6,12,0]%%}+%%{-27783069696, [2,7,7,3,5,14,1]%%}+%%{-33957085184, [2,7,7,2,4,16,2]%%}+%%{73014444032, [2,7,7,1,3,18,3]%%}+%%{42949672960, [2,7,7,0,2,20,4]%%}+%%{3556769792, [2,7,6,3,7,14,0]%%}+%%{17716740096, [2,7,6,2,6,16,1]%%}+%%{-12884901888, [2,7,6,1,5,18,2]%%}+%%{-39728447488, [2,7,6,0,4,20,3]%%}+%%{-2340421632, [2,7,5,2,8,16,0]%%}+%%{-2415919104, [2,7,5,1,7,18,1]%%}+%%{12079595520, [2,7,5,0,6,20,2]%%}+%%{603979776, [2,7,4,1,9,18,0]%%}+%%{-1207959552, [2,7,4,0,8,20,1]%%}+%%{2147483648, [1,8,10,9,3,10,1]%%}+%%{38654705664, [1,8,10,8,2,12,2]%%}+%%{51539607552, [1,8,10,7,1,14,3]%%}+%%{-274877906944, [1,8,10,6,0,16,4]%%}+%%{-536870912, [1,8,9,9,5,10,0]%%}+%%{-26843545600, [1,8,9,8,4,12,1]%%}+%%{-188978561024, [1,8,9,7,3,14,2]%%}+%%{146028888064, [1,8,9,6,2,16,3]%%}+%%{962072674304, [1,8,9,5,1,18,4]%%}+%%{-549755813888, [1,8,9,4,0,20,5]%%}+%%{4294967296, [1,8,8,8,6,12,0]%%}+%%{95026151424, [1,8,8,7,5,14,1]%%}+%%{239444426752, [1,8,8,6,4,16,2]%%}+%%{-858993459200, [1,8,8,5,3,18,3]%%}+%%{-618475290624, [1,8,8,4,2,20,4]%%}+%%{1099511627776, [1,8,8,3,1,22,5]%%}+%%{-12750684160, [1,8,7,7,7,14,0]%%}+%%{-136633647104, [1,8,7,6,6,16,1]%%}+%%{62277025792, [1,8,7,5,5,18,2]%%}+%%{936302870528, [1,8,7,4,4,20,3]%%}+%%{-549755813888, [1,8,7,3,3,22,4]%%}+%%{-549755813888, [1,8,7,2,2,24,5]%%}+%%{17985175552, [1,8,6,6,8,16,0]%%}+%%{71940702208, [1,8,6,5,7,18,1]%%}+%%{-267361714176, [1,8,6,4,6,20,2]%%}+%%{-137438953472, [1,8,6,3,5,22,3]%%}+%%{481036337152, [1,8,6,2,4,24,4]%%}+%%{-12213813248, [1,8,5,5,9,18,0]%%}+%%{7247757312, [1,8,5,4,8,20,1]%%}+%%{103079215104, [1,8,5,3,7,22,2]%%}+%%{-137438953472, [1,8,5,2,6,24,3]%%}+%%{3221225472, [1,8,4,4,10,20,0]%%}+%%{-12884901888, [1,8,4,3,9,22,1]%%}+%%{12884901888, [1,8,4,2,8,24,2]%%}+%%{-1048576, [1,6,10,5,2,4,0]%%}+%%{-8388608, [1,6,10,4,1,6,1]%%}+%%{-16777216, [1,6,10,3,0,8,2]%%}+%%{8388608, [1,6,9,4,3,6,0]%%}+%%{62914560, [1,6,9,3,2,8,1]%%}+%%{150994944, [1,6,9,2,1,10,2]%%}+%%{134217728, [1,6,9,1,0,12,3]%%}+%%{-26476544, [1,6,8,3,4,8,0]%%}+%%{-163577856, [1,6,8,2,3,10,1]%%}+%%{-301989888, [1,6,8,1,2,12,2]%%}+%%{-134217728, [1,6,8,0,1,14,3]%%}+%%{41156608, [1,6,7,2,5,10,0]%%}+%%{178257920, [1,6,7,1,4,12,1]%%}+%%{167772160, [1,6,7,0,3,14,2]%%}+%%{-31457280, [1,6,6,1,6,12,0]%%}+%%{-69206016, [1,6,6,0,5,14,1]%%}+%%{9437184, [1,6,5,0,7,14,0]%%}+%%{-402653184, [0,7,10,7,2,8,1]%%}+%%{-5637144576, [0,7,10,6,1,10,2]%%}+%%{-16106127360, [0,7,10,5,0,12,3]%%}+%%{100663296, [0,7,9,7,4,8,0]%%}+%%{4160749568, [0,7,9,6,3,10,1]%%}+%%{30198988800, [0,7,9,5,2,12,2]%%}+%%{28991029248, [0,7,9,4,1,14,3]%%}+%%{-68719476736, [0,7,9,3,0,16,4]%%}+%%{-687865856, [0,7,8,6,5,10,0]%%}+%%{-13925089280, [0,7,8,5,4,12,1]%%}+%%{-48184164352, [0,7,8,4,3,14,2]%%}+%%{49392123904, [0,7,8,3,2,16,3]%%}+%%{120259084288, [0,7,8,2,1,18,4]%%}+%%{-68719476736, [0,7,8,1,0,20,5]%%}+%%{1845493760, [0,7,7,5,6,12,0]%%}+%%{19964887040, [0,7,7,4,5,14,1]%%}+%%{11542724608, [0,7,7,3,4,16,2]%%}+%%{-113816633344, [0,7,7,2,3,18,3]%%}+%%{8589934592, [0,7,7,1,2,20,4]%%}+%%{68719476736, [0,7,7,0,1,22,5]%%}+%%{-2432696320, [0,7,6,4,7,

```

14,0]%%}+%%{-11207180288,[0,7,6,3,6,16,1]%%}+%%{28185722880,[0,7,6,2,5,
18,2]%%}+%%{34359738368,[0,7,6,1,4,20,3]%%}+%%{-60129542144,[0,7,6,0,3,
22,4]%%}+%%{1577058304,[0,7,5,3,8,16,0]%%}+%%{-201326592,[0,7,5,2,7,18,
1]%%}+%%{-14495514624,[0,7,5,1,6,20,2]%%}+%%{17179869184,[0,7,5,0,5,22,
3]%%}+%%{-402653184,[0,7,4,2,9,18,0]%%}+%%{1610612736,[0,7,4,1,8,20,1]
%%}+%%{-1610612736,[0,7,4,0,7,22,2]%%} / %%{-1024,[0,3,4,2,1,2,0]%%}+%%
{-4096,[0,3,4,1,0,4,1]%%}+%%{2560,[0,3,3,1,2,4,0]%%}+%%{4096,[0,3,3,0,
1,6,1]%%}+%%{-1536,[0,3,2,0,3,6,0]%%} Error: Bad Argument Value

```

maple [C] time = 0.02, size = 161, normalized size = 0.67

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^{(1/2)}, x$

[Out] $-1/2*e^{(1/2)}*\sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)})*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(x^2/((c*x^4 + b*x^2 + a)*\text{sqrt}(e*x^2 + d)), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int x^2/((d + e*x^2)^{(1/2)}*(a + b*x^2 + c*x^4)), x$

[Out] $\int x^2/((d + e*x^2)^{(1/2)}*(a + b*x^2 + c*x^4)), x$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

$$3.290 \quad \int \frac{1}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Optimal. Leaf size=243

$$\frac{2c \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} - \frac{2c \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Rubi [A] time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1174, 377, 205}

$$\frac{2c \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} - \frac{2c \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol]
:= With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b - r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx = \frac{(2c) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-(-2cd+(b-\sqrt{b^2-4ac})e)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-(-2cd+(b+\sqrt{b^2-4ac})e)x^2} dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{2c \tan^{-1}\left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(b-\sqrt{b^2-4ac})e}} - \frac{2c \tan^{-1}\left(\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-(b+\sqrt{b^2-4ac})e}}$$

Mathematica [A] time = 0.41, size = 229, normalized size = 0.94

$$\frac{2c \left(\frac{\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}} - \frac{\tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{b^2-4ac}+b\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}+b\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

```
[Out] (2*c*(ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/Sqrt[b^2 - 4*a*c]
```


IntegrateAlgebraic [C] time = 0.20, size = 179, normalized size = 0.74

$$-2e^{3/2}\text{RootSum}\left[\#1^4c + 4\#1^3be - 4\#1^3cd + 16\#1^2ae^2 - 8\#1^2bde + 6\#1^2cd^2 + 4\#1bd^2e - 4\#1cd^3 + cd^4, \frac{\#1\log(-\#1 - 2\sqrt{e}\sqrt{d+ex^2} + d + 2ex^2)}{\#1^3c + 3\#1^2be - 3\#1^2cd + 8\#1ae^2 - 4\#1bde + 3\#1cd^2 + bd^2e - cd^3}\right] \&$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] $-2e^{3/2}\text{RootSum}[c*d^4 - 4*c*d^3*\#1 + 4*b*d^2*e*\#1 + 6*c*d^2*\#1^2 - 8*b*d*e*\#1^2 + 16*a*e^2*\#1^2 - 4*c*d*\#1^3 + 4*b*e*\#1^3 + c*\#1^4 \& , (\text{Log}[d + 2*e*x^2 - 2*\text{Sqrt}[e]*x*\text{Sqrt}[d + e*x^2] - \#1]*\#1)/(-(c*d^3) + b*d^2*e + 3*c*d^2*\#1 - 4*b*d*e*\#1 + 8*a*e^2*\#1 - 3*c*d*\#1^2 + 3*b*e*\#1^2 + c*\#1^3) \&]$

fricas [B] time = 30.48, size = 4557, normalized size = 18.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] $1/4*\text{sqrt}(1/2)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*\text{sqrt}(1/2)*\text{sqrt}(e*x^2 + d))*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/x^2) - 1/4*\text{sqrt}(1/2)*\text{sqrt}(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\text{sqrt}((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/x^2)$

$$\begin{aligned}
& ^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 \\
& - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4))/((a*b^2*c - 4 \\
& *a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\log(- (2 \\
& *a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2* \\
& b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*\sqrt{((c^2*d^2 - 2*b*c*d*e \\
& + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3 \\
& *e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c) \\
& *d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4 \\
& a*c^2)*d*e)*x^2 - 2*\sqrt{1/2)*\sqrt{e*x^2 + d))*((2*(a^2*b^2*c^2 - 4*a^3*c^3) \\
& *d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c \\
& ^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*\sqrt{((c^2*d^2 - 2*b*c*d*e + b^2*e^ \\
& 2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^ \\
& 2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + \\
& (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c \\
&)*e^2)*x)*\sqrt{-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a \\
& b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\sqrt{((c^2*d^2 - 2*b*c*d*e + \\
& b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3* \\
& e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d \\
& *e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a \\
& ^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*\sqrt{1/2)*\sqrt{-(b*c*d \\
& - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + \\
& (a^2*b^2 - 4*a^3*c)*e^2))*\sqrt{((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 \\
& - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^ \\
& 2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5 \\
& *c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 \\
& - 4*a^3*c)*e^2))*\log(- (2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3) \\
& *d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*s \\
& \sqrt{((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2 \\
& *b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - \\
& 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4 \\
& *a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*\sqrt{1/2)*\sqrt{e*x^2 + d))*((2*(\\
& a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 \\
& - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*\sqrt{((c^2* \\
& d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - \\
& 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b \\
& ^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*a^2*c^2)* \\
& d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*\sqrt{-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2* \\
& c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\sqrt{ \\
& ((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b \\
& ^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2 \\
& *(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2 \\
& *c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) + 1/4 \\
& *\sqrt{1/2)*\sqrt{-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a \\
& *b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*\sqrt{((c^2*d^2 - 2*b*c*d*e \\
& + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3
\end{aligned}$$

```
*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*
d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*
a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e -
((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c -
4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 -
4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^
4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) +
((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2
*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4
*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*
e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2)))/x^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.02, size = 151, normalized size = 0.62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)

[Out] $-2e^{(3/2)} \sum \left(\frac{R}{R^3c+3R^2b*e-3R^2*c*d+8R_*R*a*e^2-4R_*R*b*d*e+3R_*R*c*d^2+b*d^2*e-c*d^3} \right) \ln(-R+(-e^{(1/2)})*x+(e*x^2+d)^{(1/2)})^2, R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{ex^2+d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)

[Out] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)

[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)

$$3.291 \quad \int \frac{1}{x^2 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Optimal. Leaf size=280

$$\frac{c \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) - c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) \sqrt{d+ex^2}}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} - a \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \frac{1}{adx}$$

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1303, 264, 1692, 377, 205}

$$\frac{c \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) - c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) \sqrt{d+ex^2}}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} - a \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \frac{1}{adx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx &= \int \left(\frac{1}{ax^2 \sqrt{d+ex^2}} + \frac{-b-cx^2}{a \sqrt{d+ex^2} (a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{1}{x^2 \sqrt{d+ex^2}} dx}{a} + \frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{adx} + \frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2) \sqrt{d+ex^2}} + \frac{-c+\frac{bc}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2) \sqrt{d+ex^2}} \right) dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2) \sqrt{d+ex^2}} dx}{a} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2) \sqrt{d+ex^2}} dx}{a} \\
&= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b+\sqrt{b^2-4ac} - (-2cd+(b+\sqrt{b^2-4ac})e)x^2} dx \right)}{a} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b+\sqrt{b^2-4ac} + (-2cd+(b-\sqrt{b^2-4ac})e)x^2} dx \right)}{a} \\
&= -\frac{\sqrt{d+ex^2}}{adx} - \frac{c \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})e} x}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})e}} - \frac{c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-(b+\sqrt{b^2-4ac})e} x}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a \sqrt{b+\sqrt{b^2-4ac}} \sqrt{2cd-(b+\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [A] time = 1.01, size = 271, normalized size = 0.97

$$\frac{c \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x \sqrt{e \sqrt{b^2-4ac} - be + 2cd}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{e \left(\sqrt{b^2-4ac} - b \right) + 2cd}} + \frac{c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e \left(\sqrt{b^2-4ac} + b \right)}}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b+\sqrt{b^2-4ac}} \sqrt{2cd - e \left(\sqrt{b^2-4ac} + b \right)}} + \frac{\sqrt{d+ex^2}}{dx}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -((sqrt[d + e*x^2]/(d*x) + (c*(1 + b/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - b*e + sqrt[b^2 - 4*a*c]*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d + (-b + sqrt[b^2 - 4*a*c])*e]) + (c*(1 - b/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]))/a

IntegrateAlgebraic [C] time = 36.31, size = 312, normalized size = 1.11

$$\frac{\sqrt{e} \operatorname{RootSum}\left[\#1^8 c + 4\#1^6 b e - 4\#1^5 c d + 16\#1^4 a c^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{\#1^4 c \log(-\#1 + \sqrt{d + c x^2} - \sqrt{e x}) + 4\#1^2 b e \log(-\#1 + \sqrt{d + c x^2} - \sqrt{e x}) - 2\#1^2 c d \log(-\#1 + \sqrt{d + c x^2} - \sqrt{e x}) + c d^2 \log(-\#1 + \sqrt{d + c x^2} - \sqrt{e x})}{\#1^6 (-) - 3\#1^4 b c + 3\#1^4 c d - 8\#1^2 a c^2 + 4\#1^2 b d e - 3\#1^2 c d^2 - b d^2 e + c d^3}\right]}{2a} - \frac{\sqrt{d + e x^2}}{a d x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -(sqrt[d + e*x^2]/(a*d*x)) - (sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 &, (c*d^2*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1] - 2*c*d*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1]*#1^2 + 4*b*e*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1]*#1^2 + c*Log[-(sqrt[e]*x) + sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) &])/(2*a)

fricas [B] time = 59.58, size = 6431, normalized size = 22.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] -1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4))

$$\begin{aligned}
& + ((a^2b^4c^2 - 5a^2b^2c^3 + 4a^3c^4)d^2 - (2ab^5c - 11a^2b^3c^2 + 12a^3bc^3)d^2e + (ab^6 - 6a^2b^4c + 8a^3b^2c^2)e^2)x) \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e - ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)))/x^2) - \sqrt{1/2} * a * d * x * \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e - ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)) * \log(((a^3b^2c^3 - 4a^4c^4)d^3 - (a^3b^3c^2 - 4a^4bc^3)d^2e + (a^4b^2c^2 - 4a^5c^3)d^2e^2) * x^2 \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}} + 2(a^2b^2c^3 - a^2c^4)d^2 - 2(a^2b^3c^2 - 2a^2b^3c^3)d^2e - ((b^3c^3 - ab^2c^4)d^2 - (b^4c^2 + 2ab^2c^3 - 4a^2c^4)d^2e + 4(ab^3c^2 - 2a^2b^3c^3)e^2)x^2 - 2\sqrt{1/2} * \sqrt{ex^2 + d} * (((a^4b^3c^2 - 4a^5bc^3)d^3 - 2(a^4b^4c - 5a^5b^2c^2 + 4a^6c^3)d^2e + (a^4b^5 - 5a^5b^3c + 4a^6bc^2)d^2e^2 - (a^5b^4 - 6a^6b^2c + 8a^7c^2)e^3) * x \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}} + ((a^2b^4c^2 - 5a^2b^2c^3 + 4a^3c^4)d^2 - (2ab^5c - 11a^2b^3c^2 + 12a^3bc^3)d^2e + (ab^6 - 6a^2b^4c + 8a^3b^2c^2)e^2)x) \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e - ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)))/x^2) + \sqrt{1/2} * a * d * x * \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e + ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)))/x^2) + \sqrt{1/2} * a * d * x * \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e + ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)))/x^2) + \sqrt{1/2} * a * d * x * \sqrt{-(b^3c - 3ab^2c^2)d - (b^4 - 4ab^2c + 2a^2c^2)e + ((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)} \sqrt{((b^4c^2 - 2ab^2c^3 + a^2c^4)d^2 - 2(b^5c - 3ab^3c^2 + 2a^2b^3c^3)d^2e + (b^6 - 4ab^4c + 4a^2b^2c^2)e^2)/((a^6b^2c^2 - 4a^7c^3)d^4 - 2(a^6b^3c - 4a^7bc^2)d^3e + (a^6b^4 - 2a^7b^2c - 8a^8c^2)d^2e^2 - 2(a^7b^3 - 4a^8bc)d^2e^3 + (a^8b^2 - 4a^9c)e^4)}}/((a^3b^2c - 4a^4c^2)d^2 - (a^3b^3 - 4a^4bc)d^2e + (a^4b^2 - 4a^5c)e^2)))/x^2)
\end{aligned}$$

$$\begin{aligned}
& *c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4* \\
& a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - \\
& (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*\log(-(((a^3*b^2*c^3 - \\
& 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5* \\
& c^3)*d*e^2)*x^2*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3* \\
& a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6 \\
& *b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - \\
& 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 \\
& - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^ \\
& 3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e \\
& + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*\sqrt{1/2}*\sqrt{e*x^2 + d}*(((a^ \\
& 4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^ \\
& 2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c \\
& + 8*a^7*c^2)*e^3)*x*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c \\
& - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/ \\
& ((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^ \\
& 4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8 \\
& *b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a \\
& *b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3* \\
& b^2*c^2)*e^2)*x)*\sqrt{-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^ \\
& 2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 \\
& - 4*a^5*c)*e^2)*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3* \\
& a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6 \\
& *b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - \\
& 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 \\
& - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e \\
& + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) - \sqrt{1/2}*a*d*x*\sqrt{-((b^3*c - 3*a*b* \\
& c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (\\
& a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*\sqrt{((b^4*c^2 - 2*a*b^ \\
& 2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4 \\
& *a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3* \\
& c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a \\
& ^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4* \\
& c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*\log(-(((a^ \\
& 3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c \\
& ^2 - 4*a^5*c^3)*d*e^2)*x^2*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2* \\
& (b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2) \\
& *e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + \\
& (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 \\
& + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - \\
& 2*a^2*b*c^3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a \\
& ^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*\sqrt{1/2}*\sqrt{e*x^2 \\
& + d}*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4* \\
& a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6 \\
& *a^6*b^2*c + 8*a^7*c^2)*e^3)*x*\sqrt{((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
\end{aligned}$$

$$- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))/x^2) + 4*sqrt(e*x^2 + d))/(a*d*x)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.02, size = 197, normalized size = 0.70

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)

[Out] $-(e*x^2+d)^{(1/2)}/a/d/x+1/2/a*e^{(1/2)}*\sum((c*_R^2+2*(2*b*e-c*d)*_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=RootOf(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)`

[Out] `int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2), x)`

[Out] `Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)`

$$3.292 \quad \int \frac{1}{x^4 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Optimal. Leaf size=341

$$\frac{c \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{c \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + 2$$

Rubi [A] time = 0.74, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, number of rules / integrand size = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$\frac{c \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{c \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} - \frac{\sqrt{d+ex^2}}{3adx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -sqrt[d + e*x^2]/(3*a*d*x^3) + (b*sqrt[d + e*x^2])/(a^2*d*x) + (2*e*sqrt[d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(a^2*sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b - sqrt[b^2 - 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(a^2*sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)
^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx &= \int \left(\frac{1}{ax^4 \sqrt{d+ex^2}} - \frac{b}{a^2 x^2 \sqrt{d+ex^2}} + \frac{b^2-ac+bcx^2}{a^2 \sqrt{d+ex^2} (a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx}{a^2} + \frac{\int \frac{1}{x^4 \sqrt{d+ex^2}} dx}{a} - \frac{b \int \frac{1}{x^2 \sqrt{d+ex^2}} dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + \frac{\int \left(\frac{bc + \frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{bc - \frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2 x} + \frac{\left(c \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2 x} + \frac{\left(c \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b+\sqrt{b^2-4ac}+2cx^2} dx \right)}{a^2} \\
&= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2 dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2 x} + \frac{c \left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-b^2+e}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b^2+e)}}
\end{aligned}$$

Mathematica [A] time = 0.69, size = 320, normalized size = 0.94

$$\frac{3c \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{x \sqrt{e \sqrt{b^2-4ac} - be + 2cd}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{e(\sqrt{b^2-4ac}-b)+2cd}} + \frac{3c \left(\frac{2ac-b^2}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{a(d-2ex^2)\sqrt{d+ex^2}}{d^2 x^3} + \frac{3b\sqrt{d+ex^2}}{dx}$$

$$3a^2$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] ((3*b*sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*sqrt[d + e*x^2])/(d^2*x^3) + (3*c*(b + (b^2 - 2*a*c)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - b*e + sqrt[b^2 - 4*a*c]*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d + (-b + sqrt[b^2 - 4*a*c])*e]) + (3*c*(b + (-b^2 + 2*a*c)/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e])/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2])])/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]) - (a*(d - 2*e*x^2)*sqrt[d + e*x^2])/(d^2*x^3) + (3*b*sqrt[d + e*x^2])/d)

$a*c]) * e) * x) / (\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[d + e*x^2]) / (\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]] * \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e]) / (3*a^2)$

IntegrateAlgebraic [C] time = 49.09, size = 371, normalized size = 1.09

$$\frac{\sqrt{d+ex^2}(-ad+2acx^2+3bdx^2)}{3a^2d^2x^3} - \frac{\sqrt{c}\text{RootSum}\left[\#1^8c+4\#1^6be-4\#1^6cd+16\#1^4ac^2-8\#1^4bde+6\#1^4cd^2+4\#1^2bd^2e-4\#1^2cd^3+cd^4\&, \frac{\#1^6c\log(-\#1+\sqrt{ex^2}-\sqrt{c})-4\#1^2ac\log(-\#1+\sqrt{ex^2}-\sqrt{c})+4\#1^2b^2c\log(-\#1+\sqrt{ex^2}-\sqrt{c})-2\#1^2bd\log(-\#1+\sqrt{ex^2}-\sqrt{c})+bd^2\log(-\#1+\sqrt{ex^2}-\sqrt{c})}{\#1^6c+3\#1^4be-3\#1^4cd+3\#1^2ac^2-4\#1^2bde+3\#1^2cd^2+bd^2e-cd^3}\right]}{2a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] $(\text{Sqrt}[d + e*x^2]*(-(a*d) + 3*b*d*x^2 + 2*a*e*x^2))/ (3*a^2*d^2*x^3) - (\text{Sqrt}[e]*\text{RootSum}[c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b*e*\#1^6 + c*\#1^8 \&, (b*c*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - 2*b*c*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*b^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 4*a*c*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + b*c*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(- (c*d^3) + b*d^2*e + 3*c*d^2*\#1^2 - 4*b*d*e*\#1^2 + 8*a*e^2*\#1^2 - 3*c*d*\#1^4 + 3*b*e*\#1^4 + c*\#1^6) \&])/(2*a^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.03, size = 248, normalized size = 0.73

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)

[Out] $b*(e*x^2+d)^{(1/2)}/a^2/d/x-1/2/a^2*e^{(1/2)}*\text{sum}((b*c*_R^2+2*(-2*a*c*e+2*b^2*e-b*c*d)*_R+b*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-1/3*(e*x^2+d)^{(1/2)}/a/d/x^3+2/3*e*(e*x^2+d)^{(1/2)}/a/d^2/x$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)`

[Out] `int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)`

[Out] `Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)`

$$3.293 \quad \int \frac{1}{x^6 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx$$

Optimal. Leaf size=443

$$\frac{(b^2 - ac) \sqrt{d + ex^2}}{a^3 dx} - \frac{c \left(\frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{a^3 \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}} - \frac{c \left(-\frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b + \sqrt{b^2 - 4ac})}}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{a^3 \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - e(b + \sqrt{b^2 - 4ac})}}$$

Rubi [A] time = 1.43, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1303, 271, 264, 1692, 377, 205}

$$\frac{(b^2 - ac) \sqrt{d + ex^2}}{a^3 dx} - \frac{c \left(\frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{a^3 \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}} - \frac{c \left(-\frac{b(b^2 - 3ac)}{\sqrt{b^2 - 4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd - e(b + \sqrt{b^2 - 4ac})}}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d + ex^2}} \right)}{a^3 \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - e(b + \sqrt{b^2 - 4ac})}} - \frac{2be\sqrt{d + ex^2}}{3a^2 d^2 x} + \frac{b\sqrt{d + ex^2}}{3a^2 dx^3} - \frac{8e^2 \sqrt{d + ex^2}}{15ad^3 x} + \frac{4e\sqrt{d + ex^2}}{15ad^2 x^3} - \frac{\sqrt{d + ex^2}}{5adx^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] $-\text{Sqrt}[d + e*x^2]/(5*a*d*x^5) + (b*\text{Sqrt}[d + e*x^2])/(3*a^2*d*x^3) + (4*e*\text{Sqrt}[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*\text{Sqrt}[d + e*x^2])/(a^3*d*x) - (2*b*e*\text{Sqrt}[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*\text{Sqrt}[d + e*x^2])/(15*a*d^3*x) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*x)/(\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*\text{Sqrt}[d + e*x^2])])/(a^3*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*x)/(\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*\text{Sqrt}[d + e*x^2])])/(a^3*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt{d+ex^2} (a+bx^2+cx^4)} dx &= \int \left(\frac{1}{ax^6 \sqrt{d+ex^2}} - \frac{b}{a^2 x^4 \sqrt{d+ex^2}} + \frac{b^2-ac}{a^3 x^2 \sqrt{d+ex^2}} + \frac{-b(b^2-2ac)-c(b^2-ac)}{a^3 \sqrt{d+ex^2} (a+bx^2+cx^4)} \right) dx \\
&= \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx}{a^3} + \frac{\int \frac{1}{x^6 \sqrt{d+ex^2}} dx}{a} - \frac{b \int \frac{1}{x^4 \sqrt{d+ex^2}} dx}{a^2} + \frac{(b^2-ac) \int \frac{1}{x^2 \sqrt{d+ex^2}} dx}{a^3} \\
&= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2 dx^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3 dx} + \frac{\int \left(\frac{-\frac{bc(b^2-3ac)}{\sqrt{b^2-4ac}} - c(b^2-ac)}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{a^3} \\
&= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2 dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2 x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3 dx} - \frac{2be\sqrt{d+ex^2}}{3a^2 d^2 x} \\
&= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2 dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2 x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3 dx} - \frac{2be\sqrt{d+ex^2}}{3a^2 d^2 x} \\
&= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2 dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2 x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3 dx} - \frac{2be\sqrt{d+ex^2}}{3a^2 d^2 x}
\end{aligned}$$

Mathematica [A] time = 1.64, size = 383, normalized size = 0.86

$$\frac{a^2 \sqrt{d+ex^2} (3d^2 - 4dex^2 + 8e^2 x^4)}{d^3 x^5} + \frac{15(b^2-ac)\sqrt{d+ex^2}}{dx} + \frac{15c \left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{e \sqrt{b^2-4ac} - bc + 2cd}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{e(\sqrt{b^2-4ac}-b)+2cd}} + \frac{15c \left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{5ab(d-2ex^2)\sqrt{d+ex^2}}{d^2 x^3}$$

15a³

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] -1/15*((15*(b^2 - a*c)*sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*sqrt[d + e*x^2])/(d^2*x^3) + (a^2*sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4))/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - b*e + sqrt[b^2 - 4*a*c]*e]*x)/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2]))/(sqrt[b - sqrt[b^2 - 4*a*c]]*sqrt[2*c*d + (-b + sqrt[b^2 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2*c*d - (b + sqrt[b^2 - 4*a*c])*e]*x)/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[d + e*x^2]))/(sqrt[b + sqrt[b^2 - 4*a*c]]*sqrt[2*c*d + (b + sqrt[b^2 - 4*a*c])*e]))/(d^2*x^3)

]])*Sqrt[d + e*x^2]))/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a^3

IntegrateAlgebraic [C] time = 65.03, size = 534, normalized size = 1.21

$$\frac{\sqrt{\text{RootSum}\left[e^2x^4 + 4d^2e^2x^2 - 4d^2e^2 - 16d^2e^2 - 8d^2e^2 + 4d^2e^2 + 4d^2e^2 - 4d^2e^2 + 4d^2e^2\right]} \sqrt{\text{RootSum}\left[e^2x^4 + 4d^2e^2x^2 - 4d^2e^2 - 16d^2e^2 - 8d^2e^2 + 4d^2e^2 + 4d^2e^2 - 4d^2e^2 + 4d^2e^2\right]}}{2a^3} \sqrt{d + e^2x^2} \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]

[Out] (Sqrt[d + e*x^2]*(-3*a^2*d^2 + 5*a*b*d^2*x^2 + 4*a^2*d*e*x^2 - 15*b^2*d^2*x^4 + 15*a*c*d^2*x^4 - 10*a*b*d*e*x^4 - 8*a^2*e^2*x^4))/(15*a^3*d^3*x^5) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (b^2*c*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*b^2*c*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*c^2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + 4*b^3*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 - 8*a*b*c*e*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^2 + b^2*c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4 - a*c^2*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(-(c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) &])/(2*a^3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 350, normalized size = 0.79

$$\frac{\sqrt{\text{RootSum}\left[e^2x^4 + 4d^2e^2x^2 - 4d^2e^2 - 16d^2e^2 - 8d^2e^2 + 4d^2e^2 + 4d^2e^2 - 4d^2e^2 + 4d^2e^2\right]} \sqrt{\text{RootSum}\left[e^2x^4 + 4d^2e^2x^2 - 4d^2e^2 - 16d^2e^2 - 8d^2e^2 + 4d^2e^2 + 4d^2e^2 - 4d^2e^2 + 4d^2e^2\right]}}{2a^3} \sqrt{d + e^2x^2} \sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)`

[Out]
$$-(a*c+b^2)*(e*x^2+d)^{(1/2)}/a^3/d/x-1/2/a^3*e^{(1/2)}*\text{sum}((c*(a*c-b^2)*_R^2+2*(4*a*b*c*e-a*c^2*d-2*b^3*e+b^2*c*d)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+1/3*b*(e*x^2+d)^{(1/2)}/a^2/d/x^3-2/3*b*e*(e*x^2+d)^{(1/2)}/a^2/d^2/x-1/5*(e*x^2+d)^{(1/2)}/a/d/x^5+4/15*e*(e*x^2+d)^{(1/2)}/a/d^2/x^3-8/15*e^2*(e*x^2+d)^{(1/2)}/a/d^3/x$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6 \sqrt{ex^2 + d} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)`

[Out] `int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt{d + ex^2} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)`

[Out] `Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)`

$$3.294 \quad \int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=350

$$\frac{2 \left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{c \sqrt{b-\sqrt{b^2-4ac}} \left(2cd-e(b-\sqrt{b^2-4ac}) \right)^{3/2}} + \frac{2 \left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{c \sqrt{\sqrt{b^2-4ac}+b} \left(2cd-e(\sqrt{b^2-4ac}+b) \right)^{3/2}} - \frac{e \sqrt{d+ex^2}}{e \sqrt{d+ex^2}}$$

Rubi [A] time = 4.33, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1297, 288, 217, 206, 1692, 377, 205}

$$\frac{\left(\frac{-2a^2cx-ab^2e-3abcd+e^2d}{\sqrt{b^2-4ac}} - abe - acd + b^2d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{c \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2 - bde + cd^2)} + \frac{\left(\frac{2a^2cx-ab^2e-3abcd+e^2d}{\sqrt{b^2-4ac}} - abe - acd + b^2d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{c \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2 - bde + cd^2)} - \frac{d^2 x}{e \sqrt{d+ex^2} (ae^2 - bde + cd^2)} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{e^{3/2} (ae^2 - bde + cd^2)} - \frac{(bd - ae) \tanh^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}} \right)}{c \sqrt{e} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]

[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] \text{ /; } \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 288

$\text{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \text{ :> } \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*n*(p + 1)), x] - \text{Dist}[(c^{(n*(m - n + 1))})/(b*n*(p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^{(p + 1)}, x], x] \text{ /; } \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m + 1, n] \ \&\& \ !\text{LtQ}[(m + n*(p + 1) + 1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 377

$\text{Int}[(a_) + (b_)*(x_)^{(n_)})^{(p_)}/((c_) + (d_)*(x_)^{(n_)}), x_Symbol] \text{ :> } \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$

Rule 1297

$\text{Int}[(f_)*(x_)^{(m_)}*((d_) + (e_)*(x_)^2)^{(q_)}/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] \text{ :> } \text{Dist}[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), \text{Int}[(f*x)^{(m - 4)}*(d + e*x^2)^q, x], x] - \text{Dist}[f^4/(c*d^2 - b*d*e + a*e^2), \text{Int}[(f*x)^{(m - 4)}*(d + e*x^2)^{(q + 1)}*\text{Simp}[a*d + (b*d - a*e)*x^2, x]/(a + b*x^2 + c*x^4), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1] \ \&\& \ \text{GtQ}[m, 3]$

Rule 1692

$\text{Int}[(P_x)*((d_) + (e_)*(x_)^2)^{(q_)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[P_x*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, q\}, x] \ \&\& \ \text{PolyQ}[P_x, x^2] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx &= -\frac{\int \frac{x^2(ad+(bd-ae)x^2)}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{cd^2 - bde + ae^2} + \frac{d^2 \int \frac{x^2}{(d+ex^2)^{3/2}} dx}{cd^2 - bde + ae^2} \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}} - \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{c\sqrt{d+ex^2}(a+bx^2+cx^4)} \right) dx}{cd^2 - bde + ae^2} + \dots \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} + \frac{\int \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{c(cd^2 - bde + ae^2)} + \frac{d^2 \text{Subst} \left(\int \frac{1}{\sqrt{d+ex^2}} dx \right)}{e(cd^2 - bde + ae^2)} \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{e^{3/2}(cd^2 - bde + ae^2)} + \frac{\int \left(\frac{b^2d-acd-abe+\dots}{(b-\sqrt{b^2-4ac})\dots} \right) dx}{\dots} \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{e^{3/2}(cd^2 - bde + ae^2)} - \frac{(bd-ae) \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{c\sqrt{e}(cd^2 - bde + ae^2)} \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} + \frac{d^2 \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{e^{3/2}(cd^2 - bde + ae^2)} - \frac{(bd-ae) \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{c\sqrt{e}(cd^2 - bde + ae^2)} \\
&= -\frac{d^2 x}{e(cd^2 - bde + ae^2) \sqrt{d+ex^2}} + \frac{\left(b^2d - acd - abe - \frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}} \right)}{c\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd - (b-\sqrt{b^2-4ac})x^2}}
\end{aligned}$$

Mathematica [B] time = 11.26, size = 10968, normalized size = 31.34

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] Result too large to show

IntegrateAlgebraic [C] time = 84.95, size = 593, normalized size = 1.69

$$\frac{\sqrt{\text{RootSum}\left[21^2x^2 + 441^2x + 441^2d - 441^2bd + 441^2bd^2 - 441^2bd^3 + 441^2bd^4 - 441^2bd^5 + c^2d^6, \frac{e^{\frac{1}{2}\log\left(\frac{\sqrt{d+ex^2}}{c^2}\right)}\sqrt{d+ex^2}}{2(d^2-bd+cd)}\right]}}{2(d^2-bd+cd)} \frac{d^2}{c^2} \frac{\log\left(\frac{\sqrt{d+ex^2}}{c^2}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $-\left(\frac{d^2x}{e(c^2d^2 - b^2d^2e + a^2e^2)\sqrt{d + ex^2}}\right) - \text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right]/(c^2e^{3/2}) - \left(\frac{\sqrt{e}\text{RootSum}\left[c^2d^4 - 4c^2d^3\#1^2 + 4b^2d^2e\#1^2 + 6c^2d^2\#1^4 - 8b^2d^2e\#1^4 + 16a^2e^2\#1^4 - 4c^2d\#1^6 + 4b^2e\#1^6 + c^2\#1^8 \&, (b^2d^3\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1) - a^2c^2d^3\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1) - a^2b^2d^2e\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1) + \sqrt{d + ex^2} - \#1) - 2b^2d^2\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^2 + 2a^2c^2d^2\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^2 + 6a^2b^2d^2e\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^2 - 4a^2e^2\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^2 + b^2d^2\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^4 - a^2c^2d\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^4 - a^2b^2e\text{Log}\left[-\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right) + \sqrt{d + ex^2}\right] - \#1\right)\#1^4)/(-c^2d^3 + b^2d^2e + 3c^2d^2\#1^2 - 4b^2d^2e\#1^2 + 8a^2e^2\#1^2 - 3c^2d\#1^4 + 3b^2e\#1^4 + c^2\#1^6) \&])/(2c^2(c^2d^2 - b^2d^2e + a^2e^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.25, size = 75, normalized size = 0.21

$$\frac{c^2d^2x}{(c^3d^2e - bc^2de^2 + ac^2e^3)\sqrt{x^2e + d}} - \frac{e^{\left(-\frac{3}{2}\right)} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-c^2d^2x/((c^3d^2e - b^2c^2d^2e^2 + a^2c^2e^3)\sqrt{x^2e + d}) - 1/2e^{(-3/2)}\log((xe^{(1/2)} - \sqrt{x^2e + d})^2)/c$

maple [C] time = 0.04, size = 480, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)`

[Out]
$$-1/c*x/e/(e*x^2+d)^{(1/2)}+1/c/e^{(3/2)}*\ln(e^{(1/2)}*x+(e*x^2+d)^{(1/2)})-1/c^2*b*x/d/(e*x^2+d)^{(1/2)}+2/c*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)*\sum(((a*b*e+a*c*d-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^2+b^2*d^2)*_R+a*b*d^2*e+a*c*d^3-b^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+8/c^2*e^{(3/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^{(1/2)}*(e*x^2+d)^{(1/2)}*x+2*d)*a*b+8/c*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^{(1/2)}*(e*x^2+d)^{(1/2)}*x+2*d)*a*d-8/c^2*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^{(1/2)}*(e*x^2+d)^{(1/2)}*x+2*d)*b^2*d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(ex^2 + d)^{3/2} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)`

[Out] `int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)`

[Out] `Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)`

$$3.295 \quad \int \frac{x^4}{(d+ex^2)^{3/2} (a+bx^2+cx^4)} dx$$

Optimal. Leaf size=360

$$\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) - \left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}(ae^2-bde+cd^2) - \sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}(ae^2-bde+cd^2)}$$

Rubi [A] time = 1.27, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1297, 191, 1692, 377, 205}

$$\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right) - \left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}} - ae + bd\right) \tan^{-1}\left(\frac{x\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right) + \frac{dx}{\sqrt{d+ex^2}(ae^2-bde+cd^2)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}(ae^2-bde+cd^2) - \sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]

[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1297

```
Int[(((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x]]/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
& !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx &= -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{cd^2-bde+ae^2} + \frac{d^2 \int \frac{1}{(d+ex^2)^{3/2}} dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} + \frac{bd-ae-\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{(b+\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} \right) dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \int \frac{1}{(b-\sqrt{b^2-4ac}+2cx^2)\sqrt{d+ex^2}} dx}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \text{Subst} \left(\int \frac{1}{b-\sqrt{b^2-4ac}-2cx} dx \right)}{cd^2-bde+ae^2} \\
&= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd-(b-\sqrt{b^2-4ac})e}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-(b-\sqrt{b^2-4ac})e}}
\end{aligned}$$

Mathematica [C] time = 7.90, size = 2162, normalized size = 6.01

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] x/(c*d*Sqrt[d + e*x^2]) - ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[t[-(((b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2*(d + e*x^2)))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2)] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2*(d + e*x^2)))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2)))/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]]) - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]])/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]])/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))

$$\begin{aligned}
&)) - (30*e*(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^4*\text{ArcSin}[\text{Sqrt}[-(((2*c*d + \\
&(-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] \\
&)/(d^2*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + \text{Sqrt}[b^2 \\
&- 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^{(5/2)}*\text{Sqrt}[((-b \\
&+ \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))] * \\
&\text{Hypergeometric2F1}[2, 2, 7/2, -(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d \\
&*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))] + (4*e*x^2*(-(((2*c*d + (-b + \text{Sqrt}[b \\
&^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^{(5/2)}*\text{Sqrt}[((\\
&-b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)) \\
&]*\text{Hypergeometric2F1}[2, 2, 7/2, -(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2) \\
&/ (d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))]/d)/(15*c*(b - \text{Sqrt}[b^2 - 4*a*c] \\
&)*d*(-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] \\
&- 2*c*x^2))))^{(3/2)}*(1 - (2*c*x^2)/(-b + \text{Sqrt}[b^2 - 4*a*c]))*\text{Sqrt}[d + e*x \\
&^2]*\text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] \\
&- 2*c*x^2))] - ((b - (-b^2 + 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*x*(45*\text{Sqrt}[-(((b + \\
&\text{Sqrt}[b^2 - 4*a*c])*(-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2))/(d \\
&^2*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)] + (30*e*x^2*\text{Sqrt}[-(((b + \text{Sqrt}[b^2 \\
&- 4*a*c])*(-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2))/(d^2*(b + \\
&\text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)])/d - 45*\text{ArcSin}[\text{Sqrt}[((2*c*d - (b + \text{Sqrt}[b \\
&^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))] - (30*e*x^2*\text{Ar \\
&cSin}[\text{Sqrt}[((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a* \\
&c] + 2*c*x^2)))]/d + (45*(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*\text{ArcSin}[\text{Sq \\
&rt}[((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2* \\
&c*x^2)))]/ (d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + \text{Sqr \\
&t}[b^2 - 4*a*c]))*e)*x^4*\text{ArcSin}[\text{Sqrt}[((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2 \\
&)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))]/ (d^2*(b + \text{Sqrt}[b^2 - 4*a*c] + 2* \\
&c*x^2)) + 4*(((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4 \\
&a*c] + 2*c*x^2))^{(5/2)}*\text{Sqrt}[((b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + \\
&\text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2*c*d - (b + \\
&\text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x \\
&^2*(((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2 \\
&c*x^2))^{(5/2)}*\text{Sqrt}[((b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + \text{Sqrt}[b^2 \\
&- 4*a*c] + 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2*c*d - (b + \text{Sqrt}[b^2 \\
&- 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))]/d)/(15*c*(b + \text{S \\
&qrt}[b^2 - 4*a*c])*d*(((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt} \\
&[b^2 - 4*a*c] + 2*c*x^2))^{(3/2)}*(1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 - 4*a*c]))*\text{Sq \\
&rt}[d + e*x^2]*\text{Sqrt}[((b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + \text{Sqrt}[b^2 - \\
&4*a*c] + 2*c*x^2)))]
\end{aligned}$$

IntegrateAlgebraic [C] time = 55.59, size = 405, normalized size = 1.12

$$\frac{\sqrt{e} \text{RootSum}\left[\#1^3 c + 4\#1^2 b e - 4\#1^2 c d + 16\#1^2 a e^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{-\#1^4 a c \log\left(\#1 + \sqrt{d^2 x^2 - c^2}\right) + \#1^4 b d \log\left(-\#1 + \sqrt{d^2 x^2 - c^2}\right) + 6\#1^2 a d c \log\left(-\#1 + \sqrt{d^2 x^2 - c^2}\right) - 2\#1^2 b d^2 \log\left(-\#1 + \sqrt{d^2 x^2 - c^2}\right) - a d^2 c \log\left(-\#1 + \sqrt{d^2 x^2 - c^2}\right) + b d^2 \log\left(-\#1 + \sqrt{d^2 x^2 - c^2}\right) \&}{\#1^7 c + 3\#1^5 b e - 3\#1^5 c d + 8\#1^3 a e^2 - 4\#1^2 b d e + 3\#1^2 c d^2 + b d^2 e - c d^3}\right]}{2(ae^2 - bde + cd^2)} + \frac{dx}{\sqrt{d + ex^2}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $(d*x)/((c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[d + e*x^2]) + (\text{Sqrt}[e]*\text{RootSum}[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 \& , (b*d^3*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1] - a*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1] - 2*b*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1]*#1^2 + 6*a*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1]*#1^2 + b*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1]*#1^4 - a*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - #1]*#1^4)/(-c*d^3) + b*d^2*e + 3*c*d^2*#1^2 - 4*b*d*e*#1^2 + 8*a*e^2*#1^2 - 3*c*d*#1^4 + 3*b*e*#1^4 + c*#1^6) \&])/(2*(c*d^2 - b*d*e + a*e^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a,b,c]=[-72,-7,6]Evaluation time: 0.6Unable to divide, perhaps due to rounding error%%{-2147483648, [3,8,10,8,3,10,1]%%}+%%{-12884901888, [3,8,10,7,2,12,2]%%}+%%{463856467968, [3,8,10,6,1,14,3]%%}+%%{1924145348608, [3,8,10,5,0,16,4]%%}+%%{536870912, [3,8,9,8,5,10,0]%%}+%%{20401094656, [3,8,9,7,4,12,1]%%}+%%{-150323855360, [3,8,9,6,3,14,2]%%}+%%{-3135326126080, [3,8,9,5,2,16,3]%%}+%%{-4672924418048, [3,8,9,4,1,18,4]%%}+%%{6047313952768, [3,8,9,3,0,20,5]%%}+%%{-4294967296, [3,8,8,7,6,12,0]%%}+%%{-42412802048, [3,8,8,6,5,14,1]%%}+%%{1046898278400, [3,8,8,5,4,16,2]%%}+%%{6210522710016, [3,8,8,4,3,18,3]%%}+%%{-1786706395136, [3,8,8,3,2,20,4]%%}+%%{-11544872091648, [3,8,8,2,1,22,5]%%}+%%{4398046511104, [3,8,8,1,0,24,6]%%}+%%{12750684160, [3,8,7,6,7,14,0]%%}+%%{-23890755584, [3,8,7,5,6,16,1]%%}+%%{-2103460233216, [3,8,7,4,5,18,2]%%}+%%{-3324304687104, [3,8,7,3,4,20,3]

%%}+%%{9758165696512, [3,8,7,2,3,22,4]%%}+%%{1649267441664, [3,8,7,1,2,24,5]%%}+%%{-4398046511104, [3,8,7,0,1,26,6]%%}+%%{-17985175552, [3,8,6,5,8,16,0]%%}+%%{161866579968, [3,8,6,4,7,18,1]%%}+%%{1586990415872, [3,8,6,3,6,20,2]%%}+%%{-1795296329728, [3,8,6,2,5,22,3]%%}+%%{-4123168604160, [3,8,6,1,4,24,4]%%}+%%{3848290697216, [3,8,6,0,3,26,5]%%}+%%{12213813248, [3,8,5,4,9,18,0]%%}+%%{-171798691840, [3,8,5,3,8,20,1]%%}+%%{-212600881152, [3,8,5,2,7,22,2]%%}+%%{1477468749824, [3,8,5,1,6,24,3]%%}+%%{-1099511627776, [3,8,5,0,5,26,4]%%}+%%{-3221225472, [3,8,4,3,10,20,0]%%}+%%{57982058496, [3,8,4,2,9,22,1]%%}+%%{-154618822656, [3,8,4,1,8,24,2]%%}+%%{103079215104, [3,8,4,0,7,26,3]%%}+%%{1048576, [3,6,10,4,2,4,0]%%}+%%{8388608, [3,6,10,3,1,6,1]%%}+%%{16777216, [3,6,10,2,0,8,2]%%}+%%{-5242880, [3,6,9,3,3,6,0]%%}+%%{-29360128, [3,6,9,2,2,8,1]%%}+%%{-33554432, [3,6,9,1,1,10,2]%%}+%%{9699328, [3,6,8,2,4,8,0]%%}+%%{33554432, [3,6,8,1,3,10,1]%%}+%%{16777216, [3,6,8,0,2,12,2]%%}+%%{-7864320, [3,6,7,1,5,10,0]%%}+%%{-12582912, [3,6,7,0,4,12,1]%%}+%%{2359296, [3,6,6,0,6,12,0]%%}+%%{-536870912, [2,7,10,6,2,8,1]%%}+%%{18446744062703697920, [2,7,10,5,1,10,2]%%}+%%{-18253611008, [2,7,10,4,0,12,3]%%}+%%{134217728, [2,7,9,6,4,8,0]%%}+%%{5502926848, [2,7,9,5,3,10,1]%%}+%%{36909875200, [2,7,9,4,2,12,2]%%}+%%{42949672960, [2,7,9,3,1,14,3]%%}+%%{-42949672960, [2,7,9,2,0,16,4]%%}+%%{-956301312, [2,7,8,5,5,10,0]%%}+%%{-18656264192, [2,7,8,4,4,12,1]%%}+%%{-64961380352, [2,7,8,3,3,14,2]%%}+%%{8589934592, [2,7,8,2,2,16,3]%%}+%%{85899345920, [2,7,8,1,1,18,4]%%}+%%{2642411520, [2,7,7,4,6,12,0]%%}+%%{27783069696, [2,7,7,3,5,14,1]%%}+%%{33957085184, [2,7,7,2,4,16,2]%%}+%%{-73014444032, [2,7,7,1,3,18,3]%%}+%%{-42949672960, [2,7,7,0,2,20,4]%%}+%%{-3556769792, [2,7,6,3,7,14,0]%%}+%%{-17716740096, [2,7,6,2,6,16,1]%%}+%%{12884901888, [2,7,6,1,5,18,2]%%}+%%{39728447488, [2,7,6,0,4,20,3]%%}+%%{2340421632, [2,7,5,2,8,16,0]%%}+%%{2415919104, [2,7,5,1,7,18,1]%%}+%%{-12079595520, [2,7,5,0,6,20,2]%%}+%%{-603979776, [2,7,4,1,9,18,0]%%}+%%{1207959552, [2,7,4,0,8,20,1]%%}+%%{2147483648, [1,8,10,9,3,10,1]%%}+%%{38654705664, [1,8,10,8,2,12,2]%%}+%%{51539607552, [1,8,10,7,1,14,3]%%}+%%{-274877906944, [1,8,10,6,0,16,4]%%}+%%{-536870912, [1,8,9,9,5,10,0]%%}+%%{-26843545600, [1,8,9,8,4,12,1]%%}+%%{-188978561024, [1,8,9,7,3,14,2]%%}+%%{146028888064, [1,8,9,6,2,16,3]%%}+%%{962072674304, [1,8,9,5,1,18,4]%%}+%%{-549755813888, [1,8,9,4,0,20,5]%%}+%%{4294967296, [1,8,8,8,6,12,0]%%}+%%{95026151424, [1,8,8,7,5,14,1]%%}+%%{239444426752, [1,8,8,6,4,16,2]%%}+%%{-858993459200, [1,8,8,5,3,18,3]%%}+%%{-618475290624, [1,8,8,4,2,20,4]%%}+%%{1099511627776, [1,8,8,3,1,22,5]%%}+%%{-12750684160, [1,8,7,7,7,14,0]%%}+%%{-136633647104, [1,8,7,6,6,16,1]%%}+%%{62277025792, [1,8,7,5,5,18,2]%%}+%%{936302870528, [1,8,7,4,4,20,3]%%}+%%{-549755813888, [1,8,7,3,3,22,4]%%}+%%{-549755813888, [1,8,7,2,2,24,5]%%}+%%{17985175552, [1,8,6,6,8,16,0]%%}+%%{71940702208, [1,8,6,5,7,18,1]%%}+%%{-267361714176, [1,8,6,4,6,20,2]%%}+%%{-137438953472, [1,8,6,3,5,22,3]%%}+%%{481036337152, [1,8,6,2,4,24,4]%%}+%%{-12213813248, [1,8,5,5,9,18,0]%%}+%%{7247757312, [1,8,5,4,8,20,1]%%}+%%{103079215104, [1,8,5,3,7,22,2]%%}+%%{-137438953472, [1,8,5,2,6,24,3]%%}+%%{3221225472, [1,8,4,4,10,20,0]%%}+%%{-12884901888, [1,8,4,3,9,22,1]%%}+%%{128

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84901888, [1, 8, 4, 2, 8, 24, 2]%%}+%%{-1048576, [1, 6, 10, 5, 2, 4, 0]%%}+%%{-838860
8, [1, 6, 10, 4, 1, 6, 1]%%}+%%{-16777216, [1, 6, 10, 3, 0, 8, 2]%%}+%%{8388608, [1, 6,
9, 4, 3, 6, 0]%%}+%%{62914560, [1, 6, 9, 3, 2, 8, 1]%%}+%%{150994944, [1, 6, 9, 2, 1, 10
, 2]%%}+%%{134217728, [1, 6, 9, 1, 0, 12, 3]%%}+%%{-26476544, [1, 6, 8, 3, 4, 8, 0]%%
}+%%{-163577856, [1, 6, 8, 2, 3, 10, 1]%%}+%%{-301989888, [1, 6, 8, 1, 2, 12, 2]%%}+
%%{-134217728, [1, 6, 8, 0, 1, 14, 3]%%}+%%{41156608, [1, 6, 7, 2, 5, 10, 0]%%}+%%{17
8257920, [1, 6, 7, 1, 4, 12, 1]%%}+%%{167772160, [1, 6, 7, 0, 3, 14, 2]%%}+%%{-314572
80, [1, 6, 6, 1, 6, 12, 0]%%}+%%{-69206016, [1, 6, 6, 0, 5, 14, 1]%%}+%%{9437184, [1, 6
, 5, 0, 7, 14, 0]%%}+%%{402653184, [0, 7, 10, 7, 2, 8, 1]%%}+%%{5637144576, [0, 7, 10,
6, 1, 10, 2]%%}+%%{16106127360, [0, 7, 10, 5, 0, 12, 3]%%}+%%{-100663296, [0, 7, 9, 7
, 4, 8, 0]%%}+%%{-4160749568, [0, 7, 9, 6, 3, 10, 1]%%}+%%{-30198988800, [0, 7, 9, 5,
2, 12, 2]%%}+%%{-28991029248, [0, 7, 9, 4, 1, 14, 3]%%}+%%{68719476736, [0, 7, 9, 3,
0, 16, 4]%%}+%%{687865856, [0, 7, 8, 6, 5, 10, 0]%%}+%%{13925089280, [0, 7, 8, 5, 4, 1
2, 1]%%}+%%{48184164352, [0, 7, 8, 4, 3, 14, 2]%%}+%%{-49392123904, [0, 7, 8, 3, 2, 1
6, 3]%%}+%%{-120259084288, [0, 7, 8, 2, 1, 18, 4]%%}+%%{68719476736, [0, 7, 8, 1, 0,
20, 5]%%}+%%{-1845493760, [0, 7, 7, 5, 6, 12, 0]%%}+%%{-19964887040, [0, 7, 7, 4, 5,
14, 1]%%}+%%{-11542724608, [0, 7, 7, 3, 4, 16, 2]%%}+%%{113816633344, [0, 7, 7, 2, 3
, 18, 3]%%}+%%{-8589934592, [0, 7, 7, 1, 2, 20, 4]%%}+%%{-68719476736, [0, 7, 7, 0, 1
, 22, 5]%%}+%%{2432696320, [0, 7, 6, 4, 7, 14, 0]%%}+%%{11207180288, [0, 7, 6, 3, 6, 1
6, 1]%%}+%%{-28185722880, [0, 7, 6, 2, 5, 18, 2]%%}+%%{-34359738368, [0, 7, 6, 1, 4,
20, 3]%%}+%%{60129542144, [0, 7, 6, 0, 3, 22, 4]%%}+%%{-1577058304, [0, 7, 5, 3, 8, 1
6, 0]%%}+%%{201326592, [0, 7, 5, 2, 7, 18, 1]%%}+%%{14495514624, [0, 7, 5, 1, 6, 20, 2
]%%}+%%{-17179869184, [0, 7, 5, 0, 5, 22, 3]%%}+%%{402653184, [0, 7, 4, 2, 9, 18, 0]
%%}+%%{-1610612736, [0, 7, 4, 1, 8, 20, 1]%%}+%%{1610612736, [0, 7, 4, 0, 7, 22, 2]%%
} / %%{1024, [0, 3, 4, 2, 1, 2, 0]%%}+%%{4096, [0, 3, 4, 1, 0, 4, 1]%%}+%%{-2560, [0,
3, 3, 1, 2, 4, 0]%%}+%%{-4096, [0, 3, 3, 0, 1, 6, 1]%%}+%%{1536, [0, 3, 2, 0, 3, 6, 0]%%}
Error: Bad Argument Value

```

maple [C] time = 0.03, size = 338, normalized size = 0.94

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^4/(e*x^2+d)^{(3/2)}/(c*x^4+b*x^2+a), x)$

[Out] $\frac{1}{c*x/d} \frac{1}{(e*x^2+d)^{(1/2)}} - \frac{2*e^{(1/2)}}{(4*a*e^2-4*b*d*e+4*c*d^2)} \sum \left((a*e-b*d)*_R^2+2*d*(-3*a*e+b*d)*_R+a*d^2*e-b*d^3 \right) / \left(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3 \right) * \ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z)-8/c*e^{(3/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*a+8/c*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*b*d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(ex^2 + d)^{3/2} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)

[Out] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)

$$3.296 \quad \int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=333

$$\frac{c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2 - bde + cd^2)} + \frac{c \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2 - bde + cd^2)}$$

Rubi [A] time = 0.66, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1299, 191, 1692, 377, 205}

$$\frac{c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2 - bde + cd^2)} + \frac{c \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2 - bde + cd^2)} - \frac{ex}{\sqrt{d+ex^2} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]))/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1299

Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := -Dist[(d*e*f^2)/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(d + e*x^2)^(q + 1)*Simp[a*e + c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]

Rule 1692

Int[(Px_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx &= \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx}{cd^2 - bde + ae^2} - \frac{(de) \int \frac{1}{(d+ex^2)^{3/2}} dx}{cd^2 - bde + ae^2} \\
 &= -\frac{ex}{(cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{\int \left(\frac{cd + \frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{(b - \sqrt{b^2-4ac} + 2cx^2) \sqrt{d+ex^2}} + \frac{cd - \frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{(b + \sqrt{b^2-4ac} + 2cx^2) \sqrt{d+ex^2}} \right) dx}{cd^2 - bde + ae^2} \\
 &= -\frac{ex}{(cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \int \frac{1}{(b - \sqrt{b^2-4ac} + 2cx^2) \sqrt{d+ex^2}} dx}{cd^2 - bde + ae^2} \\
 &= -\frac{ex}{(cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b - \sqrt{b^2-4ac} - (-2cdx^2)} dx \right)}{cd^2 - bde + ae^2} \\
 &= -\frac{ex}{(cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd - (b - \sqrt{b^2-4ac})}}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt{b - \sqrt{b^2-4ac}} \sqrt{2cd - (b - \sqrt{b^2-4ac})}}
 \end{aligned}$$

Mathematica [C] time = 6.72, size = 2119, normalized size = 6.36

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out]
$$\begin{aligned} & \left((1 - b/\sqrt{b^2 - 4ac}) * x * (45 \sqrt{-((-b + \sqrt{b^2 - 4ac}) * (2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2 * (d + e * x^2)) / (d^2 * (b - \sqrt{b^2 - 4ac} + 2c * x^2)^2)}) \right. \\ & + (30 * e * x^2 * \sqrt{-((-b + \sqrt{b^2 - 4ac}) * (2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2 * (d + e * x^2)) / (d^2 * (b - \sqrt{b^2 - 4ac} + 2c * x^2)^2)}) \\ & \left. \right) / d - 45 * \text{ArcSin}[\sqrt{-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))}] \\ & - (30 * e * x^2 * \text{ArcSin}[\sqrt{-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))}]) \\ & / d - (45 * (2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2 * \text{ArcSin}[\sqrt{-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))}]) \\ & + (30 * e * (2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^4 * \text{ArcSin}[\sqrt{-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))}]) \\ & \left. \right) / (d^2 * (-b + \sqrt{b^2 - 4ac} - 2c * x^2)) + 4 * (-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2)))^{5/2} \\ & * \sqrt{((-b + \sqrt{b^2 - 4ac}) * (d + e * x^2)) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))} * \text{Hypergeometric2F1}[2, 2, 7/2, -((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))] \\ & + (4 * e * x^2 * (-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2)))^{5/2} \\ & * \sqrt{((-b + \sqrt{b^2 - 4ac}) * (d + e * x^2)) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))} * \text{Hypergeometric2F1}[2, 2, 7/2, -((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))] \\ & \left. \right) / d) / (15 * (b - \sqrt{b^2 - 4ac}) * d * (-((2cd + (-b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))))^{3/2} * (1 - (2c * x^2) / (-b + \sqrt{b^2 - 4ac})) \\ & * \sqrt{d + e * x^2} * \sqrt{((-b + \sqrt{b^2 - 4ac}) * (d + e * x^2)) / (d * (-b + \sqrt{b^2 - 4ac} - 2c * x^2))} + ((1 + b / \sqrt{b^2 - 4ac}) * x * (45 \sqrt{-((b + \sqrt{b^2 - 4ac}) * (-2cd + (b + \sqrt{b^2 - 4ac}) * e) * x^2 * (d + e * x^2)) / (d^2 * (b + \sqrt{b^2 - 4ac} + 2c * x^2)^2)}) \\ & + (30 * e * x^2 * \sqrt{-((b + \sqrt{b^2 - 4ac}) * (-2cd + (b + \sqrt{b^2 - 4ac}) * e) * x^2 * (d + e * x^2)) / (d^2 * (b + \sqrt{b^2 - 4ac} + 2c * x^2)^2)}) \\ & \left. \right) / d - 45 * \text{ArcSin}[\sqrt{((2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2))}] \\ & - (30 * e * x^2 * \text{ArcSin}[\sqrt{((2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2))}]) \\ & / d + (45 * (2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2 * \text{ArcSin}[\sqrt{((2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2))}]) \\ & + (30 * e * (-2cd + (b + \sqrt{b^2 - 4ac}) * e) * x^4 * \text{ArcSin}[\sqrt{((2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2))}]) \\ & \left. \right) / (d^2 * (b + \sqrt{b^2 - 4ac} + 2c * x^2)) + 4 * ((2cd - (b + \sqrt{b^2 - 4ac}) * e) * x^2) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2)))^{5/2} \\ & * \sqrt{((b + \sqrt{b^2 - 4ac}) * (d + e * x^2)) / (d * (b + \sqrt{b^2 - 4ac} + 2c * x^2))} \end{aligned}$$

$$\frac{(d + ex^2)}{(d(b + \sqrt{b^2 - 4ac}) + 2cx^2)} \operatorname{Hypergeometric2F1}\left[2, 2, \frac{7}{2}, \frac{(2cd - (b + \sqrt{b^2 - 4ac})e)x^2}{(d(b + \sqrt{b^2 - 4ac}) + 2cx^2)}\right] + \frac{(4ex^2 \cdot ((2cd - (b + \sqrt{b^2 - 4ac})e)x^2)^{5/2} \sqrt{(b + \sqrt{b^2 - 4ac})(d + ex^2)})}{(d(b + \sqrt{b^2 - 4ac}) + 2cx^2)} \operatorname{Hypergeometric2F1}\left[2, 2, \frac{7}{2}, \frac{(2cd - (b + \sqrt{b^2 - 4ac})e)x^2}{(d(b + \sqrt{b^2 - 4ac}) + 2cx^2)}\right] \Bigg/ \frac{(15(b + \sqrt{b^2 - 4ac})d \cdot ((2cd - (b + \sqrt{b^2 - 4ac})e)x^2)^{3/2} (1 + (2cx^2)/(b + \sqrt{b^2 - 4ac}))) \sqrt{d + ex^2} \sqrt{(b + \sqrt{b^2 - 4ac})(d + ex^2)}}{(d(b + \sqrt{b^2 - 4ac}) + 2cx^2)}$$

IntegrateAlgebraic [C] time = 49.22, size = 343, normalized size = 1.03

$$\frac{\sqrt{c} \operatorname{RootSum}\left[\#1^5 c + 4\#1^4 b e - 4\#1^3 c d + 16\#1^2 a e^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{\#1^4 c d \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c x}) + 4\#1^2 a e^2 \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c x}) - 2\#1^2 c d^2 \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c x}) + c d^3 \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c x})}{\#1^5 (-) - 3\#1^4 b e + 3\#1^3 c d - 8\#1^2 a e^2 + 4\#1^2 b d e - 3\#1^2 c d^2 - b d^2 + c d^3}\right]}{2(ae^2 - bde + cd^2)} - \frac{ex}{\sqrt{d + ex^2}(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/((d + ex^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $-\frac{(ex)}{(c^2d^2 - b^2de + a^2e^2)\sqrt{d + ex^2}} + \frac{(\sqrt{e} \operatorname{RootSum}[c^2d^4 - 4c^2d^3\#1^2 + 4b^2d^2e\#1^2 + 6c^2d^2\#1^4 - 8b^2d^2e\#1^4 + 16a^2e^2\#1^4 - 4c^2d\#1^6 + 4b^2e\#1^6 + c\#1^8 \&, (c^2d^3 \operatorname{Log}[-(\sqrt{e}x) + \sqrt{d + ex^2}] - \#1) - 2c^2d^2 \operatorname{Log}[-(\sqrt{e}x) + \sqrt{d + ex^2}] - \#1] \#1^2 + 4a^2e^2 \operatorname{Log}[-(\sqrt{e}x) + \sqrt{d + ex^2}] - \#1] \#1^2 + c^2d \operatorname{Log}[-(\sqrt{e}x) + \sqrt{d + ex^2}] - \#1] \#1^4)}{(c^2d^3 - b^2d^2e - 3c^2d^2\#1^2 + 4b^2d^2e\#1^2 - 8a^2e^2\#1^2 + 3c^2d\#1^4 - 3b^2e\#1^4 - c\#1^6) \&]}{2(c^2d^2 - b^2de + a^2e^2)}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(ex^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(ex^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.57Unable to divide, perhaps due
to rounding error%%{-2147483648, [3,8,10,8,3,10,1]%%}+%%{-12884901888, [3,
8,10,7,2,12,2]%%}+%%{463856467968, [3,8,10,6,1,14,3]%%}+%%{1924145348608
, [3,8,10,5,0,16,4]%%}+%%{536870912, [3,8,9,8,5,10,0]%%}+%%{20401094656, [
3,8,9,7,4,12,1]%%}+%%{-150323855360, [3,8,9,6,3,14,2]%%}+%%{-31353261260
80, [3,8,9,5,2,16,3]%%}+%%{-4672924418048, [3,8,9,4,1,18,4]%%}+%%{6047313
952768, [3,8,9,3,0,20,5]%%}+%%{-4294967296, [3,8,8,7,6,12,0]%%}+%%{-42412
802048, [3,8,8,6,5,14,1]%%}+%%{1046898278400, [3,8,8,5,4,16,2]%%}+%%{6210
522710016, [3,8,8,4,3,18,3]%%}+%%{-1786706395136, [3,8,8,3,2,20,4]%%}+%%{-
11544872091648, [3,8,8,2,1,22,5]%%}+%%{4398046511104, [3,8,8,1,0,24,6]%%}
+%%{12750684160, [3,8,7,6,7,14,0]%%}+%%{-23890755584, [3,8,7,5,6,16,1]%%}
+%%{-2103460233216, [3,8,7,4,5,18,2]%%}+%%{-3324304687104, [3,8,7,3,4,20,3
]%%}+%%{9758165696512, [3,8,7,2,3,22,4]%%}+%%{1649267441664, [3,8,7,1,2,2
4,5]%%}+%%{-4398046511104, [3,8,7,0,1,26,6]%%}+%%{-17985175552, [3,8,6,5,
8,16,0]%%}+%%{161866579968, [3,8,6,4,7,18,1]%%}+%%{1586990415872, [3,8,6,
3,6,20,2]%%}+%%{-1795296329728, [3,8,6,2,5,22,3]%%}+%%{-4123168604160, [3
,8,6,1,4,24,4]%%}+%%{3848290697216, [3,8,6,0,3,26,5]%%}+%%{12213813248, [
3,8,5,4,9,18,0]%%}+%%{-171798691840, [3,8,5,3,8,20,1]%%}+%%{-21260088115
2, [3,8,5,2,7,22,2]%%}+%%{1477468749824, [3,8,5,1,6,24,3]%%}+%%{-10995116
27776, [3,8,5,0,5,26,4]%%}+%%{-3221225472, [3,8,4,3,10,20,0]%%}+%%{579820
58496, [3,8,4,2,9,22,1]%%}+%%{-154618822656, [3,8,4,1,8,24,2]%%}+%%{10307
9215104, [3,8,4,0,7,26,3]%%}+%%{1048576, [3,6,10,4,2,4,0]%%}+%%{8388608, [
3,6,10,3,1,6,1]%%}+%%{16777216, [3,6,10,2,0,8,2]%%}+%%{-5242880, [3,6,9,3
,3,6,0]%%}+%%{-29360128, [3,6,9,2,2,8,1]%%}+%%{-33554432, [3,6,9,1,1,10,2
]%%}+%%{9699328, [3,6,8,2,4,8,0]%%}+%%{33554432, [3,6,8,1,3,10,1]%%}+%%{
16777216, [3,6,8,0,2,12,2]%%}+%%{-7864320, [3,6,7,1,5,10,0]%%}+%%{-12582
912, [3,6,7,0,4,12,1]%%}+%%{2359296, [3,6,6,0,6,12,0]%%}+%%{-536870912, [2
,7,10,6,2,8,1]%%}+%%{18446744062703697920, [2,7,10,5,1,10,2]%%}+%%{-1825
3611008, [2,7,10,4,0,12,3]%%}+%%{134217728, [2,7,9,6,4,8,0]%%}+%%{5502926
848, [2,7,9,5,3,10,1]%%}+%%{36909875200, [2,7,9,4,2,12,2]%%}+%%{429496729
60, [2,7,9,3,1,14,3]%%}+%%{-42949672960, [2,7,9,2,0,16,4]%%}+%%{-95630131
2, [2,7,8,5,5,10,0]%%}+%%{-18656264192, [2,7,8,4,4,12,1]%%}+%%{-649613803
52, [2,7,8,3,3,14,2]%%}+%%{8589934592, [2,7,8,2,2,16,3]%%}+%%{85899345920
, [2,7,8,1,1,18,4]%%}+%%{2642411520, [2,7,7,4,6,12,0]%%}+%%{27783069696, [
2,7,7,3,5,14,1]%%}+%%{33957085184, [2,7,7,2,4,16,2]%%}+%%{-73014444032, [
2,7,7,1,3,18,3]%%}+%%{-42949672960, [2,7,7,0,2,20,4]%%}+%%{-3556769792, [
2,7,6,3,7,14,0]%%}+%%{-17716740096, [2,7,6,2,6,16,1]%%}+%%{12884901888, [
2,7,6,1,5,18,2]%%}+%%{39728447488, [2,7,6,0,4,20,3]%%}+%%{2340421632, [2,
7,5,2,8,16,0]%%}+%%{2415919104, [2,7,5,1,7,18,1]%%}+%%{-12079595520, [2,7
,5,0,6,20,2]%%}+%%{-603979776, [2,7,4,1,9,18,0]%%}+%%{1207959552, [2,7,4,
```


0,8,20,1]%%}+%%{2147483648,[1,8,10,9,3,10,1]%%}+%%{38654705664,[1,8,10,8,2,12,2]%%}+%%{51539607552,[1,8,10,7,1,14,3]%%}+%%{-274877906944,[1,8,10,6,0,16,4]%%}+%%{-536870912,[1,8,9,9,5,10,0]%%}+%%{-26843545600,[1,8,9,8,4,12,1]%%}+%%{-188978561024,[1,8,9,7,3,14,2]%%}+%%{146028888064,[1,8,9,6,2,16,3]%%}+%%{962072674304,[1,8,9,5,1,18,4]%%}+%%{-549755813888,[1,8,9,4,0,20,5]%%}+%%{4294967296,[1,8,8,8,6,12,0]%%}+%%{95026151424,[1,8,8,7,5,14,1]%%}+%%{239444426752,[1,8,8,6,4,16,2]%%}+%%{-858993459200,[1,8,8,5,3,18,3]%%}+%%{-618475290624,[1,8,8,4,2,20,4]%%}+%%{1099511627776,[1,8,8,3,1,22,5]%%}+%%{-12750684160,[1,8,7,7,7,14,0]%%}+%%{-136633647104,[1,8,7,6,6,16,1]%%}+%%{62277025792,[1,8,7,5,5,18,2]%%}+%%{936302870528,[1,8,7,4,4,20,3]%%}+%%{-549755813888,[1,8,7,3,3,22,4]%%}+%%{-549755813888,[1,8,7,2,2,24,5]%%}+%%{17985175552,[1,8,6,6,8,16,0]%%}+%%{71940702208,[1,8,6,5,7,18,1]%%}+%%{-267361714176,[1,8,6,4,6,20,2]%%}+%%{-137438953472,[1,8,6,3,5,22,3]%%}+%%{481036337152,[1,8,6,2,4,24,4]%%}+%%{-12213813248,[1,8,5,5,9,18,0]%%}+%%{7247757312,[1,8,5,4,8,20,1]%%}+%%{103079215104,[1,8,5,3,7,22,2]%%}+%%{-137438953472,[1,8,5,2,6,24,3]%%}+%%{3221225472,[1,8,4,4,10,20,0]%%}+%%{-12884901888,[1,8,4,3,9,22,1]%%}+%%{12884901888,[1,8,4,2,8,24,2]%%}+%%{-1048576,[1,6,10,5,2,4,0]%%}+%%{-8388608,[1,6,10,4,1,6,1]%%}+%%{-16777216,[1,6,10,3,0,8,2]%%}+%%{8388608,[1,6,9,4,3,6,0]%%}+%%{62914560,[1,6,9,3,2,8,1]%%}+%%{150994944,[1,6,9,2,1,10,2]%%}+%%{134217728,[1,6,9,1,0,12,3]%%}+%%{-26476544,[1,6,8,3,4,8,0]%%}+%%{-163577856,[1,6,8,2,3,10,1]%%}+%%{-301989888,[1,6,8,1,2,12,2]%%}+%%{-134217728,[1,6,8,0,1,14,3]%%}+%%{41156608,[1,6,7,2,5,10,0]%%}+%%{178257920,[1,6,7,1,4,12,1]%%}+%%{167772160,[1,6,7,0,3,14,2]%%}+%%{-31457280,[1,6,6,1,6,12,0]%%}+%%{-69206016,[1,6,6,0,5,14,1]%%}+%%{9437184,[1,6,5,0,7,14,0]%%}+%%{402653184,[0,7,10,7,2,8,1]%%}+%%{5637144576,[0,7,10,6,1,10,2]%%}+%%{16106127360,[0,7,10,5,0,12,3]%%}+%%{-100663296,[0,7,9,7,4,8,0]%%}+%%{-4160749568,[0,7,9,6,3,10,1]%%}+%%{-30198988800,[0,7,9,5,2,12,2]%%}+%%{-28991029248,[0,7,9,4,1,14,3]%%}+%%{68719476736,[0,7,9,3,0,16,4]%%}+%%{687865856,[0,7,8,6,5,10,0]%%}+%%{13925089280,[0,7,8,5,4,12,1]%%}+%%{48184164352,[0,7,8,4,3,14,2]%%}+%%{-49392123904,[0,7,8,3,2,16,3]%%}+%%{-120259084288,[0,7,8,2,1,18,4]%%}+%%{68719476736,[0,7,8,1,0,20,5]%%}+%%{-1845493760,[0,7,7,5,6,12,0]%%}+%%{-19964887040,[0,7,7,4,5,14,1]%%}+%%{-11542724608,[0,7,7,3,4,16,2]%%}+%%{113816633344,[0,7,7,2,3,18,3]%%}+%%{-8589934592,[0,7,7,1,2,20,4]%%}+%%{-68719476736,[0,7,7,0,1,22,5]%%}+%%{2432696320,[0,7,6,4,7,14,0]%%}+%%{11207180288,[0,7,6,3,6,16,1]%%}+%%{-28185722880,[0,7,6,2,5,18,2]%%}+%%{-34359738368,[0,7,6,1,4,20,3]%%}+%%{60129542144,[0,7,6,0,3,22,4]%%}+%%{-1577058304,[0,7,5,3,8,16,0]%%}+%%{201326592,[0,7,5,2,7,18,1]%%}+%%{14495514624,[0,7,5,1,6,20,2]%%}+%%{-17179869184,[0,7,5,0,5,22,3]%%}+%%{402653184,[0,7,4,2,9,18,0]%%}+%%{-1610612736,[0,7,4,1,8,20,1]%%}+%%{1610612736,[0,7,4,0,7,22,2]%%} / %%{1024,[0,3,4,2,1,2,0]%%}+%%{4096,[0,3,4,1,0,4,1]%%}+%%{-2560,[0,3,3,1,2,4,0]%%}+%%{-4096,[0,3,3,0,1,6,1]%%}+%%{1536,[0,3,2,0,3,6,0]%%}

} Error: Bad Argument Value

maple [C] time = 0.03, size = 252, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(e*x^2+d)^{(3/2)}/(c*x^4+b*x^2+a), x)$

[Out] $-2*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)*\text{sum}((_R^2*c*d+2*(2*a*e^2-c*d^2)*_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8*e^{(1/2)*d}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2/(e*x^2+d)^{(3/2)}/(c*x^4+b*x^2+a), x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^{(3/2)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(ex^2 + d)^{3/2} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/((d + e*x^2)^{(3/2)}*(a + b*x^2 + c*x^4)), x)$

[Out] $\text{int}(x^2/((d + e*x^2)^{(3/2)}*(a + b*x^2 + c*x^4)), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a), x)$

[Out] $\text{Integral}(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)$

$$3.297 \quad \int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=341

$$c \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) \quad c \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)$$

$$\frac{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2 - bde + cd^2)}{\sqrt{b^2-4ac} + b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2 - bde + cd^2)$$

Rubi [A] time = 0.77, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1172, 191, 1692, 377, 205}

$$\frac{c \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) - c \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) + \frac{e^2 x}{d \sqrt{d+ex^2} (ae^2 - bde + cd^2)}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2 - bde + cd^2) - \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b

, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1172

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x^2)^(q + 1)*(c*d - b*e - c*e*x^2))/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q] && LtQ[q, -1]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx &= \int \frac{cd - be - cex^2}{\sqrt{d+ex^2} (a+bx^2+cx^4)} dx + \frac{e^2 \int \frac{1}{(d+ex^2)^{3/2}} dx}{cd^2 - bde + ae^2} \\
 &= \frac{e^2 x}{d (cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{\int \left(\frac{-ce - \frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{(b - \sqrt{b^2-4ac} + 2cx^2) \sqrt{d+ex^2}} + \frac{-ce + \frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{(b + \sqrt{b^2-4ac} + 2cx^2) \sqrt{d+ex^2}} \right) dx}{cd^2 - bde + ae^2} \\
 &= \frac{e^2 x}{d (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{\left(c \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \right) \int \frac{1}{(b - \sqrt{b^2 - 4ac} + 2cx^2) \sqrt{d + ex^2}} dx}{cd^2 - bde + ae^2} \\
 &= \frac{e^2 x}{d (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{\left(c \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b - \sqrt{b^2 - 4ac} - (-2cd + (b - \sqrt{b^2 - 4ac}) x^2)} dx \right)}{cd^2 - bde + ae^2} \\
 &= \frac{e^2 x}{d (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{c \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac}) x^2}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{2cd - (b - \sqrt{b^2 - 4ac}) x^2}}
 \end{aligned}$$

Mathematica [C] time = 7.15, size = 2112, normalized size = 6.19

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out]
$$\begin{aligned} & (2*c*x*(45*\text{Sqrt}[-(((-b + \text{Sqrt}[b^2 - 4*a*c])*(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2)))/(d^2*(b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)]) + (30*e*x^2*\text{Sqrt}[-(((-b + \text{Sqrt}[b^2 - 4*a*c])*(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2)))/(d^2*(b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)])]/d - 45*\text{ArcSin}[\text{Sqrt}[-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] - (30*e*x^2*\text{ArcSin}[\text{Sqrt}[-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*\text{ArcSin}[\text{Sqrt}[-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))]]/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^4*\text{ArcSin}[\text{Sqrt}[-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))]]/(d^2*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*\text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))]*\text{Hypergeometric2F1}[2, 2, 7/2, -(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*\text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))]*\text{Hypergeometric2F1}[2, 2, 7/2, -(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))]/d)/(15*\text{Sqrt}[b^2 - 4*a*c]*(b - \text{Sqrt}[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + \text{Sqrt}[b^2 - 4*a*c]))*\text{Sqrt}[d + e*x^2]*\text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] - (2*c*x*(45*\text{Sqrt}[-((b + \text{Sqrt}[b^2 - 4*a*c])*(-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2)))/(d^2*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)]) + (30*e*x^2*\text{Sqrt}[-((b + \text{Sqrt}[b^2 - 4*a*c])*(-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*(d + e*x^2)))/(d^2*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)]/d - 45*\text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))] - (30*e*x^2*\text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))]/d + (45*(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2*\text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))]/d - (30*e*(-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^4*\text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))]/d^2*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2) + 4*(((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]))*e)*x^2)/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))))^(5/2)*\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))] \end{aligned}$$

$$[b^2 - 4ac] + 2cx^2)] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2cd - (b + \sqrt{b^2 - 4ac}))e)x^2 / (d(b + \sqrt{b^2 - 4ac} + 2cx^2))] + (4e^2x^2 * ((2cd - (b + \sqrt{b^2 - 4ac}))e)x^2 / (d(b + \sqrt{b^2 - 4ac} + 2cx^2)))^{5/2} * \sqrt{((b + \sqrt{b^2 - 4ac})(d + ex^2)) / (d(b + \sqrt{b^2 - 4ac} + 2cx^2))}] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2cd - (b + \sqrt{b^2 - 4ac}))e)x^2 / (d(b + \sqrt{b^2 - 4ac} + 2cx^2)))] / d) / (15\sqrt{b^2 - 4ac} * (b + \sqrt{b^2 - 4ac}) * d * (((2cd - (b + \sqrt{b^2 - 4ac}))e)x^2 / (d(b + \sqrt{b^2 - 4ac} + 2cx^2)))^{3/2} * (1 + (2cx^2) / (b + \sqrt{b^2 - 4ac}))) * \sqrt{d + ex^2} * \sqrt{((b + \sqrt{b^2 - 4ac})(d + ex^2)) / (d(b + \sqrt{b^2 - 4ac} + 2cx^2))}]$$

IntegrateAlgebraic [C] time = 62.48, size = 342, normalized size = 1.00

$$\frac{e^{3/2} \text{RootSum} \left[\#1^3 c + 4\#1^2 b e - 4\#1^2 c d + 16\#1^4 a e^2 - 8\#1^4 b d e + 6\#1^4 c d^2 + 4\#1^2 b d^2 e - 4\#1^2 c d^3 + c d^4 \&, \frac{\#1^4 \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c}) + 4\#1^2 b e \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c}) - 6\#1^2 c d \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c}) - c d^2 \log(-\#1 + \sqrt{d + ex^2} - \sqrt{c})}{\#1^4 (-c) - 3\#1^4 b e + 3\#1^4 c d - 8\#1^4 a e^2 + 4\#1^2 b d e - 3\#1^2 c d^2 - b d^2 + c d^3} \right]}{d \sqrt{d + ex^2} (ae^2 - bde + cd^2)} \frac{1}{2(ae^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((d + ex^2)^(3/2)*(a + bx^2 + cx^4)),x]

[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*sqrt[d + ex^2]) - (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 &, (c*d^2*Log[-(sqrt[e]*x) + sqrt[d + ex^2] - #1] - 6*c*d*Log[-(sqrt[e]*x) + sqrt[d + ex^2] - #1]*#1^2 + 4*b*e*Log[-(sqrt[e]*x) + sqrt[d + ex^2] - #1]*#1^2 + c*Log[-(sqrt[e]*x) + sqrt[d + ex^2] - #1]*#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4 - c*#1^6) &])/(2*(c*d^2 - b*d*e + a*e^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(ex^2+d)^(3/2)/(cx^4+bx^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(ex^2+d)^(3/2)/(cx^4+bx^2+a),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[44,93,-37]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,c]=[-72,-7,6]Evaluation time: 0.58Unable to divide, perhaps due
to rounding error%%{-2147483648, [3,8,10,8,3,10,1]%%}+%%{-12884901888, [3,
8,10,7,2,12,2]%%}+%%{463856467968, [3,8,10,6,1,14,3]%%}+%%{1924145348608
, [3,8,10,5,0,16,4]%%}+%%{536870912, [3,8,9,8,5,10,0]%%}+%%{20401094656, [
3,8,9,7,4,12,1]%%}+%%{-150323855360, [3,8,9,6,3,14,2]%%}+%%{-31353261260
80, [3,8,9,5,2,16,3]%%}+%%{-4672924418048, [3,8,9,4,1,18,4]%%}+%%{6047313
952768, [3,8,9,3,0,20,5]%%}+%%{-4294967296, [3,8,8,7,6,12,0]%%}+%%{-42412
802048, [3,8,8,6,5,14,1]%%}+%%{1046898278400, [3,8,8,5,4,16,2]%%}+%%{6210
522710016, [3,8,8,4,3,18,3]%%}+%%{-1786706395136, [3,8,8,3,2,20,4]%%}+%%{-
11544872091648, [3,8,8,2,1,22,5]%%}+%%{4398046511104, [3,8,8,1,0,24,6]%%}
+%%{12750684160, [3,8,7,6,7,14,0]%%}+%%{-23890755584, [3,8,7,5,6,16,1]%%}
+%%{-2103460233216, [3,8,7,4,5,18,2]%%}+%%{-3324304687104, [3,8,7,3,4,20,3
]%%}+%%{9758165696512, [3,8,7,2,3,22,4]%%}+%%{1649267441664, [3,8,7,1,2,2
4,5]%%}+%%{-4398046511104, [3,8,7,0,1,26,6]%%}+%%{-17985175552, [3,8,6,5,
8,16,0]%%}+%%{161866579968, [3,8,6,4,7,18,1]%%}+%%{1586990415872, [3,8,6,
3,6,20,2]%%}+%%{-1795296329728, [3,8,6,2,5,22,3]%%}+%%{-4123168604160, [3
,8,6,1,4,24,4]%%}+%%{3848290697216, [3,8,6,0,3,26,5]%%}+%%{12213813248, [
3,8,5,4,9,18,0]%%}+%%{-171798691840, [3,8,5,3,8,20,1]%%}+%%{-21260088115
2, [3,8,5,2,7,22,2]%%}+%%{1477468749824, [3,8,5,1,6,24,3]%%}+%%{-10995116
27776, [3,8,5,0,5,26,4]%%}+%%{-3221225472, [3,8,4,3,10,20,0]%%}+%%{579820
58496, [3,8,4,2,9,22,1]%%}+%%{-154618822656, [3,8,4,1,8,24,2]%%}+%%{10307
9215104, [3,8,4,0,7,26,3]%%}+%%{1048576, [3,6,10,4,2,4,0]%%}+%%{8388608, [
3,6,10,3,1,6,1]%%}+%%{16777216, [3,6,10,2,0,8,2]%%}+%%{-5242880, [3,6,9,3
,3,6,0]%%}+%%{-29360128, [3,6,9,2,2,8,1]%%}+%%{-33554432, [3,6,9,1,1,10,2
]%%}+%%{9699328, [3,6,8,2,4,8,0]%%}+%%{33554432, [3,6,8,1,3,10,1]%%}+%%
{16777216, [3,6,8,0,2,12,2]%%}+%%{-7864320, [3,6,7,1,5,10,0]%%}+%%{-12582
912, [3,6,7,0,4,12,1]%%}+%%{2359296, [3,6,6,0,6,12,0]%%}+%%{-536870912, [2
,7,10,6,2,8,1]%%}+%%{18446744062703697920, [2,7,10,5,1,10,2]%%}+%%{-1825
3611008, [2,7,10,4,0,12,3]%%}+%%{134217728, [2,7,9,6,4,8,0]%%}+%%{5502926
848, [2,7,9,5,3,10,1]%%}+%%{36909875200, [2,7,9,4,2,12,2]%%}+%%{429496729
60, [2,7,9,3,1,14,3]%%}+%%{-42949672960, [2,7,9,2,0,16,4]%%}+%%{-95630131
2, [2,7,8,5,5,10,0]%%}+%%{-18656264192, [2,7,8,4,4,12,1]%%}+%%{-649613803
52, [2,7,8,3,3,14,2]%%}+%%{8589934592, [2,7,8,2,2,16,3]%%}+%%{85899345920
, [2,7,8,1,1,18,4]%%}+%%{2642411520, [2,7,7,4,6,12,0]%%}+%%{27783069696, [
2,7,7,3,5,14,1]%%}+%%{33957085184, [2,7,7,2,4,16,2]%%}+%%{-73014444032, [
2,7,7,1,3,18,3]%%}+%%{-42949672960, [2,7,7,0,2,20,4]%%}+%%{-3556769792, [
2,7,6,3,7,14,0]%%}+%%{-17716740096, [2,7,6,2,6,16,1]%%}+%%{12884901888, [
2,7,6,1,5,18,2]%%}+%%{39728447488, [2,7,6,0,4,20,3]%%}+%%{2340421632, [2,
7,5,2,8,16,0]%%}+%%{2415919104, [2,7,5,1,7,18,1]%%}+%%{-12079595520, [2,7
,5,0,6,20,2]%%}+%%{-603979776, [2,7,4,1,9,18,0]%%}+%%{1207959552, [2,7,4,
```

```

0,8,20,1]%%}+%%{2147483648,[1,8,10,9,3,10,1]%%}+%%{38654705664,[1,8,10,
8,2,12,2]%%}+%%{51539607552,[1,8,10,7,1,14,3]%%}+%%{-274877906944,[1,8,
10,6,0,16,4]%%}+%%{-536870912,[1,8,9,9,5,10,0]%%}+%%{-26843545600,[1,8,
9,8,4,12,1]%%}+%%{-188978561024,[1,8,9,7,3,14,2]%%}+%%{146028888064,[1,
8,9,6,2,16,3]%%}+%%{962072674304,[1,8,9,5,1,18,4]%%}+%%{-549755813888,[
1,8,9,4,0,20,5]%%}+%%{4294967296,[1,8,8,8,6,12,0]%%}+%%{95026151424,[1,
8,8,7,5,14,1]%%}+%%{239444426752,[1,8,8,6,4,16,2]%%}+%%{-858993459200,[
1,8,8,5,3,18,3]%%}+%%{-618475290624,[1,8,8,4,2,20,4]%%}+%%{109951162777
6,[1,8,8,3,1,22,5]%%}+%%{-12750684160,[1,8,7,7,7,14,0]%%}+%%{-136633647
104,[1,8,7,6,6,16,1]%%}+%%{62277025792,[1,8,7,5,5,18,2]%%}+%%{936302870
528,[1,8,7,4,4,20,3]%%}+%%{-549755813888,[1,8,7,3,3,22,4]%%}+%%{-549755
813888,[1,8,7,2,2,24,5]%%}+%%{17985175552,[1,8,6,6,8,16,0]%%}+%%{719407
02208,[1,8,6,5,7,18,1]%%}+%%{-267361714176,[1,8,6,4,6,20,2]%%}+%%{-1374
38953472,[1,8,6,3,5,22,3]%%}+%%{481036337152,[1,8,6,2,4,24,4]%%}+%%{-12
213813248,[1,8,5,5,9,18,0]%%}+%%{7247757312,[1,8,5,4,8,20,1]%%}+%%{1030
79215104,[1,8,5,3,7,22,2]%%}+%%{-137438953472,[1,8,5,2,6,24,3]%%}+%%{32
21225472,[1,8,4,4,10,20,0]%%}+%%{-12884901888,[1,8,4,3,9,22,1]%%}+%%{12
884901888,[1,8,4,2,8,24,2]%%}+%%{-1048576,[1,6,10,5,2,4,0]%%}+%%{-83886
08,[1,6,10,4,1,6,1]%%}+%%{-16777216,[1,6,10,3,0,8,2]%%}+%%{8388608,[1,6
,9,4,3,6,0]%%}+%%{62914560,[1,6,9,3,2,8,1]%%}+%%{150994944,[1,6,9,2,1,1
0,2]%%}+%%{134217728,[1,6,9,1,0,12,3]%%}+%%{-26476544,[1,6,8,3,4,8,0]%%
}+%%{-163577856,[1,6,8,2,3,10,1]%%}+%%{-301989888,[1,6,8,1,2,12,2]%%}+
%%{-134217728,[1,6,8,0,1,14,3]%%}+%%{41156608,[1,6,7,2,5,10,0]%%}+%%{1
78257920,[1,6,7,1,4,12,1]%%}+%%{167772160,[1,6,7,0,3,14,2]%%}+%%{-31457
280,[1,6,6,1,6,12,0]%%}+%%{-69206016,[1,6,6,0,5,14,1]%%}+%%{9437184,[1,
6,5,0,7,14,0]%%}+%%{402653184,[0,7,10,7,2,8,1]%%}+%%{5637144576,[0,7,10
,6,1,10,2]%%}+%%{16106127360,[0,7,10,5,0,12,3]%%}+%%{-100663296,[0,7,9,
7,4,8,0]%%}+%%{-4160749568,[0,7,9,6,3,10,1]%%}+%%{-30198988800,[0,7,9,5
,2,12,2]%%}+%%{-28991029248,[0,7,9,4,1,14,3]%%}+%%{68719476736,[0,7,9,3
,0,16,4]%%}+%%{687865856,[0,7,8,6,5,10,0]%%}+%%{13925089280,[0,7,8,5,4,
12,1]%%}+%%{48184164352,[0,7,8,4,3,14,2]%%}+%%{-49392123904,[0,7,8,3,2,
16,3]%%}+%%{-120259084288,[0,7,8,2,1,18,4]%%}+%%{68719476736,[0,7,8,1,0
,20,5]%%}+%%{-1845493760,[0,7,7,5,6,12,0]%%}+%%{-19964887040,[0,7,7,4,5
,14,1]%%}+%%{-11542724608,[0,7,7,3,4,16,2]%%}+%%{113816633344,[0,7,7,2,
3,18,3]%%}+%%{-8589934592,[0,7,7,1,2,20,4]%%}+%%{-68719476736,[0,7,7,0,
1,22,5]%%}+%%{2432696320,[0,7,6,4,7,14,0]%%}+%%{11207180288,[0,7,6,3,6,
16,1]%%}+%%{-28185722880,[0,7,6,2,5,18,2]%%}+%%{-34359738368,[0,7,6,1,4
,20,3]%%}+%%{60129542144,[0,7,6,0,3,22,4]%%}+%%{-1577058304,[0,7,5,3,8,
16,0]%%}+%%{201326592,[0,7,5,2,7,18,1]%%}+%%{14495514624,[0,7,5,1,6,20,
2]%%}+%%{-17179869184,[0,7,5,0,5,22,3]%%}+%%{402653184,[0,7,4,2,9,18,0]
%%}+%%{-1610612736,[0,7,4,1,8,20,1]%%}+%%{1610612736,[0,7,4,0,7,22,2]%%
} / %%{1024,[0,3,4,2,1,2,0]%%}+%%{4096,[0,3,4,1,0,4,1]%%}+%%{-2560,[0
,3,3,1,2,4,0]%%}+%%{-4096,[0,3,3,0,1,6,1]%%}+%%{1536,[0,3,2,0,3,6,0]%%}
} Error: Bad Argument Value

```


maple [C] time = 0.02, size = 246, normalized size = 0.72

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a), x)`

[Out] $8e^{3/2}/(16ae^2-16bde+16cd^2)*\text{sum}((_R^2*c+2*(2*b*e-3*c*d)*_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{1/2})x+(e*x^2+d)^{1/2}))^2, _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))+32e^{3/2}/(16ae^2-16bde+16cd^2)/(2e*x^2-2*(e*x^2+d)^{1/2})e^{1/2}x+2*d)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a), x, algorithm="maxima")`

[Out] `integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(ex^2 + d)^{3/2} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)`

[Out] `int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a), x)`

[Out] `Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)`

$$3.298 \quad \int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=339

$$\frac{2c^2 \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a \sqrt{b-\sqrt{b^2-4ac}} \left(2cd-e(b-\sqrt{b^2-4ac}) \right)^{3/2}} - \frac{2c^2 \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a \sqrt{\sqrt{b^2-4ac}+b} \left(2cd-e(\sqrt{b^2-4ac}+b) \right)^{3/2}} + \frac{ex}{ad \sqrt{d+ex^2}}$$

Rubi [A] time = 2.84, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$\frac{c \left(\frac{2ac+e^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right)}{a \sqrt{b-\sqrt{b^2-4ac}} \sqrt{2cd-e(b-\sqrt{b^2-4ac})} (ae^2-bde+cd^2)} - \frac{c \left(-\frac{2ac+e^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a \sqrt{\sqrt{b^2-4ac}+b} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)} (ae^2-bde+cd^2)} - \frac{2e^3x}{d^2 \sqrt{d+ex^2} (ae^2-bde+cd^2)} - \frac{e^2}{dx \sqrt{d+ex^2} (ae^2-bde+cd^2)} - \frac{\sqrt{d+ex^2}(cd-be)}{adx (ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]

[Out] $-\frac{e^2}{(d*(c*d^2 - b*d*e + a*e^2)*x*\text{Sqrt}[d + e*x^2])} - \frac{(2*e^3*x)}{(d^2*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[d + e*x^2]} - \frac{((c*d - b*e)*\text{Sqrt}[d + e*x^2])}{(a*d*(c*d^2 - b*d*e + a*e^2)*x} - \frac{(c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])]*e)*x]/(\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])*\text{Sqrt}[d + e*x^2])}{(a*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)} - \frac{(c*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])]*e)*x]/(\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])*\text{Sqrt}[d + e*x^2])}{(a*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)}$

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m+1)*(a+b*x^n)^(p+1))/(a*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*(m+1)), Int[x^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n+p+1], 0] && NeQ[m, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c-(b*c-a*d)*x^n), x], x, x/(a+b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && EqQ[n*p+1, 0] && IntegerQ[n]

Rule 1301

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[e^2/(c*d^2-b*d*e+a*e^2), Int[(f*x)^m*(d+e*x^2)^q, x], x] + Dist[1/(c*d^2-b*d*e+a*e^2), Int[(f*x)^m*(d+e*x^2)^(q+1)*Simp[c*d-b*e-c*e*x^2, x]]/(a+b*x^2+c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2-4*a*c, 0] && !IntegerQ[q] && LtQ[q, -1]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(d+e*x^2)^q*(a+b*x^2+c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2-4*a*c, 0] && NeQ[c*d^2-b*d*e+a*e^2, 0] && IntegerQ[p]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a+b*x^n+c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx &= \int \frac{cd - be - cex^2}{x^2 \sqrt{d + ex^2} (a + bx^2 + cx^4)} dx + \frac{e^2 \int \frac{1}{x^2 (d + ex^2)^{3/2}} dx}{cd^2 - bde + ae^2} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \frac{\int \left(\frac{cd - be}{ax^2 \sqrt{d + ex^2}} + \frac{-bcd + b^2e - ace - c(cd - be)x^2}{a \sqrt{d + ex^2} (a + bx^2 + cx^4)} \right) dx}{cd^2 - bde + ae^2} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} + \frac{\int \frac{-bc}{a} dx}{cd^2 - bde + ae^2} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{(cd - be)}{ad (cd^2 - bde + ae^2)} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{(cd - be)}{ad (cd^2 - bde + ae^2)} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{(cd - be)}{ad (cd^2 - bde + ae^2)} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{(cd - be)}{ad (cd^2 - bde + ae^2)} \\
&= -\frac{e^2}{d (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} - \frac{2e^3 x}{d^2 (cd^2 - bde + ae^2) \sqrt{d + ex^2}} - \frac{(cd - be)}{ad (cd^2 - bde + ae^2)}
\end{aligned}$$

Mathematica [C] time = 6.77, size = 2158, normalized size = 6.37

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
x(45*Sqrt[-((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c]))*e
)*x^2*(d + e*x^2)]/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*

$$\begin{aligned}
& \text{Sqrt}[-(((-b + \text{Sqrt}[b^2 - 4ac]) * (2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2 * (d + e * x^2)) / (d^2 * (b - \text{Sqrt}[b^2 - 4ac] + 2cx^2)))] / d - 45 * \text{ArcSin}[\text{Sqrt}[-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))]] - (30 * e * x^2 * \text{ArcSin}[\text{Sqrt}[-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))]]) / d - (45 * (2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2 * \text{ArcSin}[\text{Sqrt}[-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))]]) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)) - (30 * e * (2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e) * x^4 * \text{ArcSin}[\text{Sqrt}[-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))]]) / (d^2 * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)) + 4 * (-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2))))^(5/2) * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, -(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))] + (4 * e * x^2 * (-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2))))^(5/2) * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, -(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))] / d) / (15 * a * (b - \text{Sqrt}[b^2 - 4ac]) * d * (-(((2cd + (-b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2))))^(3/2) * (1 - (2cx^2) / (-b + \text{Sqrt}[b^2 - 4ac])) * \text{Sqrt}[d + e * x^2] * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (-b + \text{Sqrt}[b^2 - 4ac] - 2cx^2)))] + ((-c + (bc) / \text{Sqrt}[b^2 - 4ac]) * x * (45 * \text{Sqrt}[-(((b + \text{Sqrt}[b^2 - 4ac]) * (-2cd + (b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2 * (d + e * x^2)) / (d^2 * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))] + (30 * e * x^2 * \text{Sqrt}[-(((b + \text{Sqrt}[b^2 - 4ac]) * (-2cd + (b + \text{Sqrt}[b^2 - 4ac]) * e)) * x^2 * (d + e * x^2)) / (d^2 * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))]) / d - 45 * \text{ArcSin}[\text{Sqrt}[((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))]] - (30 * e * x^2 * \text{ArcSin}[\text{Sqrt}[((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))]]) / d + (45 * (2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2 * \text{ArcSin}[\text{Sqrt}[((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))]]) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)) - (30 * e * (-2cd + (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^4 * \text{ArcSin}[\text{Sqrt}[((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))]]) / (d^2 * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)) + 4 * (((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))^(5/2) * \text{Sqrt}[((b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))] + (4 * e * x^2 * (((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))^(5/2) * \text{Sqrt}[((b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, ((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))] / d) / (15 * a * (b + \text{Sqrt}[b^2 - 4ac]) * d * (((2cd - (b + \text{Sqrt}[b^2 - 4ac]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2))))^(3/2) * (1 + (2cx^2) / (b + \text{Sqrt}[b^2 - 4ac])) * \text{Sqrt}[d + e * x^2] * \text{Sqrt}[((b + \text{Sqrt}[b^2 - 4ac]) * (d + e * x^2)) / (d * (b + \text{Sqrt}[b^2 - 4ac] + 2cx^2)))])
\end{aligned}$$

IntegrateAlgebraic [C] time = 109.35, size = 546, normalized size = 1.61

$$\frac{-ad^2 - 2ae^2d + b^2e + bd^2e^2 - cd^2 - ce^2d^2}{a^2e^2d + ce^2(a^2 - bde + ce^2)} \sqrt{e} \operatorname{RootSum} \left[\frac{4e^3c + 4d^3e - 4d^2cd + 16d^2a^2 - 8d^2bde + 6d^2ce^2 + 4d^2bd^2e - 4d^2cd^2 + cd^2e}{2e(a^2 - bde + ce^2)} \right]$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $(-(c*d^3) + b*d^2*e - a*d*e^2 - c*d^2*e*x^2 + b*d*e^2*x^2 - 2*a*e^3*x^2)/(a*d^2*(c*d^2 - b*d*e + a*e^2)*x*\operatorname{Sqrt}[d + e*x^2]) - (\operatorname{Sqrt}[e]*\operatorname{RootSum}[c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b*e*\#1^6 + c*\#1^8 \& , (c^2*d^3*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1] - b*c*d^2*e*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1] - 2*c^2*d^2*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 6*b*c*d*e*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 4*b^2*e^2*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*a*c*e^2*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^2 + c^2*d*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^4 - b*c*e*\operatorname{Log}[-(\operatorname{Sqrt}[e]*x) + \operatorname{Sqrt}[d + e*x^2] - \#1]*\#1^4)/(c*d^3 - b*d^2*e - 3*c*d^2*\#1^2 + 4*b*d*e*\#1^2 - 8*a*e^2*\#1^2 + 3*c*d*\#1^4 - 3*b*e*\#1^4 - c*\#1^6) \&])/(2*a*(c*d^2 - b*d*e + a*e^2))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 387, normalized size = 1.14

$$\frac{-ad^2 - 2ae^2d + b^2e + bd^2e^2 - cd^2 - ce^2d^2}{a^2e^2d + ce^2(a^2 - bde + ce^2)} \sqrt{e} \operatorname{RootSum} \left[\frac{4e^3c + 4d^3e - 4d^2cd + 16d^2a^2 - 8d^2bde + 6d^2ce^2 + 4d^2bd^2e - 4d^2cd^2 + cd^2e}{2e(a^2 - bde + ce^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)`

[Out]
$$-1/a/d/x/(e*x^2+d)^{(1/2)}-2/a*e/d^2*x/(e*x^2+d)^{(1/2)}-2/a*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)*\text{sum}((c*(b*e-c*d)*_R^2+2*(-2*a*c*e^2+2*b^2*e^2-3*b*c*d*e+c^2*d^2)*_R+b*c*d^2*e-c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2),_R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/a*e^{(3/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*b+8/a*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*c*d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (ex^2 + d)^{\frac{3}{2}} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)`

[Out] `int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)`

[Out] `Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)`

$$3.299 \quad \int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)} dx$$

Optimal. Leaf size=419

$$\frac{2c^2 \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) + 2c^2 \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right)}{a^2 \sqrt{b-\sqrt{b^2-4ac}} \left(2cd-e(b-\sqrt{b^2-4ac}) \right)^{3/2} + a^2 \sqrt{\sqrt{b^2-4ac}+b} \left(2cd-e(\sqrt{b^2-4ac}+b) \right)^{3/2}} \frac{ex(ace - \dots)}{a^2 d \sqrt{d+ex^2}}$$

Rubi [A] time = 5.57, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$\frac{\left(\frac{2bx-2ac}{\sqrt{b^2-4ac}} + ace + b^2(-c) + bcd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(b-\sqrt{b^2-4ac})}}{\sqrt{b-\sqrt{b^2-4ac}} \sqrt{d+ex^2}} \right) + \left(-\frac{2bx-2ac}{\sqrt{b^2-4ac}} + ace + b^2(-c) + bcd \right) \tan^{-1} \left(\frac{x \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}{\sqrt{\sqrt{b^2-4ac}+b} \sqrt{d+ex^2}} \right) + \frac{\sqrt{d+ex^2}(ace + b^2(-c) + bcd)}{a^2 d x (a^2 - bde + cd^2)} + \frac{3c^2 x}{3a^2 \sqrt{d+ex^2} (a^2 - bde + cd^2)} + \frac{3c^2 x \sqrt{d+ex^2}}{3a^2 \sqrt{d+ex^2} (a^2 - bde + cd^2)} + \frac{c^2}{3a^2 \sqrt{d+ex^2} (a^2 - bde + cd^2)} + \frac{2c \sqrt{d+ex^2} (cd - be)}{3a^2 d^2 (a^2 - bde + cd^2)} + \frac{\sqrt{d+ex^2} (cd - be)}{3a^2 d^2 (a^2 - bde + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $-e^2/(3*d*(c*d^2 - b*d*e + a*e^2)*x^3*\text{Sqrt}[d + e*x^2]) + (4*e^3)/(3*d^2*(c*d^2 - b*d*e + a*e^2)*x*\text{Sqrt}[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[d + e*x^2]) - ((c*d - b*e)*\text{Sqrt}[d + e*x^2])/(3*a*d*(c*d^2 - b*d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*\text{Sqrt}[d + e*x^2])/(3*a*d^2*(c*d^2 - b*d*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*\text{Sqrt}[d + e*x^2])/(a^2*d*(c*d^2 - b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*x)/(\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c])* \text{Sqrt}[d + e*x^2]])/(a^2*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c])* \text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*x)/(\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c])* \text{Sqrt}[d + e*x^2]])/(a^2*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c])* \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))$

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 1301

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && IntegerQ[q, -1]

Rule 1692

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (d + ex^2)^{3/2} (a + bx^2 + cx^4)} dx &= \int \frac{cd - be - cex^2}{x^4 \sqrt{d + ex^2} (a + bx^2 + cx^4)} dx + \frac{e^2 \int \frac{1}{x^4 (d + ex^2)^{3/2}} dx}{cd^2 - bde + ae^2} \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{\int \left(\frac{cd - be}{ax^4 \sqrt{d + ex^2}} + \frac{-bcd + b^2e - ace}{a^2 x^2 \sqrt{d + ex^2}} + \frac{b^2 cd - ac^2}{a^3 \sqrt{d + ex^2}} \right) dx}{cd^2 - bde + ae^2} \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{4e^3}{3d^2 (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \dots \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{4e^3}{3d^2 (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \dots \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{4e^3}{3d^2 (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \dots \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{4e^3}{3d^2 (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \dots \\
&= -\frac{e^2}{3d (cd^2 - bde + ae^2) x^3 \sqrt{d + ex^2}} + \frac{4e^3}{3d^2 (cd^2 - bde + ae^2) x \sqrt{d + ex^2}} + \dots
\end{aligned}$$

Mathematica [C] time = 6.80, size = 2218, normalized size = 5.29

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c)))/Sqrt[b^2 - 4*a

$$\begin{aligned}
& *c]) * x * (45 * \text{Sqrt}[-(((-b + \text{Sqrt}[b^2 - 4*a*c]) * (2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c] \\
&])) * e) * x^2 * (d + e * x^2)) / (d^2 * (b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)]) + (30 * e * \\
& x^2 * \text{Sqrt}[-(((-b + \text{Sqrt}[b^2 - 4*a*c]) * (2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c] * e) * x^2 * (d + e * x^2)) / (d^2 * (b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)))] / d - 45 * \text{ArcSin}[\\
& \text{Sqrt}[-((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] \\
&] - 2*c*x^2))] - (30 * e * x^2 * \text{ArcSin}[\text{Sqrt}[-((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c] \\
&] * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] / d - (45 * (2*c*d + (-b \\
& + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2 * \text{ArcSin}[\text{Sqrt}[-((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c] \\
&) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] / (d * (-b + \text{Sqrt}[b^2 - 4* \\
& a*c] - 2*c*x^2)) - (30 * e * (2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^4 * \text{ArcSin}[\text{Sq} \\
& \text{rt}[-((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] \\
& - 2*c*x^2)))] / (d^2 * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)) + 4 * (-((2*c*d + (- \\
& -b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^(5 \\
& / 2) * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c]) * (d + e * x^2)) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] \\
& - 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, -((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a* \\
& c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))] + (4 * e * x^2 * (-((2*c*d \\
& + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))) \\
& ^{(5/2)} * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c]) * (d + e * x^2)) / (d * (-b + \text{Sqrt}[b^2 - 4*a* \\
& c] - 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, -((2*c*d + (-b + \text{Sqrt}[b^2 - 4 \\
& *a*c]) * e) * x^2) / (d * (-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2)))] / d) / (15 * a^2 * (b - \text{S} \\
& \text{qrt}[b^2 - 4*a*c]) * d * (-((2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (-b + \\
& \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2))))^(3/2) * (1 - (2*c*x^2) / (-b + \text{Sqrt}[b^2 - 4*a*c] \\
&])) * \text{Sqrt}[d + e * x^2] * \text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c]) * (d + e * x^2)) / (d * (-b + \text{S} \\
& \text{qrt}[b^2 - 4*a*c] - 2*c*x^2))] + ((b*c - (c * (b^2 - 2*a*c)) / \text{Sqrt}[b^2 - 4*a*c] \\
&) * x * (45 * \text{Sqrt}[-(((b + \text{Sqrt}[b^2 - 4*a*c]) * (-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]) * e) \\
&) * x^2 * (d + e * x^2)) / (d^2 * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)]) + (30 * e * x^2 * \\
& \text{Sqrt}[-(((b + \text{Sqrt}[b^2 - 4*a*c]) * (-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2 * (d \\
& + e * x^2)) / (d^2 * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)^2)))] / d - 45 * \text{ArcSin}[\text{Sqrt}[\\
& ((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x \\
& ^2))] - (30 * e * x^2 * \text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d \\
& * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))] / d + (45 * (2*c*d - (b + \text{Sqrt}[b^2 - 4*a \\
& *c]) * e) * x^2 * \text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (b + \text{S} \\
& \text{qrt}[b^2 - 4*a*c] + 2*c*x^2)))] / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)) - (30 \\
& * e * (-2*c*d + (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^4 * \text{ArcSin}[\text{Sqrt}[(2*c*d - (b + \text{Sqrt} \\
& [b^2 - 4*a*c]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2)))] / (d^2 * (b + \text{S} \\
& \text{qrt}[b^2 - 4*a*c] + 2*c*x^2)) + 4 * (((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) \\
& / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))))^(5/2) * \text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c]) \\
& * (d + e * x^2)) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2 \\
& , 7/2, ((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] \\
& + 2*c*x^2))] + (4 * e * x^2 * (((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (b + \\
& \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))))^(5/2) * \text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c]) * (d + e * x \\
& ^2)) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))] * \text{Hypergeometric2F1}[2, 2, 7/2, ((\\
& 2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2 \\
&))] / d) / (15 * a^2 * (b + \text{Sqrt}[b^2 - 4*a*c]) * d * (((2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c] \\
&] * e) * x^2) / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))))^(3/2) * (1 + (2*c*x^2) / (b +
\end{aligned}$$

$\text{Sqrt}[b^2 - 4*a*c]) * \text{Sqrt}[d + e*x^2] * \text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c]) * (d + e*x^2)] / (d * (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2))$

IntegrateAlgebraic [C] time = 144.27, size = 762, normalized size = 1.82

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]

[Out] $(-(a*c*d^4) + a*b*d^3*e - a^2*d^2*e^2 + 3*b*c*d^4*x^2 - 3*b^2*d^3*e*x^2 + 4*a*c*d^3*e*x^2 - a*b*d^2*e^2*x^2 + 4*a^2*d*e^3*x^2 + 3*b*c*d^3*e*x^4 - 3*b^2*d^2*e^2*x^4 + 5*a*c*d^2*e^2*x^4 - 2*a*b*d*e^3*x^4 + 8*a^2*e^4*x^4) / (3*a^2*d^3*(c*d^2 - b*d*e + a*e^2)*x^3*\text{Sqrt}[d + e*x^2]) + (\text{Sqrt}[e]*\text{RootSum}[c*d^4 - 4*c*d^3*\#1^2 + 4*b*d^2*e*\#1^2 + 6*c*d^2*\#1^4 - 8*b*d*e*\#1^4 + 16*a*e^2*\#1^4 - 4*c*d*\#1^6 + 4*b*e*\#1^6 + c*\#1^8 \& , (- (b*c^2*d^3*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]) + b^2*c*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] - a*c^2*d^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1] + 2*b*c^2*d^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 6*b^2*c*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 6*a*c^2*d*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 + 4*b^3*e^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - 8*a*b*c*e^2*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^2 - b*c^2*d*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 + b^2*c*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4 - a*c^2*e*\text{Log}[-(\text{Sqrt}[e]*x) + \text{Sqrt}[d + e*x^2] - \#1]*\#1^4) / (- (c*d^3) + b*d^2*e + 3*c*d^2*\#1^2 - 4*b*d*e*\#1^2 + 8*a*e^2*\#1^2 - 3*c*d*\#1^4 + 3*b*e*\#1^4 + c*\#1^6) \&])$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.04, size = 541, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^4/(e*x^2+d)^{(3/2)}/(c*x^4+b*x^2+a), x)$

[Out] $1/a^2*b/d/x/(e*x^2+d)^{(1/2)}+2/a^2*b*e/d^2*x/(e*x^2+d)^{(1/2)}-2/a^2*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)*\text{sum}((c*(a*c*e-b^2*e+b*c*d)*_R^2+2*(4*a*b*c*e^2-3*a*c^2*d*e-2*b^3*e^2+3*b^2*c*d*e-b*c^2*d^2)*_R+a*c^2*d^2*e-b^2*d^2*e*c+b*c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*\ln(-_R+(-e^{(1/2)}*x+(e*x^2+d)^{(1/2)})^2), _R=\text{RootOf}(_Z^4*c+c*d^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z))-8/a^2*e^{(3/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*c+8/a^2*e^{(3/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*b^2-8/a^2*e^{(1/2)}/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*(e*x^2+d)^{(1/2)}*e^{(1/2)}*x+2*d)*b*c*d-1/3/a/d/x^3/(e*x^2+d)^{(1/2)}+4/3/a*e/d^2/x/(e*x^2+d)^{(1/2)}+8/3/a*e^2/d^3*x/(e*x^2+d)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x^4/(e*x^2+d)^{(3/2)}/(c*x^4+b*x^2+a), x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^{(3/2)}*x^4), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 (ex^2 + d)^{3/2} (cx^4 + bx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^4*(d + e*x^2)^{(3/2)}*(a + b*x^2 + c*x^4)), x)$

[Out] $\text{int}(1/(x^4*(d + e*x^2)^{(3/2)}*(a + b*x^2 + c*x^4)), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

$$3.300 \quad \int \frac{\sqrt{1 + \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1 - c^4 x^4}}{cx\sqrt{\frac{1}{c^2 x^2} + 1}}\right)}{c}$$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1448, 1252, 848, 63, 208}

$$\frac{x\sqrt{\frac{1}{c^2 x^2} + 1} \tanh^{-1}\left(\sqrt{1 - c^2 x^2}\right)}{\sqrt{c^2 x^2 + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4], x]

[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 848

Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 1252

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x]
/; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1448

```
Int[((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol]
:> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p, x], x]
/; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1 + \frac{1}{c^2 x^2}}}{\sqrt{1 - c^4 x^4}} dx &= \frac{\left(\sqrt{1 + \frac{1}{c^2 x^2}} x\right) \int \frac{\sqrt{1 + c^2 x^2}}{x \sqrt{1 - c^4 x^4}} dx}{\sqrt{1 + c^2 x^2}} \\ &= \frac{\left(\sqrt{1 + \frac{1}{c^2 x^2}} x\right) \text{Subst}\left(\int \frac{\sqrt{1 + c^2 x}}{x \sqrt{1 - c^4 x^2}} dx, x, x^2\right)}{2\sqrt{1 + c^2 x^2}} \\ &= \frac{\left(\sqrt{1 + \frac{1}{c^2 x^2}} x\right) \text{Subst}\left(\int \frac{1}{x \sqrt{1 - c^2 x}} dx, x, x^2\right)}{2\sqrt{1 + c^2 x^2}} \\ &= -\frac{\left(\sqrt{1 + \frac{1}{c^2 x^2}} x\right) \text{Subst}\left(\int \frac{1}{\frac{1}{c^2} - \frac{x^2}{c^2}} dx, x, \sqrt{1 - c^2 x^2}\right)}{c^2 \sqrt{1 + c^2 x^2}} \\ &= -\frac{\sqrt{1 + \frac{1}{c^2 x^2}} x \tanh^{-1}\left(\sqrt{1 - c^2 x^2}\right)}{\sqrt{1 + c^2 x^2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.10

$$-\frac{x \sqrt{\frac{1}{c^2 x^2} + 1} \tanh^{-1}\left(\sqrt{1 - c^2 x^2}\right)}{\sqrt{c^2 x^2 + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4], x]
```


[Out] $-\left(\sqrt{1 + 1/(c^2x^2)}\right) * x * \text{ArcTanh}\left[\sqrt{1 - c^2x^2}\right] / \sqrt{1 + c^2x^2}$

IntegrateAlgebraic [B] time = 1.09, size = 107, normalized size = 2.68

$$\frac{cx\sqrt{\frac{1}{c^2x^2} + 1} \left(\frac{\log\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{c^2x^2+1}} - 1\right)}{2c} - \frac{\log\left(\frac{c\sqrt{1-c^4x^4}}{\sqrt{c^2x^2+1}} + c\right)}{2c} \right)}{\sqrt{c^2x^2 + 1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4], x]

[Out] $\frac{(c\sqrt{1 + 1/(c^2x^2)}) * x * (\text{Log}[-1 + \sqrt{1 - c^4x^4}] / \sqrt{1 + c^2x^2}) / (2c) - \text{Log}[c + (c\sqrt{1 - c^4x^4}) / \sqrt{1 + c^2x^2}] / (2c)}{\sqrt{1 + c^2x^2}}$

fricas [B] time = 0.94, size = 120, normalized size = 3.00

$$\frac{\log\left(\frac{c^2x^2 + \sqrt{-c^4x^4 + 1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{c^2x^2+1}\right) - \log\left(-\frac{c^2x^2 - \sqrt{-c^4x^4 + 1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{c^2x^2+1}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2), x, algorithm="fricas")

[Out] $-1/2 * (\log((c^2x^2 + \sqrt{-c^4x^4 + 1}) * c * x * \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1) / (c^2x^2 + 1)) - \log(-c^2x^2 - \sqrt{-c^4x^4 + 1}) * c * x * \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1) / (c^2x^2 + 1)) / c$

giac [A] time = 0.22, size = 42, normalized size = 1.05

$$\frac{\left(\log\left(\sqrt{-c^2x^2 + 1} + 1\right) - \log\left(-\sqrt{-c^2x^2 + 1} + 1\right)\right) |c|}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2), x, algorithm="giac")

[Out] $-1/2 * (\log(\sqrt{-c^2x^2 + 1} + 1) - \log(-\sqrt{-c^2x^2 + 1} + 1)) * \text{abs}(c) / c^2$

maple [C] time = 0.06, size = 101, normalized size = 2.52

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}} \sqrt{-c^4x^4+1} x \operatorname{csgn}\left(\frac{1}{c}\right) \ln\left(\frac{2\sqrt{-\frac{c^2x^2-1}{c^2}} c \operatorname{csgn}\left(\frac{1}{c}\right)+2}{c^2x}\right)}{(c^2x^2+1) \sqrt{-\frac{c^2x^2-1}{c^2}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2), x)`

[Out] `-((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/x/c^2)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^(1/2)/c`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{c^2x^2} + 1}}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{c^2x^2} + 1}}{\sqrt{1 - c^4x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)`

[Out] `int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1 + \frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))  
, x)
```


Chapter 4

Appendix

Local contents

4.1	Download section2262
4.2	Listing of Grading functions2262

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```

```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]==Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]==Plus || Head[expn]==Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```



```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or type
(expn,'*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```



```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```